Theoretical variance, as a function of population parameters

Theoretical variance, as a function of population parameters

The variance

Note: while we focus on the theoretical variance of biased estimators (Cohen's d, Glass's d, Shieh's d and Cohen's d^*) when the normality assumption is met, it is interesting to notice that our main conclusions seem to generalize to biased estimators when samples are extracted from symmetric distributions. Moreover, unbiased estimators depend on the same factors as biased estimators, so our conclusions remain similar for unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

Cohen's *d*

When variances are equal across populations.

When $\delta_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d can be simplified as follows:

$$Var_{Cohen's\ d} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

The **variance** of Cohen's d is a function of total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The larger the total sample size, the lower the variance. The variance tends to zero when the total sample size tends to infinity (see Figure 1);

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure 2).

When $\delta_{Cohen} \neq 0$.

While the variance of Cohen's d still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size (δ_{Cohen}). The larger the population effect size, the larger the variance. Note that the effect of the population effect size decreases when sample sizes increase since

$$\lim_{n_1 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

This is illustrated in Figure 3.

In summary. The variance of Cohen's d is a function of the population effect size (δ_{Cohen}), the total sample size (N) and the sample sizes ratio ($\frac{n_2}{n_1}$):

- The variance decreases when the total sample size increases;
- The variance also decreases when the sample sizes ratio gets closer to 1;
- Finally, the variance increases when δ_{Cohen} increases. Note that the effect of δ_{Cohen} is moderated by the total sample size (the larger N, the smaller the effect of δ_{Cohen} on the variance).

Glass's *d*

When variances are equal across populations.

When $\boldsymbol{\delta_{Glass} = 0}$.

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{1}{n_e}\right)$$

In this configuration, the **variance** of Glass's d is a function of the sample sizes of both control (n_c) and experimental (n_e) groups as well as of the sample sizes allocation ratio $\left(\frac{n_c}{n_e}\right)$:

- The larger the sample sizes, the lower the variance (Figure 4);

The sample sizes ratio associated with the lowest variance is not exactly 1 (because of the term $\frac{df}{df-2}$, df depending only on n_c), but is very close to 1 (and the larger the total sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance). The further from this sample size ratio, the larger the variance (see Figure 5).

When $\boldsymbol{\delta_{Glass} \neq 0}$.

While the variance of Glass's d still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases. On the other hand, the effect of the population effect size does *not* depend on the size of the experimental group since

$$\lim_{n_c \to \infty}\left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right] = 0$$

$$\lim_{n_e \to \infty}\left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right] \neq 0$$

These limits are illustrated in Figure 6.

Note: while the sample sizes ratio associated with the lowest variance was very close to 1 with a null population effect size, this is not true anymore when the population effect size is not zero. Indeed, because of the second term in the addition, when computing the variance, one gives much more weight to the effect size of the control group (see Figure 7), especially when the effect size gets larger. For example, when $\delta_{Glass}= 4$, the lowest variance will occur when n_c is approximately 3 times larger than n_e. When $\delta_{Glass}= 7$, the lowest variance will occur when n_c is approximately 5 times larger than n_e, etc.

When variances are unequal across populations, with equal sample sizes.

When $\delta_{Glass} = 0$.

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n-1}{n(n-3)}\left(1+\frac{\sigma_e^2}{\sigma_c^2}\right)$$

where $n = N/2$ = sample size of each group. The variance is therefore a function of the total sample size and the SD-ratio ($\frac{\sigma_c}{\sigma_e}$):

- The larger the total sample size, the lower the variance (See Figure 8);

- The larger the SD-ratio (i.e. the larger is σ_c in comparison with σ_e), the lower the variance (see Figure 9). However, the effect of the SD-ratio decreases when sample sizes increase, because $\lim_{n(=n_c=n_e)\to\infty}\left[\frac{df}{n(df-2)}\right] = 0$.

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases, as previously explained and illustrated in Figure 6.

When variances are unequal across populations, with unequal sample sizes.

When $\delta_{Glass} = 0$.

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e\sigma_c^2}\right)$$

The variance of Glass's d is therefore a function of the total sample size (N), the SD-ratio and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

- For any SD and sample sizes pairing, increasing n_c and/or n_e will decrease the variance (see Figure 10);
  - The effect of the sample sizes ratio depends on the SD-ratio:
    * We previously mentioned that when $\sigma_c = \sigma_e$, the variance is minimized when sample sizes of both groups are almost identical (see Figure 5), meaning that it is more efficient, in order to reduce variance, to add subjects uniformly in both groups;
    * When $\sigma_e > \sigma_c$, more weight is given to n_e, meaning that it is more efficient, in order to reduce variance, to add subjects in the experimental group (n_e; see bottom plots in Figure 10);
    * When $\sigma_c > \sigma_e$, less weight is given to n_e, meaning that it is more efficient, in order to reduce variance, to add sujects in the control group (n_c; see top plots in Figure 10).

- Finally, there is also a main effect of the SD-ratio: the larger is σ_c in comparison with σ_e, the lower the variance, as we can observe in Figure 11. We can also notice that in Figure 10, the maximum variance is much larger in the two bottom plots (where $\sigma_c < \sigma_e$) than in the two top plots (where $\sigma_c > \sigma_e$).

Note that the effect of the SD-ratio, and the interaction effect between SD-ratio and sample sizes ratio decreases when the sample size of the control group increases (because $\frac{n_c-1}{n_c-3}$ gets closer to 1).

When $\boldsymbol{\delta_{Glass} \neq 0}$.

While the variance of Glass's d still depends on the total sample size, the SD-ratio and the interaction between the SD-ratio and the sample sizes ratio, it also depends on the population effect size (δ_{Glass}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the sample size of the control group increases, as previously explained and illustrated in Figure 6.

Note: when the population effect size was null, when $\sigma_c < \sigma_e$, it was much more efficient to add subjects in the experimental group in order to reduce the variance (because much more weight was given to n_e). When $\delta_{Glass} \neq 0$, it is important to add subjects in both groups in order to reduce the variance (because $\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2$ is only a function of the sample size of the control group). With huge population effect size, it is even always more important to add subjects in the control group (e.g. when $\delta_{Glass} = 30$).

In summary. The variance of Glass's d is a function of the population effect size (δ_{Glass}), the SD-ratio, the total sample size and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

- The variance decreases when the SD-ratio increases (i.e. when $\sigma_e >> \sigma_c$);
- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control, in the experimental, or in both groups, in order to reduce the variance, varies as a function of the SD-ratio and the population effect size. The only situation where it is optimal to

maximize the experimental group is when $\sigma_e > \sigma_c$ and $\delta_{Glass} \approx 0$. Most of the time, it is more efficient to maximize the control groups (e.g. anytime $\sigma_e < \sigma_c$, and when δ_{Glass} is very large) or to uniformly add subjects in both groups (e.g. when $\sigma_e > \sigma_c$ and δ_{Glass} is neither null nor huge);

- The variance increases when δ_{Glass} increases. Note that the effect of δ_{Glass} is moderated by the control group size (the larger n_e, the smaller the effect of δ_{Glass} on the variance).

Cohen's d^*

When variances are equal across populations.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* is computed as follows :

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{N}{n_1 n_2}$$

with

$$df = \frac{4(n_1-1)(n_2-1)}{n_1+n_2-2}$$

In this configuration, the degrees of freedom as well as the variance of Cohen's d^* depend on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure 12);

- The larger the total sample size, the lower the bias (see Figure 13).

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the sample sizes ratio get closer to 1), as illustrated in Figure 14 since

$$\lim_{df \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

When variances are unequal across populations, with equal sample sizes.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* can be simplified as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{2}{n}$$

where $n = N/2$=sample size of each group, and $df = \frac{(n-1)(\sigma_1^4+\sigma_2^4+2\sigma_1^2\sigma_2^2)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the variance of Cohen's d^* depend on the total sample size (N) and the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$:

- The further the SD-ratio is from 1, the larger the variance (see Figure 15);

- The larger the total sample size, the lower the variance (see Figure 16).

Note: for a constant SD-ratio, the size of the variance does not matter (see Figure 17).

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or the SD-ratio get closer to 1), as previously illustrated in Figure 14.

When variances are unequal across populations, with unequal sample sizes.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* can be simplified as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2}$$

with $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$. In this configuration, the degrees of freedom are a function of the total sample size (N) and the interaction between sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the total sample size, the lower the variance (illustration in Figure 18);

- The smallest variance always occurs when there is a positive pairing between variances and sample sizes, because one gives more weight to the smallest variance in the denominator of the df computation and in the numerator of the variance computation. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure 19). This can be explained by splitting the numerator and the denominator of the df computation (see the file "Theoretical Bias, as a function of population parameters").

Note: for a constant SD-ratio, the variance does not matter. (See Figure 20).

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size, the SD-ratio and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or when there is a positive pairing between the sample sizes ratio and the SD-ratio), as previously illustrated in Figure 14.

In summary. The variance of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the total sample size (N) and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;
- The variance increases when δ^*_{Cohen} increases. Note that the effect of δ^*_{Cohen} is moderated by the total sample size and the interaction between sample sizes ratio and SD-ratio.

Shieh's *d*

When variances are equal across populations.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure 21);

- The larger the total sample size, the lower the variance. It does not matter whether the sample sizes ratio is constant (see Figure 22) or not (see Figure 23).

Note: in "Theoretical Bias, as a function of population parameters," we noticed that moving the sample sizes ratio away from 1 when adding subjects in only one group could decrease the degrees of freedom. However, due to the total sample size term (N) in the denominator of the variance computation, even when degrees of freedom decrease due to the fact that one adds subjects only in one group, the variance still decreases (because the denominator of the variance computation increases; see Figure 23).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase, without increasing the sample sizes ratio, and/or the sample sizes ratio gets closer to 1), as previously illustrated in Figure 14.

Note: we previously noticed that when the effet size is zero, the variance of Shieh's d decreases, even when the sample sizes ratio increases. It is no longer true when there is a non-null effect size because the larger the sample sizes ratio, the more the variance will increase with increasing effect size.

When variances are unequal across populations, with equal sample sizes.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{(\sigma_1^2+\sigma_2^2)^2 \times (n-1)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the SD-ratio ($\frac{\sigma_2}{\sigma_1}$).

- The further the SD-ratio is from 1, the larger the variance (see Figure 24);

- The larger the total sample size, the lower the variance (see Figure 25).

Note: for a constant SD-ratio, the size of the variance does not matter (see Figure 26).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the SD-ratio gets closer to 1), as previously illustrated in Figure 14.

When variances are unequal across populations, with unequal sample sizes.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the total sample size, the lower the variance. It remains true when the sample sizes ratio is constant (see Figure 27) and when it is not (see Figure 28).

Note: When variances were equal across populations, adding subjects only in the first group had the same impact on the variance as adding subjects only in the second group (see Figure 23). When variances are unequal across groups, this is not true anymore (see Figure 28).

- The smallest variance always occurs when there is a positive pairing between variances and sample size. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure 29).

Moreover, for a constant SD-ratio, the variances do not matter (See Figure 30).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase, as previously illustrated in Figure 14.

In summary. The variance of Shieh's d is a function of the population effect size (δ_{Shieh}), the total sample size (N) and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;

- The variance increases when δ_{Shieh} increases. Note that the effect of δ_{Shieh} is moderated by the total sample size and the interaction between the sample sizes ratio and the SD-ratio.

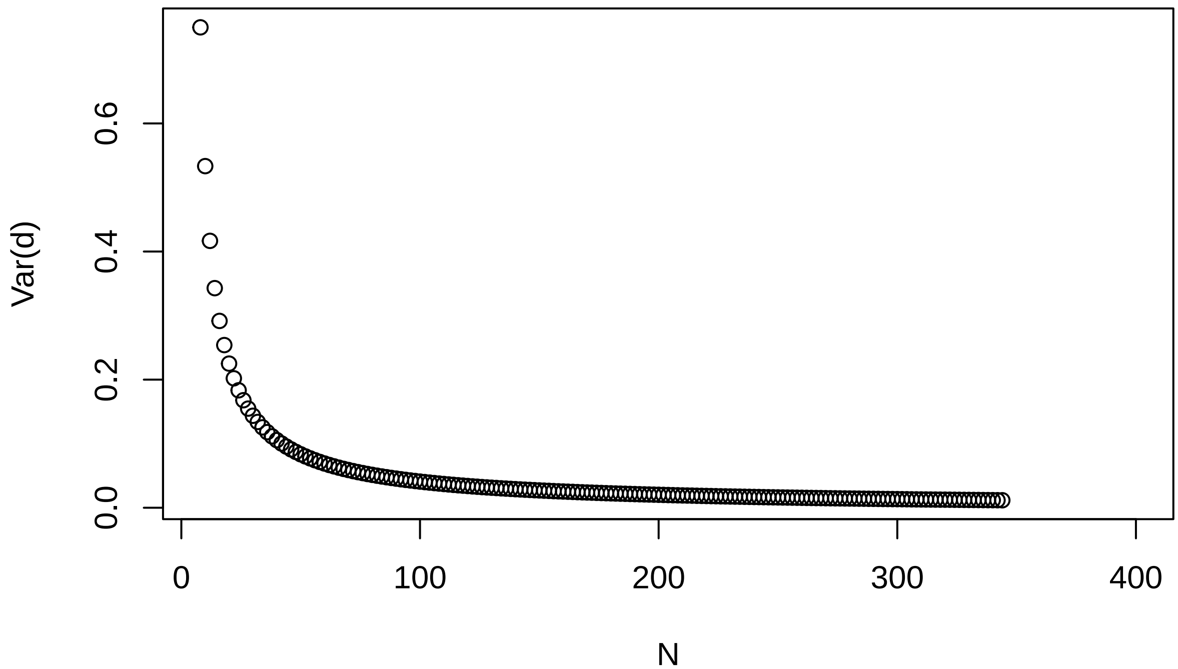

Figure 1. Variance of Cohen's d, when variances are equal across groups, as a function of the total sample size (N).

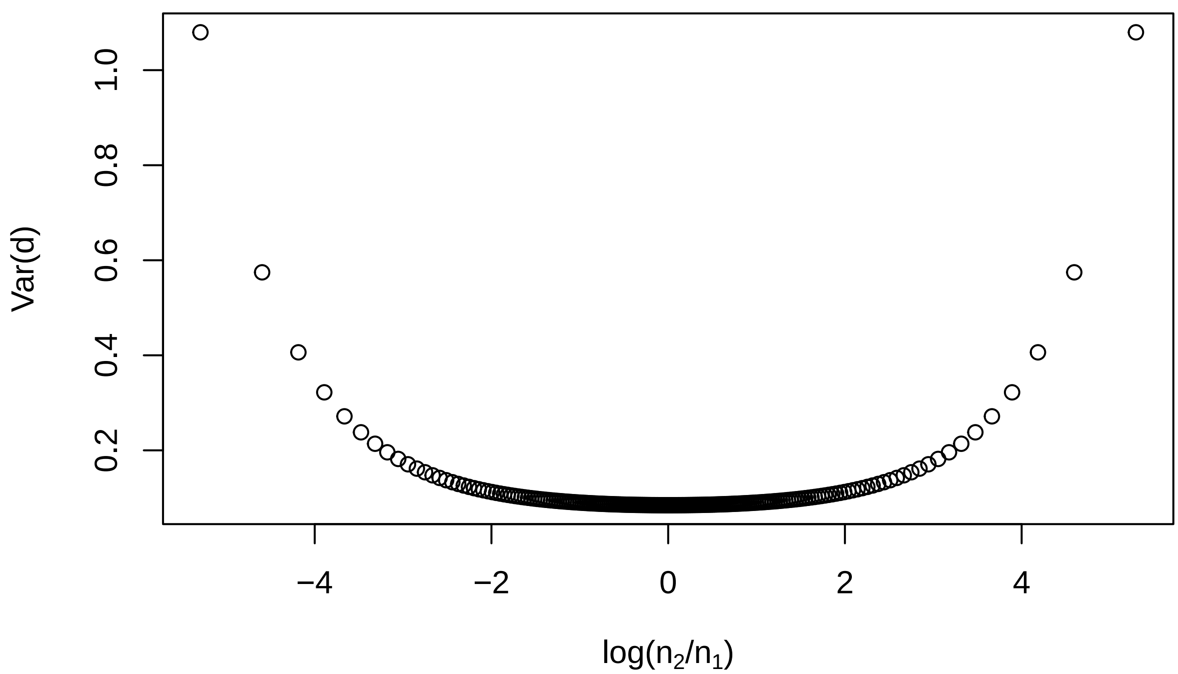

Figure 2. Variance of Cohen's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

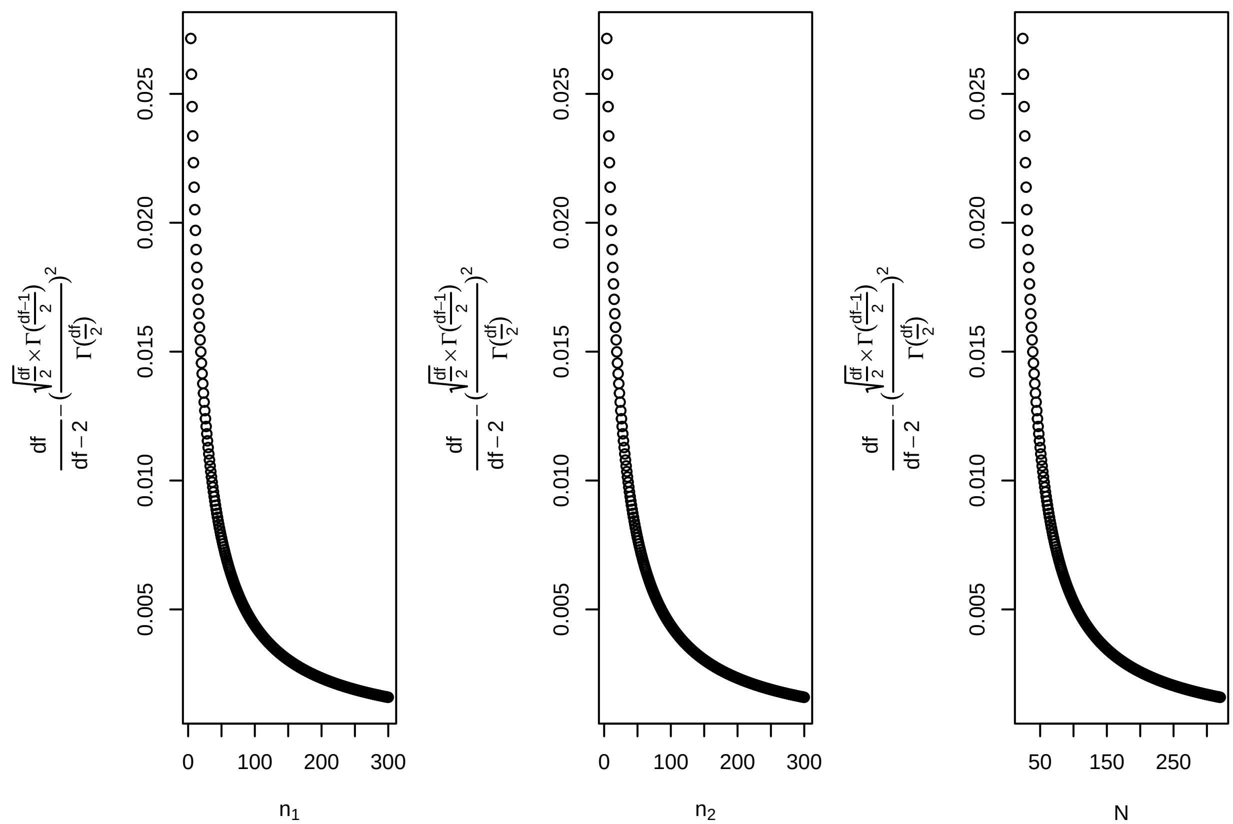

Figure 3. Effect size moderator, when computing the variance of Cohen's d, as a function of n_1 (left), n_2 (center) and $N = n_1 + n_2$ (right).

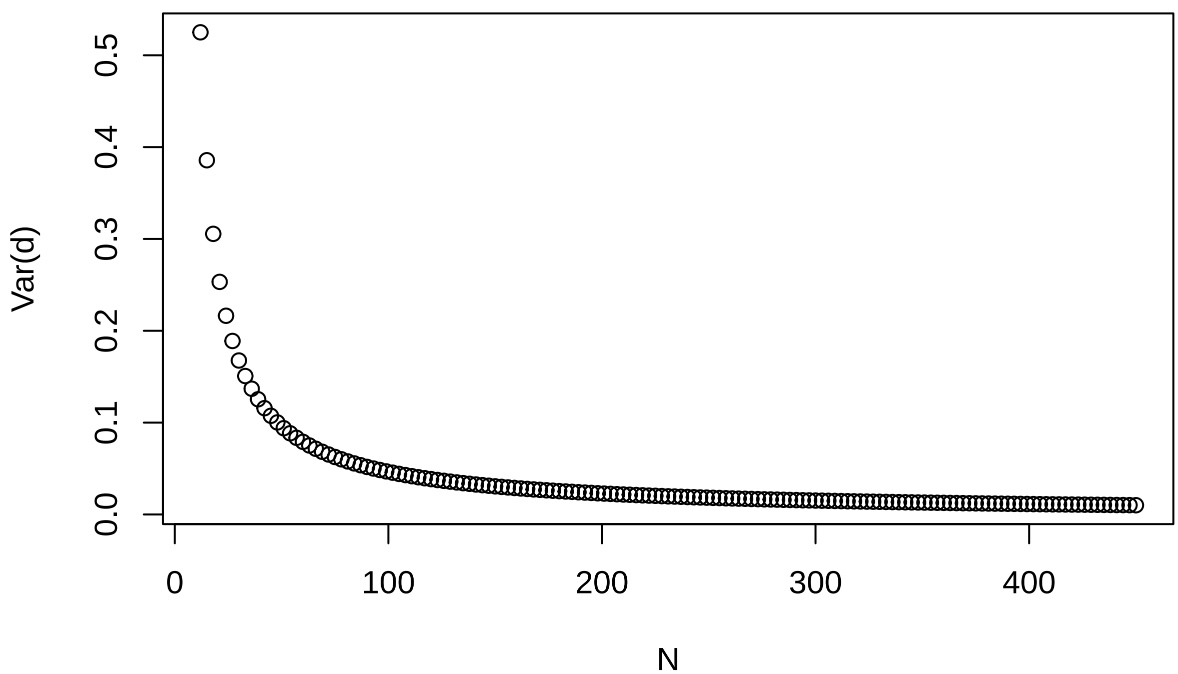

Figure 4. Variance of Glass's d, when variances are equal across groups, as a function of the total sample size (N).

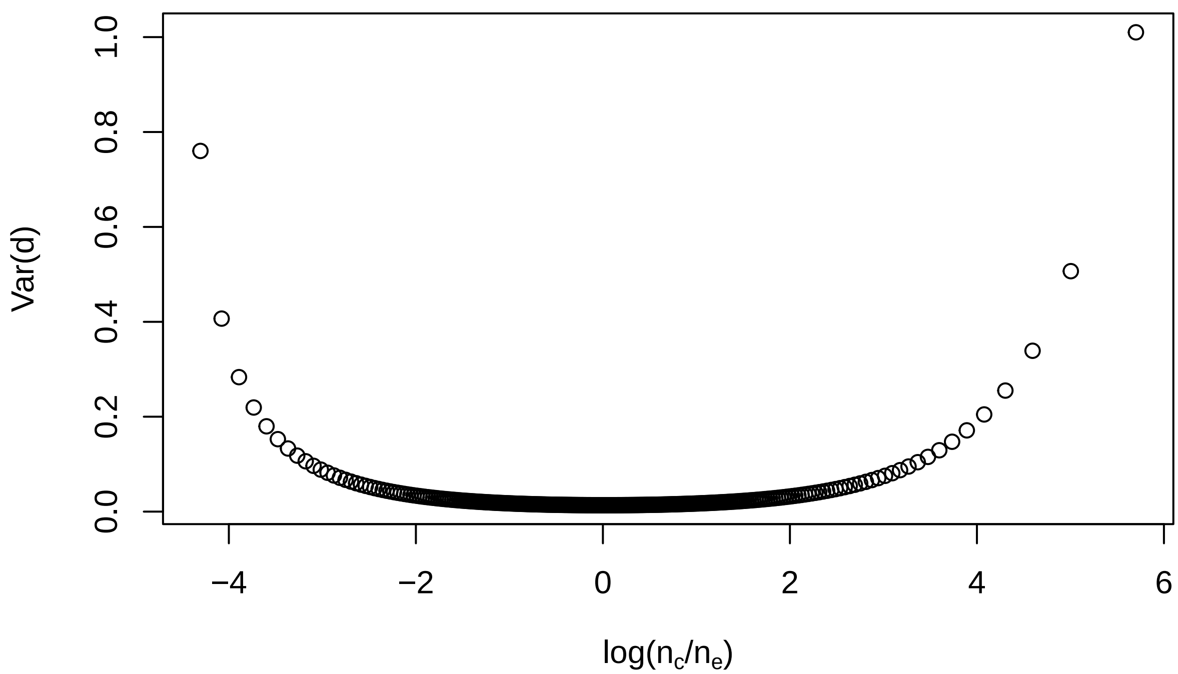

Figure 5. Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_c}{n_e}\right)$).

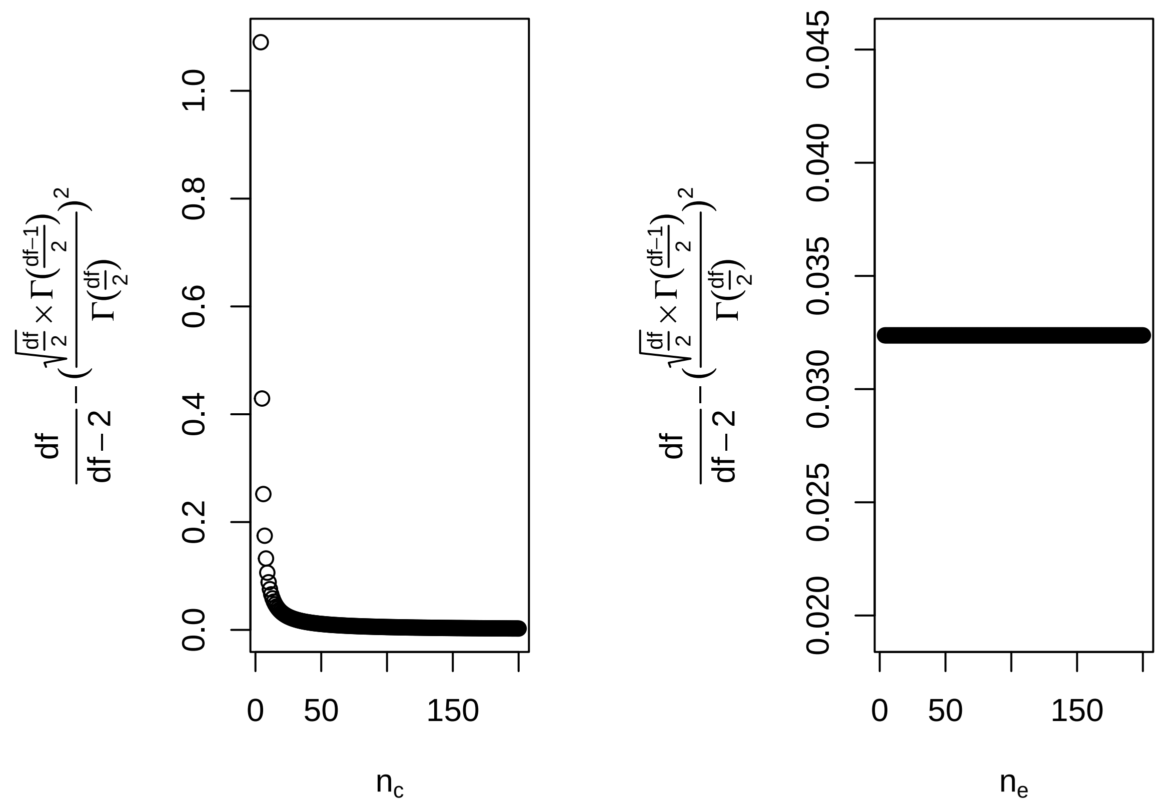

Figure 6. Effect size moderator, when computing the variance of Glass's d, as a function of the size of the control group (left) and experimental group (right).

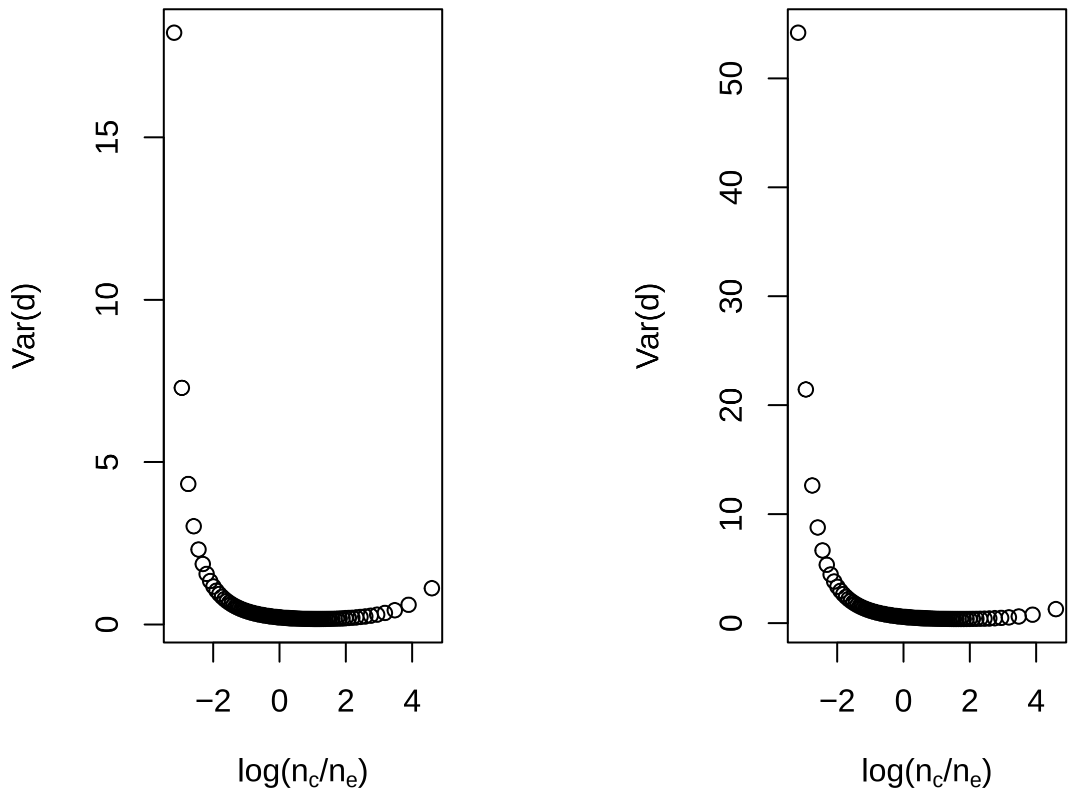

Figure 7. Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_c}{n_e}\right)$) when δ_{Glass} equals 4 (left) or 7 (right).

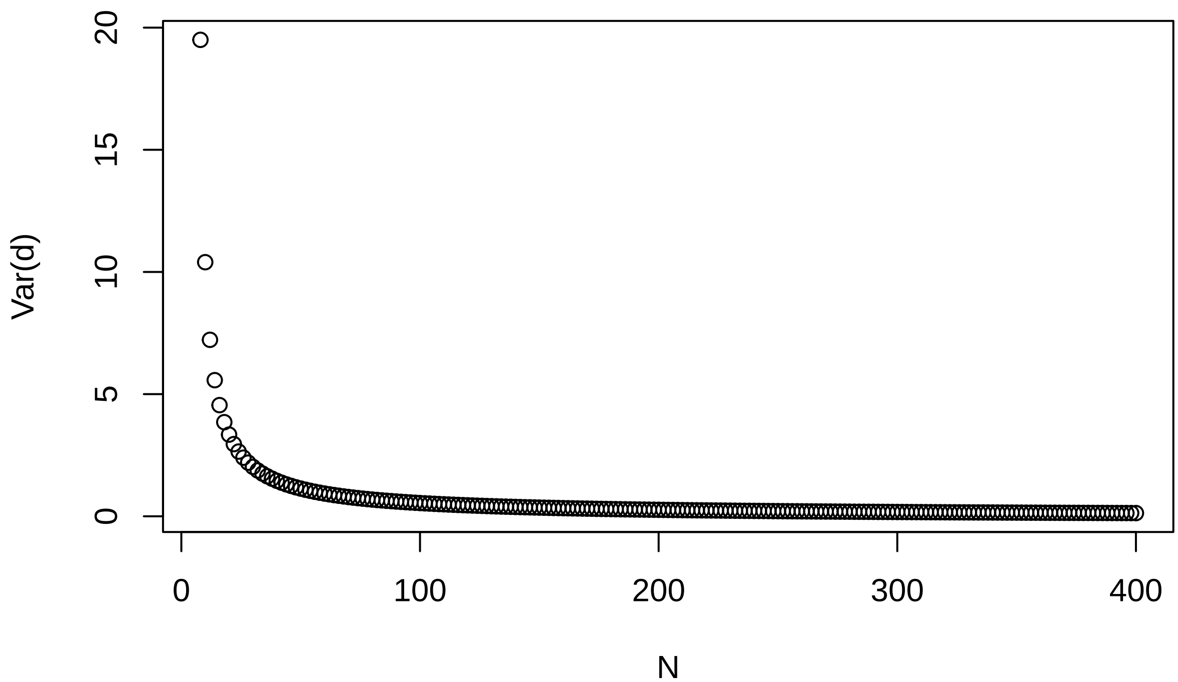

Figure 8. Variance of Glass's d, when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes (N).

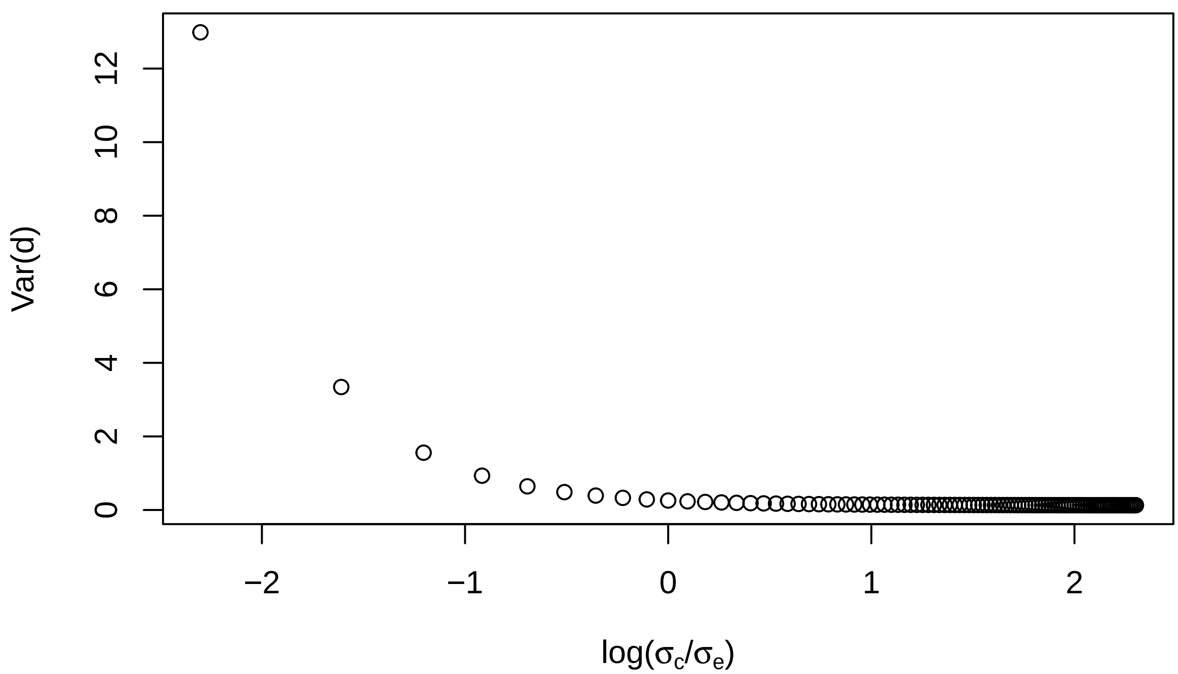

Figure 9. Variance of Glass's d, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_c}{\sigma_e}\right)$).

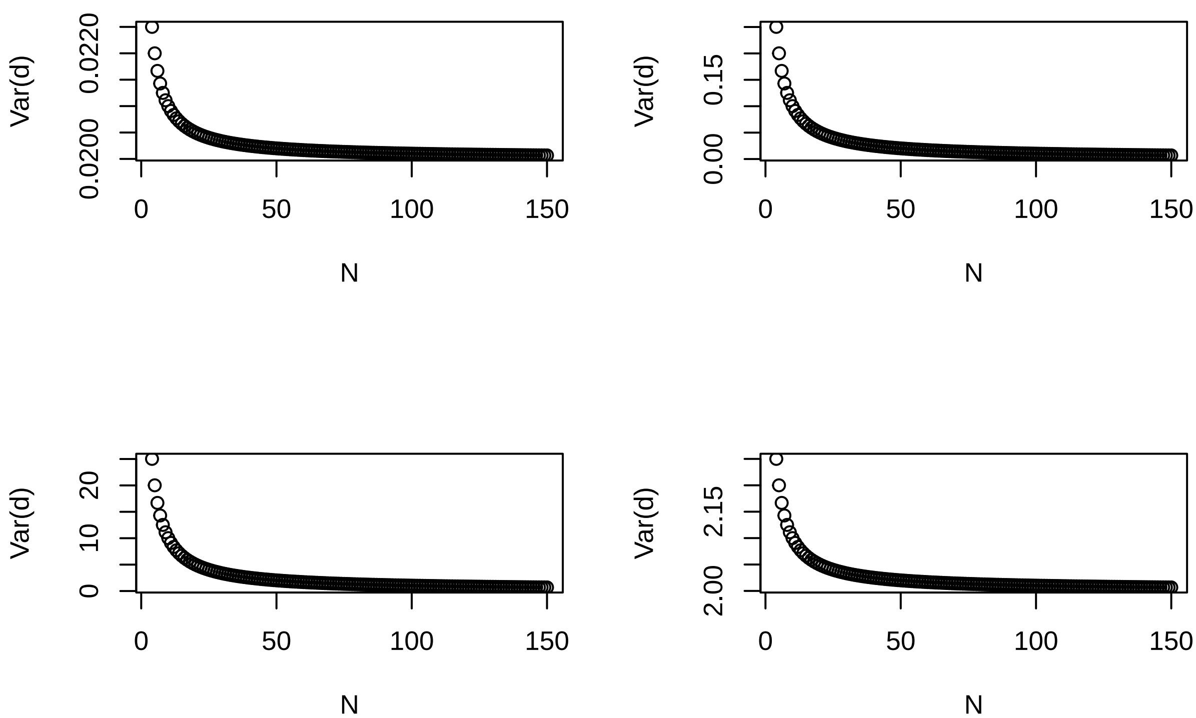

Figure 10. Variance of Glass's d, when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when $\sigma_c > \sigma_e$ (top plots) or $\sigma_c < \sigma_e$ (bottom plots).

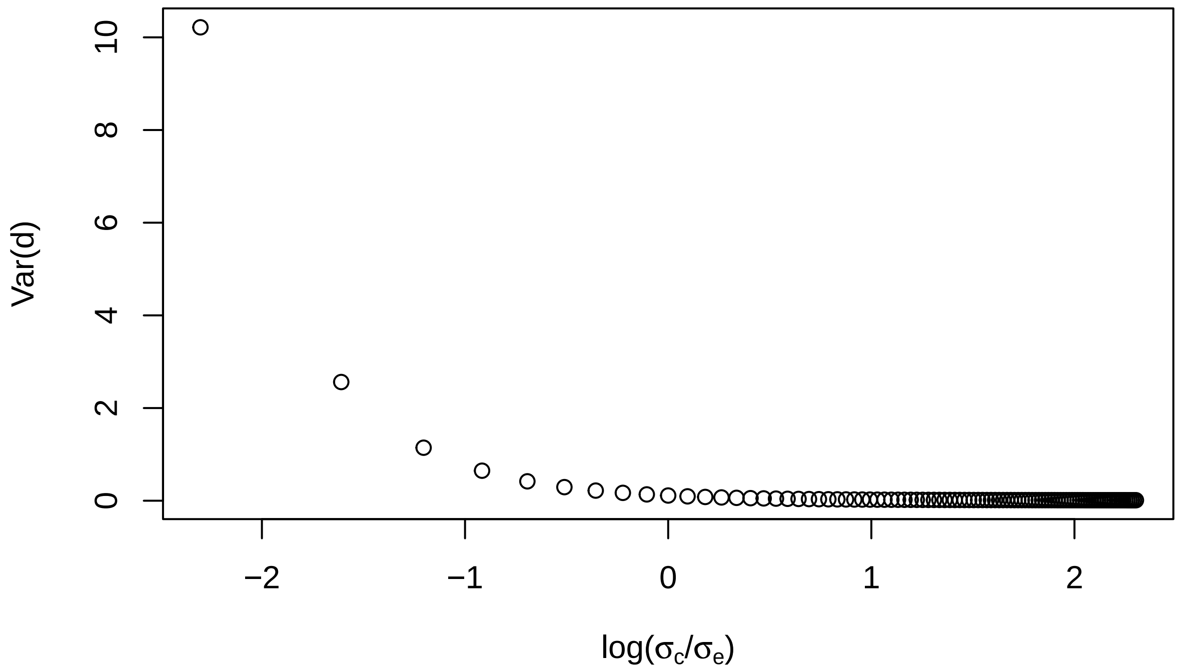

Figure 11. Variance of Glass's d, when sample sizes and variances are unequal across groups, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_c}{\sigma_e}\right)$).

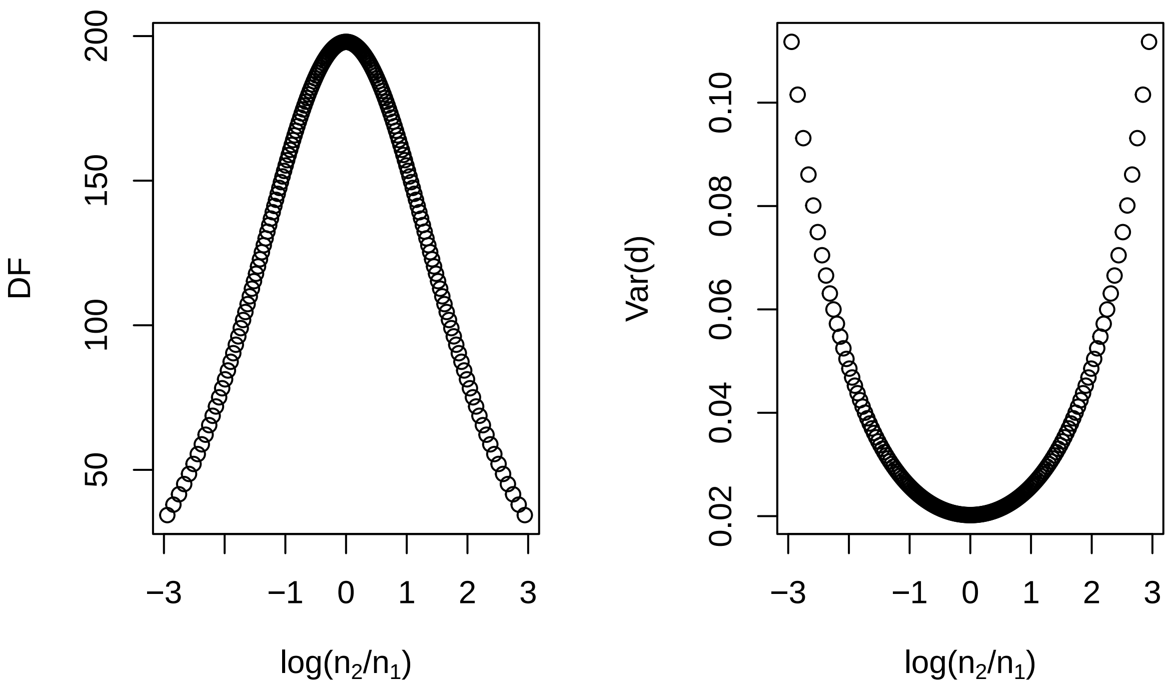

Figure 12. Variance of Cohen's d^* when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

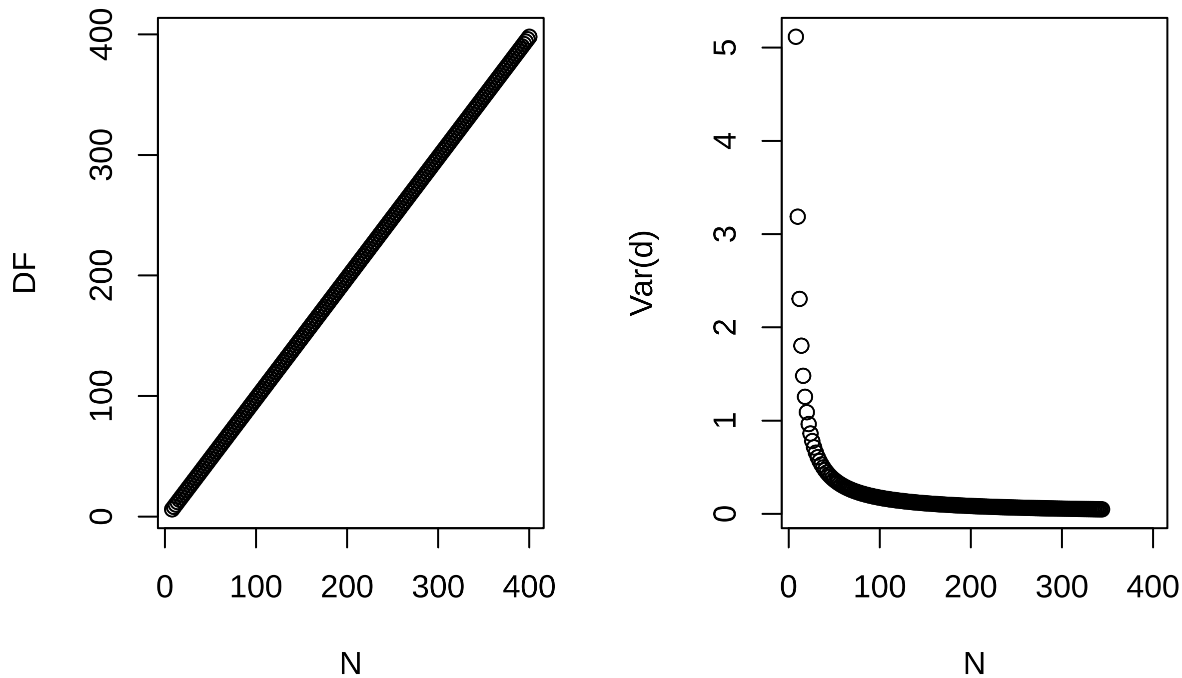

Figure 13. Variance of Cohen's d^* when variances are equal across groups, as a function of the total sample size (N).

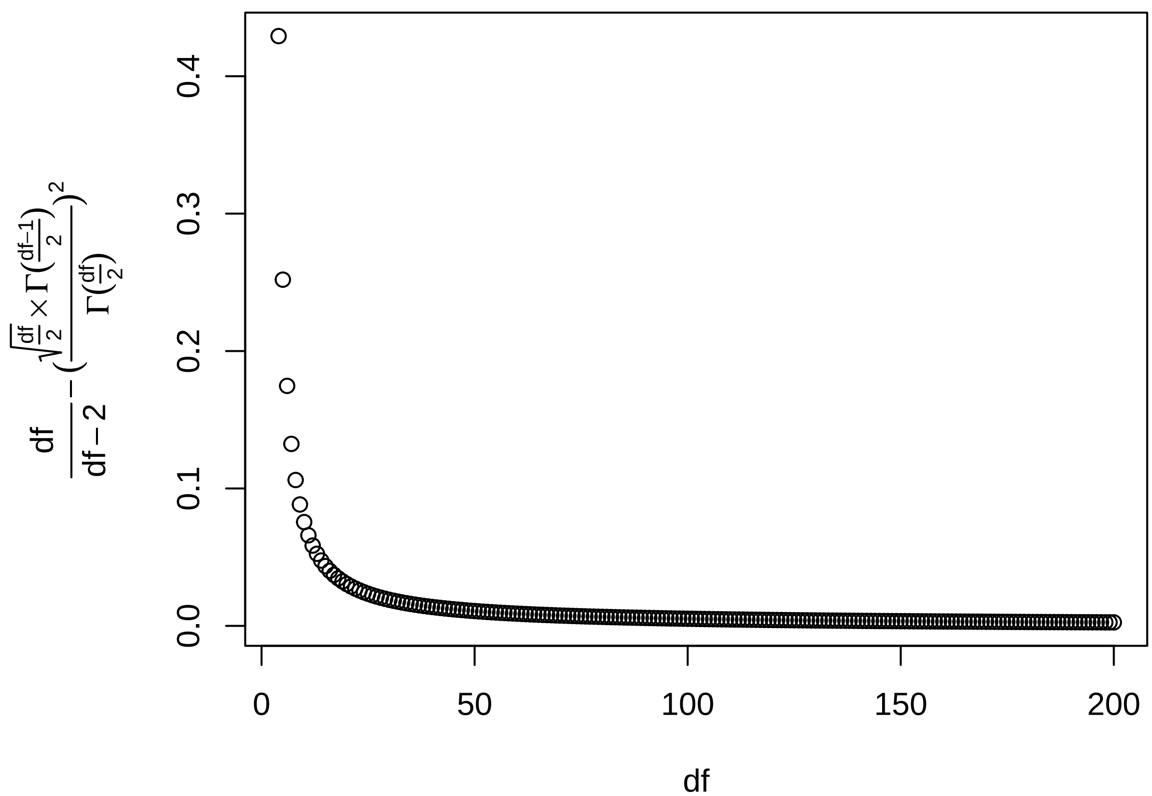

Figure 14. Effect size moderator (for all estimators), as a function of the degrees of freedom.

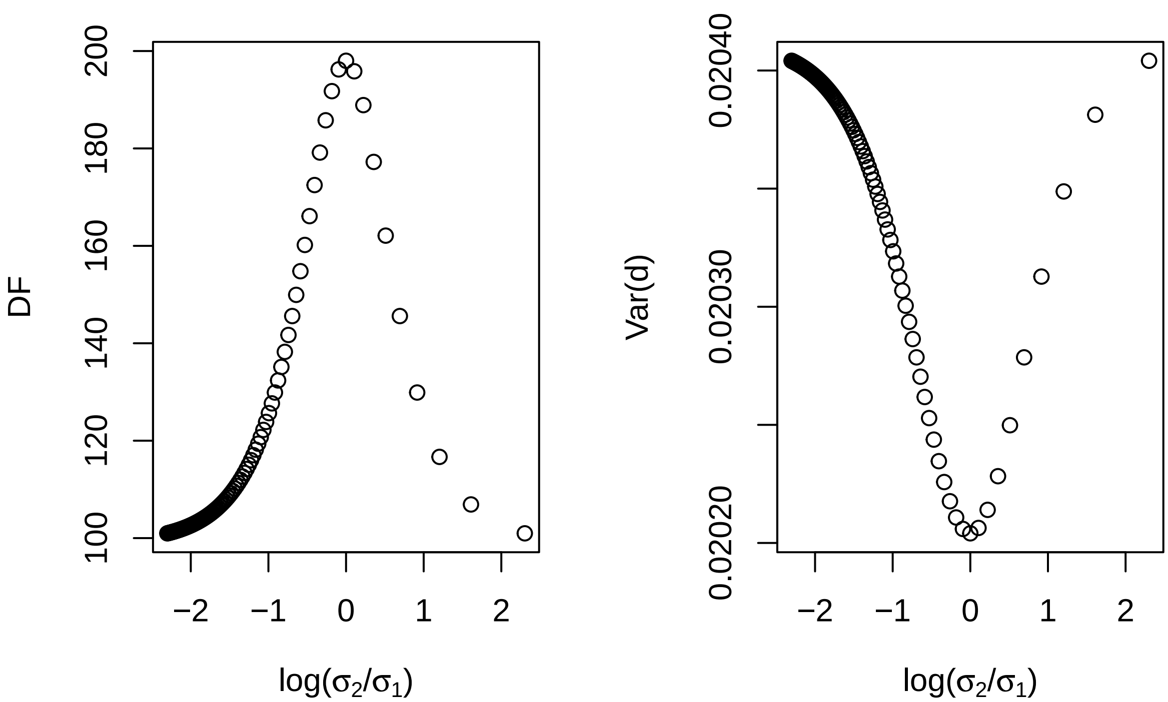

Figure 15. Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

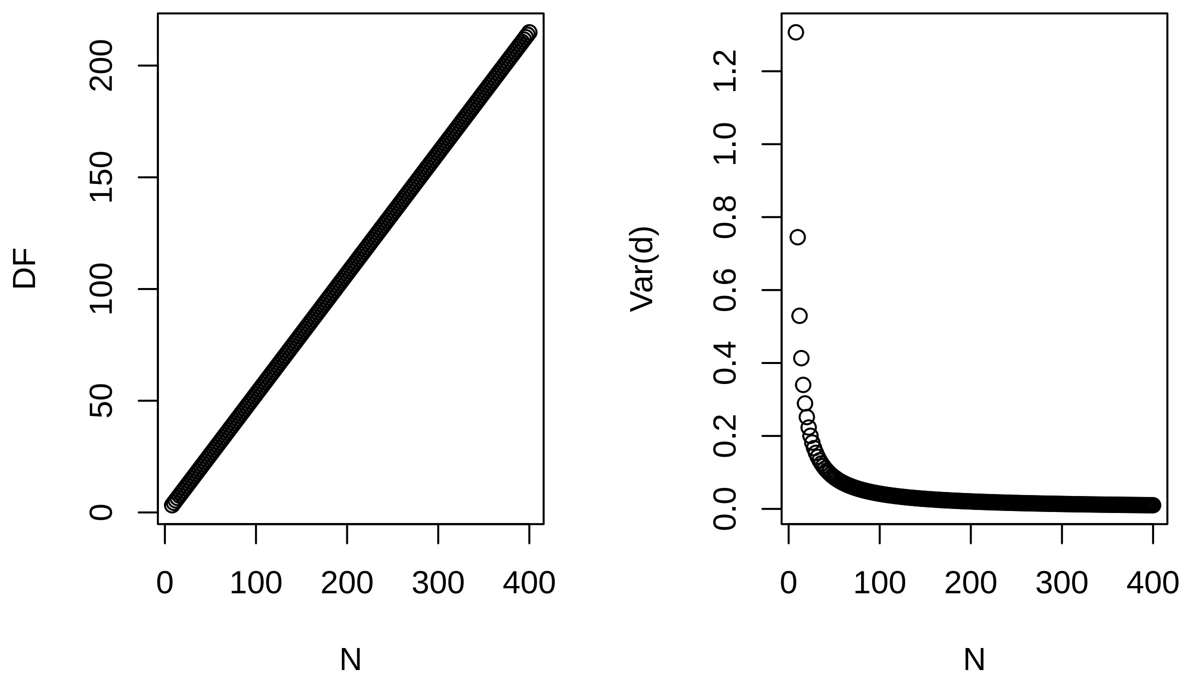

Figure 16. Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

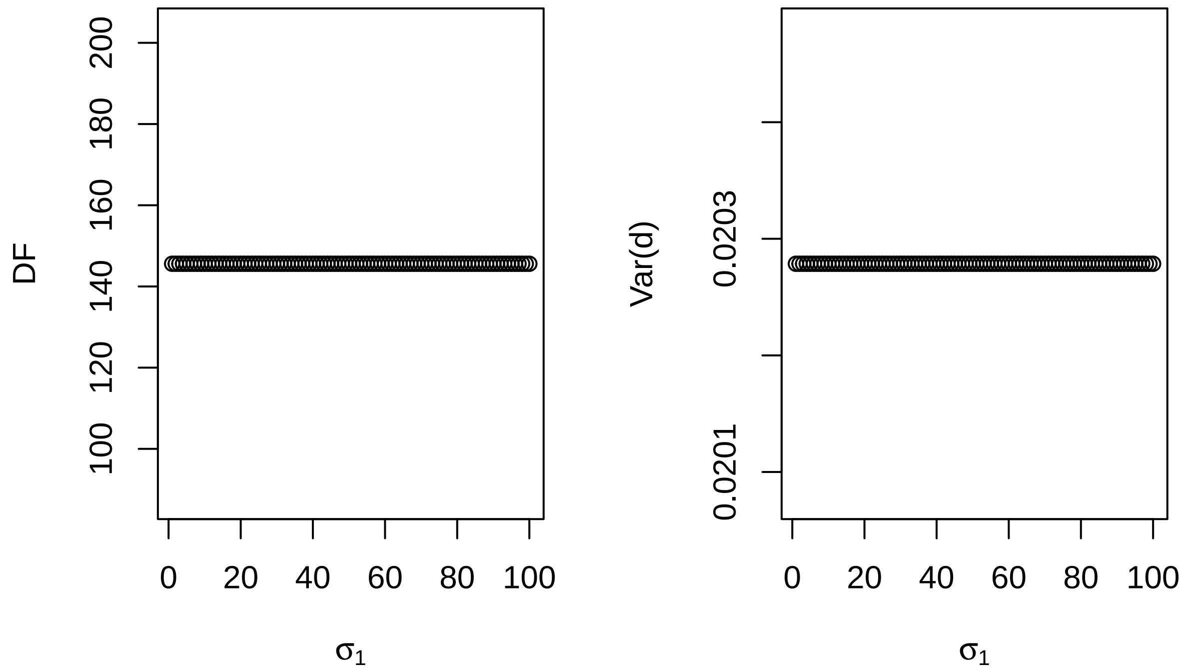

Figure 17. Variance of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant *SD*-ratio.

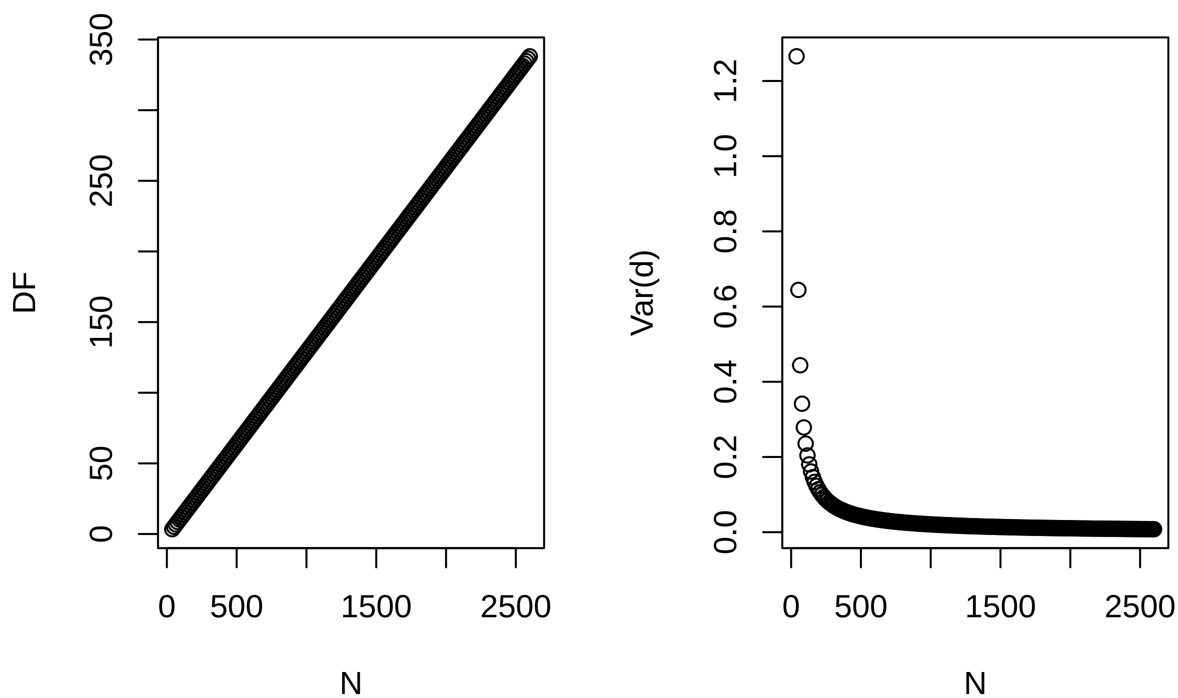

Figure 18. Variance of Cohen's d^* when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

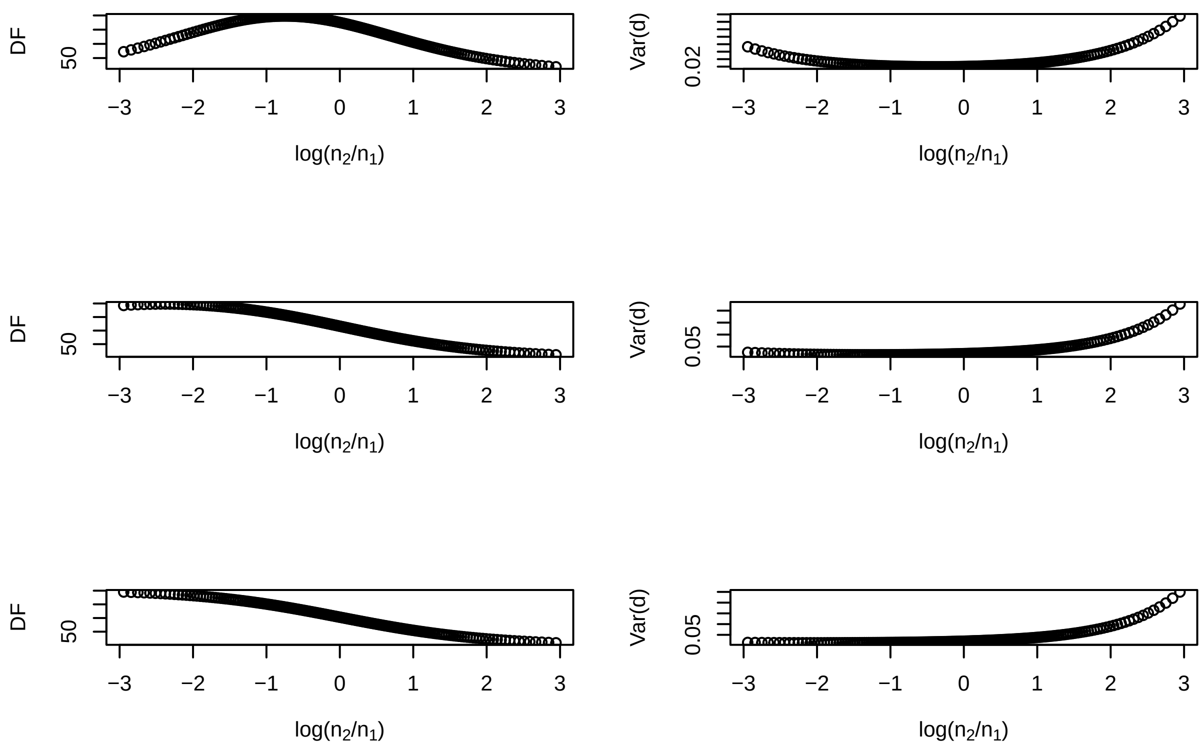

Figure 19. The variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

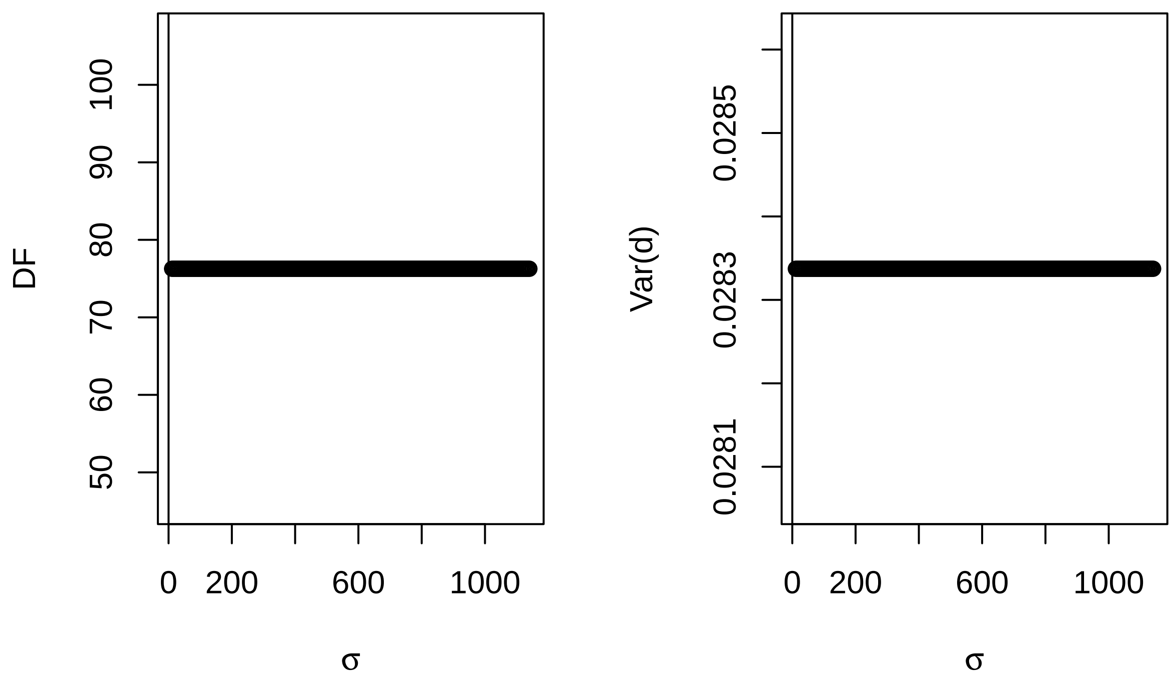

Figure 20. Variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

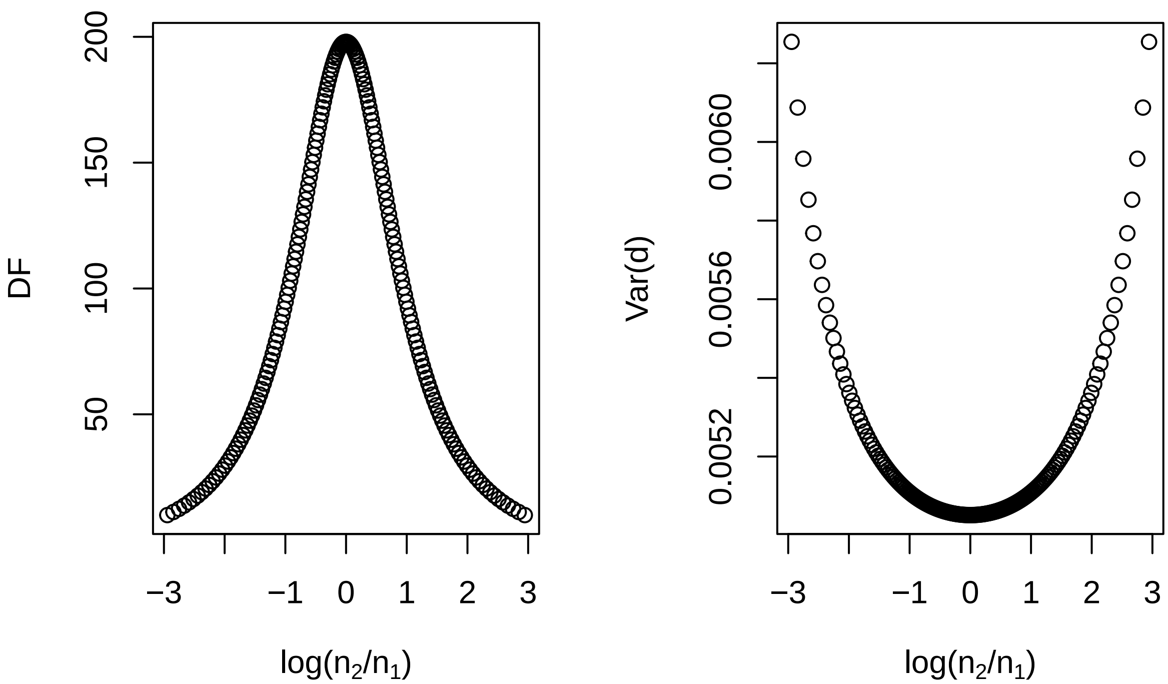

Figure 21. Variance of Shieh's d when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

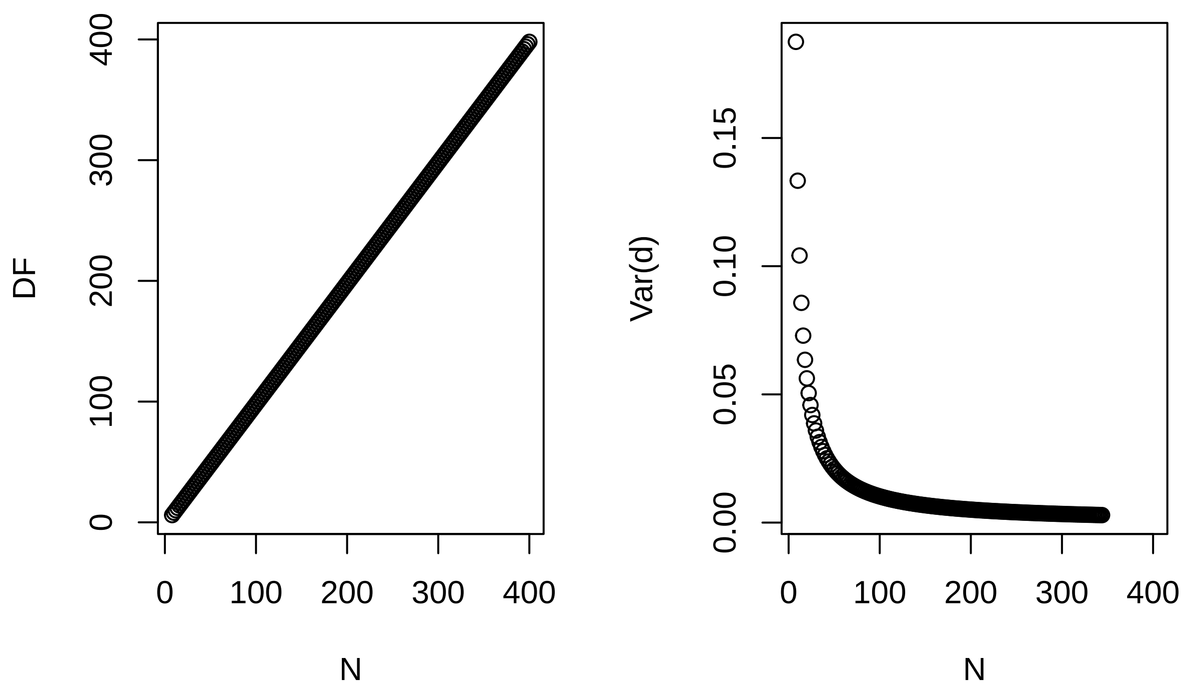

Figure 22. Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), for a constant sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

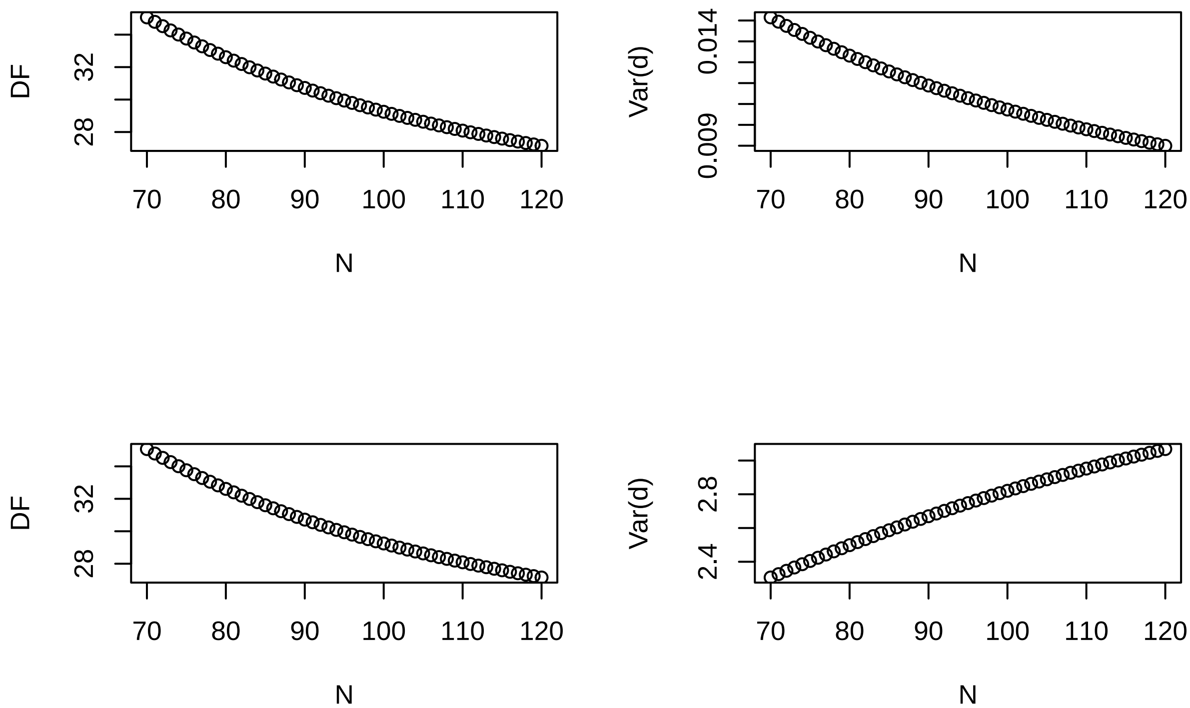

Figure 23. Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).

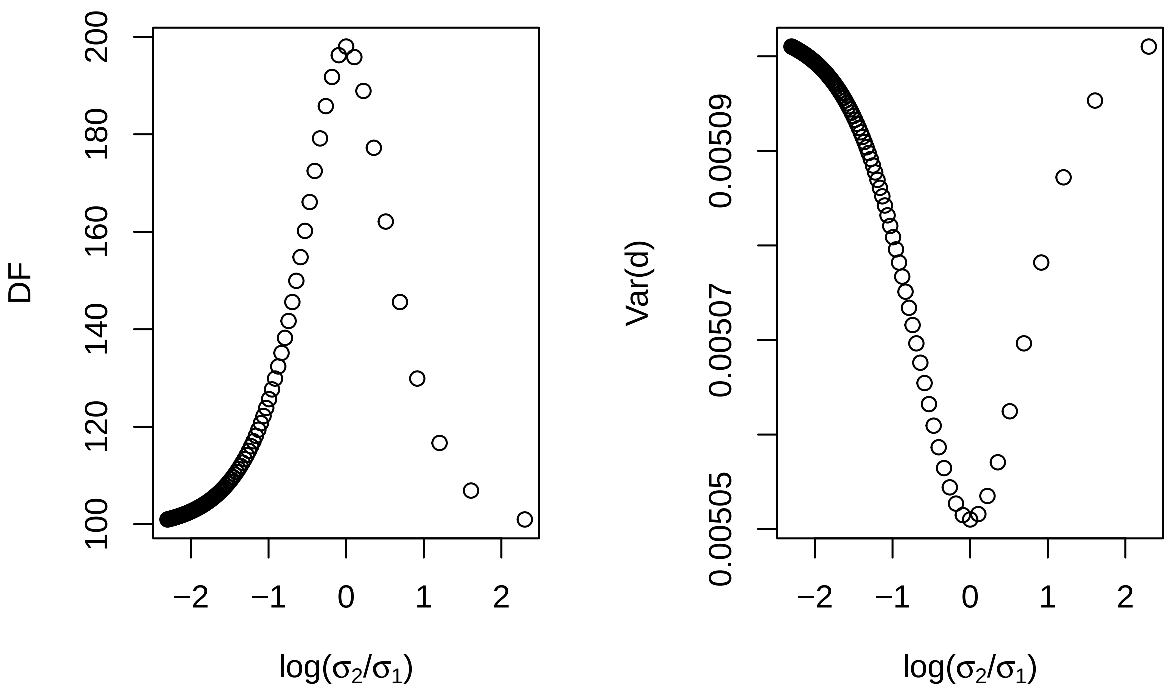

Figure 24. Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

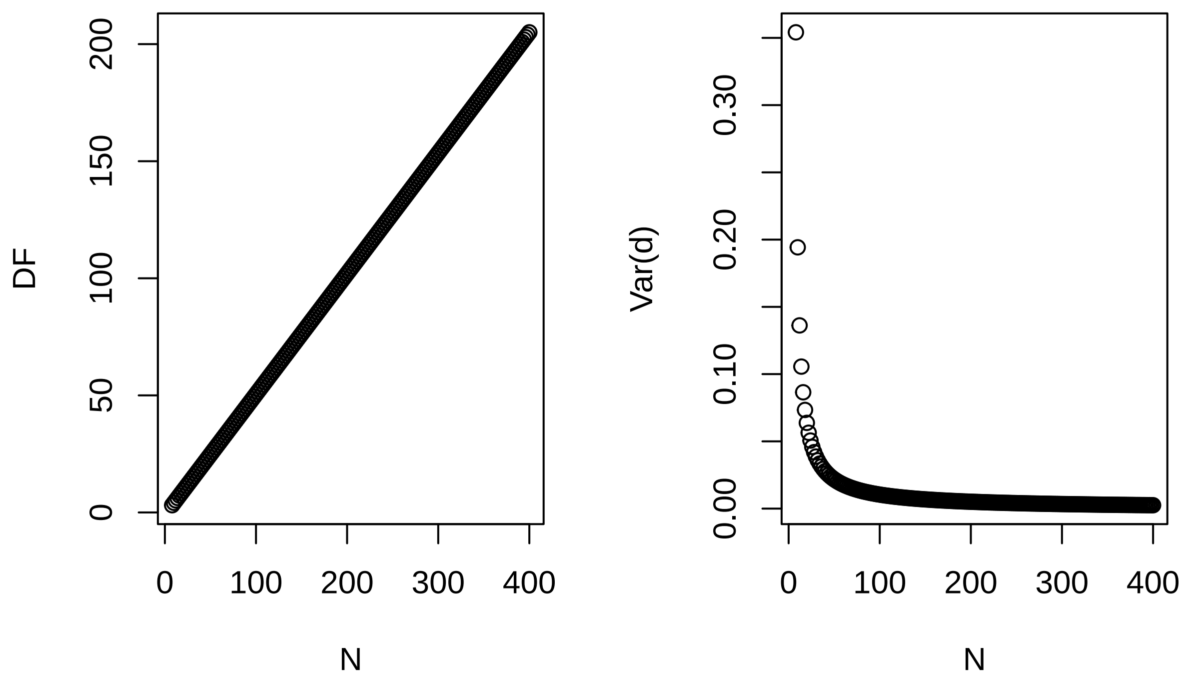

Figure 25. Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

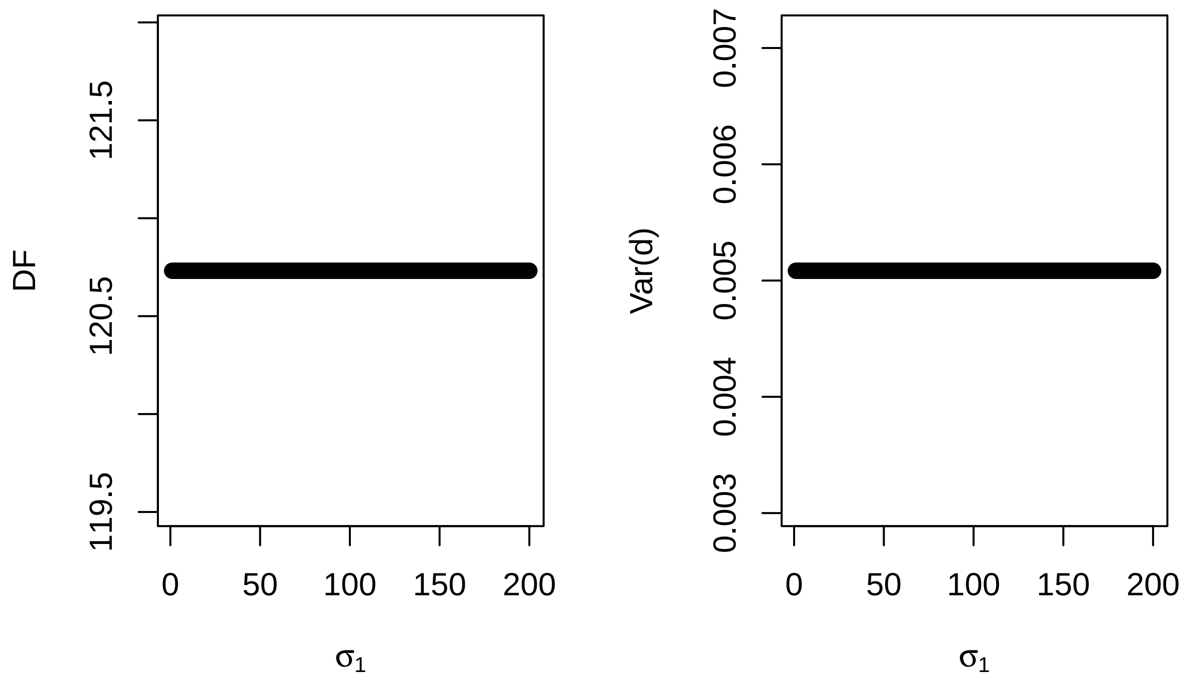

Figure 26. Variance of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

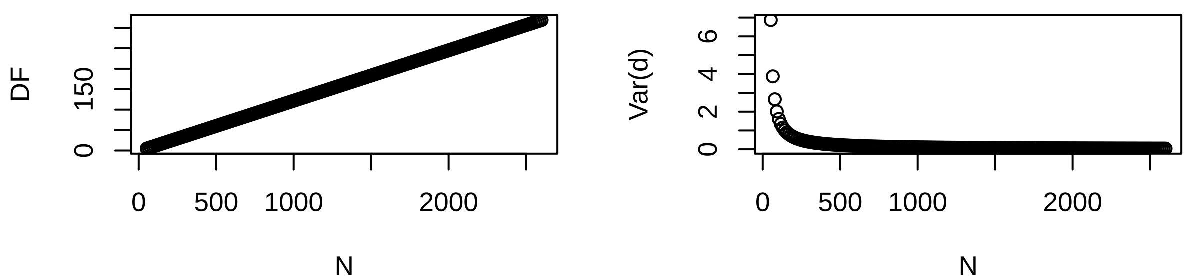

Figure 27. Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), for a constant sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

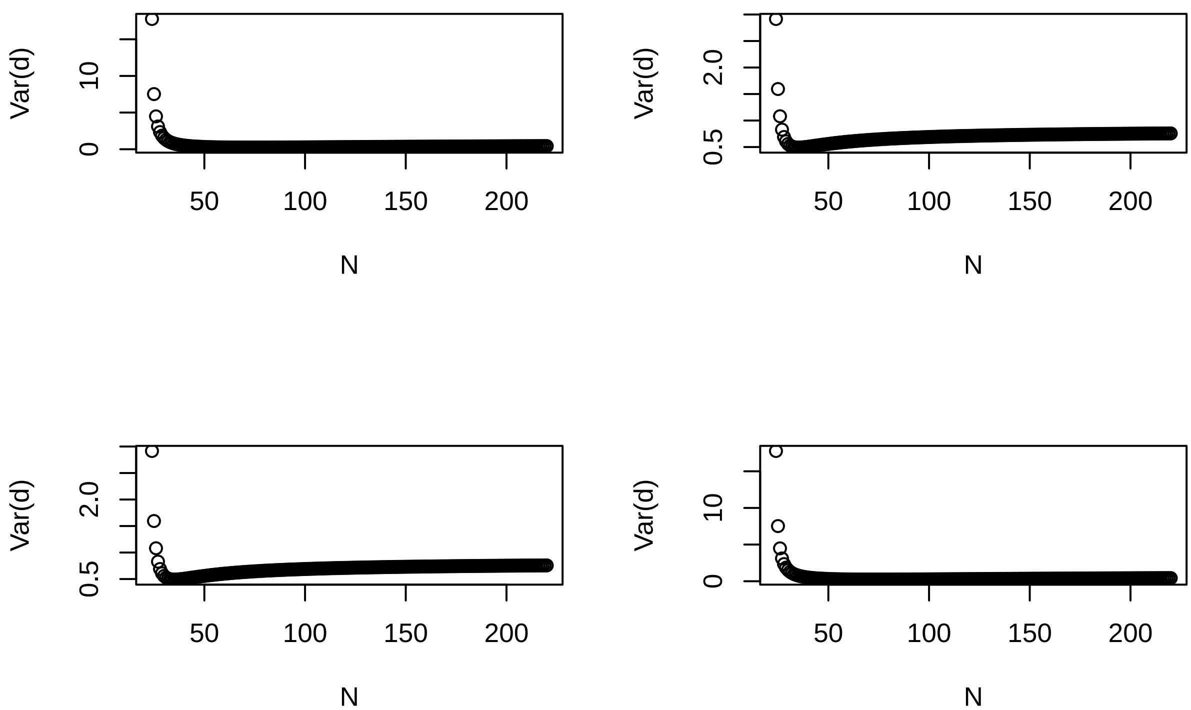

Figure 28. Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and $\sigma_1 > \sigma_2$ (top plots) or $\sigma_1 < \sigma_2$ (bottom plots).

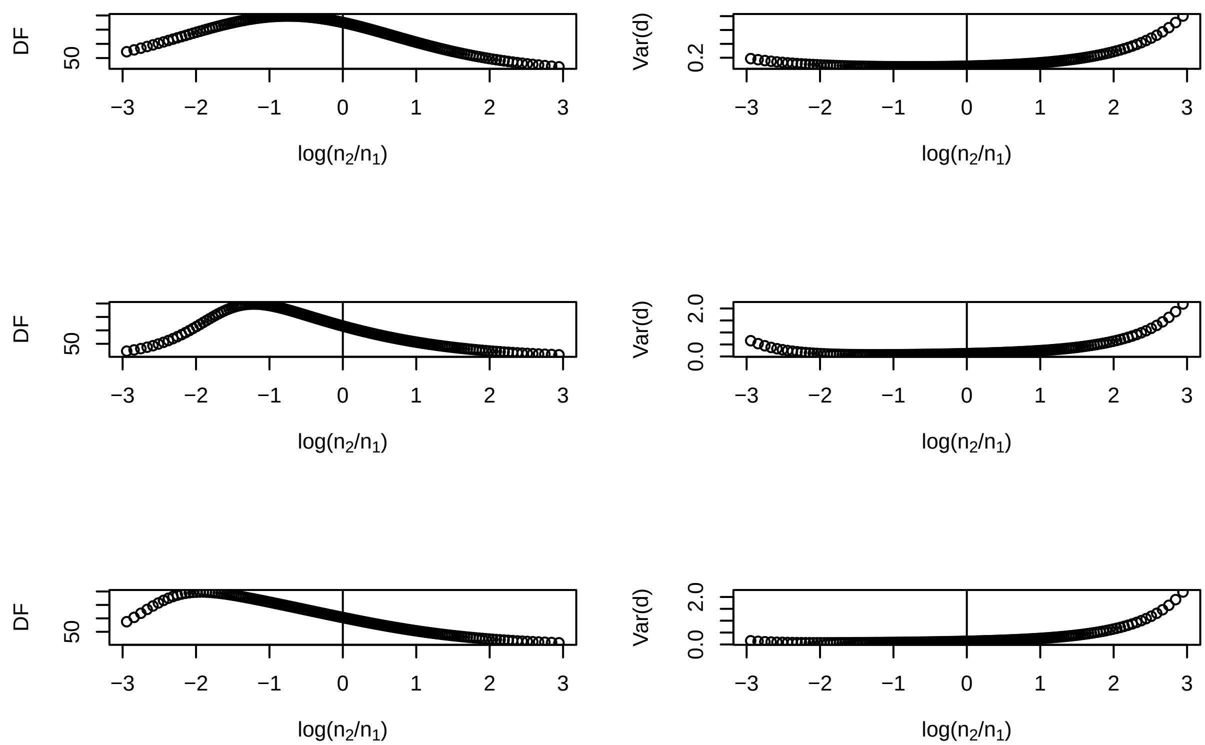

Figure 29. The variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

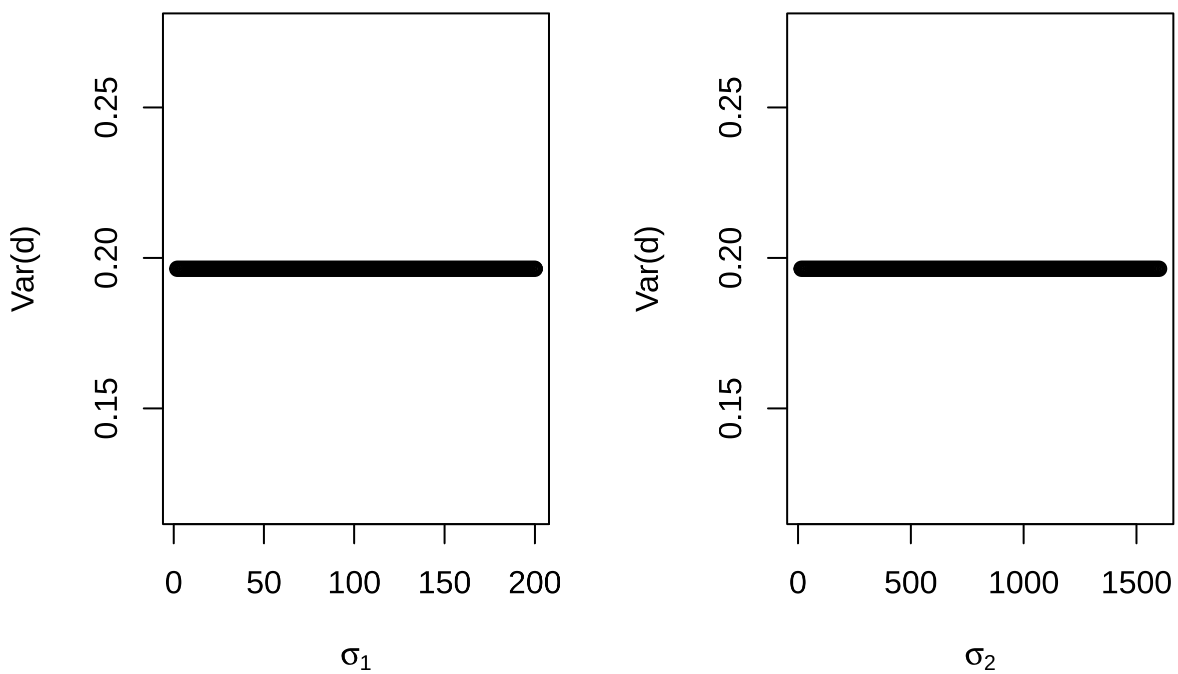

Figure 30. Variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of σ_1 (left) or σ_2 (right), for a constant SD-ratio.