Simulation checks

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## Simulation checks

## Supplemental Material 2

In order to insure the reliability of our calculation method, for all scenarios where  $G_1 = G_2 = 0$ , we compared empirical means and variances of all estimators (i.e. means and variances of all estimates) with theoretical means and variances (i.e. expected means and variances, computed based on equations in Tables 1, 2 and 3 in the main article). Because we 11 can draw exactly the same conclusions for biased (Cohen's  $d_s$ , Glass's  $d_s$  using either  $S_1$  or 12  $S_2$  as standardizer, Shieh's  $d_s$  and Cohen's  $d_s^*$ ) and **unbiased** (Hedges'  $g_s$ , Glass's  $g_s$  using 13 either  $S_1$  or  $S_2$  as standardizer, Shieh's  $g_s$  and Hedges'  $g_s^*$ ) estimators, we will simultaneously 14 present results for both categories of estimators. Results will be subdivided into 4 conditions: 15 - When population variances and sample sizes are equal across groups (condition a; see 16 Figures A2.1 and A2.5 for respectively biased and unbiased estimators); 17 - When population variances are equal across groups and sample sizes are unequal 18

(condition b; see Figures A2.2 and A2.6 for respectively biased and unbiased estimators);

- When population variances are unequal across groups and sample sizes are equal (condition c; see Figures A2.3 and A2.7 for respectively biased and unbiased estimators);

- When population variances and sample sizes are unequal across groups (condition d; see Figures A2.4 and A2.8 for respectively biased and unbiased estimators).

Because the equations of theoretical means and variances of Cohen's  $d_s$  and Hedges'  $g_s$  rely on the assumption of normality and equality of population variances, we expect empirical and theoretical parameters to be very close only in conditions a and b. For all other estimators, the equations of theoretical means and variances rely solely on the assumption of normality and therefore, we expect empirical and theoretical parameters to be very close in all conditions.

On average, empirical means (and variances) of all estimators are very close to theoretical expectations when population variances are equal across groups, with equal SIMULATION CHECKS

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sample sizes (condition a; see Tables A2.1 and A2.5) or unequal sample sizes (condition b; see Tables A2.2 and A2.6).

When population variances are unequal across groups (conditions c and d; see Tables 34 A2.3, A2.4, A2.7 and A2.8), empirical means (and variances) of Cohen's  $d_s^*$  (Hedges'  $g_s^*$ ) and 35 Shieh's  $d_s$  (Shieh's  $g_s$ ) are still very close to theoretical expectations. Regarding Glass's  $d_s$ 36 (Glass's  $g_s$ ), on average, while empirical variances remain very close to theoretical 37 expectations, one observes larger departures between empirical and theoretical means when 38 using  $S_2$  as standardizer. However, when looking at details in results for each scenario (see 39 "biased estimator\_condition C.xlsx", "biased estimator\_condition D.xlsx", "unbiased estimator condition C.xlsx" and "unbiased estimator condition C.xlsx" in Supplemental 41 Material 2), one notices that the larger the population effect size, the larger the departure 42 between empirical and theoretical means, and that relative to the population effect size, 43 departures between empirical and theoretical means are always very small. On the other hand, both empirical bias and variance of Cohen's  $d_s$  (Hedges'  $g_s$ ) highly depart from theoretical expectations, even when looking at relative departures to the population effect size, especially when sample sizes are unequal across groups (condition d; see Table A2.4 and 47 A2.8), which is not surprising, as Cohen's  $d_s$  (Hedges'  $g_s$ ) relies on the equality of population variances assumption.

Table A2.1.

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators,

when population variances and sample sizes are equal across groups (condition a).

Absolute deviation between empirical and theoretical means

Ratio between empirical and theoretical variances

Shieh's d₅	Cohen's d*s	Glass's d <sub>s,2</sub>	Glass's d <sub>s,1</sub>	Cohen's d <sub>s</sub>	Estimator $(\widehat{\delta})$	
0,006	0,012	0,023	0,022	0,012	xeM	
0,000	0,000	0,000	0,000	0,000	Min	
0,001	0,002	0,005	0,004	0,002	Mean	$ E(\widehat{\delta})$ - $\mu_{\delta} $
0,002	0,003	0,007	0,006	0,003	Standard deviation	
1,006	1,006	1,005	1,006	1,006	Max	
0,910	0,910	0,889	0,897	0,910	Min	
0,976	0,976	0,966	0,966	0,976	Mean	$S^2_{\widehat{\delta}}/\sigma_{\delta}$
0,028	0,028	0,035	0,033	0,028	Standard deviation	

Table A2.2

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when

when populatio	on variances ar Absolute de	e equal across viation betwee	s groups and sa:	when population variances are equal across groups and sample sizes are unequal (condition b).  Absolute deviation between empirical and theoretical means	condition b). Ratio b	Jetween empir	tionb ). Ratio between empirical and theoretical variances	cal variances
		_	  Ε(δ̂)-μ <sub>δ</sub>				$S^2_{\ \hat{\delta}}/\sigma_{\delta}$	
Estimator $(\widehat{\delta})$	Мах	Min	Mean Si	Standard deviation	Мах	Min	Mean St	Standard deviation
Cohen's ds	900'0	00000	0,001	0,001	1,017	0,951	586'0	0,017
Glass's d <sub>s,1</sub>	0,019	00000	0,004	900'0	1,006	0,891	996'0	0,037
Glass's d <sub>s,2</sub>	0,027	00000	0,005	0,007	1,015	0,881	896′0	0,036
Cohen's d*s	0,010	0,000	0,003	0,002	1,007	0,902	996'0	0,034
Shieh's ds	800'0	00000	0,002	0,002	1,005	0,865	0,945	0,048

Table A2.3

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators,

when population variances are unequal across groups and sample sizes are equal (condition c).

Absolute deviation between empirical and theoretical means

Ratio between empirical and theoretical variances

	Estimator (δ̂)	Cohen's ds	Glass's d <sub>s,1</sub>	Glass's d <sub>s,2</sub>	Cohen's d*,	Shieh's ds
	Max	0,080	0,037	0,230	0,036	0,018
	Min	0,000	0,000	0,000	0,000	0,000
ε(δ̂)-μ <sub>δ</sub>	Mean	0,010	0,005	0,012	0,003	0,002
	Standard deviation	0,015	0,007	0,033	0,006	0,003
	Max	1,753	1,004	1,008	1,007	1,007
	Min	1,005	0,888	0,883	0,874	0,874
$S^2_{\hat{\delta}}/\sigma_{\delta}$	Mean	1,175	0,973	0,974	0,975	0,975
	Standard deviation	0,208	0,030	0,032	0,033	0,033

Table A2.4

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are unequal across groups (condition d).

J. J.	Absolute d	eviation betw	een empirical a	Absolute deviation between empirical and theoretical means		between emp	irical and theo	Ratio between empirical and theoretical variances
			$ E(\widehat{\delta})$ - $\mu_{\delta} $				$S^{2}_{\widehat{\delta}}/\sigma_{\delta}$	
Estimator $( ilde{\delta})$	Мах	Min	Mean	Standard deviation	Мах	Min	Mean	Standard deviation
Cohen's ds	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's d <sub>s,1</sub>	0,026	0,000	0,005	900'0	1,009	0,881	0,972	0,033
Glass's d <sub>s,2</sub>	0,219	0,000	0,012	0,031	1,011	0,872	0,973	0,036
Cohen's d*s	0,030	0,000	0,003	900'0	1,011	0,860	0,974	0,034
Shieh's ds	600'0	0,000	0,001	0,002	1,011	0,867	0,970	0,036

0,008

0,000

0,002

0,002

1,006

0,908

0,975

0,029

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased Table A2.5

Hedges' g*s 0,015	Glass's g <sub>5,2</sub> 0,022	Glass's g <sub>5,1</sub> 0,021	Cohen's gs 0,011	Estimator ( $\hat{\delta}$ ) Max	estimators, when popul Abso
5 0,000	0,000	0,000	0,000	( Min	ation variances an
0,003	0,004	0,004	0,002	Mean	nd sample size ween empirical $ E(\hat{\delta}) - \mu_{\hat{\delta}} $
0,004	0,007	0,006	0,003	Standard deviation	estimators, when population variances and sample sizes are equal across groups (condition a). Absolute deviation between empirical and theoretical means Ratio $ E(\widehat{\pmb{\delta}})\mu_{\hat{\pmb{\delta}}} $
1,006	1,005	1,006	1,006	Max	(condition a,
0,908	0,889	0,897	0,911	Min	). o between em
0,975	0,966	0,966	0,976	Mean	pirical and the $S^2 {\hat g}/\sigma_{\hat g}$
0,029	0,035	0,033	0,028	Standard deviation	$m(a)$ . Ratio between empirical and theoretical variances $S^2_{ar{\delta}}/\sigma_{ar{\delta}}$

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased

Table A2.6

estimators, whe	n population	variances are	equal across	estimators, when population variances are equal across groups and sample sizes are unequal (condition b).	are unequal	(condition b)		
	Absolute o	deviation betw	een empirical	Absolute deviation between empirical and theoretical means	Rati	o between em	pirical and the	Ratio between empirical and theoretical variances
			$ E(\widehat{\delta})$ - $\mu_{\delta} $				$S^2_{\ \delta}/\sigma_{\delta}$	
Estimator $(\widehat{\delta})$	Max	Min	Mean	Standard deviation	Мах	Min	Mean	Standard deviation
Cohen's gs	900'0	000'0	0,001	0,001	1,017	0,951	0,985	0,017
Glass's g <sub>s,1</sub>	0,018	0,000	0,004	500'0	1,006	0,891	996'0	0,037
Glass's g <sub>5,2</sub>	0,026	0,000	0,004	900'0	1,015	0,881	896′0	0,036
Hedges′ g⁴s	0,010	000'0	0,003	0,003	1,007	0,925	0,972	0,027
Shieh's gs	0,007	0,000	0,002	0,002	1,007	0,900	656'0	0,037

0,017

0,000

0,002

0,003

1,008

0,890

0,978

0,029

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased

Table A2.7

estimators, whe	n population Absolute c	<i>variances an</i> leviation bet	"e unequal ac ween empirica $ E(\hat{\delta}) - \mu_{\hat{\delta}} $	estimators, when population variances are unequal across groups and sample sizes are equal (condition c). Absolute deviation between empirical and theoretical means Ratio between empirical and theoretical means $ \mathbf{E}(\hat{\boldsymbol{\delta}}) \boldsymbol{\mu}_{\boldsymbol{\delta}} $	zes are equa Rati	(condition c o between er	). $S^2 \widehat{\mathfrak{g}}/\sigma_{\mathfrak{g}}$	$qual~(condition~c).$ Ratio between empirical and theoretical variances $S^2\hat{s}/\sigma_{\delta}$
Estimator $(\widehat{\delta})$	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's g <sub>s</sub>	0,079	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's g <sub>s,1</sub>	0,036	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's g <sub>s,2</sub>	0,221	0,000	0,012	0,032	1,008	0,883	0,974	0,032
Hedges' g*s	0,034	0,000	0,003	0,006	1,008	0,890	0,978	0,029

Table A2.8

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased  $estimators, when {\it population} \ variances \ and \ sample \ sizes \ are \ unequal \ across \ groups \ (condition \ d).$ 

commune, wh	en population	ימו ומווכבי מווו	n caric aidime	estimators, mien population var tances and sample street at e anequat act oss & oups (common a).	nommina) s	.,		
	Absolute de	viation betwe	en empirical and	Absolute deviation between empirical and theoretical means	Ratio !	between empir	Ratio between empirical and theoretical variances	I variances
		_	$ E(\widehat{\delta}) \cdot \mu_{\delta} $				$S^{2}_{\ \widehat{\delta}}/\sigma_{\delta}$	
Estimator ( $\delta$ )	Мах	Min	Mean	Standard deviation	Мах	Min	Mean Star	Standard deviation
Cohen's gs	0,250	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's g <sub>5,1</sub>	0,025	0,000	0,004	0,006	1,009	0,881	0,972	0,033
Glass's g <sub>s,2</sub>	0,210	000'0	0,012	0,030	1,011	0,872	0,973	0,036
Hedges' g*s	0,029	0,000	0,003	0,005	1,011	0,882	7,60	0,029
Shieh's gs	800'0	0,000	0,001	0,002	1,011	0,881	0,973	0,032