1                                   Simulation checks

2                                   Delacre Marie[1]

3                                      [1] ULB

4                                                  Simulation checks

5                                        **Supplemental Material 2**

6          In order to insure the reliability of our calculation method, for all scenarios where

7  $G_1 = G_2 = 0$, we compared empirical means and variances of all estimators (i.e. means and

8  variances of all estimates) with theoretical means and variances (i.e. expected means and

9  variances, computed based on equations in Tables 1, 2 and 3). Because we can draw exactly

10  the same conclusions for biased and unbiased estimators, we will simultaneously present

11  results for **biased** (Cohen's $d_s$, Glass's $d_s$ using either $sd_1$ or $sd_2$ as standardizer, Shieh's $d_s$

12  and Cohen's $d'_s$) and **unbiased estimators** (Hedges' $g_s$, Glass's $g_s$ using either $sd_1$ or $sd_2$

13  as standardizer, Shieh's $g_s$ and Hedges' $g'_s$). Results will be subdivided into 4 conditions:

14          - When population variances and sample sizes are equal across groups (condition a; see

15  Figures A3.1 and A3.5 for respectively biased and unbiased estimators);

16          - When population variances are equal across groups and sample sizes are unequal

17  (condition b; see Figures A3.2 and A3.6 for respectively biased and unbiased estimators);

18          - When population variances are unequal across groups and sample sizes are equal

19  (condition c; see Figures A3.3 and A3.7 for respectively biased and unbiased estimators);

20          - When population variances and sample sizes are unequal across groups (condition d;

21  see Figures A3.4 and A3.8 for respectively biased and unbiased estimators).

22          Because the equations of theoretical means and variances of Cohen's $d_s$ and Hedges' $g_s$

23  rely on the assumption of normality and equality of population variances, we expect

24  empirical and theoretical parameters to be very close only in conditions a and b. For all

25  other estimators, the equations of theoretical means and variances rely solely on the

26  assumption of normality and therefore, we expect empirical and theoretical parameters to be

27  very close in all conditions.

28          On average, empirical means (and variances) of all estimators are very close to

29  theoretical expectations, when population variances are equal across groups, with equal

30  sample sizes (condition a; see Table A3.1) or unequal sample sizes (condition b; see Table

31  A3.2).

32       When population variances are unequal across groups (conditions c and d; see Tables

33  A3.3 and A3.4), empirical means (and variances) of Cohen's $d'_s$ (Cohen's $g'_s$) and Shieh's $d_s$

34  (Shieh's $g_s$) are still very close to theoretical expectations. Regarding Glass's $d_s$ (Glass's $g_s$),

35  while empirical variances remain very close to theoretical expectations, on average, one

36  observes a larger average departure between empirical and theoretical means when using

37  $SD_2$ as standardizer. However, when looking at details in results for each scenarios (see

38  "biased_condC.xlsx", "biased_condD.xlsx", "unbiased_condC.xlsx" and

39  "unbiased_condD.xlsx" in Supplemental Material 2), one notices that the larger the

40  population effect size, the larger the departure between empirical and theoretical means, and

41  that relative to the population effect size, departures between empirical and theoretical

42  means are always very small. On the other side, both empirical bias and variance of Cohen's

43  $d_s$ (Hedges' $g_s$) highly depart from theoretical expectations, even when looking and relative

44  departures to the population effect size, especially when sample sizes are unequal across

45  groups (condition d; see Table A3.4), which is not surprising, as Cohen's $d_s$ (Hedges' $g_s$)

46  relies on the equality of population variances assumption.

**Table A3.1**

Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_{\delta}|$ — Ratio between empirical and theoretical variances $S_{\hat{\delta}}^2/\sigma_{\delta}^2$

| Estimator ($\hat{\delta}$) | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
|---|---|---|---|---|---|---|---|---|
| Cohen's $d_s$ | 0,012 | 0,000 | 0,002 | 0,003 | 1,006 | 0,910 | 0,976 | 0,028 |
| Glass's $d_{s,1}$ | 0,022 | 0,000 | 0,004 | 0,006 | 1,006 | 0,897 | 0,966 | 0,033 |
| Glass's $d_{s,2}$ | 0,023 | 0,000 | 0,005 | 0,007 | 1,005 | 0,889 | 0,966 | 0,035 |
| Cohen's $d'_s$ | 0,012 | 0,000 | 0,002 | 0,003 | 1,006 | 0,910 | 0,976 | 0,028 |
| Shieh's $d_s$ | 0,006 | 0,000 | 0,001 | 0,002 | 1,006 | 0,910 | 0,976 | 0,028 |

**Table A3.2**

| Estimator ($\widehat{\delta}$) | Absolute deviation between empirical and theoretical means $\|E(\widehat{\delta})-\mu_\delta\|$ | | | | | Ratio between empirical and theoretical variances $S^2_{\widehat{\delta}}/\sigma_\delta$ | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | Max | Min | Mean | Standard deviation | | Max | Min | Mean | Standard deviation | |
| Cohen's $d_s$ | 0,005 | 0,000 | 0,001 | 0,001 | | 1,017 | 0,951 | 0,985 | 0,017 | |
| Glass's $d_{s,1}$ | 0,019 | 0,000 | 0,004 | 0,006 | | 1,006 | 0,891 | 0,966 | 0,037 | |
| Glass's $d_{s,2}$ | 0,027 | 0,000 | 0,005 | 0,007 | | 1,015 | 0,881 | 0,968 | 0,036 | |
| Cohen's $d'_s$ | 0,010 | 0,000 | 0,003 | 0,002 | | 1,007 | 0,902 | 0,965 | 0,034 | |
| Shieh's $d_s$ | 0,008 | 0,000 | 0,002 | 0,002 | | 1,005 | 0,865 | 0,945 | 0,048 | |

**Table A3.3**

| Estimator ($\hat{\delta}$) | Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$ | | | | Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma^2_\delta$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
| Cohen's $d_s$ | 0,080 | 0,000 | 0,010 | 0,015 | 1,753 | 1,005 | 1,175 | 0,208 |
| Glass's $d_{s,1}$ | 0,037 | 0,000 | 0,005 | 0,007 | 1,004 | 0,888 | 0,973 | 0,030 |
| Glass's $d_{s,2}$ | 0,230 | 0,000 | 0,012 | 0,033 | 1,008 | 0,883 | 0,974 | 0,032 |
| Cohen's $d'_s$ | 0,036 | 0,000 | 0,003 | 0,006 | 1,007 | 0,874 | 0,975 | 0,033 |
| Shieh's $d_s$ | 0,018 | 0,000 | 0,002 | 0,003 | 1,007 | 0,874 | 0,975 | 0,033 |

**Table A3.4**

| Estimator ($\hat{\delta}$) | Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$ | | | | Ratio between empirical and theoretical variances $S^2_{\hat\delta}/\sigma_\delta$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
| Cohen's $d_s$ | 0,252 | 0,000 | 0,015 | 0,034 | 5,624 | 0,208 | 1,638 | 1,357 |
| Glass's $d_{s,1}$ | 0,026 | 0,000 | 0,005 | 0,006 | 1,009 | 0,881 | 0,972 | 0,033 |
| Glass's $d_{s,2}$ | 0,219 | 0,000 | 0,012 | 0,031 | 1,011 | 0,872 | 0,973 | 0,036 |
| Cohen's $d'_s$ | 0,030 | 0,000 | 0,003 | 0,006 | 1,011 | 0,860 | 0,974 | 0,034 |
| Shieh's $d_s$ | 0,009 | 0,000 | 0,001 | 0,002 | 1,011 | 0,867 | 0,970 | 0,036 |

**Table A3.5**

Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_{\hat{\delta}}|$ and Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma^2_{\hat{\delta}}$

| Estimator ($\hat{\delta}$) | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
|---|---|---|---|---|---|---|---|---|
| Cohen's $d_s$ | 0,011 | 0,000 | 0,002 | 0,003 | 1,006 | 0,911 | 0,976 | 0,028 |
| Glass's $d_{s,1}$ | 0,021 | 0,000 | 0,004 | 0,006 | 1,006 | 0,897 | 0,966 | 0,033 |
| Glass's $d_{s,2}$ | 0,022 | 0,000 | 0,004 | 0,007 | 1,005 | 0,889 | 0,966 | 0,035 |
| Cohen's $d'_s$ | 0,015 | 0,000 | 0,003 | 0,004 | 1,006 | 0,908 | 0,975 | 0,029 |
| Shieh's $d_s$ | 0,008 | 0,000 | 0,002 | 0,002 | 1,006 | 0,908 | 0,975 | 0,029 |

**Table A3.6**

| Estimator ($\hat{\delta}$) | Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$ | | | | Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
| Cohen's $d_s$ | 0,005 | 0,000 | 0,001 | 0,001 | 1,017 | 0,951 | 0,985 | 0,017 |
| Glass's $d_{s,1}$ | 0,018 | 0,000 | 0,004 | 0,005 | 1,006 | 0,891 | 0,966 | 0,037 |
| Glass's $d_{s,2}$ | 0,026 | 0,000 | 0,004 | 0,006 | 1,015 | 0,881 | 0,968 | 0,036 |
| Cohen's $d'_s$ | 0,010 | 0,000 | 0,003 | 0,003 | 1,007 | 0,925 | 0,972 | 0,027 |
| Shieh's $d_s$ | 0,007 | 0,000 | 0,002 | 0,002 | 1,007 | 0,900 | 0,959 | 0,037 |

**Table A3.7**

Absolute deviation between empirical and theoretical means

$|E(\hat{\delta})-\mu_{\delta}|$

Ratio between empirical and theoretical variances

$S^2_{\hat{\delta}}/\sigma^2_{\hat{\delta}}$

| Estimator ($\hat{\delta}$) | Max | Min | Mean | Standard deviation | Max | Min | Mean | Standard deviation |
|---|---|---|---|---|---|---|---|---|
| Cohen's $d_s$ | 0,079 | 0,000 | 0,010 | 0,015 | 1,753 | 1,005 | 1,175 | 0,208 |
| Glass's $d_{s,1}$ | 0,036 | 0,000 | 0,005 | 0,007 | 1,004 | 0,888 | 0,973 | 0,030 |
| Glass's $d_{s,2}$ | 0,221 | 0,000 | 0,012 | 0,032 | 1,008 | 0,883 | 0,974 | 0,032 |
| Cohen's $d'_s$ | 0,034 | 0,000 | 0,003 | 0,006 | 1,008 | 0,890 | 0,978 | 0,029 |
| Shieh's $d_s$ | 0,017 | 0,000 | 0,002 | 0,003 | 1,008 | 0,890 | 0,978 | 0,029 |

**Table A3.8**

| Estimator ($\widehat{\delta}$) | Absolute deviation between empirical and theoretical means $\|E(\widehat{\delta}) - \mu_\delta\|$ | | | | | Ratio between empirical and theoretical variances $S^2_{\widehat{\delta}} / \sigma_\delta$ | | | | |
| | Max | Min | Mean | Standard deviation | | Max | Min | Mean | Standard deviation |
| Cohen's $d_s$ | 0,250 | 0,000 | 0,015 | 0,034 | | 5,624 | 0,208 | 1,638 | 1,357 |
| Glass's $d_{s,1}$ | 0,025 | 0,000 | 0,004 | 0,006 | | 1,009 | 0,881 | 0,972 | 0,033 |
| Glass's $d_{s,2}$ | 0,210 | 0,000 | 0,012 | 0,030 | | 1,011 | 0,872 | 0,973 | 0,036 |
| Cohen's $d'_s$ | 0,029 | 0,000 | 0,003 | 0,005 | | 1,011 | 0,882 | 0,977 | 0,029 |
| Shieh's $d_s$ | 0,008 | 0,000 | 0,001 | 0,002 | | 1,011 | 0,881 | 0,973 | 0,032 |