Bayesian Networks

Problem 1: Host's Console Crash (modeling, storage size, inference, independence)

The host's game console *crashes* (C=1) only due to either a *power problem* (P) or a *buggy game build* (G). The music playlist app (M) keeps playing if power is fine. Assume P and G are a priori independent.

- $P(P{=}1) = 0.97$ (*power OK* with prob. 0.97);
- $P(G{=}1) = 0.35$ (*game is buggy* with prob. 0.35);
- $P(M{=}1 \mid P{=}1) = 0.92, \quad P(M{=}1 \mid P{=}0) = 0.12$;
- Crash distribution $P(C{=}1 \mid P, G)$:

P	G	$P(C{=}1 \mid P, G)$
0	1	0.98
1	1	0.45
0	0	0.88
1	0	0.08

a) Draw a BN over $\{P, G, M, C\}$ matching the story and write the factorization of $P(P, G, M, C)$.

b) **Storage:** How many numbers in the *full joint* over four binary variables? How many *parameters* in your factorization?

c) **Inference (use 0/1):** Compute by hand:

(i) $P(G{=}1 \mid C{=}1)$;

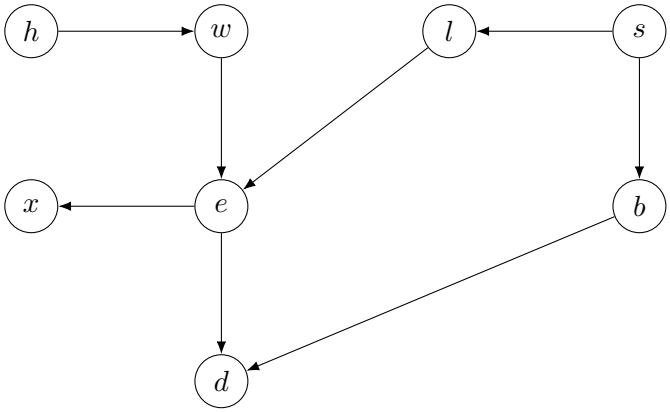
(ii) $P(G{=}1 \mid C{=}1, M{=}1)$.

d) **Independence:** List all pairs among $\{P, G, M, C\}$ that are independent; and all pairs that are conditionally independent *given exactly one* other variable.

Problem 2: Party Noise Network

We use the following network:

- h: **Hot weather**; w: **Windows open**;
- s: **Loud guest**; l: **Loud music set**;
- e: **Major noise source** (deterministic OR of w or l);
- b: **Beer pong running**; d: **Noise complaint**; x: **Noise meter peaks**.



Parameters (binary, shown for = 1; otherwise use complement):

$P(h{=}1)$	=	0.20	$P(s{=}1)$	=	0.50
$P(w{=}1 \mid h{=}1)$	=	0.50	$P(l{=}1 \mid s{=}1)$	=	0.30
$P(w{=}1 \mid h{=}0)$	=	0.10	$P(l{=}1 \mid s{=}0)$	=	0.05
$P(b{=}1 \mid s{=}1)$	=	0.60	$P(b{=}1 \mid s{=}0)$	=	0.20
$P(x{=}1 \mid e{=}1)$	=	0.95	$P(x{=}1 \mid e{=}0)$	=	0.10

Deterministic node: $e{=}1$ iff $w{=}1$ *or* $l{=}1$ (logical OR).

e	b	$P(d{=}1 \mid e, b)$
1	1	0.90
1	0	0.80
0	1	0.70
0	0	0.10

a) True/False using d-separation (no numbers):
(i) $h \perp s$; (ii) $h \perp l$; (iii) $h \perp x$; (iv) $w \perp s$; (v) $x \perp b \mid e$.

b) Compute (by enumeration or grouped terms):

(i) $P(d{=}1)$, (ii) $P(d{=}1 \mid s{=}1)$, (iii) $P(d{=}1 \mid s{=}0)$.

Problem 3: D-Separation Quick Test

For each tiny network, decide whether $X \perp Y$ and whether $X \perp Y \mid Z$. Give a one-line reason.

(1) $X \to Z \to Y$ chain
(2) $X \leftarrow Z \to Y$ common cause
(3) $X \to Z \leftarrow Y$ collider
(4) $X \leftarrow Z \leftarrow Y$ chain