1　　　　　What measure of effect size using when performing a Welch's t-test?

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Abstract

Keywords: keywords

Word count: X

15   What measure of effect size using when performing a Welch's t-test?

16   **Intro**

17   During decades, researchers in social science (Henson & Smith, 2000) and education

18 (Fan, 2001) have overestimated the ability of the null hypothesis (H0) testing to determine

19 the importance of their results. The standard for researchers in social science is to define H0

20 as the absence of effect (Meehl, 1990). For example, when comparing the mean of two

21 groups, researchers commonly test the H0 that there is no mean difference between groups

22 (Steyn, 2000). Any effect that is significantly different from zero will be seen as sole support

23 for a theory.

24   Such an approach has faced many criticisms among which the most relevant to our

25 concern is that the null hypothesis testing highly depends on sample size: for a given alpha

26 level and a given difference between groups, the larger the sample size, the higher the

27 probability of rejecting the null hypothesis (Fan, 2001; Kirk, 2009; Olejnik & Algina, 2000;

28 Sullivan & Feinn, 2012). It implies that even tiny differences could be detected as

29 statistically significant with very large sample sizes (McBride, Loftis, & Adkins, 1993)[1].

30   Facing this argument, it has become an adviced practice to report the *p*-value assorted

31 by a measure of the effect size, that is, a quantitative measure of the magnitude of the

32 experimental effect (Cohen, 1965; Fan, 2001; Hays, 1963). This practice is also highly

33 endorsed by the American Psychological Association (APA) and the American Educational

34 Research Association (AERA) (American Educational Research Association, 2006; American

35 Psychological Association, 2010). However, only a limited number of studies have properly

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[1] Tiny differences might be due to sampling error, or to other factors than the one of interest: even under
the assumption of random assignent (which is a necessary but not sufficient condition), it is almost
impossible to be sure that the only difference between two conditions is the one defined by the factor of
interest. Other tiny factors of no theoretical interest might slightly influence results, making the probability
of getting an actual zero effect very low. This is what Meehl (1990) calls 'systematic noise'.

reported effect size in the last decades.

Generally, there is a high confusion between the effect size and other related concepts such as the Clinical significance. Moreover, there are several situations that call for effect size measures and, in the current litterature, it is not always easy to know which measure to use in which context. We will therefore begin this paper with 3 sections in which we will:

1. Clearly define what is a measure of effect size;

2. List the different situations that call for effect sizes measures;

3. Define required properties of the effect size estimators depending on the specific situation.

Moreover, it is highly recommended to compute a confidence interval around the point effect size. In a fourth section, we will therefore summarize in how far it is an added value to mention the confidence interval around the effect size.

After these general adjustments, we will focus our attention on "between-subject" designs where individuals are randomly assigned into one of two independent groups and group scores are compared based on their means[2]. Because it has been widely argued that there are many fields in psychology where the assumption of equal variances between two populations is ecologically unlikely (Delacre, Lakens, & Leys, 2017; Erceg-Hurn & Mirosevich, 2008; Grissom, 2000), it is becoming more common in statistical software to present a t-test that does not hold under this assumption by default, namely the Welch's t-test (e.g., R, Minitab). However, similar issues for the measures of effect sizes have received less attention (Shieh, 2013), and Cohen's d_s remains persistent[3]. One possible reason is that researchers cannot find a consensus on which alternative should be used (Shieh, 2013). We will limit our study to the standardized mean difference, called the d-family, because it is the

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[2] We made this choice because *t*-tests are still the most commonly used tests in the field of Psychology.

[3] For example, in Jamovi, Cohen's ds is provided, independently of whether one performs Student's or Welch's t-test.

⁵⁹ dominant family of estimators of effect size when comparing two groups based on their

⁶⁰ means (Peng, Chen, Chiang, & Chiang, 2013; Shieh, 2013), and we will see that even in this

⁶¹ very specific context, there is little agreement between researchers as to which is the most

⁶² suitable estimator. According to us, the main reason is that it is difficult, based on currently

⁶³ existing measures, to optimally serve all the purposes of an effect size measure. Throughout

⁶⁴ this section, we will:

⁶⁵   1. Present the main measures of the d-family that are proposed in the

⁶⁶ literature,related to the purpose they serve, and introduce a new one, namely the

⁶⁷ "transformed Shieh's d" that should help at reaching all the purposes simultaneously;

⁶⁸   2. Present and discuss the results of simulations we performed, in order to compare

⁶⁹ existing measures and our newly introduced one;

⁷⁰   3. Summarize our conclusions in practical recommendations. In this section, we will

⁷¹ provide useful tools (i.e., an R package) to compute relevant measures of effect sizes and

⁷² related information.

Measure of effect size: what it is, what it is not

⁷⁴   The effect size is commonly referred to as the practical significance of a test. Grissom

⁷⁵ & Kim (2005) define the effect size as the extent to which results differ from what is implied

⁷⁶ by the null hypothesis. In the context of the comparison of two groups based on their means,

⁷⁷ depending on the defined null hypothesis (considering the absence of effect as the null

⁷⁸ hypothesis), we could define the effect size either as the magnitude of differences between

⁷⁹ parameters of two populations groups are extracted from (e.g. the mean; Peng & Chen, 2014)

⁸⁰ or as the magnitude of the relation between one dichotomous factor and one dependent

⁸¹ variable (American Educational Research Association, 2006). Both definitions refer to the

⁸² most famous families of measures of effect sizes (Rosenthal, 1994): the d-family and the

⁸³ r-family.

⁸⁴   Very often, the contribution of the measures of effect size is overestimated. First,

benchmarks about what should be a small, medium or large effect size might have
contributed to viewing the effect size as a measure of the importance or the relevance of an
effect in real life, but it is not (Stout & Ruble, 1995). The effect size is only a mathematical
indicator of the magnitude of a difference, which depends on the way a variable is converted
into numerical indicator. In order to assess the meaningfulness of an effect, we should be
able to relate this effect with behaviors/meaningful consequences in the real world
(Andersen, McCullagh, & Wilson, 2007). For example, let us imagine a sample of students in
serious school failure who are randomly divided into two groups: an experimental group
following a training program and a control group. At the end of the training, students in the
experimental group have on average significantly higher scores on a test than students in the
control group, and the difference is large (e.g. 30 percents). Does it automatically mean that
students in the experimental condition will be able to pass to the next grade and to continue
normal schooling? Whether the computed magnitude of difference is an important,
meaningful change in everyday life refers to the interpretation of treatment outcomes and is
neither a statistical nor mathematical concept, but is related to the underlying theory that
posits an empirical hypothesis. This concept is sometimes called *Clinical significance*
(Grissom & Kim, 2012; Thompson, 2002) or *Social significance* (Tyler, 1931) in the current
literature. However, in our conception, we should use a more general term and we propose to
rename this concept to *Applied significance*[4].

      Second, in the context of the comparison of two groups based on their means, the effect
size should not replace the null hypothesis testing. Statistical testing allows the researcher to
determine whether the oberved departure from H0 occured by chance or not (Stout & Ruble,
1995), while effect size estimators allow to assess the practical signficance of an effect, and as
reminds Fan (2001): *"a practically meaningful outcome may also have occured by chance,*

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[4] In our conception Applied significance encompasses all what refers to the relevance of an effect in real life,
such as for instance clinical, personnal, social, professionnal relevance

and consequently, is not trustworthy" (p.278). For this reason, the use of confidence intervals around the effect size estimate is highly recommended (Bothe & Richardson, 2011).

Different purposes of effect size measures

Effect size measures can be used in an *inferential* perspective:

- The effect sizes from previous studies can be used in a prior power analysis when planning a new study (Lakens, 2013; Prentice & Miller, 1990; Stout & Ruble, 1995; Sullivan & Feinn, 2012; Wilkinson & the Task Force on Statistical Inference, 1999);

- We can compute confidence limits around the point estimator (Shieh, 2013) in order to replace conventional hypothesis testing : if the null hypothesis area is out of the confidence interval, we can conclude that the null hypothesis is false.

Measures of effect size can also be used in a *comparative* perspective, that is, to assess the stability of results across designs, analysis, samples sizes (Wilkinson & the Task Force on Statistical Inference, 1999). This includes

- the comparison of results from 2 or more studies (Prentice & Miller, 1990);

- the incorporation of results in meta-analysis (Lakens, 2013; Li, 2016; Nakagawa & Cuthill, 2007; Stout & Ruble, 1995; Wilkinson & the Task Force on Statistical Inference, 1999).

Finally, effect size measures can be used for *interpretative* purposes, namely to assess the practical significance of a result (beyond statistical significance; Lakens, 2013; American Psychological Association, 2010; Prentice & Miller, 1990).

Properties of a good effect size estimator

The empirical value of an estimator (called estimate) depends on the sampling, in other words, different samples extracted from the same population will of course lead to different estimates for a same estimator. The *sampling distribution* of the estimator is the distribution of all estimates, based on all possible samples of size n extracted from one

134   population. Studying the sampling distribution is very useful, as it allows us to assess the

135   qualities of estimator. More specifically, three desirable properties a good estimator should

136   possess for inferential purposes are: **unbiasedness**, **consistency** and **efficiency** (Wackerly,

137   Mendenhall, & Scheaffer, 2008).

138   An estimator is unbiased if the distribution of estimates is centered around the true

139   population parameter. On the other hand, an estimator is positively (or negatively) biased if

140   the distribution is centered around a value that is higher (or smaller) than the true

141   population parameter (see Figure 1). In other words, the bias tells us if estimates are good,

142   on average. The *bias* of a point estimator $\hat{\delta}$ can be computed as

$$\hat{\delta}_{bias} = E(\hat{\delta}) - \delta \tag{1}$$

143   where $E(\hat{\delta})$ is the expectation of the sampling distribution of the estimator (i.e. the

144   population average) and δ is the true (population) parameter.
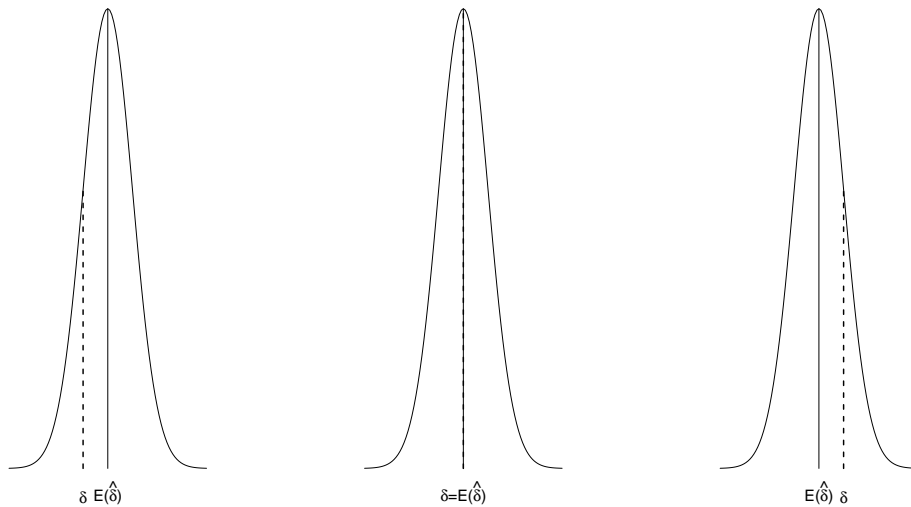

Figure 1. Samplig distribution for a positively biased (left), an unbiased (center) and a
negatively biased estimator (right)

145   Moreover, since there is a strong relationship between the bias and the size of any

146 estimator (the larger an estimator, the larger the bias), it might be interesting to also define

147 the *relative bias* as the ratio between the bias and the population parameter:

$$\hat{\delta}_{relative\ bias} = \frac{E(\hat{\delta}) - \delta}{\delta} \tag{2}$$

148   While the bias informs us about the quality of estimates on average, in particular their

149 capacity of lying close to the true value, it says nothing about individual estimates. Imagine

150 a situation where the distribution of estimates is centered around the real parameter but

151 with such a large variance that some point estimates are very far from the center. This would

152 be problematic, since we then do not know if this estimate, based on the sample at hand, is

153 close to the truth or far off. Therefore it is not only essential for an estimator to be unbiased,

154 but the variability of its sampling distribution should also ideally be small. Put simply, we

155 hope that *all* possible estimates are close enough of the true population parameter, in order

156 to be sure that for *any* estimate, one has a correct estimation of the real parameter. Among

157 two unbiased estimators $\hat{\delta}_1$ and $\hat{\delta}_2$, we therefore say that $\hat{\delta}_1$ is **more efficient** than $\hat{\delta}_2$ if

$$Var(\hat{\delta}_1) \leq Var(\hat{\delta}_2) \tag{3}$$

158   Where $Var(\hat{\delta})$ is the variance of the sampling distribution of the estimator $\hat{\delta}$. Among

159 all unbiased estimators, the more efficient will be the one with the smallest variance [5].

160 Again, the variance of an estimator $\hat{\delta}$ is a function of its size (the larger the estimator, the

161 larger the variance) and therefore, we might be interested in reducing the effect size impact

162 in computing the *relative variance* as the ratio between the variance and the square of the

163 population estimator:

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[5] The famous Cramer-Rao inequality provides a theoretical lower bound for the variance of unbiased
estimators. An estimator reaching this bound is therefore most efficient.

$$\hat{\delta}_{relative\ variance} = \frac{Var(\hat{\delta})}{\delta^2} \qquad (4)$$

₁₆₄   This doesn't entirely remove the impact of the effect size, but greatly reduces it. SEE

₁₆₅ TABLES 1 –> 4: ONLY THE SECOND TERM IN THE ADDITION CONTAINS δ^2, SO

₁₆₆ WHEN DIVIDING THE VARIANCE BY δ^2, THE FIRST TERM IN THE ADDITION

₁₆₇ DEPENDS ON THE EFFECT SIZE. Note that both unbiasedness and efficiency are very

₁₆₈ important. An unbiased estimator with such a large variance that somes estimates are

₁₆₉ extremely far from the real parameter is as undesirable as a parameter which is highly

₁₇₀ biased. In some situations, it is better to have a slightly biased estimator with a tight shape

₁₇₁ around the biased value (so that each estimate remains relatively close to the true parameter

₁₇₂ and one can apply bias correction techniques) rather than an unbiased estimator with a large

₁₇₃ variance (Raviv, 2014).

₁₇₄   Finally, the last property of a good point estimator is **consistency**: consistency means

₁₇₅ that the bigger the sample size, the closer the estimate is to the population parameter. In

₁₇₆ other words, the estimates *converge* to the true population parameter.

₁₇₇   Beyond the inferential properties, Cumming (2013) reminds that an effect size

₁₇₈ estimator needs to have a constant value across designs in order to be easily interpretable

₁₇₉ and to be included in meta-analysis. In other words, it should achieve the property of

₁₈₀ **generality**.

₁₈₁   **Confidence interval around a point estimator**

₁₈₂   We already mentioned that confidence interval around a point estimate could replace

₁₈₃ conventional hypothesis testing. A confidence interval contains all the information that a

₁₈₄ *p*-value of a test based on the same estimator does: if the area of the null hypothesis is out of

₁₈₅ the $(1 - \alpha)$-confidence interval, then the hypothesis test would also result in a *p*-value below

₁₈₆ the nominal alpha level. Hypothesis tests and confidence intervals based on the same

187 statistical quantity (this is an essential requirement) are thus directly related. At the same

188 time, the intervals provide extra information about the precision of the sample estimate for

189 inferential purposes, and therefore on how confident we can be in the observed results

190 (Altman, 2005; Ellis, 2015): the narrower the interval, the higher the precision. On the other

191 hand, the wider the confidence interval, the more the data lacks precision (for example,

192 because the sample size is too small).

Different measures of effect sizes

194   The d-family effect sizes are commonly used with "between-subject" designs where

195 individuals are randomly assigned into one of two independent groups and groups scores

196 means are compared. The population effect size is defined as

$$\delta = \frac{\mu_1 - \mu_2}{\sigma} \tag{5}$$

197   where both populations follow a normal distribution with mean μ_j in the jth

198 population (j=1,2) and common standard deviation σ. They exist different estimators of this

199 effect size measure. For all of them, the mean difference is estimated by the difference

200 $\bar{X}_1 - \bar{X}_2$ of both sample means. When the equality of variances assumption is assumed, σ is

201 estimated by pooling both samples standard deviations (S_1 and S_2). When the equality of

202 variances assumption cannot be assumed, alternatives to the common standard deviation are

203 available. Throughout this section, we will present some of these estimators, separately

204 depending on whether they rely on the assumption of equality of variances or not. For each

205 of them, we will provide information about their theoretical bias, variance and consistency.

When variances are equal between groups

207   When we have good reasons to assume equality of variances between groups, then the

208 most common estimator of δ is Cohen's d_s where the sample mean difference is divided by a

209 pooled error term (Cohen, 1965):

$$Cohen's\ d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1) \times S_1^2 + (n_2-1) \times S_2^2}{n_1+n_2-2}}} \tag{6}$$

210 Where S_j is the standard deviation and n_j the sample size of the jth sample (j=1,2).

211 The reasoning behind this measure is to make use of the fact that both samples share the

212 same population variance (Keselman, Algina, Lix, Deering, & Wilcox, 2008), hence we

213 achieve a more accurate estimation of the population variance by pooling both estimates of

214 this parameter (i.e S_1 and S_2). Since the larger the sample size, the more accurate the

215 estimate, we give more weight to the estimate based on the larger sample size. Cohen's d_s is

216 directly related with Student's t-statistic:

$$cohen's\ d_s = t_{student} \times \sqrt{\frac{n_1 + n_2}{n_1 n_2}} \tag{7}$$

217 Under the assumption of normality and equal variances between groups, Student's

218 t-statistic follows a t-distribution with known degrees of freedom and noncentrality

219 parameter [6]:

$$df_{student} = n_1 + n_2 - 2 \tag{8}$$

$$ncp_{student} = \frac{\mu_1 - \mu_2}{\sigma_{pooled}} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}, \quad where\ \sigma_{pooled} = \sqrt{\frac{(n_1 - 1) \times \sigma_1^2 + (n_2 - 1) \times \sigma_2^2}{n_1 + n_2 - 2}} \tag{9}$$

220 The relationship described in equation 7 and the theoretical distribution of Student's

221 t-statistic allow us to theoretically determine the sampling distribution of Cohen's d_s, and

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[6] Under the null hypothesis of no differences between sample means, Student's t-statistic will follow a central

t-distribution with $n_1 + n_2 - 2$ degrees of freedom. However, when the null hypothesis is false, the

distribution of this quantity will not be centered, and noncentral t-distribution will arise.

222 therefore, its theoretical expectancy and variance when the assumptions of normality and

223 equal variances are met. All these equations are provided in Table 1. For interested readers,

224 bias and variance of Cohen's d_s have been thoroughly studied, based on Table 1, so as to

225 determine the way different parameters influence them, and results are detailed and available

226 on Github (see Appendix 1 in https://github.com/mdelacre/Effect-sizes/; it will be the

227 same for all estimators described later).

228   While Cohen's d_s is a consistent estimator, its bias and variance are substantial with

229 small sample sizes, even under the assumptions of normality and equal variances (Lakens,

230 2013). In order to compensate for Cohen's d_s bias with small sample sizes, Hedges & Olkin

231 (1985) has defined a bias-corrected version:

$$Hedges'\ g_s = Cohen's\ d_s \times \frac{\Gamma(\frac{df_{Student}}{2})}{\sqrt{\frac{df_{Student}}{2}} \times \Gamma(\frac{df_{Student}-1}{2})} \tag{10}$$

232   Where $df_{Student}$ has been defined in equation 8,and $\Gamma()$ is the gamma function. This

233 equation can be approximated as follows:

$$Hedges'\ g_s = Cohen's\ d_s \times \left(1 - \frac{3}{4N - 9}\right) \tag{11}$$

234   Hedges' g_s is theoretically unbiased when the assumptions of normality and equal

235 variances are met (see Table 1), and it has a smaller variance than Cohen's d_s, especially

236 with small sample sizes [7]. As Cohen's d_s, its variance depends on the total sample size (N),

237 the sample sizes ratio $\left(\frac{n_1}{n_2}\right)$ and the population effect size (δ_{Cohen}). How these parameters

238 influence the variance of Hedges' g_s will be summarized in a later section in which we will

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[7] $.52 \leq \left[\frac{\Gamma(\frac{df}{2})}{\sqrt{\frac{df}{2}} \times \Gamma(\frac{df-1}{2})}\right]^2 < 1$ for $3 \leq df < \infty$. The larger the total sample size, the smaller the difference between the variance of Cohen's d_s and Hedges' g_s.

239   compare different estimators through Monte Carlo simulations (see the section "Monte Carlo

240   Simulations").

241   While the pooled error term is the best choice when variances are equal between

242   groups (Grissom & Kim, 2001), it may not be well advised for use with data that violate this

243   assumption (Cumming, 2013; Grissom & Kim, 2001, 2005; Kelley, 2005, 2005; Shieh, 2013).

244   When variances are unequal between groups, the expression in equation 5 is no longer valid

245   because both groups don't share a common population variance. If we pool the estimates of

246   two unequal population variances, the estimator of effect size will be lower as it should be in

247   case of positive pairing (i.e. the group with the larger sample size is extracted from the

248   population with the larger variance) and larger as it should be in case of negative pairing

249   (i.e. the group with the larger sample size is extracted from the population with the smaller

250   variance). Because the assumption of equal variances across populations is very rare in

251   practice (Cain, Zhang, & Yuan, 2017; Delacre et al., 2017; Delacre, Leys, Mora, & Lakens,

252   2019; Erceg-Hurn & Mirosevich, 2008; Glass, Peckham, & Sanders, 1972; Grissom, 2000;

253   Micceri, 1989; Yuan, Bentler, & Chan, 2004), both Cohen's d_s and Hedges' g_s should be

254   abandoned in favor of an alternative robust to unequal population variances.

Table 1

Expentency, bias and variance of Cohen's d_s and Hedges' g_s under the assumptions that independent residuals are normally distributed with equal variances across groups.

	df	Expectancy	Variance
Cohen's d_s	$N-2$	$\delta_{cohen} \times \dfrac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$	$\dfrac{N \times df}{n_1 n_2 \times (df-2)} + \delta^2_{Cohen}\left[\dfrac{df}{df-2} - \left(\dfrac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right]$
		$\approx \dfrac{\delta_{Cohen}}{\left(1-\frac{3}{4N-9}\right)}$	$\approx \dfrac{N \times df}{n_1 n_2 \times (df-2)} + \delta^2_{Cohen}\left[\dfrac{df}{df-2} - \left(\dfrac{1}{1-\frac{3}{4N-9}}\right)^2\right]$
Hedges's g_s	$N-2$	δ_{Cohen}	$Var(Cohen's\ d_s) \times \left[\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}\right]^2$
			$\approx Var(Cohen's\ d_s) \times \left[1-\dfrac{3}{4N-9}\right]^2$

Note. Cohen's d_s is a biased estimator, because its expectation differ from the population effect size. On the other hand, Hedges' g_s is an unbiased estimator, because its expectation equals the population effect size; equations in Table 1 require $df \geq 3$ (i.e. $N \geq 5$).

258 **When variances are unequal between populations**

259   In his review, Shieh (2013) mentions three options available in the literature to deal

260 with the case of unequal variances: the sample mean difference divided by (A) the Glass's d_s,

261 (B) the Shieh's d_s and (C) the non pooled average of both variance estimates.

262   **Glass's d_s.**   When comparing one control group with one experimental group, Glass,

263 McGav, & Smith (2005) recommend using the standard deviation SD of the control group

264 as standardizer. It is also advocated by Cumming (2013), because, according to him, it is

265 what makes the most sense, conceptually speaking. This yields

$$Glass's\ d_s = \frac{\bar{X}_e - \bar{X}_c}{S_c} \tag{12}$$

266   Where $\bar{X}_e\ and\ \bar{X}_c$ are respectively the sample means of the experimental and control

267 groups, and S_c is the sample SD of the control group. One argument in favour of using the

268 SD of the control group as standardizer is the fact that it is not affected by the experimental

269 treatment. When it is easy to identify which group is the "control" one, it is therefore

270 convenient to compare the effect size estimation of different designs studying the same effect.

271 However, defining this group is not always obvious (Coe, 2002). This could induce large

272 ambiguity because depending of the chosen SD as standardizer, measures could be

273 substantially different (Shieh, 2013).

274   The distribution of Glass's d_s is defined as following (Algina, Keselman, & Penfield,

275 2006):

$$Glass's\ d_s \sim \sqrt{\frac{1}{n_c} + \frac{S_e^2}{n_e \times S_c^2}} \times t_{df,ncp} \tag{13}$$

276   Where n_c and n_e are respectively the sample sizes of the control and experimental

277 groups, and df and ncp are defined as follows:

$$df = n_c - 1 \tag{14}$$

$$ncp = \frac{\mu_c - \mu_e}{\sigma_c \times \sqrt{\frac{1}{n_c} + \frac{\sigma_e^2}{n_e \times \sigma_c^2}}} \tag{15}$$

Where μ_c and μ_e are respectively the mean of the populations control and experimental groups are extracted from. Thanks to equation 13, we can compute its theoretical expectancy variance when the assumptions of normality is met (See Table 2), and therefore determine which factors influence bias and variance, and how they do so (see Appendix 1).

Table 2

Expentency, bias and variance of Glass's d_s and Cohen's d'_s and Shieh's d_s under the assumptions that independent residuals are normally distributed.

	df	Expectancy	Variance
Glass's d_s	$n_c - 1$	$\delta_{glass} \times c_f$	$\frac{df}{df-2} \times \left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e \sigma_c^2}\right) + \delta_{Glass}^2 \left(\frac{df}{df-2} - c_f^2\right)$
Cohen's d'_s	$\frac{(n_1-1)(n_2-1)(s_1^2+s_2^2)^2}{(n_2-1)s_1^4+(n_1-1)s_2^4}$	$\delta'_{Cohen} \times c_f$	$\frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2} + (\delta'_{Cohen})^2 \left(\frac{df}{df-2} - c_f^2\right)$
		$\approx \delta'_{Cohen} \times \frac{4df-1}{4(df-1)}$	$\approx \frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2} + (\delta'_{Cohen})^2 \left[\frac{df}{df-2} - \left(\frac{4df-1}{4(df-1)}\right)^2\right]$
Shieh's d_s	$\approx \frac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$	$\delta_{Shieh} \times c_f$	$\frac{df}{(df-2)N} + \delta_{Shieh}^2 \left(\frac{df}{df-2} - c_f^2\right)$

Note. $c_f = \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$; all estimators are biased estimators, because their expectations differ from the population effect size δ; equations require $df \geq 3$ and at least 2 subjects per group.

²⁸⁴ **Shieh's d_s.** Kulinskaya & Staudte (2007) were the first to advice the use of a

²⁸⁵ standardizer that takes the sample sizes allocation ratios into account, in addition to the

²⁸⁶ variance of both samples. Shieh (2013), following Kulinskaya & Staudte (2007), proposed a

²⁸⁷ modification of the exact SD of the sample mean difference:

$$Shieh's\ d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_1^2/q_1 + S_2^2/q_2}}; \quad q_j = \frac{n_j}{N}(j = 1, 2) \tag{16}$$

²⁸⁸ where $N = n_1 + n_2$. Shieh's d_s is directly related with Welch's t-statistic:

$$t_{welch} = Shieh's\ d_s \times \sqrt{N} \tag{17}$$

²⁸⁹ Where $N = n_1 + n_2$. The exact distribution of Welch's t-statistic is more complicated

²⁹⁰ than the exact distribution of Student's t-statistic, but it follows a t-distribution with

²⁹¹ degrees of freedom and noncentrality parameters that can be approximated as follows, under

²⁹² the assumption of normality (Shieh, 2013; Welch, 1938):

$$df_{Welch} \approx \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1} + \frac{(\sigma_2^2/n_2)^2}{n_2-1}} \tag{18}$$

$$ncp_{Welch} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{n_1/N} + \frac{\sigma_2^2}{n_2/N}}} \times \sqrt{N} \tag{19}$$

²⁹³ The relationship described in equation 17 and the theoretical distribution of Welch's

²⁹⁴ t-statistic allow us to theoretically approximate the sampling distribution of Shieh's d_s.

²⁹⁵ Based on the sampling distribution of Shieh's d_s, we can estimate its theoretical expectancy

²⁹⁶ and variance under the assumption of normality (see Table 2), and therefore and therefore

²⁹⁷ determine which factors influence bias and variance, and how they do so (see Appendix 1).

298   It can be demonstrated that when variances and sample sizes are equal across groups,

299 the biases and variances of Shieh's d_s and Cohen's d_s are identical except for a constant, as

300 shown in equations 20 and 21:

$$Shieh's \ d_{s,bias} = 2 \times Cohen's \ d_{s,bias} \quad (considering \ \sigma_1 = \sigma_2 \ and \ n_1 = n_2) \qquad (20)$$

$$Shieh's \ d_{s,variance} = 4 \times Cohen's \ d_{s,variance} \quad (considering \ \sigma_1 = \sigma_2 \ and \ n_1 = n_2) \qquad (21)$$

301   Due to the relation described in equation 22 when sample sizes are equal between

302 groups (as explained in Appendix 2), such proportions mean that relative to their respective

303 true effect size, Cohen's d_s and Shieh's d_s are equally good. This is a good illustration of the

304 fact that biases and variances should always be studied relative to the population effect size

305 (and not in absolute terms), as we will do later.

$$Shieh's \ \delta_{n_1=n_2} = \frac{Cohen's \ \delta_{n_1=n_2}}{2} \qquad (22)$$

306   Except for this very specific situation, according to the statistical properties of Welch's

307 statistic under heteroscedasticity, Shieh's d_s accounts for the sample sizes allocation ratio.

308 The lack of generality caused by taking this specificity of the design into account has led

309 Cumming (2013) to question its usefulness in terms of interpretability: when keeping

310 constant the mean difference $(\bar{X}_1 - \bar{X}_2)$ as well as SD_1 and SD_2, Shieh's d_s will vary as a

311 function of the sample sizes allocation ratio (the dependency of Shieh's d_s value on the

312 sample sizes allocation ratio is illustrated in the following shiny application:

313 https://mdelacre.shinyapps.io/ShiehvsCohen/).

314   **Cohen's d'_s.** The sample mean difference, divided by the non pooled average of both

315 variance estimates was suggested by Welch (1938). This yields:

$$Cohen's\ d'_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(S_1^2 + S_2^2)}{2}}} \tag{23}$$

316   Where $\bar{X}_j$ is the mean and S_j is the standard deviation of the jth sample (j = 1,2). We

317 know the distribution of Cohen's d'_s (Huynh, 1989):

$$cohen's\ d'_s \sim \sqrt{\frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)(\sigma_1^2 + \sigma_2^2)}{2}} \times t_{df^*,ncp^*} \tag{24}$$

318   Where df^* and ncp^* are defined as follows:

$$df^* = \frac{(n_1 - 1)(n_2 - 1)(s_1^2 + s_2^2)^2}{(n_2 - 1)s_1^4 + (n_1 - 1)s_2^4} \tag{25}$$

$$ncp^* = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}}} \times \sqrt{\frac{n_1 n_2 (\sigma_1^2 + \sigma_2^2)}{2(n_2 \sigma_1^2 + n_1 \sigma_2^2)}} \tag{26}$$

319   Thanks to equation 26, we can compute its theoretical expectancy variance when the

320 assumptions of normality is met (See Table 2), and therefore determine which factors

321 influence bias and variance, and how they do so (see Appendix 1). This estimator has been

322 widely criticized, because:

323   - it results in a variance term of an artificial population and is therefore very difficult

324 to interpret (Grissom & Kim, 2001);

325   - unless both sample sizes are equal, the variance term does not correspond to the

326 variance of the mean difference (Shieh, 2013).

327 However, we will show throughout the simulation section that this estimator show very

328 good inferential properties.

329 **Glass's g_s, Shieh's g_s and Hedges' g'_s.** As for Cohen's d_s, an Hedges' correction

330 can be applied in order to compensate for the bias of Glass's d_s, Shieh's d_s and Cohen's d'_s

331 with small sample sizes (see Table 2). This correction has the following general form:

$$g_s = d_s \times \frac{\Gamma(\frac{\nu}{2})}{\sqrt{\frac{\nu}{2}} \times \Gamma(\frac{\nu-1}{2})} \tag{27}$$

332 Where ν are provided in equation 15 for Glass's g_s, in equation 25 for Hedges' g'_s and

333 in equation 18 for Shieh's g_s. The three corrected estimators are theoretically unbiased when

334 the assumptions of normality is met. Their variance are a function of the same factors as

335 their biased equivalent, however, due to the correction, they have a smaller variance,

336 especially with small sample size, as shown in Table 3. In summary:

- 337 The variances of Hedges' g'_s and Shieh's g_s depend on the total sample size (N), their
- 338 respective population effect size (δ), and the interaction between the sample sizes ratio
- 339 and the SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$.
- 340 The variance of Glass's g_s also depends on N, δ and $\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}$. In addition, there is also
- 341 a main effect of the SD-ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$ on its variance.

342 How these parameters influence the variance of the estimators will be summarized and

343 illustrated in the section dedicated to the Monte Carlo simulations.

Table 3

Expentency, bias and variance of Glass's d_s and Cohen's d'_s and Shieh's d_s under the assumptions that independent residuals are normally distributed.

	df	Expectancy	Variance
Glass's g_s	$n_c - 1$	δ_{glass}	$Var(Glass's\ d_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)} \right)^2$
Cohen's g'_s	$\dfrac{(n_1-1)(n_2-1)(s_1^2+s_2^2)^2}{(n_2-1)s_1^4+(n_1-1)s_2^4}$	δ'_{Cohen}	$Var(Cohen's\ d'_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)} \right)^2$
Shieh's g_s	$\approx \dfrac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$	δ_{Shieh}	$Var(Shieh's\ d_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)} \right)^2$

Note. $c_f = \dfrac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$; all estimators are unbiased estimators, because their expectations equal the population effect size δ; equations require $df \geq 3$ and at least 2 subjects per group.

344

345

Monte Carlo Simulations: assessing the bias, efficiency and consistency of 5

estimators

Method. We performed Monte Carlo simulations using R (version 3.5.0) to assess

the bias, efficiency and consistency of Cohen's g_s, Glass's g_s (using respectively the sample

SD of the first or second group as a standardizer), Hedges' g'_s and Shieh's g_s.

A set of 100,000 datasets were generated for 1,008 scenarios as a function of different

criterions that will be explained below. In 252 scenarios, samples were extracted from a

normally distributed population (in order to insure the reliability of our calculation method)

and in 756 scenarios, samples were extracted from non normal population distributions. In

order to assess the quality of estimators under realistic deviations from the normality

assumption, we referred to the review of Cain et al. (2017). Cain et al. (2017) investigated

1,567 univariate distributions from 194 studies published by authors in Psychological Science

(from January 2013 to June 2014) and the American Education Research Journal (from

January 2010 to June 2014). For each distribution, they computed the Fisher's skewness

(G1) and kurtosis (G2):

$$G_1 = \frac{\sqrt{n(n-1)}}{n-2} \frac{m_3}{\sqrt{(m_2)^3}} \tag{28}$$

with s = standard deviation, n = sample size, m_2 = second centered moment and m_3

= third centered moment.

$$G_2 = \frac{n-1}{(n-2)(n-3)} \times [(n+1)(\frac{m_4}{(m_2)^2} - 3) + 6] \tag{29}$$

with s = standard deviation, n = sample size and m_3=third centered moment. They

found values of kurtosis from G2 = -2.20 to 1,093.48. According to their suggestions,

throughout our simulations, we kept constant the population kurtosis value at the 99th

percentile of their distribution of kurtosis, i.e. G2=95.75. Regarding skewness, we simulated

367 population parameter values which correspond to the 1st and 99th percentile of their

368 distribution of skewness, i.e. respectively G1 = -2.08 and G1 = 6.32. We also simulated

369 samples extracted from population where G1 = 0, in order to assess the main effect of high

370 kurtosis on the quality of estimators. All possible combinations of population skewness and

371 kurtosis and the number of scenarios for each combination are summarized in Table 5.

Table 4

Number of Combinations of skewness and kurtosis in our simulations.

		Kurtosis		
		0	95.75	**TOTAL**
	0	252	252	**504**
Skewness	-2.08	/	252	**252**
	6.32	/	252	**252**
	TOTAL	**252**	**756**	**1008**

[372] *Note.* Fisher's skewness (G1) and kurtosis (G2) are presented in Table 5. The 252

[373] combinations where both G1 and G2 equal 0 correspond to the normal case.

[374]   For the 4 resulting combinations of skewness and kurtosis (see Table 5), all other

[375] parameter values were chosen in order to illustrate the consequences of factors identified as

[376] playing a key role on the variance of unbiased estimators. We manipulated the population

[377] mean difference $(\mu_1 - \mu_2)$, the sample sizes (n), the sample size ratio $(n\text{-ratio} = \frac{n_1}{n_2})$, the

[378] population *SD*-ratio (i.e. $\frac{\sigma_1}{\sigma_2}$), and the sample size and population variance pairing $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$.

[379] In our scenarios, μ_2 was always 0 and μ_1 varied from 1 to 4, in step of 1 (so does $\mu_1 - \mu_2$)[8].

[380] Moreover, σ_1 always equals 1, and σ_2 equals .1, .25, .5, 1, 2, 4 or 10 (so does $\frac{\sigma_1}{\sigma_2}$). The

[8] In the original plan, we had added 252 simulations in which μ_1 and μ_2 were both null. We decided to not present the results of these simulations, because the relative bias and the relative variance appeared to us to be very useful to fully understand the estimators comparison, and computing them is impossible when the real mean difference is zero.

simulations for which both σ_1 and σ_2 equal 1 are the particular case of homoscedasticity

(i.e. equal population variances across groups). Sample size of both groups (n_1 and n_2) were

20, 50 or 100. When sample sizes of both groups are equal, the n-ratio equals 1 (it is known

as a balanced design). All possible combinations of n-ratio and population SD-ratio were

performed in order to distinguish scenarios where both sample sizes and population variances

are unequal across groups (with positive pairing when the group with the largest sample size

is extracted from the population with the largest SD, and negative pairing when the group

with the smallest sample size is extracted from the population with the smallest SD) and

scenarios with no pairing between sample sizes and variances (sample sizes and/or

population SD are equal across all groups). In sum, the simulations grouped over different

sample sizes yield 4 conditions based on the n-ratio, population SD-ratio, and sample size

and population variance pairing, as summarized in Table 6.

Table 5

4 conditions based on the n-ratio, SD-ratio, and sample size and variance pairing.

		n-ratio		
		1	**>1**	**<1**
	1	a	b	b
SD-ratio	**>1**	c	d	d
	<1	c	d	d

Note. The n-ratio is the sample size of the first group (n_1) divided by the sample size

of the second group (n_2). When all sample sizes are equal across groups, the n-ratio equals 1.

When $n_1 > n_2$, n-ratio > 1, and when $n_1 < n_2$, n-ratio < 1. SD-ratio is the population SD

396   of the first group (σ_1) divided by the population *SD* of the second group (σ_2). When

397   $\sigma_1 = \sigma_2$, *SD*-ratio $= 1$. When $\sigma_1 > \sigma_2$, *SD*-ratio > 1. Finally, when $\sigma_1 < \sigma_2$, *SD*-ratio < 1.

398   ***Results.***   Before detailing estimators comparison for each condition, it might be

399   interesting to make some general comments.

400   1) We previously introduced the fact that raw bias and variances are sometimes

401   misleading. They can give the illusion of huge differences between two estimators, even

402   if these differences only reflect a change of unit (i.e. different population effect sizes).

403   To better understand this, imagine a sample of 15 people for whom we know the height

404   (in meters) and we compute a sample variance of 0.06838. If we convert sizes to

405   centimeters and compute the sample variance again, we find a measure of 683.8

406   (i.e. 10^3 larger). Both measures represent the same amount of variability, but they are

407   expressed in different units. Similar things occur when comparing the estimates of

408   different population measures. To avoid this possible confusion, we will only present

409   the relative bias and relative variance in all Figures (and anytime we will mention the

410   biases and variances in the results section, we will be referring to relative bias and

411   variance). For interested reader, the raw bias and variance are available on Github.

412   2) In a purpose of readability, we used different scales for the ordinate axis of our plots,

413   for both relative bias and relative variance, when $G_1 = 6.32$ and $G_2 = 95.75$, in

414   comparisons with all other conditions. Indeed, relative biases and variances are much

415   larger for this specific combination G_1/G_2.

416   3) Throughout this section, we will **compare** the quality of different estimators. We

417   chose very extreme (although realistic) conditions, and we know that none of the

418   parametric measures of effect size will be robust against such extreme conditions. Our

419   goal is therefore to study the robustness of the estimators against normality violations

420   only in comparison with the robustness of other indicators, but not in absolute terms.

421   After these general remarks, we will analyze each condition separately. In all Figures

422   presented below, averaged relative bias and relative variance for each sub-condition are

423   presented under five different configurations of distributions, using the legend described in

424   Figure 2. When describing the Glass's g_s estimators, we will systematically call "control

425   group" the group the standardizer is computed from (i.e. the first group when using SD_1 as

426   standardizer, the second group when using SD_2 as standardizer). The other group will be

427   called "experimental group".

■ Hedges' g_s ■ Glass's $g_s(\sigma = S_1)$ ■ Glass's $g_s(\sigma = S_2)$ □ Shieh's g_s □ Hedges' g_s'

Figure 2. Legend

428   *When variances are equal across groups.*   Figures 3 and 4 represent configurations

429   where the equality of variances assumption is met. According to our expectations, one

430   observes that the bias of all estimators is approximately zero as long as the normality

431   assumption is met (first column in both Figures)[9]. However, the further from the normality

432   assumption (i.e. when moving from left to right in Figures), the larger the bias.

433   Figure 3 illustrates scenarios where both population variances and sample sizes are

434   equal across groups (condition a). One can first notice that all estimators are consistent, as

435   their bias and variance decrease when the total sample size increase. For any departure from

436   the normality assumption, the biases of Hedges' g_s, Hedges' g_s' and Shieh's d_s are similar[10]

---

[9] When looking at the relative bias for all estimators, the maximum departure from zero is 0.0064 when
sample sizes are equal across groups, and 0.0065 with unequal sample sizes.

[10] While the biases of Cohen's d_s, Cohen's d_s' and Shieh's d_s are identical, the bias of Hedges' g_s is
marginally different than the biases of Hedges' g_s' and Shieh's g_s (these last two being identical). Indeed,
because of the sampling error, difference remain between sample variances, even when population variances
are equal groups. Because the Hedges' correction applied to Cohen's d_s does not imply the sample variances
(unlike the one applied on both other estimators), the bias of Hedges' g_s is slighly different than the bias of
Hedges' g_s' and Shieh's g_s

437   and smaller than the bias rate of both glass's g_s estimates (i.e. using S_1 and S_2), because the

438   three firsts depend on the total sample size (N; the larger the total sample size, the lower

439   the bias), while Glass's g_s only depends on the control group size (n_c; the larger the control

440   group size, the lower the bias). Moreover, when samples are extracted from skewed

441   distributions, Glass's g_s will show unequal biases as a function of the chosen standardizer (S_1

442   or S_2), even if both S_1 and S_2 are estimates of the same population variance, based on the

443   same sample size. This is due to non-null correlations of opposite sign between the mean

444   difference ($\bar{X}_1 - \bar{X}_2$) and respectively S_1 and S_2. For interested reader, when a non nul

445   correlation occur between the sample means difference ($\bar{X}_1 - \bar{X}_2$) and the estimators

446   standardizers as well as the way they impact the biases and variances of estimators is

447   detailed in Appendix 3.

448   **VARIANCE** : TEXTE THEORIQUE, DE QUOI DEPEND CHAQUE

449   ESTIMATEUR: voir annexe 1. Pour le reste, en discuter avec Christophe. Ensuite, dire ce

450   qui diffère quand on s'écarte de la normalité.

451   Figure 4 illustrates scenarios where population variances are equal across groups and

452   sample sizes are unequal (condition b). For any departures from the normality assumptions,

453   Hedges' g_s shows the smallest bias. Hedges' g_s and Hedges' g_s' are consistent estimators

454   (i.e. the larger the sample sizes, the lower the bias), unlike Shieh's g_s and Glass's g_s. The

455   bias of Glass's g_s does not depend either on the size of the experimental group or on the

456   total sample size. The only way to decrease the bias of Glass's d_s is therefore to add subjects

457   in the control group[11]. Regarding Shieh's g_s, while increasing sample sizes will decrease the

458   bias for a given sample size ratio, there is a large impact of the sample sizes ratio in order

459   that when the sample sizes ratio moves away from 1 by adding subjects, bias increases. On

460   the other side, when the sample sizes ratio moves closer to 1 by adding subjects, the bias will

─────

[11] e.g. when there are 50 subjects in the control group, the bias will remain identical when there are
respectively 20 and 100 subjects in the experimental group

461   decrease[12].

462   When samples are extracted from skewed distributions and have unequal sizes

463   (i.e. $n_1 \neq n_2$, the two last columns in Figure 4), for a constant total sample size, $Glass'sg_s$,

464   Shieh's g_s and Hedge's g_s' will show unequal biases depending on which group is the largest

465   one (e.g. when distributions are right-skewed, the bias of all estimator when n_1 and n_2 are

466   respectively 50 and 20 are not the same as their bias when n_1 and n_2 are respectively 20 and

467   50). This is due to non-null correlations of opposite sign between the mean difference

468   $(\bar{X}_1 - \bar{X}_2)$ and their respective standardizers depending on which group is the largest one, as

469   detailed in Appendix 3.

470   **VARIANCE**: the variance of Glass's d_s depends on the sample size of both control

471   and experimental groups, however, Glass's d_s will be less variable when there are more

472   subjects in the control group than when there are more subjects in the experimental group.

473   Finally, as previously observed in Figure 3, the bias and variances of Glass's estimates also

474   depends on the correlation between $\bar{X}_1 - \bar{X}_2$ and the standardizer, when distribution are

475   skewed. [13]

476   In conclusion, Glass's g_s should always be avoided when the equality of variance

─────

[12] e.g. when there are 20 subjects in one group, the bias of Shieh's d_s is larger when there are 100 subjects in

the other group than when there are only 50 subjects in the other group, because despite the increasing total

sample size, the sample size ratio increases. On the other side, when there are 100 subjects in one group, the

bias and variance of Shieh's d_s are smaller when there are 50 subjects in the other gruop than when there are

only 20 subjects in the other group, because the sample size ratio decreases

[13] In our simulations, the worst configuration will occure when the standardizer is computed with the smaller

group and is negatively correlated with the sample means difference (i.e. when choosing SD2 when

distributions are right-skewed, and when choosing SD1 when distributions are left-skewed). Again, we should

remind that in all our simulations, the population mean difference is positive. If mean difference were

negative, glass's d_s would be more biased and variable when the chosen standardizer is positively correlated

with the mean difference and associated with the smaller sample size.

⁴⁷⁷ assumption is met. Hedge's g_s, Hedges' g'_s and Shieh's g_s are equally performant as long as

⁴⁷⁸ the sample size ratio is close to 1. However, when designs are highly unbalanced, Shieh's g_s

⁴⁷⁹ is not consistent anymore. While Hedge's g'_s is consistent, Hedges's g_s remains a better

⁴⁸⁰ estimator.

⁴⁸¹   *When variances are unequal across groups.*   Figures 5 to 10 represent configurations

⁴⁸² where the equality of variances assumption is not met. According to our expectations, one

⁴⁸³ observes that the bias of all estimators is approximately zero as long as the normality

⁴⁸⁴ assumption is met (first column in the three Figures), and the further from the normality

⁴⁸⁵ assumption (i.e. when moving from left to right in Figures), the larger the bias[14]. You might

⁴⁸⁶ find it surprising that the bias of Hedges' g_s remain very small troughout these conditions.

⁴⁸⁷ As reminded in the section "Different measures of effect size", Hedges' g_s should be avoided

⁴⁸⁸ when population variances and sample sizes are unequal across groups, because of the pooled

⁴⁸⁹ error term. When pooling the estimates of two unequal population variances, the resulting

⁴⁹⁰ estimator will be lower (in case of positive pairing) or larger (in case of negative pairing) as it

⁴⁹¹ should be. At the same time, when pooling two unequal population variances, the population

⁴⁹² effect size will also be lower (in case of positive pairing) or larger (in case of negative pairing)

⁴⁹³ as it should be. As a consequence, the distorsion cannot be seen through the difference

⁴⁹⁴ between the expected estimator and the population effect size measure. For this reason, the

⁴⁹⁵ bias and variance of Hedges' g_s will not be taken into account in the following comparisons.

⁴⁹⁶   Figures 5 and 6 are dedicated to scenarios where population variances are unequal

⁴⁹⁷ between groups and sample sizes are equal (condition c). In Figure 5, scenarios are

⁴⁹⁸ subdivided as a function of the sample sizes and one can notice that all estimators are

⁴⁹⁹ consistent, as their bias and variance decrease when the total sample size increases. In

⁵⁰⁰ Figure 6, scenarios are subdivided as a function of the SD-ratio. Because the comparison

⁵⁰¹ pattern remains very similar for all sample sizes, we present only scenarios when sample sizes

---

[14] When looking at the relative bias for all estimators, the maximum departure from zero is 0.0173 when

sample sizes are equal across groups, and 0.0274 when both sample sizes and variances differ across groups.

502 equal 20. One observes then that the biases of Shieh's g_s and Hedges' g_s' are identical, for

503 any departures from the normality assumption. When distributions are symmetric (second

504 column in Figure 6), the bias of both Shieh's g_s and Hedges' g_s' depend on the SD-ratio in

505 order that the larger the difference between σ_1 and σ_2, the larger the bias. On the other side,

506 the bias of Glass's g_s does not depend on the SD-ratio, but it remains larger than the bias of

507 Shieh's g_s and Hedges' g_s', even when the SD-ratio is very large $\left(i.e. \ \frac{\sigma_1}{\sigma_2} = 10 \ or \ 0.1\right)$.

508 However, when samples are extracted from skewed distributions, the bias of Glass's g_s is

509 sometimes smaller and sometimes larger than the bias of Shieh's g_s and Hedge's g_s'. This is

510 mainly due to the fact that when two samples of same sizes are extracted from two skewed

511 distributions with unequal variances (i.e. $\sigma_1 \neq \sigma_2$, the two last columns in Figure 6), there

512 will be non-null correlations of opposite sign between the mean difference $(\bar{X}_1 - \bar{X}_2)$ and the

513 standardizer of *all* estimators, depending on which population variance is the largest one [15].

514   **VARIANCE**: Glass's more variable than Shieh's g_s and Hedge's g_s'. Moreover, the

515 relative variance of Glass's d_s is larger when computing the standardizer based on the sample

516 extracted from the less variable population, as shown in Figures 6 and **??**.

517   Figures 7 to 10 are dedicated to scenarios where both samples sizes and population

518 variances differ across groups. Due to a high number of combinations between the sample

519 sizes-ratio and the variances-ratio in our simulations, we decided to present only some

520 conditions. Because equations in Table 3 revealed an interaction effect between the sample

521 sizes ratio and the SD-ratio on biases and variances of Hedge's g_s' and Shieh's g_s (see

522 Appendix 1), we chose to present all configurations where the total sample sizes is either 150

523 (Figures 7 and 8), or 120 (Figures 9 and 10), in order to compare the effect of the SD-ratio

---

[15] When population variances are unequal, a non-null correlation occurs between standardizers estimates and
$\bar{X}_1 - \bar{X}_2$. The sign of the correlation between the standardizer and the means difference will be the same as
the sign of the correlation between the mean difference and the estimate of the larger population variance.
For interested readers, this is detailed in Appendix 3.

524   on biases and variances when the sample sizes ratio is .2, .5, 2 or 5.

525   When distributions are symmetric, we can observe that for a constant total sample size
526   (N), the bias of Glass's g_s only depends on the size of the control group and is therefore not
527   impacted by the SD-ratio. On the other side, the biases of Hedge's g'_s and Shieh's g_s are
528   always smaller when there is a positive pairing between sample sizes and variances. Moreover,
529   they are also impacted by an interaction between the SD-ratio and the sample sizes ratio:
530   - Regarding Hedge's g'_s, the closer the SD-ratio is from 1, the smaller the bias;
531   - Regarding Shieh's g'_s, the further the SD-ratio is from 1, the further from 1 will also
532   be the sample sizes ratio associated with the smallest bias [16].

533   When distributions are skewed,

534   Revoir ce texte ci-dessous. Figure **??** shows that when variances are unequal, and the
535   largest group is associated with largest variance, the more biased and variable estimator is
536   Glass's d_s when choosing the standard deviation of the smallest group as standardizer. REM:
537   AGAIN ONE OBSERVE THE SAME INTERACTION EFFECT BETWEEN
538   STANDARDISER IN GLASS MEASURE AND SENSE OF ASYMMETRY AS
539   OBSERVED FOR FIGURE 3 (IN SAME DIRECTION: WITH NEGATIVE SKEWNESS,
540   WORST WHEN CHOOSING SD1 AND WHEN POSITIVE SKEWNESS, WORST WHEN
541   CHOOSING SD2). Glass's d_s when choosing the standard deviation of the largest group as
542   standardizer, Shieh's d_s and transformed Shieh's d^*_s perform very similarly, both in terms of
543   bias and efficiency.

544   Figure **??** shows that when variances are unequal, and the largest group is associated
545   with smallest variance, as in all other configurations, the more biased and variable estimator

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[16] When the sample sizes ratio equals 2 (or .5), the smallest bias occurs when the SD-ratio equals 2 (or .5;
see Figures 7 and 8). On the other side, when the sample sizes ratio equals 5 (or .2), the smallest bias occurs
when the SD-ratio equals 4 (or .25; see Figures 9 and 10).

is Glass's d_s when choosing the standard deviation of the smallest group as standardizer (sauf quand asymetrie négative... not true anymore when there is asymmetry... explain it).

In summary, Cohen's d_s remains the best measure when the asssumption of equal variances is met. When variances are unequal across populations, Cohen's d_s performs exactly as well as Shieh's d_s and transformed Shieh's d_s^*, as long as sample sizes are equal across groups. However, when variances and sample sizes are both unequal across groups, Cohen's d_s becomes irrelevant. Glass's d_s is most of the time the more biased and variable measure. Only under very specific conditions (when there is a negative correlation between sample sizes and variances and the sample size of the control group is larger than the sample size of the experimental group), Glass's d_s performs the best in comparison with all other estimators.

Conclusion.   Algina, J., Keselman, H. J., & Penfield, R. D. (2006). Confidence intervals for an effect size when variances are not equal. *Journal of Modern Applied Statistical Methods*, *5*(1), 1–13. doi:10.22237/jmasm/1146456060

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Figure 3. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are equal across groups (condition a)
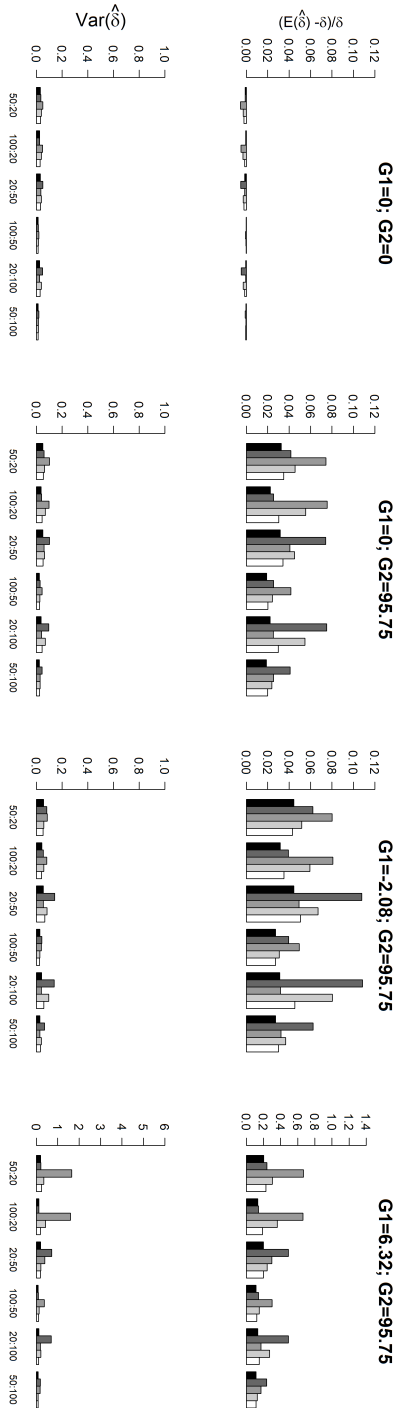
Figure 4. Bias and efficiency of estimators of standardized mean difference, when variances are equal across groups and sample sizes are unequal (condition b)
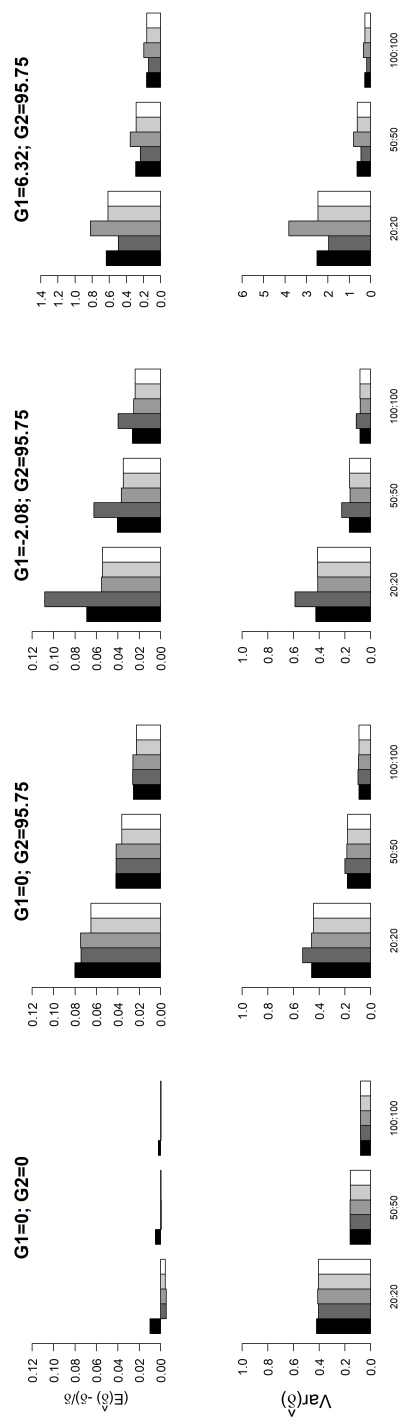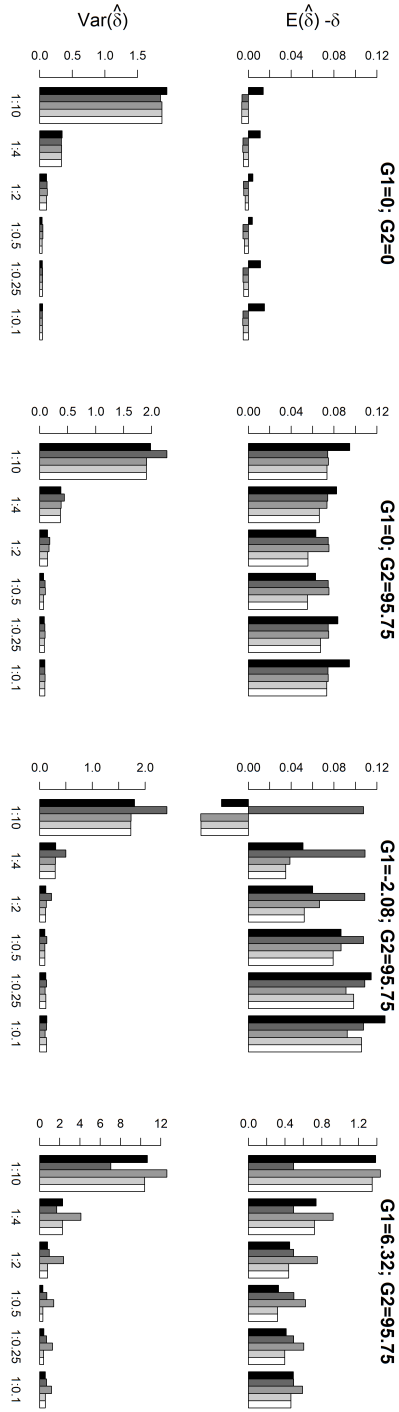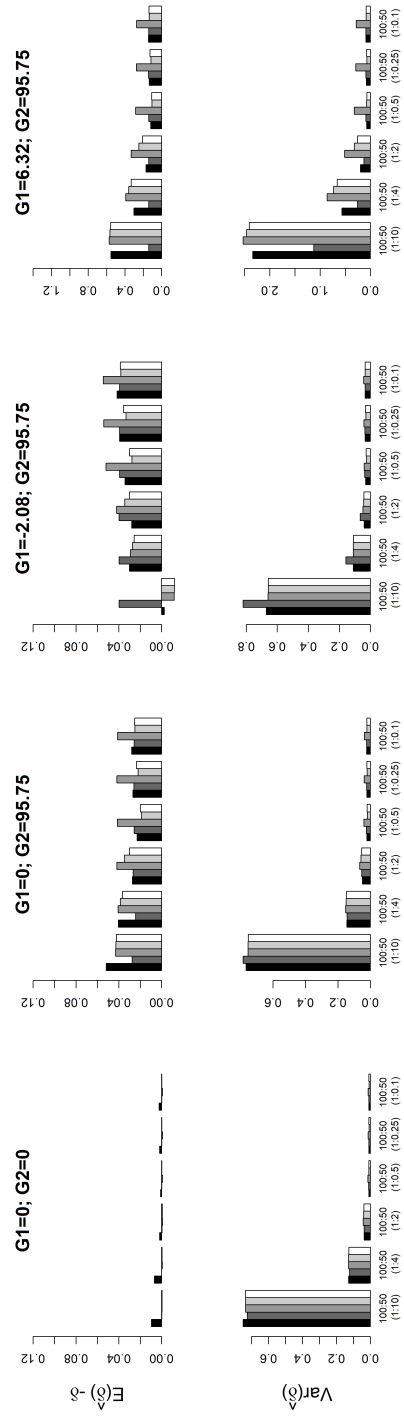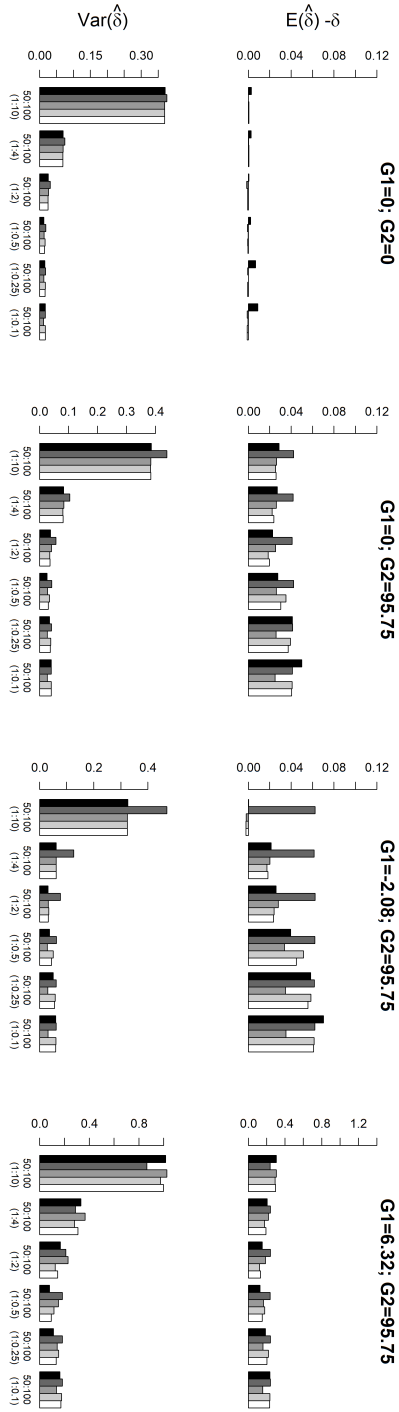
Figure 5. Bias and efficiency of estimators of standardized mean difference, when variances are unequal across groups and sample sizes are equal (condition c), as a function of *n*-ratio

Figure 6. Bias and efficiency of estimators of standardized mean difference, when variances are unequal across groups and sample sizes are equal (condition c) as a function of the SD-ratio (when $n_1 = n_2 = 20$)

Figure 7. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 150, and $n_1 > n_2$

Figure 8. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 150, and $n_1 < n_2$
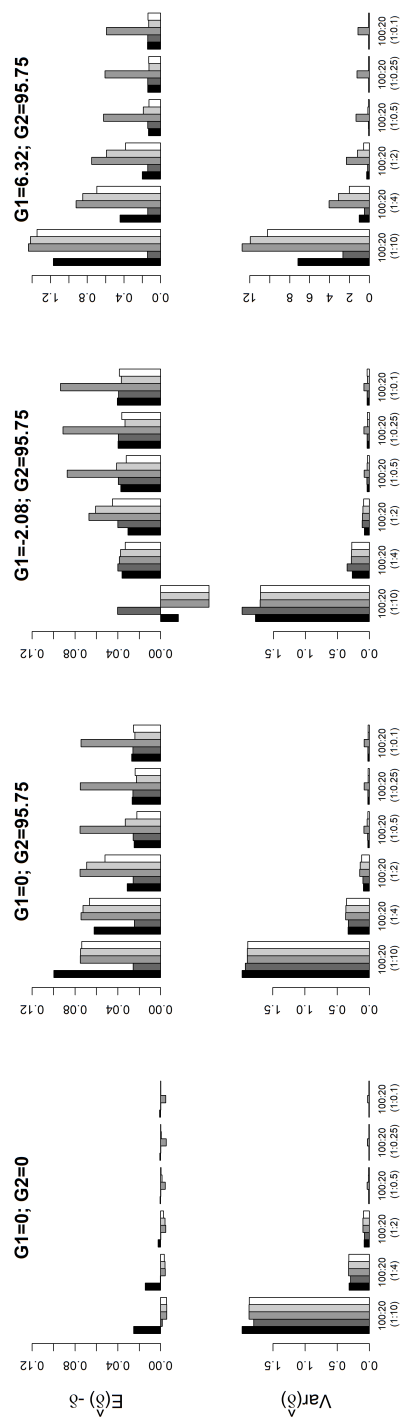
Figure 9. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 120, and $n_1 > n_2$
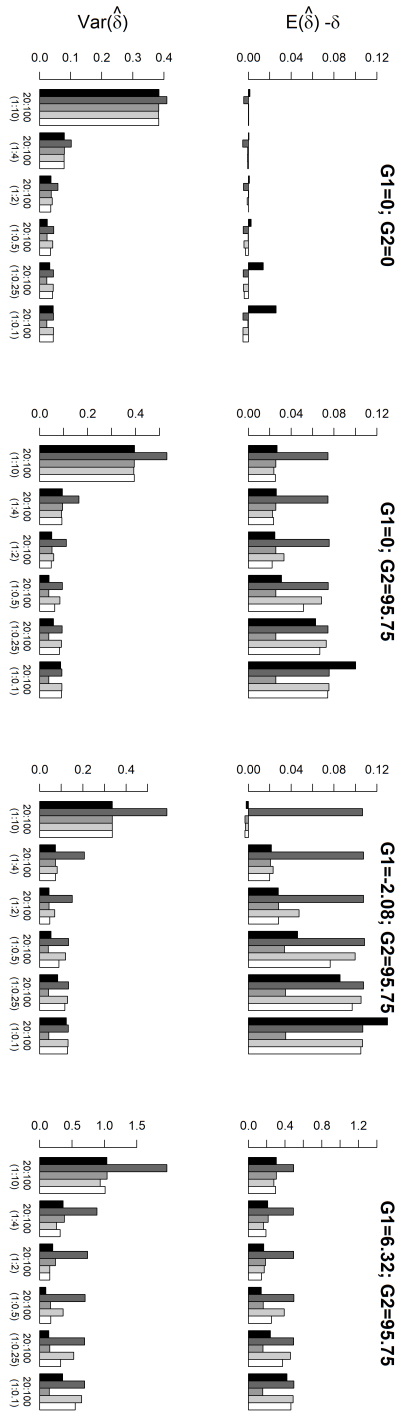
Figure 10. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 120, and $n_1 < n_2$

Appendix

Appendix 1: The mathematical study of Shieh's δ

Paste Appendix 1 when it will be finished

Appendix 2: Confidence intervals

Paste Appendix 2 when it will be finished

Appendix 3: a priori power analyses

Paste Appendix 3 when it will be finished (Cumming & Finch, 2001)

Cumming, G., & Finch, S. (2001). A primer on the understanding, use, and calculation of confidence intervales that are based on central and noncentral distributions. *Educational and Psychological Measurement*, *61*(532), 532–574.