1   Theoretical variance, as a function of population parameters

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11   Theoretical variance, as a function of population parameters

12   **The variance**

13   Note: while we focus on the theoretical variance of biased estimators (Cohen's d_s,

14   Glass's d_s, Shieh's d_s and Cohen's d_s^*) when the normality assumption is met, it is interesting

15   to notice that our main conclusions seem to generalize to biased estimators when samples are

16   extracted from symmetric distributions. Moreover, unbiased estimators depend on the same

17   factors as biased estimators, so our conclusions remain similar for unbiased estimators when

18   samples are extracted from heavy-tailed symmetric distributions.

19   **Cohen's d_s**

20   **When variances are equal across populations.**

21   ***When $\delta_{Cohen} = 0$.***   When the population effect size is zero, the variance of Cohen's

22   d_s can be simplified as follows:

$$Var_{Cohen's \ d_s} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

23   The **variance** of Cohen's d_s is a function of total sample size (N) and the sample sizes

24   allocation ratio ($\frac{n_1}{n_2}$):

25   • The larger the total sample size, the lower the variance. The variance tends to zero

26   when the total sample size tends to infinity (see Figure 1);

27   • The further the sample sizes allocation ratio is from 1, the larger the variance (see

28   Figure 2).

29   ***When $\delta_{Cohen} \neq 0$.***   While the variance of Cohen's d_s still depends on the total

30   sample size and the sample sizes allocation ratio, it also depends on the population effect

31   size (δ_{Cohen}). The larger the population effect size, the larger the variance. Note that the

32 effect of the population effect size decreases when sample sizes increase since

$$\lim_{n_1 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

33   This is illustrated in Figure 3.

34   **In summary.**   The variance of Cohen's d_s is a function of the population effect size

35 (δ_{Cohen}), the total sample size (N) and the sample sizes ratio ($\frac{n_1}{n_2}$):

36   • The variance decreases when the total sample size increases;

37   • The variance also decreases when the sample sizes ratio gets closer to 1;

38   • Finally, the variance increases when δ_{Cohen} increases. Note that the effect of δ_{Cohen} is

39   moderated by the total sample size (the larger N, the smaller the effect of δ_{Cohen} on

40   the variance).

41 **Glass's d_s**

42   **When variances are equal across populations.**

43   ***When $\delta_{Glass} = 0$.***   When the population effect size is zero, the variance of Glass's d_s

44 can be simplified as follows:

$$Var_{Glass's\ d_s} = \frac{n_c - 1}{n_c - 3} \left(\frac{1}{n_c} + \frac{1}{n_e} \right)$$

45 . In this configuration, the **variance** of Glass's d_s is a function of the sample sizes of both

46 control (n_c) and experimental (n_e) groups as well as of the sample sizes allocation ratio ($\frac{n_c}{n_e}$):

47   • The larger the sample sizes, the lower the variance (Figure 4);

48   The sample sizes ratio associated with the lowest variance is not exactly 1 (because of

49   the term $\frac{df}{df-2}$, df depending only on n_c), but is very close to 1 (and the larger the total

50   sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance).

51   The further from this sample size ratio, the larger the variance (see Figure 5).

52   ***When $\delta_{Glass} \neq 0$.***   While the variance of Glass's d_s still depends on the total sample

53   size and the sample sizes allocation ratio, it also depends on the population effect size

54   (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of

55   the population effect size decreases when the control group increases. On the other hand, the

56   effect of the population effect size does *not* depend on the size of the experimental group

57   since

$$\lim_{n_c \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

58

$$\lim_{n_e \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] \neq 0$$

59   These limits are illustrated in Figure 6.

60   Note: while the sample sizes ratio associated with the lowest variance was very close to

61   1 with a null population effect size, this is not true anymore when the population effect size

62   is not zero. Indeed, because of the second term in the adddition, when computing the

63   variance, one gives much more weight to the effect size of the control group (see Figure 7),

64   especially when the effect size gets larger. For example, when $\delta_{Glass}= 4$, the lowest variance

65   will occur when n_c is approximately 3 times larger than n_e. When $\delta_{Glass}= 7$, the lowest

66   variance will occur when n_c is approximately 5 times larger than n_e, etc.

67   **When variances are unequal across populations, with equal sample sizes.**

⁶⁸ **_When $\delta_{Glass} = 0$._** When the population effect size is zero, the variance of Glass's d_s

⁶⁹ can be simplified as follows:

$$Var_{Glass's\ d_s} = \frac{n-1}{n(n-3)}\left(1 + \frac{\sigma_e^2}{\sigma_c^2}\right)$$

⁷⁰ where $n = N/2 =$ sample size of each group. The variance is therefore a function of the total

⁷¹ sample size and the SD-ratio $\left(\frac{\sigma_c}{\sigma_e}\right)$:

⁷² • The larger the total sample size, the lower the variance (See Figure 8);

⁷³ • The larger the SD-ratio (i.e. the larger is σ_c in comparison with σ_e), the lower the

⁷⁴ variance (see Figure 9). However, the effect of the SD-ratio decreases when sample

⁷⁵ sizes increase, because $\lim_{n(=n_c=n_e)\to\infty}\left[\frac{df}{n(df-2)}\right] = 0$.

⁷⁶ **_When $\delta_{Glass} \neq 0$._** While the variance of Glass's d_s still depends on the total sample

⁷⁷ size and the SD-ratio, it also depends on the population effect size (δ_{Glass}). The larger the

⁷⁸ population effect size, the larger the variance. However, the effect of the population effect

⁷⁹ size decreases when the control group increases, as previously explained and illustrated in

⁸⁰ Figure 6.

⁸¹ **When variances are unequal across populations, with unequal sample sizes.**

⁸² **_When $\delta_{Glass} = 0$._** When the population effect size is zero, the variance of Glass's d_s

⁸³ can be simplified as follows:

$$Var_{Glass's\ d_s} = \frac{n_c-1}{n_c-3}\left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e\sigma_c^2}\right)$$

⁸⁴ The variance of Glass's d_s is therefore a function of the total sample size (N), the

⁸⁵ SD-ratio and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

⁸⁶ • For any SD and sample sizes pairing, increasing n_c and/or n_e will decrease the

⁸⁷ variance (see Figure 10);

88   – The effect of the sample sizes ratio depends on the SD-ratio:

89   * We previously mentioned that when $\sigma_c = \sigma_e$, the variance is minimized when

90   sample sizes of both groups are almost identical (see Figure 5), meaning that

91   it is more efficient, in order to reduce variance, to add subjects uniformly in

92   both groups;

93   * When $\sigma_e > \sigma_c$, more weight is given to n_e, meaning that it is more efficient,

94   in order to reduce variance, to add subjects in the experimental group (n_e;

95   see bottom plots in Figure 10);

96   * When $\sigma_c > \sigma_e$, less weight is given to n_e, meaning that it is more efficient, in

97   order to reduce variance, to add sujects in the control group (n_c; see top plots

98   in Figure 10).

99   • Finally, there is also a main effect of the SD-ratio: the larger is σ_c in comparison with

100   σ_e, the lower the variance, as we can observe in Figure 11. We can also notice that in

101   Figure 10, the maximum variance is much larger in the two bottom plots (where

102      $\sigma_c < \sigma_e$) than in the two top plots (where $\sigma_c > \sigma_e$).

103   Note that the effect of the SD-ratio, and the interaction effect between SD-ratio and

104 sample sizes ratio decreases when the sample size of the control group increases (because

105 $\frac{n_c-1}{n_c-3}$ gets closer to 1).

106   **When $\delta_{Glass} \neq 0$.**   While the variance of Glass's d_s still depends on the total sample

107 size, the SD-ratio and the interaction between the SD-ratio and the sample sizes ratio, it

108 also depends on the population effect size (δ_{Glass}): the larger the population effect size, the

109 larger the variance. However, the effect of the population effect size decreases when the

110 sample size of the control group increases, as previously explained and illustrated in Figure 6.

111   Note: when the population effect size was null, when $\sigma_c < \sigma_e$, it was much more

112 efficient to add subjects in the experimental group in order to reduce the variance (because

113 much more weight was given to n_e). When $\delta_{Glass} \neq 0$, it is important to add subjects in both

114   groups in order to reduce the variance (because $\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2$ is only a function of

115   the sample size of the control group). With huge population effect size, it is even always

116   more important to add subjects in the control group (e.g. when $\delta_{Glass} = 30$).

117   **In summary.**   The variance of Glass's d_s is a function of the population effect size

118   (δ_{Glass}), the SD-ratio, the total sample size and the interaction between sample sizes ratio

119   and SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

120   - The variance decreases when the SD-ratio increases (i.e. when $\sigma_e >> \sigma_c$);
121   - The variance always decreases when the control and/or the experimental group
122   increases. The benefit of adding subjects rather in the control, in the experimental, or
123   in both groups, in order to reduce the variance, varies as a function of the SD-ratio
124   and the population effect size. The only situation where it is optimal to maximize the
125   experimental group is when $\sigma_e > \sigma_c$ and $\delta_{Glass} \approx 0$. Most of the time, it is more
126   efficient to maximize the control groups (e.g. anytime $\sigma_e < \sigma_c$, and when δ_{Glass} is very
127   large) or to uniformly add subjects in both groups (e.g. when $\sigma_e > \sigma_c$ and δ_{Glass} is
128   neither null nor huge);
129   - The variance increases when δ_{Glass} increases. Note that the effect of δ_{Glass} is moderated
130   by the control group size (the larger n_e, the smaller the effect of δ_{Glass} on the variance).

131   ## Cohen's d_s^*

132   **When variances are equal across populations.**

133   ***When $\delta_{Cohen}^* = 0$.***   When the population effect size is zero, the variance of Cohen's

134   d_s^* is computed as follows :

$$Var_{Cohen's\ d_s^*} = \frac{df}{df - 2} \times \frac{N}{n_1 n_2}$$

135   with

$$df = \frac{4(n_1 - 1)(n_2 - 1)}{n_1 + n_2 - 2}$$

136 In this configuration, the degrees of freedom as well as the variance of Cohen's d_s^* depend on

137 the total sample size (N) and the sample sizes allocation ratio $\left(\frac{n_1}{n_2}\right)$:

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure 12);

- The larger the total sample size, the lower the bias (see Figure 13).

141 **When $\delta_{Cohen}^* \neq 0$.**   While the variance of Cohen's d_s^* still depends on the total

142 sample size and the sample sizes ratio, it also depends on the population effect size (δ_{Cohen}^*):

143 the larger the population effect size, the larger the variance. However, the effect of the

144 population effect size decreases when the degrees of freedom increase (i.e. when sample sizes

145 increase and/or the sample sizes ratio get closer to 1), as illustrated in Figure 14 since

$$\lim_{df \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

146 **When variances are unequal across populations, with equal sample sizes.**

147 **When $\delta_{Cohen}^* = 0$.**   When the population effect size is zero, the variance of Cohen's

148 d_s^* can be simplified as follows:

$$Var_{Cohen's\ d_s^*} = \frac{df}{df - 2} \times \frac{2}{n}$$

149 where $n = N/2$=sample size of each group, and $df = \frac{(n-1)(\sigma_1^4 + \sigma_2^4 + 2\sigma_1^2\sigma_2^2)}{\sigma_1^4 + \sigma_2^4}$. In this configuration,

150 the degrees of freedom as well as the variance of Cohen's d_s depend on the total sample size

151 (N) and the SD-ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$:

- The further the SD-ratio is from 1, the larger the variance (see Figure 15);

- The larger the total sample size, the lower the variance (see Figure 16).

154 Note: for a constant SD-ratio, the size of the variance does not matter (see Figure 17).

155 **When $\delta_{Cohen}^* \neq 0$.**   While the variance of Cohen's d_s^* still depednds on the total

156 sample size and the SD-ratio, it also depends on the population effect size (δ_{Cohen}^*): the

₁₅₇ larger the population effect size, the larger the variance. However, the effect of the population

₁₅₈ effect size decreases when the degrees of freedom increase (i.e. when the total sample size

₁₅₉ increases and/or the SD-ratio get closer to 1), as previously illustrated in Figure 14.

₁₆₀ **When variances are unequal across populations, with unequal sample sizes.**

₁₆₁ ***When $\delta^*_{Cohen} = 0$.*** When the population effect size is zero, the variance of Cohen's

₁₆₂ d^*_s can be simplified as follows:

$$Var_{Cohen's\ d^*_s} = \frac{df}{df - 2} \times \frac{2\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2 + \sigma_2^2}$$

₁₆₃ with $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$. In this configuration, the degrees of freedom are a function of

₁₆₄ the total sample size (N) and the interaction between sample sizes ratio and the SD-ratio

₁₆₅ $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

₁₆₆ • The larger the total sample size, the lower the variance (illustration in Figure 18);

₁₆₇ • The smallest variance always occurs when there is a positive pairing between variances

₁₆₈ and sample sizes, because one gives more weight to the smallest variance in the

₁₆₉ denominator of the df computation and in the numerator of the variance computation.

₁₇₀ Moreover, the further the SD-ratio is from 1, the further from 1 will also be the

₁₇₁ sample sizes ratio associated with the smallest variance (see Figure 19). This can be

₁₇₂ explained by splitting the numerator and the denominator of the df computation (see

₁₇₃ the file "Theoretical Bias, as a function of population parameters").

₁₇₄ Note: for a constant SD-ratio, the variance does not matter. (See Figure 20).

₁₇₅ ***When $\delta^*_{Cohen} \neq 0$.*** While the variance of Cohen's d^*_s still depends on the total

₁₇₆ sample size, the SD-ratio and the interaction between the sample sizes ratio and the

₁₇₇ SD-ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population

₁₇₈ effect size, the larger the variance. However, the effect of the population effect size decreases

₁₇₉ when the degrees of freedom increase (i.e. when the total sample size increases and/or when

₁₈₀ there is a positive pairing between the sample sizes ratio and the SD-ratio), as previously

181 illustrated in Figure 14.

182   **In summary.**   The variance of Cohen's d_s^* is a function of the population effect size

183 (δ_{Cohen}^*), the total sample size (N) and the interaction between sample sizes ratio and

184 SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

185   • The variance always decreases when the control and/or the experimental group

186   increases. The benefit of adding subjects rather in the control or in the experimental

187   group depends on the SD-ratio. Indeed, the smallest variance always occurs when

188   there is a positive pairing between variances and sample sizes. Moreover, the further

189   the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated

190   with the smallest variance;

191   • The variance increases when δ_{Cohen}^* increases. Note that the effect of δ_{Cohen}^* is

192   moderated by the total sample size and the interaction between sample sizes ratio and

193   SD-ratio.

194 **Shieh's d$_s$**

195   **When variances are equal across populations.**

196   ***When $\delta_{Shieh} = 0$.***   When the population effect size is zero, the variance of Shieh's d_s

197 can be simplified as follows:

$$Var_{Shieh's\,d_s} = \frac{df}{(df-2)N}$$

198 with $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$. In this configuration, the degrees of freedom as well as the

199 variance of Shieh's d_s depend on the total sample size (N) and the sample sizes allocation

200 ratio $\left(\frac{n_1}{n_2}\right)$:

201   • The further the sample sizes allocation ratio is from 1, the larger the variance (see

202   Figure 21);

203   • The larger the total sample size, the lower the variance. It does not matter whether

204   the sample sizes ratio is constant (see Figure 22) or not (see Figure 23).

205   Note: in "Theoretical Bias, as a function of population parameters", we noticed that

206 increasing the sample sizes ratio when adding subjects in only one group could decrease the

207 degrees of freedom. However, due to the total sample size term (N) in the denominator of

208 the variance computation, even when degrees of freedom decrease due to the fact that one

209 adds subjects only in one group, the variance still decreases (because the denominator of the

210 variance computation increases; see Figure 23).

211   ***When $\delta_{Shieh} \neq 0$.***   While the variance of Shieh's d_s still depends on the total sample

212 size and the sample sizes ratio, it also depends on the population effect size (δ_{Shieh}): the

213 larger the population effect size, the larger the variance. However, the effect of the

214 population effect size decreases when the degrees of freedom increase (i.e. when sample sizes

215 increase, without increasing the sample sizes ratio, and/or the sample sizes ratio gets closer

216 to 1), as previously illustrated in Figure 14.

217   Note: we previously noticed that when the effet size is zero, the variance of Shieh's d_s

218 decreases, even when the sample sizes ratio increases. It is no longer true when there is a

219 non-null effect size because the larger the sample sizes ratio, the more the variance will

220 increase with increasing effect size.

221   **When variances are unequal across populations, with equal sample sizes.**

222   ***When $\delta_{Shieh} = 0$.***   When the population effect size is zero, the variance of Shieh's d_s

223 can be simplified as follows:

$$Var_{Shieh's\ d_s} = \frac{df}{(df-2)N}$$

224 with $df = \frac{(\sigma_1^2+\sigma_2^2)^2 \times (n-1)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the variance

225 of Shieh's d_s depend on the total sample size (N) and the SD-ratio ($\frac{\sigma_1}{\sigma_2}$).

226   • The further the SD-ratio is from 1, the larger the variance (see Figure 24);

227   • The larger the total sample size, the lower the variance (see Figure 25).

228   Note: for a constant SD-ratio, the size of the variance does not matter (see Figure 26).

²²⁹   ***When δ$_{Shieh}$ ≠ 0.***   While the variance of Shieh's d_s still depends on the total sample

²³⁰   size and the SD-ratio, it also depends on the population effect size ($δ_{Shieh}$): the larger the

²³¹   population effect size, the larger the variance. However, the effect of the population effect

²³²   size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or

²³³   the SD-ratio gets closer to 1), as previously illustrated in Figure 14.

²³⁴   **When variances are unequal across populations, with unequal sample sizes.**

²³⁵   ***When δ$_{Shieh}$ = 0.***   When the population effect size is zero, the variance of Shieh's d_s

²³⁶   can be simplified as follows:

$$Var_{Shieh's\ d_s} = \frac{df}{(df - 2)N}$$

²³⁷   with $df = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1 - 1} + \frac{(\sigma_2^2/n_2)^2}{n_2 - 1}}$. In this configuration, the degrees of freedom as well as the

²³⁸   variance of Shieh's d_s depend on the total sample size (N) and the interaction between the

²³⁹   sample sizes ratio and the SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

²⁴⁰   • The larger the total sample size, the lower the variance. It remains true when the

²⁴¹   sample sizes ratio is constant (see Figure 27) and when it is not (see Figure 28).

²⁴²   Note: When variances were equal across populations, adding subjects only in the first

²⁴³   group had the same impact on the variance as adding subjects only in the second group (see

²⁴⁴   Figure 23). When variances are unequal across groups, this is not true anymore (see Figure

²⁴⁵   28).

²⁴⁶   • The smallest variance always occurs when there is a positive pairing between variances

²⁴⁷   and sample size. Moreover, the further the SD-ratio is from 1, the further from 1 will

²⁴⁸   also be the sample sizes ratio associated with the smallest variance (see Figure 29).

²⁴⁹   Moreover, for a constant SD-ratio, the variances do not matter (See Figure 30).

²⁵⁰   ***When δ$_{Shieh}$ ≠ 0.***   While the variance of Shieh's d_s still depends on the total sample

²⁵¹   size and the interaction between the sample sizes ratio and the SD-ratio, it also depends on

²⁵²   the population effect size ($δ_{Shieh}$): the larger the population effect size, the larger the

253 variance. However, the effect of the population effect size decreases when the degrees of

254 freedom increase, as previously illustrated in Figure 14.

255   **In summary.**   The variance of Shieh's d_s is a function of the population effect size

256 (δ_{Shieh}), the total sample size (N) and the interaction between sample sizes ratio and

257 SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

258 • The variance always decreases when the control and/or the experimental group

259   increases. The benefit of adding subjects rather in the control or in the experimental

260   group depends on the SD-ratio. Indeed, the smallest variance always occurs when

261   there is a positive pairing between variances and sample sizes. Moreover, the further

262   the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated

263   with the smallest variance;

264 • The variance increases when δ_{Shieh} increases. Note that the effect of δ_{Shieh} is

265   moderated by the total sample size and the interaction between the sample sizes ratio

266   and the SD-ratio.

Figure 1. Variance of Cohen's d_s, when variances are equal across groups, as a function of the total sample size (N).
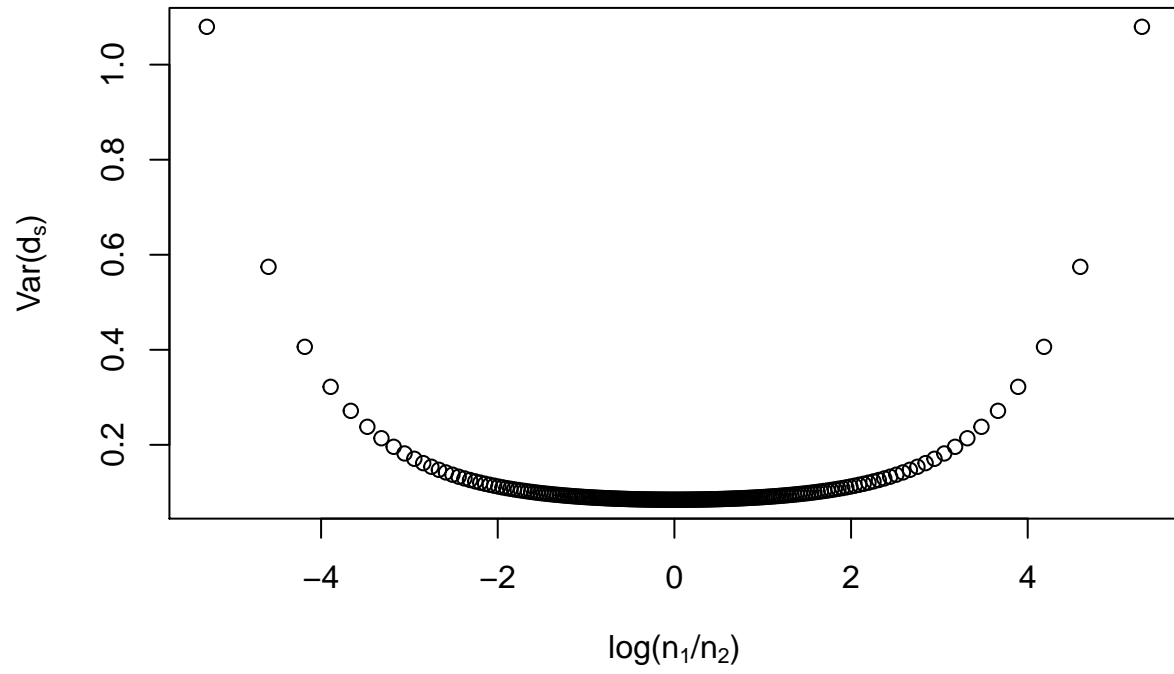
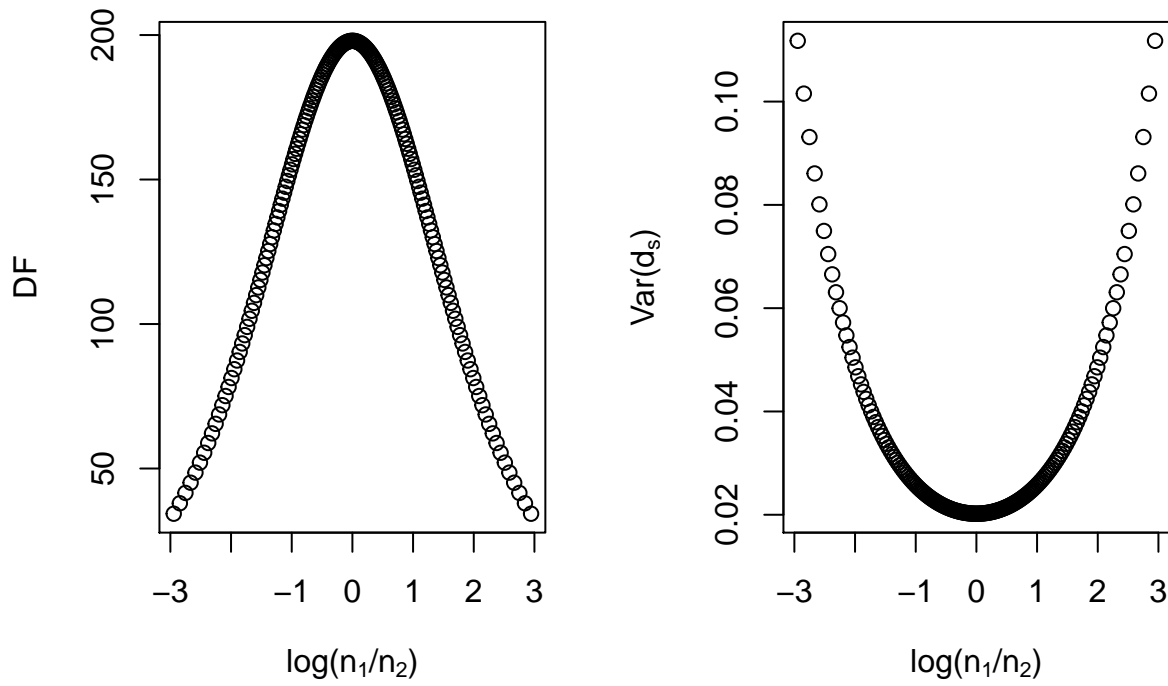*Figure 2*. Variance of Cohen's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_1}{n_2}\right)$).
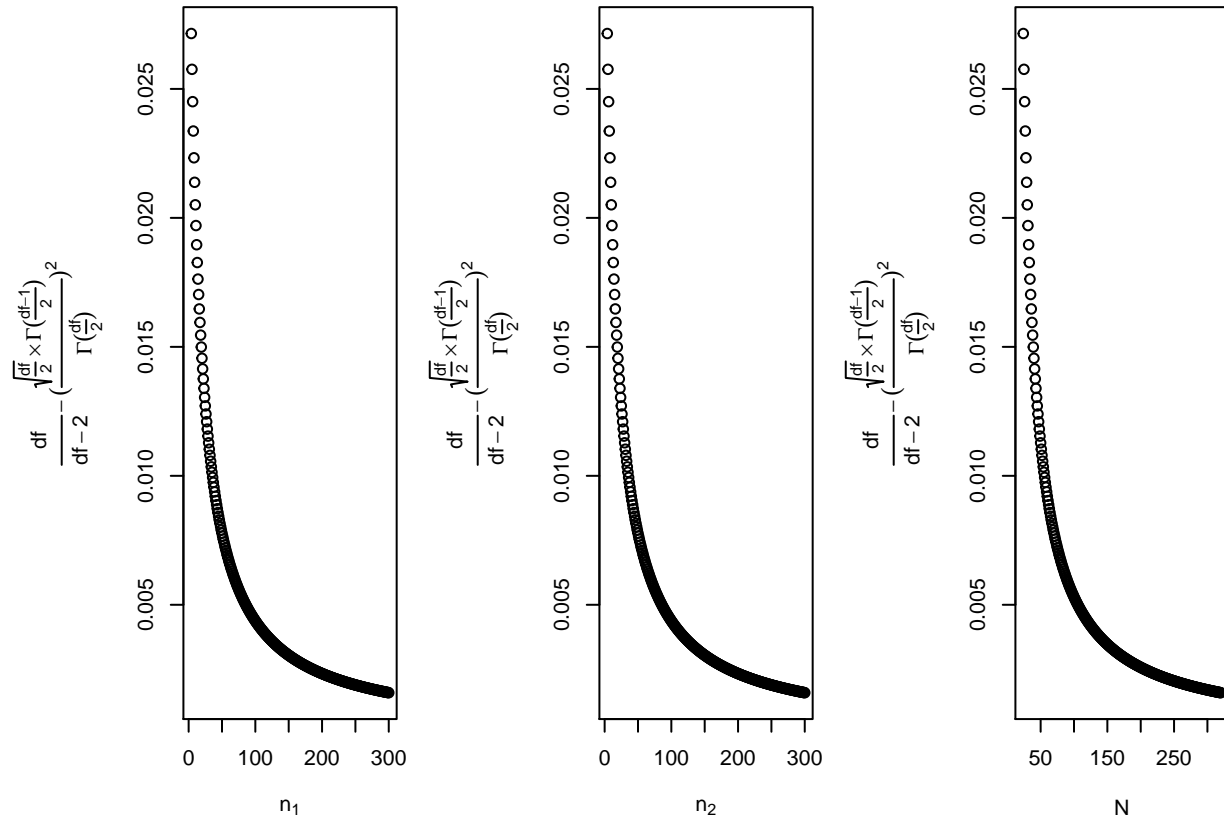
Figure 3. Effect size moderator, when computing the variance of Cohen's d_s, as a function of n_1 (left), n_2 (center) and $N = n_1 + n_2$ (right).
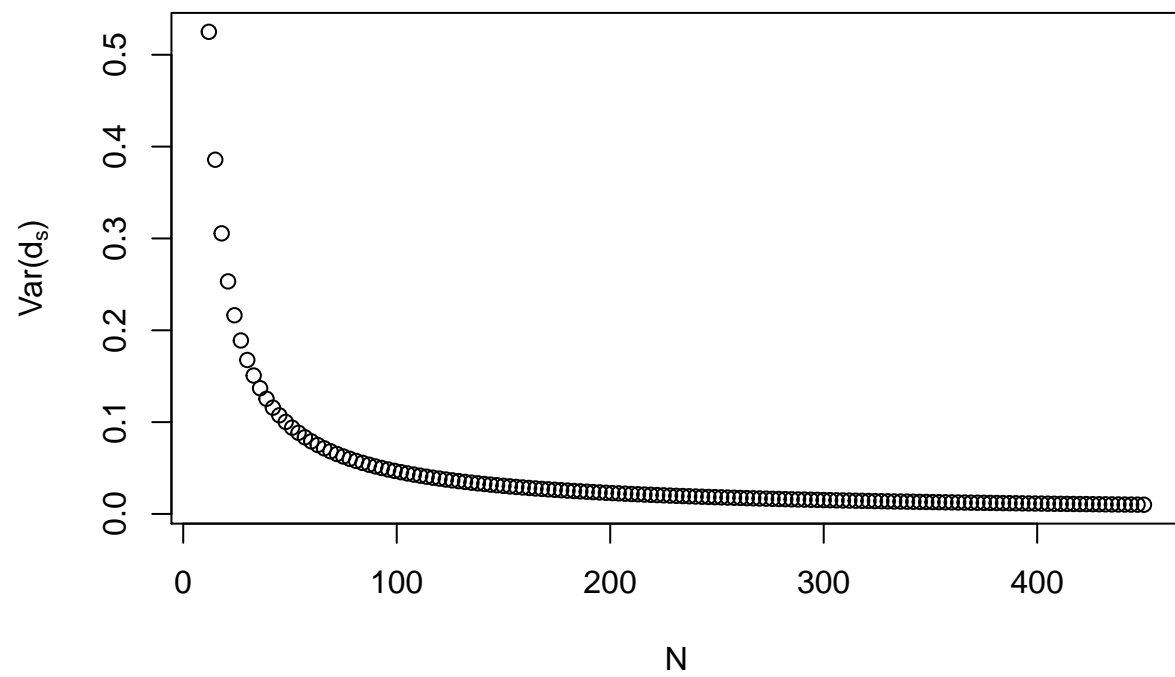
Figure 4. Variance of Glass's d_s, when variances are equal across groups, as a function of the total sample size (N).
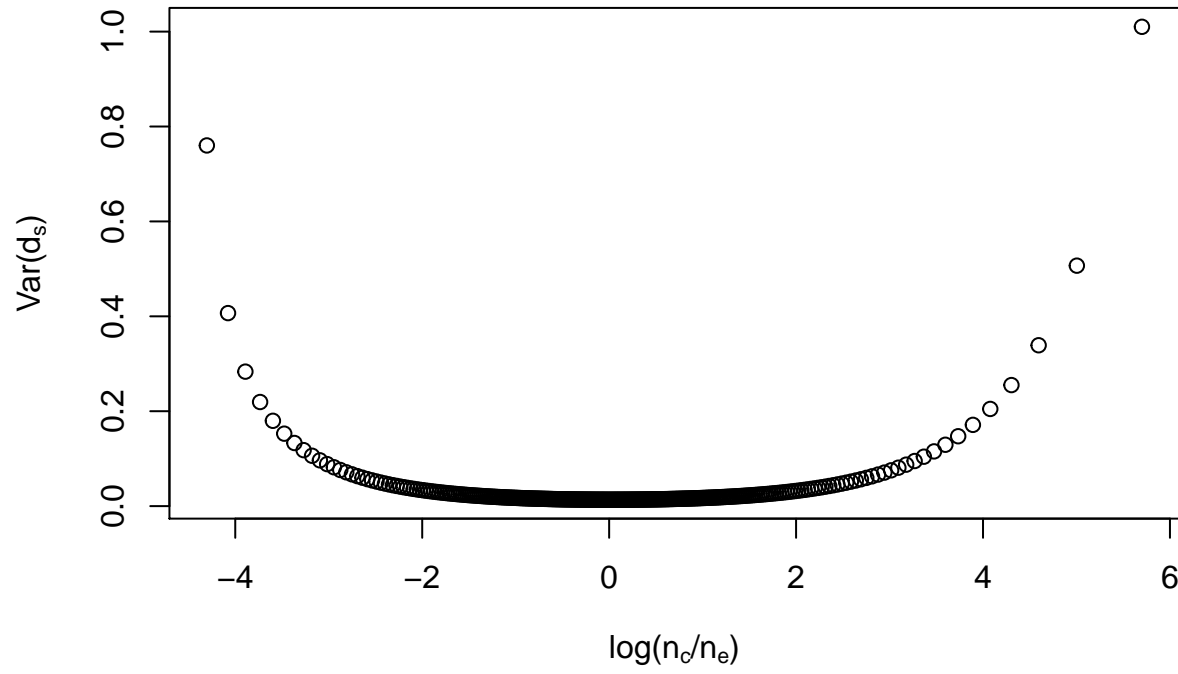
Figure 5. Variance of Glass's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_c}{n_e}\right)\right)$.
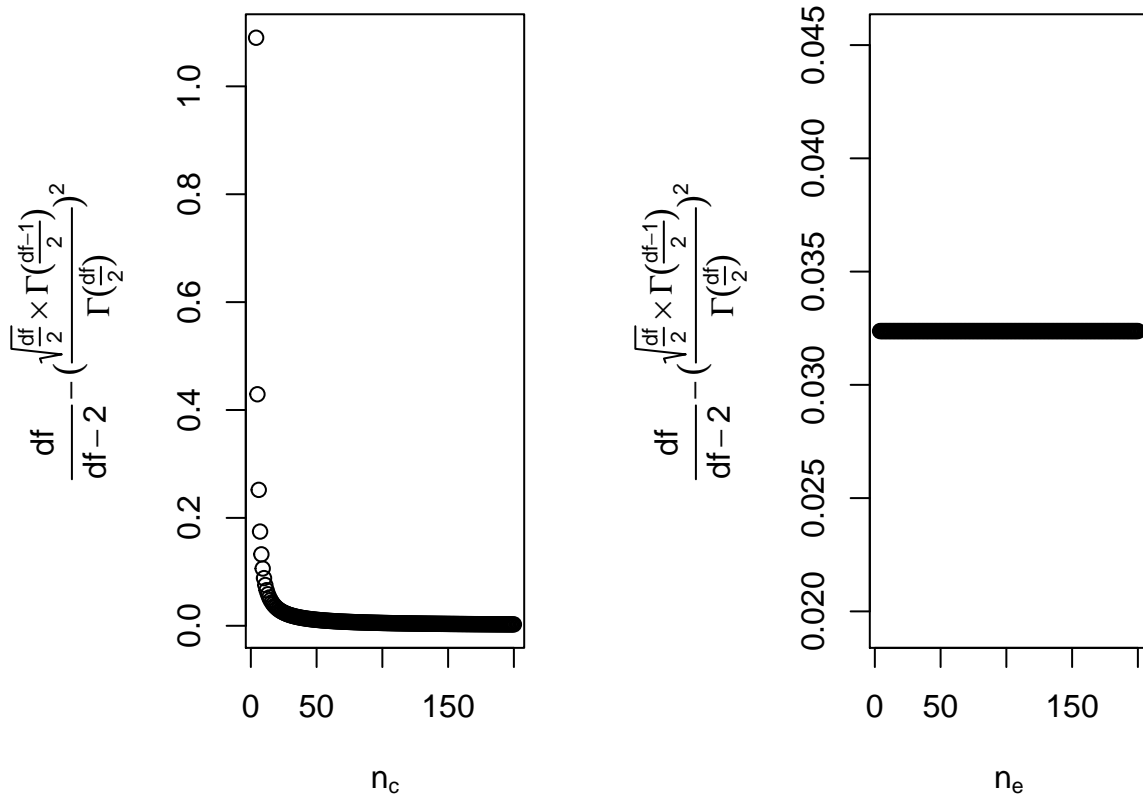
Figure 6. Effect size moderator, when computing the variance of Glass's d_s, as a function of the size of the control group (left) and experimental group (right).
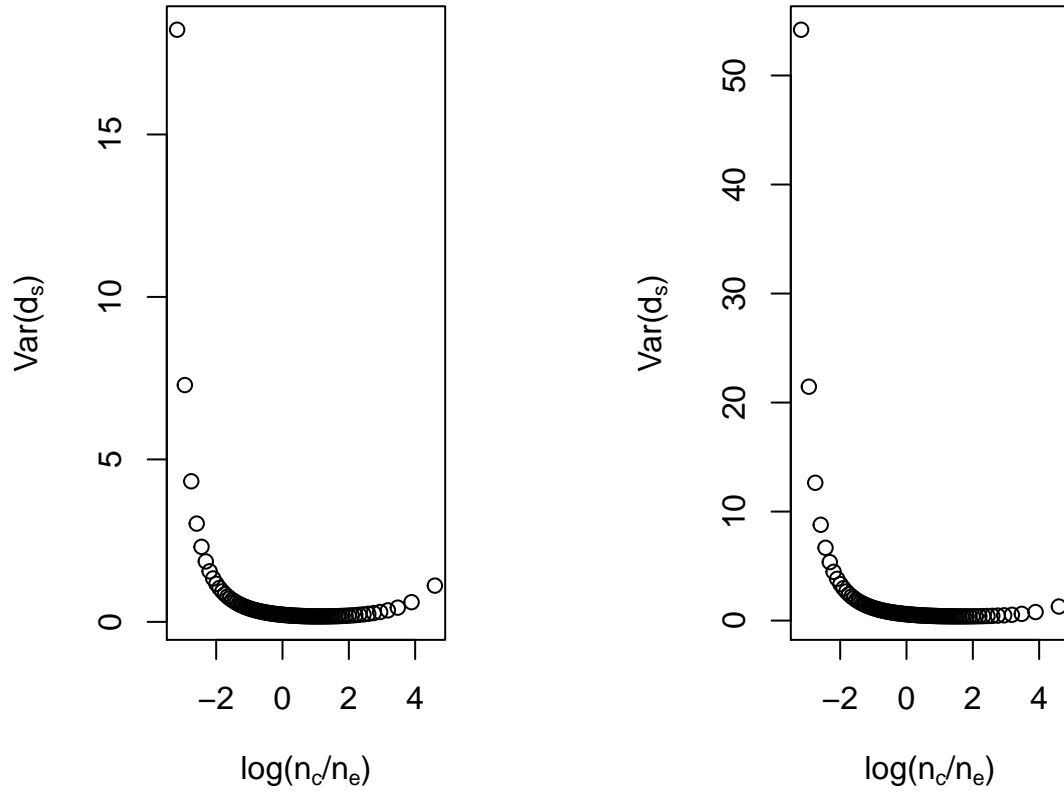
Figure 7. Variance of Glass's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_c}{n_e}\right))$ when δ_{Glass} equals 4 (left) or 7 (right).
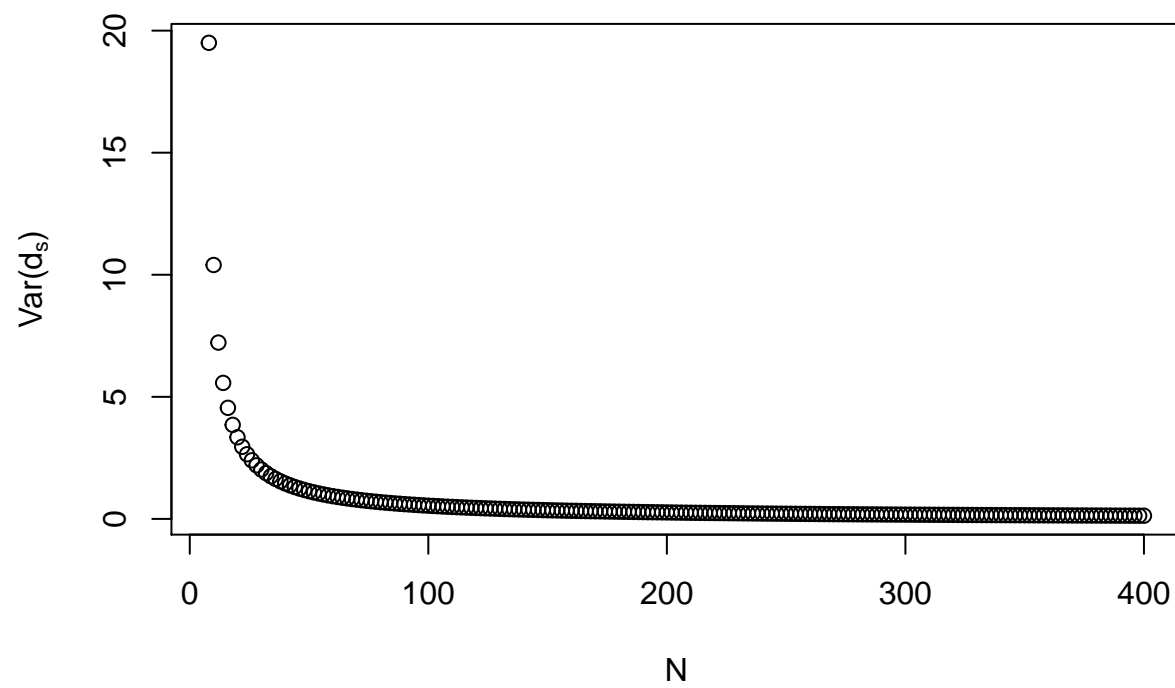
Figure 8. Variance of Glass's d_s, when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes (N).
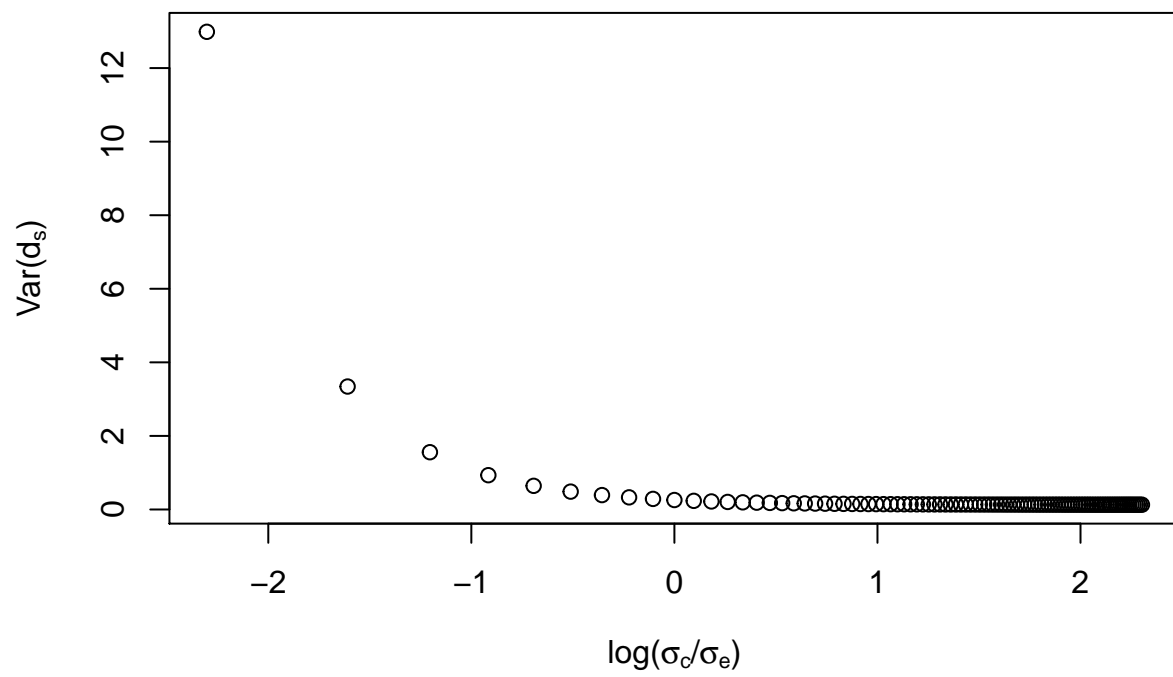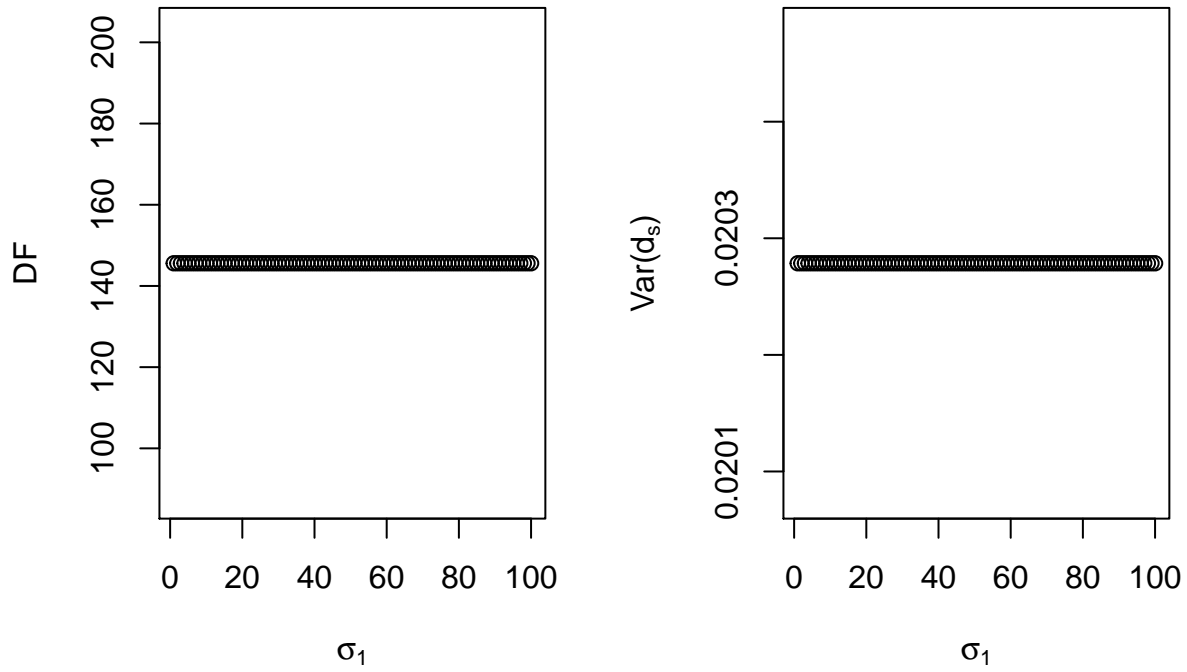
Figure 9. Variance of Glass's d_s, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
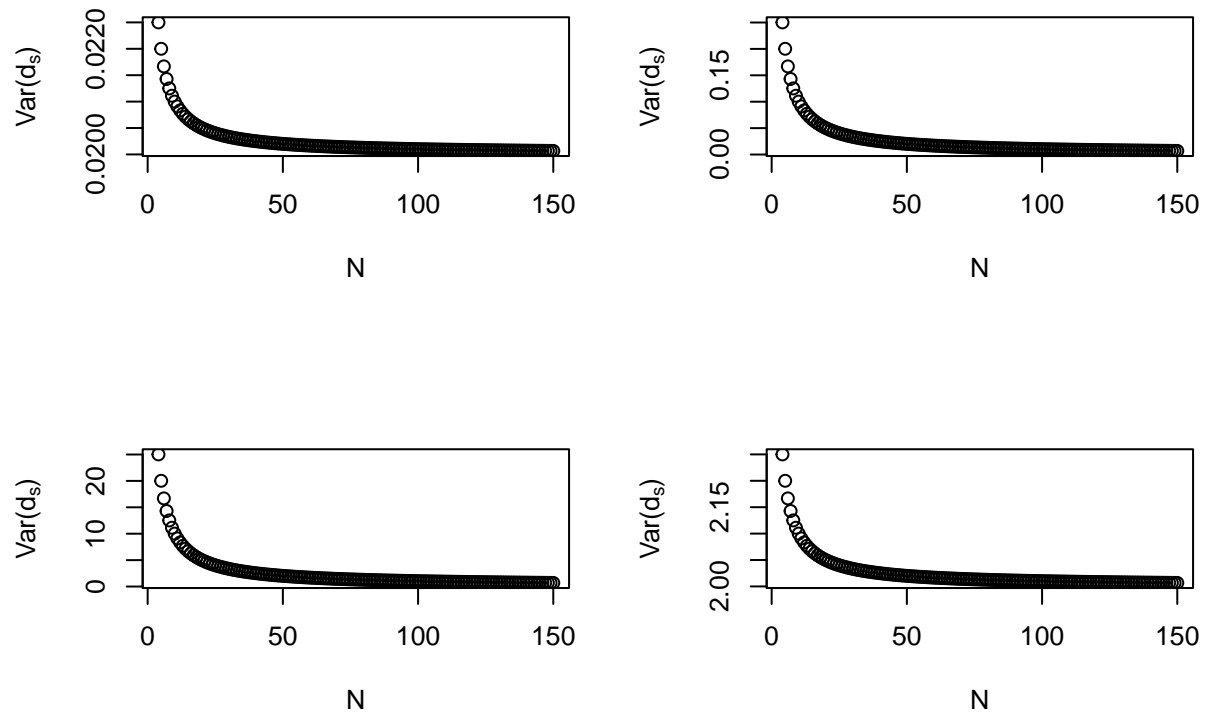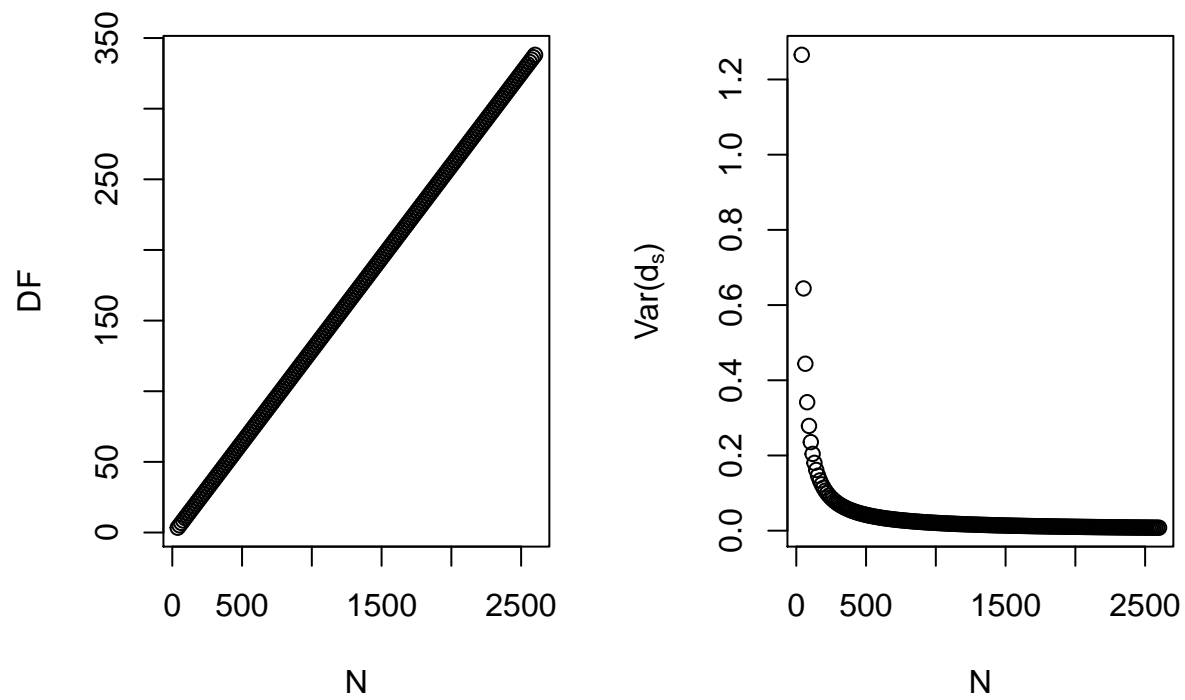
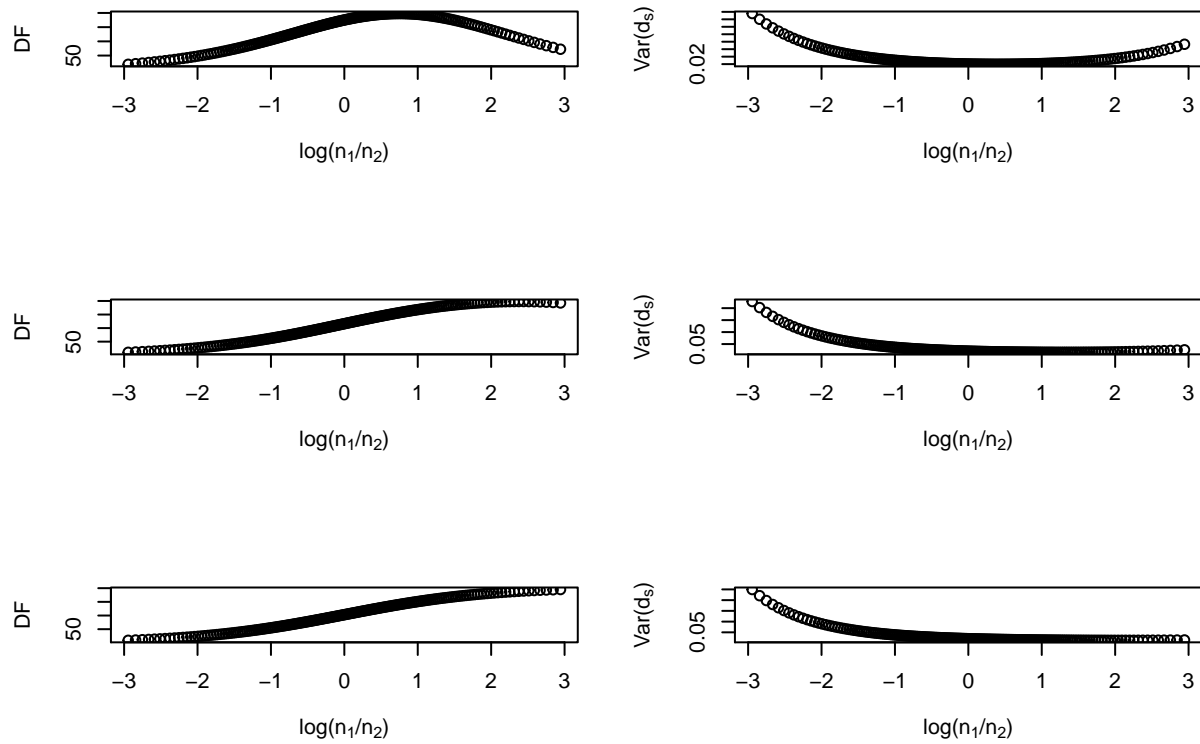*Figure 10*. Variance of Glass's d_s, when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when $\sigma_c > \sigma_e$ (top plots) or $\sigma_c < \sigma_e$ (bottom plots).
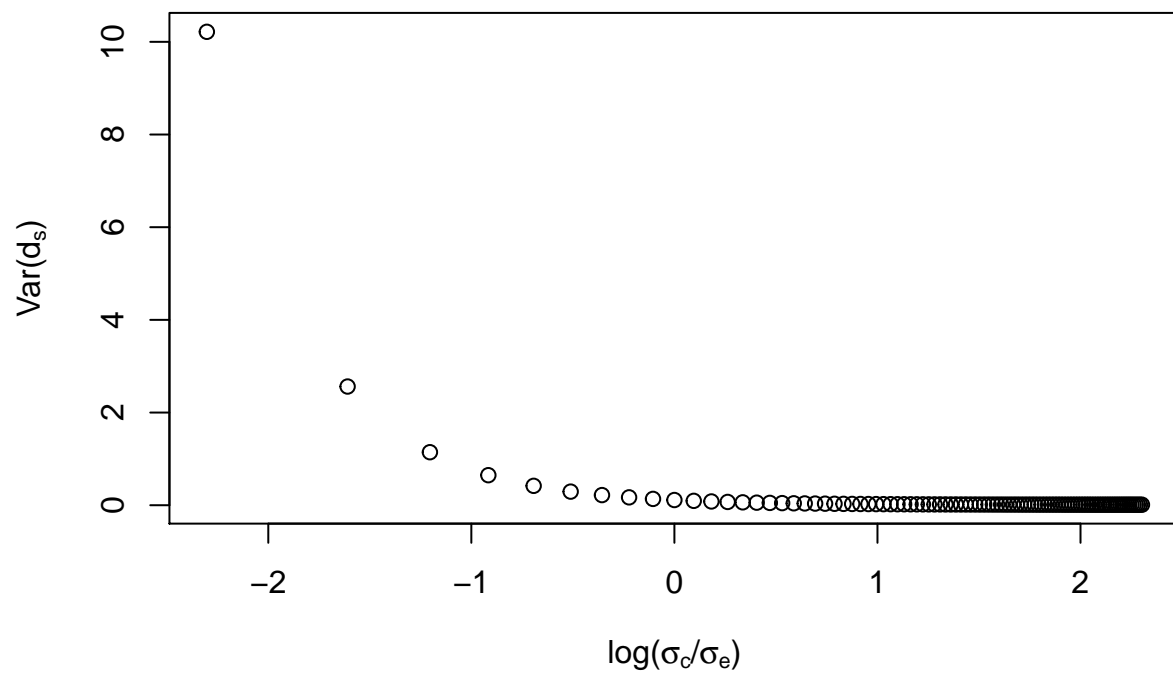
Figure 11. Variance of Glass's d_s, when sample sizes and variances are unequal across groups, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
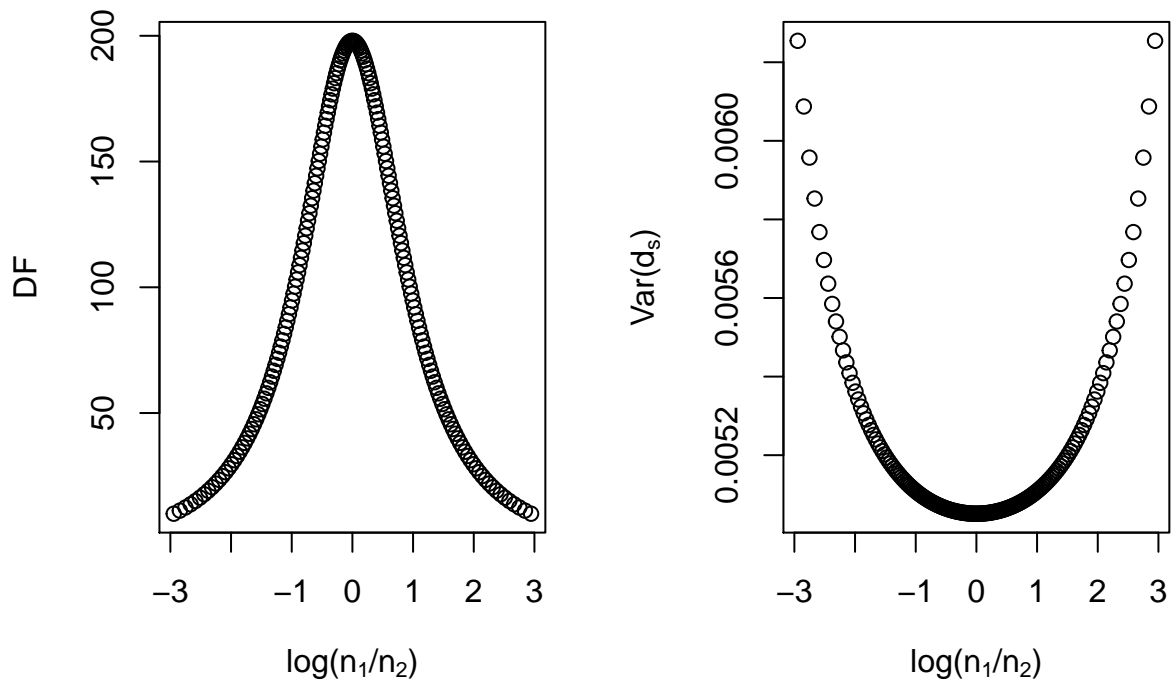
Figure 12. Variance of Cohen's d_s^* when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_1}{n_2}\right))$.
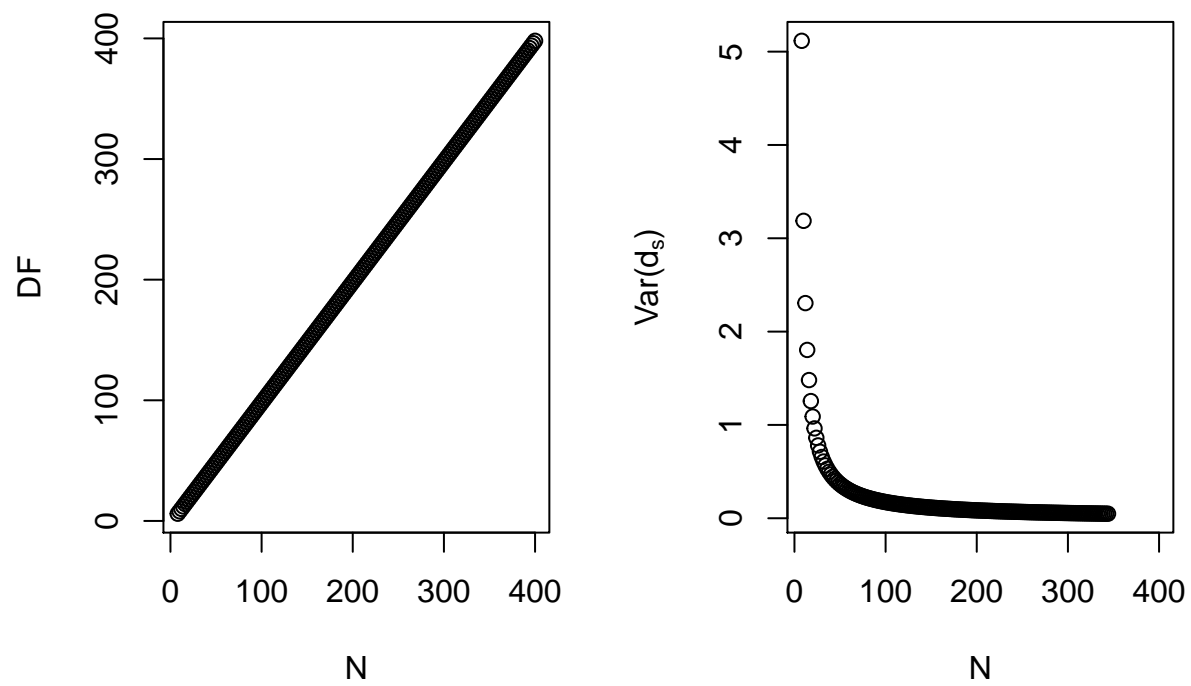
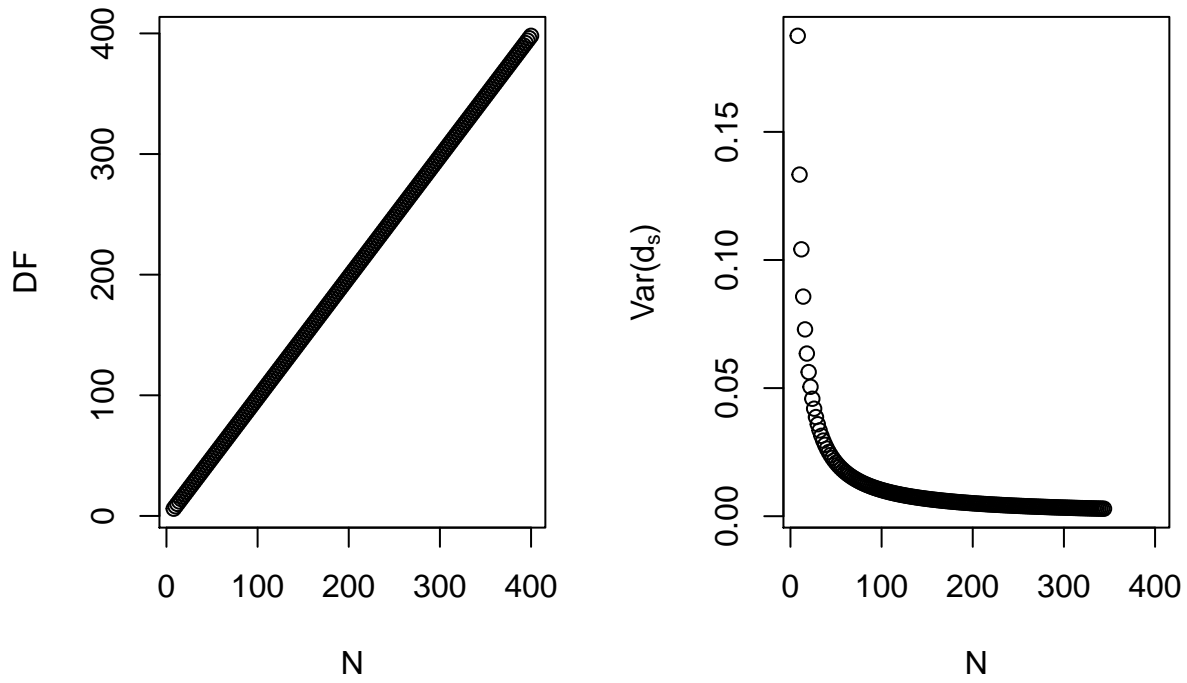*Figure 13*. Variance of Cohen's d_s^* when variances are equal across groups, as a function of the total sample size (N).
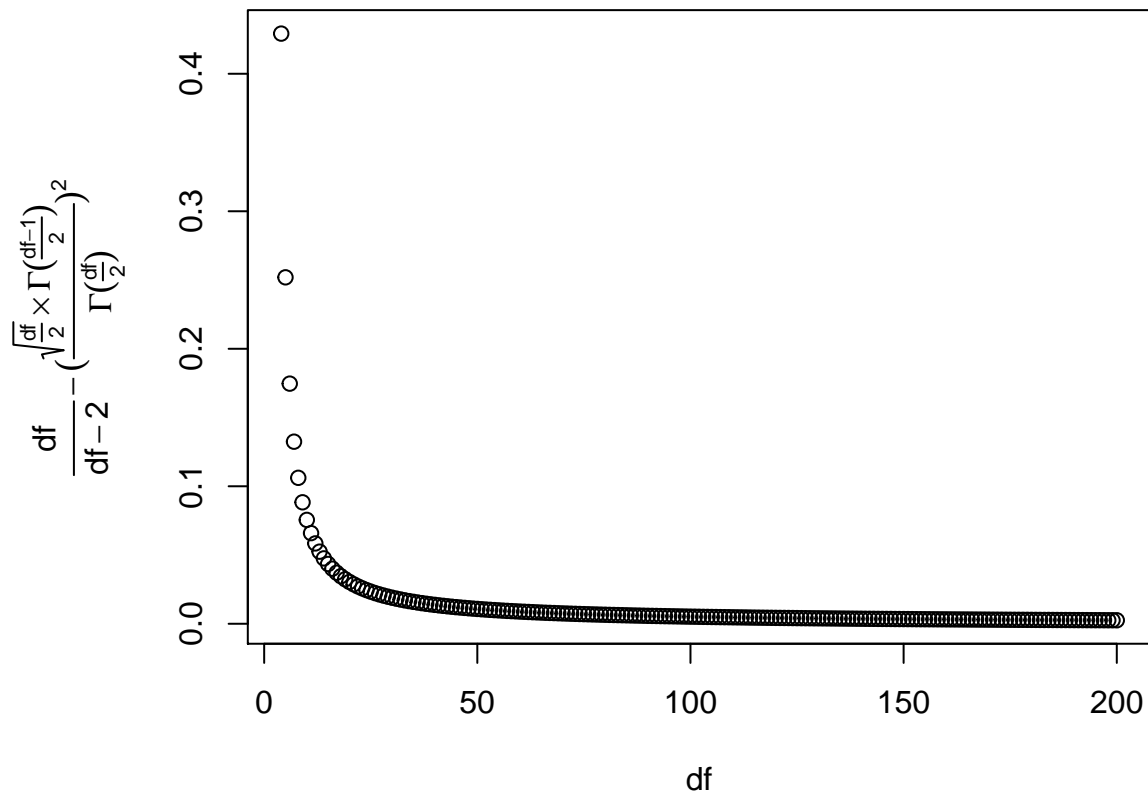
Figure 14. Effect size moderator (for all estimators), as a function of the degrees of freedom.
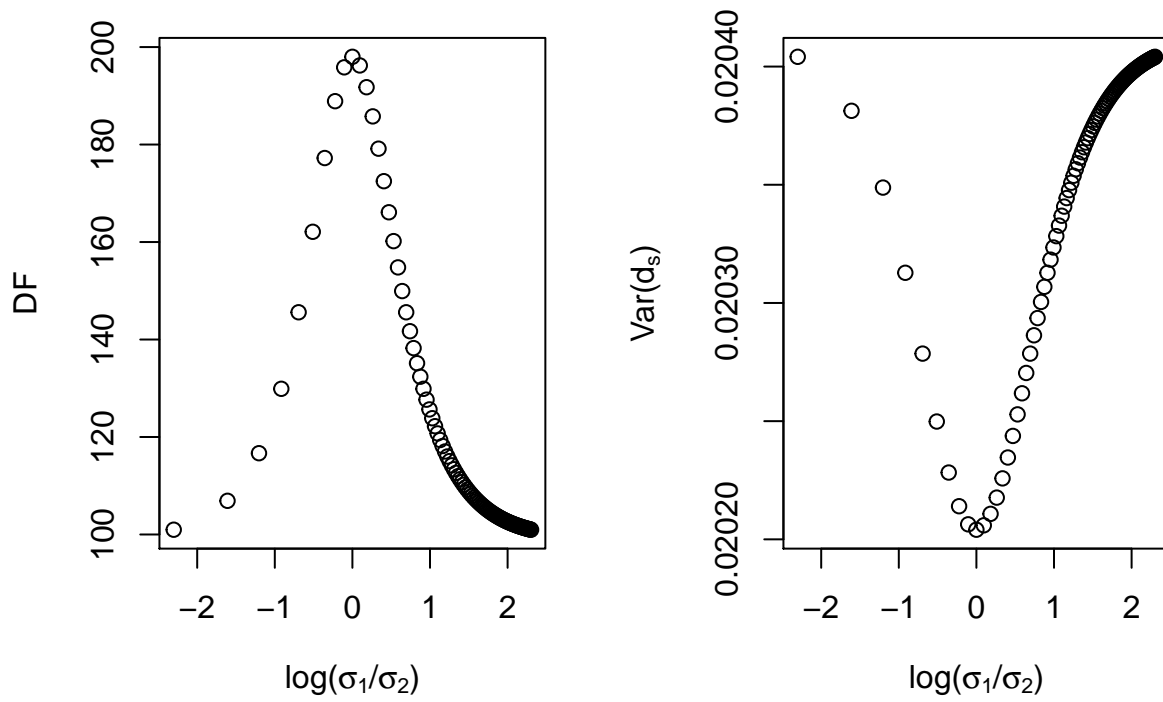
Figure 15. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $(log\left(\frac{\sigma_1}{\sigma_2}\right))$.
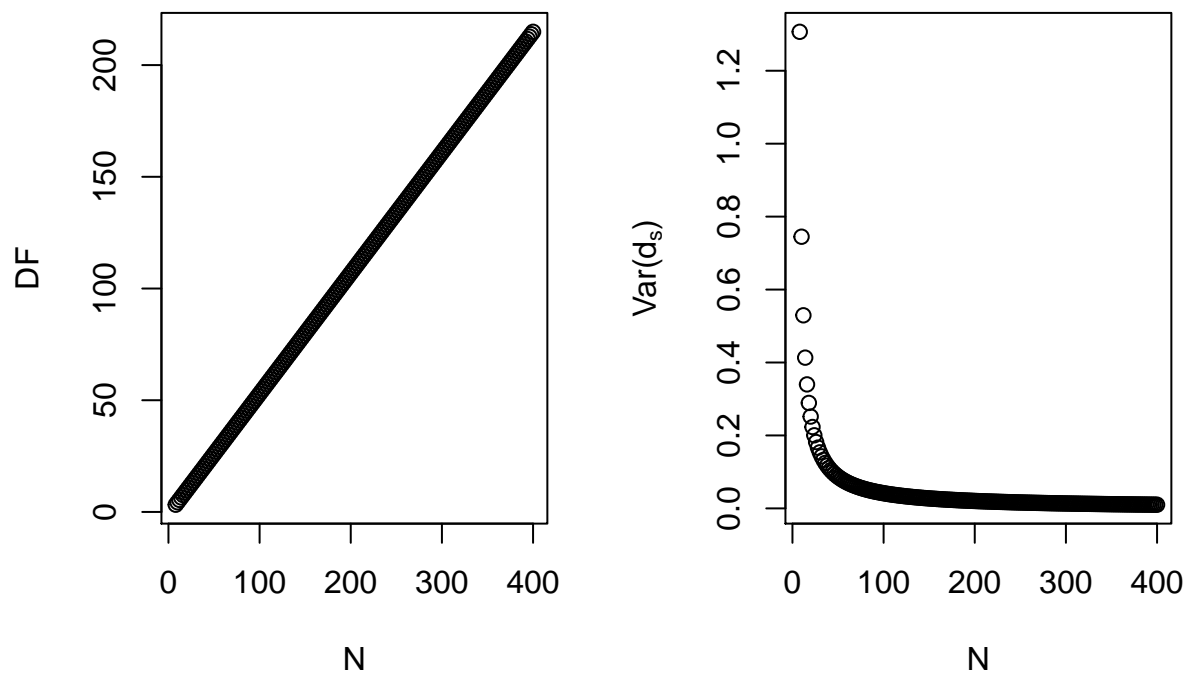
Figure 16. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 17. Variance of Cohen's d_s^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

Figure 18. Variance of Cohen's d_s^* when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

Figure 19. The variance of Cohen's d_s^*, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$, when SD-ratio equals 1.46 (first row), 3.39 (second row) or 7 (third row).
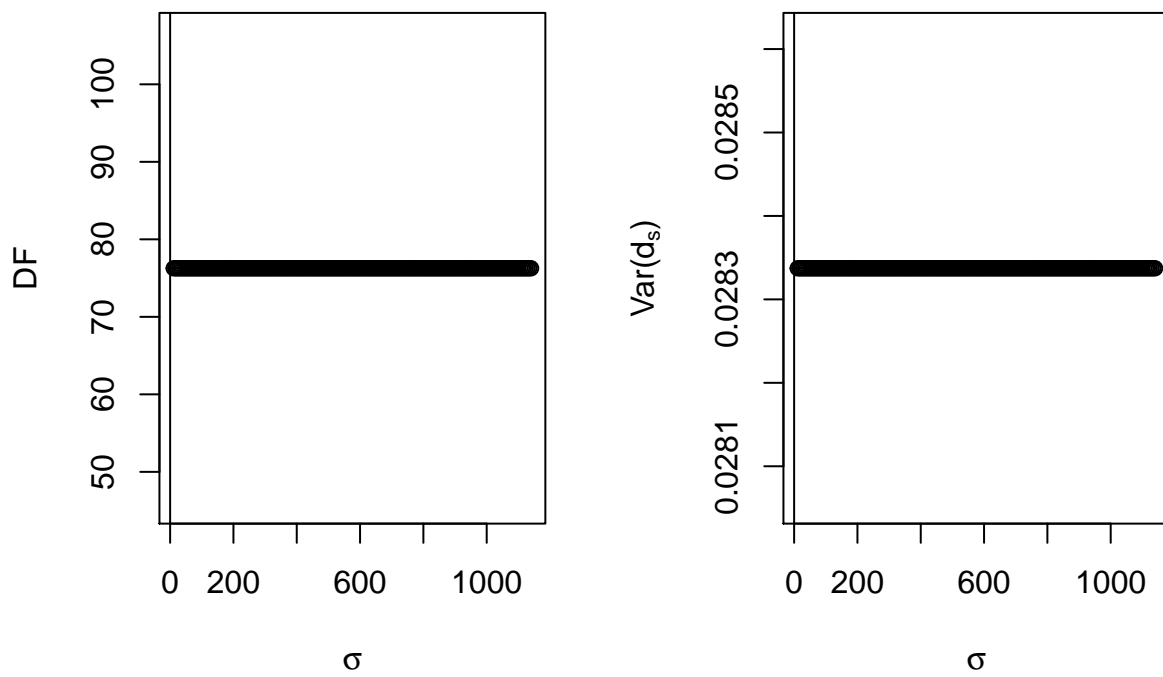
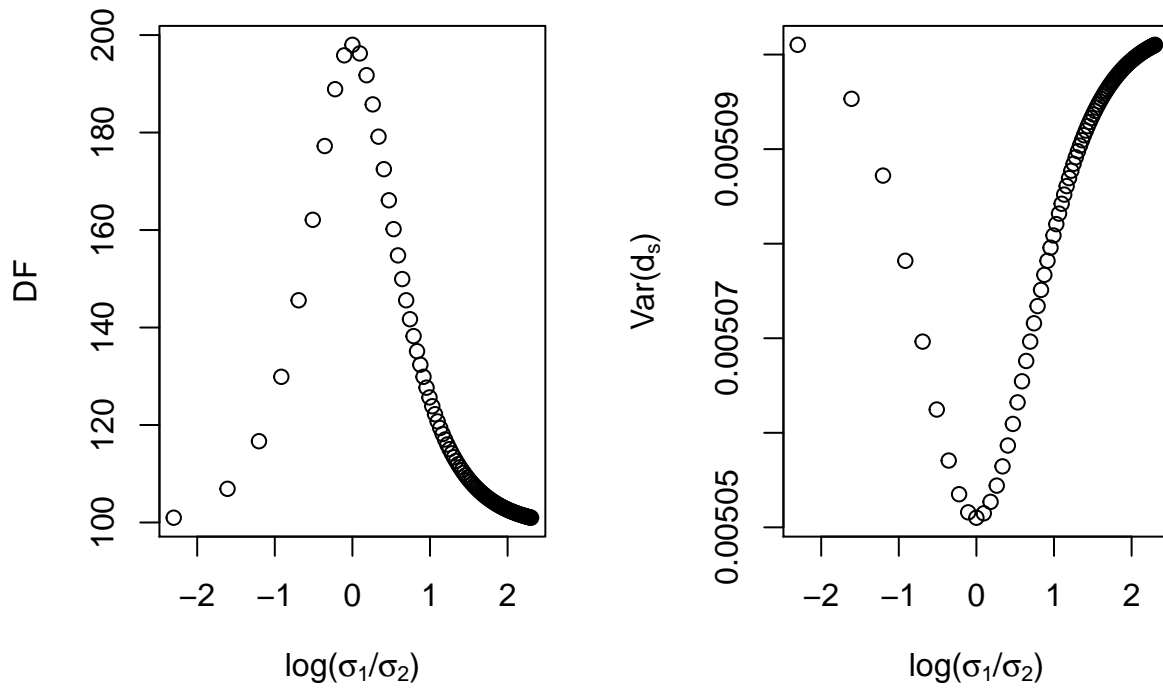*Figure 20*. Variance of Cohen's d_s^*, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

Figure 21. Variance of Shieh's d_s when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_1}{n_2}\right)$).

Figure 22. Variance of Shieh's d_s when variances are equal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.
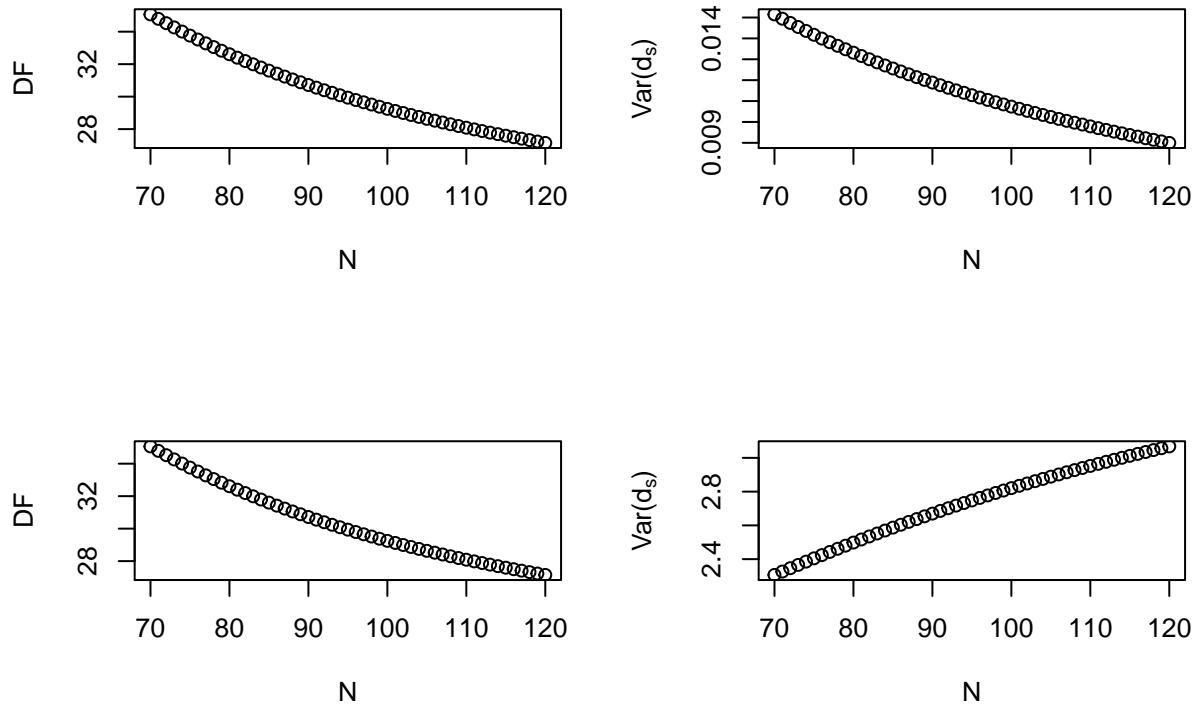
Figure 23. Variance of Shieh's d_s when variances are equal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).
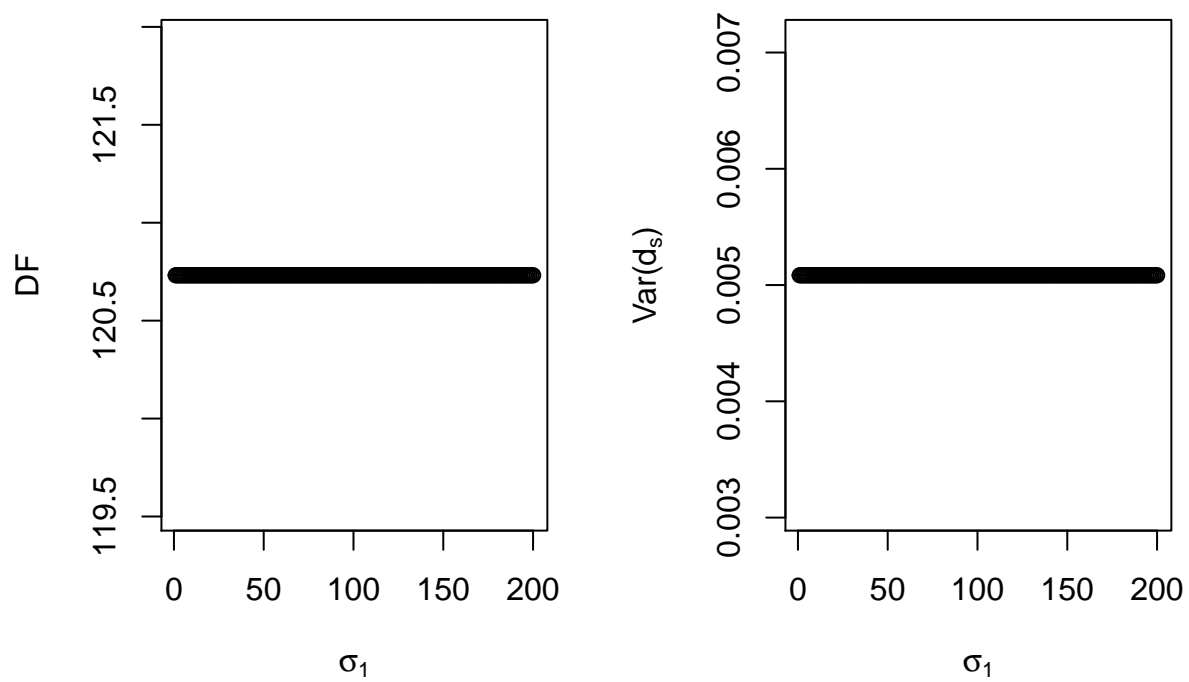
Figure 24. Variance of Shieh's d_s when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_1}{\sigma_2}\right)\right)$.
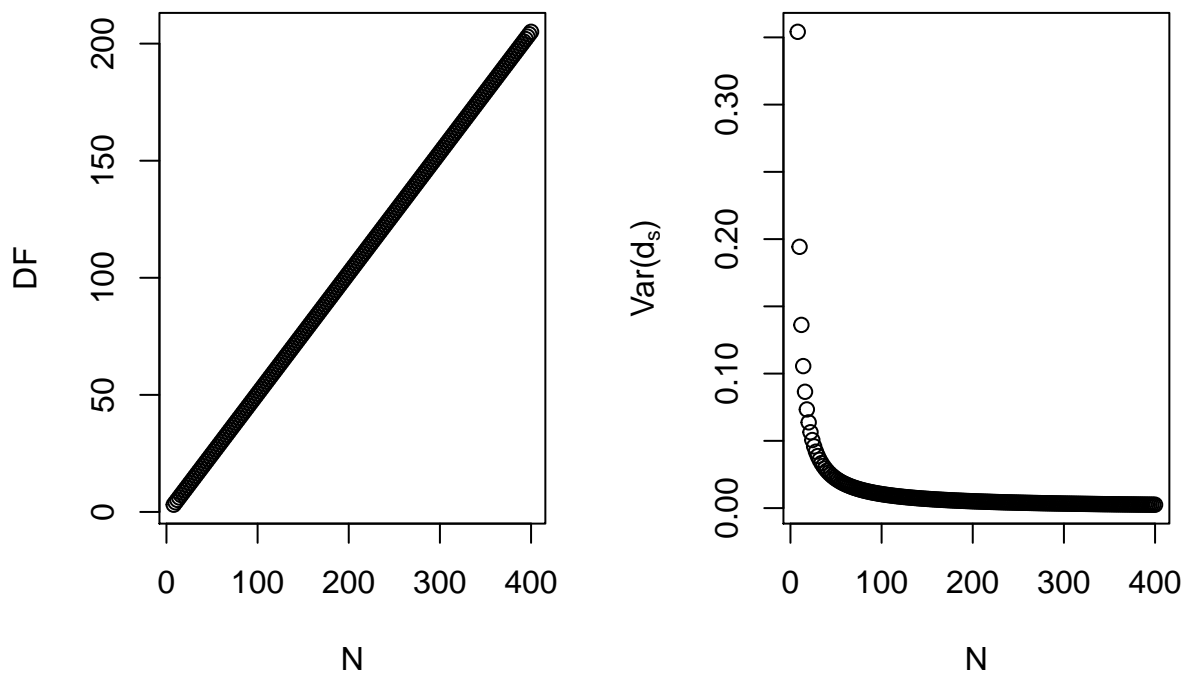
Figure 25. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 26. Variance of Shieh's d_s, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.
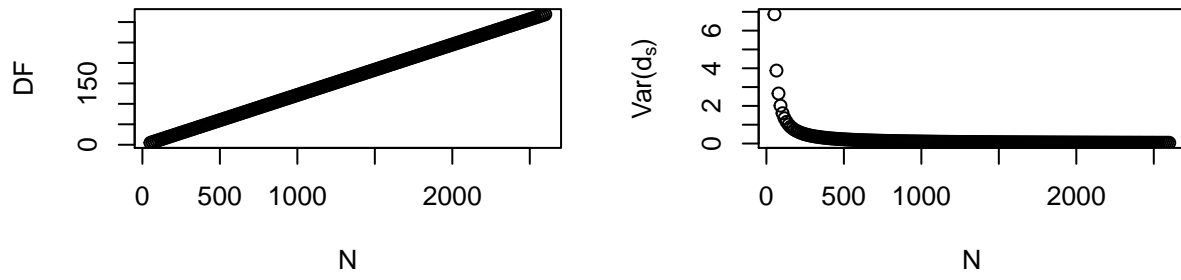
Figure 27. Variance of Shieh's d_s when variances and sample sizes are unequal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.
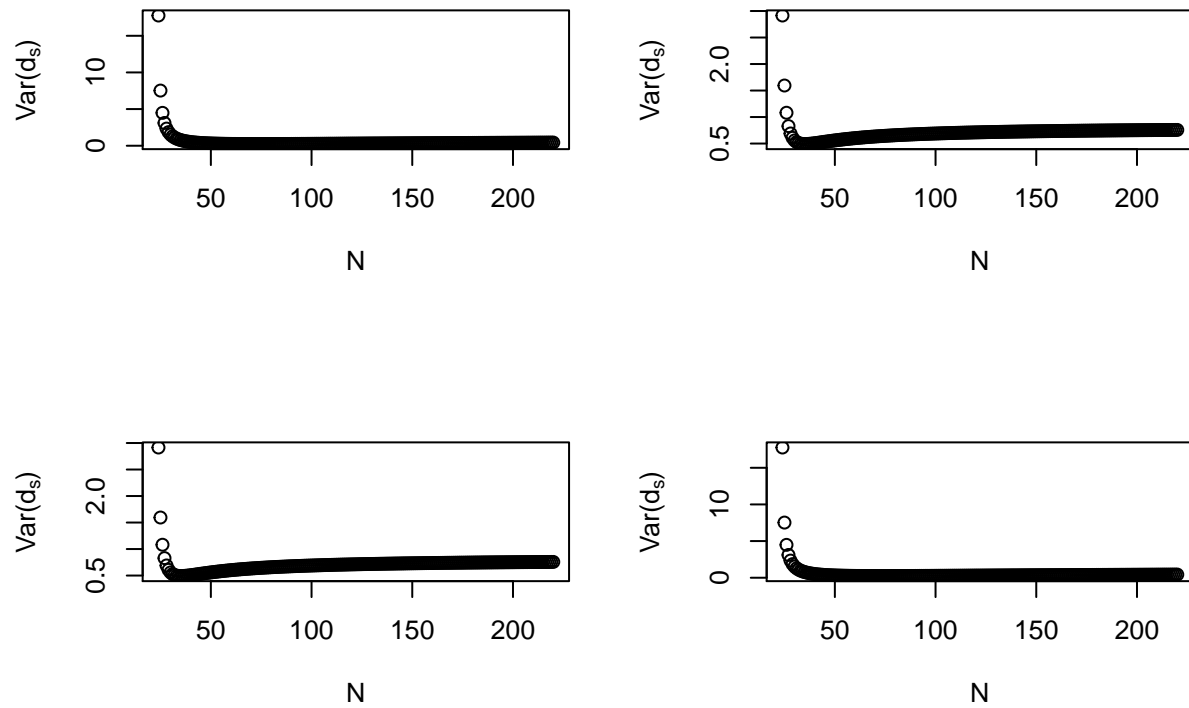
Figure 28. Variance of Shieh's d_s when variances and sample sizes are unequal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and $\sigma_1 > \sigma_2$ (top plots) or $\sigma_1 < \sigma_2$ (bottom plots).
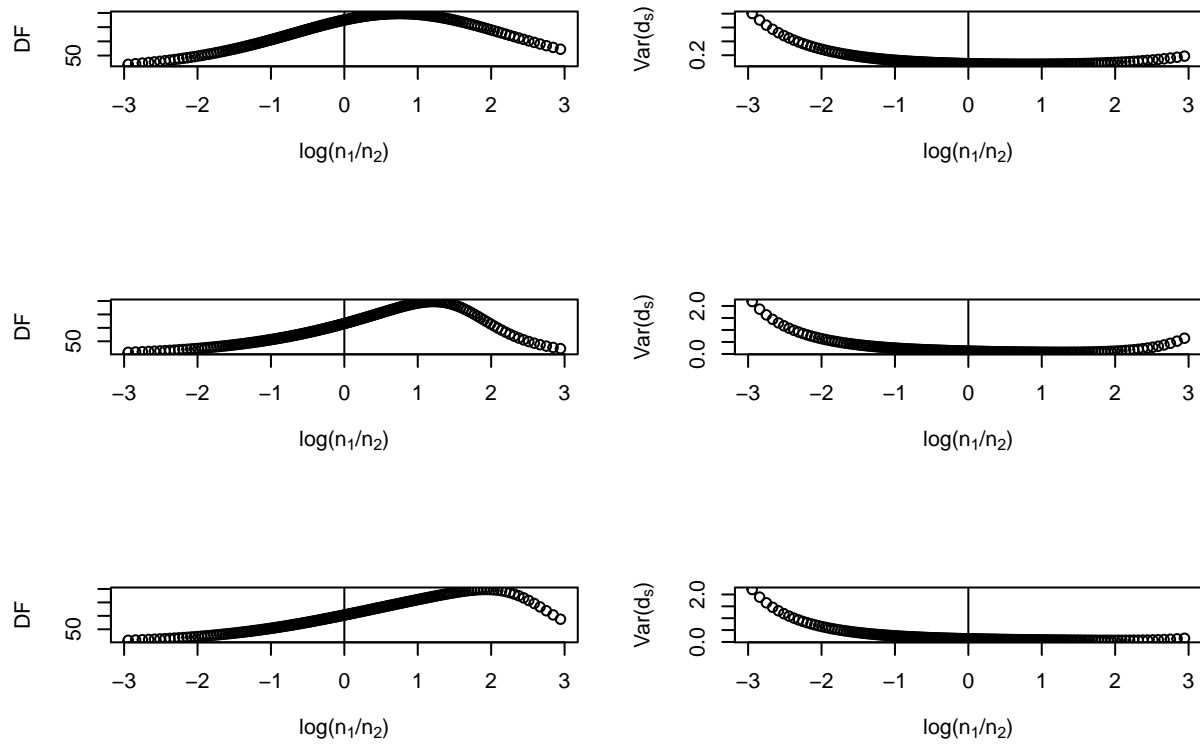
Figure 29. The variance of Shieh's d_s, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_1}{n_2}\right))$, when SD-ratio equals 1.46 (first row), 3.39 (second row) or 7 (third row).
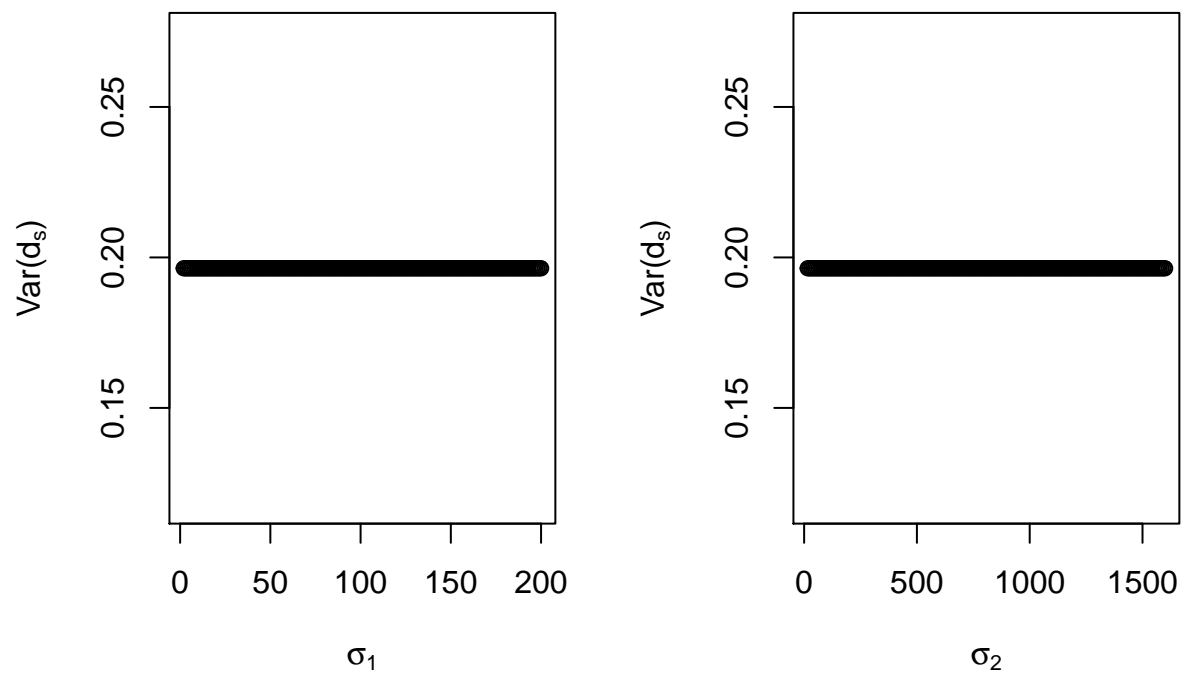
Figure 30. Variance of Shieh's d_s, when variances and sample sizes are unequal across groups, as a function of σ_1 (left) or σ_2 (right), for a constant SD-ratio.