Secant Method

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Secant Method Formula

The secant method is a root-finding method that uses recurrence to solve for a given function.

$$x_n = x_{n-1} - f(x_{n-1})\frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})}$$

The method begins with two values, x_0 and x_1, which will converge to the true value of the root if they are sufficiently close to it. Thus iteration of the method begins with the following:

$$x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

The following secant_method function generates a list of the iterative values, and a plot of x_n versus $f(x_n)$. The secant_method function takes in a given function for which we wish to estimate roots, initial x_0 and x_1 values, and the number of desired iterations.

```
library(ggplot2)

secant_method <- function(f, x0, x1, iter = 5) {
  df_points <- data.frame(x1 = numeric(0),
                          y1 = numeric(0),
                          x2 = numeric(0),
                          y2 = numeric(0))

  xold <- x0
  xnew <- x1
  cat("Starting values are:", xold, " and ", xnew, "\n")

  # the algorithm which generates each xn value
  for (i in 1:iter) {
    fx_new <- f(xnew)
    fx_old <- f(xold)
    xn_1 <- xnew - fx_new*(xnew - xold)/(fx_new - fx_old)
    f_xn_1 <- f(xn_1)
    df_points[2*i - 1,] <- c(xold, fx_old , xnew, fx_new)
    df_points[2*i,] <- c(xn_1, 0 , xn_1, f_xn_1)

    xold <- xnew
    xnew <- xn_1
    cat("Next x values are:", xold, " and ", xnew, "\n")
  }

  # determines the limits for the graph
  x_start <- min(df_points$x1, df_points$x2, x0 - .1) # start is the min of these values
  x_end <- max(df_points$x1, df_points$x2, x0 + .1)  # end is the max of these values
  x <- seq(x_start, x_end, length.out = 200)
  fx <- rep(NA, length(x))
```

```
  for (i in seq_along(x)) {
    fx[i] <- f(x[i])
  }
  function_data <- data.frame(x, fx)

  # uses ggplot to plot the function and the segments for each iteration
  p <- ggplot(function_data, aes(x = x, y = fx)) +
    geom_line(colour = "blue", size = 1) +
    geom_segment(aes(x = x1, y = y1, xend = x2, yend = y2), data = df_points) +
    geom_abline(intercept = 0, slope = 1) +
    geom_abline(intercept = 0, slope = 0) +
    labs(y = "f(x)", title = "Secant Method for f(x) = log(x) - exp(-x)")

  print(p)
  return(cat("The estimated root for the function is", xnew))
}
```

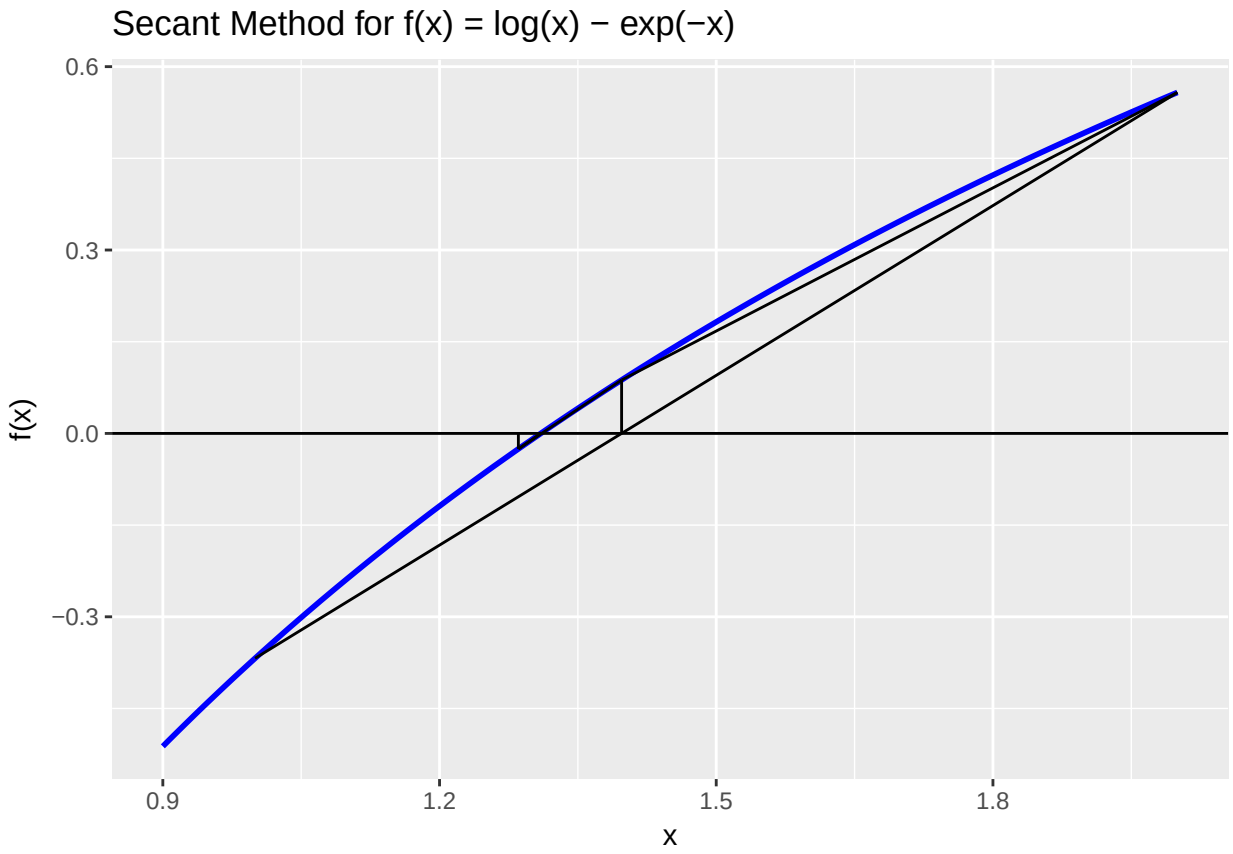
Example

This example estimates the zero for the function $f(x) = log(x) - e^{-x}$ using the initial values of x_0 and x_1.

```
f <- function(x) log(x) - exp(-x)

secant_method(f, 1, 2, iter = 5)
```

```
## Starting values are: 1  and  2
## Next x values are: 2  and  1.39741
## Next x values are: 1.39741  and  1.285476
## Next x values are: 1.285476  and  1.310677
## Next x values are: 1.310677  and  1.309808
## Next x values are: 1.309808  and  1.3098
```




```
## The estimated root for the function is 1.3098
```