Simulation checks

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 1 ULB

Simulation checks

Supplemental Material 2

In order to insure the reliability of our calculation method, for all scenarios where 6 $G_1 = G_2 = 0$, we compared empirical means and variances of all estimators (i.e. means and variances of all estimates) with theoretical means and variances (i.e. expected means and variances, computed based on equations in Tables 1, 2 and 3). Because we can draw exactly the same conclusions for **biased** (Cohen's d_s , Glass's d_s using either sd_1 or sd_2 as 10 standardizer, Shieh's d_s and Cohen's d'_s) and **unbiased** (Hedges' g_s , Glass's g_s using either 11 sd_1 or sd_2 as standardizer, Shieh's g_s and Hedges' g'_s) estimators, we will simultaneously 12 present results for both categories of estimators. Results will be subdivided into 4 conditions: 13 - When population variances and sample sizes are equal across groups (condition a; see 14 Figures A2.1 and A2.5 for respectively biased and unbiased estimators); 15 - When population variances are equal across groups and sample sizes are unequal 16 (condition b; see Figures A2.2 and A2.6 for respectively biased and unbiased estimators); 17 - When population variances are unequal across groups and sample sizes are equal 18 (condition c; see Figures A2.3 and A2.7 for respectively biased and unbiased estimators); 19 - When population variances and sample sizes are unequal across groups (condition d; 20 see Figures A2.4 and A2.8 for respectively biased and unbiased estimators). 21 Because the equations of theoretical means and variances of Cohen's d_s and Hedges' g_s 22

Because the equations of theoretical means and variances of Cohen's d_s and Hedges' g_s rely on the assumption of normality and equality of population variances, we expect empirical and theoretical parameters to be very close only in conditions a and b. For all other estimators, the equations of theoretical means and variances rely solely on the assumption of normality and therefore, we expect empirical and theoretical parameters to be very close in all conditions.

On average, empirical means (and variances) of all estimators are very close to theoretical expectations when population variances are equal across groups, with equal SIMULATION CHECKS

sample sizes (condition a; see Tables A2.1 and A2.5) or unequal sample sizes (condition b; see Tables A2.2 and A2.6).

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When population variances are unequal across groups (conditions c and d; see Tables 32 A2.3, A2.4, A2.7 and A2.8), empirical means (and variances) of Cohen's d'_s (Cohen's g'_s) and 33 Shieh's d_s (Shieh's g_s) are still very close to theoretical expectations. Regarding Glass's d_s (Glass's g_s), on average, while empirical variances remain very close to theoretical 35 expectations, one observes larger departures between empirical and theoretical means when 36 using SD_2 as standardizer. However, when looking at details in results for each scenarios 37 (see "biased estimator_condition C.xlsx", "biased estimator_condition D.xlsx", "unbiased 38 estimator condition C.xlsx" and "unbiased estimator condition C.xlsx" in Supplemental 39 Material 2), one notices that the larger the population effect size, the larger the departure 40 between empirical and theoretical means, and that relative to the population effect size, 41 departures between empirical and theoretical means are always very small. On the other 42 side, both empirical bias and variance of Cohen's d_s (Hedges' g_s) highly depart from 43 theoretical expectations, even when looking at relative departures to the population effect size, especially when sample sizes are unequal across groups (condition d; see Table A2.4 and A2.8), which is not surprising, as Cohen's d_s (Hedges' g_s) relies on the equality of population variances assumption.

Table A2.1

Ratio between empirical and theoretical variances

Absolute deviation between empirical and theoretical means

Shieh's ds	Cohen's d's	Glass's d _{s,2}	Glass's d _{s,1}	Cohen's d _s	Estimator $(\widehat{\delta})$	
0,006	0,012	0,023	0,022	0,012	Max	
0,000	0,000	0,000	0,000	0,000	Min	
0,001	0,002	0,005	0,004	0,002	Mean	$ E(\widehat{\delta}) - \mu_{\widehat{\delta}} $
0,002	0,003	0,007	0,006	0,003	Standard deviation	
1,006	1,006	1,005	1,006	1,006	Max	
0,910	0,910	0,889	0,897	0,910	Min	
0,976	0,976	0,966	0,966	0,976	Mean	$S^2 \hat{\delta}/\sigma_{\delta}$
0,028	0,028	0,035	0,033	0,028	Standard deviation	

Table A2.2

Ratio between empirical and theoretical variances		Standard deviation	0,017	0,037	0,036	0,034	0,048
npirical and th	$S^2_{\hat{\delta}}/\sigma_{\delta}$	Mean	986'0	996'0	896'0	996'0	0,945
io between er		Min	0,951	0,891	0,881	0,902	0,865
Rat		Max	1,017	1,006	1,015	1,007	1,005
Absolute deviation between empirical and theoretical means		Standard deviation	0,001	900'0	0,007	0,002	0,002
ween empirica	$ E(\widehat{\delta})$ - $\mu_{\delta} $	Mean	0,001	0,004	0,005	0,003	0,002
deviation bet		Min	00000	0,000	0,000	00000	0,000
Absolute		Max	900'0	0,019	0,027	0,010	0,008
		Estimator $(\widehat{\delta})$	Cohen's ds	Glass's d _{s,1}	Glass's d _{s,2}	Cohen's d's	Shieh's ds

Table A2.3

Absolute deviation between empirical and theoretical means

Ratio between empirical and theoretical variances

Shieh's ds	Cohen's d's	Glass's d _{s,2}	Glass's d _{s,1}	Cohen's ds	Estimator $(\widehat{\delta})$	
0,018	0,036	0,230	0,037	0,080	Max	
0,000	0,000	0,000	0,000	0,000	Min	
0,002	0,003	0,012	0,005	0,010	Mean	$ E(\widehat{\delta})$ - $\mu_{\delta} $
0,003	0,006	0,033	0,007	0,015	Standard deviation	
1,007	1,007	1,008	1,004	1,753	Max	
0,874	0,874	0,883	0,888	1,005	Min	
0,975	0,975	0,974	0,973	1,175	Mean	$S^2 \hat{g}/\sigma_{\delta}$
0,033	0,033	0,032	0,030	0,208	Standard deviation	

Table A2.4

	Absolute	deviation betw	reen empirical $ E(\widehat{\delta}) - \mu_{\delta} $	Absolute deviation between empirical and theoretical means $ E(\widehat{\delta}) - \mu_{\delta} $	Rativ	o between emp	virical and theo $S^{2}_{\widehat{\delta}}/\sigma_{\delta}$	Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\hat{\delta}}$
Estimator (δ)	Max	Min	Mean	Standard deviation	Мах	Min	Mean	Standard deviation
Cohen's ds	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's d _{s,1}	0,026	0,000	0,005	900'0	1,009	0,881	276'0	0,033
Glass's d _{s,2}	0,219	0,000	0,012	0,031	1,011	0,872	0,973	980'0
Cohen's d's	0,030	00000	0,003	900'0	1,011	098'0	0,974	0,034
Shieh's ds	600'0	0,000	0,001	0,002	1,011	0,867	0,970	980'0

Table A2.5

Absolute deviation between empirical and theoretical means

Ratio between empirical and theoretical variances

Estimator ($\hat{\delta}$) Max Min Mean Standard deviation Max Min Mean Standard deviation Cohen's dis 0,011 0,000 0,002 0,003 1,006 0,911 0,976 Standard deviation Glass's di, 2 0,021 0,000 0,004 0,005 1,006 0,897 0,966 0,033 Glass's di, 2 0,022 0,000 0,004 0,007 1,005 0,897 0,966 0,033 Cohen's d's 0,015 0,003 0,004 0,004 0,004 0,005 0,989 0,966 0,035 Cohen's d's 0,015 0,000 0,003 0,004 1,006 0,989 0,966 0,035 Shieh's di 0,008 0,000 0,002							
Min Mean Standard deviation Max Min Mean 0,000 0,002 0,003 1,006 0,911 0,976 0,000 0,004 0,007 1,005 0,897 0,966 0,000 0,003 0,004 1,006 0,989 0,966 0,000 0,003 0,004 1,006 0,908 0,975 0,000 0,003 0,002 1,006 0,908 0,975	Shieh's ds	Cohen's d's	Glass's d _{s,2}	Glass's d _{s,1}	Cohen's d _s	Estimator $(\widehat{\delta})$	
$ E(\tilde{\delta}) + \mu_{\delta} $ $S^2_{\delta}/\sigma_{\delta}$ Mean Standard deviation Max Min Mean 0,002 0,003 1,006 0,911 0,976 0,004 0,007 1,006 0,897 0,966 0,003 0,004 1,005 0,889 0,966 0,003 0,004 1,006 0,908 0,975 0,002 0,002 1,006 0,908 0,975	0,008	0,015	0,022	0,021	0,011	Max	
Standard deviation Max Min Mean $S^2_{\tilde{e}}/\sigma_{\tilde{e}}$ Standard deviation Max Min Mean $0,003$ $1,006$ $0,911$ $0,976$ $0,006$ $0,897$ $0,966$ $0,007$ $1,005$ $0,889$ $0,966$ $0,004$ $1,006$ $0,908$ $0,975$	0,000	0,000	0,000	0,000	0,000	Min	
$S^2 \hat{\epsilon}/\sigma_{\hat{\epsilon}}$ Max Min Mean 1,006 0,911 0,976 1,005 0,889 0,966 1,006 0,908 0,975	0,002	0,003	0,004	0,004	0,002	Mean	$ E(\widehat{\delta})$ - $\mu_{\delta} $
$S^2 \hat{s}/\sigma_{\hat{s}}$ Min Mean 0,911 0,976 0,897 0,966 0,889 0,966 0,908 0,975	0,002	0,004	0,007	0,006	0,003	Standard deviation	
$S^2 \hat{_{\mathcal{S}}}/\sigma_{\mathcal{S}}$ Mean 0,976 0,966 0,966 0,975	1,006	1,006	1,005	1,006	1,006	Max	
	0,908	0,908	0,889	0,897	0,911	Min	
Standard deviation 0,028 0,033 0,035 0,029 0,029	0,975	0,975	0,966	0,966	0,976	Mean	$S^2\hat{\delta}/\sigma_{\delta}$
	0,029	0,029	0,035	0,033	0,028	Standard deviation	

Table A2.6

Absolute deviation between empirical and theoretical means Ratio between empirical and theoretical variances $ {\sf E}(\hat{\delta}) - \mu_{\delta} $	Min Mean Standard deviation Max Min Mean Standard deviation	0,000 0,001 0,001 1,017 0,951 0,985 0,017	0,000 0,004 0,005 1,006 0,891 0,966 0,037	0,000 0,004 0,006 1,015 0,881 0,968 0,036	0,000 0,003 0,003 1,007 0,925 0,972 0,027	
on between empirical and t $ {\sf E}(\widehat{\delta}) \cdot \mu_{\delta} $	Mean					
Absolute deviatic	Max Mi	0,005 0,00	0,018 0,00	0,026 0,00	0,010 0,00	1000
	Estimator $(\widehat{\delta})$	Cohen's ds	Glass's d _{s,1}	Glass's d _{5,2}	Cohen's d's	Shieh's de

Table A2.7

Shieh's ds	Cohen's d's	Glass's d _{s,2}	Glass's d _{s,1}	Cohen's d _s	Estimator $(\widehat{\delta})$	
0,017	0,034	0,221	0,036	0,079	Max	Absolute o
0,000	0,000	0,000	0,000	0,000	Min	deviation betv
0,002	0,003	0,012	0,005	0,010	Mean	veen empirica $ E(\widehat{\delta}) \cdot \mu_{\widehat{\delta}} $
0,003	0,006	0,032	0,007	0,015	Standard deviation	Absolute deviation between empirical and theoretical means $ E[\widehat{\delta}] \cdot \mu_{\delta} $
1,008	1,008	1,008	1,004	1,753	Max	Rati
0,890	0,890	0,883	0,888	1,005	Min	o between er
0,978	0,978	0,974	0,973	1,175	Mean	npirical and th $S^2 {\hat g}/\sigma_{\delta}$
0,029	0,029	0,032	0,030	0,208	Standard deviation	Ratio between empirical and theoretical variances $S^2\hat{_\delta}/\sigma_\delta$

Table A2.8

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Ratio between empirical and theoretical variances $S^2{}_{\hat{\delta}}/\sigma_{\hat{\delta}}$	Standard deviation	1,357	0,033	0,036	0,029	0,032
mpirical and th $S^2_{\hat{\delta}}/\sigma_{\delta}$	Mean	1,638	0,972	0,973	716'0	0,973
tio between eı	Min	0,208	0,881	0,872	0,882	0,881
Rat	Мах	5,624	1,009	1,011	1,011	1,011
Absolute deviation between empirical and theoretical means $ E[\hat{\delta}]\cdot\mu_{\delta} $	Standard deviation	0,034	900'0	0,030	0,005	0,002
tween empiric $ E(\widehat{\delta})$ - $\mu_{\delta} $	Mean	0,015	0,004	0,012	0,003	0,001
e deviation be	Min	000'0	0,000	0,000	0,000	000'0
Absolute	Мах	0,250	0,025	0,210	0,029	0,008
	Estimator $(\widehat{\delta})$	Cohen's ds	Glass's d _{s,1}	Glass's d _{s,2}	Cohen's d's	Shieh's ds