Simulation checks

Simulation checks

Supplemental Material 2

In order to insure the reliability of our calculation method, for all scenarios where $G_1 = G_2 = 0$, we compared empirical means and variances of all estimators (i.e. means and variances of all estimates) with theoretical means and variances (i.e. expected means and variances, computed based on equations in Tables 1, 2 and 3 in the main article). Because we can draw exactly the same conclusions for **biased** (Cohen's d, Glass' d using either S_1 or S_2 as standardizer, Shieh's d and Cohen's d^*) and **unbiased** (Hedges' g, Glass' g using either S_1 or S_2 as standardizer, Shieh's g and Hedges' g^*) estimators, we will simultaneously present results for both categories of estimators. Results will be subdivided into 4 conditions:

- When population variances and sample sizes are equal across groups (condition a; see Figures A2.1 and A2.5 for respectively biased and unbiased estimators);

- When population variances are equal across groups and sample sizes are unequal (condition b; see Figures A2.2 and A2.6 for respectively biased and unbiased estimators);

- When population variances are unequal across groups and sample sizes are equal (condition c; see Figures A2.3 and A2.7 for respectively biased and unbiased estimators);

- When population variances and sample sizes are unequal across groups (condition d; see Figures A2.4 and A2.8 for respectively biased and unbiased estimators).

Because the equations of theoretical means and variances of Cohen's d and Hedges' g rely on the assumption of normality and equality of population variances, we expect empirical and theoretical parameters to be very close only in conditions a and b. For all other estimators, the equations of theoretical means and variances rely solely on the assumption of normality and therefore, we expect empirical and theoretical parameters to be very close in all conditions.

On average, empirical means (and variances) of all estimators are very close to theoretical expectations when population variances are equal across groups, with equal sample sizes (condition a; see Tables A2.1 and A2.5) or unequal sample sizes (condition b; see Tables A2.2 and A2.6).

When population variances are unequal across groups (conditions c and d; see Tables A2.3, A2.4, A2.7 and A2.8), empirical means (and variances) of Cohen's d^* (Hedges' g^*) and Shieh's d (Shieh's g) are still very close to theoretical expectations. Regarding Glass' d (Glass' g), on average, while empirical variances remain very close to theoretical expectations, one observes larger departures between empirical and theoretical means when using S_2 as standardizer. However, when looking at details in results for each scenario (see "biased estimator_condition C.xlsx," "biased estimator_condition D.xlsx," "unbiased estimator_condition C.xlsx" and "unbiased estimator_condition C.xlsx" in Supplemental Material 2), one notices that the larger the population effect size, the larger the departure between empirical and theoretical means, and that relative to the population effect size, departures between empirical and theoretical means are always very small. On the other hand, both empirical bias and variance of Cohen's d (Hedges' g) highly depart from theoretical expectations, even when looking at relative departures to the population effect size, especially when sample sizes are unequal across groups (condition d; see Table A2.4 and A2.8), which is not surprising, as Cohen's d (Hedges' g) relies on the equality of population variances assumption.

Table A2.1.

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are equal across groups (condition a).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Glass' d_1	0,022	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass' d_2	0,023	0,000	0,005	0,007	1,005	0,889	0,966	0,035
Cohen's d^*	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Shieh's d	0,006	0,000	0,001	0,002	1,006	0,910	0,976	0,028

Table A2.2

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta} \rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's d	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass' d_1	0,019	0,000	0,004	0,006	1,006	0,891	0,966	0,037
Glass' d_2	0,027	0,000	0,005	0,007	1,015	0,881	0,968	0,036
Cohen's d^*	0,010	0,000	0,003	0,002	1,007	0,902	0,965	0,034
Shieh's d	0,008	0,000	0,002	0,002	1,005	0,865	0,945	0,048

Table A2.3

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_\delta \rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's *d*	0,080	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass' d_1	0,037	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass' d_2	0,230	0,000	0,012	0,033	1,008	0,883	0,974	0,032
Cohen's d^*	0,036	0,000	0,003	0,006	1,007	0,874	0,975	0,033
Shieh's *d*	0,018	0,000	0,002	0,003	1,007	0,874	0,975	0,033

Table A2.4

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are unequal across groups (condition d).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's *d*	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass'd_1	0,026	0,000	0,005	0,006	1,009	0,881	0,972	0,033
Glass'd_2	0,219	0,000	0,012	0,031	1,011	0,872	0,973	0,036
Cohen's *d**	0,030	0,000	0,003	0,006	1,011	0,860	0,974	0,034
Shieh's *d*	0,009	0,000	0,001	0,002	1,011	0,867	0,970	0,036

Table A2.5

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are equal across groups (condition a).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's g	0,011	0,000	0,002	0,003	1,006	0,911	0,976	0,028
Glass' g_1	0,021	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass' g_2	0,022	0,000	0,004	0,007	1,005	0,889	0,966	0,035
Cohen's g^*	0,015	0,000	0,003	0,004	1,006	0,908	0,975	0,029
Shieh's g	0,008	0,000	0,002	0,002	1,006	0,908	0,975	0,029

Table A2.6

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_\delta \rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's g	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass' g_1	0,018	0,000	0,004	0,005	1,006	0,891	0,966	0,037
Glass' g_2	0,026	0,000	0,004	0,006	1,015	0,881	0,968	0,036
Cohen's g^*	0,010	0,000	0,003	0,003	1,007	0,925	0,972	0,027
Shieh's g	0,007	0,000	0,002	0,002	1,007	0,900	0,959	0,037

Table A2.7

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's g	0,079	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass' g_1	0,036	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass' g_2	0,221	0,000	0,012	0,032	1,008	0,883	0,974	0,032
Cohen's g^*	0,034	0,000	0,003	0,006	1,008	0,890	0,978	0,029
Shieh's g	0,017	0,000	0,002	0,003	1,008	0,890	0,978	0,029

Table A2.8

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are unequal across groups (condition d).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta} \rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's g	0,250	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass' g_1	0,025	0,000	0,004	0,006	1,009	0,881	0,972	0,033
Glass' g_2	0,210	0,000	0,012	0,030	1,011	0,872	0,973	0,036
Cohen's g^*	0,029	0,000	0,003	0,005	1,011	0,882	0,977	0,029
Shieh's g	0,008	0,000	0,001	0,002	1,011	0,881	0,973	0,032