1   Theoretical variance, as a function of population parameters

2   Theoretical variance, as a function of population parameters

3   **The variance**

4   Note: while we focus on the theoretical variance of biased estimators (Cohen's d_s,

5 Glass's d_s, Shieh's d_s and Cohen's d_s^*) when the normality assumption is met, it is

6 interesting to notice that our main conclusions seem to generalize to biased estimators

7 when samples are extracted from symmetric distributions. Moreover, unbiased estimators

8 depend on the same factors as biased estimators, so our conclusions remain similar for

9 unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

10 **Cohen's d_s**

11   **When variances are equal across populations.**

12   ***When $\delta_{Cohen} = 0$.***

13   When the population effect size is zero, the variance of Cohen's d_s can be simplified

14 as follows:

$$Var_{Cohen's\ d_s} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

15   The **variance** of Cohen's d_s is a function of total sample size (N) and the sample

16 sizes allocation ratio $\left(\frac{n_1}{n_2}\right)$:

17   • The larger the total sample size, the lower the variance. The variance tends to zero

18   when the total sample size tends to infinity (see Figure 1);

19   • The further the sample sizes allocation ratio is from 1, the larger the variance (see

20   Figure 2).

21   ***When $\delta_{Cohen} \neq 0$.***

22   While the variance of Cohen's d_s still depends on the total sample size and the

23 sample sizes allocation ratio, it also depends on the population effect size (δ_{Cohen}). The

24 larger the population effect size, the larger the variance. Note that the effect of the

25 population effect size decreases when sample sizes increase since

$$\lim_{n_1 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

26   This is illustrated in Figure 3.

27   **In summary.**   The variance of Cohen's d_s is a function of the population effect size

28 (δ_{Cohen}), the total sample size (N) and the sample sizes ratio ($\frac{n_1}{n_2}$):

29   • The variance decreases when the total sample size increases;

30   • The variance also decreases when the sample sizes ratio gets closer to 1;

31   • Finally, the variance increases when δ_{Cohen} increases. Note that the effect of δ_{Cohen} is

32   moderated by the total sample size (the larger N, the smaller the effect of δ_{Cohen} on

33   the variance).

34 **Glass's d_s**

35   **When variances are equal across populations.**

36 **When $\delta_{Glass} = 0$.**

37 When the population effect size is zero, the variance of Glass's d_s can be simplified as

38 follows:

$$Var_{Glass's\ d_s} = \frac{n_c - 1}{n_c - 3} \left(\frac{1}{n_c} + \frac{1}{n_e} \right)$$

39 . In this configuration, the **variance** of Glass's d_s is a function of the sample sizes of both

40 control (n_c) and experimental (n_e) groups as well as of the sample sizes allocation ratio

41 $\left(\frac{n_c}{n_e} \right)$:

42 • The larger the sample sizes, the lower the variance (Figure 4);

43 The sample sizes ratio associated with the lowest variance is not exactly 1 (because of

44 the term $\frac{df}{df-2}$, df depending only on n_c), but is very close to 1 (and the larger the total

45 sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance).

46 The further from this sample size ratio, the larger the variance (see Figure 5).

47 **When $\delta_{Glass} \neq 0$.**

48 While the variance of Glass's d_s still depends on the total sample size and the sample

49 sizes allocation ratio, it also depends on the population effect size (δ_{Glass}). The larger the

50 population effect size, the larger the variance. However, the effect of the population effect

51 size decreases when the control group increases. On the other hand, the effect of the

52 population effect size does *not* depend on the size of the experimental group since

$$\lim_{n_c \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2} \right)}{\Gamma\left(\frac{df}{2} \right)} \right)^2 \right] = 0$$

53

$$\lim_{n_e \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2} \right)}{\Gamma\left(\frac{df}{2} \right)} \right)^2 \right] \neq 0$$

54 These limits are illustrated in Figure 6.

Note: while the sample sizes ratio associated with the lowest variance was very close to 1 with a null population effect size, this is not true anymore when the population effect size is not zero. Indeed, because of the second term in the adddition, when computing the variance, one gives much more weight to the effect size of the control group (see Figure 7), especially when the effect size gets larger. For example, when $\delta_{Glass}= 4$, the lowest variance will occur when n_c is approximately 3 times larger than n_e. When $\delta_{Glass}= 7$, the lowest variance will occur when n_c is approximately 5 times larger than n_e, etc.

When variances are unequal across populations, with equal sample sizes.

When $\delta_{Glass} = 0$.

When the population effect size is zero, the variance of Glass's d_s can be simplified as follows:

$$Var_{Glass's\ d_s} = \frac{n-1}{n(n-3)}\left(1 + \frac{\sigma_e^2}{\sigma_c^2}\right)$$

where $n = N/2 =$ sample size of each group. The variance is therefore a function of the total sample size and the SD-ratio $\left(\frac{\sigma_c}{\sigma_e}\right)$:

- The larger the total sample size, the lower the variance (See Figure 8);

- The larger the SD-ratio (i.e. the larger is σ_c in comparison with σ_e), the lower the variance (see Figure 9). However, the effect of the SD-ratio decreases when sample sizes increase, because $\lim_{n(=n_c=n_e)\to\infty}\left[\frac{df}{n(df-2)}\right] = 0$.

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d_s still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases, as previously explained and illustrated in Figure 6.

₇₈   **When variances are unequal across populations, with unequal sample sizes.**

₇₉   ***When $\delta_{Glass} = 0$.***

₈₀   When the population effect size is zero, the variance of Glass's d_s can be simplified as

₈₁ follows:

$$Var_{Glass's\ d_s} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e\sigma_c^2}\right)$$

₈₂   The variance of Glass's d_s is therefore a function of the total sample size (N), the

₈₃ SD-ratio and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

₈₄   • For any SD and sample sizes pairing, increasing n_c and/or n_e will decrease the

₈₅   variance (see Figure 10);

₈₆   − The effect of the sample sizes ratio depends on the SD-ratio:

₈₇   ∗ We previously mentioned that when $\sigma_c = \sigma_e$, the variance is minimized

₈₈   when sample sizes of both groups are almost identical (see Figure 5),

₈₉   meaning that it is more efficient, in order to reduce variance, to add

₉₀   subjects uniformly in both groups;

₉₁   ∗ When $\sigma_e > \sigma_c$, more weight is given to n_e, meaning that it is more efficient,

₉₂   in order to reduce variance, to add subjects in the experimental group $(n_e;$

₉₃   see bottom plots in Figure 10);

₉₄   ∗ When $\sigma_c > \sigma_e$, less weight is given to n_e, meaning that it is more efficient, in

₉₅   order to reduce variance, to add sujects in the control group $(n_c;$ see top

₉₆   plots in Figure 10).

₉₇   • Finally, there is also a main effect of the SD-ratio: the larger is σ_c in comparison

₉₈   with σ_e, the lower the variance, as we can observe in Figure 11. We can also notice

that in Figure 10, the maximum variance is much larger in the two bottom plots

(where $\sigma_c < \sigma_e$) than in the two top plots (where $\sigma_c > \sigma_e$).

Note that the effect of the SD-ratio, and the interaction effect between SD-ratio and

sample sizes ratio decreases when the sample size of the control group increases (because

$\frac{n_c-1}{n_c-3}$ gets closer to 1).

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d_s still depends on the total sample size, the SD-ratio

and the interaction between the SD-ratio and the sample sizes ratio, it also depends on the

population effect size (δ_{Glass}): the larger the population effect size, the larger the variance.

However, the effect of the population effect size decreases when the sample size of the

control group increases, as previously explained and illustrated in Figure 6.

Note: when the population effect size was null, when $\sigma_c < \sigma_e$, it was much more

efficient to add subjects in the experimental group in order to reduce the variance (because

much more weight was given to n_e). When $\delta_{Glass} \neq 0$, it is important to add subjects in

both groups in order to reduce the variance (because $\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2$ is only a

function of the sample size of the control group). With huge population effect size, it is

even always more important to add subjects in the control group (e.g. when $\delta_{Glass} = 30$).

In summary.   The variance of Glass's d_s is a function of the population effect size

(δ_{Glass}), the SD-ratio, the total sample size and the interaction between sample sizes ratio

and SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e} \right)$:

- The variance decreases when the SD-ratio increases (i.e. when $\sigma_e >> \sigma_c$);
- The variance always decreases when the control and/or the experimental group

   increases. The benefit of adding subjects rather in the control, in the experimental,

   or in both groups, in order to reduce the variance, varies as a function of the

   SD-ratio and the population effect size. The only situation where it is optimal to

124   maximize the experimental group is when $\sigma_e > \sigma_c$ and $\delta_{Glass} \approx 0$. Most of the time, it

125   is more efficient to maximize the control groups (e.g. anytime $\sigma_e < \sigma_c$, and when

126   δ_{Glass} is very large) or to uniformly add subjects in both groups (e.g. when $\sigma_e > \sigma_c$

127   and δ_{Glass} is neither null nor huge);

128 • The variance increases when δ_{Glass} increases. Note that the effect of δ_{Glass} is

129   moderated by the control group size (the larger n_e, the smaller the effect of δ_{Glass} on

130   the variance).

Cohen's d_s^*

131

When variances are equal across populations.

132

When $\delta_{Cohen}^* = 0$.

133

134   When the population effect size is zero, the variance of Cohen's d_s^* is computed as

135 follows :

$$Var_{Cohen's\ d_s^*} = \frac{df}{df - 2} \times \frac{N}{n_1 n_2}$$

136 with

$$df = \frac{4(n_1 - 1)(n_2 - 1)}{n_1 + n_2 - 2}$$

137 In this configuration, the degrees of freedom as well as the variance of Cohen's d_s^* depend

138 on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_1}{n_2}$):

139 • The further the sample sizes allocation ratio is from 1, the larger the variance (see

140   Figure 12);

141 • The larger the total sample size, the lower the bias (see Figure 13).

142   ***When $\delta^*_{Cohen} \neq 0$.***

143   While the variance of Cohen's d^*_s still depends on the total sample size and the

144   sample sizes ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the

145   population effect size, the larger the variance. However, the effect of the population effect

146   size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or

147   the sample sizes ratio get closer to 1), as illustrated in Figure 14 since

$$\lim_{df \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

148   **When variances are unequal across populations, with equal sample sizes.**

149   ***When $\delta^*_{Cohen} = 0$.***

150   When the population effect size is zero, the variance of Cohen's d^*_s can be simplified

151   as follows:

$$Var_{Cohen's\ d^*_s} = \frac{df}{df - 2} \times \frac{2}{n}$$

152   where $n = N/2$=sample size of each group, and $df = \frac{(n-1)(\sigma_1^4 + \sigma_2^4 + 2\sigma_1^2\sigma_2^2)}{\sigma_1^4 + \sigma_2^4}$. In this

153   configuration, the degrees of freedom as well as the variance of Cohen's d_s depend on the

154   total sample size (N) and the SD-ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$:

155   • The further the SD-ratio is from 1, the larger the variance (see Figure 15);

156   • The larger the total sample size, the lower the variance (see Figure 16).

157   Note: for a constant SD-ratio, the size of the variance does not matter (see Figure

158   17).

159   ***When $\delta^*_{Cohen} \neq 0$.***

160   While the variance of Cohen's d^*_s still depednds on the total sample size and the

161   SD-ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population

162   effect size, the larger the variance. However, the effect of the population effect size

163   decreases when the degrees of freedom increase (i.e. when the total sample size increases

164   and/or the SD-ratio get closer to 1), as previously illustrated in Figure 14.

165   **When variances are unequal across populations, with unequal sample sizes.**

166   ***When $\delta^*_{Cohen} = 0$.***

167   When the population effect size is zero, the variance of Cohen's d^*_s can be simplified

168   as follows:

$$Var_{Cohen's\ d^*_s} = \frac{df}{df - 2} \times \frac{2\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2 + \sigma_2^2}$$

169   with $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$. In this configuration, the degrees of freedom are a function of

170   the total sample size (N) and the interaction between sample sizes ratio and the SD-ratio

171   $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

172   • The larger the total sample size, the lower the variance (illustration in Figure 18);

173   • The smallest variance always occurs when there is a positive pairing between

174   variances and sample sizes, because one gives more weight to the smallest variance in

175   the denominator of the df computation and in the numerator of the variance

176   computation. Moreover, the further the SD-ratio is from 1, the further from 1 will

177   also be the sample sizes ratio associated with the smallest variance (see Figure 19).

178   This can be explained by splitting the numerator and the denominator of the df

179   computation (see the file "Theoretical Bias, as a function of population parameters").

180   Note: for a constant SD-ratio, the variance does not matter. (See Figure 20).

181   ***When $\delta^*_{Cohen} \neq 0$.***

182   While the variance of Cohen's d^*_s still depends on the total sample size, the SD-ratio

183 and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the

184 population effect size (δ^*_{Cohen}): the larger the population effect size, the larger the variance.

185 However, the effect of the population effect size decreases when the degrees of freedom

186 increase (i.e. when the total sample size increases and/or when there is a positive pairing

187 between the sample sizes ratio and the SD-ratio), as previously illustrated in Figure 14.

188   **In summary.**   The variance of Cohen's d^*_s is a function of the population effect size

189 (δ^*_{Cohen}), the total sample size (N) and the interaction between sample sizes ratio and

190 SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2} \right)$:

191   • The variance always decreases when the control and/or the experimental group

192   increases. The benefit of adding subjects rather in the control or in the experimental

193   group depends on the SD-ratio. Indeed, the smallest variance always occurs when

194   there is a positive pairing between variances and sample sizes. Moreover, the further

195   the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio

196   associated with the smallest variance;

197   • The variance increases when δ^*_{Cohen} increases. Note that the effect of δ^*_{Cohen} is

198   moderated by the total sample size and the interaction between sample sizes ratio

199   and SD-ratio.

200 **Shieh's d_s**

201   **When variances are equal across populations.**

202   ***When $\delta_{Shieh} = 0$.***

203   When the population effect size is zero, the variance of Shieh's d_s can be simplified as

204 follows:

$$Var_{Shieh's \ d_s} = \frac{df}{(df - 2)N}$$

205 with $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$. In this configuration, the degrees of freedom as well as the

206 variance of Shieh's d_s depend on the total sample size (N) and the sample sizes allocation

207 ratio $\left(\frac{n_1}{n_2}\right)$:

208 • The further the sample sizes allocation ratio is from 1, the larger the variance (see

209   Figure 21);

210 • The larger the total sample size, the lower the variance. It does not matter whether

211   the sample sizes ratio is constant (see Figure 22) or not (see Figure 23).

212 Note: in "Theoretical Bias, as a function of population parameters," we noticed that

213 increasing the sample sizes ratio when adding subjects in only one group could decrease the

214 degrees of freedom. However, due to the total sample size term (N) in the denominator of

215 the variance computation, even when degrees of freedom decrease due to the fact that one

216 adds subjects only in one group, the variance still decreases (because the denominator of

217 the variance computation increases; see Figure 23).

218 ***When $\delta_{Shieh} \neq 0$.***

219 While the variance of Shieh's d_s still depends on the total sample size and the sample

220 sizes ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population

221 effect size, the larger the variance. However, the effect of the population effect size

222 decreases when the degrees of freedom increase (i.e. when sample sizes increase, without

223 increasing the sample sizes ratio, and/or the sample sizes ratio gets closer to 1), as

224 previously illustrated in Figure 14.

225 Note: we previously noticed that when the effet size is zero, the variance of Shieh's d_s

226 decreases, even when the sample sizes ratio increases. It is no longer true when there is a

227 non-null effect size because the larger the sample sizes ratio, the more the variance will

228 increase with increasing effect size.

229   **When variances are unequal across populations, with equal sample sizes.**

230   ***When $\delta_{Shieh} = 0$.***

231   When the population effect size is zero, the variance of Shieh's d_s can be simplified as

232 follows:

$$Var_{Shieh's\ d_s} = \frac{df}{(df-2)N}$$

233 with $df = \frac{(\sigma_1^2 + \sigma_2^2)^2 \times (n-1)}{\sigma_1^4 + \sigma_2^4}$. In this configuration, the degrees of freedom as well as the

234 variance of Shieh's d_s depend on the total sample size (N) and the SD-ratio ($\frac{\sigma_1}{\sigma_2}$).

235   • The further the SD-ratio is from 1, the larger the variance (see Figure 24);

236   • The larger the total sample size, the lower the variance (see Figure 25).

237   Note: for a constant SD-ratio, the size of the variance does not matter (see Figure

238 26).

239   ***When $\delta_{Shieh} \neq 0$.***

240   While the variance of Shieh's d_s still depends on the total sample size and the

241 SD-ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population

242 effect size, the larger the variance. However, the effect of the population effect size

243 decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the

244 SD-ratio gets closer to 1), as previously illustrated in Figure 14.

245   **When variances are unequal across populations, with unequal sample sizes.**

246   ***When $\delta_{Shieh} = 0$.***

247   When the population effect size is zero, the variance of Shieh's d_s can be simplified as

248 follows:

$$Var_{Shieh's\ d_s} = \frac{df}{(df-2)N}$$

249   with $df = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1} + \frac{(\sigma_2^2/n_2)^2}{n_2-1}}$. In this configuration, the degrees of freedom as well as the

250   variance of Shieh's d_s depend on the total sample size (N) and the interaction between the

251   sample sizes ratio and the SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

252   • The larger the total sample size, the lower the variance. It remains true when the

253   sample sizes ratio is constant (see Figure 27) and when it is not (see Figure 28).

254   Note: When variances were equal across populations, adding subjects only in the first

255   group had the same impact on the variance as adding subjects only in the second group

256   (see Figure 23). When variances are unequal across groups, this is not true anymore (see

257   Figure 28).

258   • The smallest variance always occurs when there is a positive pairing between

259   variances and sample size. Moreover, the further the SD-ratio is from 1, the further

260   from 1 will also be the sample sizes ratio associated with the smallest variance (see

261   Figure 29).

262   Moreover, for a constant SD-ratio, the variances do not matter (See Figure 30).

263   **When $\delta_{Shieh} \neq 0$.**

264   While the variance of Shieh's d_s still depends on the total sample size and the

265   interaction between the sample sizes ratio and the SD-ratio, it also depends on the

266   population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance.

267   However, the effect of the population effect size decreases when the degrees of freedom

268   increase, as previously illustrated in Figure 14.

269   **In summary.**   The variance of Shieh's d_s is a function of the population effect size

270   (δ_{Shieh}), the total sample size (N) and the interaction between sample sizes ratio and

271   SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;

- The variance increases when δ_{Shieh} increases. Note that the effect of δ_{Shieh} is moderated by the total sample size and the interaction between the sample sizes ratio and the SD-ratio.
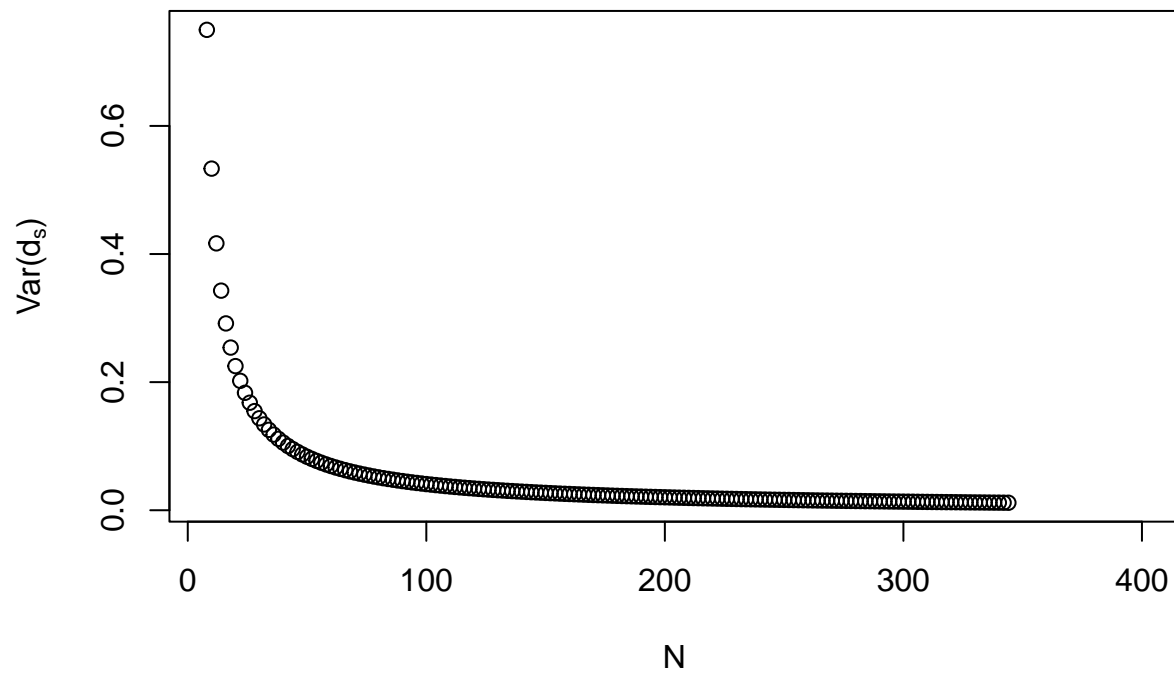
Figure 1. Variance of Cohen's d_s, when variances are equal across groups, as a function of the total sample size (N).
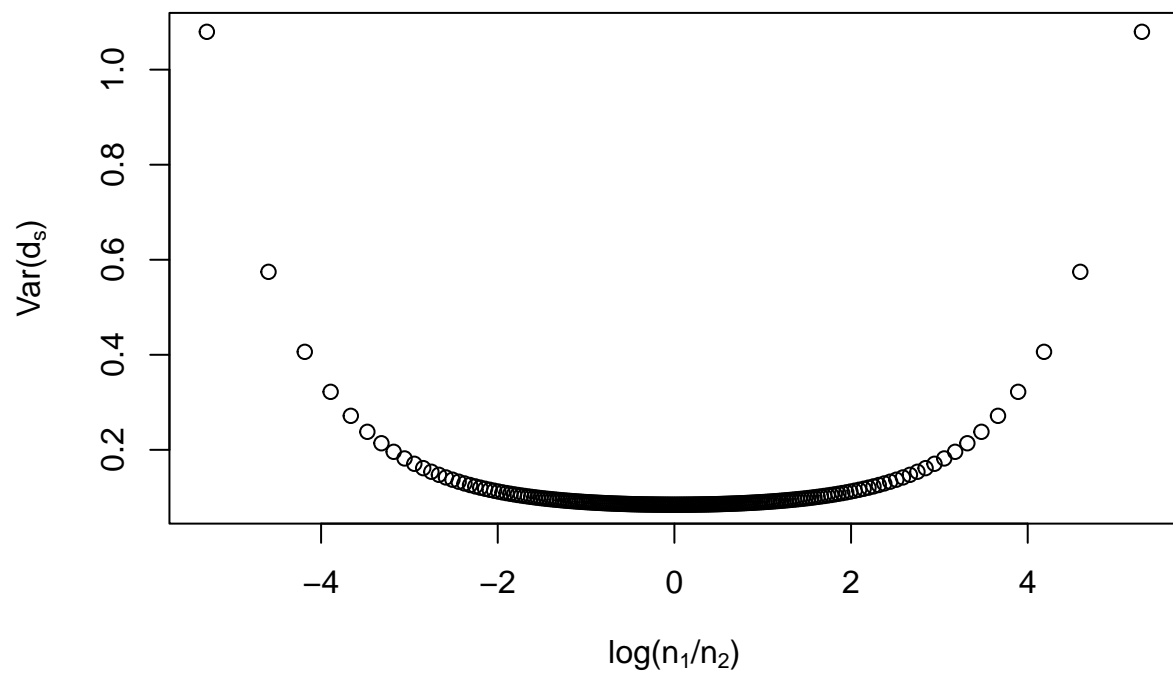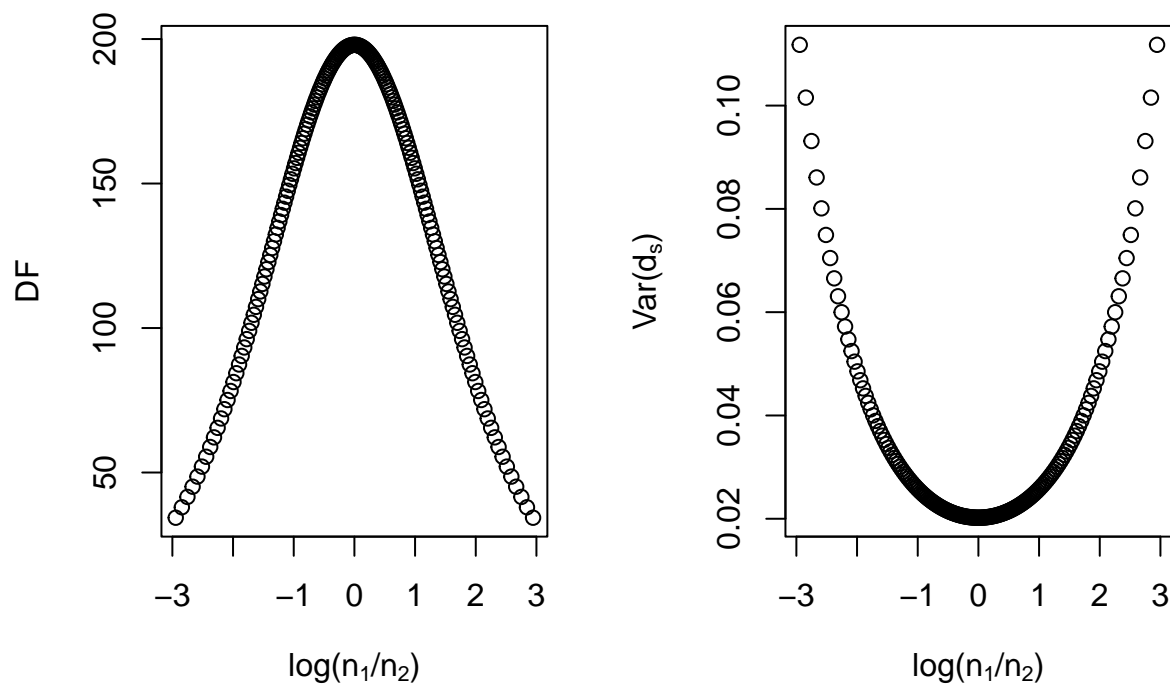
Figure 2. Variance of Cohen's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_1}{n_2}\right)$).
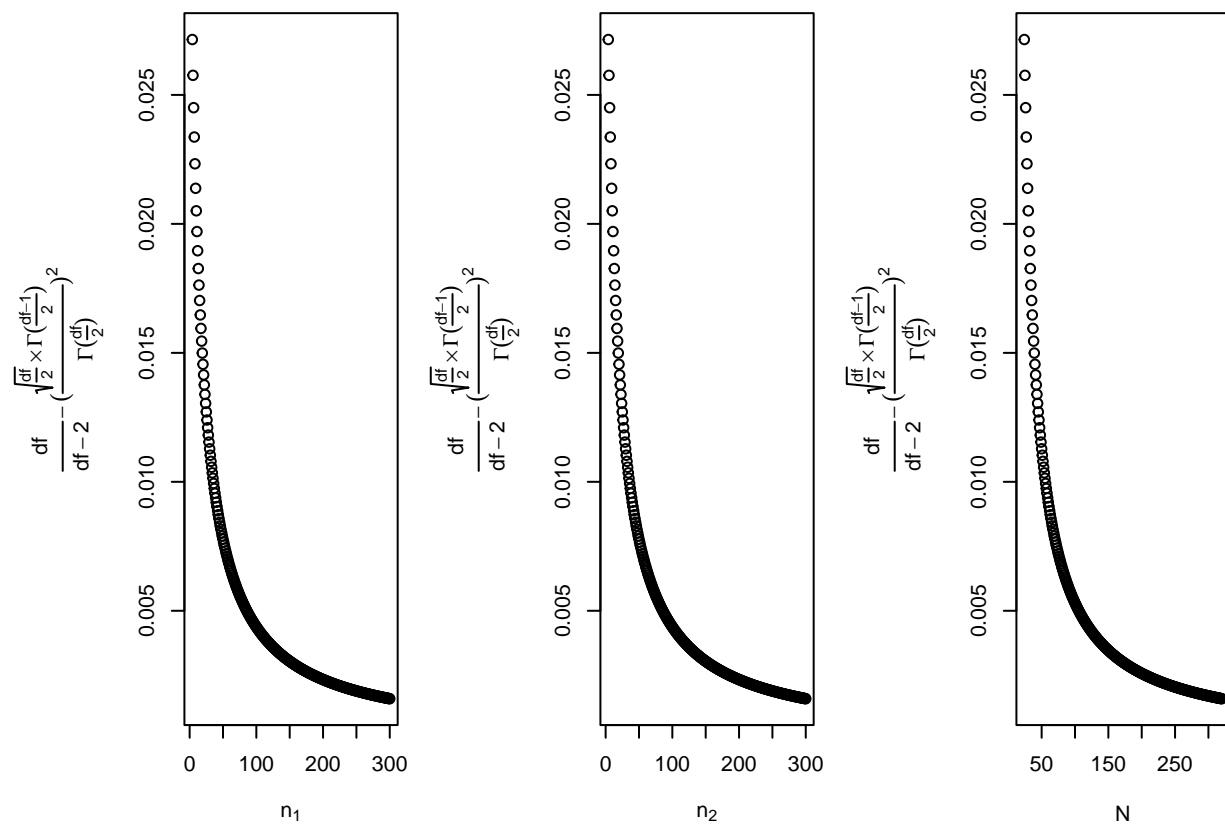
Figure 3. Effect size moderator, when computing the variance of Cohen's d_s, as a function of n_1 (left), n_2 (center) and $N = n_1 + n_2$ (right).
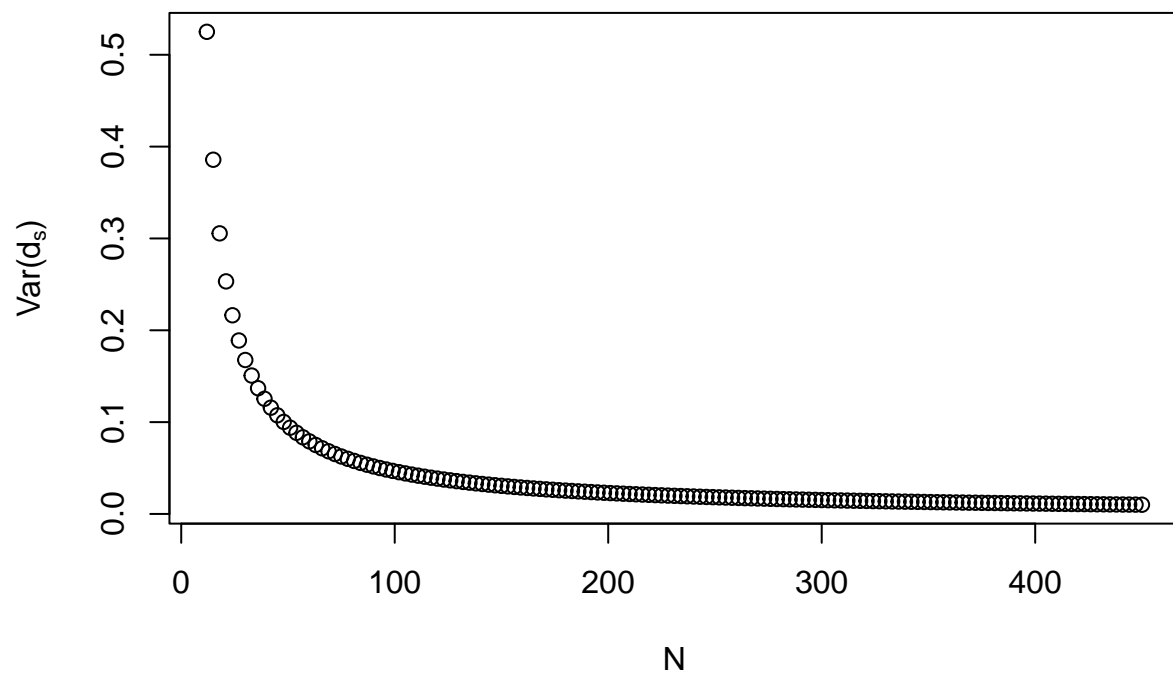
Figure 4. Variance of Glass's d_s, when variances are equal across groups, as a function of the total sample size (N).
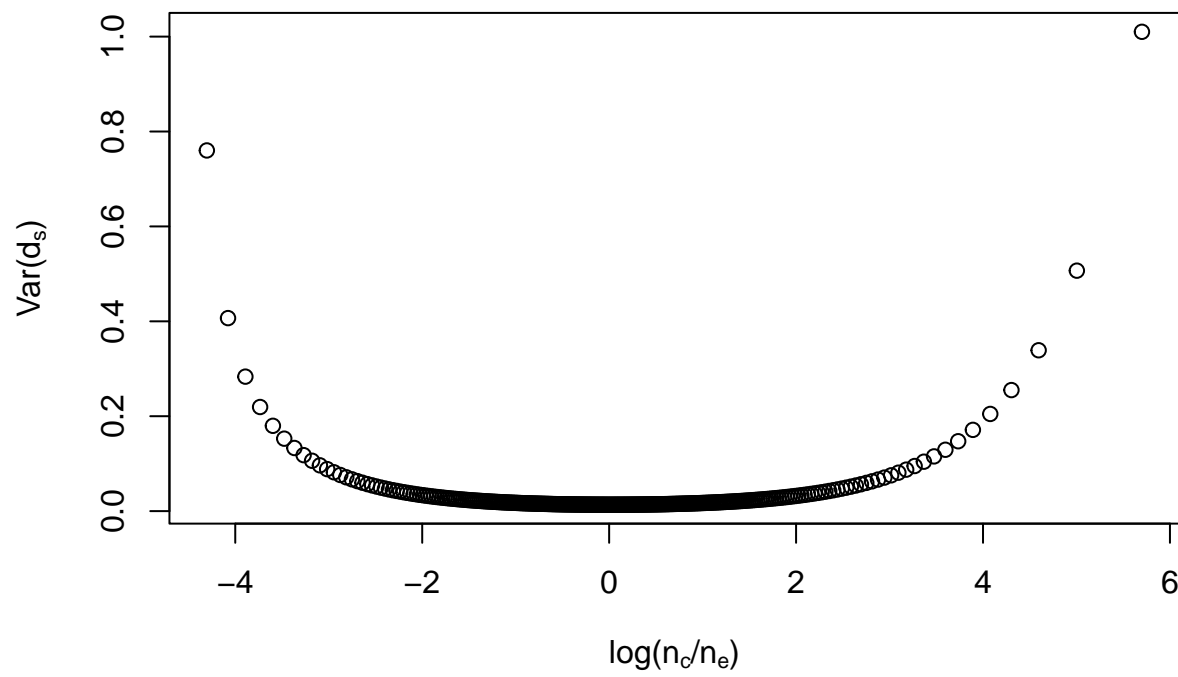
Figure 5. Variance of Glass's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_c}{n_e}\right))$.
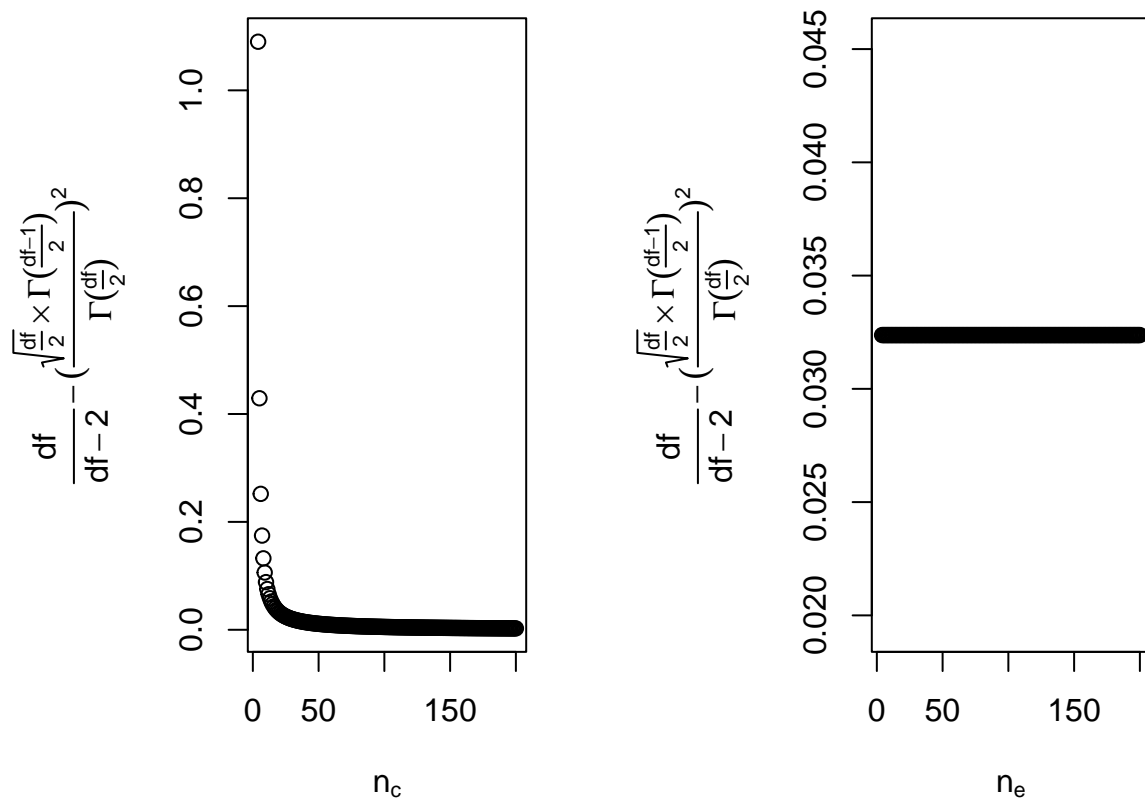
Figure 6. Effect size moderator, when computing the variance of Glass's d_s, as a function of the size of the control group (left) and experimental group (right).
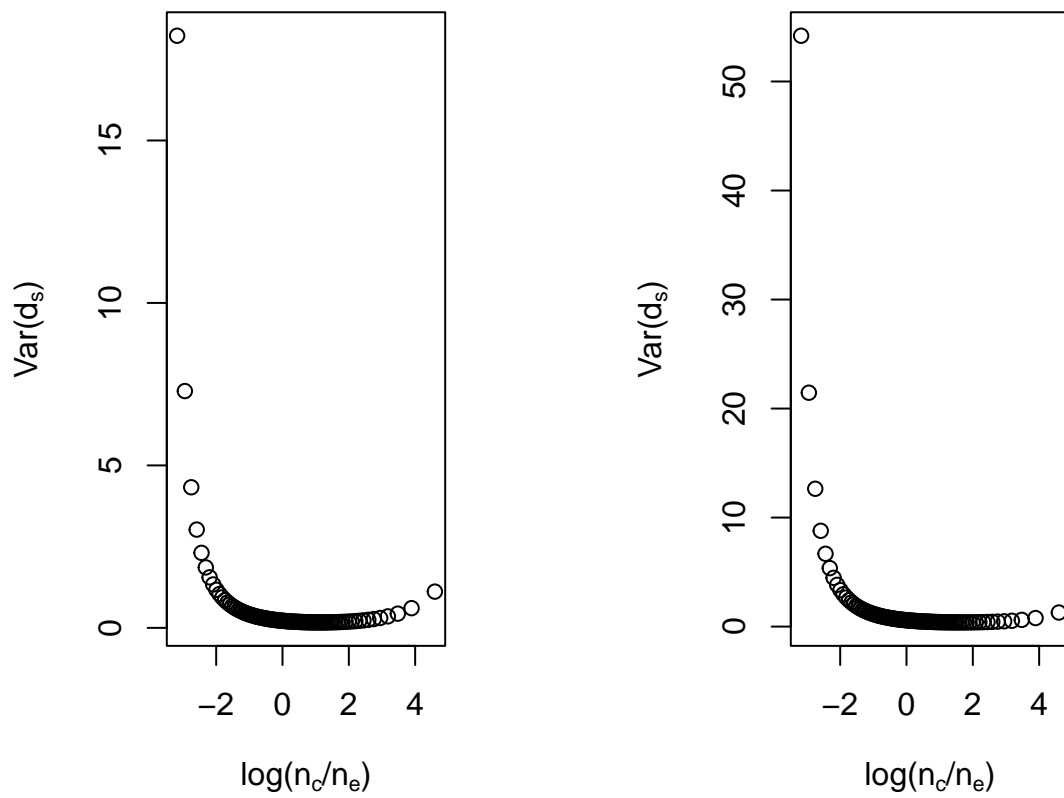
Figure 7. Variance of Glass's d_s, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_c}{n_e}\right)\right)$ when δ_{Glass} equals 4 (left) or 7 (right).
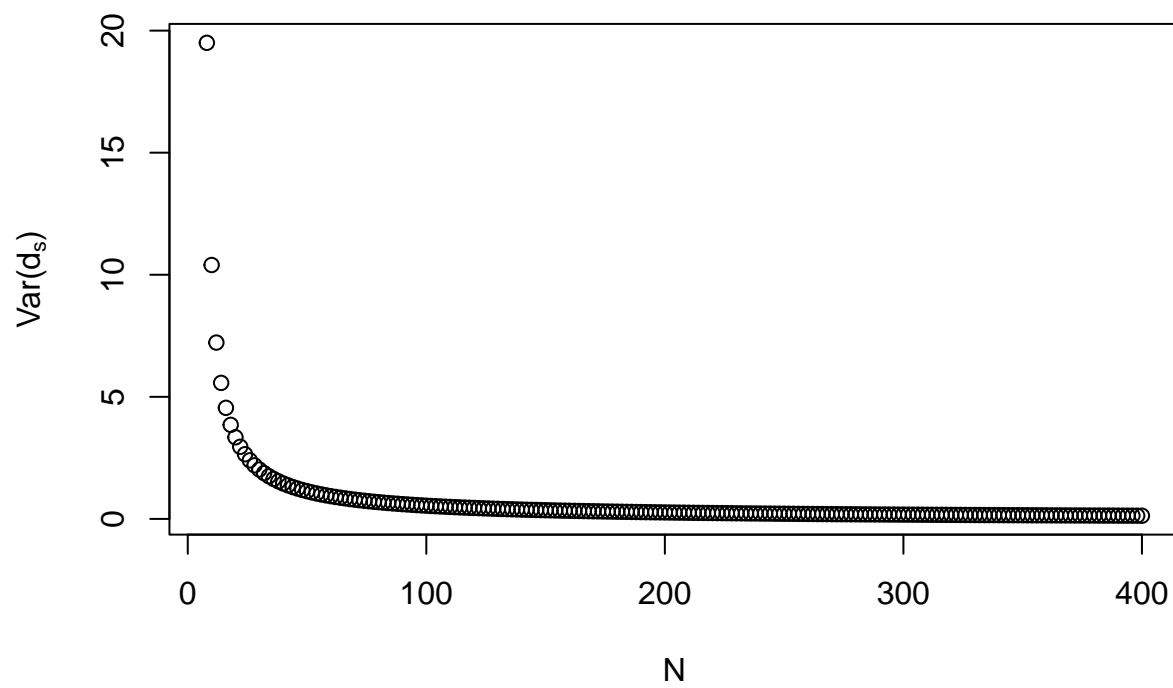
Figure 8. Variance of Glass's d_s, when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes (N).
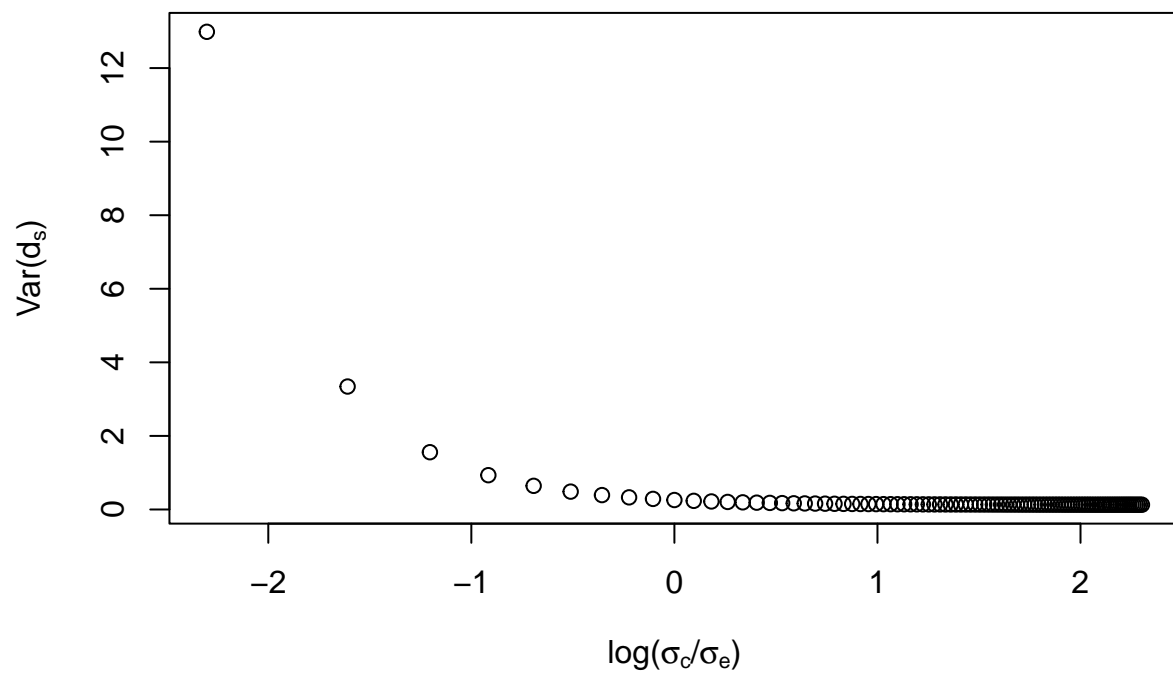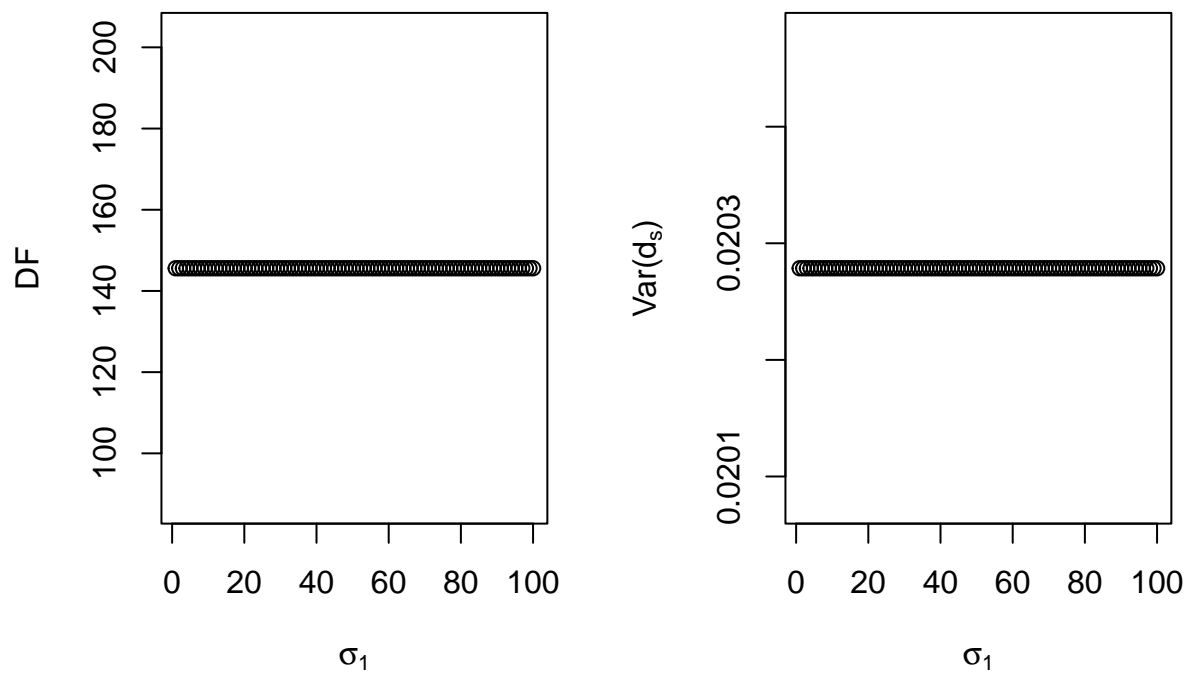
Figure 9. Variance of Glass's d_s, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
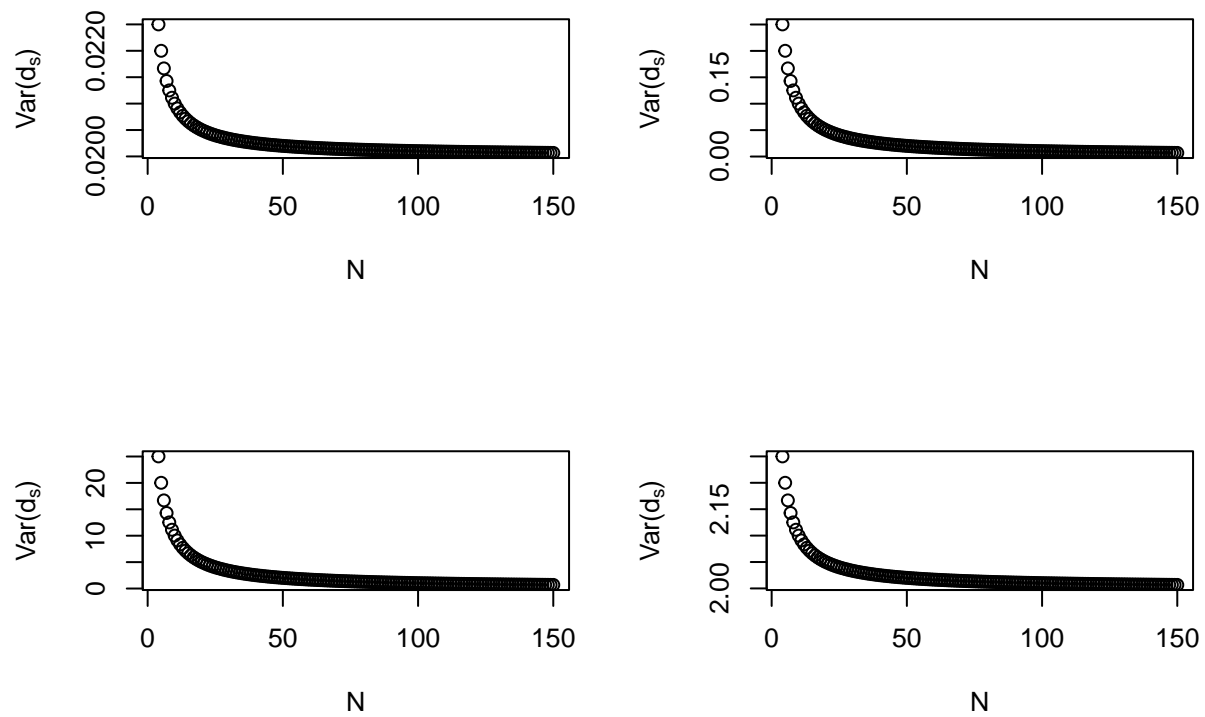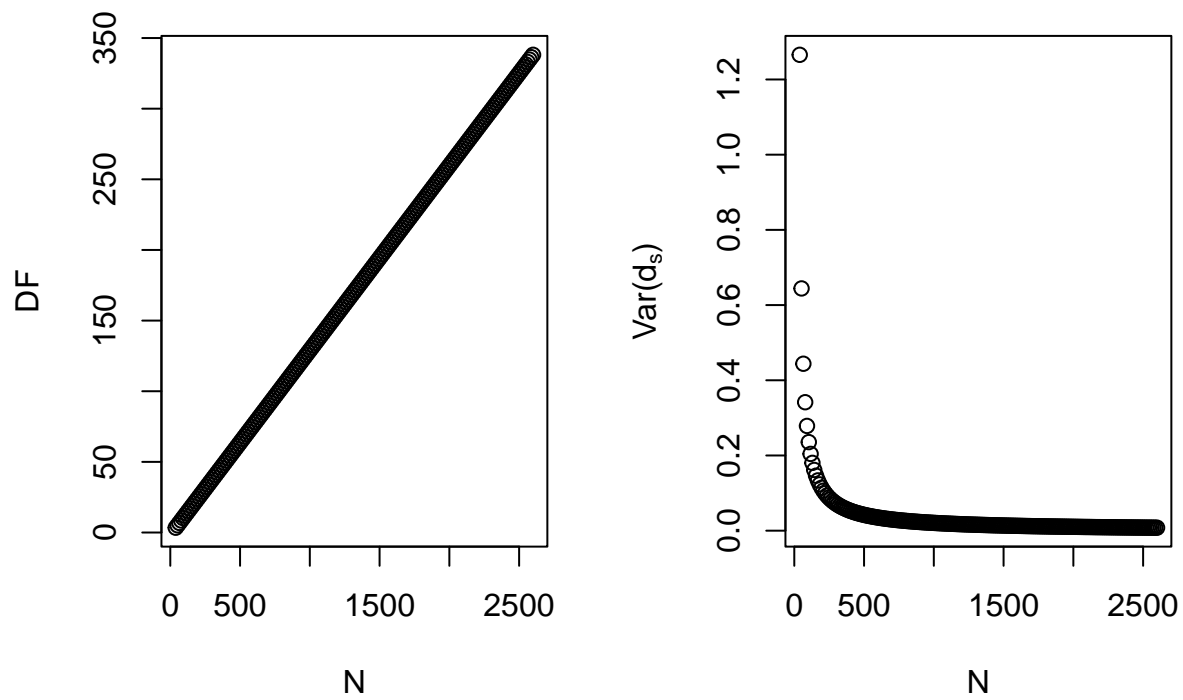
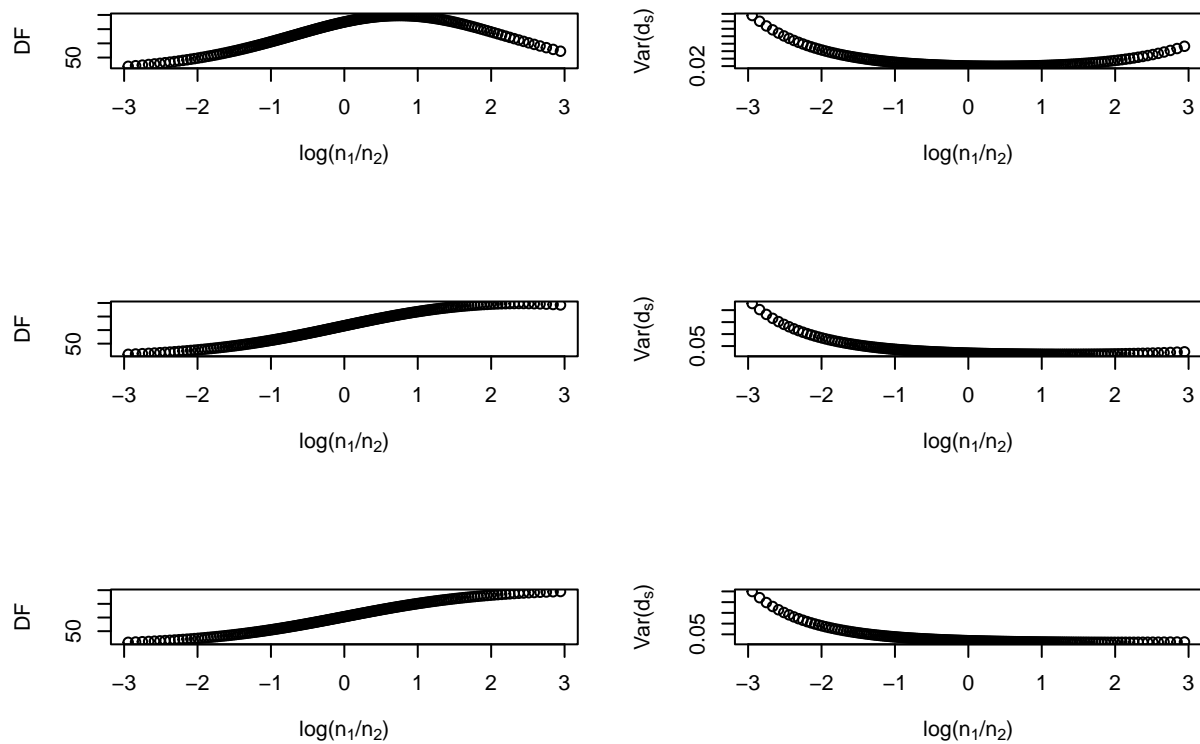*Figure 10.* Variance of Glass's d_s, when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when $\sigma_c > \sigma_e$ (top plots) or $\sigma_c < \sigma_e$ (bottom plots).
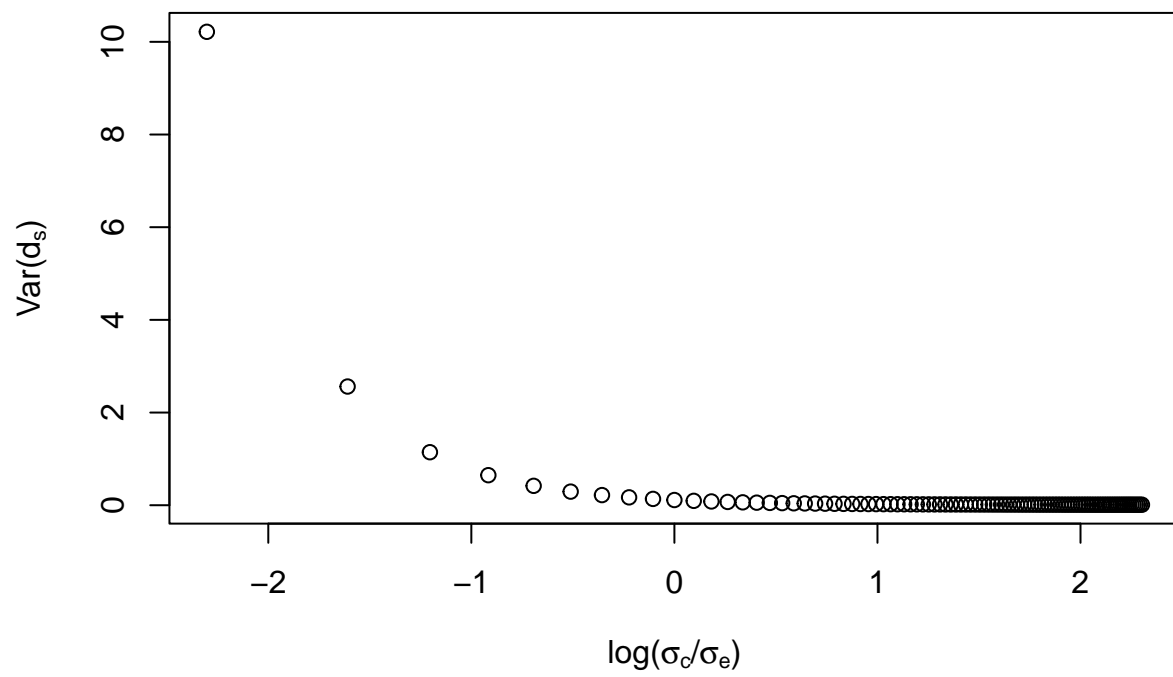
Figure 11. Variance of Glass's d_s, when sample sizes and variances are unequal across groups, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
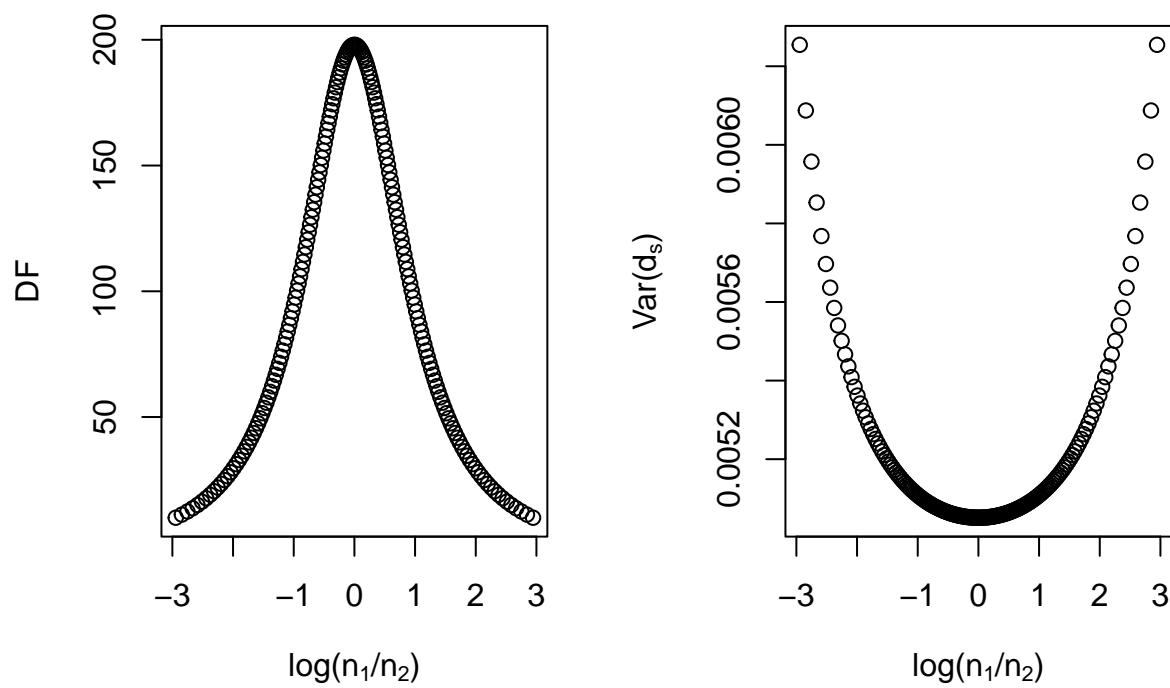
Figure 12. Variance of Cohen's d_s^* when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.
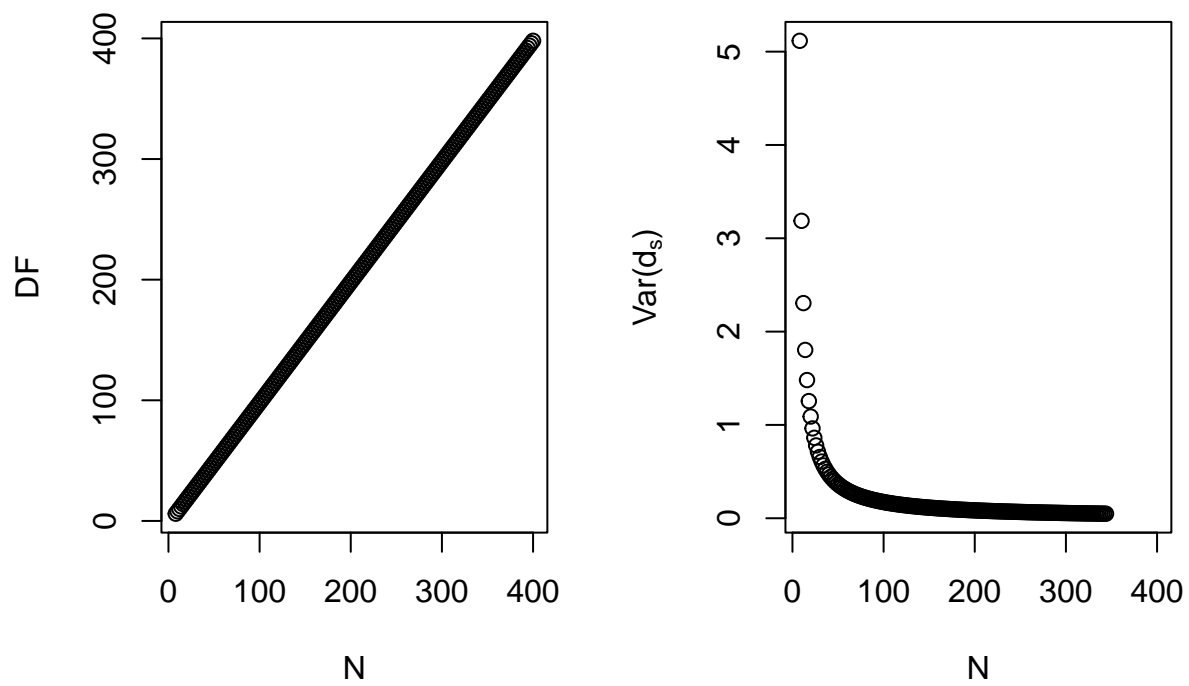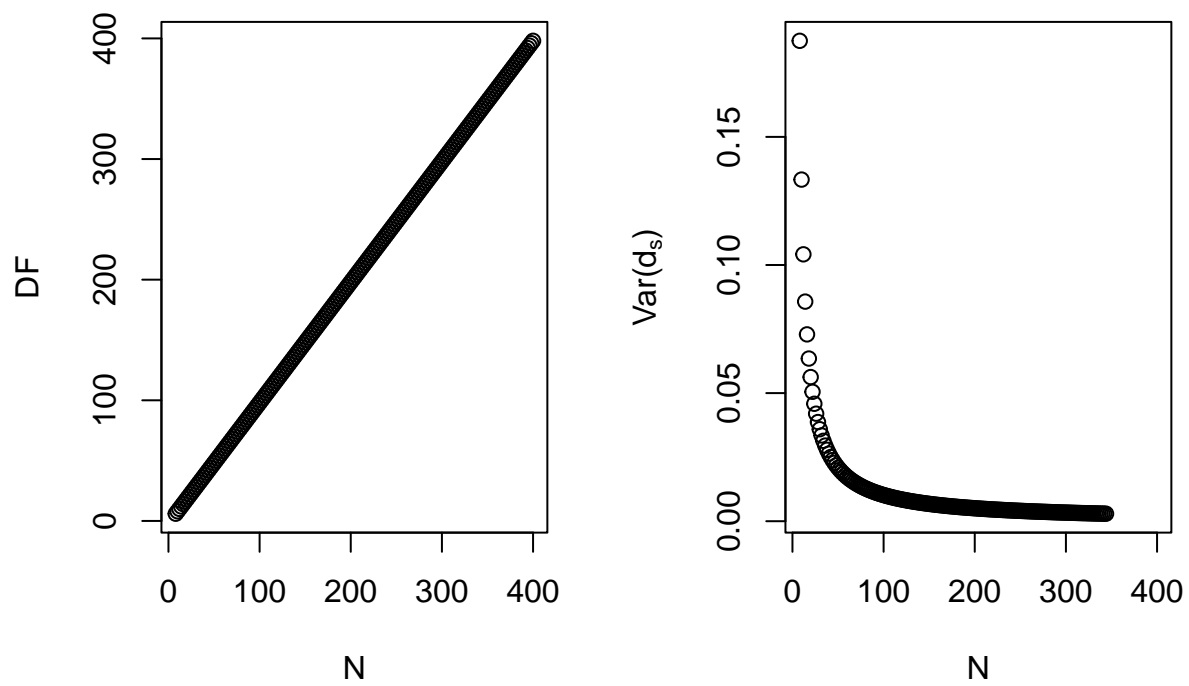
Figure 13. Variance of Cohen's d_s^* when variances are equal across groups, as a function of the total sample size (N).
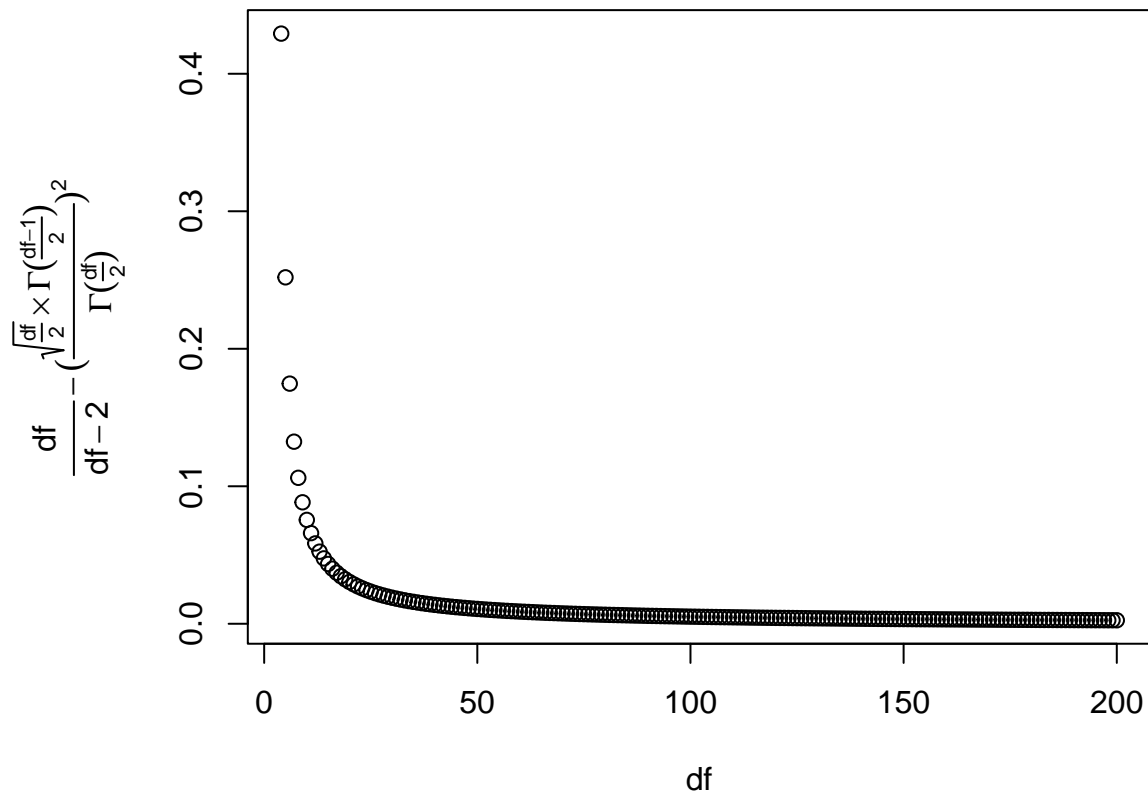
Figure 14. Effect size moderator (for all estimators), as a function of the degrees of freedom.
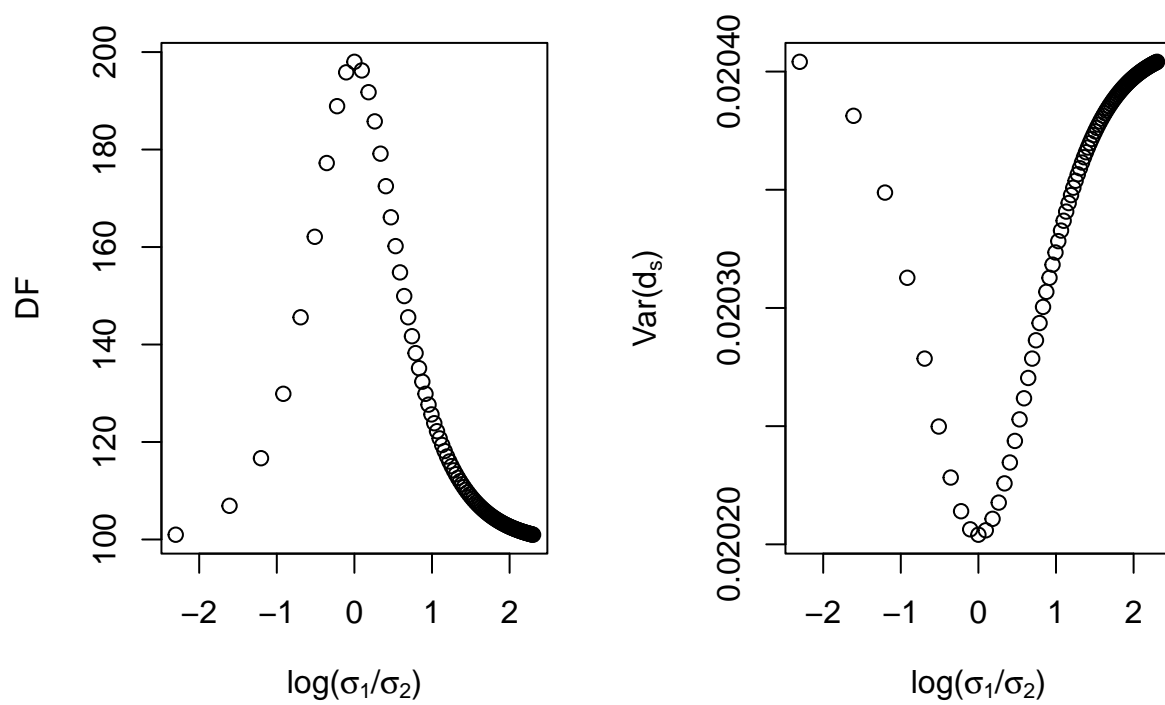
Figure 15. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_1}{\sigma_2}\right)\right)$.
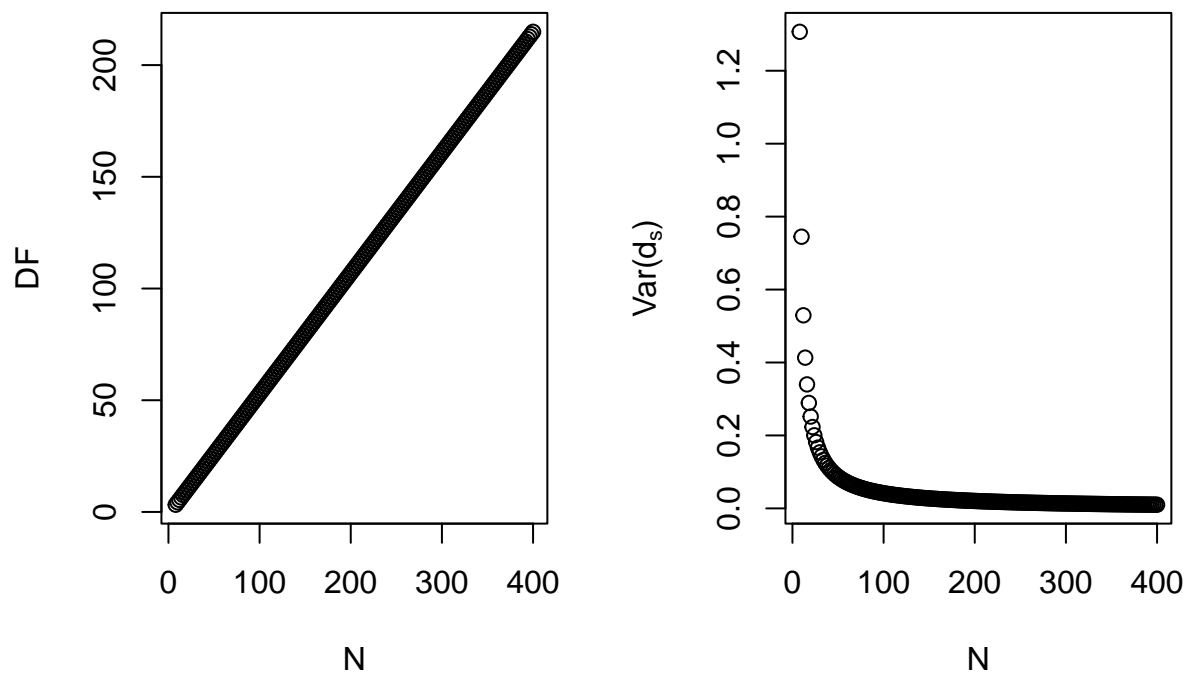
Figure 16. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 17. Variance of Cohen's d_s^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

Figure 18. Variance of Cohen's d_s^* when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

Figure 19. The variance of Cohen's d_s^*, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_1}{n_2}\right))$, when SD-ratio equals 1.46 (first row), 3.39 (second row) or 7 (third row).
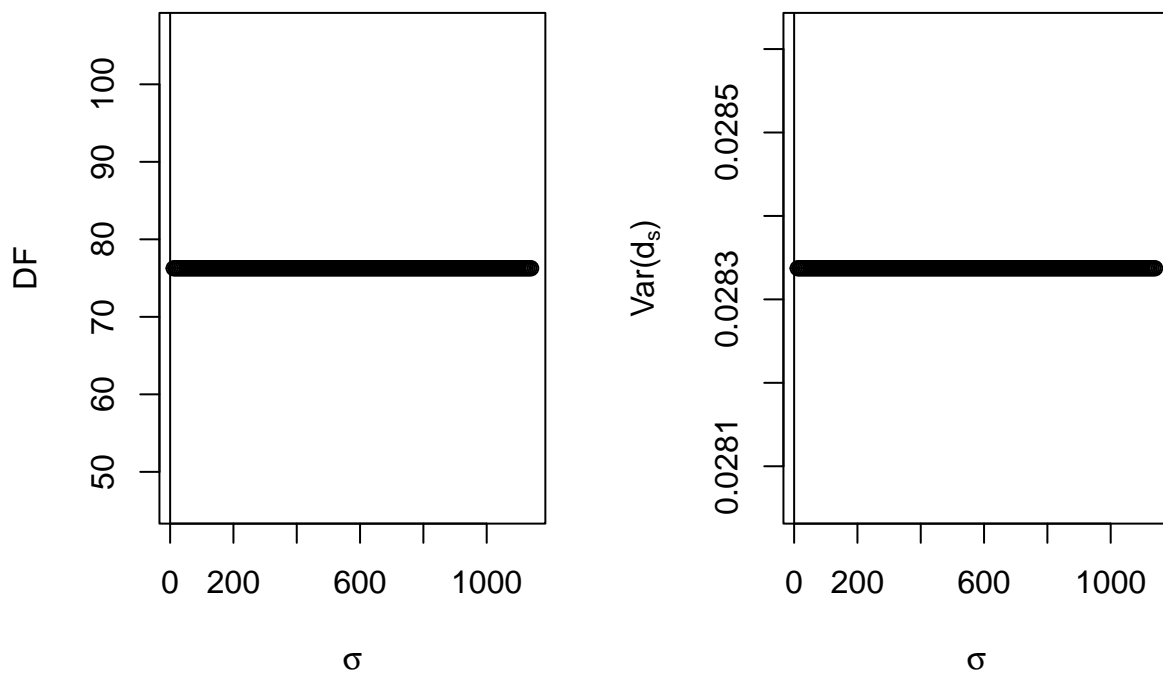
Figure 20. Variance of Cohen's d_s^*, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

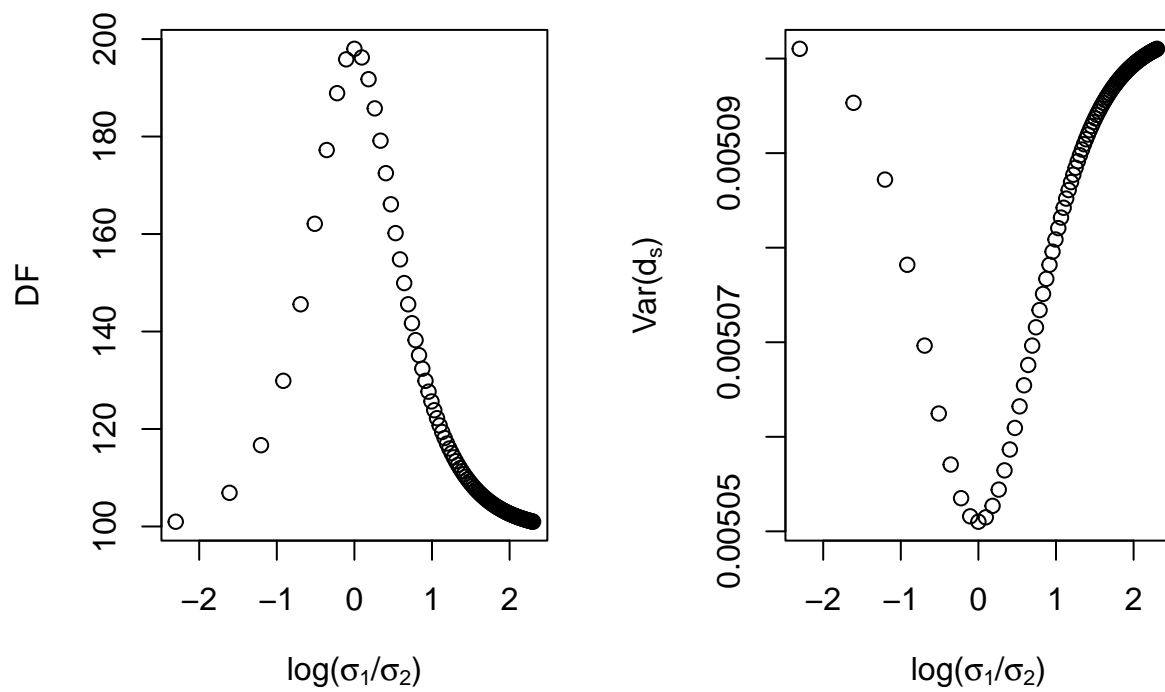*Figure 21*. Variance of Shieh's d_s when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.

Figure 22. Variance of Shieh's d_s when variances are equal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.
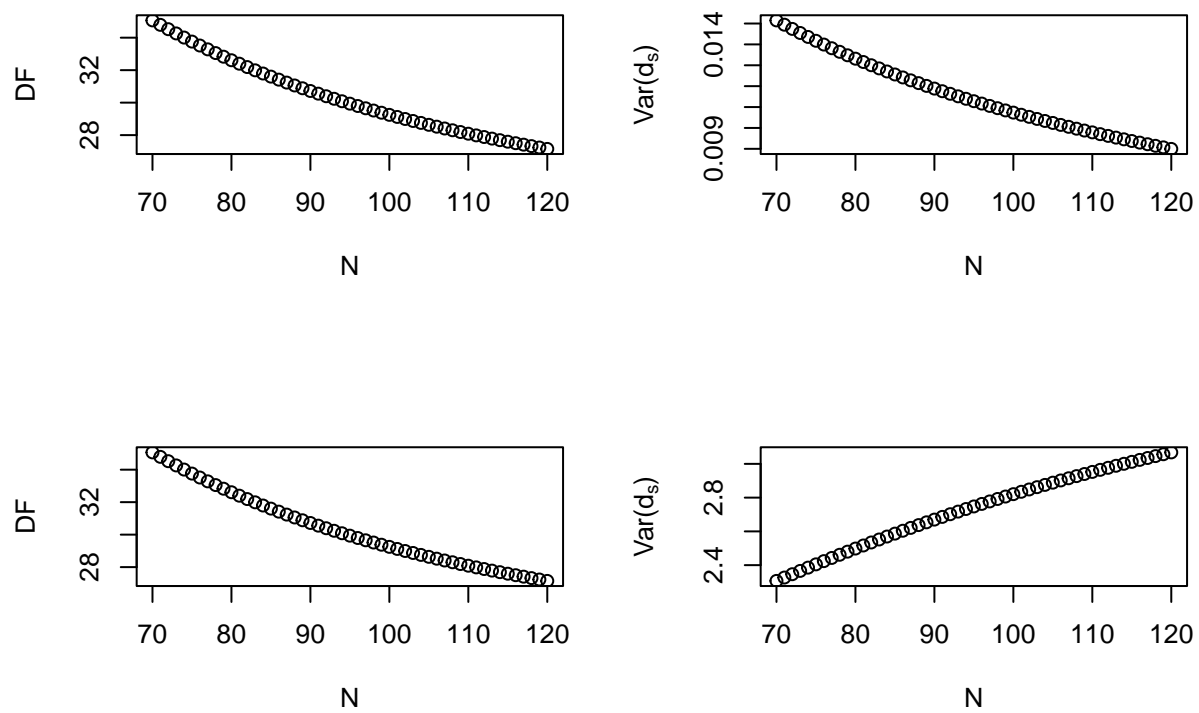
Figure 23. Variance of Shieh's d_s when variances are equal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).

Figure 24. Variance of Shieh's d_s when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $(log\left(\frac{\sigma_1}{\sigma_2}\right))$.
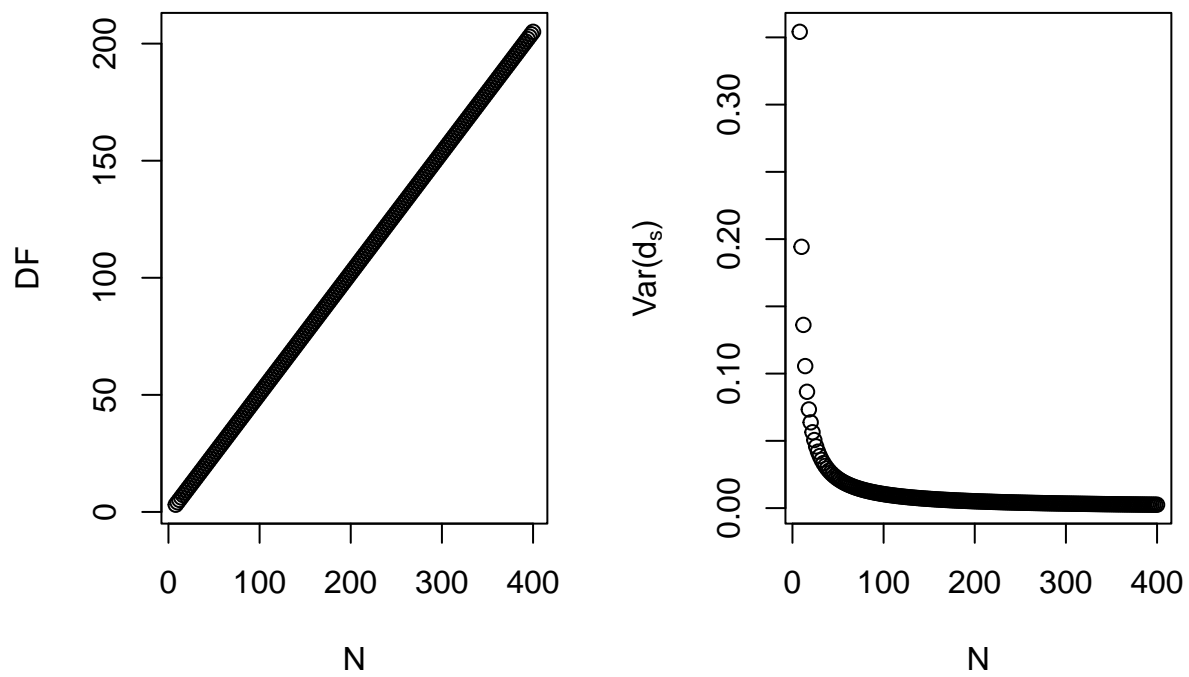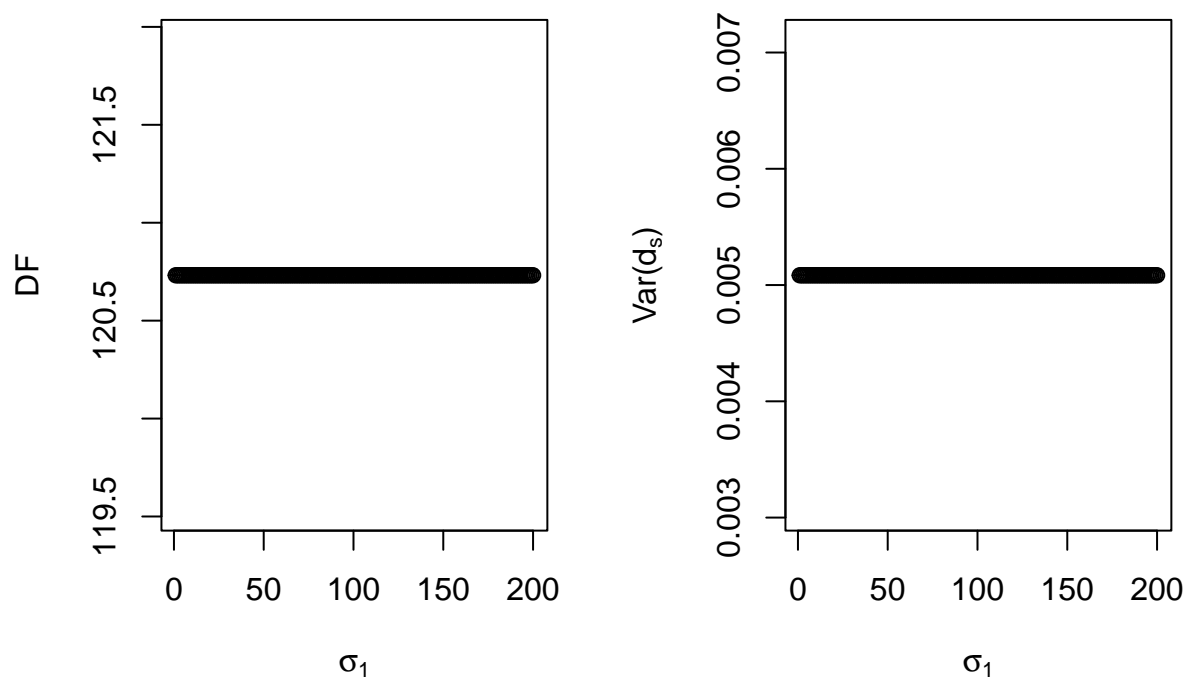
Figure 25. Variance of Cohen's d_s^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 26. Variance of Shieh's d_s, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.
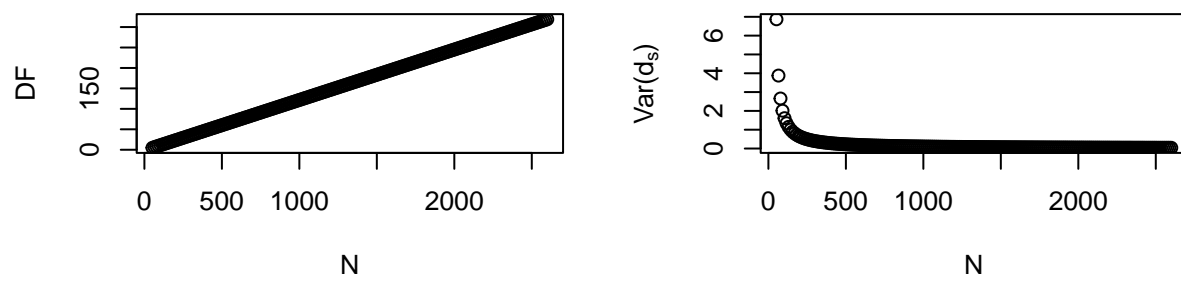
Figure 27. Variance of Shieh's d_s when variances and sample sizes are unequal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$.
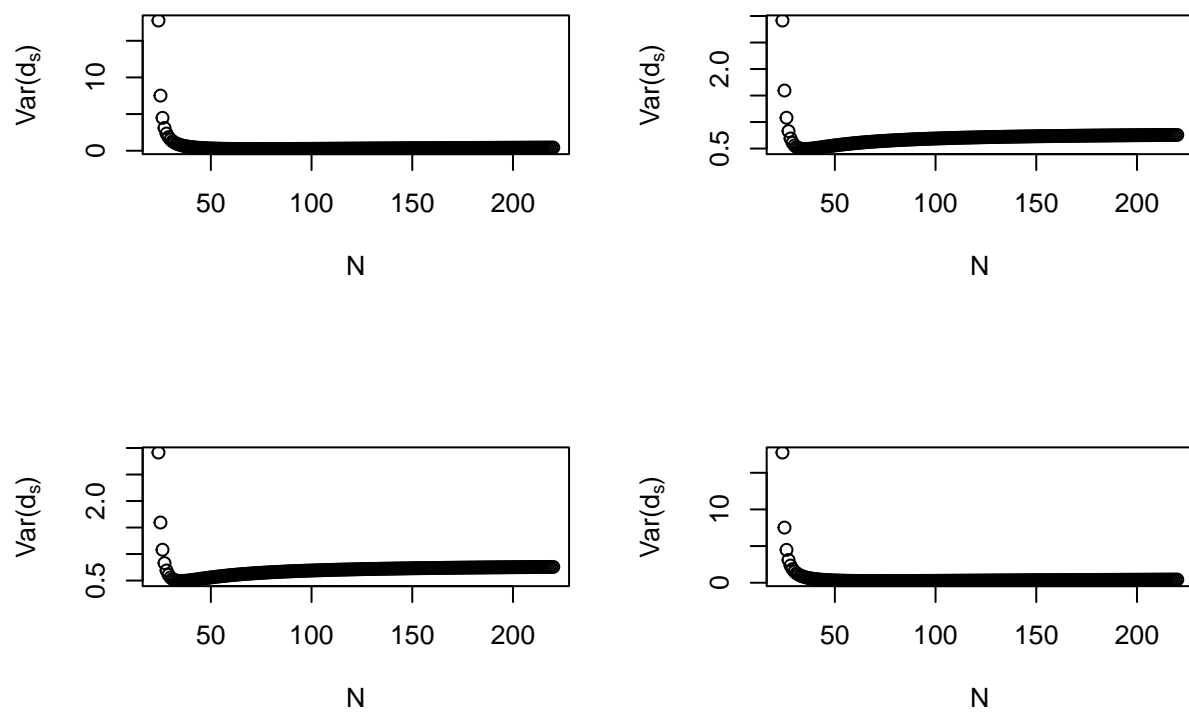
Figure 28. Variance of Shieh's d_s when variances and sample sizes are unequal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and $\sigma_1 > \sigma_2$ (top plots) or $\sigma_1 < \sigma_2$ (bottom plots).
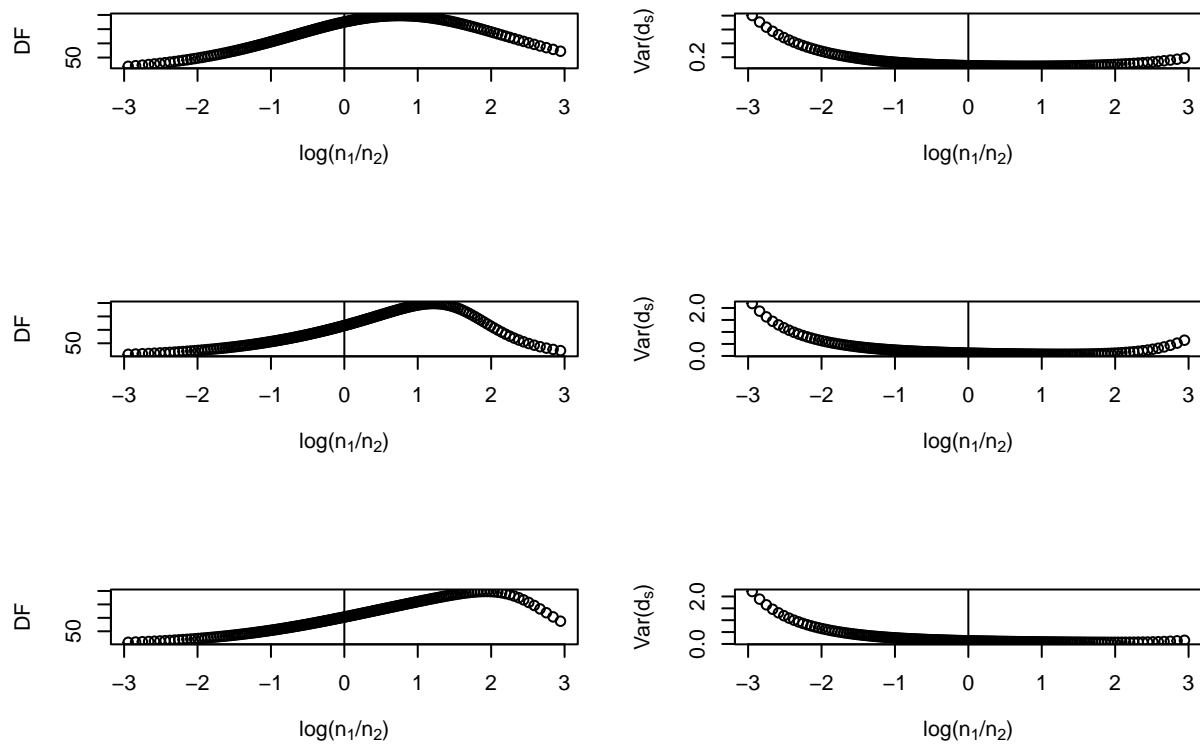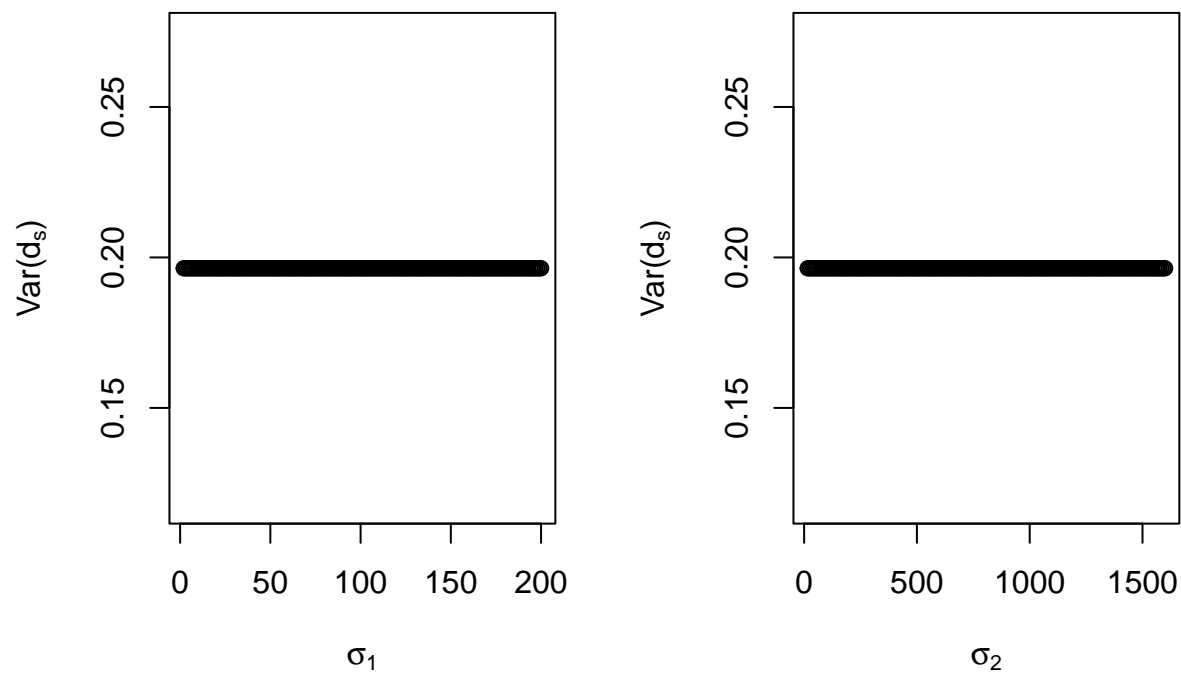
Figure 29. The variance of Shieh's d_s, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_1}{n_2}\right)\right)$, when SD-ratio equals 1.46 (first row), 3.39 (second row) or 7 (third row).

Figure 30. Variance of Shieh's d_s, when variances and sample sizes are unequal across groups, as a function of σ_1 (left) or σ_2 (right), for a constant SD-ratio.