Task 7: HMM

A 1:

$$\alpha_t(x) = P(e_{1:t}, X_t = x)$$

$$P(X_t = x \mid e_{1:t}) = \frac{\alpha_t(x)}{\sum_{x'} \alpha_t(x')}$$

t = 1

$$\alpha_1(x) = P(X_1 = x) \cdot P(e_1 \mid X_1 = x)$$

R: $\underbrace{P(X_1 = R)}_{0,5} \cdot \underbrace{P(U \mid R)}_{0,9} = 0,45$

S: $\underbrace{P(X = S)}_{0,5} \cdot \underbrace{P(U \mid S)}_{0,2} = 0,1$

Normalisieren

$$R: P(R \mid e_1) = \frac{\alpha_1(R)}{\alpha_1(R) + \alpha_1(S)} = \frac{0,45}{0,55} = 0,82$$

($\alpha_1(R) = 0,45$, $\alpha_1(S) = 0,1$)

$$S: P(S \mid e_1) = \frac{0,1}{0,55} = 0,18$$

$t = 2$

$$\alpha_2(R) = P(U|R) \cdot [P(R|R) \cdot \alpha_1(R) + P(R|S) \alpha_1(S)]$$

(0,9; 0,7; 0,45; 0,2; 0,10)

$$= 0,3015$$

$$\alpha_2(S) = P(U|S) \cdot [P(S|R) \cdot \alpha_1(R) + P(S|S) \alpha_1(S)]$$

(0,2; 0,3; 0,45; 0,8; 0,1)

$$= 0,043$$

Normalisieren

$$P(R|e_{1:2}) = \frac{0,3015}{0,3445} = 0,87$$

$$P(S|e_{1:2}) = \frac{0,043}{0,3445} = 0,13$$

$t = 3$

$$\alpha_3(R) = P(N|R) \cdot [P(R|R) \cdot \alpha_2(R) + P(R|S) \cdot \alpha_2(S)]$$

(0,1; 0,7; 0,3015; 0,2; 0,043)

$$= 0,022$$

$$\alpha_3(S) = P(N|S) \cdot [P(S|R) \cdot \alpha_2(R) + P(S|S) \cdot \alpha_2(S)]$$

(0,8; 0,3; 0,3015; 0,8; 0,043)

$$= 0,010$$

Normalisatie

$$P(R \mid e_{1:3}) = \frac{0,022}{0,12} = 0,18$$

$$P(S \mid e_{1:3}) = \frac{0,099}{0,12} = 0,82$$

t = 4

$$\alpha_4(R) = P(U_4 \mid R) \cdot [P(R \mid R) \cdot \alpha_3(R) + P(R \mid S) \cdot \alpha_3(S)]$$

($0,9$ / $0,7$ / $0,022$ / $0,2$ / $0,099$)

$$= 0,032$$

$$\alpha_4(S) = P(U_4 \mid S) \cdot [P(S \mid R) \cdot \alpha_3(R) + P(S \mid S) \cdot \alpha_3(S)]$$

($0,1$ / $0,3$ / $0,022$ / $0,8$ / $0,099$)

$$= 0,017$$

Normaliseren

$$P(R \mid e_{1:4}) = \frac{0,032}{0,049} = 0,65$$

$$R(S \mid e_{1:4}) = \frac{0,017}{0,049} = 0,35$$

t	Observatie (et)	αt I(R)	αt (S)	P(Xt =R|e1:t)	P(Xt =S|e1:t)
1	Paraplu (U)	0,450	0,100	0,818	0,182
2	Paraplu (U)	0,302	0,043	0,875	0,125
3	Geen (N)	0,022	0,100	0,180	0,820
4	Paraplu (U)	0,032	0,017	0,649	0,351