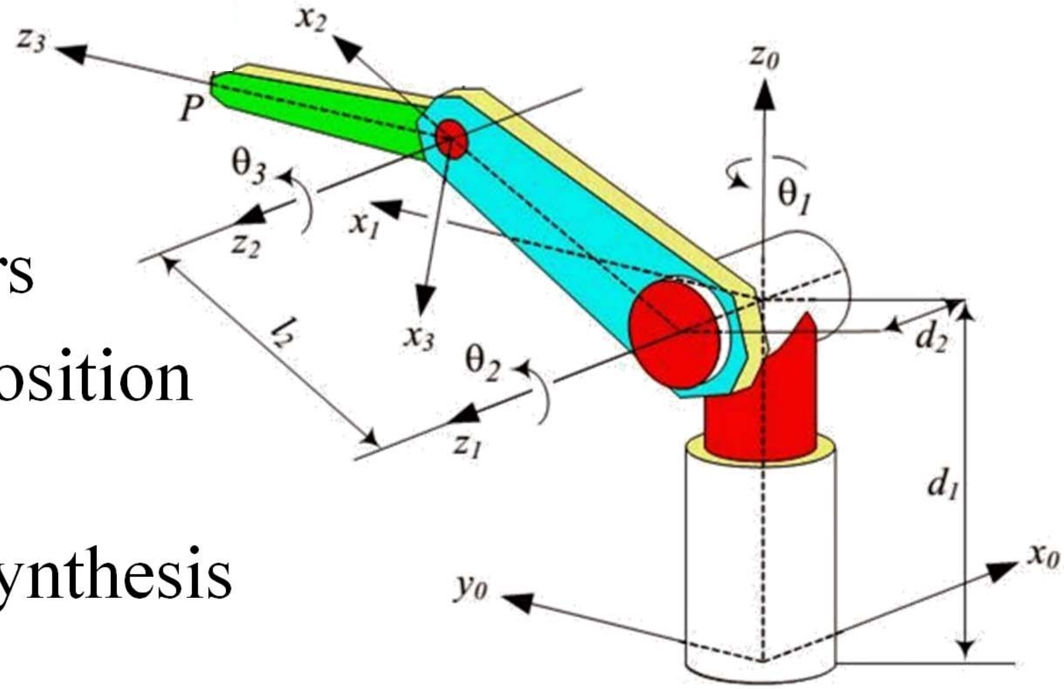
# ME 5243:
# ADVANCED MECHANISM DESIGN

## Class #24

## DH Parameters

Source: irobotkits.blogspot.com




- Setting Up Parameters
- DH Parameters for Position Analysis
- DH Parameters for Synthesis

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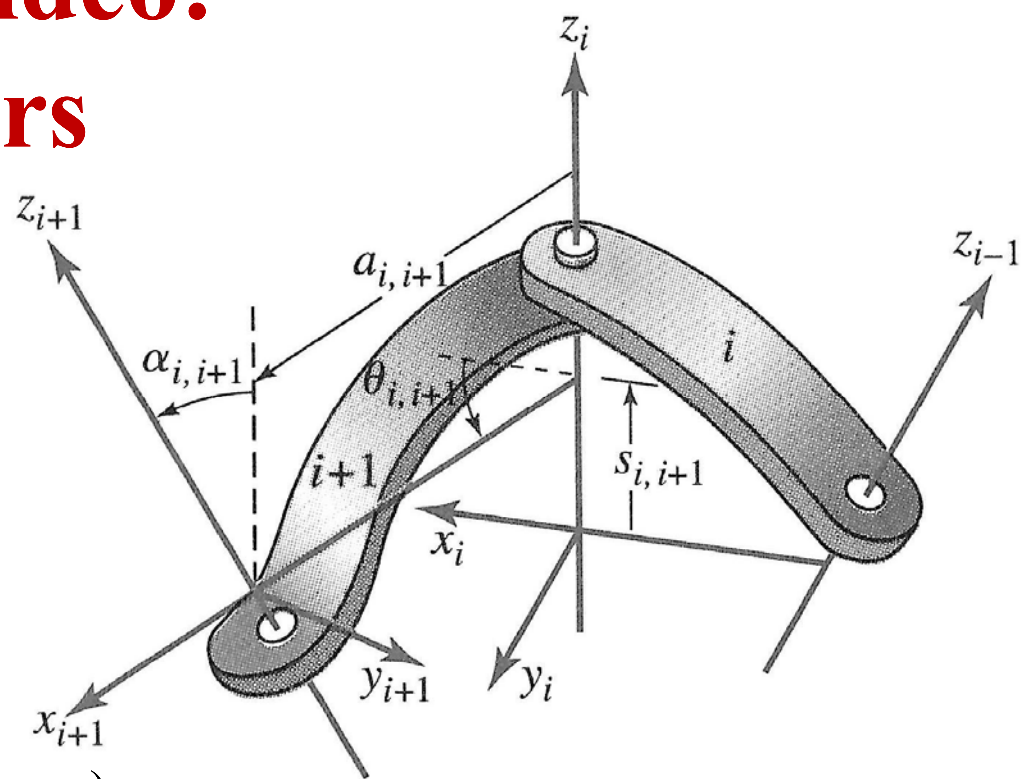
# Notes

- Report Deliverables
  - Draft Report Due Today
    - Submit as PDF to Ryan: fossx231@umn.edu
  - Peer Review Due Thursday, 12/7, by 9am
  - Final Paper Due 12/12
- Oral Presentations: Dec 5 & 7
- Peer Evaluation Due 12/19

# Questions from Video: DH Parameters

1. Number joints consecutively, start at input
2. Joint axes: $x_i$ is perp to $z_{i-1}$ and $z_i$
3. Origin $x_i$, $y_i$, $z_i$ is fixed in link w/ joints $i$-1 and $i$




$a_{i,i+1}$: dist along $x_{i+i}$ from $z_i$ to $z_{i+1}$

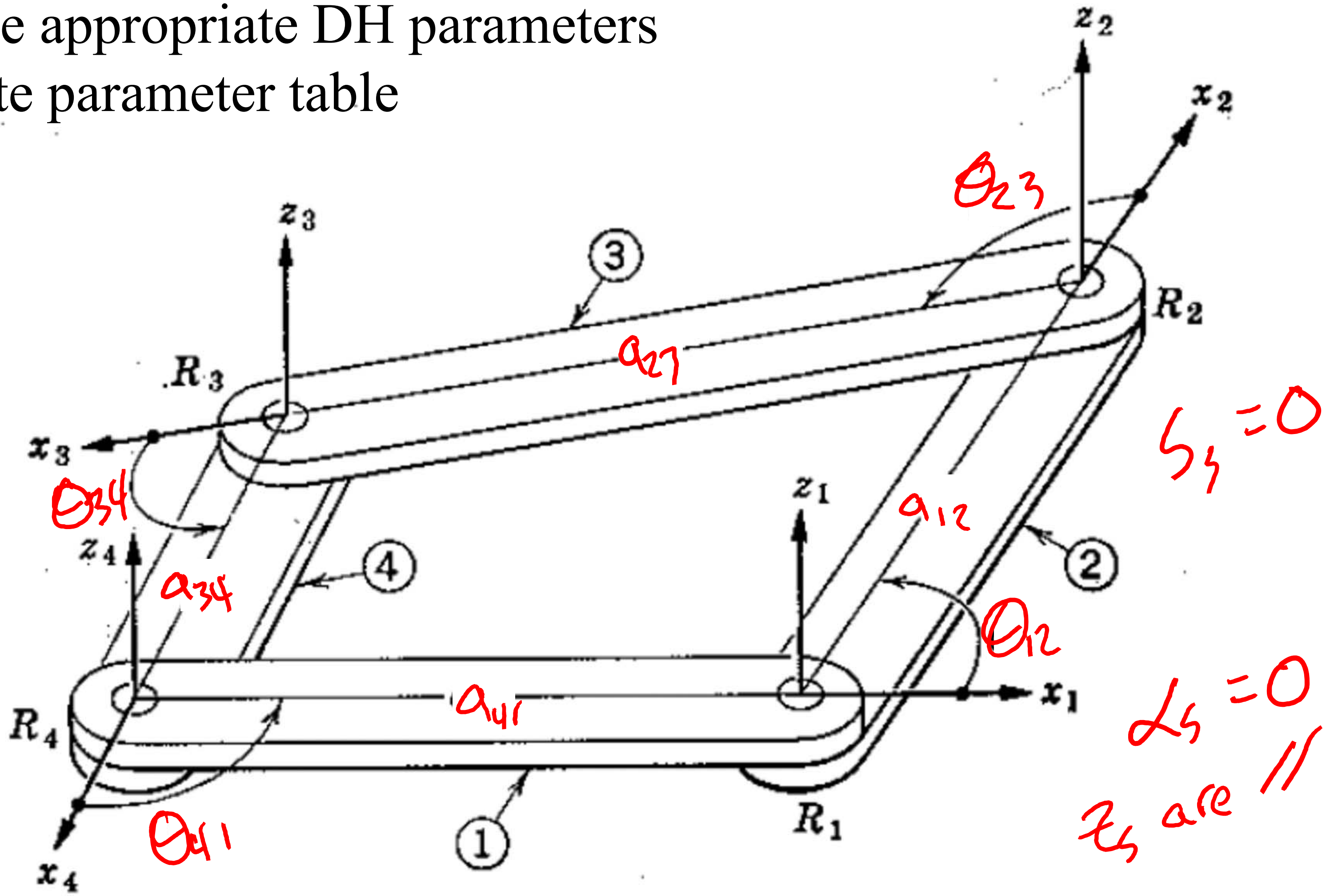
$\alpha_{i,i+1}$: angle from $z_i$ to $z_{i+1}$ (as seen from $x_{i+1}$)

$\theta_{i,i+1}$: angle from $x_i$ to $x_{i+1}$ (as seen from $z_i$)

$s_{i,i+1}$: dist along $z_i$ from $x_i$ to $x_{i+1}$

$$T_{i,i+1} = \begin{bmatrix} \cos\theta_{i,i+1} & -\cos\alpha_{i,i+1}\sin\theta_{i,i+1} & \sin\alpha_{i,i+1}\sin\theta_{i,i+1} & a_{i,i+1}\cos\theta_{i,i+1} \\ \sin\theta_{i,i+1} & \cos\alpha_{i,i+1}\cos\theta_{i,i+1} & -\sin\alpha_{i,i+1}\cos\theta_{i,i+1} & a_{i,i+1}\sin\theta_{i,i+1} \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & s_{i,i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Label the appropriate DH parameters
- Complete parameter table



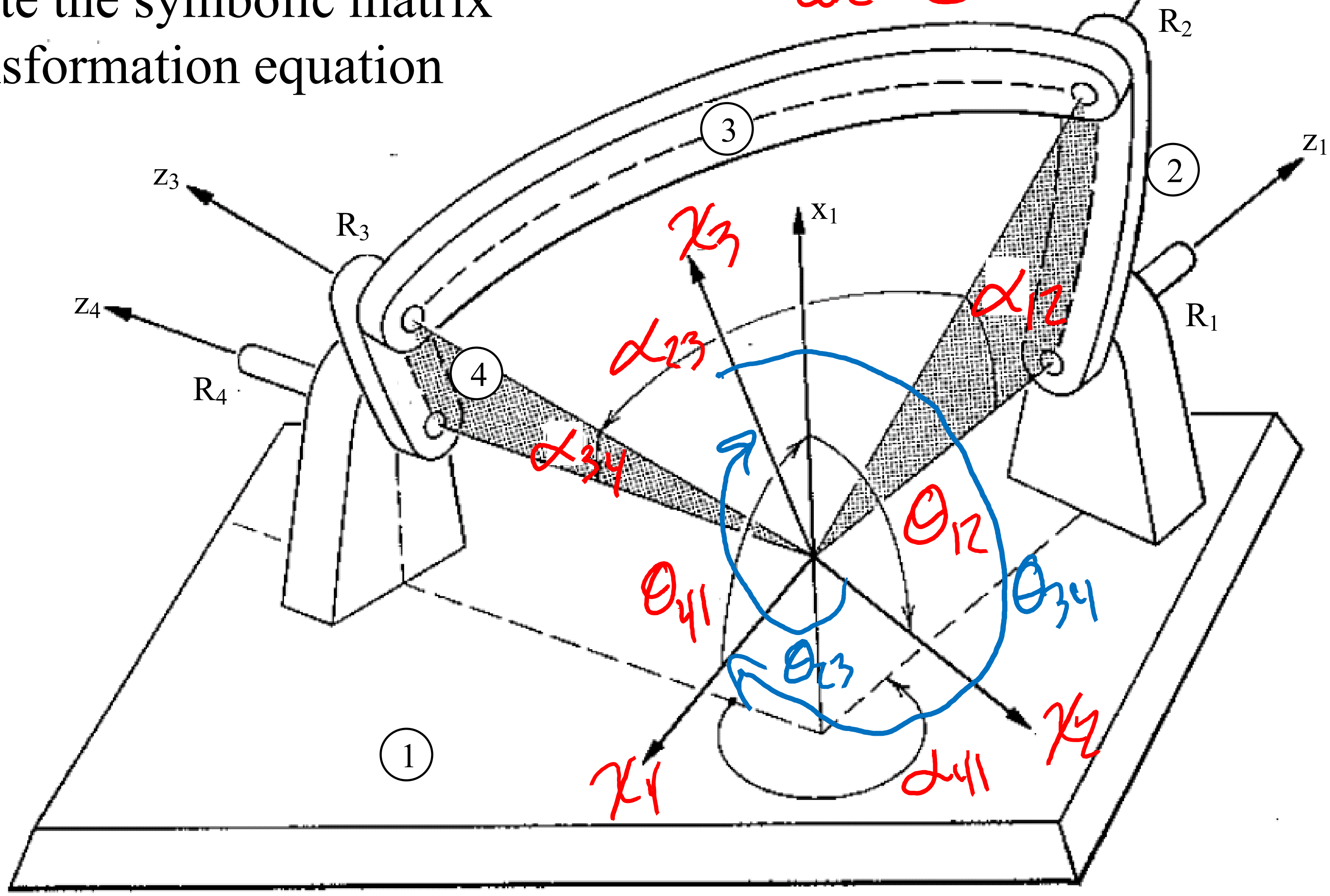

| DH Parameter Table | | | | |
|---|---|---|---|---|
| Joint | a | $\alpha$ | $\theta$ | $s$ |
| 0 | $a_{0,1} =$ | $\alpha_{0,1} =$ | $\theta_{0,1} =$ | $s_{0,1} =$ |
| 1 | $a_{1,2} =$ | $\alpha_{1,2} =$ | $\theta_{1,2} =$ | $s_{1,2} =$ |
| 2 | $a_{2,3} =$ | $\alpha_{2,3} =$ | $\theta_{2,3} =$ | $s_{2,3} =$ |

# Spherical Four-Bar

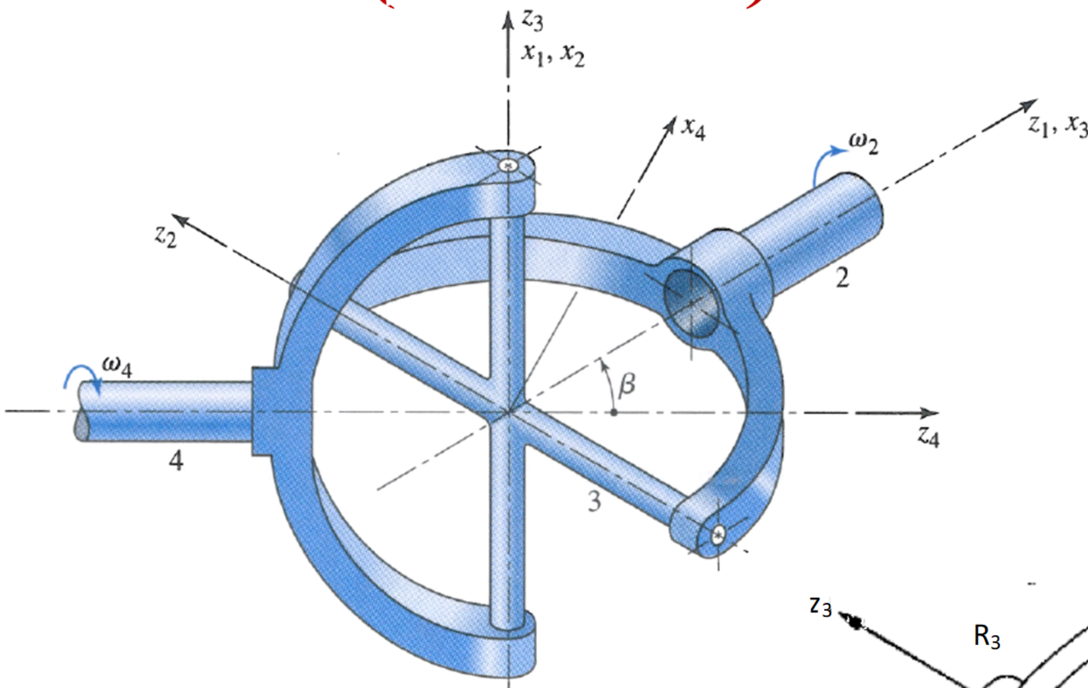
1. Label the x-axes
2. Draw the appropriate DH parameters
3. Write the symbolic matrix transformation equation

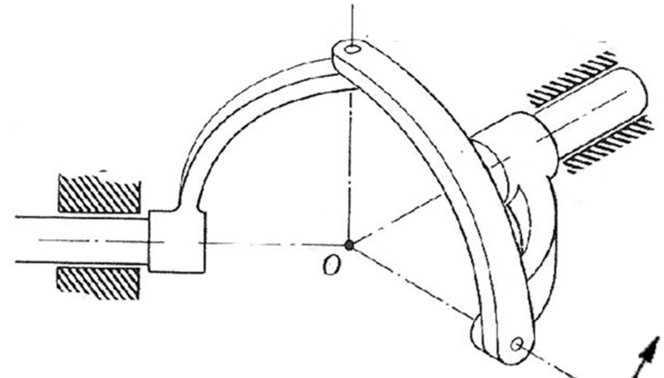
$a = s = 0$

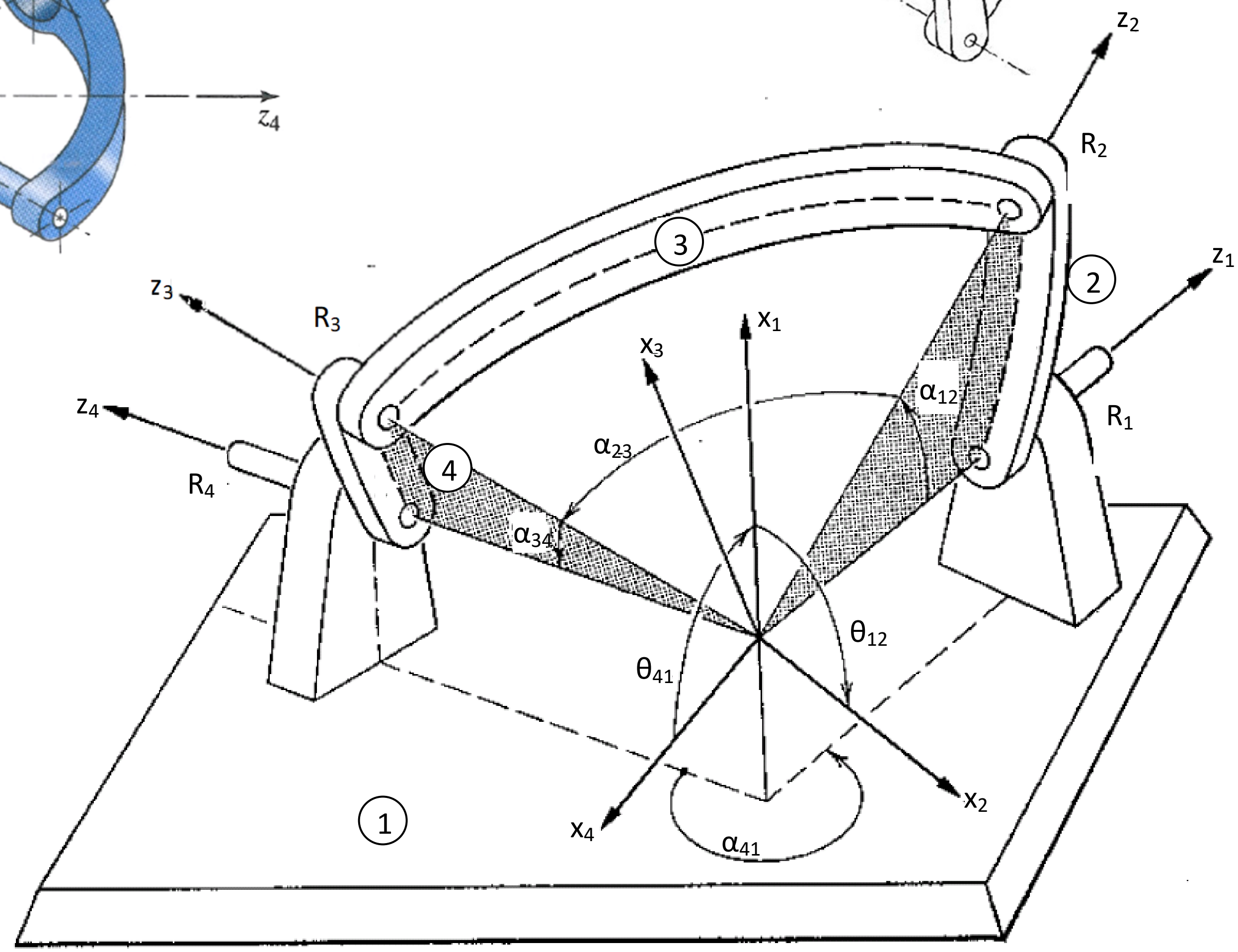
coordinate systems are coincident



# Hooke (Cardan) Joint Analysis

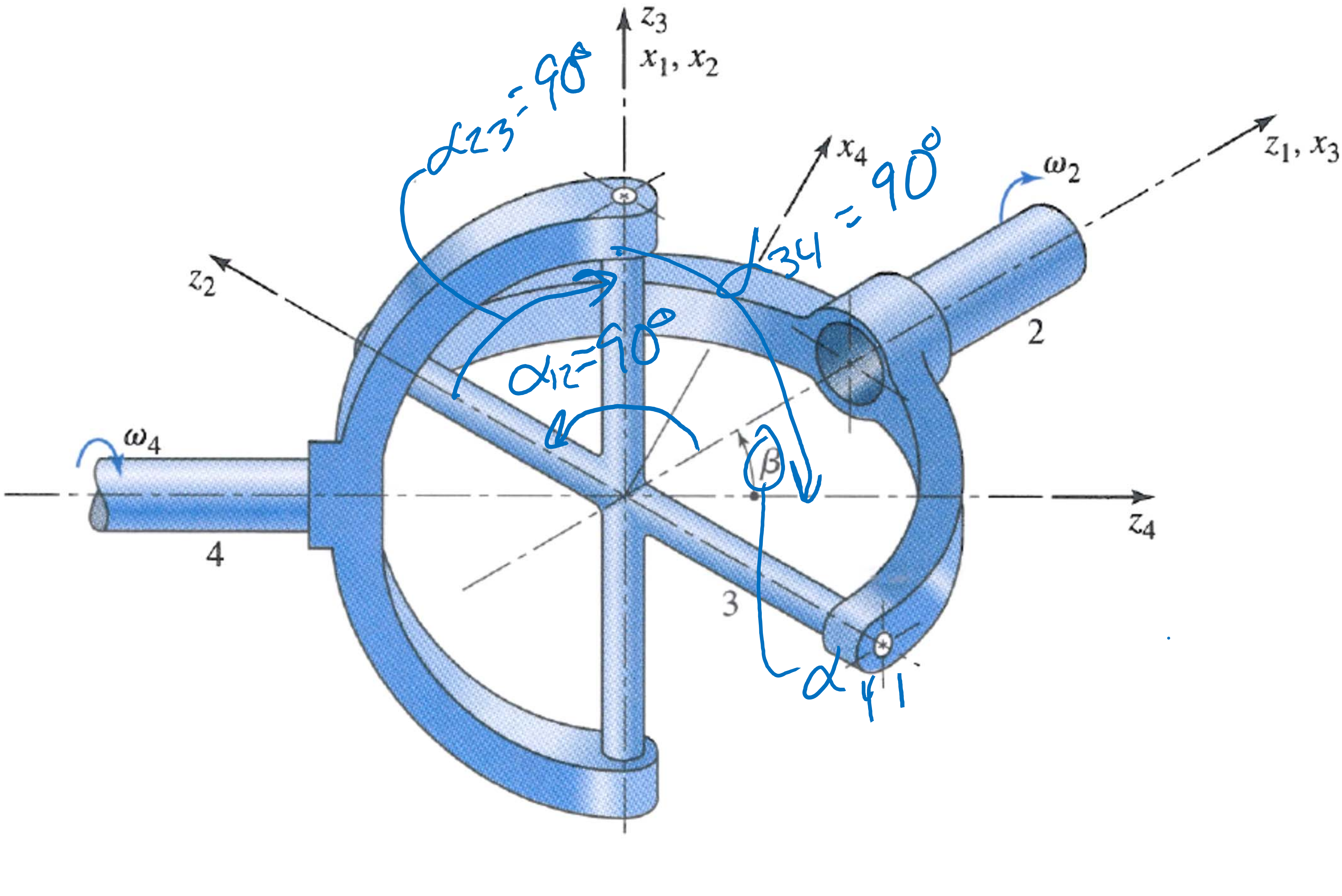


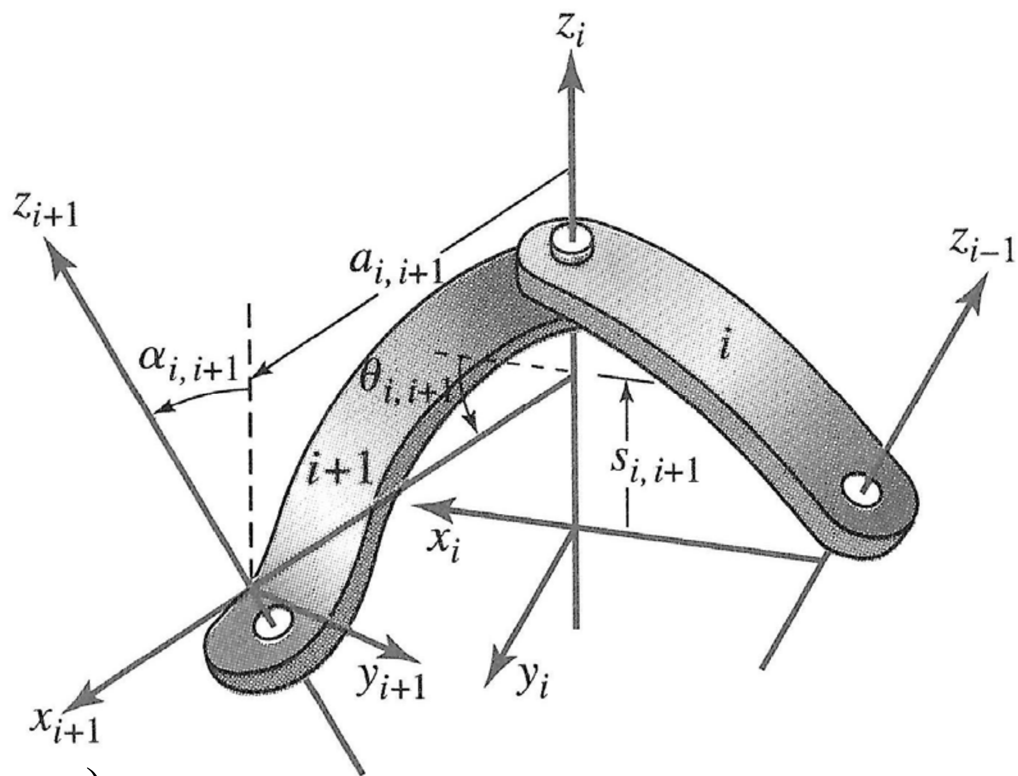

1. Write symbolic matrix transformation equation around closed chain
2. Write 4x4 $T_{12}$

# Hooke / Cardan Joint



# DH Parameters

1. Number joints consecutively, start at input
2. Joint axes: $x_i$ is perp to $z_{i\text{-}1}$ and $z_i$
3. Origin $x_i$, $y_i$, $z_i$ is fixed in link w/ joints $i$-1 and $i$




$a_{i,i+1}$: dist along $x_{i+i}$ from $z_i$ to $z_{i+1}$

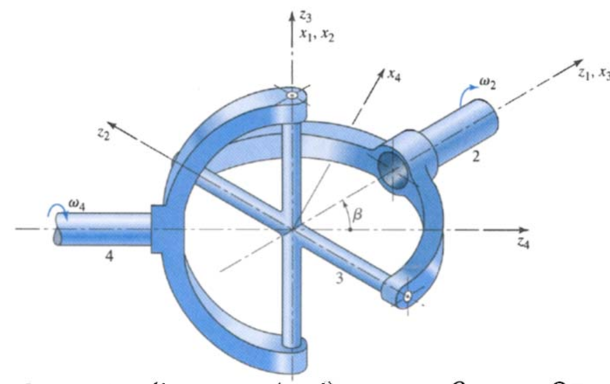
$\alpha_{i,i+1}$: angle from $z_i$ to $z_{i+1}$ (as seen from $x_{i+1}$)

$\theta_{i,i+1}$: angle from $x_i$ to $x_{i+1}$ (as seen from $z_i$)

$s_{i,i+1}$: dist along $z_i$ from $x_i$ to $x_{i+1}$

$$T_{i,i+1} = \begin{bmatrix} \cos\theta_{i,i+1} & -\cos\cancel{\alpha_{i,i+1}}\sin\theta_{i,i+1} & \sin\alpha_{i,i+1}\sin\theta_{i,i+1} & \cancel{a_{i,i+1}}\cos\theta_{i,i+1} \\ \sin\theta_{i,i+1} & \cos\cancel{\alpha_{i,i+1}}\cos\theta_{i,i+1} & -\sin\alpha_{i,i+1}\cos\theta_{i,i+1} & \cancel{a_{i,i+1}}\sin\theta_{i,i+1} \\ 0 & \sin\alpha_{i,i+1} & \cos\cancel{\alpha_{i,i+1}} & \cancel{s_{i,i+1}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Hooke Joint Analysis




$$T_{1,2}T_{2,3}T_{3,4} = T_{4,1}^{-1}$$

den

num

$$\begin{bmatrix} \cos\theta_{1,2}\cos\theta_{2,3}\cos\theta_{3,4} + \sin\theta_{1,2}\sin\theta_{3,4} & \cos\theta_{1,2}\sin\theta_{2,3} & -\cos\theta_{1,2}\cos\theta_{2,3}\sin\theta_{3,4} - \sin\theta_{1,2}\cos\theta_{3,4} & 0 \\ \sin\theta_{1,2}\cos\theta_{2,3}\cos\theta_{3,4} - \cos\theta_{1,2}\sin\theta_{3,4} & \sin\theta_{1,2}\sin\theta_{2,3} & \sin\theta_{1,2}\cos\theta_{2,3}\sin\theta_{3,4} + \cos\theta_{1,2}\cos\theta_{3,4} & 0 \\ \sin\theta_{2,3}\cos\theta_{3,4} & -\cos\theta_{2,3} & \sin\theta_{2,3}\sin\theta_{3,4} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

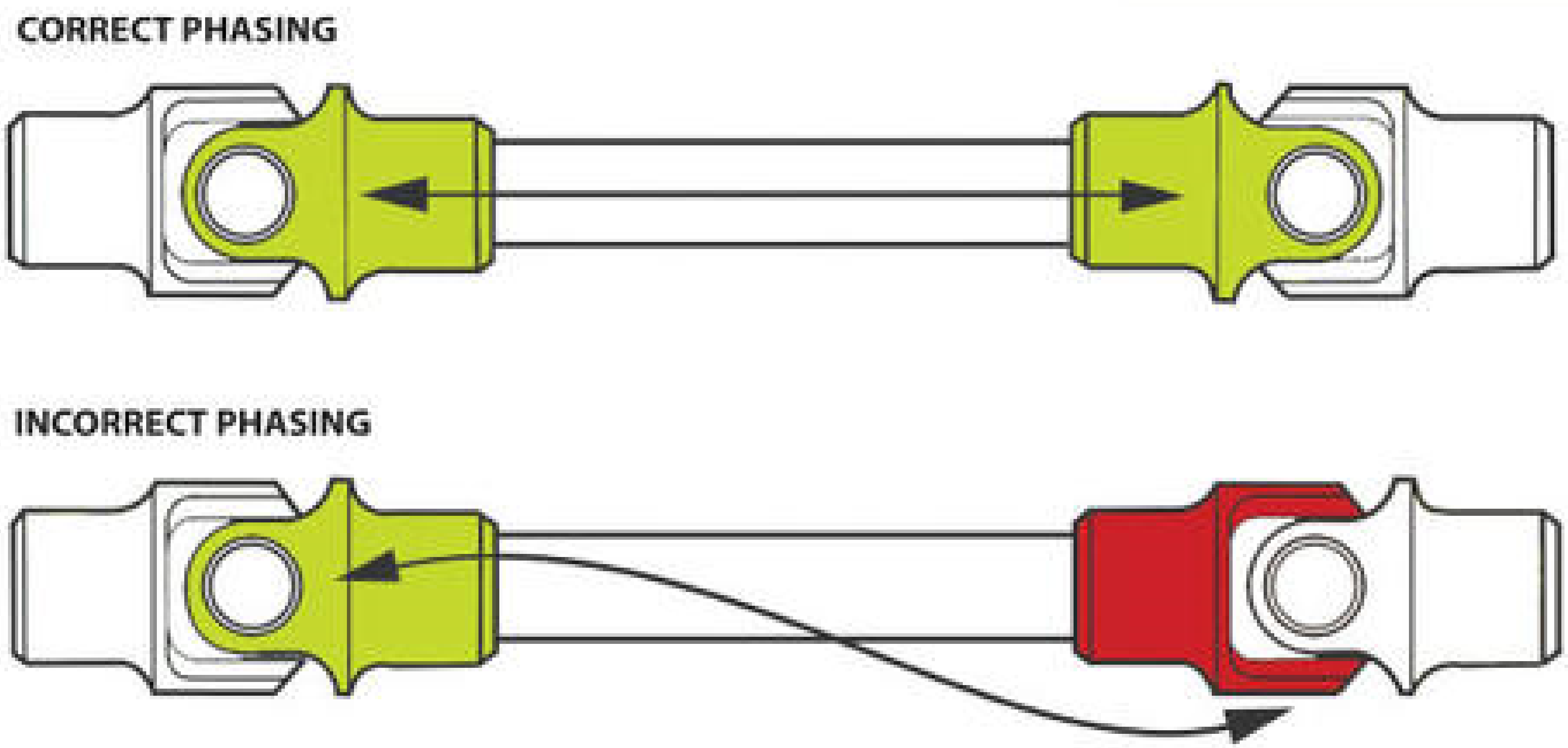
den

num

$$= \begin{bmatrix} \cos\theta_{4,1} & \sin\theta_{4,1} & 0 & 0 \\ -\cos\beta\sin\theta_{4,1} & \cos\beta\cos\theta_{4,1} & \sin\beta & 0 \\ \sin\beta\sin\theta_{4,1} & -\sin\beta\cos\theta_{4,1} & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

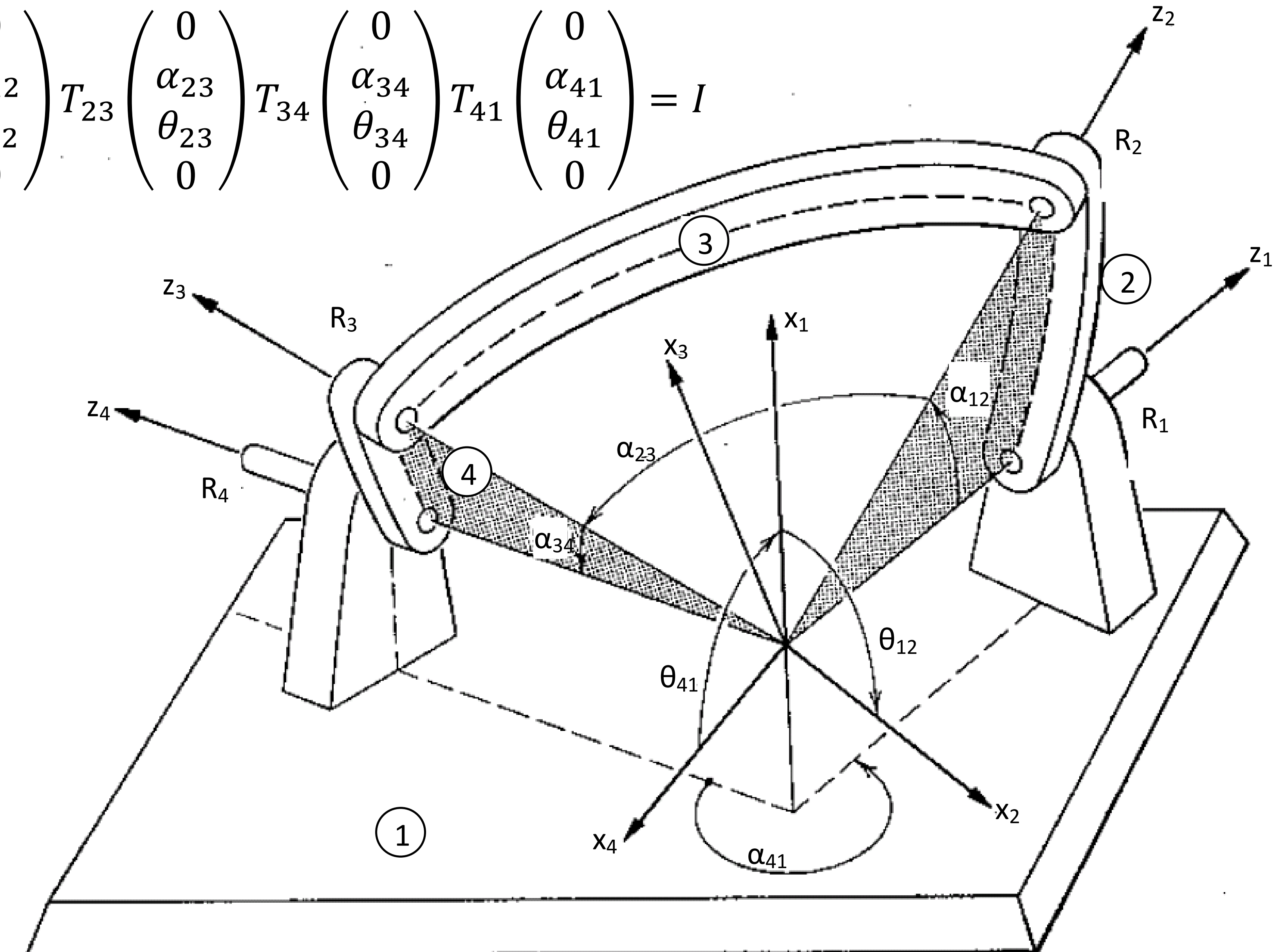
3. Develop an expression for the angular position of the output link as a function of the angular position of the input link and β

# Driveshaft Phasing



# Spherical 4-bar 3PP Function Generator Synthesis

$$T_{12}\begin{pmatrix}0\\ \alpha_{12}\\ \theta_{12}\\ 0\end{pmatrix}T_{23}\begin{pmatrix}0\\ \alpha_{23}\\ \theta_{23}\\ 0\end{pmatrix}T_{34}\begin{pmatrix}0\\ \alpha_{34}\\ \theta_{34}\\ 0\end{pmatrix}T_{41}\begin{pmatrix}0\\ \alpha_{41}\\ \theta_{41}\\ 0\end{pmatrix}=I$$



# Spherical 4-bar 3PP Function Generator Synthesis

$$T_{12}\begin{pmatrix}0\\ \alpha_{12}\\ \theta_{12}\\ 0\end{pmatrix} T_{23}\begin{pmatrix}0\\ \alpha_{23}\\ \theta_{23}\\ 0\end{pmatrix} T_{34}\begin{pmatrix}0\\ \alpha_{34}\\ \theta_{34}\\ 0\end{pmatrix} T_{41}\begin{pmatrix}0\\ \alpha_{41}\\ \theta_{41}\\ 0\end{pmatrix} = I$$

Develop Expression for $\theta_{12} = f(\theta_{41})$

$$\overbrace{\sin\alpha_{41}\cot\alpha_{12}}^{K_1}\cos\theta_{41} + \overbrace{\sin\alpha_{41}\cot\alpha_{41}}^{K_2}\cos\theta_{12} + \overbrace{\frac{\cos\alpha_{23}}{\sin\alpha_{12}\sin\alpha_{34}} - \cos\alpha_{41}\cot\alpha_{12}\cot\alpha_{34}}^{K_3}$$
$$= \sin\theta_{41}\sin\theta_{12} - \cos\alpha_{41}\cos\theta_{41}\cos\theta_{12}$$

Pick $\alpha_{41}$, Unknowns: $\alpha_{12}$, $\alpha_{23}$, $\alpha_{34}$

$$K_2\cos\theta_{41} + K_2\cos\theta_{12} + K_3 = \sin\theta_{41}\sin\theta_{12} - \cos\alpha_{41}\cos\theta_{41}\cos\theta_{12}$$

Where:

$$K_1 = \sin\alpha_{41}\cot\alpha_{12}$$
$$K_2 = \sin\alpha_{41}\cot\alpha_{41}$$
$$K_3 = \frac{\cos\alpha_{23}}{\sin\alpha_{12}\sin\alpha_{34}} - \cos\alpha_{41}\cot\alpha_{12}\cot\alpha_{34}$$

# Spherical 4-bar 3PP Function Generator Synthesis

$$K_2 \cos\theta_{41} + K_2 \cos\theta_{12} + K_3 = \sin\theta_{41}\sin\theta_{12} - \cos\alpha_{41}\cos\theta_{41}\cos\theta_{12}$$

Example: function $y = \log x$ for $1 \leq x \leq 10$

$$\theta_{41,initial} = 45°, \qquad \Delta\theta_{41} = 60°$$

$$\theta_{12,initial} = -45°, \qquad \Delta\theta_{12} = 90°$$

| **PP** | **1** | **2** | **3** |
|---|---|---|---|
| $x$ | 1 | 3 | 10 |
| $y$ | 0 | 0.477 | 1 |
| $\theta_{41}$ | 45° | 58.3° | 105° |
| $\theta_{12}$ | -45° | -2.1° | 45° |

Plug $\theta_{41}$ and $\theta_{12}$ into 3 simultaneous equations, solve for $K_1$, $K_2$, $K_3$

# Spherical 4-bar 3PP Function Generator Synthesis

$$K_2 \cos\theta_{41} + K_2 \cos\theta_{12} + K_3 = \sin\theta_{41}\sin\theta_{12} - \cos\alpha_{41}\cos\theta_{41}\cos\theta_{12}$$

Example: function $y = \log x$ for $1 \leq x \leq 10$

$$\theta_{41,initial} = 45°, \qquad \Delta\theta_{41} = 60°$$

$$\theta_{12,initial} = -45°, \qquad \Delta\theta_{12} = 90°$$

$K_1$ = -1.225

$K_2$ = 0.842

$K_3$ = -0.230

Where:

$$K_1 = \sin\alpha_{41}\cot\alpha_{12}$$

$$K_2 = \sin\alpha_{41}\cot\alpha_{41}$$

$$K_3 = \frac{\cos\alpha_{23}}{\sin\alpha_{12}\sin\alpha_{34}} - \cos\alpha_{41}\cot\alpha_{12}\cot\alpha_{34}$$

Solve for central angles:

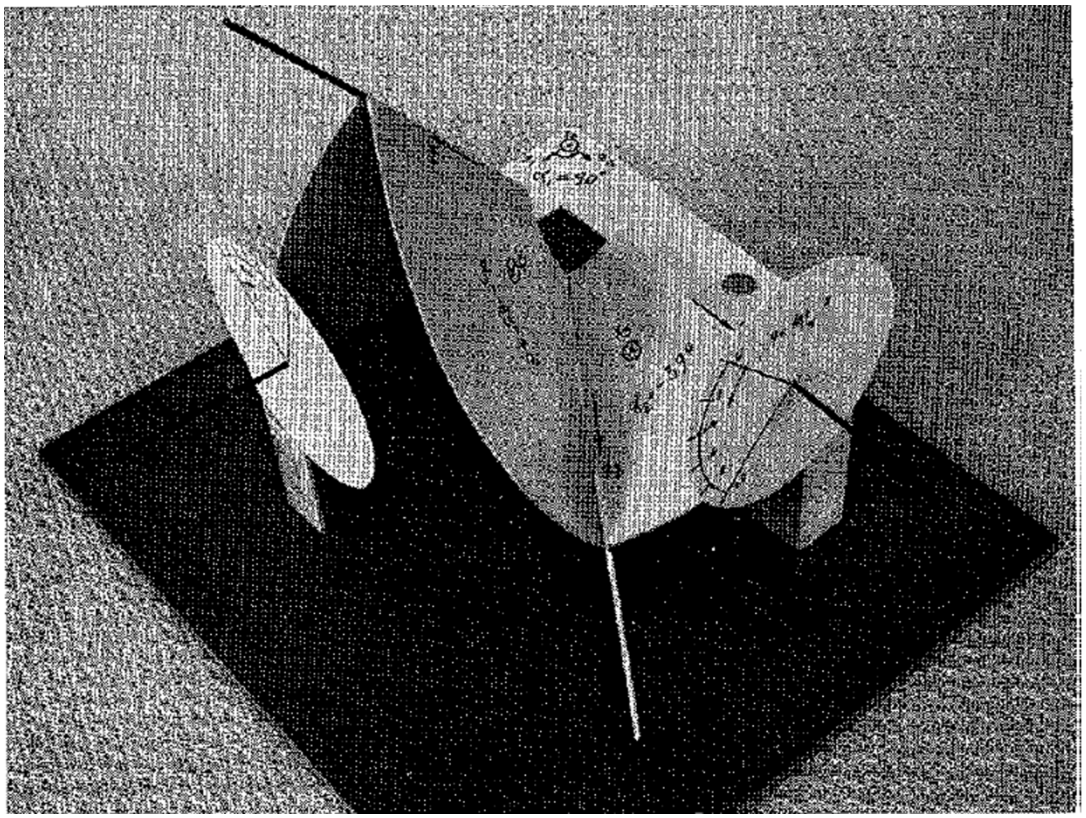
$$\alpha_{12} = -39.3°, \qquad \alpha_{23} = 83.7°, \qquad \alpha_{34} = 49.8°$$

# Spherical 4-bar 3PP Function Generator Synthesis

Example: function $y = \log x$ for $1 \leq x \leq 10$

$$\theta_{41,initial} = 45°, \qquad \Delta\theta_{41} = 60°$$

$$\theta_{12,initial} = -45°, \qquad \Delta\theta_{12} = 90°$$



# Survey of Spatial Synthesis Methods

# Anonymous Feedback

chimein.cla.umn.edu or text 1-503-770-6789

(text 22554 plus your answer)

1. What topics did you find most interesting/useful?
2. What topics would you like to see in the course review?