Machines and Mechanisms: Applied Kinematic Analysis, 4/e

Chapter 1

PEARSON

Chap 1 Introduction

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. **Explain the need for kinematic analysis of mechanisms.**
2. **Define the basic components that comprise a mechanism.**
3. **Draw a kinematic diagram from a view of a complex machine.**
4. **Compute the number of degrees of freedom of a mechanism.**
5. **Identify a four-bar mechanism and classify it according to its possible motion.**
6. **Identify a slider-crank mechanism.**

1.1 INTRODUCTION

- Determine appropriate movement of the wipers
  - View range
  - Tandem or opposite
  - Wipe angle
  - Location of pivots
- Timing of wipers
- Wiping velocity
- The force acting on the machine

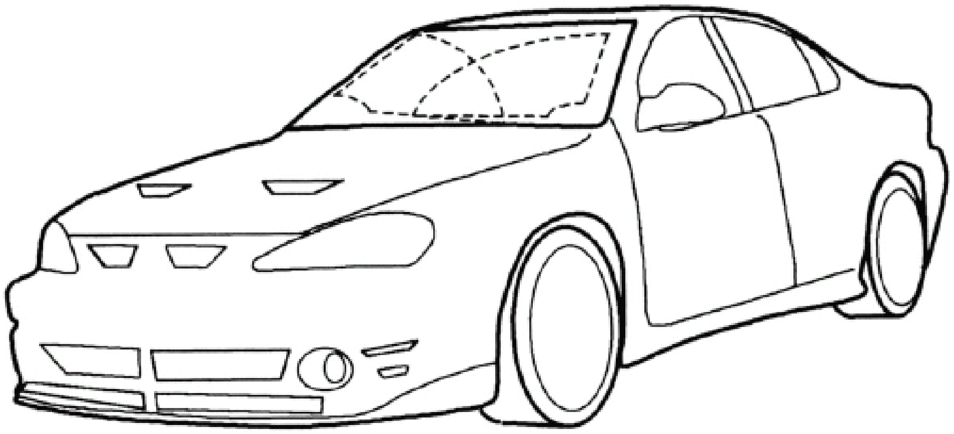


Figure 1.1 Proposed windshield wiper movements.

1.2 MACHINES AND MECHANISMS

- Machine
  - Devices used to alter, transmit, and direct forces to accomplish a specific objective
- Mechanism
  - Mechanical portion of a machine that has the function of transferring motion and forces from a power source to an output

FIGURE 1.2 Adjustable height platform (Courtesy Advance Lifts).

1.3 KINEMATICS

To illustrate the importance of such analysis, refer to the lift platform in Figure 1.2. Kinematic analysis provides insight into significant design questions, such as:

- What is the significance of the length of the legs that support the platform?
- Is it necessary for the support legs to cross and be connected at their midspan, or is it better to arrange the so that they cross closer to the platform?
- How far must the cylinder extend to raise the platform 8 in.?

Dynamics

As a second step, dynamic force analysis of the platform could provide insight into another set of important design questions:

- What capacity (maximum force) is required of the hydraulic cylinder?
- Is the platform free of any tendency to tip over?
- What cross-sectional size and material are required of the support legs so they don't fail?

Kinematics

- Kinematics
  - Deal with the way things move
- Kinematic analysis
  - Determine
    - Position, displacement, rotation, speed, velocity, acceleration
  - Provide
    - Geometry dimensions of the mechanism
    - Operation range
- Dynamic analysis
  - Power capacity, stability, member load
- Planar mechanism – motion in 2D space

1.4 MECHANISM TERMINOLOGY
Mechanism

- Synthesis is the process of developing mechanism to satisfy a set of performance requirements for the machine.
- Analysis ensures that the mechanism will exhibit motion to accomplish the requirements.

FIGURE 1.3 Elliptical trainer exercise machine (photo from www.precor.com).

- Linkage
- Frame
- Links– rigid body
- Joint
- Primary joint (full joint)
  - Revolute joint (pin or hinge joint)– pure rotation
  - Sliding joint (piston or prism joint)– linear sliding

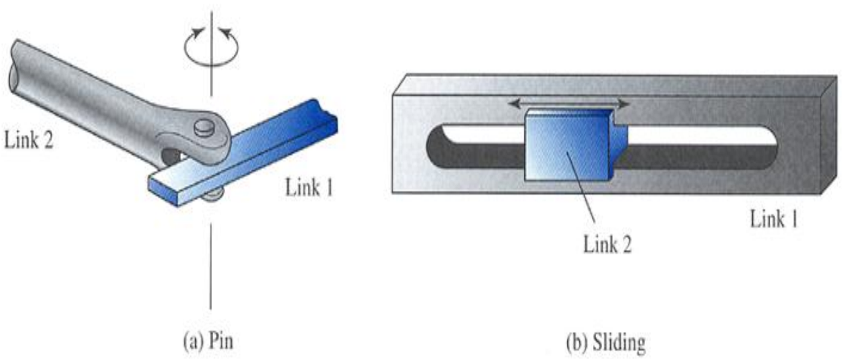


FIGURE 1.4 Primary joints: (a) Pin and (b) Sliding.

- Higher-order joint (half joint)
  - Allow rotation and sliding
  - Cam joint
  - Gear connection

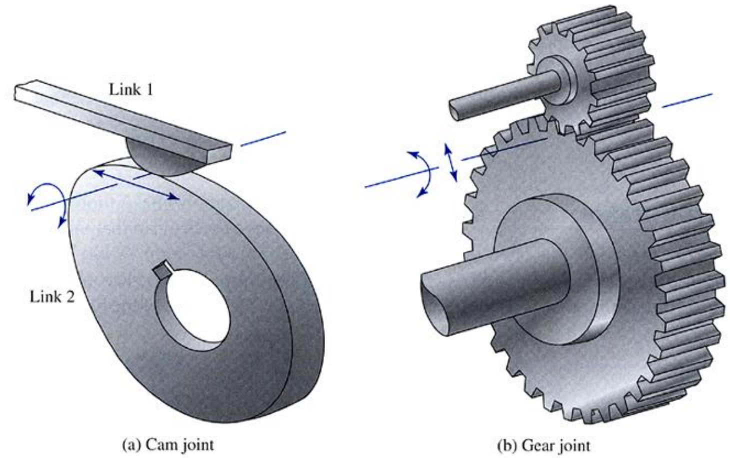


FIGURE 1.5 Higher-order joints: (a) Cam joint and (b) Gear joint.

- Simple link
  - A rigid body contains only two joints
  - Crank
  - Rocker
- Complex link
  - A rigid body contains more than two joints
  - Rocker arm
  - Bellcrank
- Point of interest
- Actuator
  - A power source link

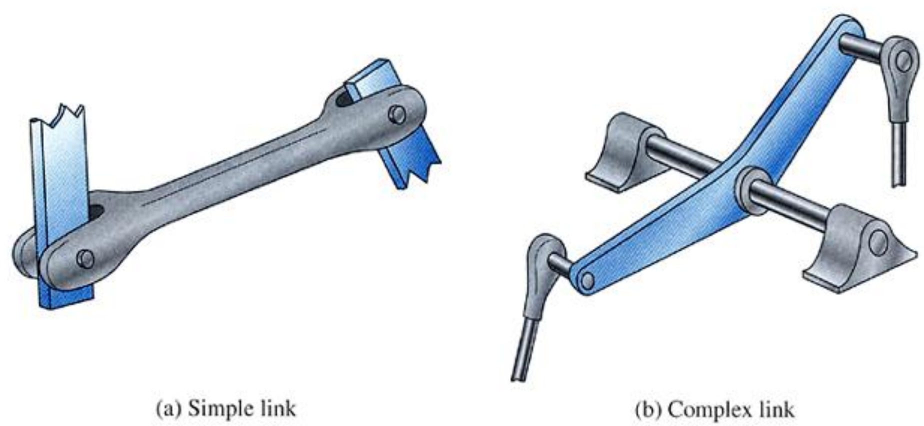
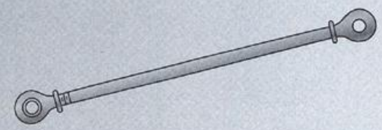
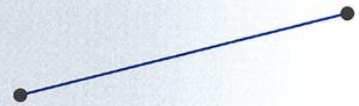
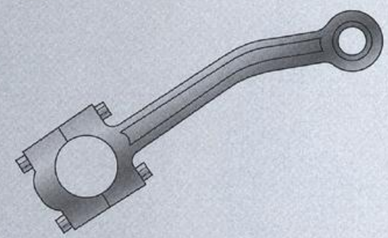
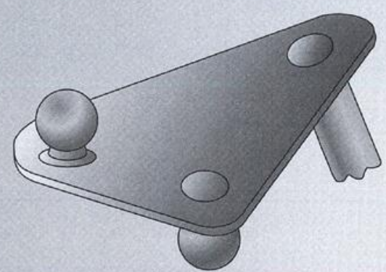
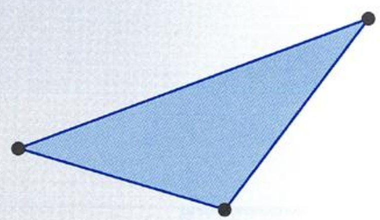
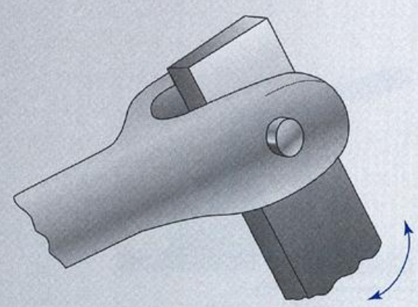
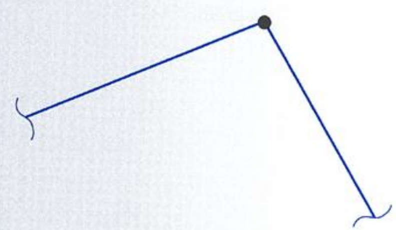


FIGURE 1.6 Links: (a) Simple link and (b) Complex link.

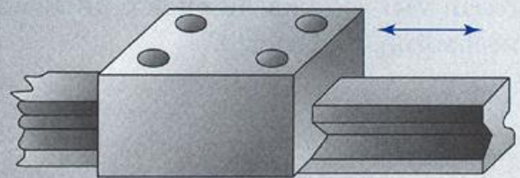
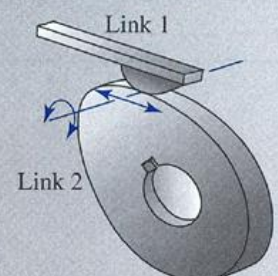
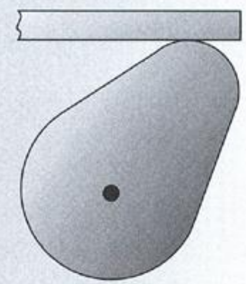
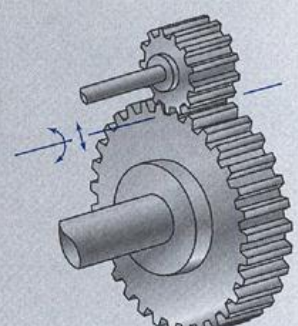
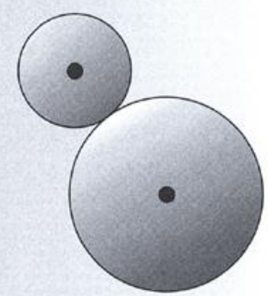
1.5 Kinematic Diagram

TABLE 1.1 Symbols Used in Kinematic Diagrams

Component	Typical Form	Kinematic Representation
Simple Link		
Simple Link (with point of interest)		
Complex Link		
Pin Joint		

Kinematic Diagram

TABLE 1.1 (Continued)

Component	Typical Form	Kinematic Representation
Slider Joint		
Cam Joint		
Gear Joint		

EXAMPLE PROBLEM 1.1

Figure 1.9 shows a shear that is used to cut and trim electronic circuit board laminates. Draw a kinematic diagram.

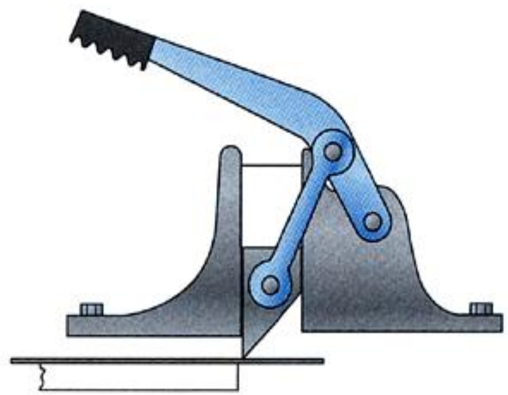


FIGURE 1.9 Shear press for Example Problem 1.1.

SOLUTION: In this problem, the large base that is bolted to the table is designated as the frame. The motion of all other links is determined relative to the base. The base is numbered as link 1.

2. ***Identify All Other Links***

   Link 2: Handle

   Link 3: Cutting blade

   Link 4: Bar that connects the cutter with the handle

3. ***Identify the Joints***

   Pin joints are used to connect link 1 to 2, link 2 to 3, and link 3 to 4. These joints are lettered *A* through *C*. In addition, the cutter slides up and down, along the base. This sliding joint connects link 4 to 1, and is lettered *D*.

4. ***Identify Any Points of Interest***

   Finally, the motion of the end of the handle is desired. This is designated as *point of interest X*.

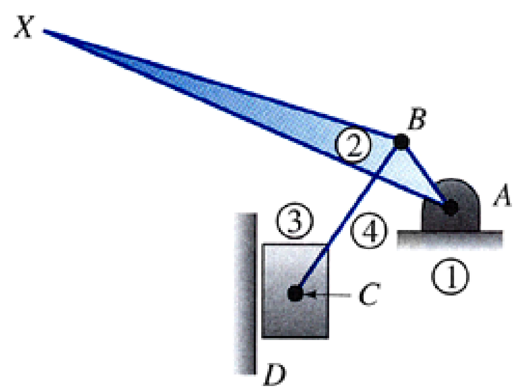


FIGURE 1.10 Kinematic diagram for Example Problem 1.1.

EXAMPLE PROBLEM 1.2

Figure 1.11 shows a pair of vise grips. Draw a kinematic diagram.

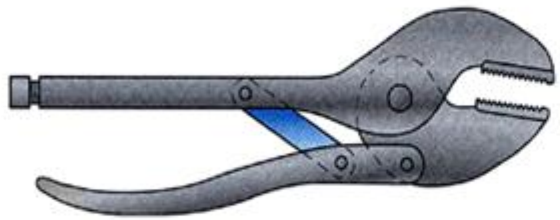


FIGURE 1.11 Vise grips for Example Problem 1.2.

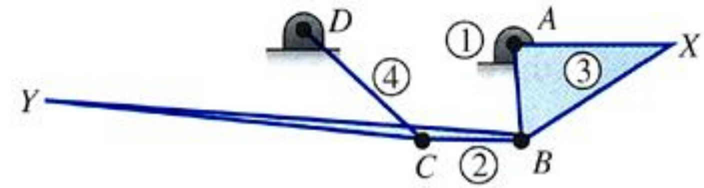


FIGURE 1.12 Kinematic diagram for Example Problem 1.2.

SOLUTION:

1. ***Identify the Frame***

   Is rather arbitrary.

   The top handle is numbered as link 1.

2. ***Identify All Other Links***

   Link 2: Bottom handle

   Link 3: Bottom jaw

   Link 4: Bar that connects the top and bottom handle

3. ***Identify the Joints***

   Four pin joints are used to connect these different links (link 1 to 2, 2 to 3, 3 to 4, and 4 to 1). These joints are lettered *A* through *D*.

4. ***Identify Any Points of Interest***

   The motion of the end of the bottom jaw is desired. This is designated as point of interest *X*. Finally, the motion of the end of the lower handle is also desired. This is designated as point of interest *Y*.

1.7 MOBILITY

1.7.1 Gruebler's Equation

$$M = \text{degrees of freedom} = 3(n - 1) - 2j_p - j_h$$

where:

n = total number of links in the mechanism

j_p = total number of primary joints (pins or sliding joints)

j_h = total number of higher-order joints (cam or gear joints)

- Constrained mechanism
  - one degree of freedom
- Locked mechanism
  - Zero or negative degrees of freedom
- Unconstrained mechanism
  - More than one degree of freedom

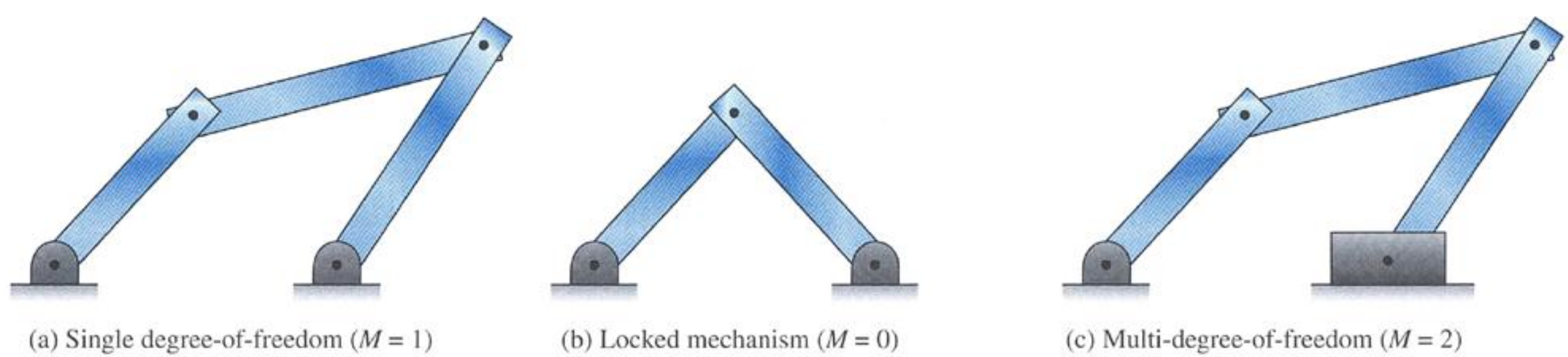


FIGURE 1.13 Mechanisms and structures with varying mobility.

EXAMPLE PROBLEM 1.3

Figure 1.14 shows a toggle clamp. Draw a kinematic diagram, using the clamping jaw and the handle as points of interest. Also compute the degrees of freedom for the clamp.

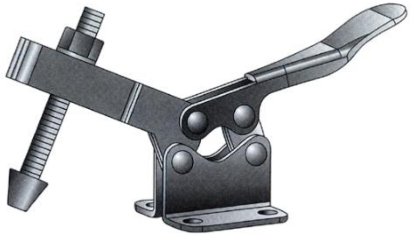


FIGURE 1.14 Toggle clamp for Example Problem 1.3.

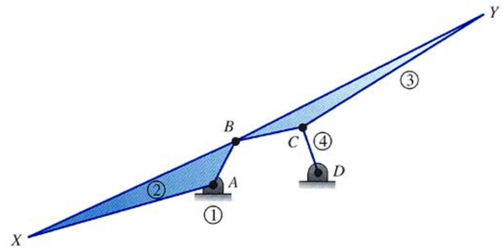


FIGURE 1.15 Kinematic diagram for Example Problem 1.3.

SOLUTION:

1. ***Identify the Frame***

   The component that is bolted to the table is designated as the frame. The frame is numbered as link 1.

2. ***Identify All Other Links***

   Link 2: Handle

   Link 3: Arm that serves as the clamping jaw

   Link 4: Bar that connects the clamping arm and handle

3. ***Identify the Joints***

   Four pin joints are used to connect these different links (link 1 to 2, 2 to 3, 3 to 4, and 4 to 1). These joints are lettered *A* through *D*.

4. ***Identify Any Points of Interest***

   The motion of the clamping jaw is desired. This is designated as point of interest *X*. Finally, the motion of the end of the handle is also desired. This is designated as point of interest *Y*.

5. ***Draw the Kinematic Diagram***

   The kinematic diagram is detailed in Figure 1.15.

6. ***Calculate Mobility***

   Having four links and four pin joints,

$$n = 4, j_p = 4 \text{ pins}, j_h = 0$$

and

$$M = 3(n - 1) - 2j_p - j_h = 3(4 - 1) - 2(4) - 0 = 1$$

With one degree of freedom, the clamp mechanism is constrained. Moving only one link, the handle, precisely positions all other links in the clamp.

EXAMPLE PROBLEM 1.4

Figure 1.16 shows a beverage can crusher used to reduce the size of cans for easier storage prior to recycling. Draw a kinematic diagram, using the end of the handle as a point of interest. Also compute the degrees of freedom for the device.

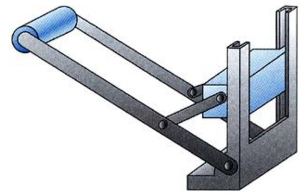


FIGURE 1.16 Can crusher for Example Problem 1.4.

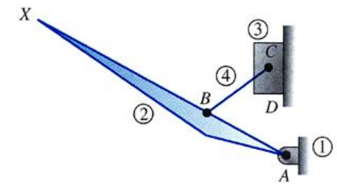


FIGURE 1.17 Kinematic diagram for Example Problem 1.4.

SOLUTION:

1. ***Identify the Frame***

   The back portion of the device serves as a base and can be attached to a wall. This component is designated as the frame. numbered as link 1.

2. ***Identify All Other Links***

   Link 2: Handle
   Link 3: Block that serves as the crushing surface
   Link 4: Bar that connects the crushing block and handle

3. ***Identify the Joints***

   Three pin joints are used to connect these different parts. One pin connects the handle to the base. This joint is labeled as *A*. A second pin is used to connect link 4 to the handle. This joint is labeled *B*. The third pin connects the crushing block and link 4. This joint is labeled *C*.

   The crushing block slides vertically during operation; therefore, a sliding joint connects the crushing block to the base. This joint is labeled *D*.

4. ***Identify Any Points of Interest***

   The motion of the handle end is desired. This is designated as point of interest *X*.

5. ***Draw the Kinematic Diagram***

   The kinematic diagram is given in Figure 1.17.

6. ***Calculate Mobility***

   It was determined that there are four links in this mechanism. There are also three pin joints and one slider joint. Therefore,

$$n = 4, j_p = (3 \text{ pins} + 1 \text{ slider}) = 4, j_h = 0$$

and

$$M = 3(n - 1) - 2j_p - j_h = 3(4 - 1) - 2(4) - 0 = 1$$

With one degree of freedom, the can crusher mechanism is constrained. Moving only one link, the handle, precisely positions all other links and crushes a beverage can placed under the crushing block.

EXAMPLE PROBLEM 1.5

Figure 1.18 shows another device that can be used to shear material. Draw a kinematic diagram, using the end of the handle and the cutting edge as points of interest. Also, compute the degrees of freedom for the shear press.

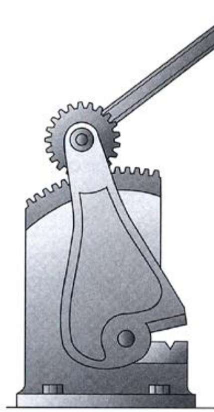


FIGURE 1.18 Shear press for Example Problem 1.5.

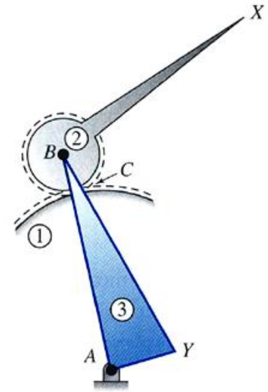


FIGURE 1.19 Kinematic diagram for Example Problem 1.5.

SOLUTION:

1. ***Identify the Frame***

   The base is bolted to a working surface and can be designated as the frame. The frame is numbered as link 1.

2. ***Identify All Other Links***

   Link 2: Gear/handle

   Link 3: Cutting lever

3. ***Identify the Joints***

4. ***Identify Any Points of Interest***

5. ***Draw the Kinematic Diagram***

   The kinematic diagram is given in Figure 1.19.

6. ***Calculate Mobility***

$$n = 3 \quad j_p = (2 \text{ pins}) = 2 \quad j_h = (1 \text{ gear connection}) = 1$$

and

$$M = 3(n - 1) - 2j_p - j_h = 3(3 - 1) - 2(2) - 1 = 1$$

Actuators and Drivers

- Electric motors (AC)
- Electric motors (DC)
- Engines
- Servomotors
- Air or hydraulic motors
- Hydraulic or pneumatic cylinders
- Screw actuators
- Manual

1.7.2 Actuators and Drivers

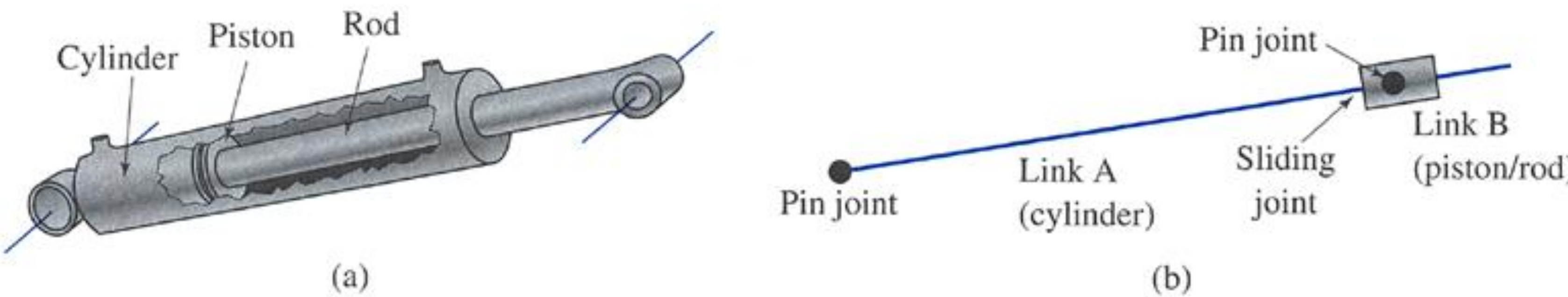


FIGURE 1.20 Hydraulic cylinder.

EXAMPLE PROBLEM 1.6

Figure 1.21 shows an outrigger foot to stabilize a utility truck. Draw a kinematic diagram, using the bottom of the stabilizing foot as a point of interest. Also compute the degrees of freedom.

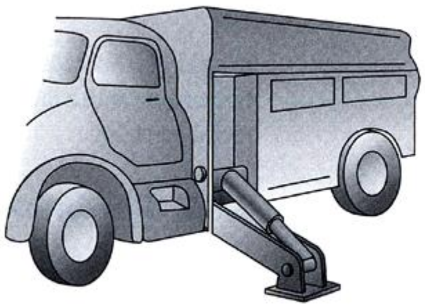


FIGURE 1.21 Outrigger for Example Problem 1.6.

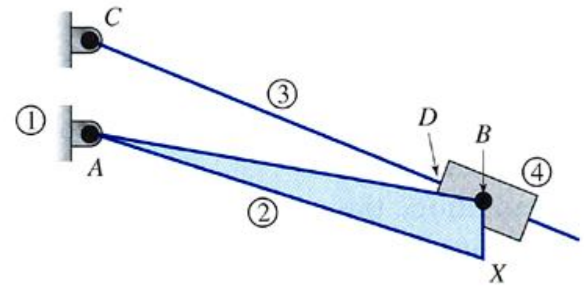


FIGURE 1.22 Kinematic diagram for Example Problem 1.6.

SOLUTION:

1. ***Identify the Frame***
2. ***Identify All Other Links***

   Link 2: Outrigger leg

   Link 3: Cylinder

   Link 4: Piston/rod
3. ***Identify the Joints***

   as joint *D*.
4. ***Identify Any Points of Interest***

   The stabilizer foot is part of link 2, and a point of interest located on the bottom of the foot is labeled as point of interest *X*.
5. ***Draw the Kinematic Diagram***

   The resulting kinematic diagram is given in Figure 1.22.
6. ***Calculate Mobility***

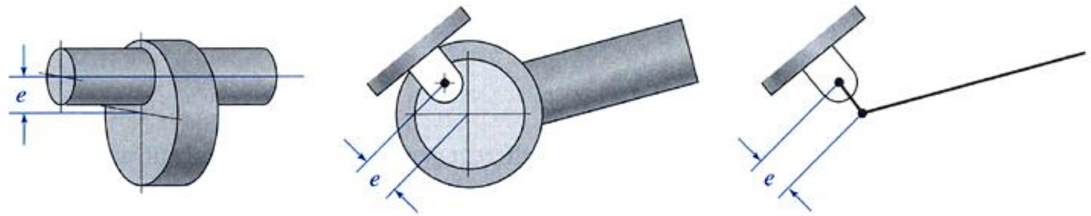
   $$n = 4, j_p = (3 \text{ pins} + 1 \text{ slider}) = 4, j_h = 0$$

   and

   $$M = 3(n - 1) - 2j_p - j_h = 3(4 - 1) - 2(4) - 0 = 1$$

1.8 COMMONLY USED LINKS AND JOINTS

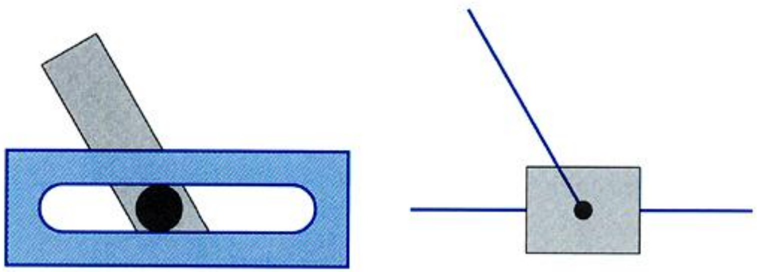
1.8.1 Eccentric Crank



(a) Eccentric crankshaft (b) Eccentric crank (c) Eccentric crank model

FIGURE 1.23 Eccentric crank.

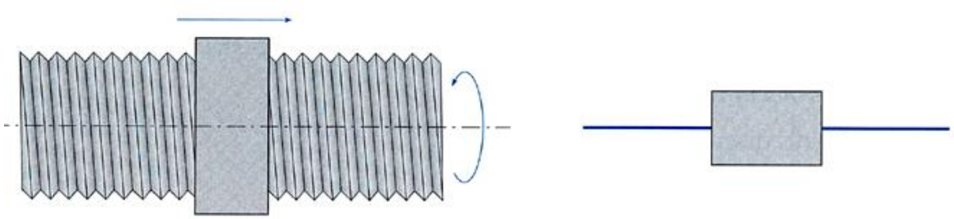
1.8.2 Pin-in-a-Slot Joint



(a) Actual pin-in-a-slot joint (b) Pin-in-a-slot model

FIGURE 1.24 Pin-in-a-slot joint.

1.8.3 Screw Joint



(a) Actual screw joint (b) Screw modeled as a slider

FIGURE 1.25 Screw joint.

EXAMPLE PROBLEM 1.7

Figure 1.26 presents a lift table used to adjust the working height of different objects. Draw a kinematic diagram and compute the degrees of freedom.

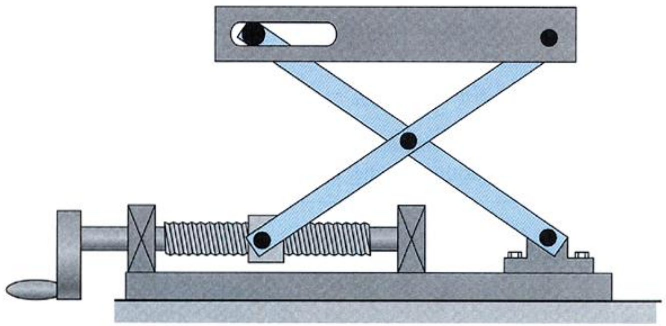


FIGURE 1.26 Lift table for Example Problem 1.7.

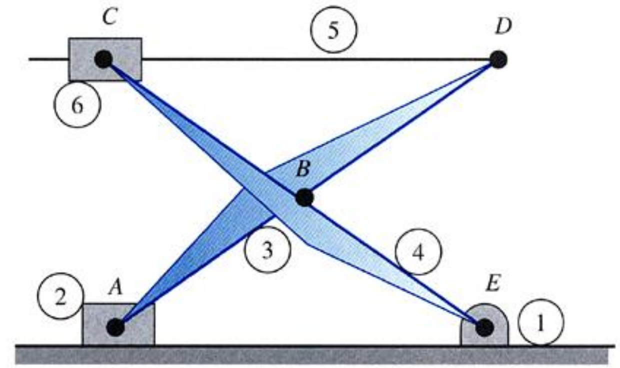


FIGURE 1.27 Kinematic diagram for Example Problem 1.7.

SOLUTION:

1. ***Identify the Frame***
2. ***Identify All Other Links***
   Link 2: Nut
   Link 3: Support arm that ties the nut to the table
   Link 4: Support arm that ties the fixed bearing to the slot in the table
   Link 5: Table
   Link 6: Extra link used to model the pin in slot joint with separate pin and slider joints
3. ***Identify the Joints***
4. ***Draw the Kinematic Diagram***

   The kinematic diagram is given in Figure 1.27.

5. ***Calculate Mobility***

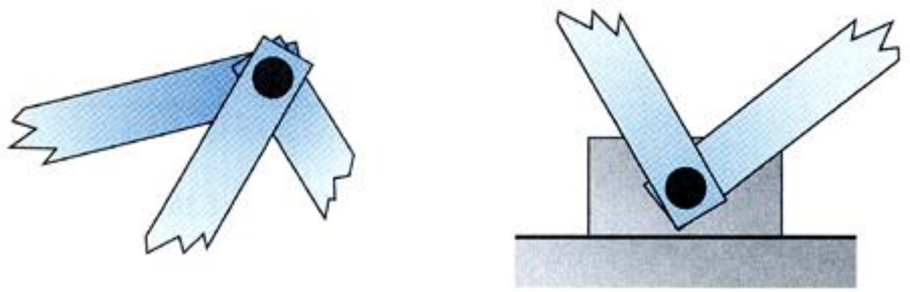
$$n = 6 \quad j_p = (5 \text{ pins} + 2 \text{ sliders}) = 7 \quad j_h = 0$$

and

$$M = 3(n-1) - 2j_p - j_h = 3(6-1) - 2(7) - 0 = 15 - 14 = 1$$

1.9 SPECLAL CASES OF THE MOBILITY EQUATION

1.9.1 Coincident Joints



(a) Three rotating links (b) Two rotating and one sliding link

FIGURE 1.28 Three links connected at a common pin joint.

1.9.2

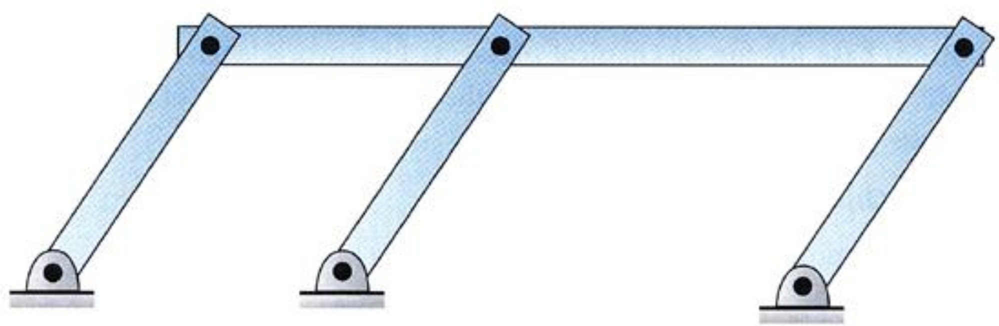


FIGURE 1.31 Mechanism that violates the Gruebler's equation.

- One degree of freedom actually if pivoted links are the same size

EXAMPLE PROBLEM 1.8

Figure 1.29 shows a mechanical press used to exert large forces to insert a small part into a larger one. Draw a kinematic diagram, using the end of the handle as a point of interest. Also compute the degrees of freedom.

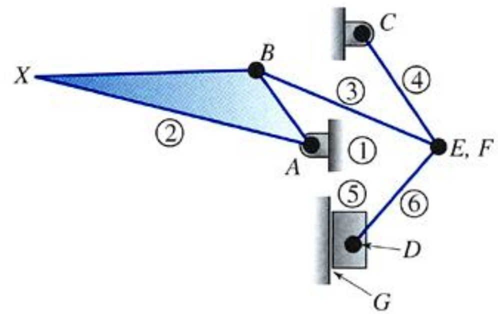


FIGURE 1.30 Kinematic diagram for Example Problem 1.8.

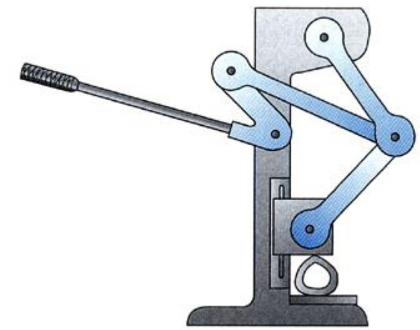


FIGURE 1.29 Mechanical press for Example Problem 1.8.

SOLUTION:

1. ***Identify the Frame***
2. ***Identify All Other Links***

   Link 2: Handle

   Link 3: Arm that connects the handle to the other arms

   Link 4: Arm that connects the base to the other arms

   Link 5: Press head

   Link 6: Arm that connects the head to the other arms

3. ***Identify the Joints***
4. ***Identify Any Points of Interest***
5. ***Draw the Kinematic Diagram***
6. ***Calculate Mobility***

   To calculate the mobility, it was determined that there are six links in this mechanism, as well as six pin joints and one slider joint. Therefore,

   $$n = 6, j_p = (6 \text{ pins} + 1 \text{ slider}) = 7, j_h = 0$$

   and

   $$M = 3(n - 1) - 2j_p - j_h = 3(6 - 1) - 2(7) - 0 = 15 - 14 = 1$$

1.10 THE FOUR-BAR MECHANISM

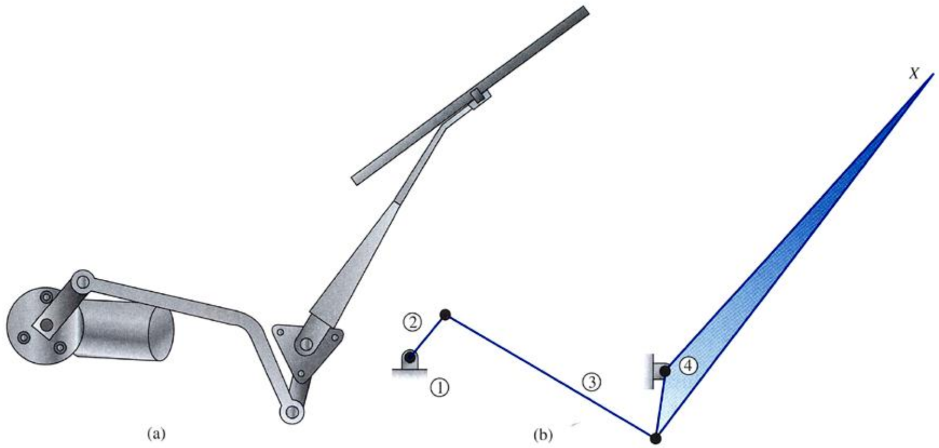


FIGURE 1.33 Rear-window wiper mechanism.

The mobility of a four-bar mechanism consists of the following:

$$n = 4,\ j_p = 4 \text{ pins},\ j_h = 0$$

and

$$M = 3(n-1) - 2j_p - j_h = 3(4-1) - 2(4) - 0 = 1$$

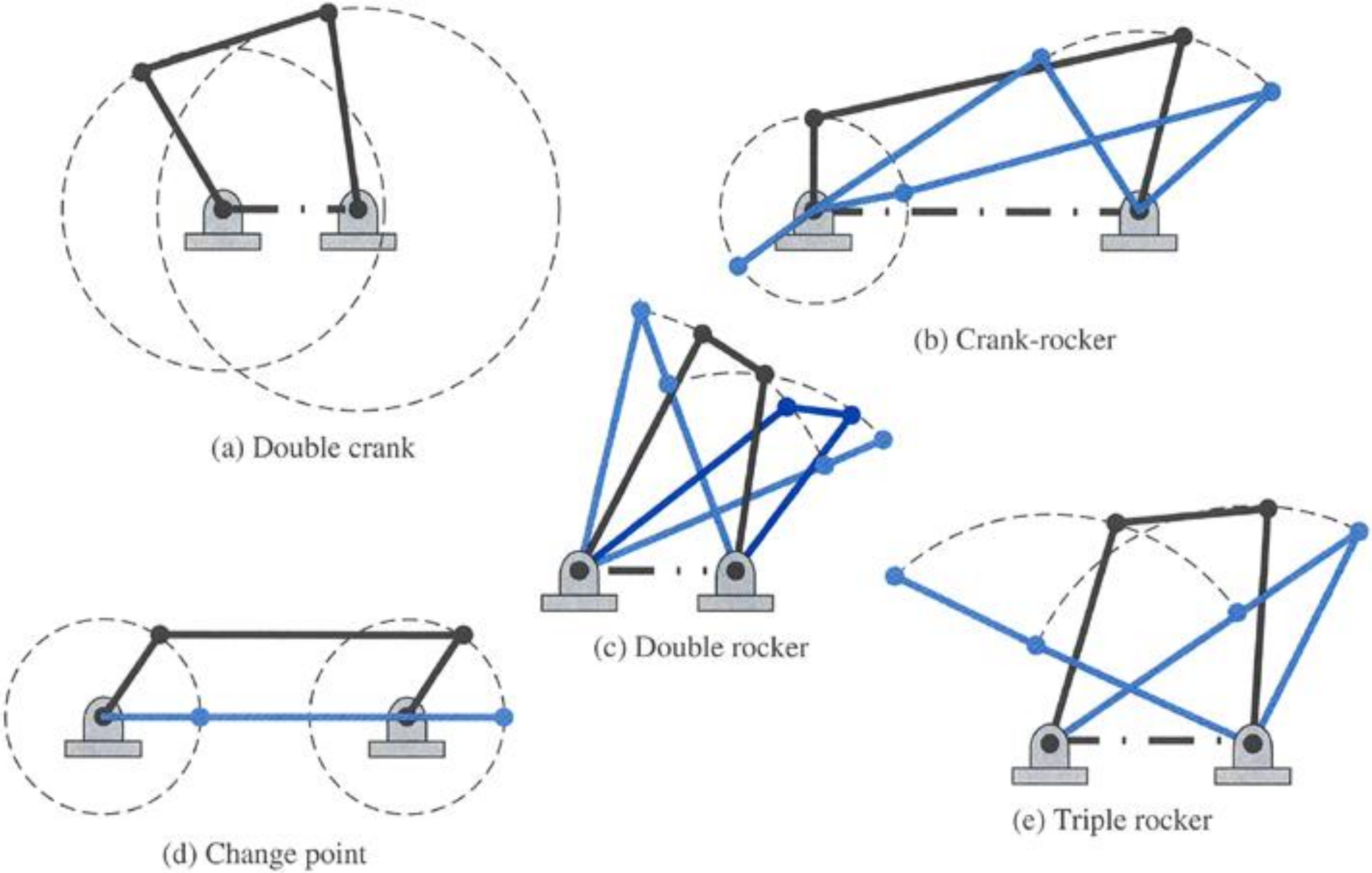


FIGURE 1.34 Categories of four-bar mechanisms.

1.10.1 Degree-of-Freedom

A four-bar mechanism has at least one revolving link if: $s + l \leq p + q$
Conversely, the three nonfixed links will merely rock if: $s + l > p + q$

s : short link
l : long link
p , q : intermediate link

TABLE 1.2 Categories of Four-Bar Mechanisms

Case	Criteria	Shortest Link	Category
1	$s + l < p + q$	Frame	Double crank
2	$s + l < p + q$	Side	Crank-rocker
3	$s + l < p + q$	Coupler	Double rocker
4	$s + l = p + q$	Any	Change point
5	$s + l > p + q$	Any	Triple rocker

EXAMPLE PROBLEM 1.9

A nosewheel assembly for a small aircraft is shown in Figure 1.35. Classify the motion of this four-bar mechanism based on the configuration of the links.

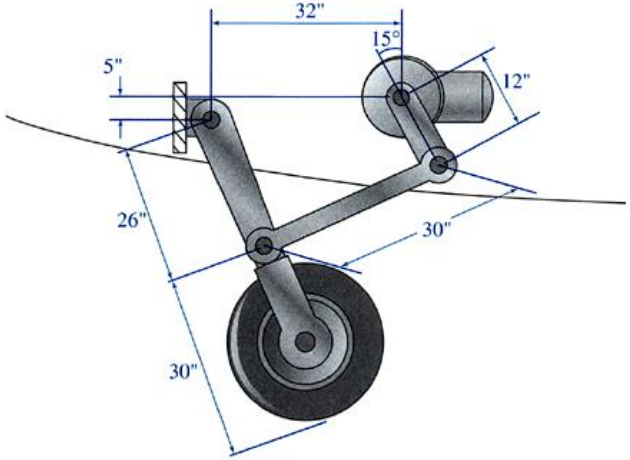


FIGURE 1.35 Nosewheel assembly for Example Problem 1.9.

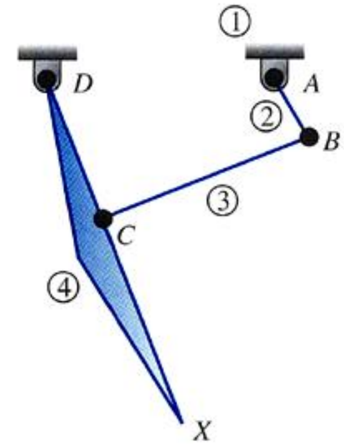


FIGURE 1.36 Kinematic diagram for Example Problem 1.9.

SOLUTION:

1. ***Distinguish the Links Based on Length***

   The lengths of the links are: $s = 12$ in.; $l = 32$ in.; $p = 30$ in.; $q = 26$ in.

2. ***Compare to Criteria***

$$s + l < p + q$$

$$(12 + 32) < (30 + 26)$$

$$44 < 56 \quad \rightarrow \quad \{\text{yes}\}$$

The nosewheel assembly is a crank-rocker mechanism.

1.11 SLIDER-CRANK MECHANISM

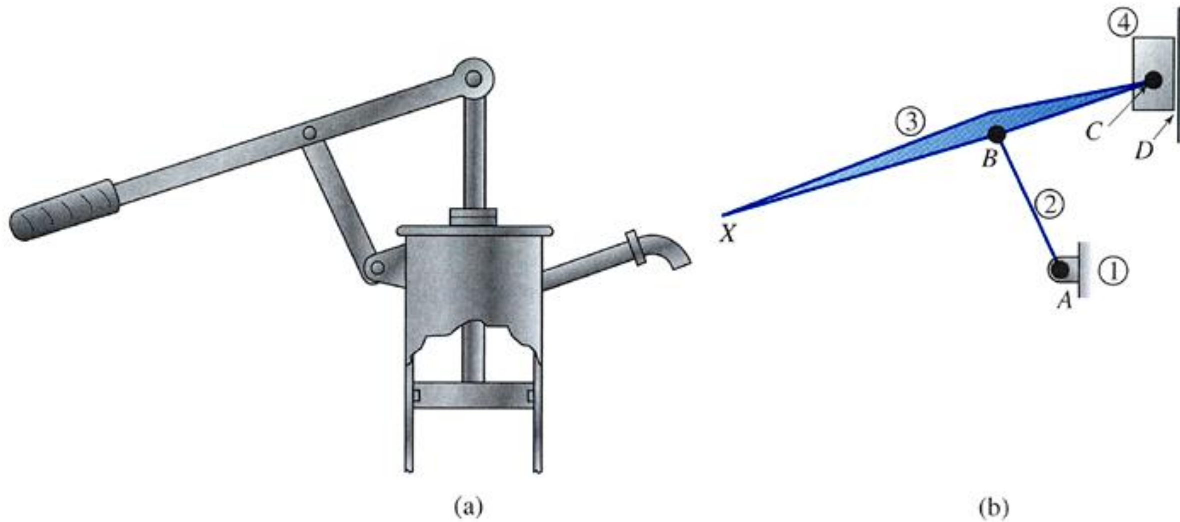


FIGURE 1.37 Pump mechanism for a manual water pump: (a) Mechanism and (b) Kinematic diagram.

The mobility of a slider-crank mechanism is represented by the following:

$$n = 4, j_p = (3 \text{ pins} + 1 \text{ sliding}) = 4, j_h = 0$$

and

$$M = 3(n - 1) - 2j_p - j_h = 3(4 - 1) - 2(4) - 0 = 1.$$

1.12 SPECIAL PURPOSE MECHANISMS

1.12.1 Straight-Line Mechanisms

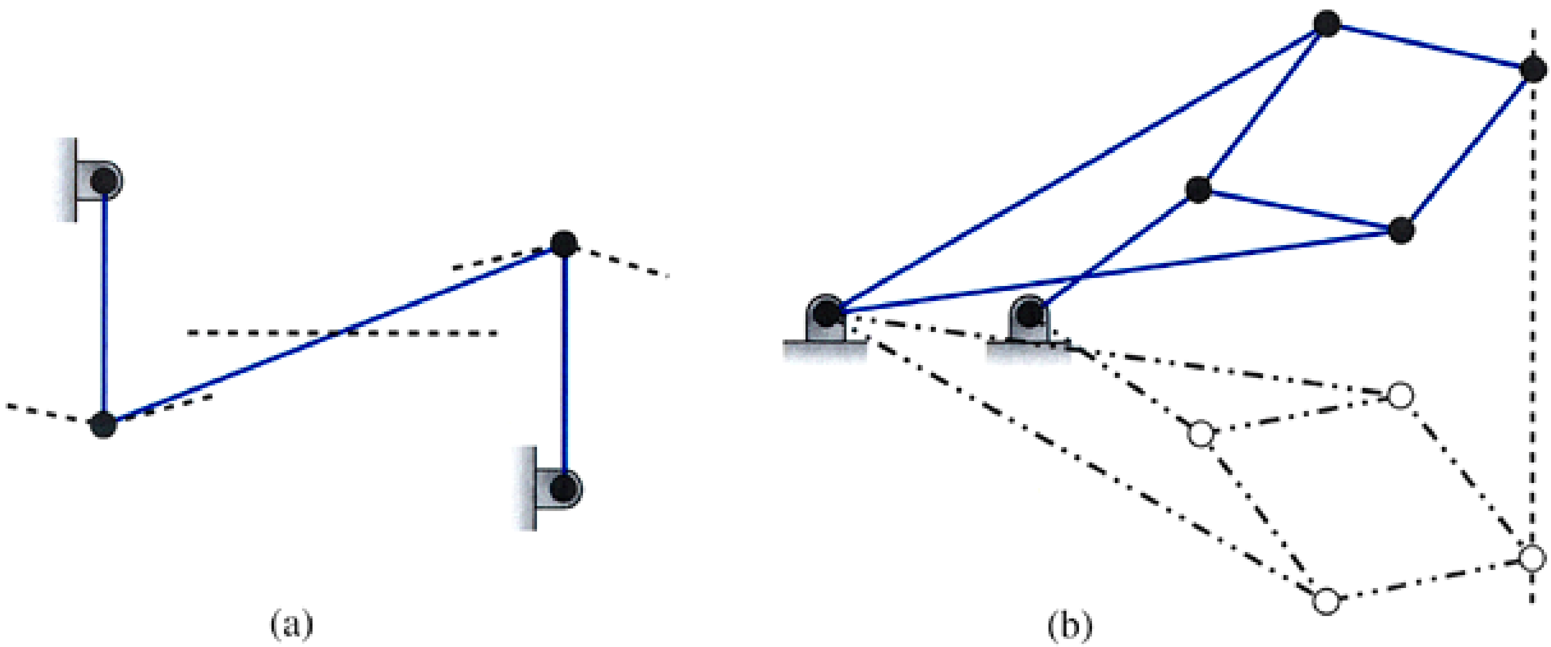


FIGURE 1.38 Straight-line mechanisms

1.12.2 Parallelogram Mechanisms

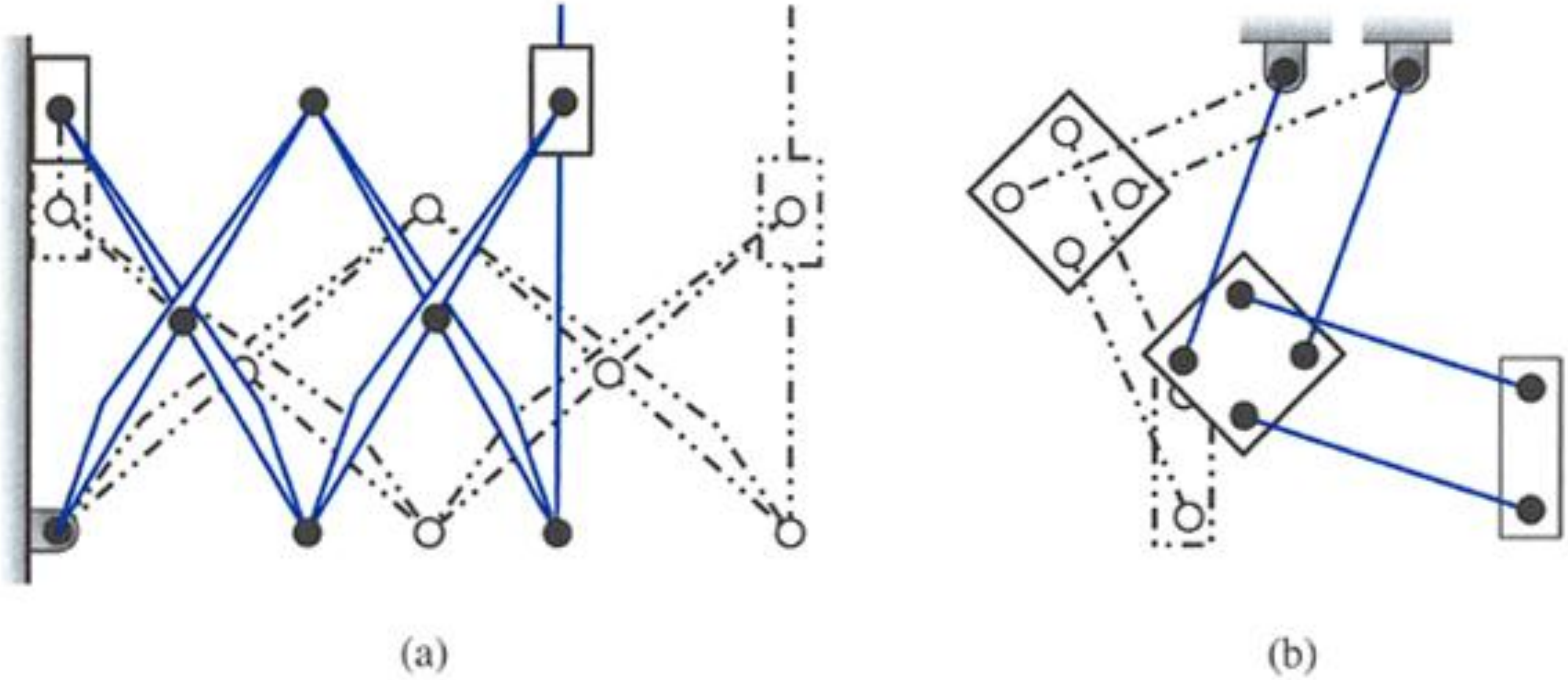


FIGURE 1.39 Parallelogram mechanisms.

1.12.3 Quick-Return Mechanisms

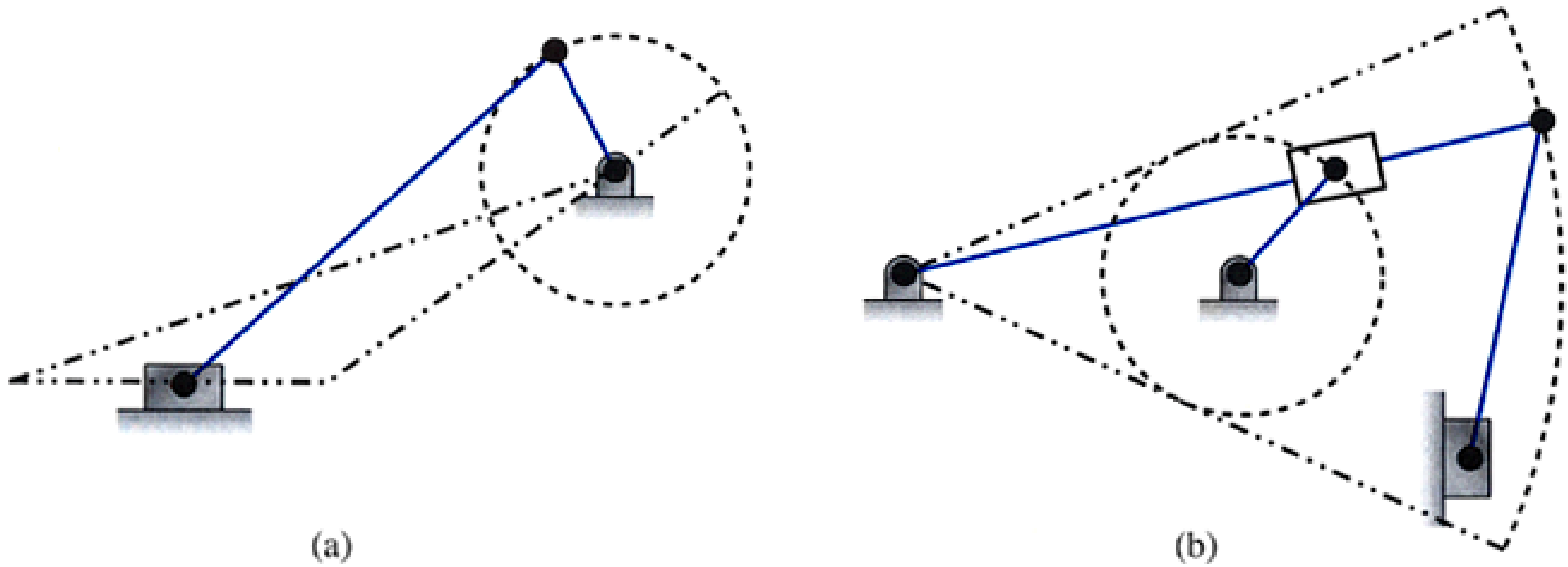


FIGURE 1.40 Quick-return mechanisms.

1.12.4 Scotch Yoke Mechanism

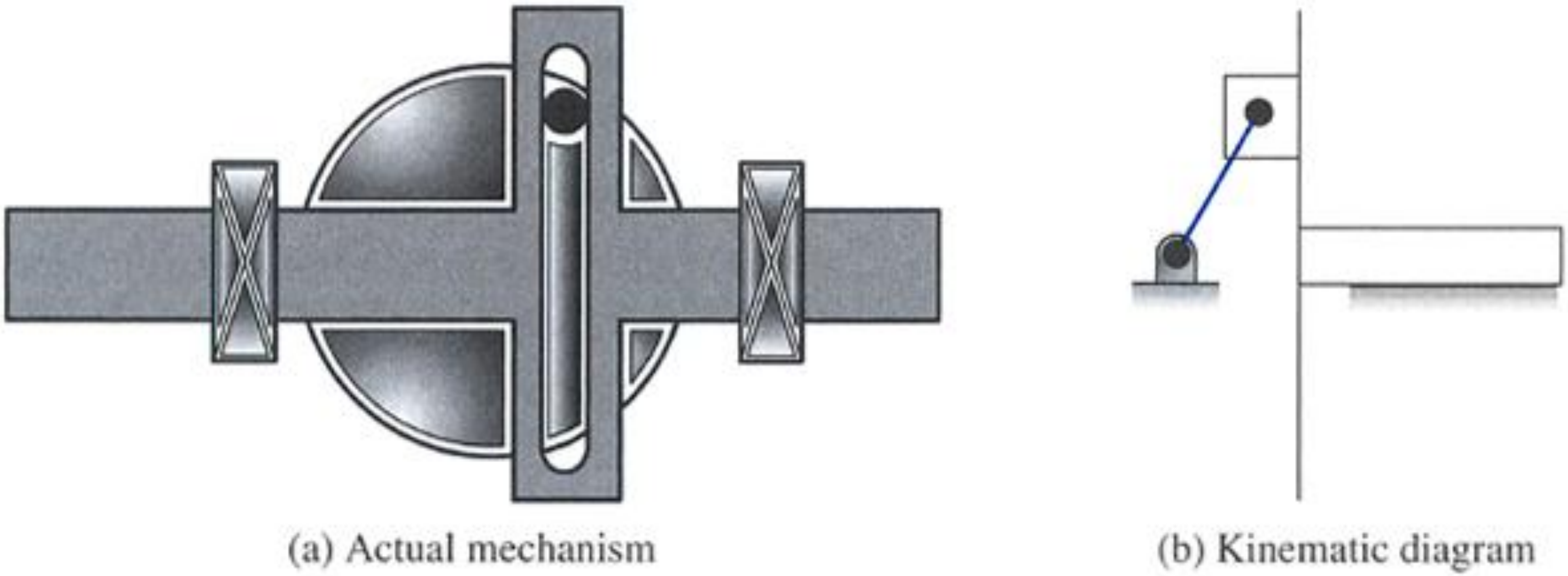


FIGURE 1.41 Scotch yoke mechanism.

Machines and Mechanisms: Applied Kinematic Analysis, 4/e

Chapter 4 Displacement Analysis

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Define position and displacement of a point.
2. Graphically and analytically determine the position of all links in a mechanism as the driver links are displaced.
3. Graphically and analytically determine the limiting positions of a mechanism.
4. Graphically and analytically determine the position of all links for an entire cycle of mechanism motion.
5. Plot a displacement diagram for various points on a mechanism as a function of the motion of other points on the mechanism.

PEARSON

4.2 POSITION

4.2.1 Position of a Point

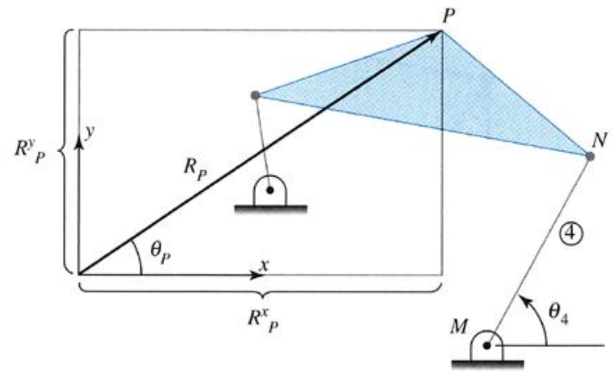


FIGURE 4.2 Position vector for point P.

4.3 DISPLACEMENT

4.3.1 Linear Displacement

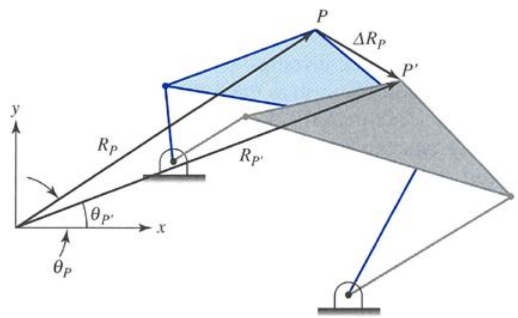


FIGURE 4.3 Displacement vector for point P.

$$\Delta \mathbf{R}_P = \mathbf{R}_{P'} -> \mathbf{R}_P \qquad (4.1)$$

4.3.2 Angular Displacement

$$\Delta\theta_3 = \theta_{3'} - \theta_3 \qquad (4.2)$$

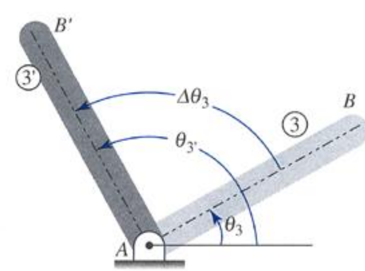


FIGURE 4.4 Angular displacement.

4.4 DISPLACEMENT ANALYSIS

- Locate the positions of all links as driver link is displaced
- Configuration
  - Positions of all the links
- One degree of freedom
  - Moving one link will precisely position all other links

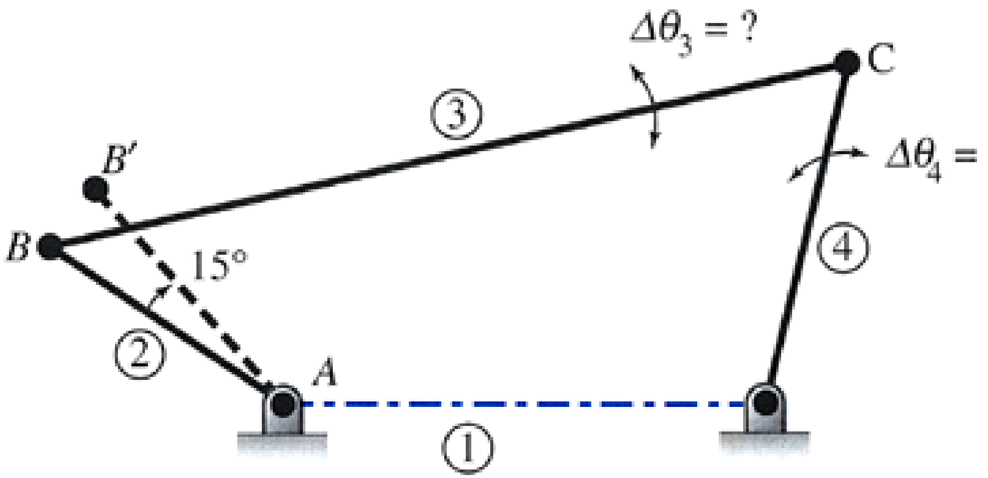


FIGURE 4.5 Typical position analysis.

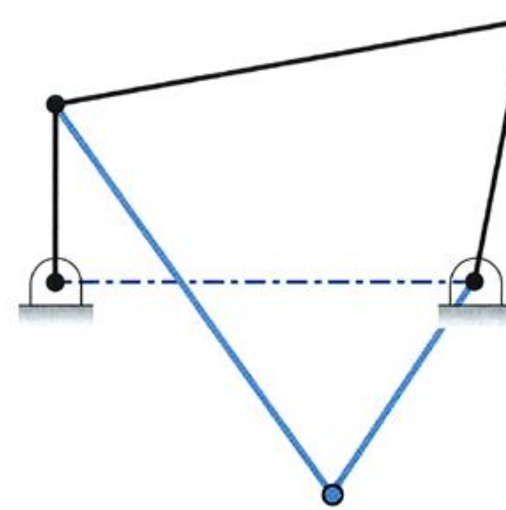


FIGURE 4.6 Two geometric inversions of a four-bar mechanism.

4.5 DISPLACEMENT:GRAPHICAL ANALYSIS

4.5.1 Displacement of a Single Driving Link

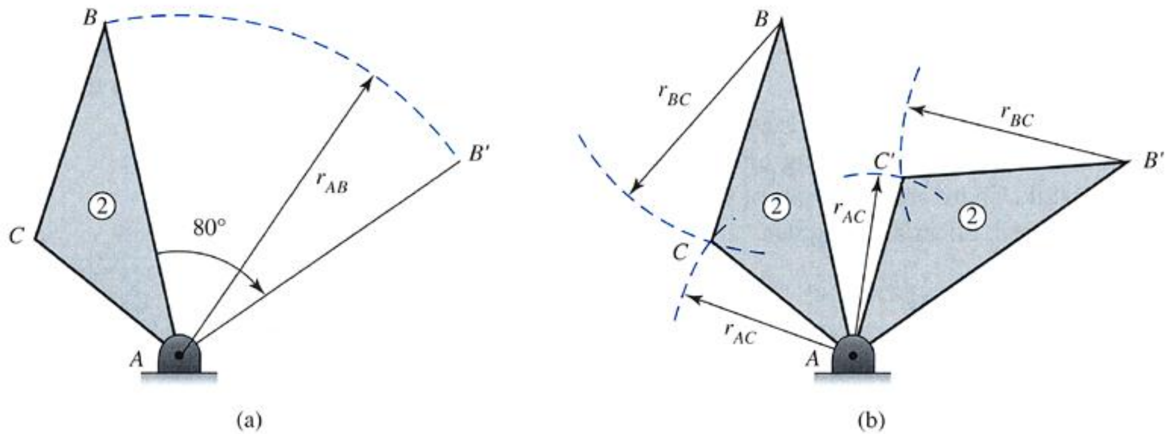


FIGURE 4.7 Rotating a complex link.

4.5.2 Displacement of the Remaining Slave Links

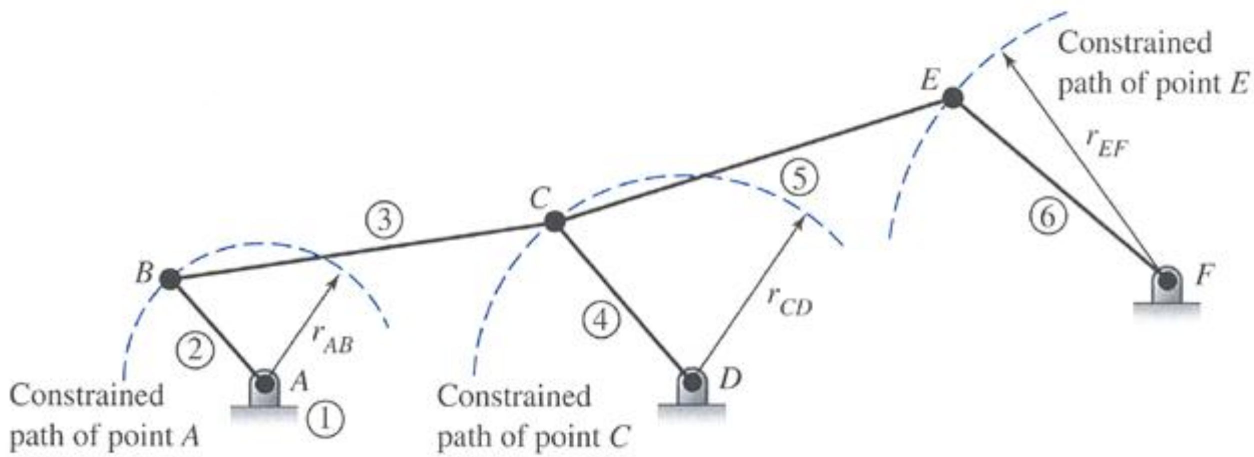


FIGURE 4.8 Constrained paths of points on a link pinned to the frame.

4.5.2 Displacement of the Remaining Slave Links

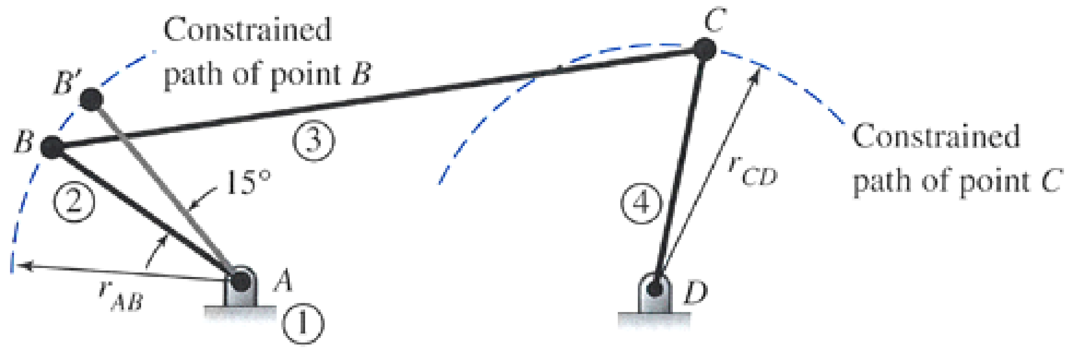


FIGURE 4.9 Constructing the constrained path of C.

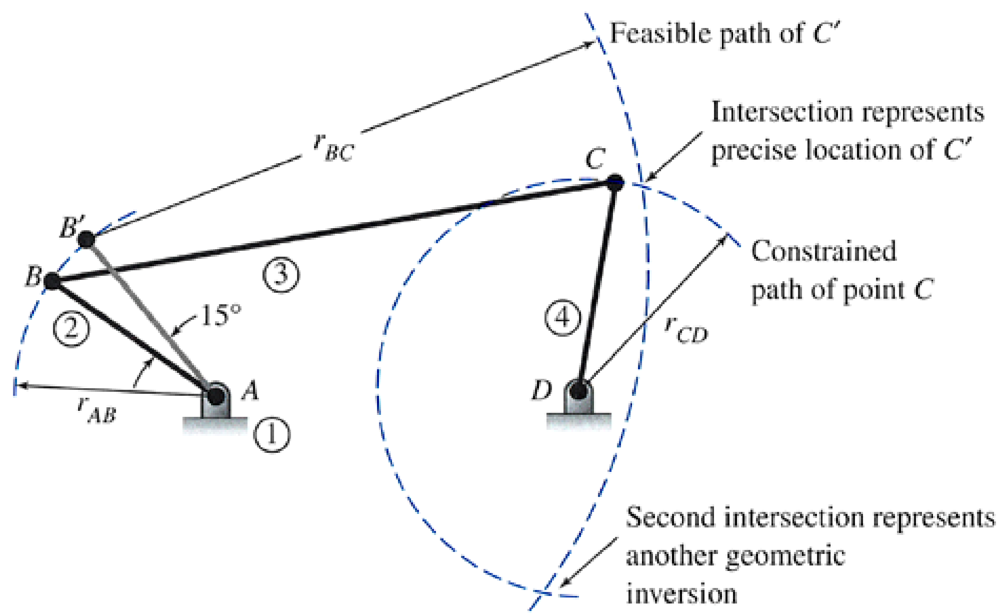


FIGURE 4.10 Locating the position of C'.

EXAMPLE PROBLEM 4.1

Figure 4.11 shows a kinematic diagram of a mechanism that is driven by moving link 2. Graphically reposition the links of the mechanism as link 2 is displaced 30° counterclockwise. Determine the resulting angular displacement of link 4 and the linear displacement of point *E*.

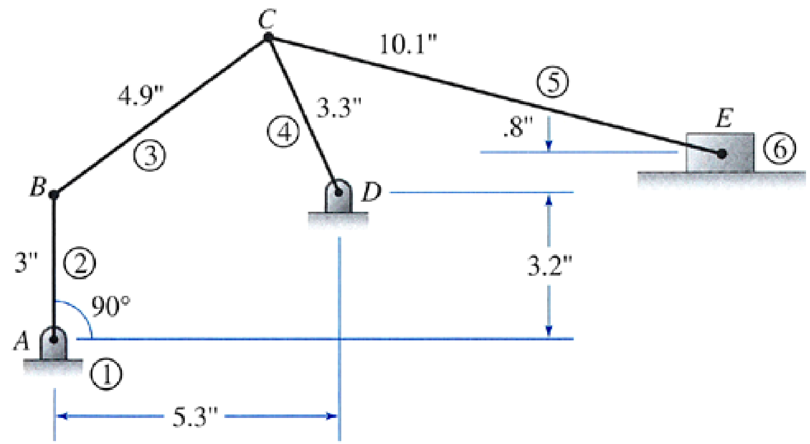


FIGURE 4.11 Kinematic diagram for Example Problem 4.1.

SOLUTION:

1. ***Calculate Mobility***

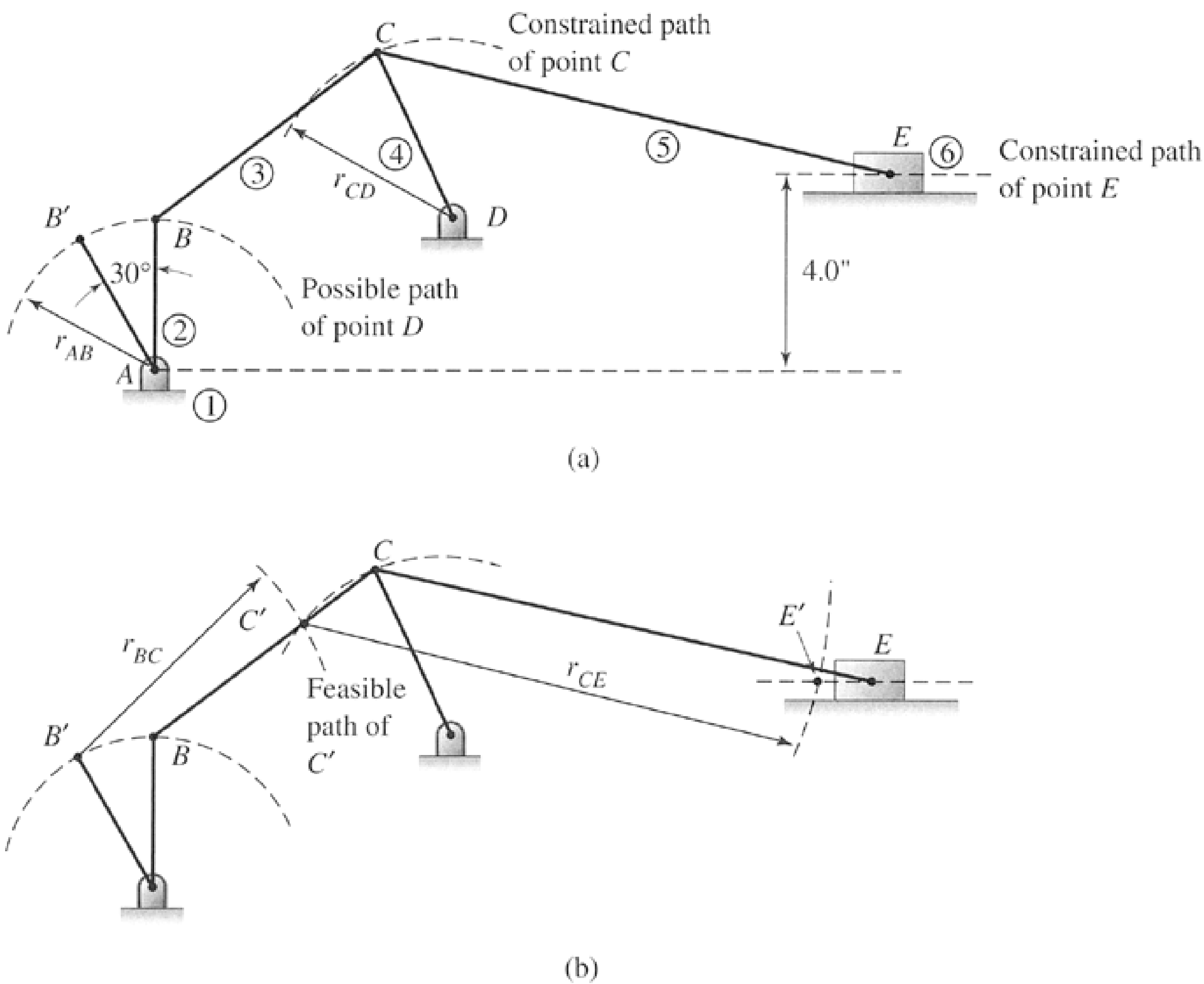
$$n = 6\ j_p = (6 \text{ pins} + 1 \text{ sliding}) = 7\ j_h = 0$$

and

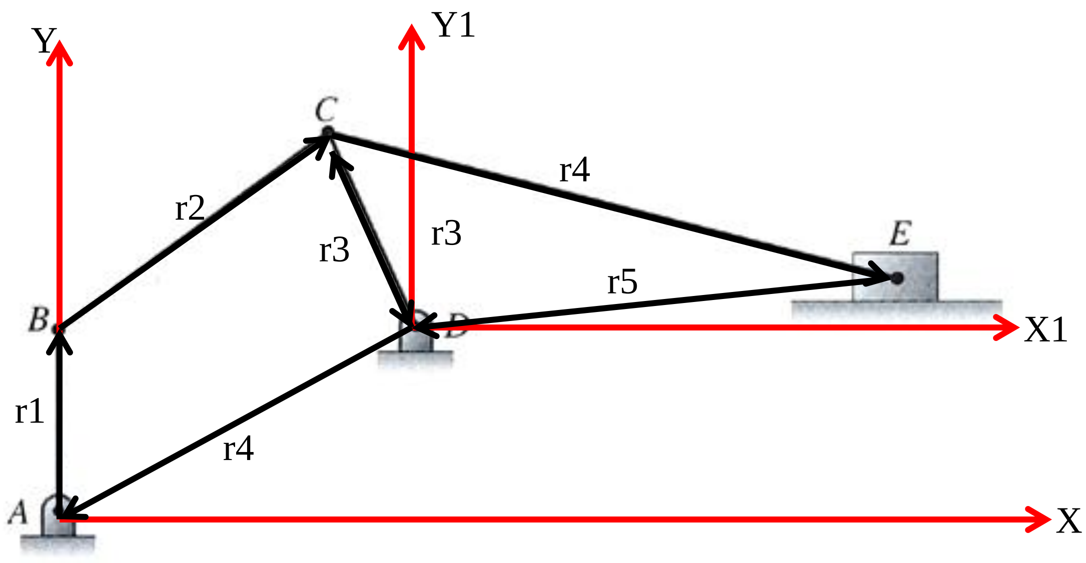
$$M = 3(n-1) - 2j_p - j_h = 3(6-1) - 2(7) - 0 = 15 - 14 = 1$$

2. ***Reposition the Driving Link***

Link 2 is graphically rotated 30° counterclockwise, locating the position of point B'. This is shown in Figure 4.12a



4.1 Vector Analysis of Displacement



$$(1)\quad \underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\begin{bmatrix} -r_1 s\theta_1 \\ +r_1 c\theta_1 \end{bmatrix}+\begin{bmatrix} r_2 c\theta_2 \\ r_2 s\theta_2 \end{bmatrix}+\begin{bmatrix} r_3 c\theta_3 \\ r_3 s\theta_3 \end{bmatrix}+\begin{bmatrix} -5.3 \\ -3.2 \end{bmatrix}=0$$

$$r_1=3,\ \theta_1=30,\ r_2=4.9,\ r_3=3.3$$

2 *equations* for 2 *unknows* $\theta_2,\ \theta_3$

$$(2)\quad -\underset{\sim}{r}_3+\underset{\sim}{r}_4+\underset{\sim}{r}_5=0$$

$$\begin{bmatrix} -r_3 c\theta_3 \\ -r_3 s\theta_3 \end{bmatrix}+\begin{bmatrix} r_4 c\theta_4 \\ r_4 s\theta_4 \end{bmatrix}+\begin{bmatrix} x_1 \\ 0.8 \end{bmatrix}=0$$

$$r_1=10.1$$

2 *equations* for 2 *unknows* θ_4 *and* x_1

EXAMPLE PROBLEM 4.2

Compound-lever snips, as shown in Figure 4.13, are often used in place of regular tinner snips when large cutting forces are required. Using the top handle as the frame, graphically reposition the components of the snips when the jaw is opened 15°. Determine the resulting displacement of the lower handle.

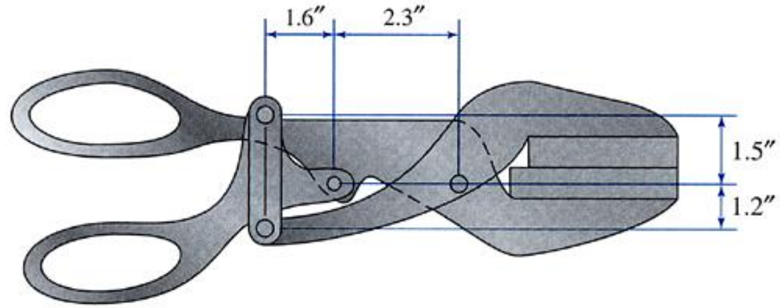
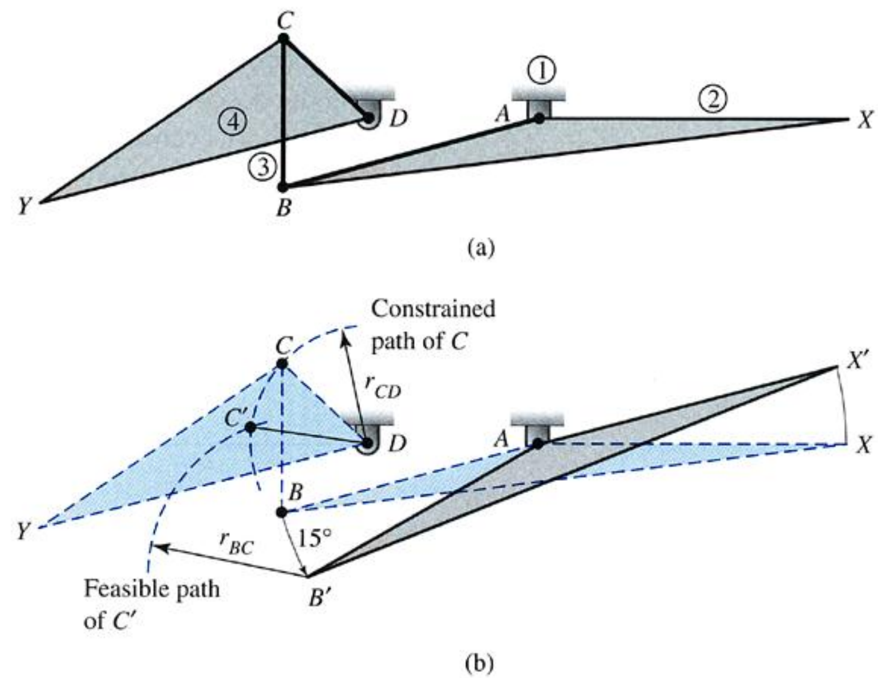
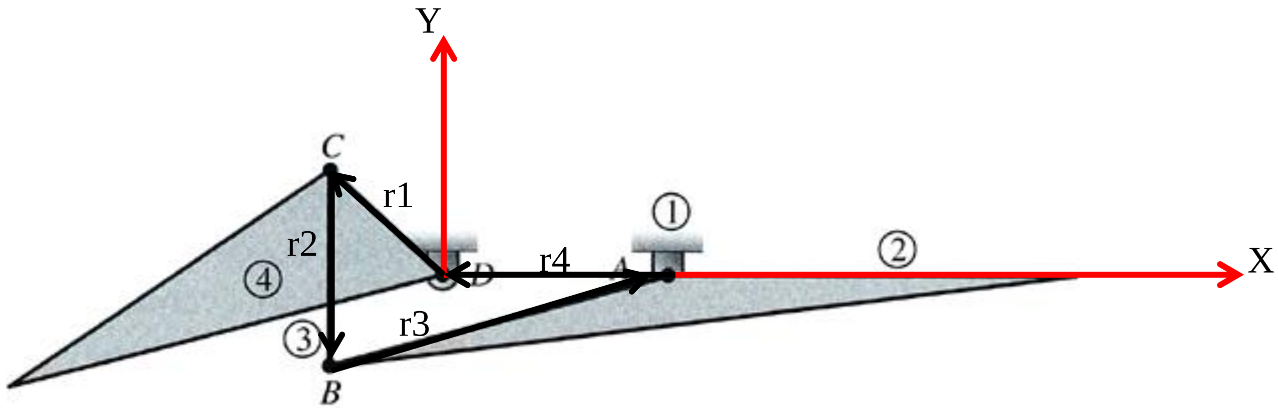


FIGURE 4.13 Cutting snips for Example Problem 4.2.





$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\begin{bmatrix}-1.6\\1.5\end{bmatrix}+\begin{bmatrix}3c\theta_2\\3s\theta_2\end{bmatrix}+\begin{bmatrix}r_3c\theta_3\\r_3s\theta_3\end{bmatrix}+\begin{bmatrix}-2.3\\0\end{bmatrix}=0$$

$$r_3=(3.9^2+1.2^2)^{1/2}$$

$$solve \;\; for \;\; \theta_2 \;\; and \;\; \theta_3$$

EXAMPLE PROBLEM 4.3

Figure 4.15 shows a toggle clamp used to securely hold parts. Analytically determine the displacement of the clamp surface as the handle rotates downward, 15°.

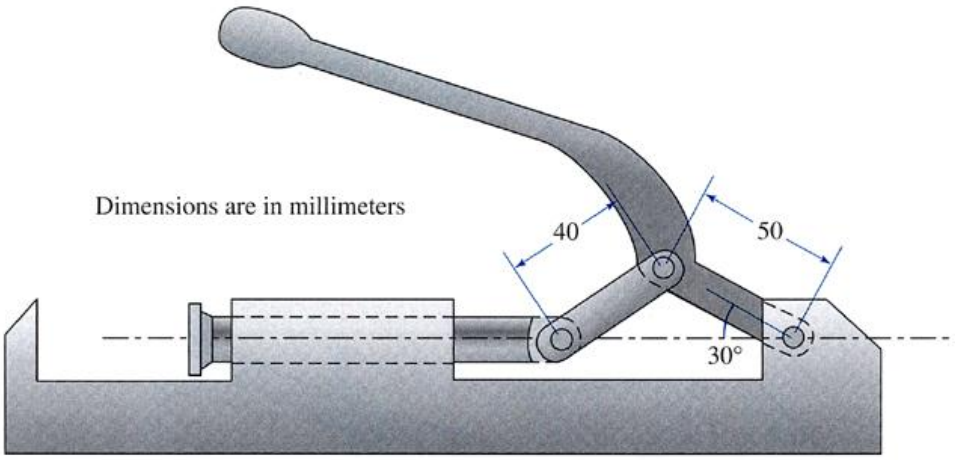


FIGURE 4.15 Toggle clamp for Example Problem 4.3.

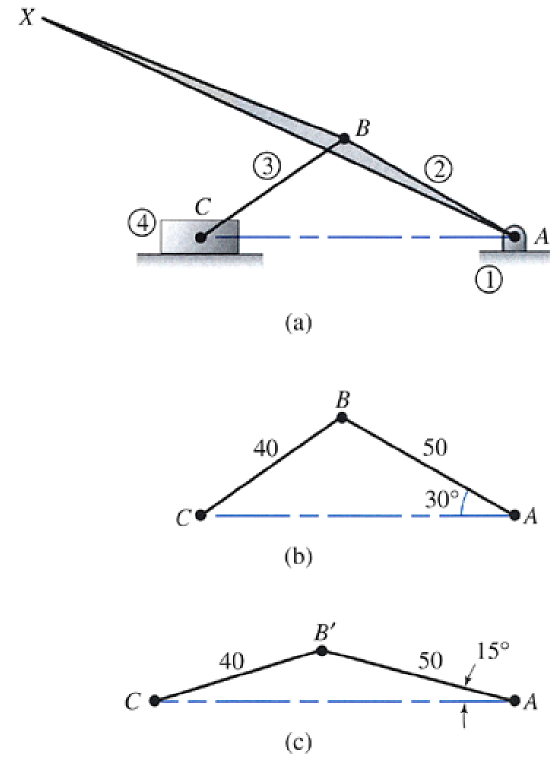
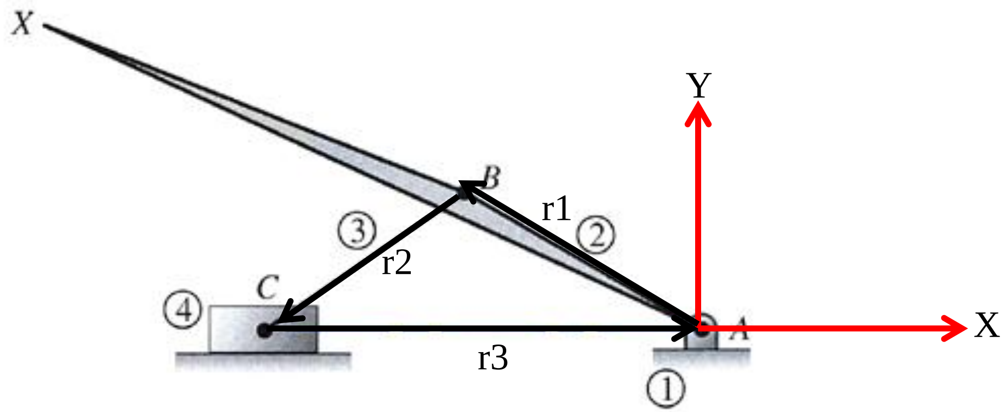


FIGURE 4.16 Mechanism for Example Problem 4.3.

SOLUTION: 1. ***Draw a Kinematic Diagram***

The kinematic diagram is given in Figure 4.16a.
The end of the handle was labeled as point of interest *X*.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$\begin{bmatrix}50c\theta_1\\50s\theta_1\end{bmatrix}+\begin{bmatrix}40c\theta_2\\40s\theta_2\end{bmatrix}+\begin{bmatrix}d_1\\0\end{bmatrix}=0$$

$$\theta_1=240°,\ solve\ \ for\ \ \theta_2\ \ and\ \ d_1$$

$$when\ \ rotate\ 15°,\ \theta_1=255°$$

$$\begin{bmatrix}50c\theta_1\\50s\theta_1\end{bmatrix}+\begin{bmatrix}40c\theta_2\\40s\theta_2\end{bmatrix}+\begin{bmatrix}d_2\\0\end{bmatrix}=0$$

$$solve\ \ for\ \ \theta_2\ \ and\ \ d_2$$

$$\Delta d=\left|d_1-d_2\right|$$

EXAMPLE PROBLEM 4.4

Figure 4.18 shows a concept for a hand pump used for increasing oil pressure in a hydraulic line. Analytically determine the displacement of the piston as the handle rotates 15° counterclockwise.

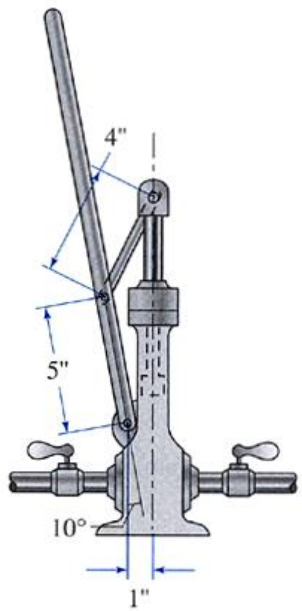


FIGURE 4.18 Toggle clamp for Example Problem 4.4.

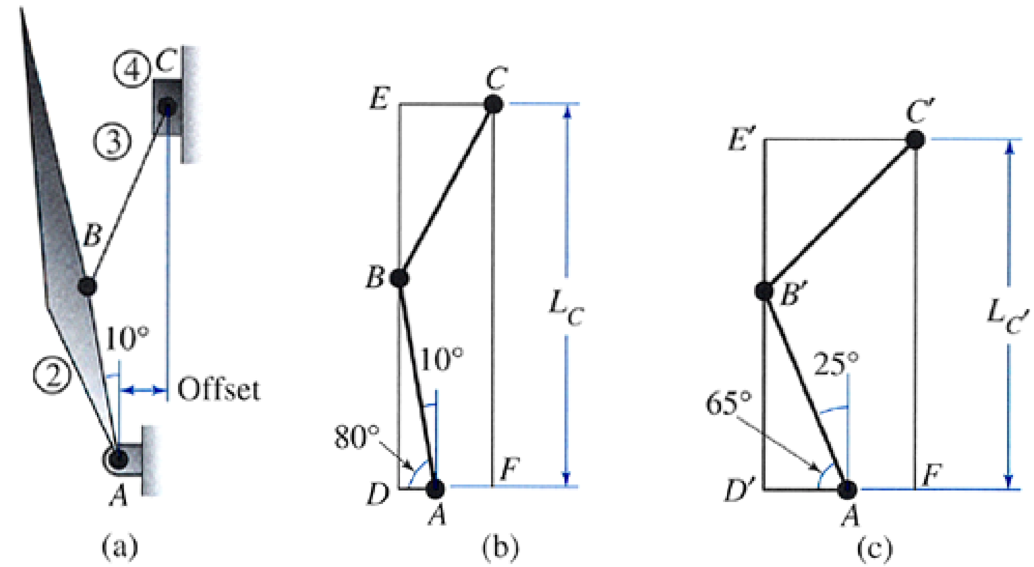


FIGURE 4.19 Mechanism diagrams for Example Problem 4.4.

SOLUTION: 1. ***Draw a Kinematic Diagram***

The kinematic diagram is given in Figure 4.19a. The end of the handle was labeled as point of interest *X*.

4.6.1 Closed-Form Position Analysis Equations for an In-Line Slider-Crank

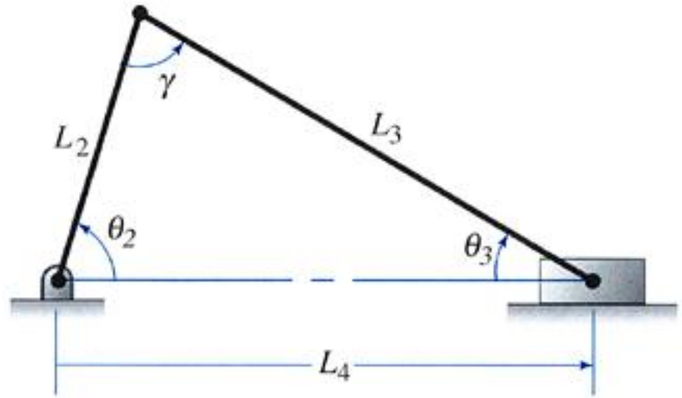


FIGURE 4.17 In-line slider-crank mechanism.

The equations used in Example Problem 4.3 are summarized in terms of L_2, L_3, and θ_2:

$$\theta_3 = \sin^{-1}\left[\frac{L_2}{L_3}\sin\theta_2\right] \quad \textbf{(4.3)}$$

$$\gamma = 180^\circ - (\theta_2 + \theta_3) \quad \textbf{(4.4)}$$

$$L_4 = \sqrt{L_2^2 + L_3^2 - 2(L_2)(L_3)\cos\gamma} \quad \textbf{(4.5)}$$

These equations can be used to determine the position of the links in any configuration of an in-line slider-crank mechanism.

4.6.2 Closed-Form Position Analysis Equations for an Offset Slider-Crank

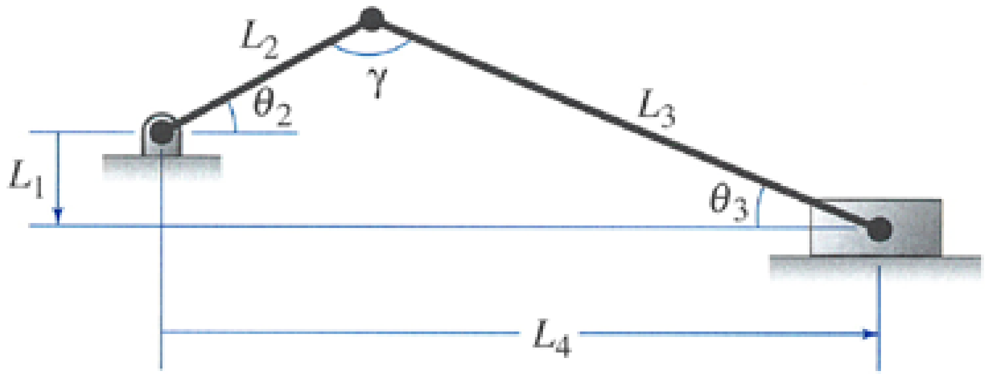


FIGURE 4.20 Offset slider-crank mechanism.

$$\theta_3 = \sin^{-1}\left[\frac{L_1 + L_2 \sin \theta_2}{L_3}\right] \quad (4.6)$$

$$L_4 = L_2 \cos \theta_2 + L_3 \cos \theta_3 \quad (4.7)$$

$$\gamma = 180° - (\theta_2 + \theta_3) \quad (4.8)$$

EXAMPLE PROBLEM 4.5

Figure 4.21 shows a toggle clamp used for securing a workpiece during a machining operation. Analytically determine the angle that the handle must be displaced in order to lift the clamp arm 30° clockwise.

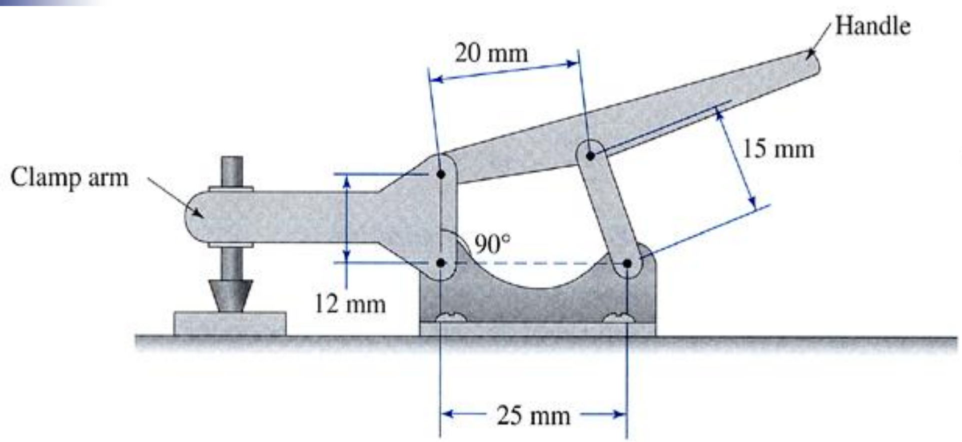


FIGURE 4.21 Clamp for Example Problem 4.5.

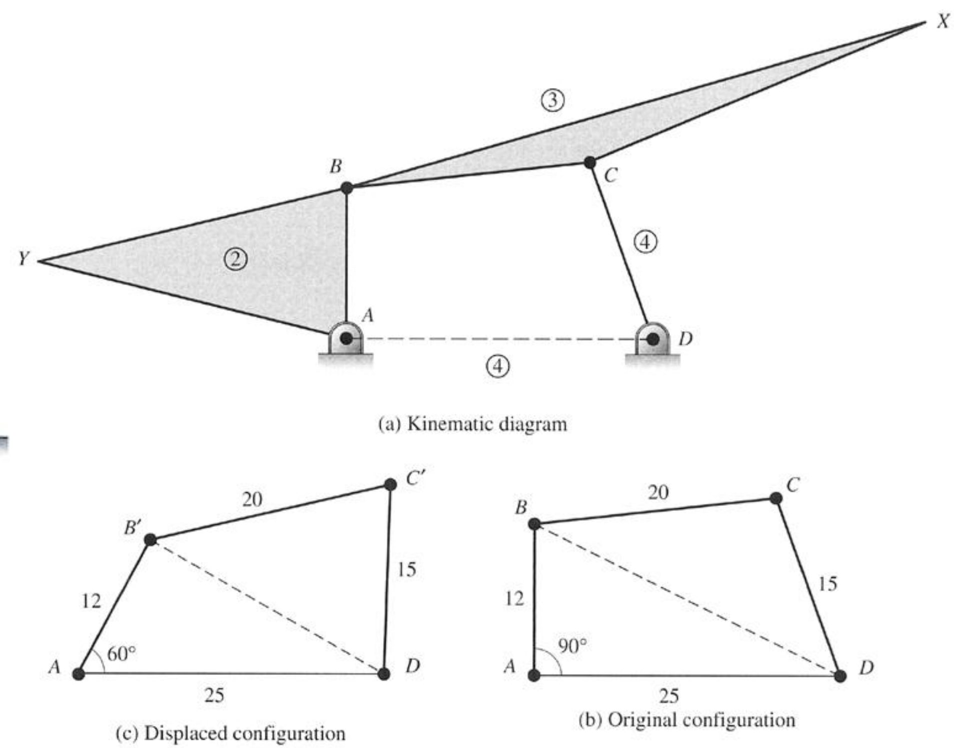
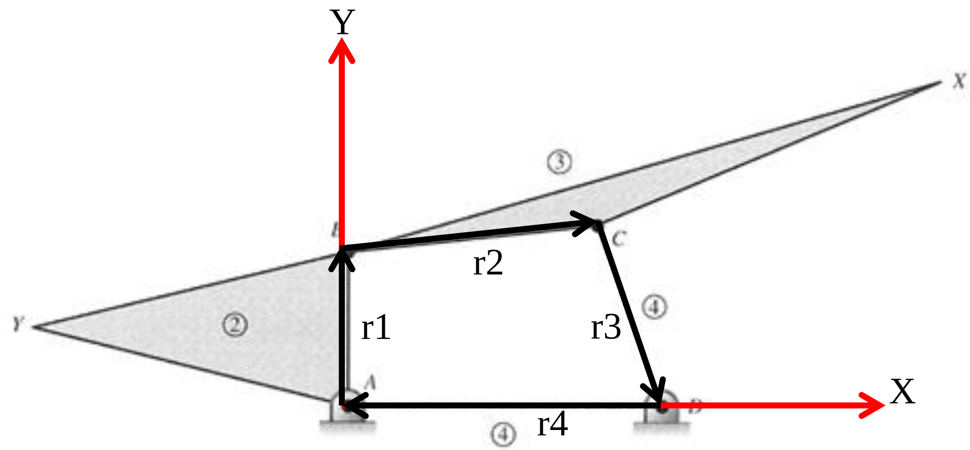


FIGURE 4.22 Mechanism for Example Problem 4.5.



$$\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3 + \underset{\sim}{r}_4 = 0$$

$$\begin{bmatrix} 12c\theta_1 \\ 12s\theta_1 \end{bmatrix} + \begin{bmatrix} 20c\theta_2 \\ 20s\theta_2 \end{bmatrix} + \begin{bmatrix} 15c\theta_3 \\ 15s\theta_3 \end{bmatrix} + \begin{bmatrix} -25 \\ 0 \end{bmatrix} = 0$$

$(1)\ \ \theta_1 = 90°,\ 2\ eqs.\ solve\ \ for\ \ \theta_2\ \ and\ \ \theta_3$

$(2)\ \ \theta_1 = 60°,\ 2\ eqs.\ solve\ \ for\ \ \theta_2\ \ and\ \ \theta_3$

Calculate the difference of θ_2 *in* (1) *and* (2).

EXAMPLE PROBLEM 4.6

The mechanism shown in Figure 4.26 is the driving linkage for a reciprocating saber saw. Determine the configurations of the mechanism that places the saw blade in its limiting positions.

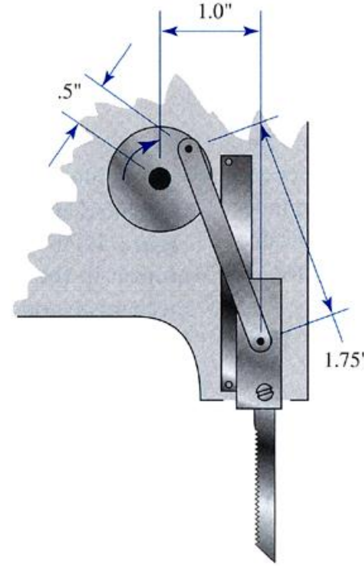
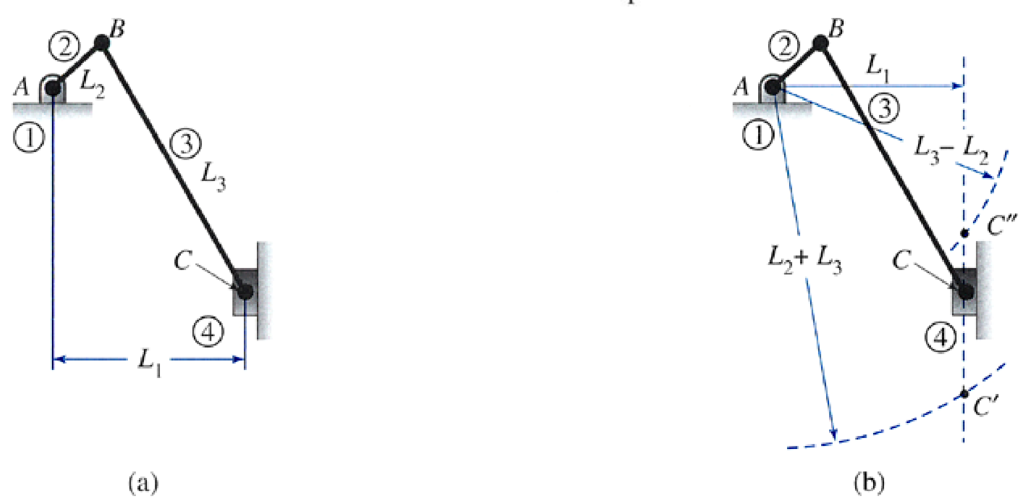
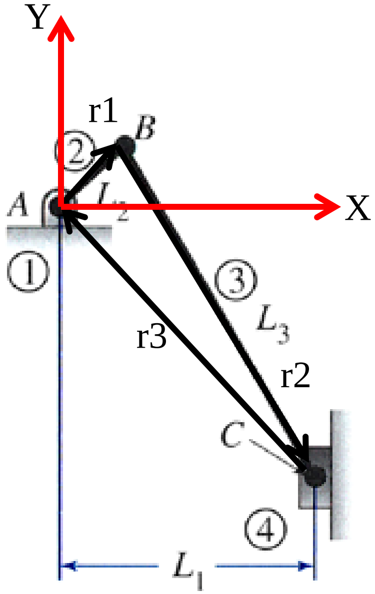
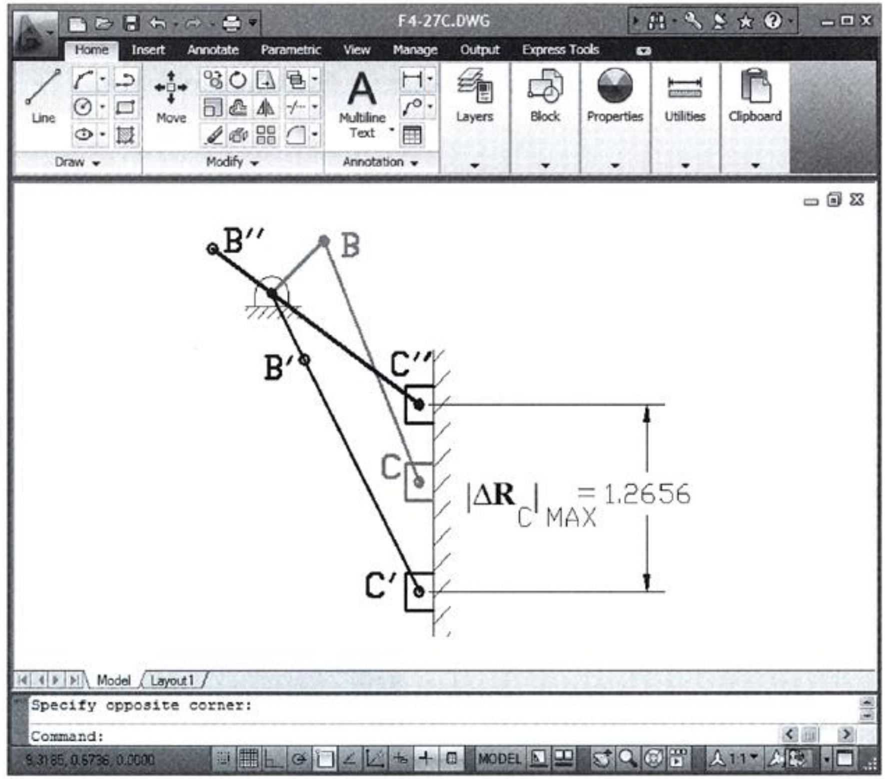


FIGURE 4.26 Saber saw mechanism for Example Problem 4.6.







(c)

FIGURE 4.27 Extreme positions for Example Problem 4.6.

$$\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3 = 0$$

$$\begin{bmatrix} 0.5c\theta_1 \\ 0.5s\theta_1 \end{bmatrix} + \begin{bmatrix} 1.75c\theta_2 \\ 1.75s\theta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ y \end{bmatrix} = 0$$

$$\textit{for} \quad \theta_1 = \theta_2, \qquad \textit{solve} \quad \textit{for} \quad \theta_1 \quad \textit{and} \quad y_{\max}$$

$$\textit{for} \quad \theta_1 = \theta_2 + \pi, \quad \textit{solve} \quad \textit{for} \quad \theta_1 \quad \textit{and} \quad y_{\min}$$

4.7 LIMITING POSITIONS:

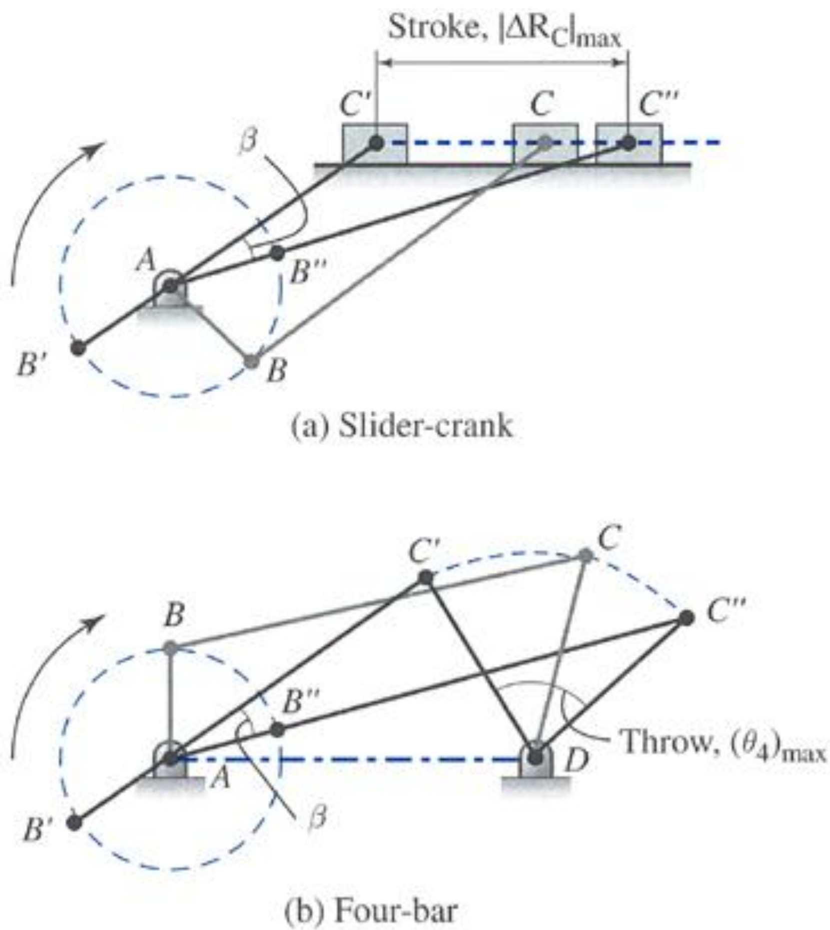


FIGURE 4.25 Limiting positions.

EXAMPLE PROBLEM 4.7

Figure 4.28 illustrates a linkage that operates a water nozzle at an automatic car wash. Determine the limiting positions of the mechanism that places the nozzle in its extreme positions.

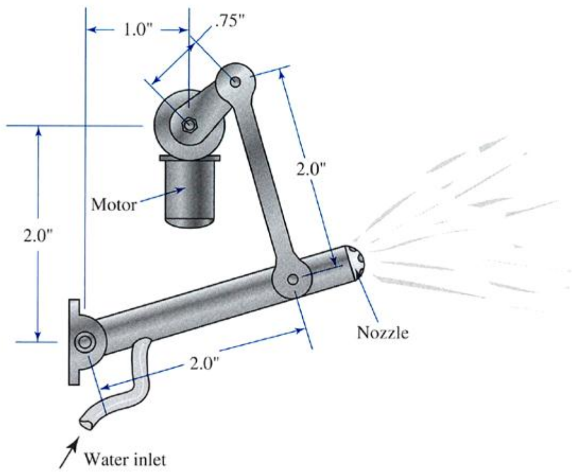
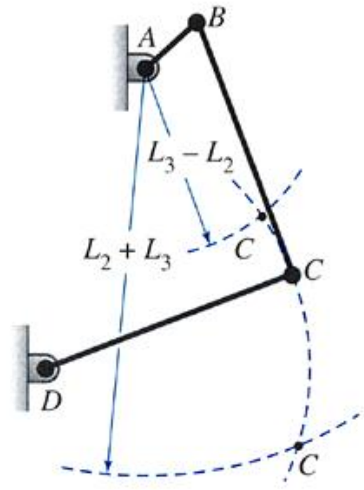
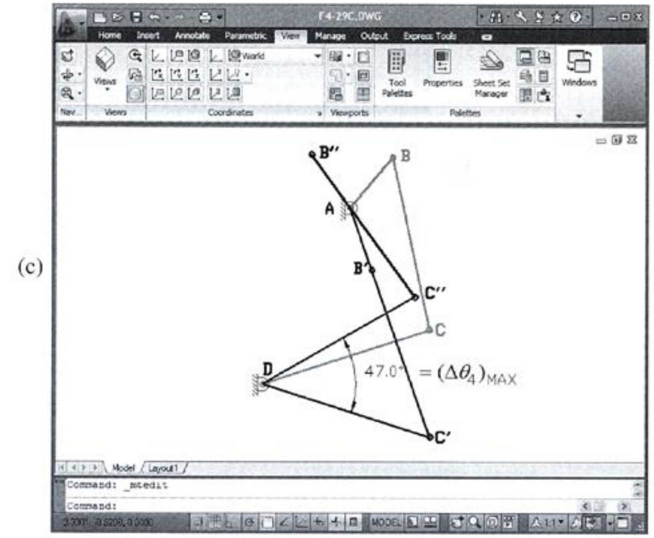


FIGURE 4.28 Water nozzle linkage for Example Problem 4.7.

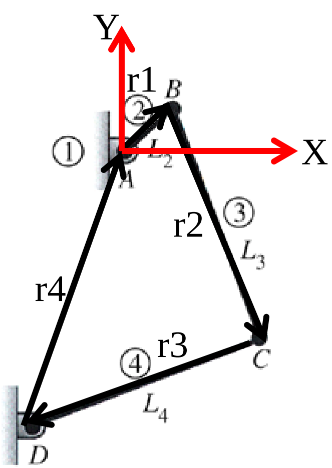


(b)



(c)

FIGURE 4.29 Extreme positions for Example Problem 4.7.



(a)

$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$for \;\; \theta_1=\theta_2, \qquad solve \;\; for \;\; \theta_1 \;\; and \;\; \theta_3)_{\max}$$

$$for \;\; \theta_1=\theta_2+\pi, \; solve \;\; for \;\; \theta_1 \;\; and \;\; \theta_3)_{\min}$$

EXAMPLE PROBLEM 4.8

Figure 4.30 shows a conveyor transfer mechanism. Its function is to feed packages to a shipping station at specific intervals. Analytically determine the extreme positions of the lifting conveyor segment.

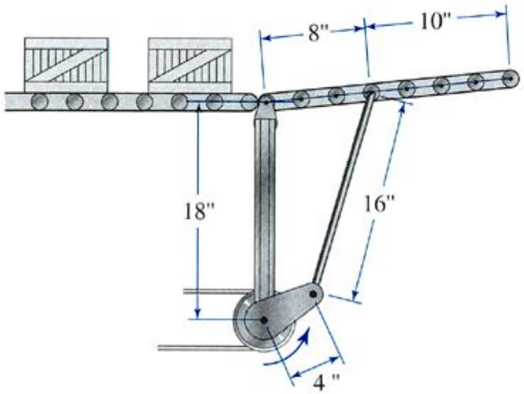


FIGURE 4.30 Conveyor feed for Example Problem 4.8.

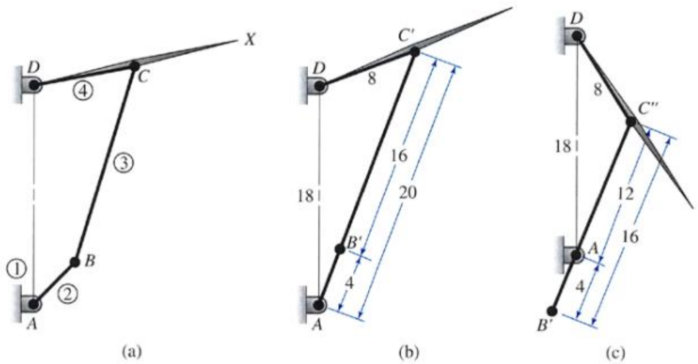
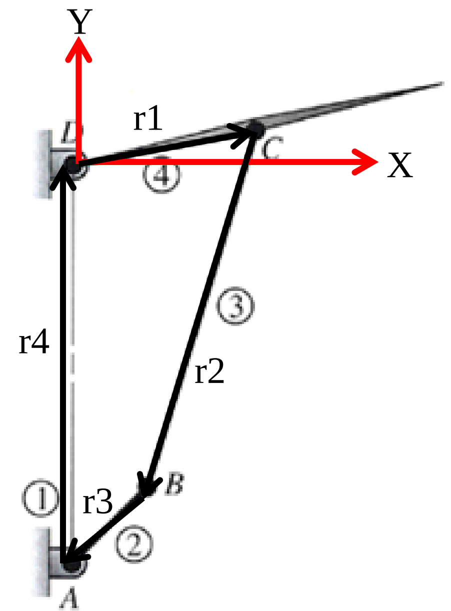


FIGURE 4.31 Mechanism for Example Problem 4.8.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$for\ \ \theta_2=\theta_3, \qquad solve\ \ for\ \ \theta_1)_{\max}\ \ and\ \theta_2$$

$$for\ \ \theta_2=\theta_3+\pi,\ solve\ \ for\ \ \theta_1)_{\min}$$

4.9 TRANSMISSION ANGLE

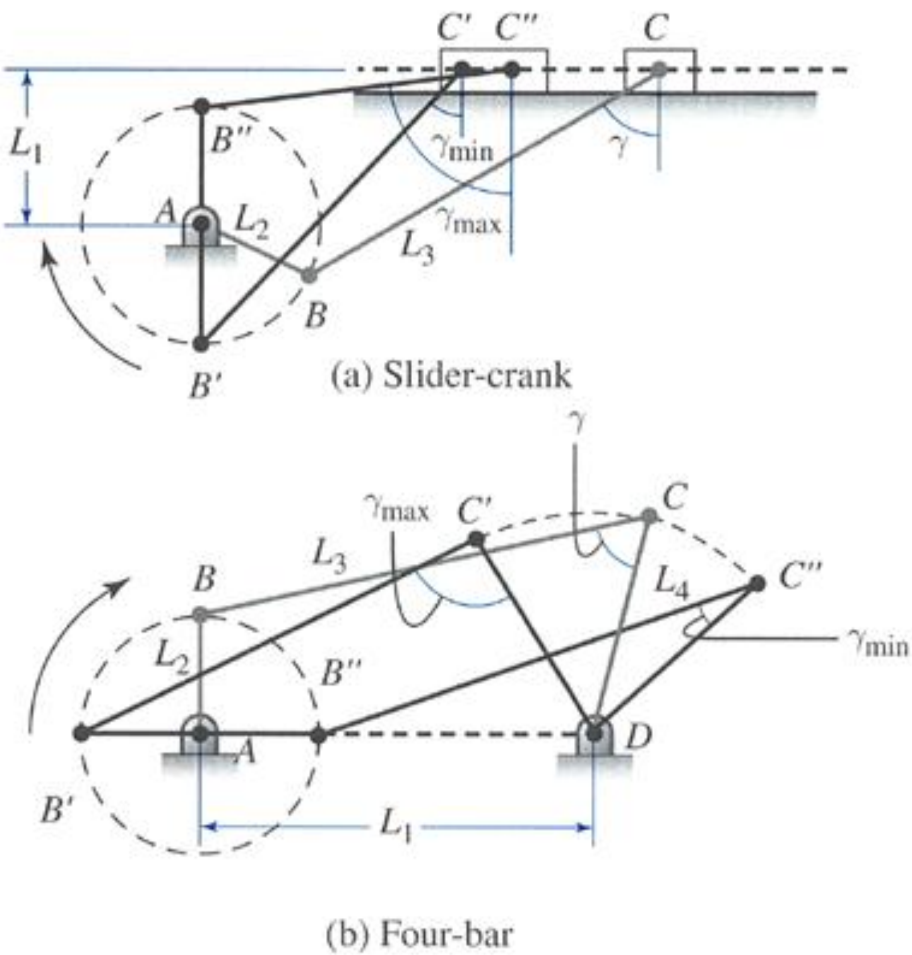


FIGURE 4.32 Transmission angles.

$$\gamma_{min} = \cos^{-1}\left[\frac{L_1 + L_2}{L_3}\right] \tag{4.15}$$

$$\gamma_{max} = \cos^{-1}\left[\frac{L_1 - L_2}{L_3}\right] \tag{4.16}$$

$$\gamma_{min} = \cos^{-1}\left[\frac{L_3^2 + L_4^2 - (L_1 - L_2)^2}{2L_3L_4}\right] \tag{4.17}$$

$$\gamma_{max} = \cos^{-1}\left[\frac{L_3^2 + L_4^2 - (L_1 + L_2)^2}{2L_3L_4}\right] \tag{4.18}$$

EXAMPLE PROBLEM 4.9

Figure 4.33 shows the driving mechanism of handheld grass shears. The mechanism operates by rotating the large disc as shown. Graphically determine the position of the driving mechanism at several phases of its operating cycle.

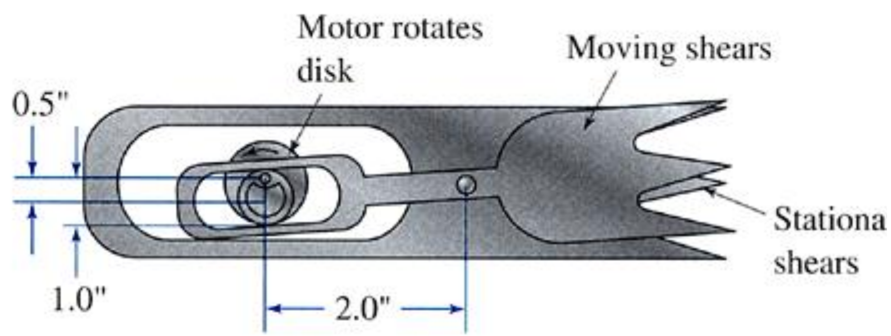
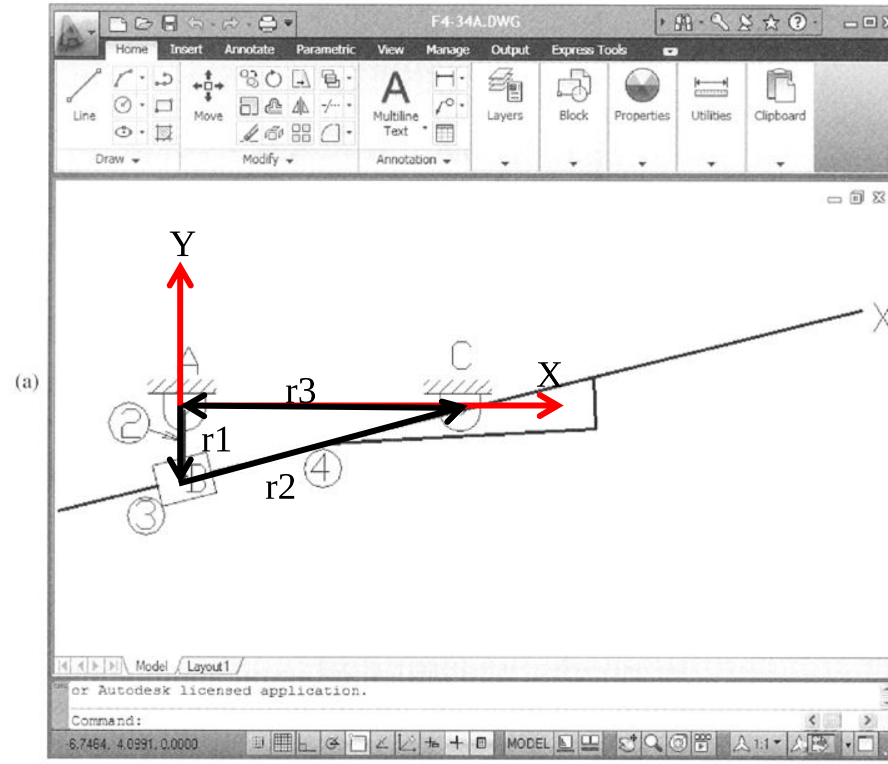
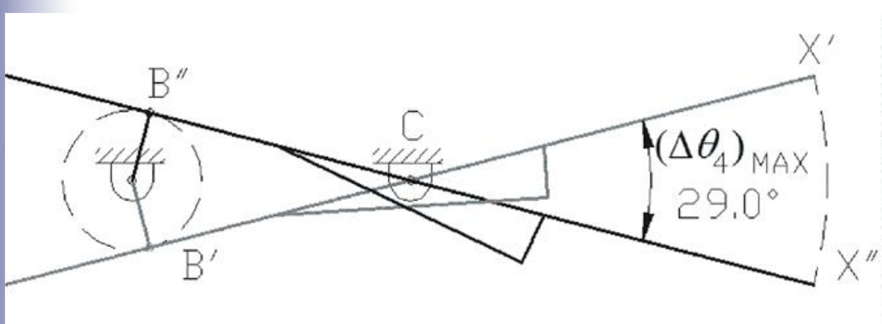


FIGURE 4.33 Grass shears for Example Problem 4.9.

SOLUTION:



(a)



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\begin{bmatrix}0.5c\theta_1\\0.5s\theta_1\end{bmatrix}+\begin{bmatrix}lc\theta_2\\ls\theta_2\end{bmatrix}+\begin{bmatrix}-2.0\\0\end{bmatrix}=0$$

given θ_1 *solve for* l *and* θ_2

EXAMPLE PROBLEM 4.10

Figure 4.35 shows a mechanism that is designed to push parts from one conveyor to another. During the transfer, the parts must be rotated as shown. Analytically determine the position of the pusher rod at several phases of its motion.

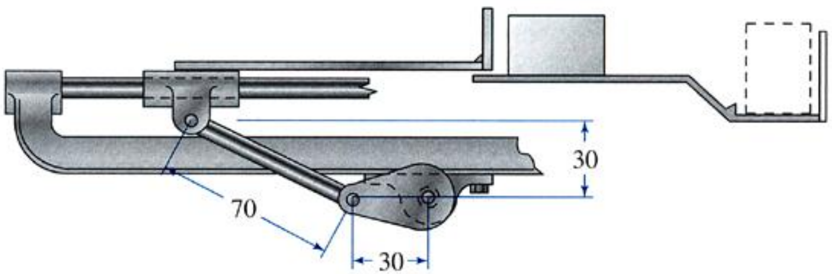


FIGURE 4.35 Conveyor feed for Example Problem 4.10.

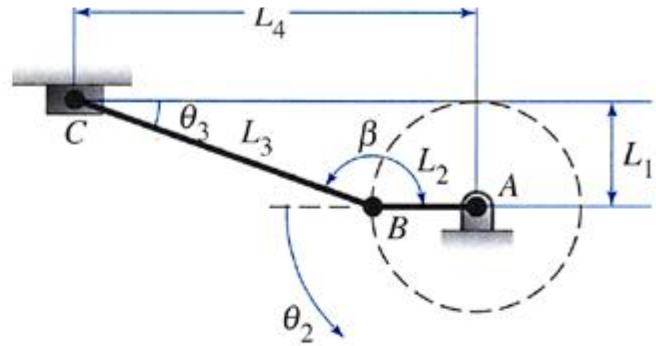


FIGURE 4.36 Kinematic diagram for Example Problem 4.10.

SOLUTION:

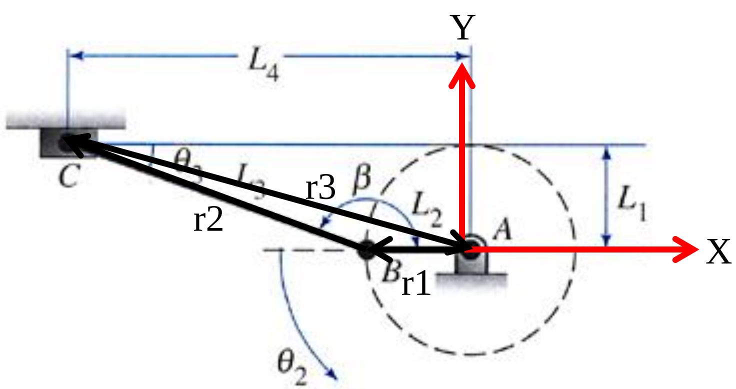


FIGURE 4.36 Kinematic diagram for Example Problem 4.10.

$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$r_1=30,\ r_2=70,\ \underset{\sim}{r}_3=\begin{bmatrix} x \\ -30 \end{bmatrix}=0$$

$$given\ \ \theta_1,\ solve\ \ for\ \ \theta_2\ \ and\ \ x$$

Machines and Mechanisms: Applied Kinematic Analysis, 4/e

Chapter 6 Velocity Analysis

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Define linear, rotational, and relative velocities.
2. Convert between linear and angular velocities.
3. Use the relative velocity method to graphically solve for the velocity of a point on a link, knowing the velocity of another point on that link.
4. Use the relative velocity method to graphically and analytically determine the velocity of a point of interest on a floating link.
5. Use the relative velocity method to analytically solve for the velocity of a point on a link, knowing the velocity of another point on that link.
6. Use the instantaneous center method to graphically and analytically determine the velocity of a point.
7. Construct a velocity curve to locate extreme velocity values.

PEARSON

6.2 LINEAR AND ANGULAR VELOCITY

Mathematically, linear velocity of a point is expressed as

$$\mathbf{V} = \lim_{\Delta t \to 0} \frac{d\mathbf{R}}{dt} \qquad (6.1)$$

and for short time periods as

$$\mathbf{V} \cong \frac{\Delta \mathbf{R}}{\Delta t} \qquad (6.2)$$

$$\omega = \lim_{\Delta t \to o} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \qquad (6.4)$$

$$\omega \cong \frac{\Delta \theta}{\Delta t} \qquad (6.5)$$

6.2.2 Linear Velocity of a General Point

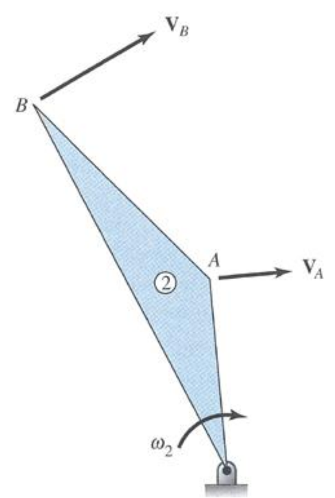


FIGURE 6.2 Linear velocities of points on a link.

EXAMPLE PROBLEM 6.5

Figure 6.7 shows a cargo lift mechanism for a delivery truck. At this instant, point *A* has a velocity of 12 in./s in the direction shown, and point *B* has a velocity of 10.4 in./s, also in the direction shown. Determine the angular velocity of the lower link and the relative velocity of point *B* relative to point *A*.

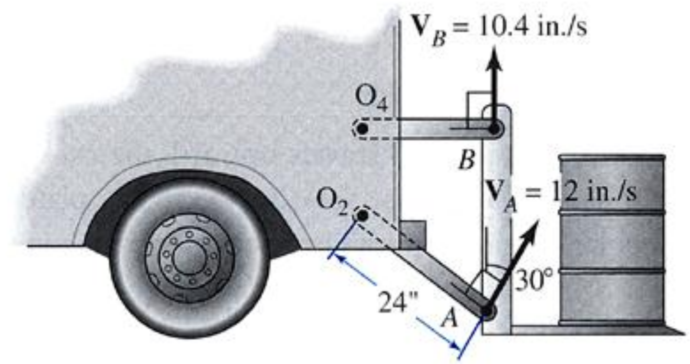


FIGURE 6.7 Mechanism for Example Problem 6.5.

SOLUTION:

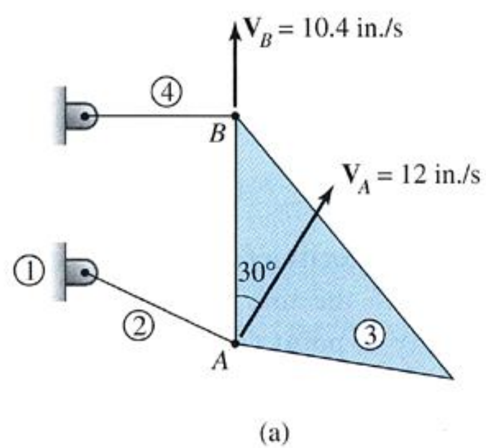


FIGURE 6.8 Kinematic diagram for Example Problem 6.5.

$$V_A \cos\theta = V_B \quad along \quad AB \ \ link$$

$$w_{AB} = V_A \sin\theta / (link \ \ length)$$

EXAMPLE PROBLEM 6.6

Figure 6.11 shows a rock-crushing mechanism. It is used in a machine where large rock is placed in a vertical hopper and falls into this crushing chamber. Properly sized aggregate, which passes through a sieve, is discharged at the bottom. Rock not passing through the sieve is reintroduced into this crushing chamber.

Determine the angular velocity of the crushing ram, in the shown configuration, as the 60-mm crank rotates at 120 rpm, clockwise.

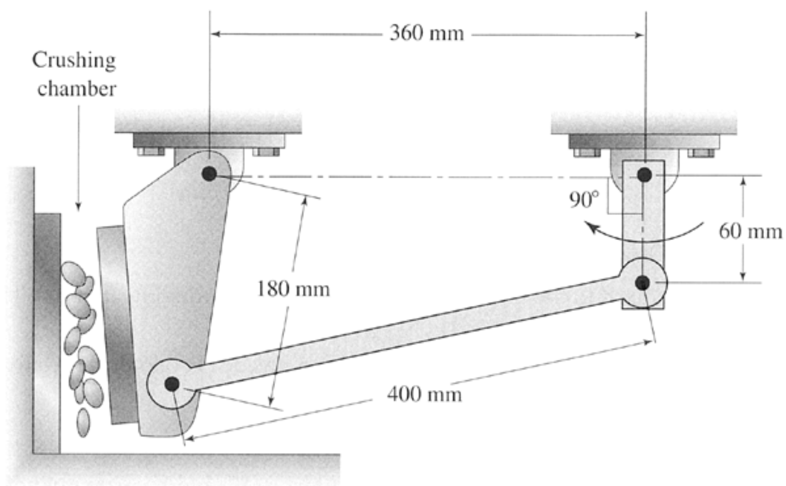
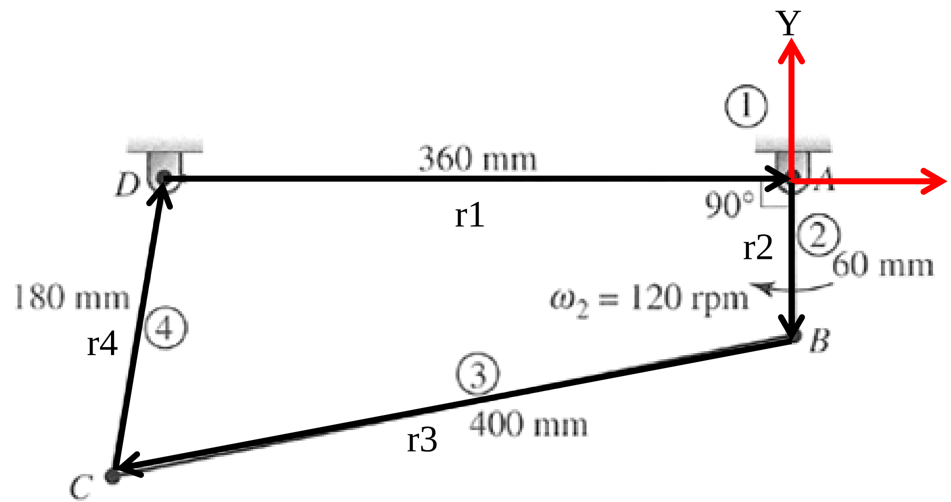


FIGURE 6.11 Mechanism for Example Problem 6.6.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3+\underset{\sim}{\omega}_4\times\underset{\sim}{r}_4=0$$

$$2 \ eqs \ \ for \ \ 2 \ unknowns \ \ \omega_3 \ \ and \ \ \omega_4$$

6.6 GRAPHICAL VELOCITY ANALYSIS:RELATIVE VELOCITY METHOD

6.6.1 Points on Links Limited to Pure Rotation or Rectilinear Translation

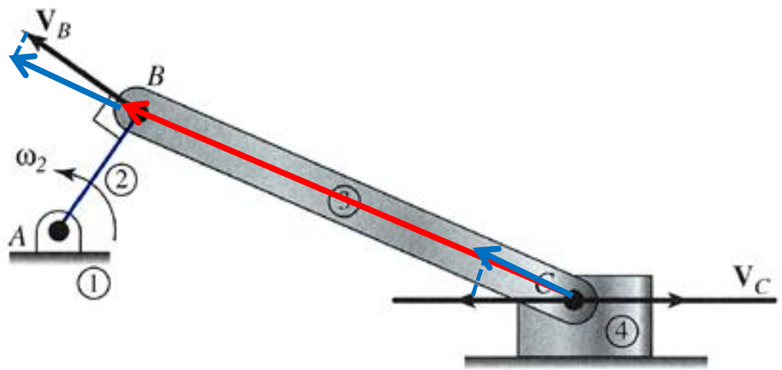
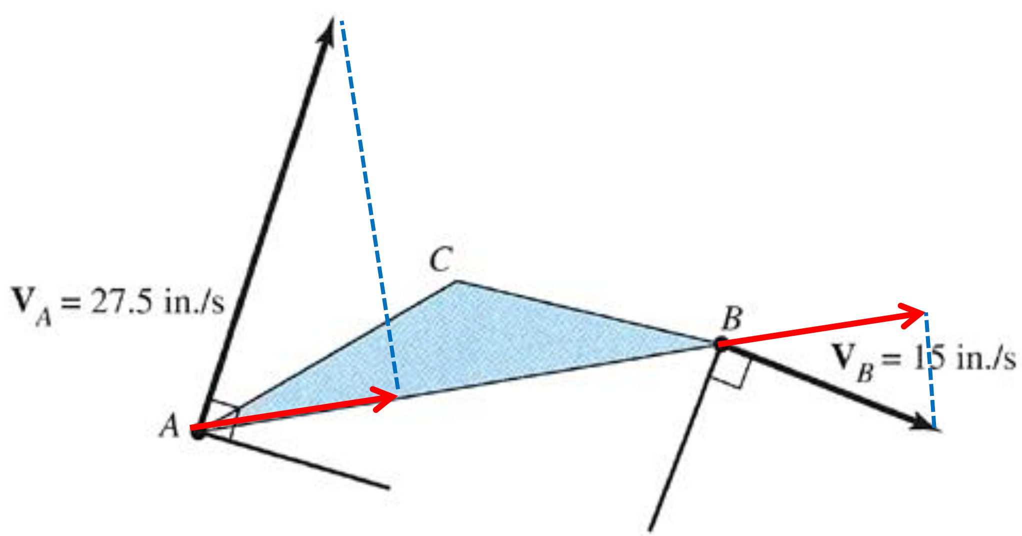


FIGURE 6.10 Links constrained to pure rotation and rectilinear translation.

6.6.2 General Points on a Floating Link



EXAMPLE PROBLEM 6.7

Figure 6.14 illustrates a mechanism that extends reels of cable from a delivery truck. It is operated by a hydraulic cylinder at A. At this instant, the cylinder retracts at a rate of 5 mm/s. Determine the velocity of the top joint, point E.

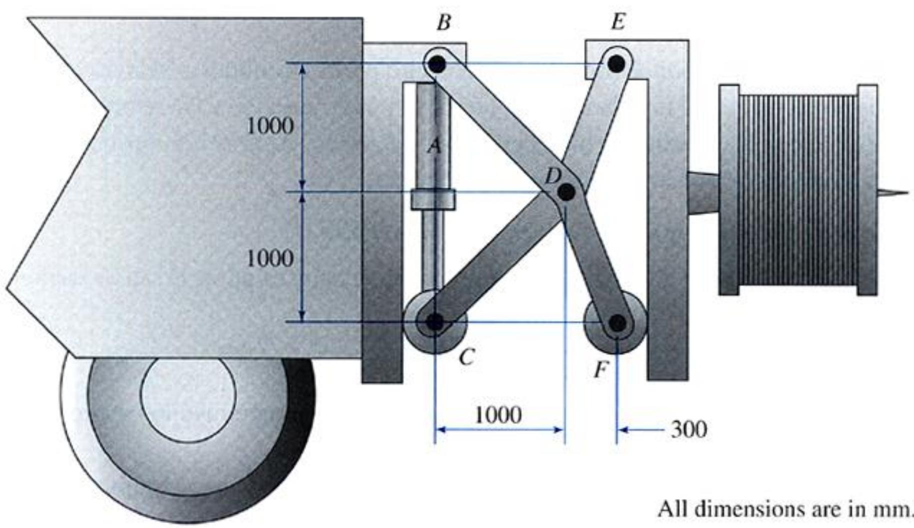
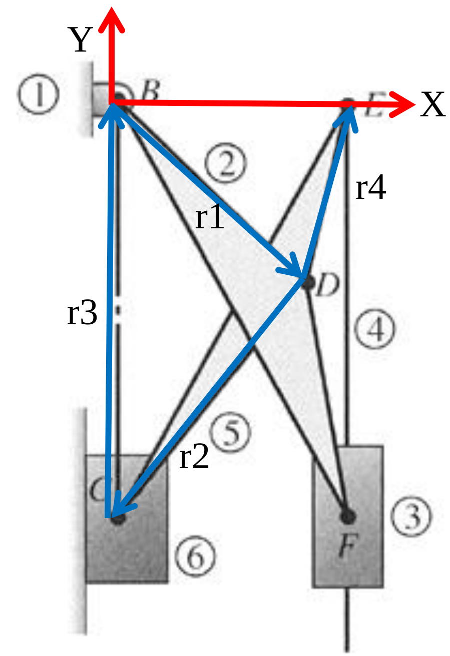


FIGURE 6.14 Mechanism for Example Problem 6.7.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\begin{bmatrix}0\\5\end{bmatrix}=0$$

$$2\ \ eqs\ \ for\ \ \omega_1\ \ and\ \ \omega_2$$

$$\underset{\sim}{r}=\underset{\sim}{r}_1+\underset{\sim}{r}_4$$

$$\dot{\underset{\sim}{r}}=\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_4$$

EXAMPLE PROBLEM 6.8

Figure 6.16 shows a mechanism that tips the bed of a dump truck. Determine the required speed of the hydraulic cylinder in order to tip the truck at a rate of 5 rad/min.

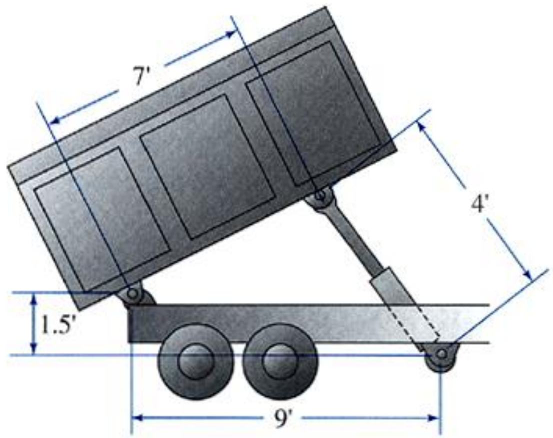
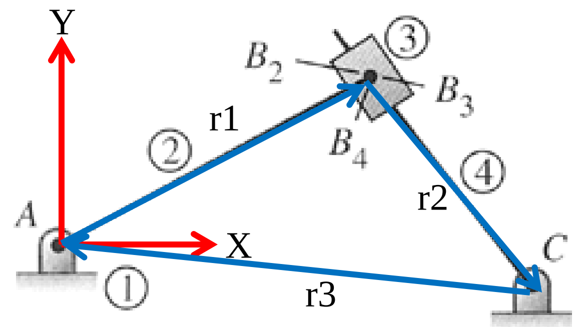


FIGURE 6.16 Dump truck mechanism for Example Problem 6.8.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$\text{solve for } \theta_1 \text{ and } \theta_2$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\dot{r}_2=0$$

$$\omega_1=5\,rad/\min\,,\ \dot{\underset{\sim}{r}}_2=\begin{bmatrix} vc\theta_2 \\ vs\theta_2 \end{bmatrix}$$

$$2 \text{ eqs for } 2 \text{ unknowns } \omega_2 \text{ and } v$$

EXAMPLE PROBLEM 6.9

Figure 6.19 shows a primitive well pump that is common in undeveloped areas. To maximize water flow, the piston should travel upward at a rate of 50 mm/s. In the position shown, determine the angular velocity that must be imposed on the handle to achieve the desired piston speed.

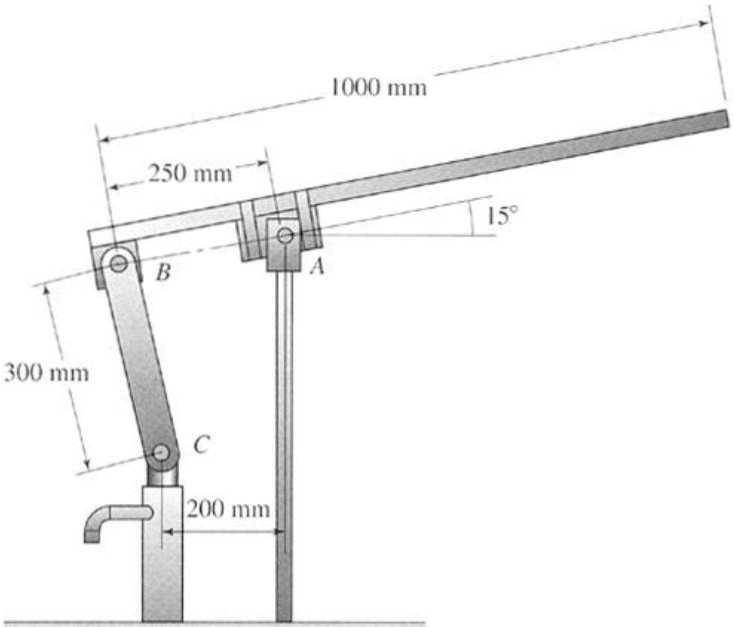
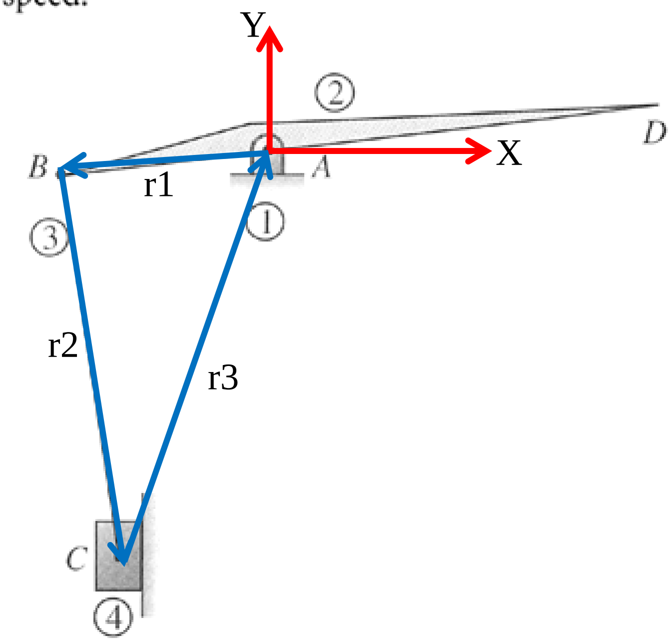


FIGURE 6.19 Well pump for Example Problem 6.9.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\dot{\underset{\sim}{r}}_2=0$$

$$\dot{\underset{\sim}{r}}_2=\begin{bmatrix}0\\50\end{bmatrix}$$

$$2\ \ eqs\ \ for\ \ 2\ \ unknowns\ \ \underset{\sim}{\omega}_1\ \ and\ \ \underset{\sim}{\omega}_2$$

EXAMPLE PROBLEM 6.10

Figure 6.21 illustrates a roofing material delivery truck conveyor. Heavy roofing materials can be transported on the conveyor to the roof. The conveyor is lifted into place by extending the hydraulic cylinder. At this instant, the cylinder is extending at a rate of 8 fpm (ft/min). Determine the rate that the conveyor is being lifted.

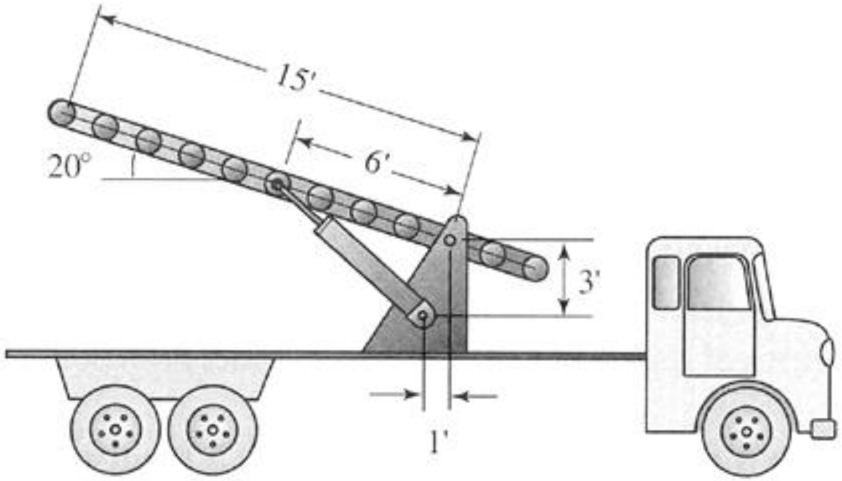
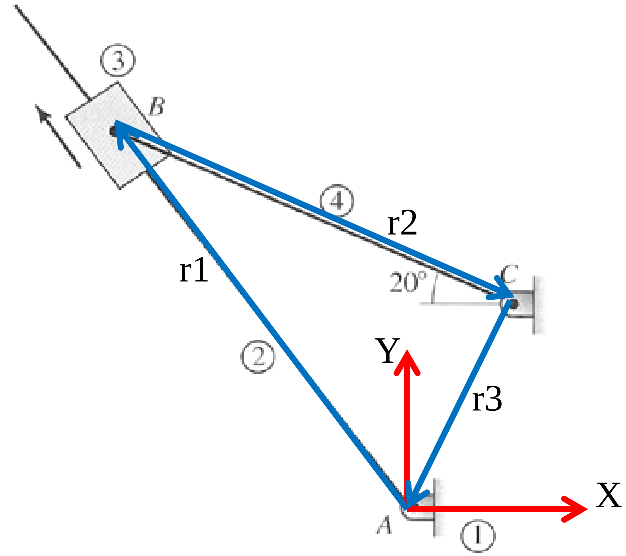


FIGURE 6.21 Conveyor for Example Problem 6.10.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$r_2=6,\ \theta_2=340^\circ,\ \underset{\sim}{r}_3=\begin{bmatrix}-1\\-3\end{bmatrix}$$

$$2\ \textit{eqs}\ \ \textit{for}\ \ 2\ \textit{unknowns}\ \ r_1\ \ \textit{and}\ \ \theta_1$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\dot{r}}_2=0$$

$$\underset{\sim}{\dot{r}}_2=\begin{bmatrix}8c\theta_2\\8s\theta_2\end{bmatrix}$$

$$2\ \textit{eqs}\ \ \textit{for}\ \ 2\ \textit{unknowns}\ \ \omega_1\ \ \textit{and}\ \ \omega_2$$

6.9 ALGEBRAIC SOLUTIONS

6.9.1 Slider-Crank Mechanism

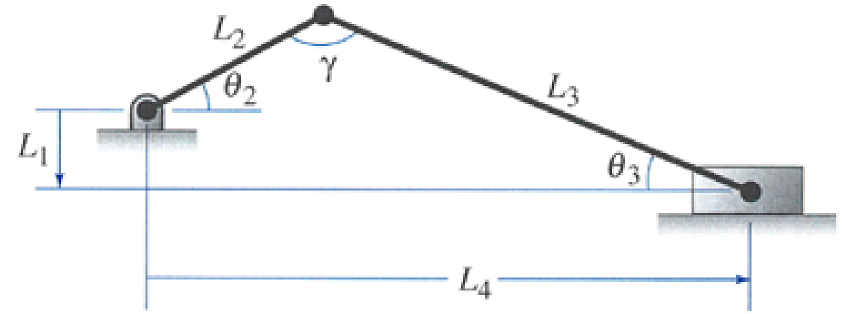


FIGURE 4.20 Offset slider-crank mechanism.

$$\theta_3 = \sin^{-1}\left\{\frac{L_1 + L_2 \sin\ \theta_2}{L_3}\right\} \tag{4.6}$$

$$L_4 = L_2 \cos(\theta_2) + L_3 \cos(\theta_3) \tag{4.7}$$

$$\omega_3 = -\omega_2\left(\frac{L_2 \cos \theta_2}{L_3 \cos \theta_3}\right) \tag{6.12}$$

$$v_4 = -\omega_2 L_2 \sin\ \theta_2 + \omega_3 L_3 \sin \theta_3 \tag{6.13}$$

6.9.2 Four-Bar Mechanism

$$BD = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos\ \theta_2} \qquad \textbf{(4.9)}$$

$$\gamma = \cos^{-1}\left[\frac{(L_3)^2 + (L_4)^2 - (BD)^2}{2(L_3)(L_4)}\right] \qquad \textbf{(4.10)}$$

$$\theta_3 = 2\tan^{-1}\left[\frac{-L_2 \sin\theta_2 + L_4 \sin\gamma}{L_1 + L_3 - L_2\cos\theta_2 - L_4\cos\gamma}\right] \qquad \textbf{(4.11)}$$

$$\theta_4 = 2\tan^{-1}\left[\frac{L_2 \sin\theta_2 - L_3 \sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right] \qquad \textbf{(4.12)}$$

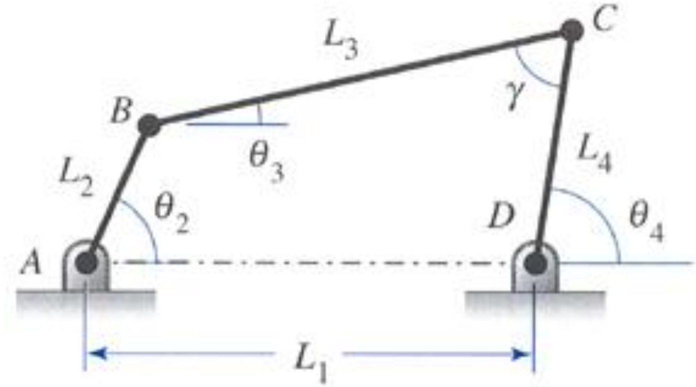


FIGURE 4.23 The four-bar mechanism.

$$\omega_3 = -\omega_2\left[\frac{L_2\sin(\theta_4 - \theta_2)}{L_3 \sin\gamma}\right] \qquad \textbf{(6.14)}$$

$$\omega_4 = -\omega_2\left[\frac{L_2\sin(\theta_3 - \theta_2)}{L_4 \sin\gamma}\right] \qquad \textbf{(6.15)}$$

6.10 INSTANTANEOUS CENTER OF ROTATION

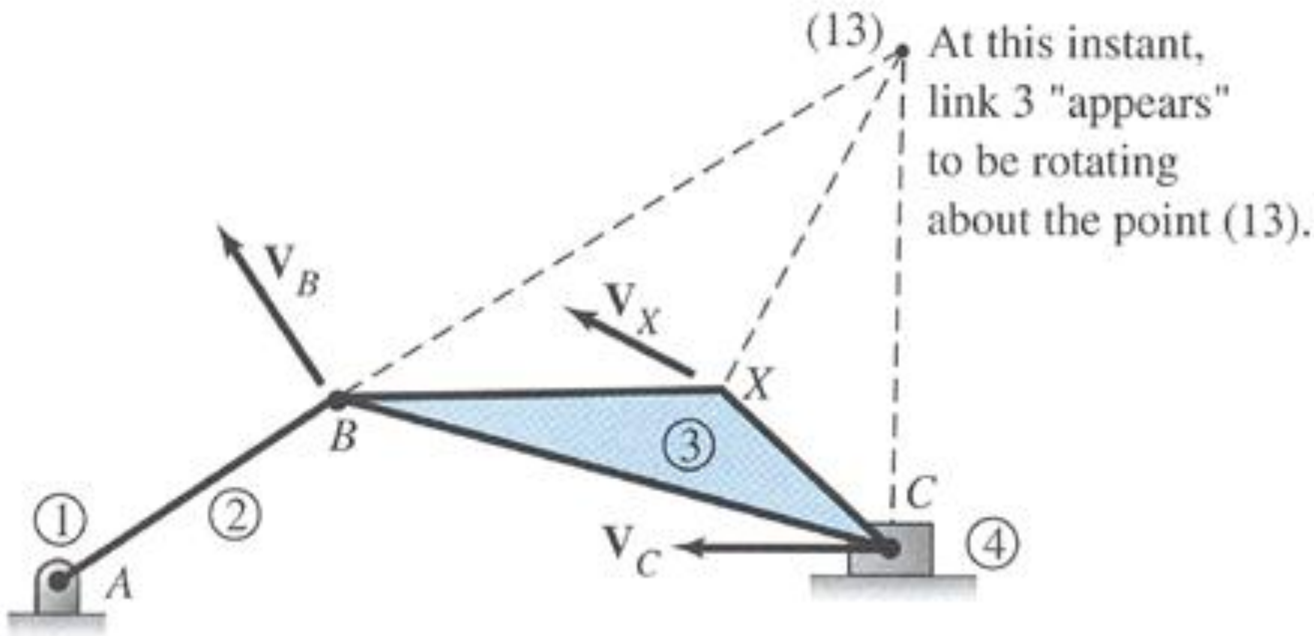


FIGURE 6.23 Instantaneous center.

$$\text{Total number of instant centers} = \frac{n(n-1)}{2} \qquad \textbf{(6.16)}$$

6.11 LOCATING INSTANT CENTERS

6.11.1 Primary Centers

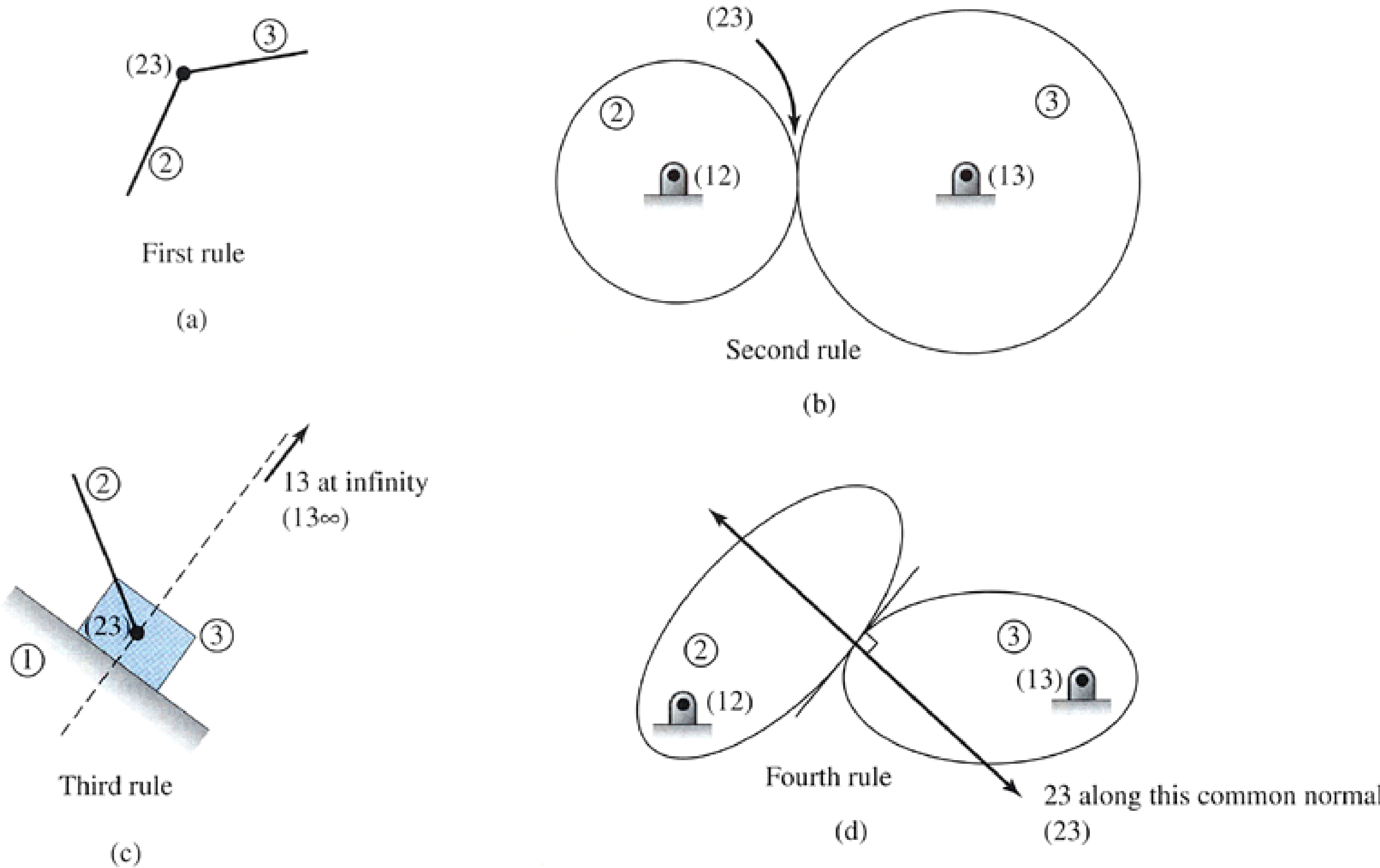


FIGURE 6.24 Locating primary centers.

6.12 GRAPHICAL VELOCITY ANALYSIS: INSTANT CENTER METHOD

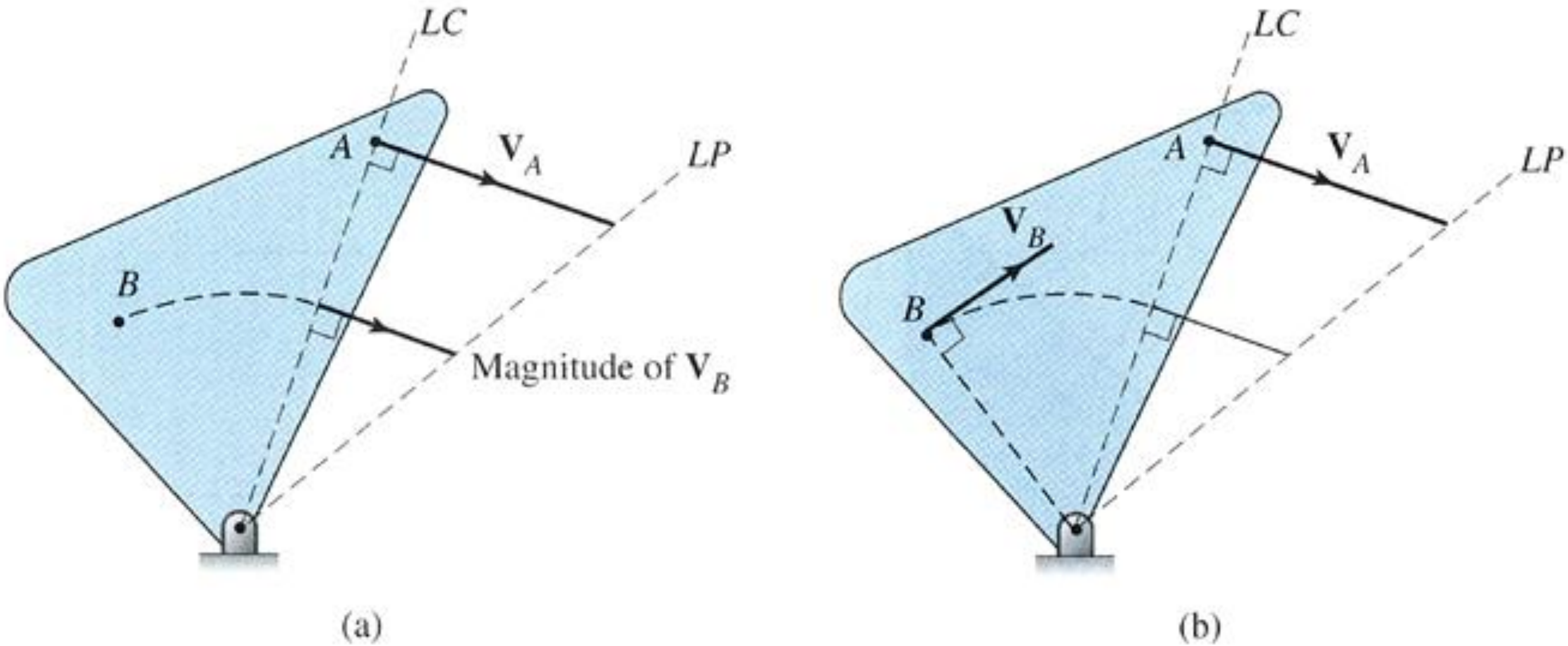


FIGURE 6.35 Using a line of centers and line of proportion.

EXAMPLE PROBLEM 6.14

Figure 6.29 illustrated an automated, self-locking brace for a platform used on shipping docks. Example Problem 6.12 located all instant centers for the mechanism. Determine the angular velocity of link 4, knowing that link 2 is rising at a constant rate of 3 rad/s.

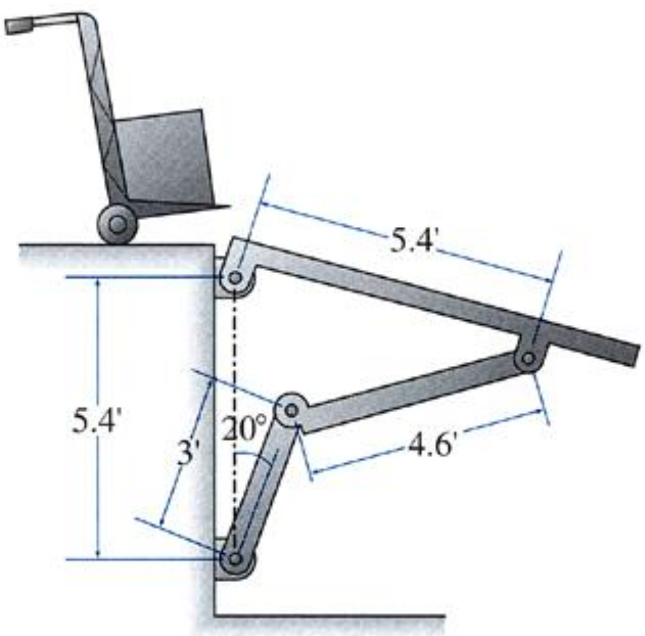
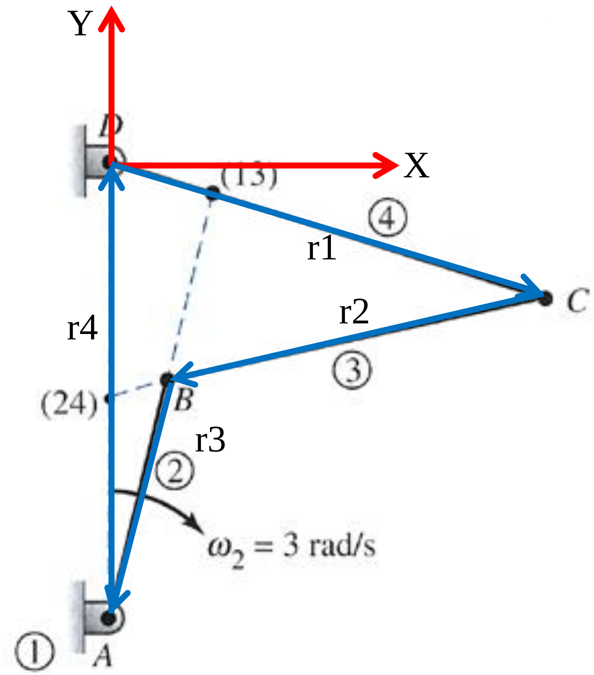


FIGURE 6.29 Locking brace for Example Problem 6.12.

SOLUTION:



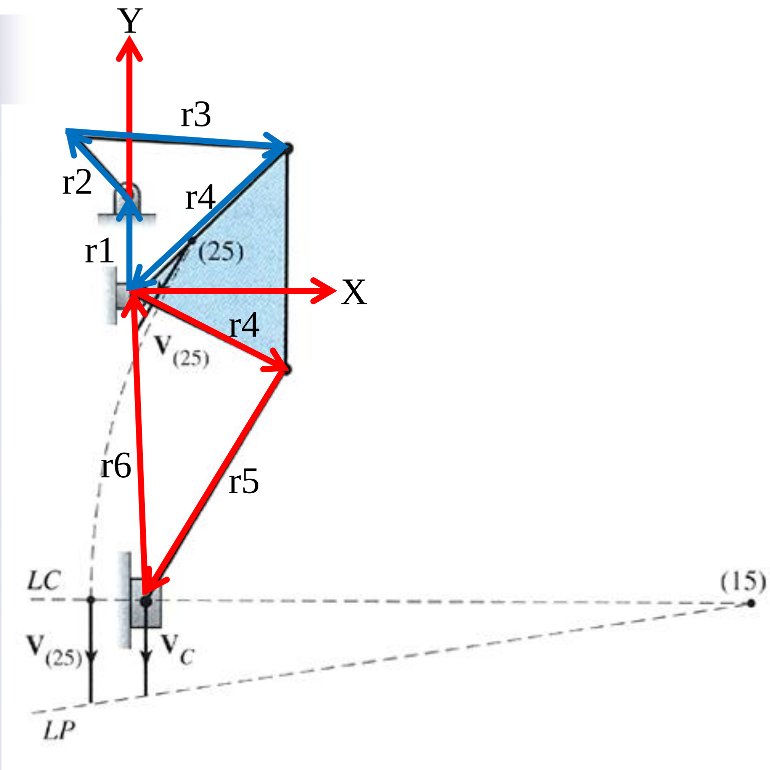
$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3=0$$

$$\textit{given}\ \ \omega_3,\ \textit{solve}\ \ \textit{for}\ \ \omega_1\ \ \textit{and}\ \ \omega_2$$

EXAMPLE PROBLEM 6.15

Figure 6.32 illustrates a rock-crushing device. Example Problem 6.13 located all instant centers for the mechanism. In the position shown, determine the velocity of the crushing ram when the crank is rotating at a constant rate of 60 rpm clockwise.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3+\underset{\sim}{\omega}_4\times\underset{\sim}{r}_4=0$$

$$\omega_2=60\,rpm.\ \ solve\ \ for\ \ \omega_3\ \ and\ \ \omega_4$$

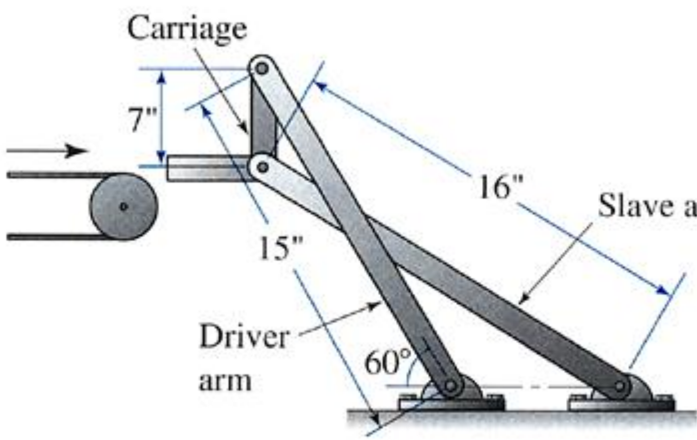
$$\underset{\sim}{r}_4+\underset{\sim}{r}_5+\underset{\sim}{r}_6=0$$

$$\underset{\sim}{\omega}_4\times\underset{\sim}{r}_4+\underset{\sim}{\omega}_5\times\underset{\sim}{r}_5+\begin{bmatrix}0\\v\end{bmatrix}=0$$

$$2\ \ eqs\ \ for\ \ 2\ \ unknowns\ \ \omega_5\ \ and\ \ v$$

EXAMPLE PROBLEM 6.16

Figure 6.38 shows a mechanism used in a production line to turn over cartons so that labels can be glued to the bottom of the carton. The driver arm is 15 in. long and, at the instant shown, it is inclined at a 60° angle with a clockwise angular velocity of 5 rad/s. The follower link is 16 in. long. The distance between the pins on the carriage is 7 in., and they are currently in vertical alignment. Determine the angular velocity of the carriage and the slave arm.



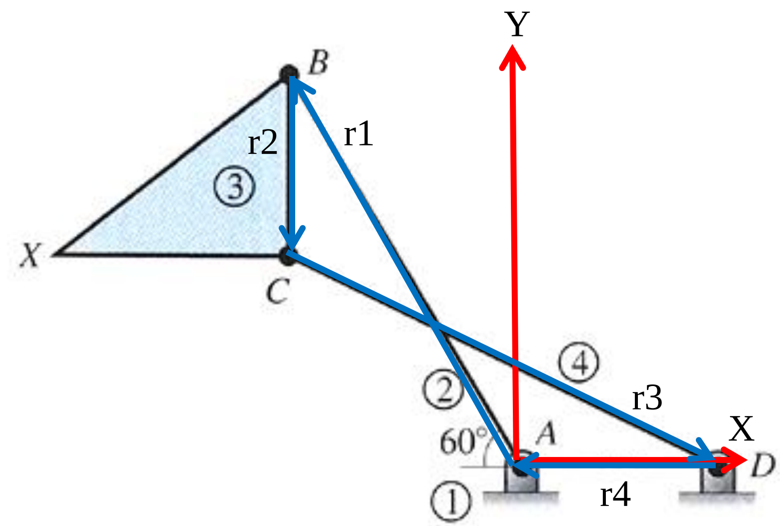


FIGURE 6.38 Turnover mechanism for Example Problem 6.16.

$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3=0$$

$$given\ \ \underset{\sim}{\omega}_1\ \ find\ \ \omega_2\ \ and\ \ \omega_3$$

Chapter 7 Acceleration Analysis

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Define linear, rotational, normal, tangential, Coriolis, and relative accelerations.
2. Use the relative acceleration method to graphically solve for the acceleration of a point on a link, knowing the acceleration of another point on that link.
3. Use the relative acceleration method to graphically determine the acceleration of a point of interest on a floating link.
4. Understand when the Coriolis acceleration is present, and include it in the analysis.
5. Use the relative acceleration method to analytically solve for the acceleration of a point.
6. Use the relative acceleration method to analytically determine the acceleration of a point of interest on a floating link.
7. Construct an acceleration curve to locate extreme acceleration values.

7.2 LINEAR ACCELERATION

7.2.1 Linear Acceleration of Rectilinear Points

$$\mathbf{A} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{V}}{\Delta t} = \frac{\mathbf{dv}}{\mathbf{dt}} \quad (7.1)$$

$$\mathbf{V} = \frac{d\mathbf{R}}{dt}$$

$$\mathbf{A} = \frac{d^2\mathbf{R}}{dt^2} \quad (7.2)$$

$$\mathbf{A} \cong \frac{\Delta \mathbf{V}}{\Delta t} \quad (7.3)$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \quad (7.7)$$

$$\alpha = \frac{d^2\theta}{dt^2} \quad (7.8)$$

$$\alpha \cong \frac{\Delta \omega}{\Delta t} \quad (7.9)$$

EXAMPLE PROBLEM 7.7

The mechanism shown in Figure 7.11 is designed to move parts along a conveyor tray and then rotate and lower those parts to another conveyor. The driving wheel rotates with a constant angular velocity of 12 rpm. Determine the angular acceleration of the rocker arm that rotates and lowers the parts.

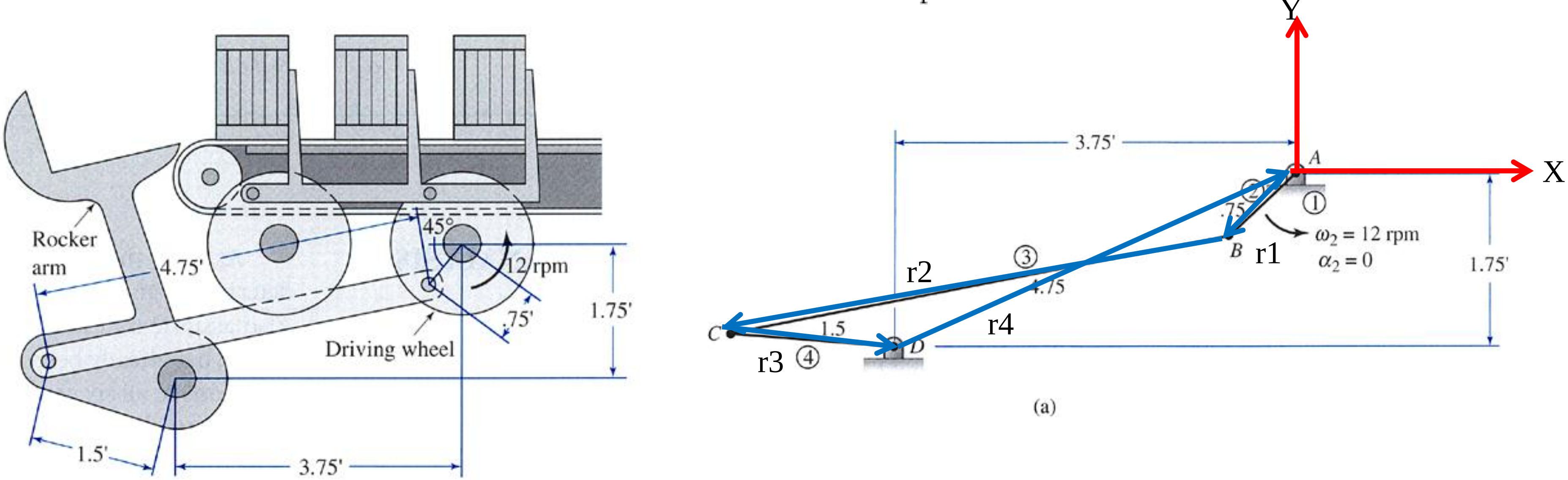


FIGURE 7.11 Mechanism for Example Problem 7.7.

$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3+\underset{\sim}{r}_4=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3=0$$

$$given\ \ \omega_1\ \ find\ \ \omega_2\ \ and\ \ \omega_3$$

$$\underset{\sim}{\omega}_1\times\left(\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1\right)+\dot{\underset{\sim}{\omega}}_2+\underset{\sim}{\omega}_2\times\left(\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2\right)+\dot{\underset{\sim}{\omega}}_3+\underset{\sim}{\omega}_3\times\left(\underset{\sim}{\omega}_3\times\underset{\sim}{r}_3\right)=0$$

$$solve\ \ for\ \ \dot{\omega}_2\ \ and\ \ \underset{\sim}{\omega}_3$$

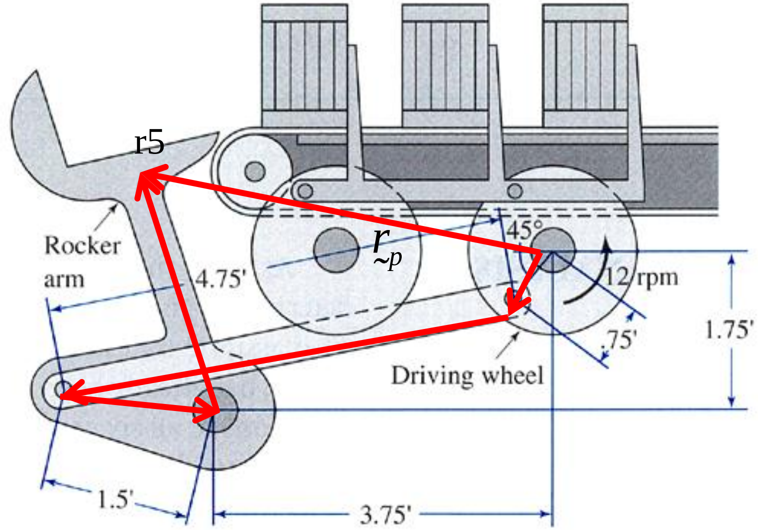


FIGURE 7.11 Mechanism for Example Problem 7.7.

$$r_p = \underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3 + \underset{\sim}{r}_5 = 0$$

$$\dot{r}_p = \underset{\sim}{\omega}_1 \times \underset{\sim}{r}_1 + \underset{\sim}{\omega}_2 \times \underset{\sim}{r}_2 + \underset{\sim}{\omega}_3 \times \left(\underset{\sim}{r}_3 + \underset{\sim}{r}_5\right)$$

$$\ddot{r}_p =$$

EXAMPLE PROBLEM 7.8

The mechanism shown in Figure 7.13 is a common punch press designed to perform successive stamping operations. The machine has just been powered and at the instant shown is coming up to full speed. The driveshaft rotates clockwise with an angular velocity of 72 rad/s and accelerates at a rate of 250 rad/s^2. At the instant shown, determine the acceleration of the stamping die, which will strike the workpiece.

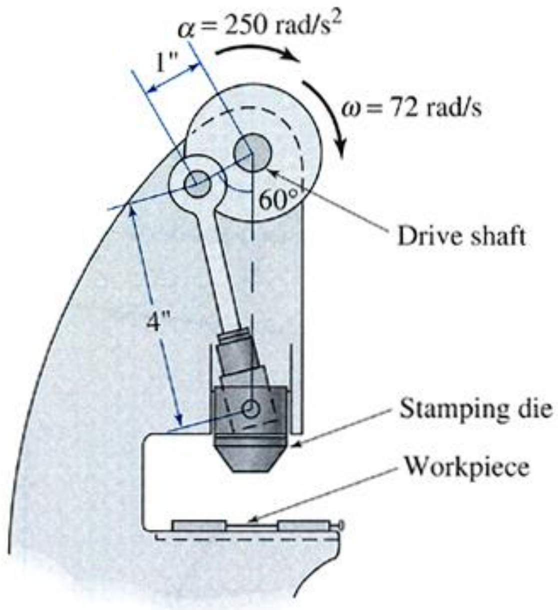


FIGURE 7.13 Mechanism for Example Problem 7.8.

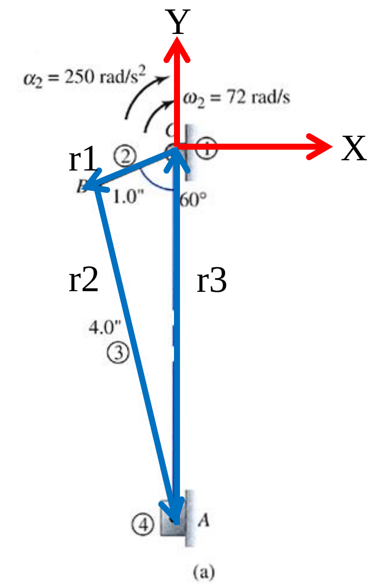


FIGURE 7.14 Diagrams for Example Problem 7.8.

$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2+\begin{bmatrix}0\\v\end{bmatrix}=0$$

$$solve\ \ for\ \ \omega_2\ \ and\ \ v$$

$$\dot{\underset{\sim}{\omega}}_1+\underset{\sim}{\omega}_1\times\left(\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1\right)+\dot{\underset{\sim}{\omega}}_2+\underset{\sim}{\omega}_2\times\left(\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2\right)+\begin{bmatrix}0\\a\end{bmatrix}=0$$

$$solve\ \ for\ \ \dot{\omega}_2\ \ and\ \ a$$

EXAMPLE PROBLEM 7.9

The mechanism shown in Figure 7.15 is used to feed cartons to a labeling machine and, at the same time, to prevent the stored cartons from moving down. At full speed, the driveshaft rotates clockwise with an angular velocity of 200 rpm. At the instant shown, determine the acceleration of the ram and the angular acceleration of the connecting rod.

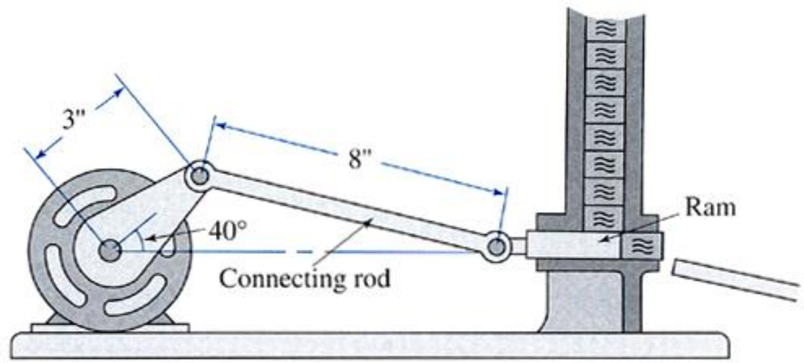
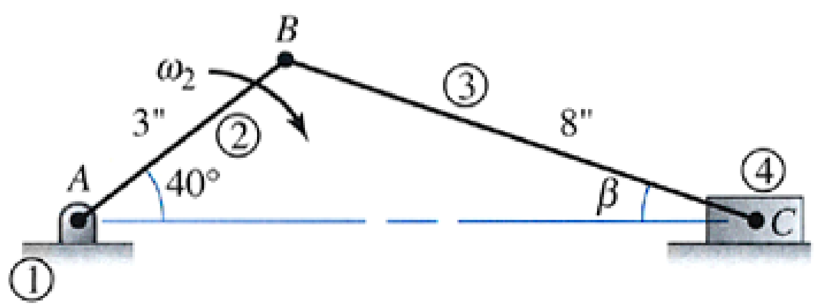


FIGURE 7.15 Mechanism for Example Problem 7.9.



7.8 ALGEBRAIC SOLUTIONS

7.8.1 Slider-Crank Mechanism

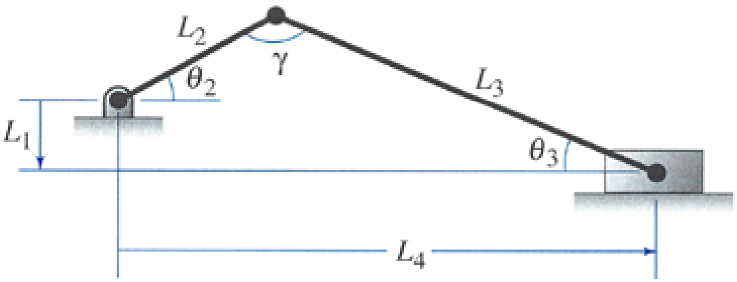


FIGURE 4.20 Offset slider-crank mechanism.

$$\theta_3 = \sin^{-1}\left(\frac{L_1 + L_2 \sin\theta_2}{L_3}\right) \qquad (4.6)$$

$$L_4 = L_2 \cos(\theta_2) + L_3 \cos(\theta_3) \qquad (4.7)$$

$$\omega_3 = -\omega_2\left(\frac{L_2 \cos\theta_2}{L_3 \cos\theta_3}\right) \qquad (6.12)$$

$$v_4 = -\omega_2 L_2 \sin\theta_2 + \omega_3 L_3 \sin\theta_3 \qquad (6.13)$$

$$\alpha_3 = \frac{\omega_2^2 L_2 \sin\theta_2 + \omega_2^3 L_3 \sin\theta_3 - \alpha_2 L_2 \cos\theta_2}{L_3 \cos\theta_3} \qquad (7.21)$$

$$\begin{aligned}\alpha_4 = {} & -\alpha_2 L_2 \sin\theta_2 - \alpha_3 L_3 \sin\theta_3 \\ & - \omega_2^2 L_2 \cos\theta_2 - \omega_3^2 L_3 \cos\theta_3 \end{aligned} \qquad (7.22)$$

7.8.2 Four-Bar Mechanism

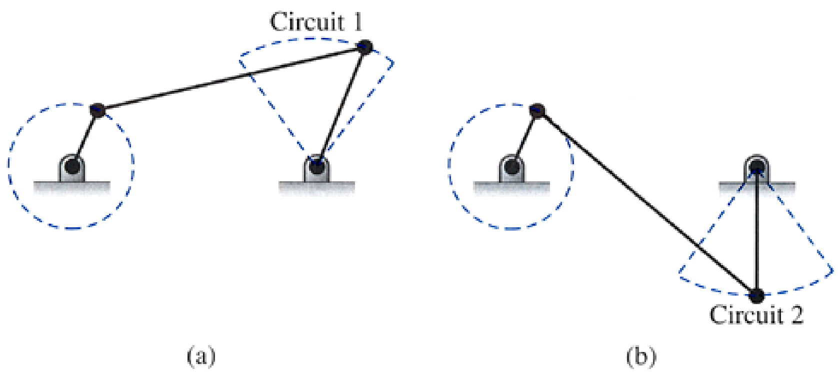


FIGURE 4.24 Circuits of a four-bar mechanism.

$$BD = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos(\theta_2)} \tag{4.9}$$

$$\gamma = \cos^{-1}\left(\frac{L_3^2 + L_4^2 - BD^2}{2(L_3)(L_4)}\right) \tag{4.10}$$

$$\theta_3 = 2\tan^{-1}\left[\frac{-L_2 \sin\theta_2 + L_4 \sin\gamma}{L_1 + L_3 - L_2\cos\theta_2 - L_4\cos\gamma}\right] \tag{4.11}$$

$$\theta_4 = 2\tan^{-1}\left[\frac{L_2 \sin\theta_2 - L_3 \sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right] \tag{4.12}$$

$$\omega_3 = -\omega_2\left[\frac{L_2 \sin(\theta_4 - \theta_2)}{L_3 \sin\gamma}\right] \tag{6.14}$$

$$\omega_4 = -\omega_2\left[\frac{L_2 \sin(\theta_3 - \theta_2)}{L_4 \sin\gamma}\right] \tag{6.15}$$

$$\alpha_3 = \frac{\alpha_2 L_2 \sin(\theta_2 - \theta_4) + \omega_2^2 L_2 \cos(\theta_2 - \theta_4) - \omega_4^2 L_4 + \omega_3^2 L_3 \cos(\theta_4 - \theta_3)}{L_3 \sin(\theta_4 - \theta_3)} \tag{7.23}$$

$$\alpha_4 = \frac{\alpha_2 L_2 \sin(\theta_2 - \theta_3) + \omega_2^2 L_2 \cos(\theta_2 - \theta_3) - \omega_3^2 L_4 \cos(\theta_4 - \theta_3) + \omega_3^2 L_3}{L_4 \sin(\theta_4 - \theta_3)} \tag{7.24}$$

EXAMPLE PROBLEM 7.10

The mechanism shown in Figure 7.18 is used to pull movie film through a projector. The mechanism is driven by the drive wheel rotating at a constant 560 rpm. At the instant shown, graphically determine the acceleration of the claw, which engages with the film.

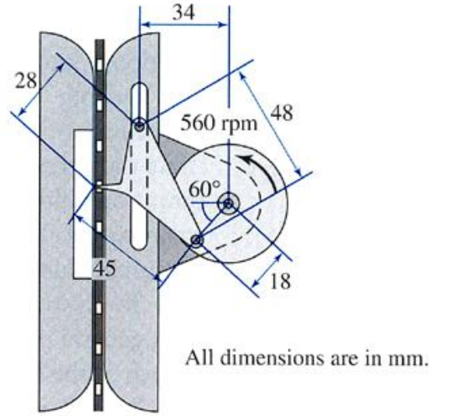


FIGURE 7.18 Film advance mechanism for Example Problem 7.10.

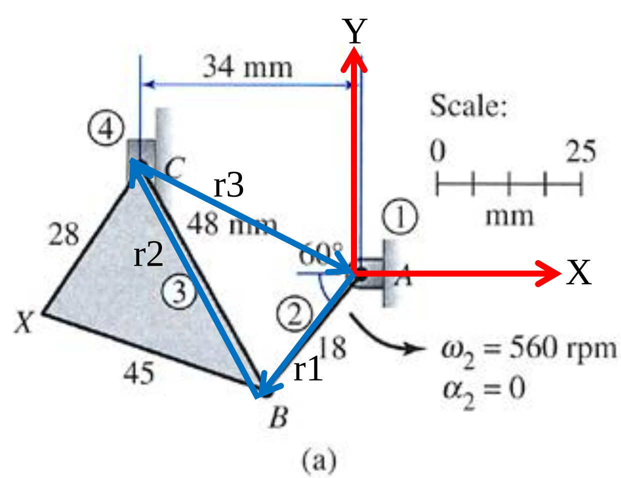


FIGURE 7.19 Diagrams for Example Problem 7.10.

7.10 ACCELERATION IMAGE (Useless!)

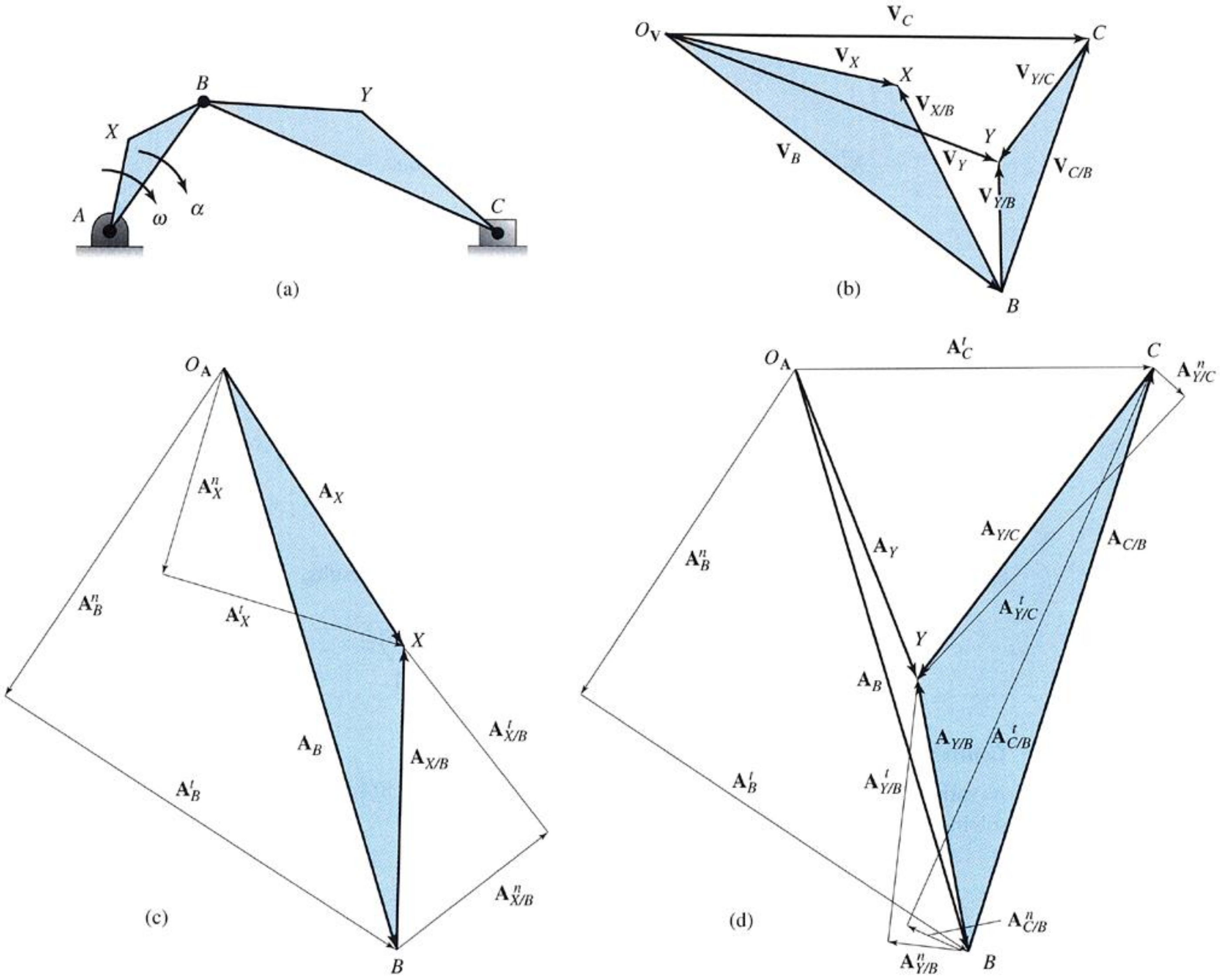


FIGURE 7.20 Acceleration image.

7.11 CORIOLIS ACCELERATION

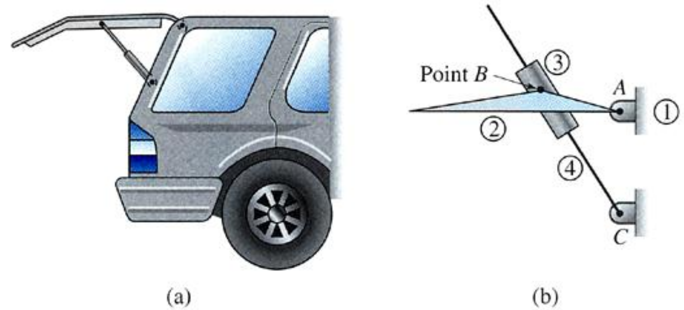


FIGURE 7.21 Case where Coriolis acceleration is encountered.

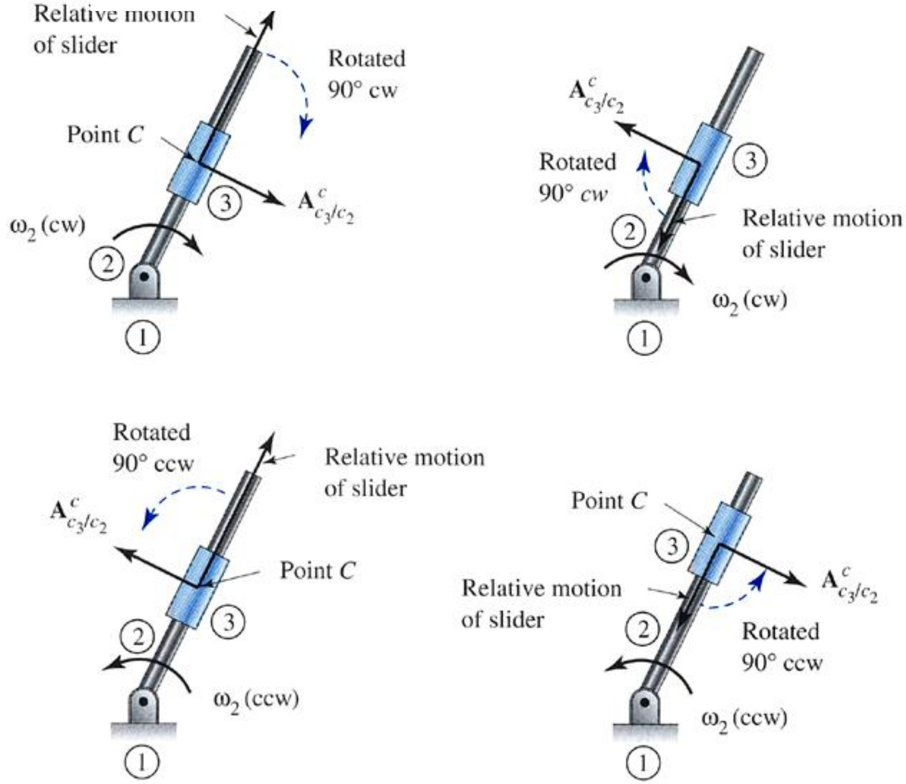


FIGURE 7.22 Directions of the Coriolis acceleration component.

EXAMPLE PROBLEM 7.11

Figure 7.23 illustrates handheld grass shears, used for trimming areas that are hard to reach with mowers or weed whackers. The drive wheel rotates counterclockwise at 400 rpm. Determine the angular acceleration of the oscillating blades at the instant shown.

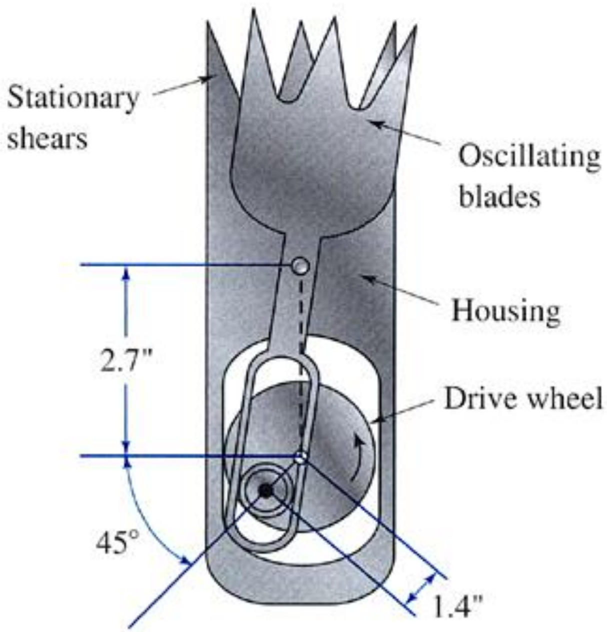
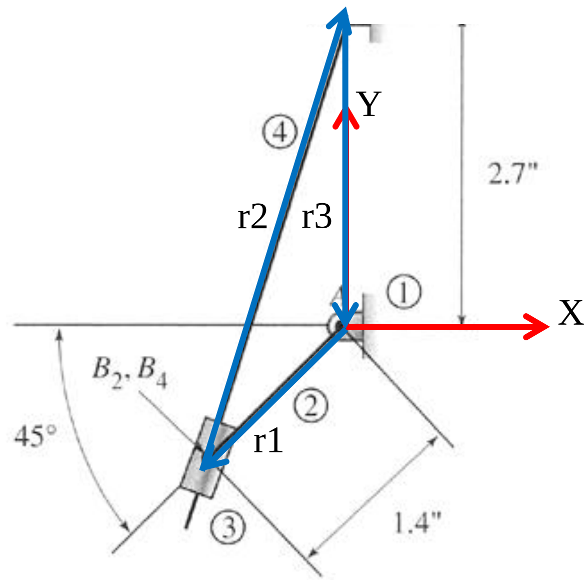


FIGURE 7.23 Grass shears for Example Problem 7.11.



$$\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3=0$$

$$solve \quad for \quad \theta_1 \quad and \quad \theta_2$$

$$\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1+\dot{\underset{\sim}{r}}_2+\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2=0$$

$$\omega_1=400, \ \dot{\underset{\sim}{r}}_2=\begin{bmatrix} vc\theta_2 \\ vs\theta_2 \end{bmatrix}, \ solve \quad for \quad \omega_2 \quad and \quad v$$

$$\underset{\sim}{\omega}_1\times\left(\underset{\sim}{\omega}_1\times\underset{\sim}{r}_1\right)+\dot{\underset{\sim}{\omega}}_2+\underset{\sim}{\omega}_2\times\left(\underset{\sim}{\omega}_2\times\underset{\sim}{r}_2\right)+\begin{bmatrix} ac\theta_2 \\ as\theta_2 \end{bmatrix}=0$$

$$solve \quad for \quad \dot{\omega}_2 \quad and \quad a$$

7.12 EQUIVALENT LINKAGES

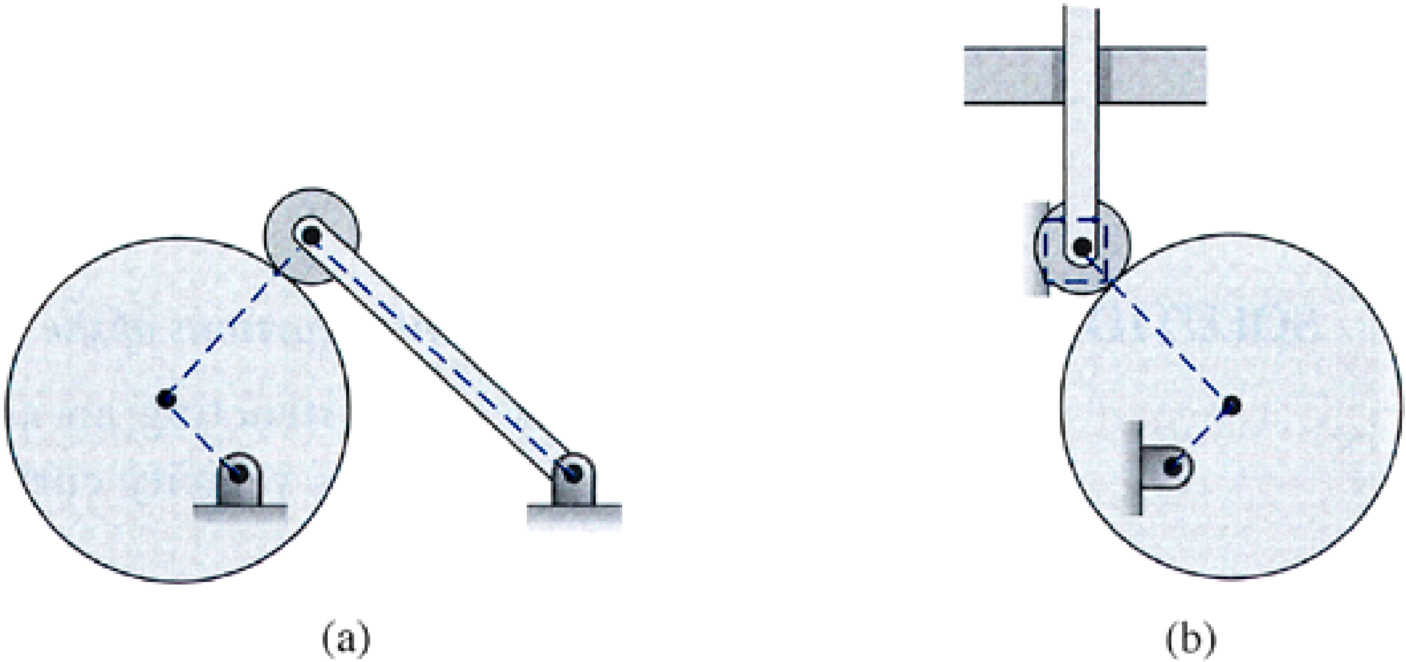


FIGURE 7.25 Equivalent linkages.

Chapter 9 Cams

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Identify the different types of cams and cam followers.
2. Create a follower displacement diagram from prescribed follower motion criteria.
3. Understand the benefits of different follower motion schemes.
4. Use equations to construct cam follower displacement diagrams.
5. Geometrically construct cam follower displacement diagrams.
6. Graphically and analytically construct disk cam profiles with several types of followers.
7. Graphically and analytically construct cylindrical cam profiles.

9.1 INTRODUCTION

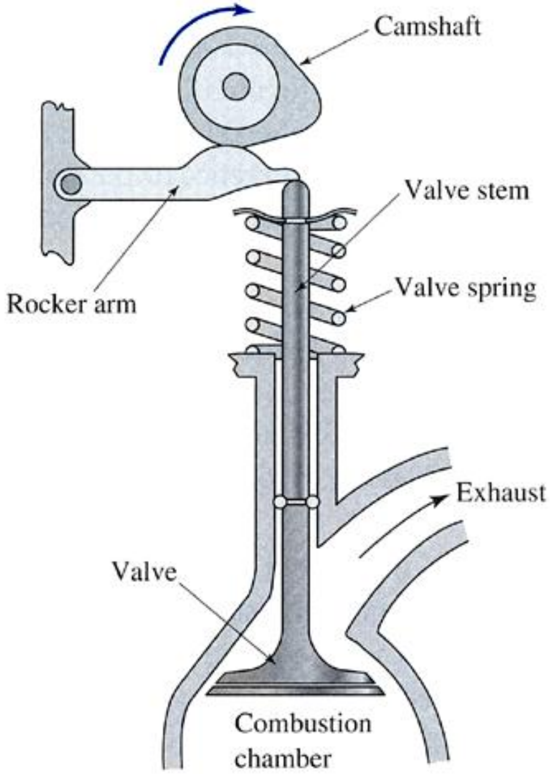
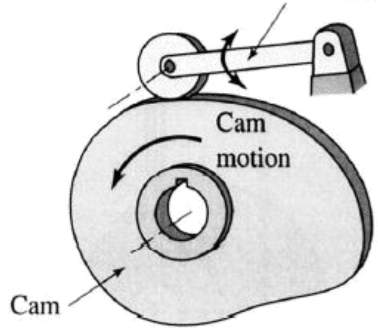


FIGURE 9.1 Engine valve train.

FIGURE 9.2 Various custom cams. (Courtesy of DE-STA-Co CAMCO Products.)

Plate cam

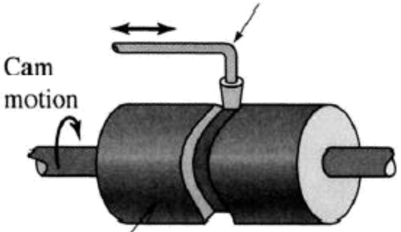
Plate or disk cams are the simplest and most common type of cam. A plate cam is illustrated in Fig. 9.2a. This type of cam is formed on a disk or plate. The radial distance from the center of the disk is varied throughout the circumference of the cam. Allowing a follower to ride on this outer edge gives the follower a radial motion.



(a) Plate Cam

Cylindrical cam

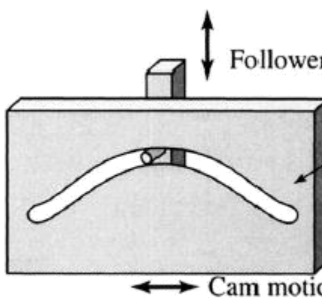
A *cylindrical or drum cam* is illustrated in Fig. 9.2b. This type of cam is formed on a cylinder. A groove is cut into the cylinder, which varies along the axis of rotation. Attaching a follower that rides in the groove gives the follower motion along the axis of rotation.



(b) Cylindical Cam

Linear cam

A *linear cam* is illustrated in Fig. 9.2c. This type of cam is formed on a translated block. A groove is cut into the block with a distance that varies from the plane of translation. Attaching a follower that rides in the groove gives the follower motion perpendicular to the plane of translation.



(c) Linear Cam

9.2 TYPES OF CAMS

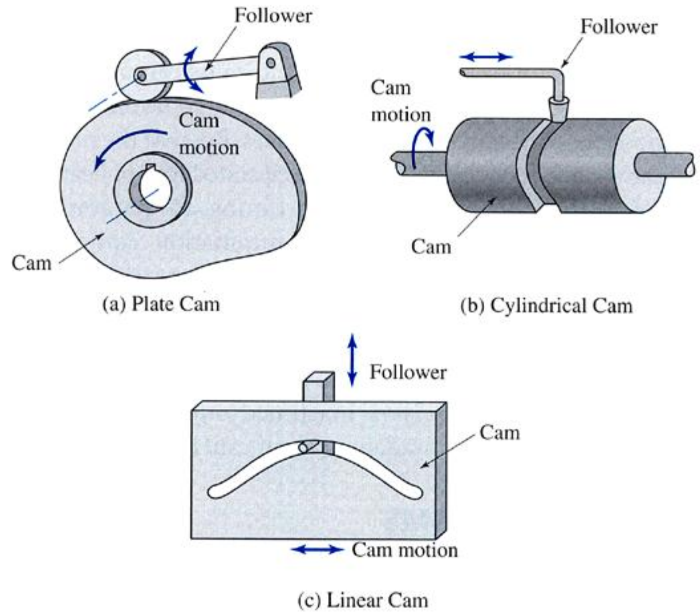
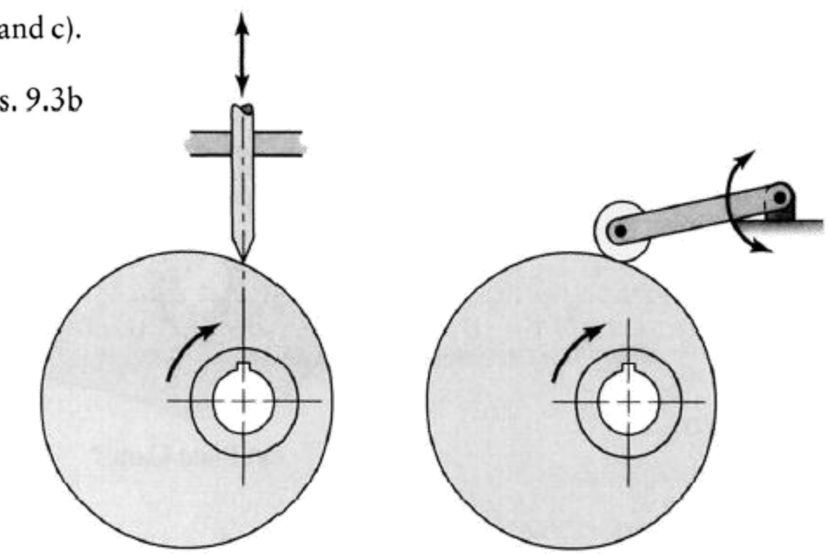


FIGURE 9.3 Cam types.

Follower motion

Translating followers are constrained to motion in a straight line (Figs. 9.3a and c).

Swinging arm or pivoted followers are constrained to rotational motion (Figs. 9.3b and d).

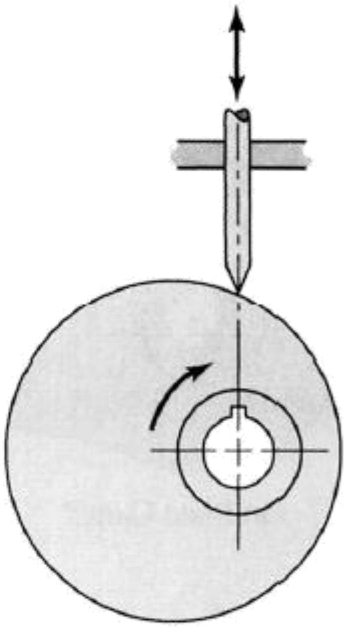


(a) In-line, knife-edge follower (b) Pivoted, roller follower

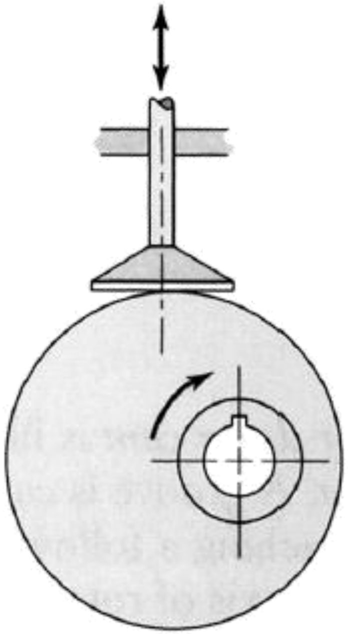
Follower position

An *in-line follower* exhibits straight line motion, such that the line of translation extends through the center of rotation of the cam (Fig. 9.3a).

An *offset follower* exhibits straight line motion, such that the line of the motion is offset from the center of rotation of the cam (Fig. 9.3c).

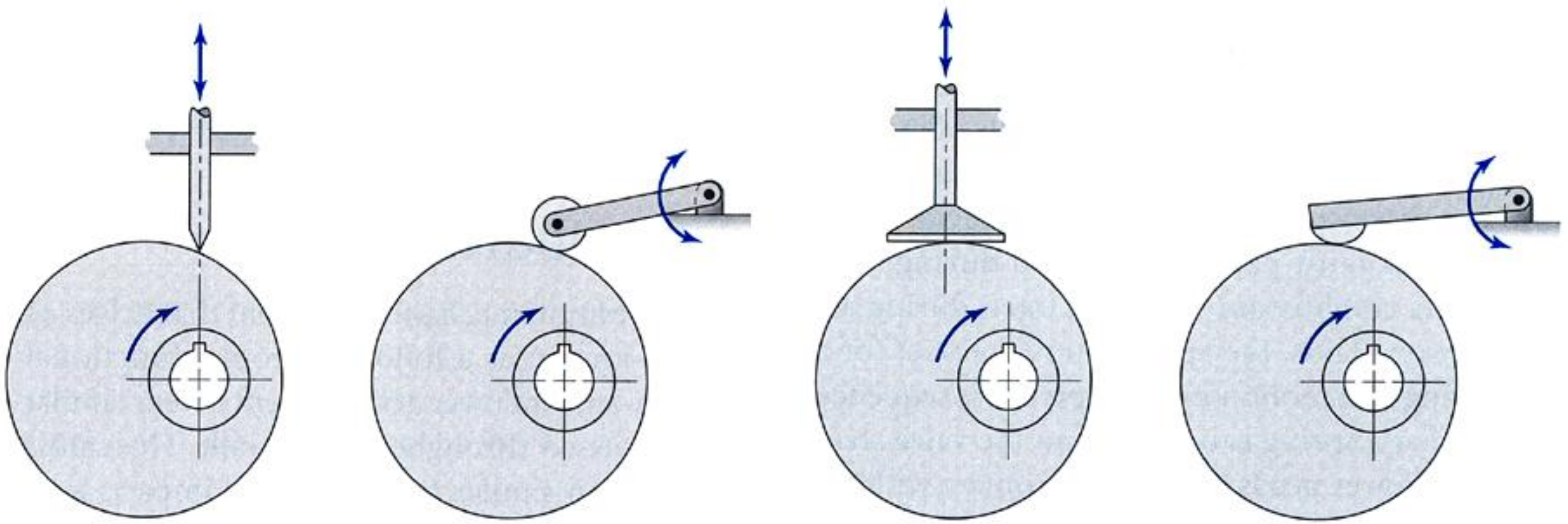


(a) In-line, knife-edge follower



(c) Offset, flat-face follower

9.3 TYPES OF FOLLOWERS



(a) In-line, knife-edge follower (b) Pivoted, roller follower (c) Offset, flat-face follower (d) Pivoted, spherical-face follower

FIGURE 9.4 Follower types.

9.11 THE 4-STATION GENEVA MECHANISM

Constant rotation producing index motion

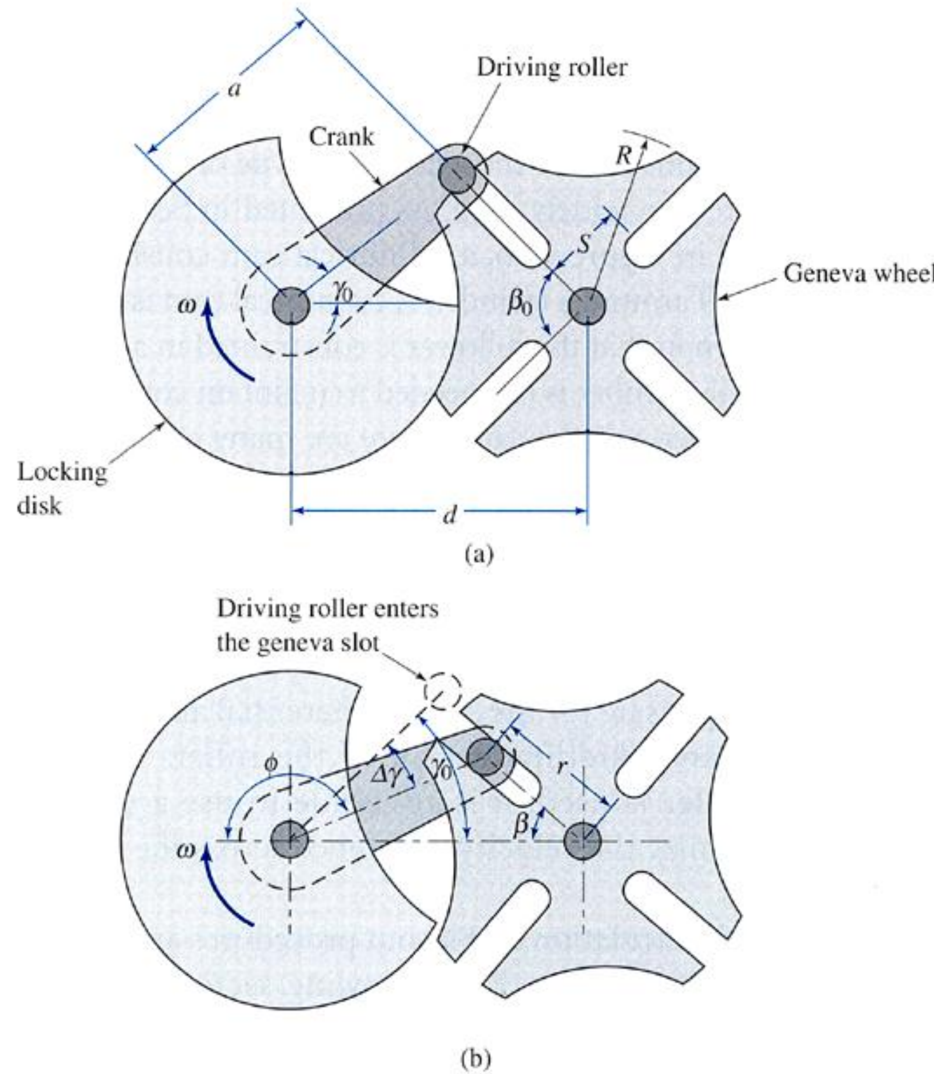


FIGURE 9.36 Four-station geneva mechanism.

Chapter 13 STATIC FORCE ANALYSIS

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Define and identify a force.
2. Calculate the moment of a force.
3. Understand the difference between mass and weight.
4. Understand and apply Newton's three laws of motion.
5. Create a free-body diagram of a general machine component.
6. Identify and use the special conditions for equilibrium of a two-force member.
7. Calculate sliding frictional force and identify its direction.
8. Determine the forces acting throughout a mechanism.

13.3 MOMENTS AND TORQUES

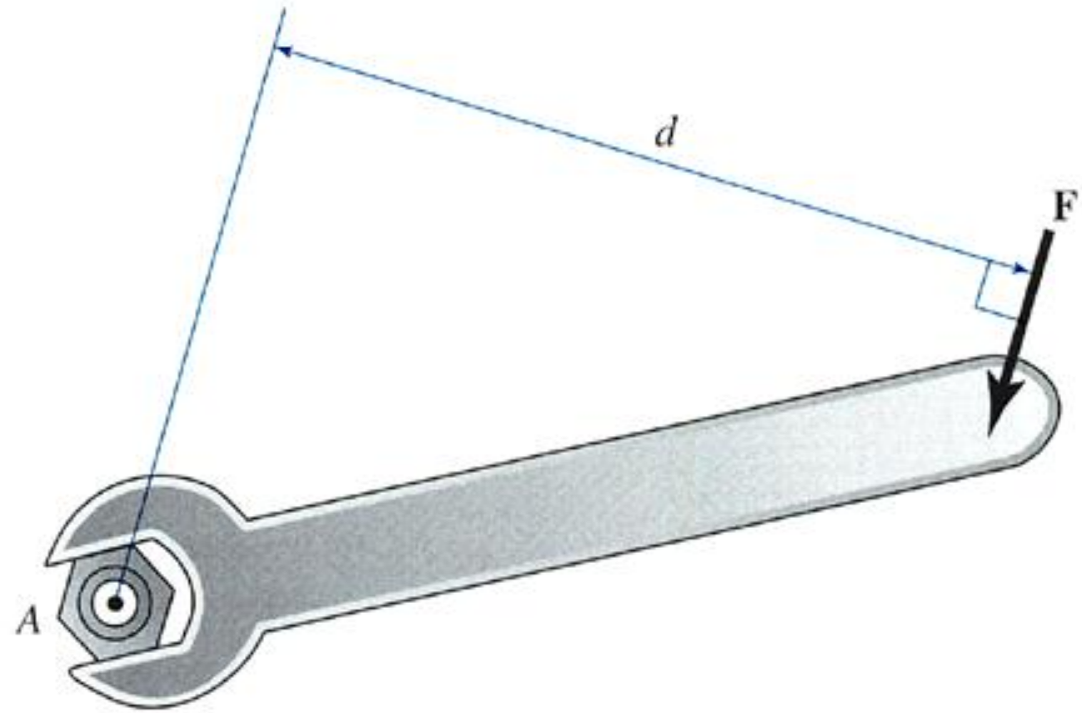


FIGURE 13.1 The definition of a moment or torque.

$$M_A = (F)(d) \tag{13.1}$$

13.5 FREE-BODY DIAGRAMS

13.5.1 Drawing a Free-Body Diagram

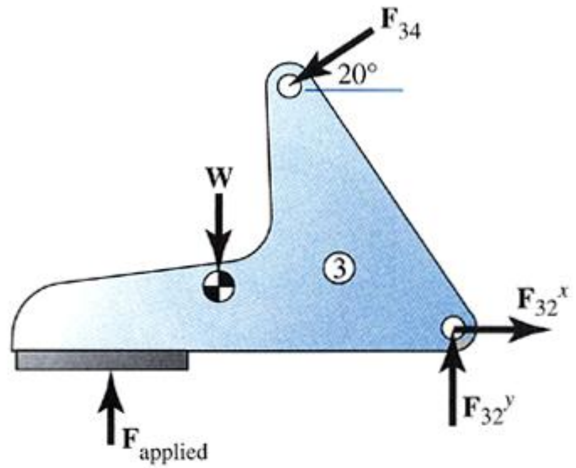


FIGURE 13.5 Free-body diagram.

13.5.2 Characterizing Contact Forces

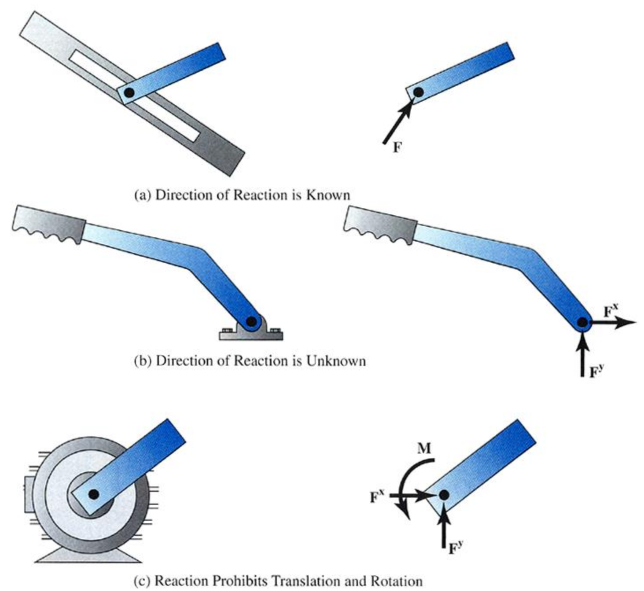


FIGURE 13.6 Reaction forces.

EXAMPLE PROBLEM 13.2

An engine hoist is shown in Figure 13.7. The engine being raised weighs 250 lb. Draw a free-body diagram of the entire hoist.

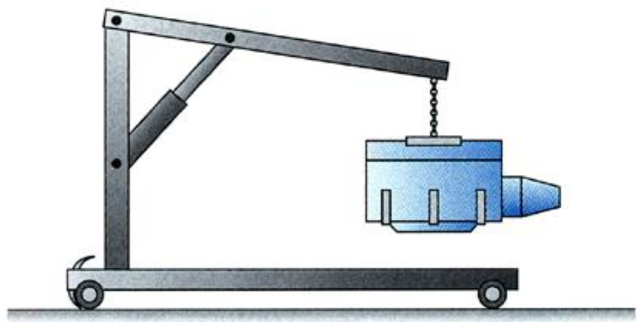


FIGURE 13.7 Engine hoist for Example Problem 13.2.

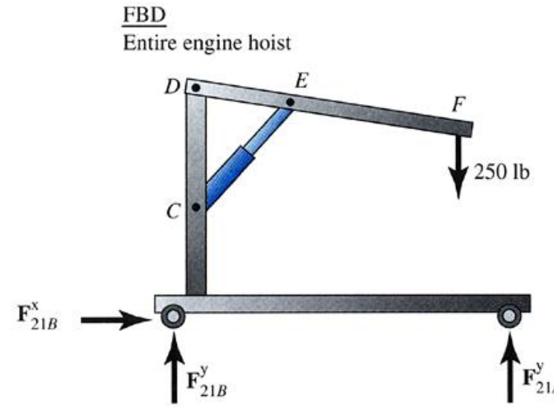


FIGURE 13.8 Free-body diagram for Example Problem 13.2.

13.6 STATIC EQUILIBRIUM

$$\Sigma F = 0 \qquad (13.2)$$

13.7 ANALYSIS OF A TWO-FORCE MEMBER

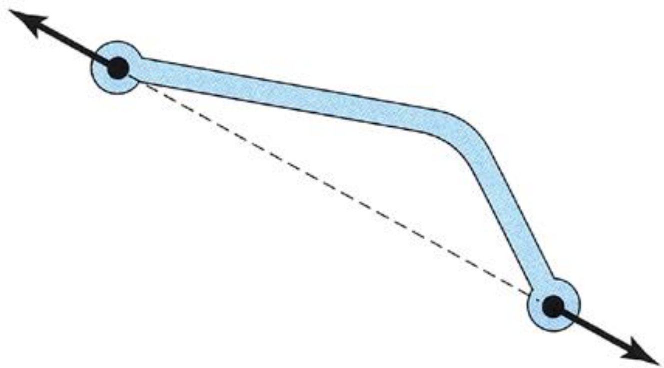


FIGURE 13.9 Two-force member.

EXAMPLE PROBLEM 13.3

A novelty nutcracker is shown in Figure 13.10. A force of 5 lb is applied to the top handle, as shown, and the mechanism does not move (static). Draw a free-body diagram and determine the forces on each link. For this analysis, the weight of each link can be considered negligible.

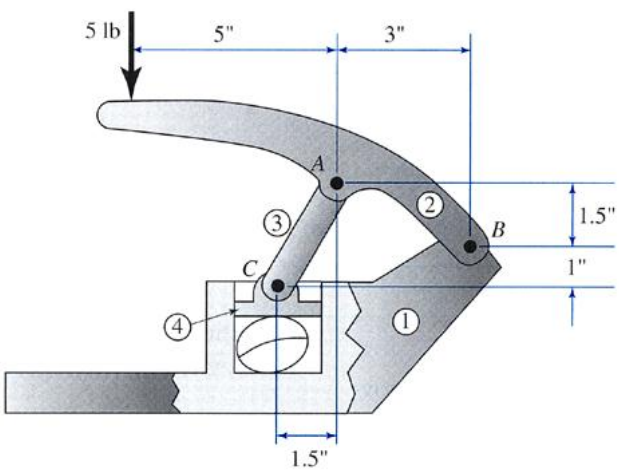


FIGURE 13.10 Nutcracker for Example Problem 13.3.

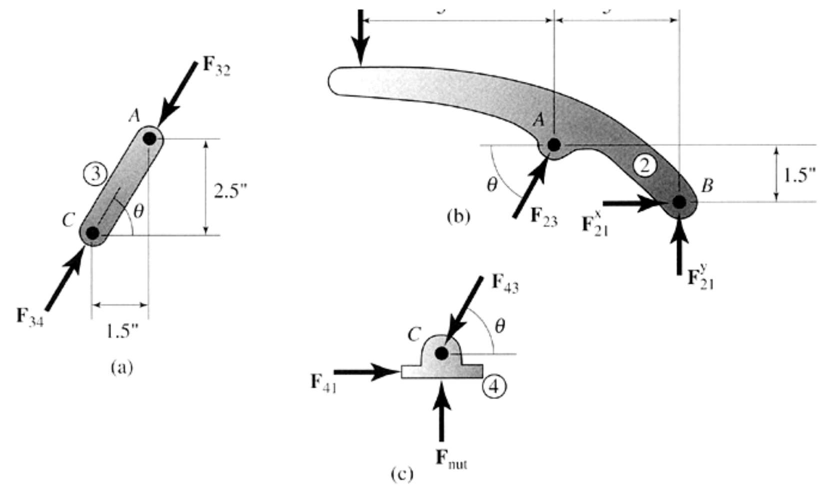


FIGURE 13.11 Free-body diagrams for Example Problem 13.3.

EXAMPLE PROBLEM 13.4

Figure 13.12 shows a mechanism used to crush rocks. The 60-mm mechanism crank is moving slowly, and inertial forces can be neglected. In the position shown, determine the torque required to drive the 60-mm crank and crush the rocks.

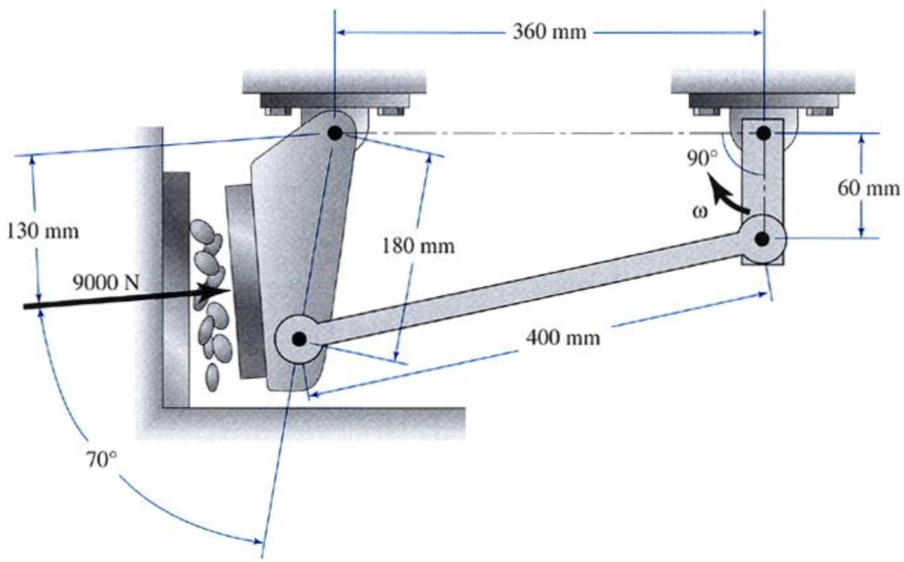


FIGURE 13.12 Rock crusher for Example Problem 13.4.

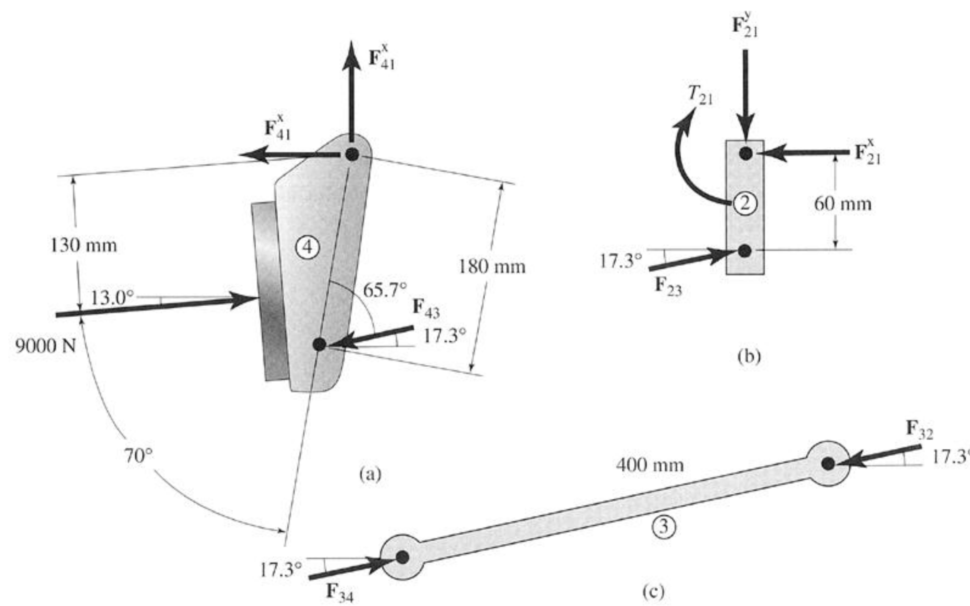


FIGURE 13.14 Free-body diagrams for Example Problem 13.4.

Chapter 14 DYNAMIC FORCE ANALYSIS

OBJECTIVES

Upon completion of this chapter, the student will be able to:

1. Understand the difference between mass and weight.
2. Calculate the mass moment of inertia of an object either by assuming a similarity to a basic shape or from the radius of gyration.
3. Transfer the mass moment of inertia to an alternative reference axis.
4. Calculate inertial forces and torques.
5. Determine the forces, including inertia, acting throughout a mechanism.

EXAMPLE PROBLEM 14.5

The compressor mechanism shown in Figure 14.7 is driven clockwise by a DC electric motor at a constant rate of 600 rpm. In the position shown, the cylinder pressure is 45 psi. The piston weighs 0.5 lb, and the coefficient of friction between the piston and the compressor cylinder is 0.1. The weight of all other links is negligible. At the instant shown, determine the torque required from the motor to operate the compressor.

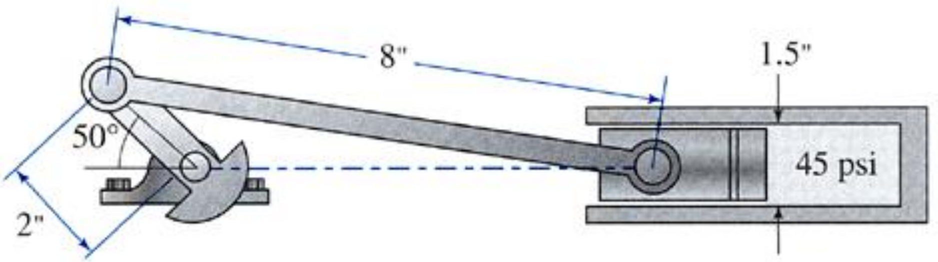


FIGURE 14.7 Mechanism for Example Problem 14.5.

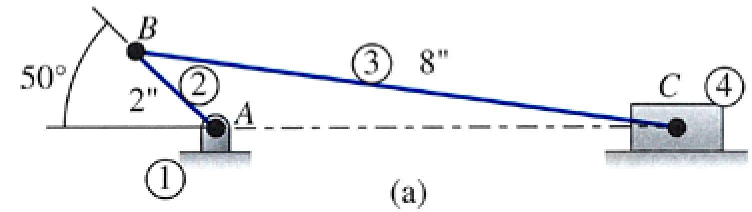
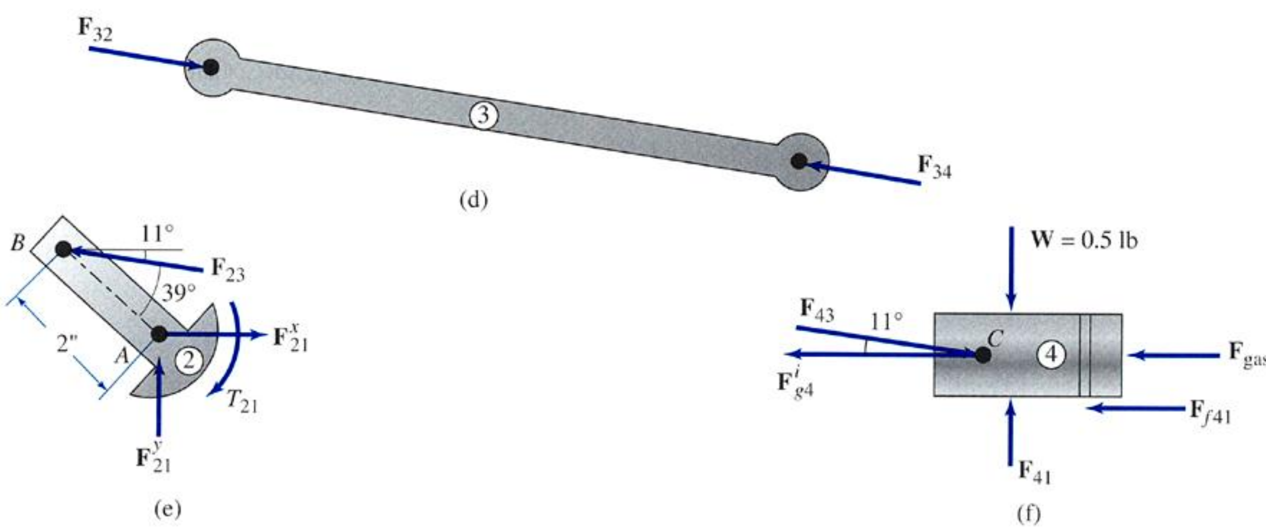


FIGURE 14.8 Diagrams for Example Problem 14.5.



EXAMPLE PROBLEM 14.6

The mechanism shown in Figure 14.9 is used to lower and retract the landing gear on small airplanes. The wheel assembly link weighs 100 lb, with a center of gravity as shown. The radius of gyration of the assembly, relative to the center of gravity, has been experimentally determined as 1.2 ft. The motor link is rotating counterclockwise at 3 rad/s and accelerating at 10 rad/s^2. For mass property estimation, the motor crank will weigh approximately 15 lb and will be 2 ft long, 1 ft wide, and 0.25 ft thick. The connecting link is estimated to weigh 20 lb and can be modeled as a 3.5-ft slender rod. Determine all forces acting on the joints of all links and the torque required to drive the motor link.

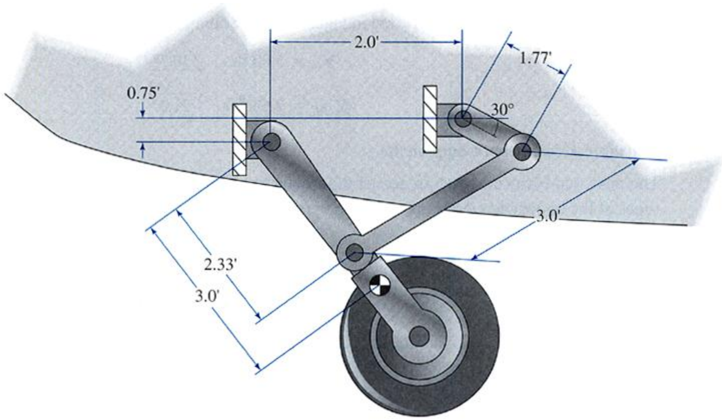
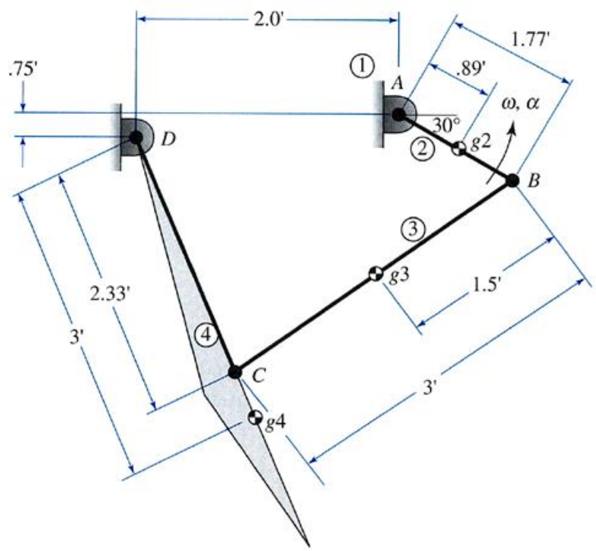


FIGURE 14.9 Landing gear for Example Problem 14.6.



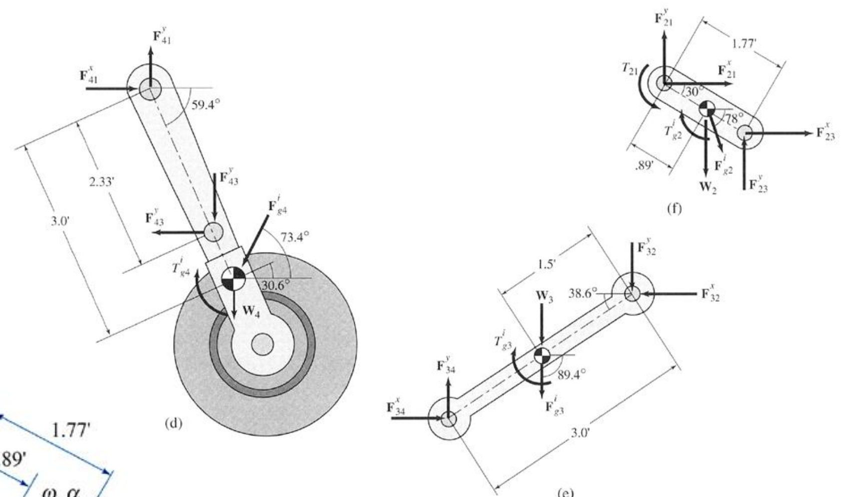


FIGURE 14.10 Continued