1   What measure of effect size using when performing a Welch's t-test?

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Abstract

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16   **Intro**

17   During decades, researchers in social science (Henson & Smith, 2000) and education

18 (Fan, 2001) have overestimated the ability of the null hypothesis (H0) testing to determine

19 the importance of their results. The standard for researchers in social science is to define H0

20 as the absence of effect (Meehl, 1990). For example, when comparing the mean of two

21 groups, researchers commonly test the H0 that there is no mean difference between groups

22 (Steyn, 2000). Any effect that is significantly different from zero will be seen as sole support

23 for a theory.

24   Such an approach has faced many criticisms among which the most relevant to our

25 concern is that the null hypothesis testing highly depends on sample size: for a given alpha

26 level and a given difference between groups, the larger the sample size, the higher the

27 probability of rejecting the null hypothesis (Fan, 2001; Kirk, 2009; Olejnik & Algina, 2000;

28 Sullivan & Feinn, 2012). It implies that even tiny differences could be detected as

29 statistically significant with very large sample sizes (McBride, Loftis, & Adkins, 1993)[1].

30   Facing this argument, it has become an adviced practice to report the *p*-value assorted

31 by a measure of the effect size, that is, a quantitative measure of the magnitude of the

32 experimental effect (Cohen, 1965; Fan, 2001; Hays, 1963). This practice is also highly

33 endorsed by the American Psychological Association (APA) and the American Educational

34 Research Association (AERA) (American Educational Research Association, 2006; American

35 Psychological Association, 2010). However, only a limited number of studies have properly

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[1] Tiny differences might be due to sampling error, or to other factors than the one of interest: even under

the assumption of random assignent (which is a necessary but not sufficient condition), it is almost

impossible to be sure that the only difference between two conditions is the one defined by the factor of

interest. Other tiny factors of no theoretical interest might slightly influence results, making the probability

of getting an actual zero effect very low. This is what Meehl (1990) calls 'systematic noise'.

36   reported effect size in the last decades.

37   Generally, there is a high confusion between the effect size and other related concepts

38   such as the Clinical significance. Moreover, there are several situations that call for effect

39   size measures and, in the current litterature, it is not always easy to know which measure to

40   use in which context. We will therefore begin this paper with 3 sections in which we will:

41   1. Clearly define what is a measure of effect size;

42   2. List the different situations that call for effect sizes measures;

43   3. Define required properties of the effect size estimators depending on the specific

44   situation.

45   Moreover, it is highly recommended to compute a confidence interval around the point

46   effect size. In a fourth section, we will therefore summarize in how far it is an added value to

47   mention the confidence interval around the effect size.

48   After these general adjustments, we will focus our attention on "between-subject"

49   designs where individuals are randomly assigned into one of two independent groups and

50   group scores are compared based on their means[2]. Because it has been widely argued that

51   there are many fields in psychology where the assumption of equal variances between two

52   populations is ecologically unlikely (Delacre, Lakens, & Leys, 2017; Erceg-Hurn &

53   Mirosevich, 2008; Grissom, 2000), it is becoming more common in statistical software to

54   present a t-test that does not hold under this assumption by default, namely the Welch's

55   t-test (e.g., R, Minitab). However, similar issues for the measures of effect sizes have received

56   less attention (Shieh, 2013), and Cohen's d_s remains persistent[3]. One possible reason is that

57   researchers cannot find a consensus on which alternative should be used (Shieh, 2013). We

58   will limit our study to the standardized mean difference, called the d-family, because it is the

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[2] We made this choice because *t*-tests are still the most commonly used tests in the field of Psychology.

[3] For example, in Jamovi, Cohen's ds is provided, independently of whether one performs Student's or Welch's t-test.

59 dominant family of estimators of effect size when comparing two groups based on their

60 means (Peng, Chen, Chiang, & Chiang, 2013; Shieh, 2013), and we will see that even in this

61 very specific context, there is little agreement between researchers as to which is the most

62 suitable estimator. According to us, the main reason is that it is difficult, based on currently

63 existing measures, to optimally serve all the purposes of an effect size measure. Throughout

64 this section, we will:

65 1. Present the main measures of the d-family that are proposed in the

66 literature,related to the purpose they serve, and introduce a new one, namely the

67 "transformed Shieh's d" that should help at reaching all the purposes simultaneously;

68 2. Present and discuss the results of simulations we performed, in order to compare

69 existing measures and our newly introduced one;

70 3. Summarize our conclusions in practical recommendations. In this section, we will

71 provide useful tools (i.e., an R package) to compute relevant measures of effect sizes and

72 related information.

Measure of effect size: what it is, what it is not

74 The effect size is commonly referred to as the practical significance of a test. Grissom

75 & Kim (2005) define the effect size as the extent to which results differ from what is implied

76 by the null hypothesis. In the context of the comparison of two groups based on their means,

77 depending on the defined null hypothesis (considering the absence of effect as the null

78 hypothesis), we could define the effect size either as the magnitude of differences between

79 parameters of two populations groups are extracted from (e.g. the mean; Peng & Chen, 2014)

80 or as the magnitude of the relation between one dichotomous factor and one dependent

81 variable (American Educational Research Association, 2006). Both definitions refer to the

82 most famous families of measures of effect sizes (Rosenthal, 1994): the d-family and the

83 r-family.

84 Very often, the contribution of the measures of effect size is overestimated. First,

benchmarks about what should be a small, medium or large effect size might have contributed to viewing the effect size as a measure of the importance or the relevance of an effect in real life, but it is not (Stout & Ruble, 1995). The effect size is only a mathematical indicator of the magnitude of a difference, which depends on the way a variable is converted into numerical indicator. In order to assess the meaningfulness of an effect, we should be able to relate this effect with behaviors/meaningful consequences in the real world (Andersen, McCullagh, & Wilson, 2007). For example, let us imagine a sample of students in serious school failure who are randomly divided into two groups: an experimental group following a training program and a control group. At the end of the training, students in the experimental group have on average significantly higher scores on a test than students in the control group, and the difference is large (e.g. 30 percents). Does it automatically mean that students in the experimental condition will be able to pass to the next grade and to continue normal schooling? Whether the computed magnitude of difference is an important, meaningful change in everyday life refers to the interpretation of treatment outcomes and is neither a statistical nor mathematical concept, but is related to the underlying theory that posits an empirical hypothesis. This concept is sometimes called *Clinical significance* (Grissom & Kim, 2012; Thompson, 2002) or *Social significance* (Tyler, 1931) in the current literature. However, in our conception, we should use a more general term and we propose to rename this concept to *Applied significance*[4].

Second, in the context of the comparison of two groups based on their means, the effect size should not replace the null hypothesis testing. Statistical testing allows the researcher to determine whether the oberved departure from H0 occured by chance or not (Stout & Ruble, 1995), while effect size estimators allow to assess the practical signficance of an effect, and as reminds Fan (2001): *"a practically meaningful outcome may also have occured by chance,*

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[4] In our conception Applied significance encompasses all what refers to the relevance of an effect in real life, such as for instance clinical, personnal, social, professionnal relevance

109 *and consequently, is not trustworthy"* (p.278). For this reason, the use of confidence intervals

110 around the effect size estimate is highly recommended (Bothe & Richardson, 2011).

Different purposes of effect size measures

112 Effect size measures can be used in an *inferential* perspective:

113 - The effect sizes from previous studies can be used in a prior power analysis when

114 planning a new study (Lakens, 2013; Prentice & Miller, 1990; Stout & Ruble, 1995; Sullivan

115 & Feinn, 2012; Wilkinson & the Task Force on Statistical Inference, 1999);

116 - We can compute confidence limits around the point estimator (Shieh, 2013) in order

117 to replace conventional hypothesis testing : if the null hypothesis area is out of the

118 confidence interval, we can conclude that the null hypothesis is false.

119 Measures of effect size can also be used in a *comparative* perspective, that is, to assess

120 the stability of results across designs, analysis, samples sizes (Wilkinson & the Task Force on

121 Statistical Inference, 1999). This includes

122 - the comparison of results from 2 or more studies (Prentice & Miller, 1990);

123 - the incorporation of results in meta-analysis (Lakens, 2013; Li, 2016; Nakagawa &

124 Cuthill, 2007; Stout & Ruble, 1995; Wilkinson & the Task Force on Statistical Inference,

125 1999).

126 Finally, effect size measures can be used for *interpretative* purposes, namely to assess

127 the practical significance of a result (beyond statistical significance; Lakens, 2013; American

128 Psychological Association, 2010; Prentice & Miller, 1990).

Properties of a good effect size estimator

130 The empirical value of an estimator (called estimate) depends on the sampling, in

131 other words, different samples extracted from the same population will of course lead to

132 different estimates for a same estimator. The *sampling distribution* of the estimator is the

133 distribution of all estimates, based on all possible samples of size n extracted from one

₁₃₄ population. Studying the sampling distribution is very useful, as it allows us to assess the

₁₃₅ qualities of estimator. More specifically, three desirable properties a good estimator should

₁₃₆ possess for inferential purposes are: **unbiasedness**, **consistency** and **efficiency** (Wackerly,

₁₃₇ Mendenhall, & Scheaffer, 2008).

₁₃₈   An estimator is unbiased if the distribution of estimates is centered around the true

₁₃₉ population parameter. On the other hand, an estimator is positively (or negatively) biased if

₁₄₀ the distribution is centered around a value that is higher (or smaller) than the true

₁₄₁ population parameter (see Figure 1). In other words, the bias tells us if estimates are good,

₁₄₂ on average. The *bias* of a point estimator $\hat{\delta}$ can be computed as

$$\hat{\delta}_{bias} = E(\hat{\delta}) - \delta \tag{1}$$

₁₄₃   where $E(\hat{\delta})$ is the expectation of the sampling distribution of the estimator (i.e. the

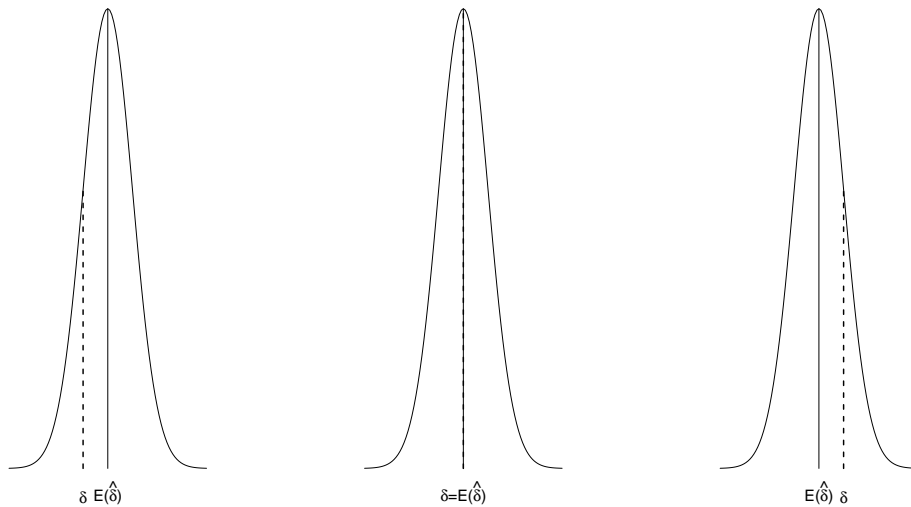₁₄₄ population average) and δ is the true (population) parameter.

Figure 1. Samplig distribution for a positively biased (left), an unbiased (center) and a
negatively biased estimator (right)

145   Moreover, since there is a strong relationship between the bias and the size of any

146 estimator (the larger an estimator, the larger the bias), it might be interesting to also define

147 the *relative bias* as the ratio between the bias and the population parameter:

$$\hat{\delta}_{relative\ bias} = \frac{E(\hat{\delta}) - \delta}{\delta} \tag{2}$$

148   While the bias informs us about the quality of estimates on average, in particular their

149 capacity of lying close to the true value, it says nothing about individual estimates. Imagine

150 a situation where the distribution of estimates is centered around the real parameter but

151 with such a large variance that some point estimates are very far from the center. This would

152 be problematic, since we then do not know if this estimate, based on the sample at hand, is

153 close to the truth or far off. Therefore it is not only essential for an estimator to be unbiased,

154 but the variability of its sampling distribution should also ideally be small. Put simply, we

155 hope that *all* possible estimates are close enough of the true population parameter, in order

156 to be sure that for *any* estimate, one has a correct estimation of the real parameter. Among

157 two unbiased estimators $\hat{\delta}_1$ and $\hat{\delta}_2$, we therefore say that $\hat{\delta}_1$ is **more efficient** than $\hat{\delta}_2$ if

$$Var(\hat{\delta}_1) \leq Var(\hat{\delta}_2) \tag{3}$$

158   Where $Var(\hat{\delta})$ is the variance of the sampling distribution of the estimator $\hat{\delta}$. Among

159 all unbiased estimators, the more efficient will be the one with the smallest variance [5].

160 Again, the variance of an estimator $\hat{\delta}$ is a function of its size (the larger the estimator, the

161 larger the variance) and therefore, we might be interested in reducing the effect size impact

162 in computing the *relative variance* as the ratio between the variance and the square of the

163 population estimator:

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[5] The famous Cramer-Rao inequality provides a theoretical lower bound for the variance of unbiased

estimators. An estimator reaching this bound is therefore most efficient.

$$\hat{\delta}_{relative\ variance} = \frac{Var(\hat{\delta})}{\delta^2} \tag{4}$$

Note that both unbiasedness and efficiency are very important. An unbiased estimator with such a large variance that somes estimates are extremely far from the real parameter is as undesirable as a parameter which is highly biased. In some situations, it is better to have a slightly biased estimator with a tight shape around the biased value (so that each estimate remains relatively close to the true parameter and one can apply bias correction techniques) rather than an unbiased estimator with a large variance (Raviv, 2014).

Finally, the last property of a good point estimator is **consistency**: consistency means that the bigger the sample size, the closer the estimate is to the population parameter. In other words, the estimates *converge* to the true population parameter.

Beyond the inferential properties, Cumming (2013) reminds that an effect size estimator needs to have a constant value across designs in order to be easily interpretable and to be included in meta-analysis. In other words, it should achieve the property of **generality**.

Confidence interval around a point estimator

We already mentioned that confidence interval around a point estimate could replace conventional hypothesis testing. A confidence interval contains all the information that a *p*-value of a test based on the same estimator does: if the area of the null hypothesis is out of the $(1 - \alpha)$-confidence interval, then the hypothesis test would also result in a *p*-value below the nominal alpha level. Hypothesis tests and confidence intervals based on the same statistical quantity (this is an essential requirement) are thus directly related. At the same time, the intervals provide extra information about the precision of the sample estimate for inferential purposes, and therefore on how confident we can be in the observed results (Altman, 2005; Ellis, 2015): the narrower the interval, the higher the precision. On the other

187 hand, the wider the confidence interval, the more the data lacks precision (for example,

188 because the sample size is too small).

Different measures of effect sizes

190 The d-family effect sizes are commonly used with "between-subject" designs where

191 individuals are randomly assigned into one of two independent groups and groups scores

192 means are compared. The population effect size is defined as

$$\delta = \frac{\mu_1 - \mu_2}{\sigma} \tag{5}$$

193 where both populations follow a normal distribution with mean μ_j in the jth

194 population (j=1,2) and common standard deviation σ. They exist different estimators of this

195 effect size measure. For all of them, the mean difference is estimated by the difference

196 $\bar{X}_1 - \bar{X}_2$ of both sample means. When the equality of variances assumption is assumed, σ is

197 estimated by pooling both samples standard deviations (S_1 and S_2). When the equality of

198 variances assumption cannot be assumed, alternatives to the common standard deviation are

199 available. Throughout this section, we will present some of these estimators, separately

200 depending on whether they rely on the assumption of equality of variances or not. For each

201 of them, we will provide information about their theoretical bias, variance and consistency.

202 **When variances are equal between groups**

203 When we have good reasons to assume equality of variances between groups, then the

204 most common estimator of δ is Cohen's d_s where the sample mean difference is divided by a

205 pooled error term (Cohen, 1965):

$$Cohen's\ d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1) \times S_1^2 + (n_2-1) \times S_2^2}{n_1 + n_2 - 2}}} \tag{6}$$

206   Where S_j is the standard deviation and n_j the sample size of the jth sample (j=1,2).

207   The reasoning behind this measure is to make use of the fact that both samples share the

208   same population variance (Keselman, Algina, Lix, Deering, & Wilcox, 2008), hence we

209   achieve a more accurate estimation of the population variance by pooling both estimates of

210   this parameter (i.e S_1 and S_2). Since the larger the sample size, the more accurate the

211   estimate, we give more weight to the estimate based on the larger sample size. Cohen's d_s is

212   directly related with Student's t-statistic:

$$cohen's\ d_s = t_{student} \times \sqrt{\frac{n_1 + n_2}{n_1 n_2}} \tag{7}$$

213   Under the assumption of normality and equal variances between groups, Student's

214   t-statistic follows a t-distribution with known degrees of freedom and noncentrality

215   parameter [6]:

$$df_{student} = n_1 + n_2 - 2 \tag{8}$$

$$ncp_{student} = \frac{\mu_1 - \mu_2}{\sigma_{pooled}} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}, \quad where\ \sigma_{pooled} = \sqrt{\frac{(n_1 - 1) \times \sigma_1^2 + (n_2 - 1) \times \sigma_2^2}{n_1 + n_2 - 2}} \tag{9}$$

216   The relationship described in equation 7 and the theoretical distribution of Student's

217   t-statistic allow us to theoretically determine the sampling distribution of Cohen's d_s, and

218   therefore, its theoretical expectancy and variance when the assumptions of normality and

219   equal variances are met. All these equations are provided in Table 1. For interested readers,

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[6] Under the null hypothesis of no differences between sample means, Student's t-statistic will follow a central
t-distribution with $n_1 + n_2 - 2$ degrees of freedom. However, when the null hypothesis is false, the
distribution of this quantity will not be centered, and noncentral t-distribution will arise.

220 bias and variance of Cohen's d_s have been thoroughly studied, based on Table 1, so as to

221 determine the way different parameters influence them, and results are detailed and available

222 on Github (see Appendix 1 in https://github.com/mdelacre/Effect-sizes/; it will be the

223 same for all estimators described later).

224   While Cohen's d_s is a consistent estimator, its bias and variance are substantial with

225 small sample sizes, even under the assumptions of normality and equal variances (Lakens,

226 2013). In order to compensate for Cohen's d_s bias with small sample sizes, Hedges & Olkin

227 (1985) has defined a bias-corrected version:

$$Hedges'\ g_s = Cohen's\ d_s \times \frac{\Gamma(\frac{df_{Student}}{2})}{\sqrt{\frac{df_{Student}}{2}} \times \Gamma(\frac{df_{Student}-1}{2})} \tag{10}$$

228   Where $df_{Student}$ has been defined in equation 8, and $\Gamma()$ is the gamma function. This

229 equation can be approximated as follows:

$$Hedges'\ g_s = Cohen's\ d_s \times \left(1 - \frac{3}{4N-9}\right) \tag{11}$$

230   Hedges' g_s is theoretically unbiased when the assumptions of normality and equal

231 variances are met (see Table 1), and it has a smaller variance than Cohen's d_s, especially

232 with small sample sizes [7]. As Cohen's d_s, its variance depends on the total sample size (N),

233 the sample sizes ratio $\left(\frac{n_1}{n_2}\right)$ and the population effect size (δ_{Cohen}). How these parameters

234 influence the variance of Hedges' g_s will be summarized in a later section in which we will

235 compare different estimators through Monte Carlo simulations (see the section "Monte Carlo

236 Simulations").

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[7] $.52 \leq \left[\frac{\Gamma(\frac{df}{2})}{\sqrt{\frac{df}{2}} \times \Gamma(\frac{df-1}{2})}\right]^2 < 1$ for $3 \leq df < \infty$. The larger the total sample size, the smaller the difference
between the variance of Cohen's d_s and Hedges' g_s.

₂₃₇   While the pooled error term is the best choice when variances are equal between

₂₃₈ groups (Grissom & Kim, 2001), it may not be well advised for use with data that violate this

₂₃₉ assumption (Cumming, 2013; Grissom & Kim, 2001, 2005; Kelley, 2005, 2005; Shieh, 2013).

₂₄₀ When variances are unequal between groups, the expression in equation 5 is no longer valid

₂₄₁ because both groups don't share a common population variance. If we pool the estimates of

₂₄₂ two unequal population variances, the estimator of effect size will be lower as it should be in

₂₄₃ case of positive pairing (i.e. the group with the larger sample size is extracted from the

₂₄₄ population with the larger variance) and larger as it should be in case of negative pairing

₂₄₅ (i.e. the group with the larger sample size is extracted from the population with the smaller

₂₄₆ variance). Because the assumption of equal variances across populations is very rare in

₂₄₇ practice (Cain, Zhang, & Yuan, 2017; Delacre et al., 2017; Delacre, Leys, Mora, & Lakens,

₂₄₈ 2019; Erceg-Hurn & Mirosevich, 2008; Glass, Peckham, & Sanders, 1972; Grissom, 2000;

₂₄₉ Micceri, 1989; Yuan, Bentler, & Chan, 2004), both Cohen's d_s and Hedges' g_s should be

₂₅₀ abandoned in favor of an alternative robust to unequal population variances.

Table 1

Expentency, bias and variance of Cohen's d_s and Hedges' g_s under the assumptions that independent residuals are normally distributed with equal variances across groups.

	df	Expectancy	Variance
Cohen's d_s	$N-2$	$\delta_{cohen} \times \dfrac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$	$\dfrac{N \times df}{n_1 n_2 \times (df-2)} + \delta_{Cohen}^2 \left[\dfrac{df}{df-2} - \left(\dfrac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right]$
		$\approx \dfrac{\delta_{Cohen}}{\left(1 - \frac{3}{4N-9}\right)}$	$\approx \dfrac{N \times df}{n_1 n_2 \times (df-2)} + \delta_{Cohen}^2 \left[\dfrac{df}{df-2} - \left(\dfrac{1}{1 - \frac{3}{4N-9}} \right)^2 \right]$
Hedges's g_s	$N-2$	δ_{Cohen}	$Var(Cohen's\ d_s) \times \left[\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)} \right]^2$
			$\approx Var(Cohen's\ d_s) \times \left[1 - \frac{3}{4N-9} \right]^2$

Note. Cohen's d_s is a biased estimator, because its expectation differ from the population effect size. On the other hand, Hedges' g_s is an unbiased estimator, because its expectation equals the population effect size; equations in Table 1 require $df \geq 3$ (i.e. $N \geq 5$).

When variances are unequal between populations

In his review, Shieh (2013) mentions three options available in the literature to deal with the case of unequal variances: the sample mean difference divided by (A) the Glass's d_s, (B) the Shieh's d_s and (C) the non pooled average of both variance estimates.

Glass's d_s. When comparing one control group with one experimental group, Glass, McGav, & Smith (2005) recommend using the standard deviation SD of the control group as standardizer. It is also advocated by Cumming (2013), because, according to him, it is what makes the most sense, conceptually speaking. This yields

$$Glass's\ d_s = \frac{\bar{X}_e - \bar{X}_c}{S_c} \tag{12}$$

Where $\bar{X}_e\ and\ \bar{X}_c$ are respectively the sample means of the experimental and control groups, and S_c is the sample SD of the control group. One argument in favour of using the SD of the control group as standardizer is the fact that it is not affected by the experimental treatment. When it is easy to identify which group is the "control" one, it is therefore convenient to compare the effect size estimation of different designs studying the same effect. However, defining this group is not always obvious (Coe, 2002). This could induce large ambiguity because depending of the chosen SD as standardizer, measures could be substantially different (Shieh, 2013).

The distribution of Glass's d_s is defined as following (Algina, Keselman, & Penfield, 2006):

$$Glass's\ d_s \sim \sqrt{\frac{1}{n_c} + \frac{S_e^2}{n_e \times S_c^2}} \times t_{df,ncp} \tag{13}$$

Where n_c and n_e are respectively the sample sizes of the control and experimental groups, and df and ncp are defined as follows:

$$df = n_c - 1 \tag{14}$$

$$ncp = \frac{\mu_c - \mu_e}{\sigma_c \times \sqrt{\frac{1}{n_c} + \frac{\sigma_e^2}{n_e \times \sigma_c^2}}} \tag{15}$$

274 Where μ_c and μ_e are respectively the mean of the populations control and experimental

275 groups are extracted from. Thanks to equation 13, we can compute its theoretical

276 expectancy variance when the assumptions of normality is met (See Table 2), and therefore

277 determine which factors influence bias and variance, and how they do so (see Appendix 1).

Table 2

Expentency, bias and variance of Glass's d_s and Cohen's d'_s and Shieh's d_s under the assumptions that independent residuals are normally distributed.

	df	Expectancy	Variance
Glass's d_s	$n_c - 1$	$\delta_{glass} \times c_f$	$\frac{df}{df-2} \times \left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e \sigma_c^2}\right) + \delta_{Glass}^2 \left(\frac{df}{df-2} - c_f^2\right)$
Cohen's d'_s	$\frac{(n_1-1)(n_2-1)(s_1^2+s_2^2)^2}{(n_2-1)s_1^4+(n_1-1)s_2^4}$	$\delta'_{Cohen} \times c_f$	$\frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2} + (\delta'_{Cohen})^2 \left(\frac{df}{df-2} - c_f^2\right)$
		$\approx \delta'_{Cohen} \times \frac{4df-1}{4(df-1)}$	$\approx \frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2} + (\delta'_{Cohen})^2 \left[\frac{df}{df-2} - \left(\frac{4df-1}{4(df-1)}\right)^2\right]$
Shieh's d_s	$\approx \frac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$	$\delta_{Shieh} \times c_f$	$\frac{df}{(df-2)N} + \delta_{Shieh}^2\left(\frac{df}{df-2} - c_f^2\right)$

Note. $c_f = \frac{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$; all estimators are biased estimators, because their expectations differ from the population effect size δ; equations require $df \geq 3$ and at least 2 subjects per group.

280   **Shieh's d_s.**   Kulinskaya & Staudte (2007) were the first to advice the use of a

281   standardizer that takes the sample sizes allocation ratios into account, in addition to the

282   variance of both samples. Shieh (2013), following Kulinskaya & Staudte (2007), proposed a

283   modification of the exact SD of the sample mean difference:

$$Shieh's\ d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_1^2/q_1 + S_2^2/q_2}}; \quad q_j = \frac{n_j}{N}(j = 1, 2) \tag{16}$$

284   where $N = n_1 + n_2$. Shieh's d_s is directly related with Welch's t-statistic:

$$t_{welch} = Shieh's\ d_s \times \sqrt{N} \tag{17}$$

285   Where $N = n_1 + n_2$. The exact distribution of Welch's t-statistic is more complicated

286   than the exact distribution of Student's t-statistic, but it follows a t-distribution with

287   degrees of freedom and noncentrality parameters that can be approximated as follows, under

288   the assumption of normality (Shieh, 2013; Welch, 1938):

$$df_{Welch} \approx \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1} + \frac{(\sigma_2^2/n_2)^2}{n_2-1}} \tag{18}$$

$$ncp_{Welch} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{n_1/N} + \frac{\sigma_2^2}{n_2/N}}} \times \sqrt{N} \tag{19}$$

289   The relationship described in equation 17 and the theoretical distribution of Welch's

290   t-statistic allow us to theoretically approximate the sampling distribution of Shieh's d_s.

291   Based on the sampling distribution of Shieh's d_s, we can estimate its theoretical expectancy

292   and variance under the assumption of normality (see Table 2), and therefore and therefore

293   determine which factors influence bias and variance, and how they do so (see Appendix 1).

294   It can be demonstrated that when variances and sample sizes are equal across groups,

295   the biases and variances of Shieh's d_s and Cohen's d_s are identical except for a constant, as

296   shown in equations 20 and 21:

$$Shieh's\ d_{s,bias} = 2 \times Cohen's\ d_{s,bias} \quad (considering\ \sigma_1 = \sigma_2\ and\ n_1 = n_2) \tag{20}$$

$$Shieh's\ d_{s,variance} = 4 \times Cohen's\ d_{s,variance} \quad (considering\ \sigma_1 = \sigma_2\ and\ n_1 = n_2) \tag{21}$$

297   Due to the relation described in equation 22 when sample sizes are equal between

298   groups (as explained in Appendix 2), such proportions mean that relative to their respective

299   true effect size, Cohen's d_s and Shieh's d_s are equally good. This is a good illustration of the

300   fact that biases and variances should always be studied relative to the population effect size

301   (and not in absolute terms), as we will do later.

$$Shieh's\ \delta_{n_1=n_2} = \frac{Cohen's\ \delta_{n_1=n_2}}{2} \tag{22}$$

302   Except for this very specific situation, according to the statistical properties of Welch's

303   statistic under heteroscedasticity, Shieh's d_s accounts for the sample sizes allocation ratio.

304   The lack of generality caused by taking this specificity of the design into account has led

305   Cumming (2013) to question its usefulness in terms of interpretability: when keeping

306   constant the mean difference $(\bar{X}_1 - \bar{X}_2)$ as well as SD_1 and SD_2, Shieh's d_s will vary as a

307   function of the sample sizes allocation ratio (the dependency of Shieh's d_s value on the

308   sample sizes allocation ratio is illustrated in the following shiny application:

309   https://mdelacre.shinyapps.io/ShiehvsCohen/).

310 **Cohen's d'_s.**   The sample mean difference, divided by the non pooled average of both

311 variance estimates was suggested by Welch (1938). This yields:

$$Cohen's\ d'_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(S_1^2 + S_2^2)}{2}}} \tag{23}$$

312 Where $\bar{X}_j$ is the mean and S_j is the standard deviation of the jth sample (j = 1,2). We

313 know the distribution of Cohen's d'_s (Huynh, 1989):

$$cohen's\ d'_s \sim \sqrt{\frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)(\sigma_1^2 + \sigma_2^2)}{2}} \times t_{df^*,ncp^*} \tag{24}$$

314 Where df^* and ncp^* are defined as follows:

$$df^* = \frac{(n_1 - 1)(n_2 - 1)(s_1^2 + s_2^2)^2}{(n_2 - 1)s_1^4 + (n_1 - 1)s_2^4} \tag{25}$$

$$ncp^* = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}}} \times \sqrt{\frac{n_1 n_2(\sigma_1^2 + \sigma_2^2)}{2(n_2\sigma_1^2 + n_1\sigma_2^2)}} \tag{26}$$

315 Thanks to equation 26, we can compute its theoretical expectancy variance when the

316 assumptions of normality is met (See Table 2), and therefore determine which factors

317 influence bias and variance, and how they do so (see Appendix 1). This estimator has been

318 widely criticized, because:

319   - it results in a variance term of an artificial population and is therefore very difficult

320 to interpret (Grissom & Kim, 2001);

321   - unless both sample sizes are equal, the variance term does not correspond to the

322 variance of the mean difference (Shieh, 2013).

³²³ However, we will show throughout the simulation section that this estimator show very
³²⁴ good inferential properties.

³²⁵ **Glass's g_s, Shieh's g_s and Hedges' g'_s.** As for Cohen's d_s, an Hedges' correction
³²⁶ can be applied in order to compensate for the bias of Glass's d_s, Shieh's d_s and Cohen's d'_s
³²⁷ with small sample sizes (see Table 2). This correction has the following general form:

$$g_s = d_s \times \frac{\Gamma(\frac{\nu}{2})}{\sqrt{\frac{\nu}{2}} \times \Gamma(\frac{\nu-1}{2})} \tag{27}$$

³²⁸ Where ν are provided in equation 15 for Glass's g_s, in equation 25 for Hedges' g'_s and
³²⁹ in equation 18 for Shieh's g_s. The three corrected estimators are theoretically unbiased when
³³⁰ the assumptions of normality is met. Their variance are a function of the same factors as
³³¹ their biased equivalent, however, due to the correction, they have a smaller variance,
³³² especially with small sample size, as shown in Table 3. In summary:

- ³³³ The variances of Hedges' g'_s and Shieh's g_s depend on the total sample size (N), their
  ³³⁴ respective population effect size (δ), and the interaction between the sample sizes ratio
  ³³⁵ and the SD-ratio $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$.
- ³³⁶ The variance of Glass's g_s also depends on N, δ and $\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}$. In addition, there is also
  ³³⁷ a main effect of the SD-ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$ on its variance.

³³⁸ How these parameters influence the variance of the estimators will be summarized and
³³⁹ illustrated in the section dedicated to the Monte Carlo simulations.

Table 3

Expentency, bias and variance of Glass's d_s and Cohen's d'_s and Shieh's d_s under the assumptions that independent residuals are normally distributed.

	df	Expectancy	Variance
Glass's g_s	$n_c - 1$	δ_{glass}	$Var(Glass's\ d_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}\right)^2$
Cohen's g'_s	$\dfrac{(n_1-1)(n_2-1)(s_1^2+s_2^2)^2}{(n_2-1)s_1^4+(n_1-1)s_2^4}$	δ'_{Cohen}	$Var(Cohen's\ d'_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}\right)^2$
Shieh's g_s	$\approx \dfrac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$	δ_{Shieh}	$Var(Shieh's\ d_s) \times \left(\dfrac{\Gamma\left(\frac{df}{2}\right)}{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}\right)^2$

Note. $c_f = \dfrac{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}$; all estimators are unbiased estimators, because their expectations equal the population effect size δ; equations require $df \geq 3$ and at least 2 subjects per group.

Monte Carlo Simulations: assessing the bias, efficiency and consistency of 5 estimators

Method. We performed Monte Carlo simulations using R (version 3.5.0) to assess the bias, efficiency and consistency of Cohen's g_s, Glass's g_s (using respectively the sample SD of the first or second group as a standardizer), Hedges' g'_s and Shieh's g_s.

A set of 100,000 datasets were generated for 1,008 scenarios as a function of different criterions that will be explained below. In 252 scenarios, samples were extracted from a normally distributed population (in order to insure the reliability of our calculation method) and in 756 scenarios, samples were extracted from non normal population distributions. In order to assess the quality of estimators under realistic deviations from the normality assumption, we referred to the review of Cain et al. (2017). Cain et al. (2017) investigated 1,567 univariate distributions from 194 studies published by authors in Psychological Science (from January 2013 to June 2014) and the American Education Research Journal (from January 2010 to June 2014). For each distribution, they computed the Fisher's skewness (G1) and kurtosis (G2):

$$G_1 = \frac{\sqrt{n(n-1)}}{n-2} \frac{m_3}{\sqrt{(m_2)^3}} \tag{28}$$

with s = standard deviation, n = sample size, m_2 = second centered moment and m_3 = third centered moment.

$$G_2 = \frac{n-1}{(n-2)(n-3)} \times [(n+1)(\frac{m_4}{(m_2)^2} - 3) + 6] \tag{29}$$

with s = standard deviation, n = sample size and m_3=third centered moment. They found values of kurtosis from G2 = -2.20 to 1,093.48. According to their suggestions, throughout our simulations, we kept constant the population kurtosis value at the 99th percentile of their distribution of kurtosis, i.e. G2=95.75. Regarding skewness, we simulated

363 population parameter values which correspond to the 1st and 99th percentile of their

364 distribution of skewness, i.e. respectively G1 = -2.08 and G1 = 6.32. We also simulated

365 samples extracted from population where G1 = 0, in order to assess the main effect of high

366 kurtosis on the quality of estimators. All possible combinations of population skewness and

367 kurtosis and the number of scenarios for each combination are summarized in Table 4.

Table 4

Number of Combinations of skewness and kurtosis in our simulations.

		Kurtosis		
		0	95.75	**TOTAL**
	0	252	252	**504**
Skewness	-2.08	/	252	**252**
	6.32	/	252	**252**
	TOTAL	**252**	**756**	**1008**

368   *Note.* Fisher's skewness (G1) and kurtosis (G2) are presented in Table 4. The 252

369 combinations where both G1 and G2 equal 0 correspond to the normal case.

370   For the 4 resulting combinations of skewness and kurtosis (see Table 4), all other

371 parameter values were chosen in order to illustrate the consequences of factors identified as

372 playing a key role on the variance of unbiased estimators. We manipulated the population

373 mean difference $(\mu_1 - \mu_2)$, the sample sizes (n), the sample size ratio (n-ratio $= \frac{n_1}{n_2}$), the

374 population *SD*-ratio (i.e. $\frac{\sigma_1}{\sigma_2}$), and the sample size and population variance pairing $\left(\frac{n_1}{n_2} \times \frac{\sigma_1}{\sigma_2}\right)$.

375 In our scenarios, μ_2 was always 0 and μ_1 varied from 1 to 4, in step of 1 (so does $\mu_1 - \mu_2$)[8].

376 Moreover, σ_1 always equals 1, and σ_2 equals .1, .25, .5, 1, 2, 4 or 10 (so does $\frac{\sigma_1}{\sigma_2}$). The

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[8] In the original plan, we had added 252 simulations in which μ_1 and μ_2 were both null. We decided to not

present the results of these simulations, because the relative bias and the relative variance appeared to us to

be very useful to fully understand the estimators comparison, and computing them is impossible when the

real mean difference is zero.

$_{377}$ simulations for which both σ_1 and σ_2 equal 1 are the particular case of homoscedasticity

$_{378}$ (i.e. equal population variances across groups). Sample size of both groups (n_1 and n_2) were

$_{379}$ 20, 50 or 100. When sample sizes of both groups are equal, the n-ratio equals 1 (it is known

$_{380}$ as a balanced design). All possible combinations of n-ratio and population SD-ratio were

$_{381}$ performed in order to distinguish scenarios where both sample sizes and population variances

$_{382}$ are unequal across groups (with positive pairing when the group with the largest sample size

$_{383}$ is extracted from the population with the largest SD, and negative pairing when the group

$_{384}$ with the smallest sample size is extracted from the population with the smallest SD) and

$_{385}$ scenarios with no pairing between sample sizes and variances (sample sizes and/or

$_{386}$ population SD are equal across all groups). In sum, the simulations grouped over different

$_{387}$ sample sizes yield 4 conditions based on the n-ratio, population SD-ratio, and sample size

$_{388}$ and population variance pairing, as summarized in Table 5.

Table 5

4 conditions based on the n-ratio, SD-ratio, and sample size and variance pairing.

		n-ratio		
		1	**>1**	**<1**
	1	a	b	b
SD-ratio	**>1**	c	d	d
	<1	c	d	d

$_{389}$ *Note.* The n-ratio is the sample size of the first group (n_1) divided by the sample size

$_{390}$ of the second group (n_2). When all sample sizes are equal across groups, the n-ratio equals 1.

$_{391}$ When $n_1 > n_2$, n-ratio > 1, and when $n_1 < n_2$, n-ratio < 1. SD-ratio is the population SD

392   of the first group (σ_1) divided by the population SD of the second group (σ_2). When

393   $\sigma_1 = \sigma_2$, SD-ratio $= 1$. When $\sigma_1 > \sigma_2$, SD-ratio > 1. Finally, when $\sigma_1 < \sigma_2$, SD-ratio < 1.

394   ***Results.***   Before detailing estimators comparison for each condition, it might be

395   interesting to make some general comments.

396   1) We previously introduced the fact that raw bias and variances are sometimes

397   misleading. They can give the illusion of huge differences between two estimators, even

398   if these differences only reflect a change of unit (i.e. different population effect sizes).

399   To better understand this, imagine a sample of 15 people for whom we know the height

400   (in meters) and we compute a sample variance of 0.06838. If we convert sizes to

401   centimeters and compute the sample variance again, we find a measure of 683.8

402   (i.e. 10^3 larger). Both measures represent the same amount of variability, but they are

403   expressed in different units. Similar things occur when comparing the estimates of

404   different population measures. To avoid this possible confusion, we will only present

405   the relative bias and relative variance in all Figures (and anytime we will mention the

406   biases and variances in the results section, we will be referring to relative bias and

407   variance). For interested reader, illustrations of the raw bias and variance are available

408   on Github.

409   2) In a purpose of readability, we used different scales for the ordinate axis of our plots,

410   for both relative bias and relative variance, when $G_1 = 6.32$ and $G_2 = 95.75$, in

411   comparisons with all other conditions. Indeed, relative biases and variances are much

412   larger for this specific combination G_1/G_2. In the same way of thinking, we use

413   different scales for the ordinate axis of our plots, for the relative variance, when

414   sub-conditions are divided as a function of only the sample sizes ratio (Figures 2, 3 and

415   4) or the SD-ratio (Figure 5) and as a function of both sample sizes ratio and

416   SD-ratio (Figures 6 to 9).

417   3) Throughout this section, we will **compare** the quality of different estimators. We

⁴¹⁸   chose very extreme (although realistic) conditions, and we know that none of the

⁴¹⁹   parametric measures of effect size will be robust against such extreme conditions. Our

⁴²⁰   goal is therefore to study the robustness of the estimators against normality violations

⁴²¹   only in comparison with the robustness of other indicators, but not in absolute terms.

⁴²²   After these general remarks, we will analyze each condition separately. In all Figures

⁴²³ presented below, for different sub-conditions, the averaged relative bias and relative variance

⁴²⁴ of five estimators are presented. When describing the Glass's g_s estimators, we will

⁴²⁵ systematically call "control group" the group the standardizer is computed from (i.e. the first

⁴²⁶ group when using SD_1 as standardizer, the second group when using SD_2 as standardizer).

⁴²⁷ The other group will be called "experimental group".

⁴²⁸   *When variances are equal across groups.*   Figures 2 and 3 represent configurations

⁴²⁹ where the equality of variances assumption is met. According to our expectations, one

⁴³⁰ observes that the bias of all estimators is approximately zero as long as the normality

⁴³¹ assumption is met (first column in both Figures)[9]. However, the further from the normality

⁴³² assumption (i.e. when moving from left to right in Figures), the larger the bias.

⁴³³   Figure 2 illustrates scenarios where both population variances and sample sizes are

⁴³⁴ equal across groups (condition a). One can first notice that all estimators are consistent, as

⁴³⁵ their bias and variance decrease when the total sample size increase. For any departure from

⁴³⁶ the normality assumption, both bias and variance of Hedges' g_s, Shieh's d_s and Hedges' g'_s

⁴³⁷ are similar[10] and smaller than the bias and variance of glass's g_s estimates using either S_1 or

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[9] When looking at the relative bias for all estimators, the maximum departure from zero is 0.0064 when
sample sizes are equal across groups, and 0.0065 with unequal sample sizes.

[10] While the bias and variance of Cohen's d_s, Cohen's d'_s and Shieh's d_s are identical, the bias and variance
of Hedges' g_s is marginally different than the bias and variance of Hedges' g'_s and Shieh's g_s (these last two
having identical bias and variance). Indeed, because of the sampling error, differences remain between sample
variances, even when population variances are equal groups. Because the Hedges' correction applied to
Cohen's d_s does not imply the sample variances (unlike the one applied on both other estimators), the bias

S_2 as standardizer. Moreover, when samples are extracted from skewed distributions, Glass's

g_s will show different bias and variance as a function of the chosen standardizer (S_1 or S_2),

even if both S_1 and S_2 are estimates of the same population variance, based on the same

sample size. This is due to non-null correlations of opposite sign between the mean difference

$(\bar{X}_1 - \bar{X}_2)$ and respectively S_1 and S_2. For interested reader, when a non nul correlation

occurs between the sample means difference $(\bar{X}_1 - \bar{X}_2)$ and the standardizer of compared

estimators as well as the way this correlation impacts the bias and variance of estimators is

detailed in Appendix 3.

Figure 3 illustrates scenarios where population variances are equal across groups and

sample sizes are unequal (condition b). For any departures from the normality assumptions,

Hedges' g_s shows the smallest bias and variance. Hedges' g_s and Hedges' g'_s are consistent

estimators (i.e. the larger the sample sizes, the lower the bias and the variance), unlike

Shieh's g_s and Glass's g_s. The bias of Glass's g_s does not depend either on the size of the

experimental group or on the total sample size. The only way to decrease the bias of Glass's

g_s is therefore to add subjects in the control group. On the other hand, the variance of

Glass's g_s depends on both sample sizes, but not in an equivalent way: in order to reduce the

variance, it is much more efficient to add subjects in the control group and when the size of

the experimental group decreases so does the variance, even when the total sample size is

increased. Regarding Shieh's g_s, for a given sample size ratio,the bias and variance will

decrease when sample sizes increase. However, there is a large effect of the sample sizes ratio

in order that when the sample sizes ratio moves away from 1 by adding subjects, bias and

variance might increase.[11] On the other side, when the sample sizes ratio moves closer to 1

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and variance of Hedges' g_s is slighly different than the bias and variance of Hedges' g'_s and Shieh's g_s

[11] In Appendix 1, we mentioned that when the population effect size is nul, the larger the total sample size,

the lower the variance, whatever the sample sizes ratio is constant or not. We also mentioned that this is no

longer true when the population effect size is not zero and in our simulations, the effect size is never zero.

The effect size effect is partially visible in Figure 3 because we do not entirely remove the effect size effect

460 by adding subjects, the bias will decrease.

461   When samples are extracted from skewed distributions and have unequal sizes

462 (i.e. $n_1 \neq n_2$, the two last columns in Figure 3), for a constant total sample size, *Glass's g_s*,

463 Shieh's g_s and Hedge's g'_s will show different bias and variance depending on which group is

464 the largest one (e.g. when distributions are right-skewed, the bias and variance of all these

465 estimators when n_1 and n_2 are respectively 50 and 20 are not the same as their bias and

466 variance when n_1 and n_2 are respectively 20 and 50). This is due to a non-null correlations

467 of opposite sign between the mean difference $(\bar{X}_1 - \bar{X}_2)$ and their respective standardizers

468 depending on which group is the largest one, as detailed in Appendix 3. One observes that

469 under these configurations, the bias and variance of Glass's g_s are sometimes a bit smaller

470 and sometimes much larger than the bias and variance of Shieh's g_s and Cohen's d'_s. [12]

471   In conclusion, Glass's g_s should always be avoided when the equality of variance

472 assumption is met. Hedge's g_s, Hedges' g'_s and Shieh's g_s are equally performant as long as

473 the sample size ratio is close to 1. However, when designs are highly unbalanced, Shieh's g_s

474 is not consistent anymore. While Hedge's g'_s is consistent, Hedges's g_s remains a better

475 estimator.

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when we divide the variance by δ^2. This is due to the fact that one term, in the equation of the variance

computation, does not depend on the effect size.

[12] We learn from appendix 3 that when the $\mu_1 - \mu_2 > 0$ (like in our simulations), all other parameters being

equal, an estimator is always less biased and variable when choosing a standardizer that is positively

correlated with $\bar{X}_1 - \bar{X}_2$. We also learn from appendix 3 that the smaller n_c, the larger the magnitude of

correlation between s_c and $\bar{X}_1 - \bar{X}_2$. When $cor(S_c, \bar{X}_1 - \bar{X}_2)$ is positive, the positive effect of increasing the

magnitude of the correlation is counterbalanced by the negative effect of reducing n_c. On the other hand,

when $cor(S_c, \bar{X}_1 - \bar{X}_2)$ is negative, the negative effect of increasing the magnitude of the correlation is

amplified by the negative effect of decreasing n_c. This explain why the difference between Glass's g_s and

other estimators is larger when Glass's g_s is the least efficient estimator.

476   *When variances are unequal across groups.*   Figures 4 to 9 represent configurations

477 where the equality of variances assumption is not met. According to our expectations, one

478 observes that the bias of all estimators is approximately zero as long as the normality

479 assumption is met (first column in the three Figures), and the further from the normality

480 assumption (i.e. when moving from left to right in Figures), the larger the bias[13]. You might

481 find it surprising that the bias of Hedges' g_s remain very small troughout these conditions.

482 As reminded in the section "Different measures of effect size", Hedges' g_s should be avoided

483 when population variances and sample sizes are unequal across groups, because of the pooled

484 error term. When pooling the estimates of two unequal population variances, the resulting

485 estimator will be lower (in case of positive pairing) or larger (in case of negative pairing) as it

486 should be. At the same time, when pooling two unequal population variances, the population

487 effect size will also be lower (in case of positive pairing) or larger (in case of negative pairing)

488 as it should be. As a consequence, the distorsion cannot be seen through the difference

489 between the expected estimator and the population effect size measure. For this reason, the

490 bias and variance of Hedges' g_s will not be taken into account in the following comparisons.

491   Figures 4 and 5 are dedicated to scenarios where population variances are unequal

492 between groups and sample sizes are equal (condition c). In Figure 4, scenarios are

493 subdivided as a function of the sample sizes and one can notice that all estimators are

494 consistent, as their bias and variance decrease when the total sample size increases. In

495 Figure 5, scenarios are subdivided as a function of the SD-ratio. Because the comparison

496 pattern remains very similar for all sample sizes, we present only scenarios when sample sizes

497 equal 100.[14] One observes that the biases of Shieh's g_s and Hedges' g_s' are identical, for any

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[13] When looking at the relative bias for all estimators, the maximum departure from zero is 0.0173 when

sample sizes are equal across groups, and 0.0274 when both sample sizes and variances differ across groups.

[14] Remind that when dividing the variance by δ^2, we do not entirely remove the effect of the effect size on

variance. However, when the sample sizes increase, the effect size effect is reduced, because the term that

does not depend on δ^2, in the equation of the variance computation, get closer to 0. This explains why we

departures from the normality assumption. When distributions are symmetric (second column in Figure 5), the bias of both Shieh's g_s and Hedges' g'_s depends on the SD-ratio in order that the larger the difference between σ_1 and σ_2, the larger the bias. On the other side, the bias of Glass's g_s does not depend on the SD-ratio. It is always a bit larger than the bias of Shieh's g_s and Hedges' g'_s, but the difference decrease when SD-ratio get larger $\left(i.e. \ \frac{\sigma_1}{\sigma_2} = 10 \ or \ 0.1\right)$. IDEM VARIANCE. When samples are extracted from skewed distributions, the bias and variance of Glass's g_s is sometimes smaller and sometimes larger than the bias of Shieh's g_s and Hedge's g'_s. This is mainly due to the fact that when two samples of same sizes are extracted from two skewed distributions with unequal variances (i.e. $\sigma_1 \neq \sigma_2$, the two last columns in Figure 5), there will be non-null correlations of opposite sign between the mean difference $(\bar{X}_1 - \bar{X}_2)$ and the standardizer of *all* estimators, depending on which population variance is the largest one [15].

VARIANCE: Glass's more variable than Shieh's g_s and Hedge's g'_s. Moreover, the relative variance of Glass's d_s is larger when computing the standardizer based on the sample extracted from the less variable population. MAIS IL FAUT VRAIMENT LE VOIR...

Figures 6 to 9 are dedicated to scenarios where both samples sizes and population variances differ across groups. Due to a high number of combinations between the sample sizes-ratio and the variances-ratio in our simulations, we decided to present only some conditions. Because equations in Table 3 revealed an interaction effect between the sample sizes ratio and the SD-ratio on biases and variances of Hedge's g'_s and Shieh's g_s (see Appendix 1), we chose to present all configurations where the total sample sizes is either 150

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choose to select only the scenario with the larger sample sizes.

[15] When population variances are unequal, a non-null correlation occurs between standardizers estimates and $\bar{X}_1 - \bar{X}_2$. The sign of the correlation between the standardizer and the means difference will be the same as the sign of the correlation between the mean difference and the estimate of the larger population variance. For interested readers, this is detailed in Appendix 3.

519 (Figures 6 and 7), or 120 (Figures 8 and 9), in order to compare the effect of the SD-ratio on

520 biases and variances when the sample sizes ratio is .2, .5, 2 or 5.

521   When distributions are symmetric, we can observe that for a constant total sample size

522 (N), the bias of Glass's g_s only depends on the size of the control group and is therefore not

523 impacted by the SD-ratio. On the other side, the biases of Hedge's g'_s and Shieh's g_s are

524 always smaller when there is a positive pairing between sample sizes and variances. Moreover,

525 they are also impacted by an interaction between the SD-ratio and the sample sizes ratio:

526   - Regarding Hedge's g'_s, the closer the SD-ratio is from 1, the smaller the bias;

527   - Regarding Shieh's g'_s, the further the SD-ratio is from 1, the further from 1 will also

528 be the sample sizes ratio associated with the smallest bias [16].

529   When distributions are skewed... VOIR NOTES MANUSCRITES

530   In summary, Hedges's g_s remains the best measure when the asssumption of equal

531 variances is met. When variances are unequal across populations, Glass's g_s is sometimes

532 better but also sometimes much worst than respectively Shieh's g_s and Cohen's g'_s. The

533 performance of Glass's g_s highly depends on parameters that we cannot control (i.e. an triple

534 interaction effect between the sense of asymmetry, which group is the largest one and which

535 group has the larger SD) and for this reason, we do not recommend using it. When designs

536 are not "too unbalanced", Shieh's g_s and Cohen's g'_s are both appropriate but the further the

537 sample sizes ratio is from 1, the larger the bias of Shieh's g_s in order that in the end, our

538 favourite measure is Cohen's g'_s.

539 ***Conclusion.*** TO DO

540   Algina, J., Keselman, H. J., & Penfield, R. D. (2006). Confidence intervals for an effect

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[16] When the sample sizes ratio equals 2 (or .5), the smallest bias occurs when the SD-ratio equals 2 (or .5;

see Figures 6 and 7). On the other side, when the sample sizes ratio equals 5 (or .2), the smallest bias occurs

when the SD-ratio equals 4 (or .25; see Figures 8 and 9).

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796   Yuan, K.-H., Bentler, P. M., & Chan, W. (2004). Structural equation modeling with

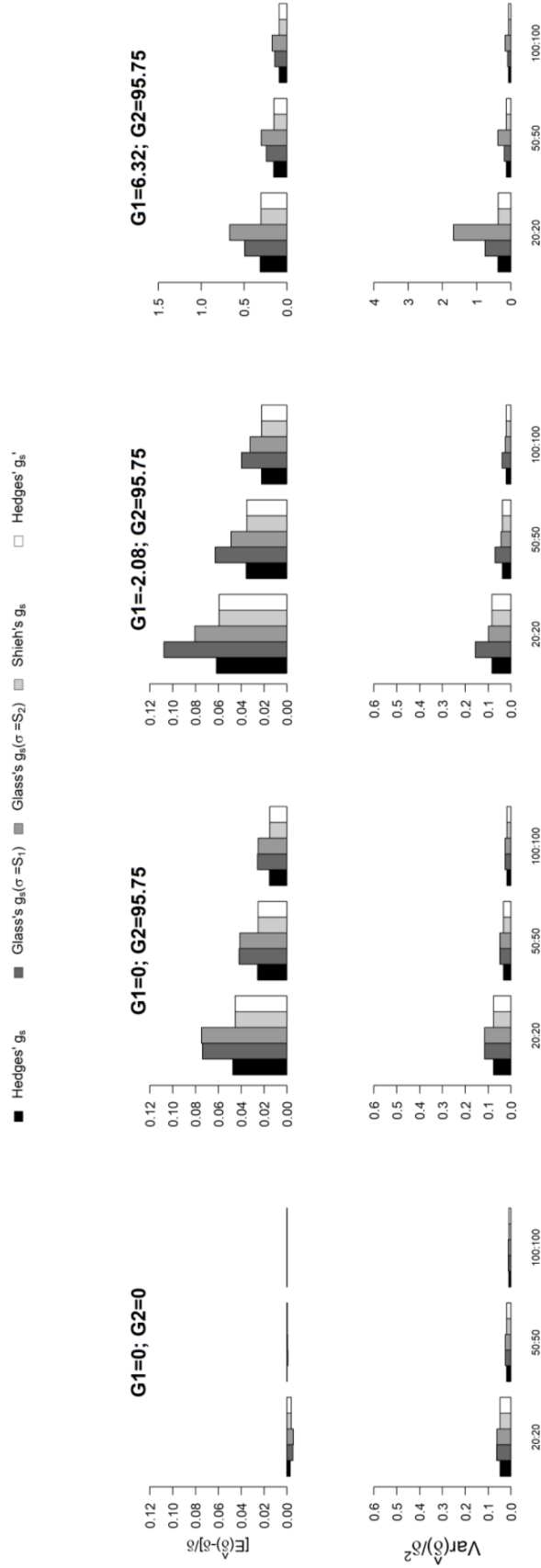797   heavy tailed distributions. *Psychometrika*, *69*(3), 421–436. doi:10.1007/bf02295644

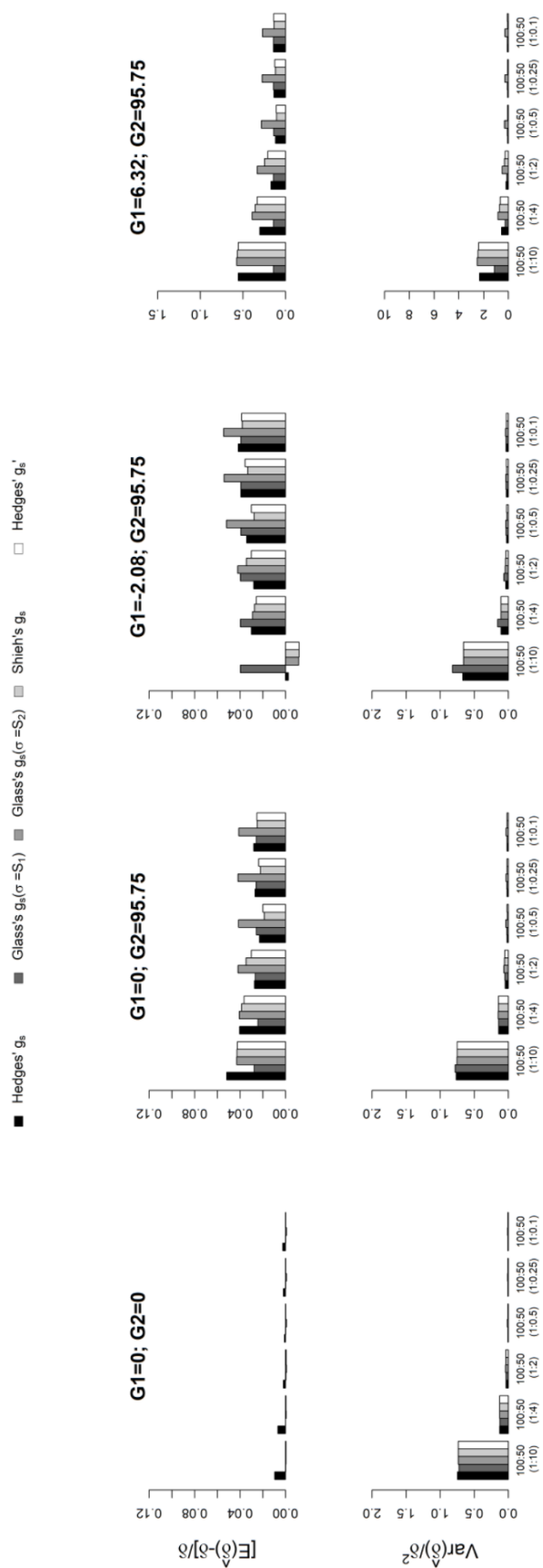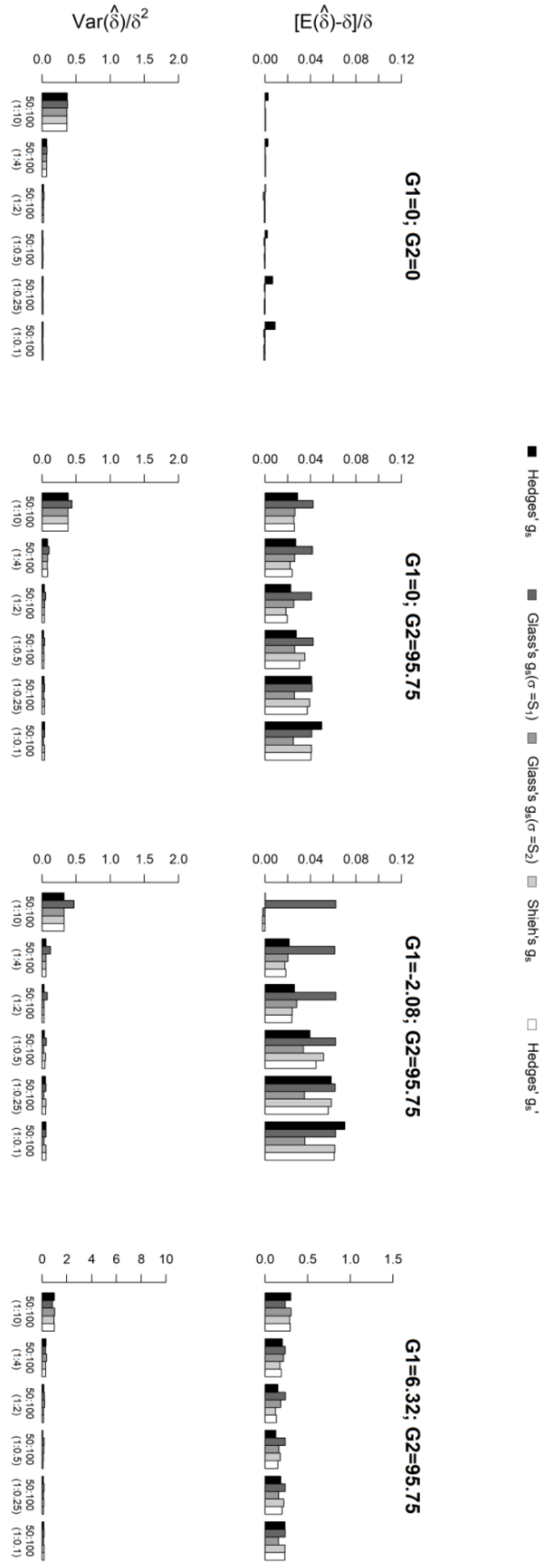*Figure 2.* Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are equal across groups (condition a)
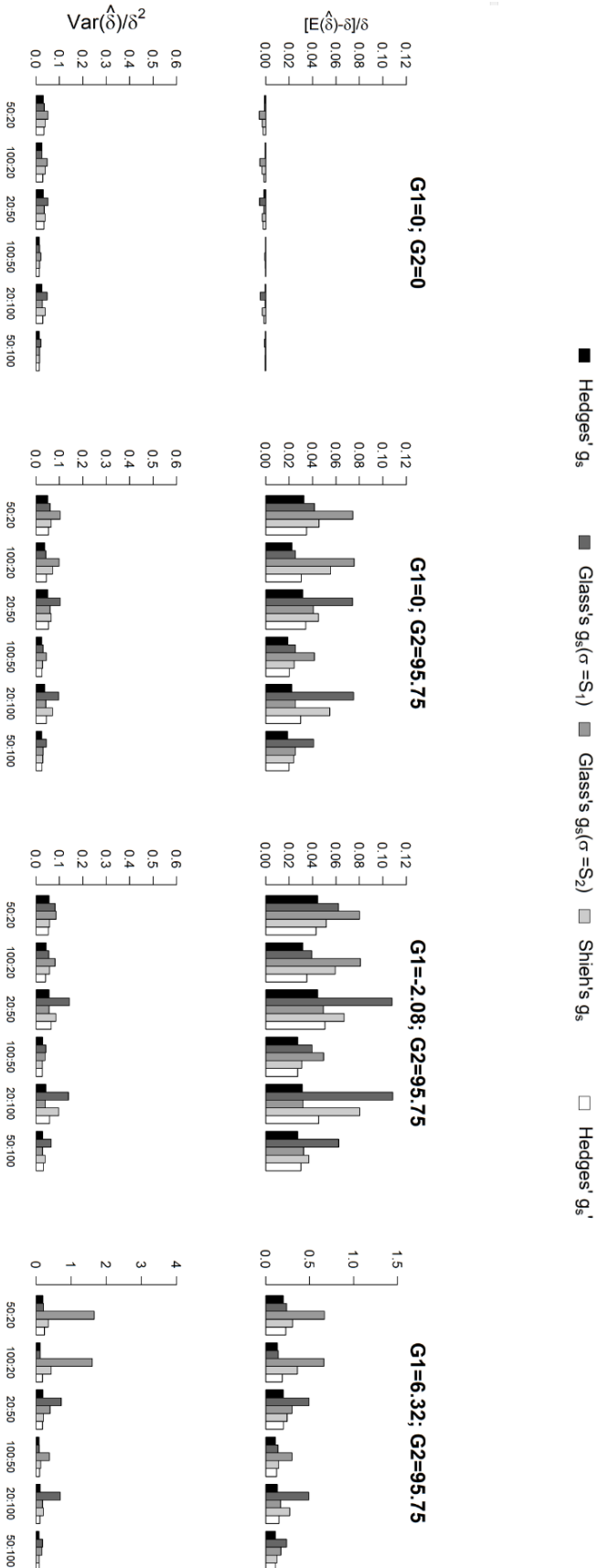
Figure 3. Bias and efficiency of estimators of standardized mean difference, when variances are equal across groups and sample sizes are unequal (condition b)
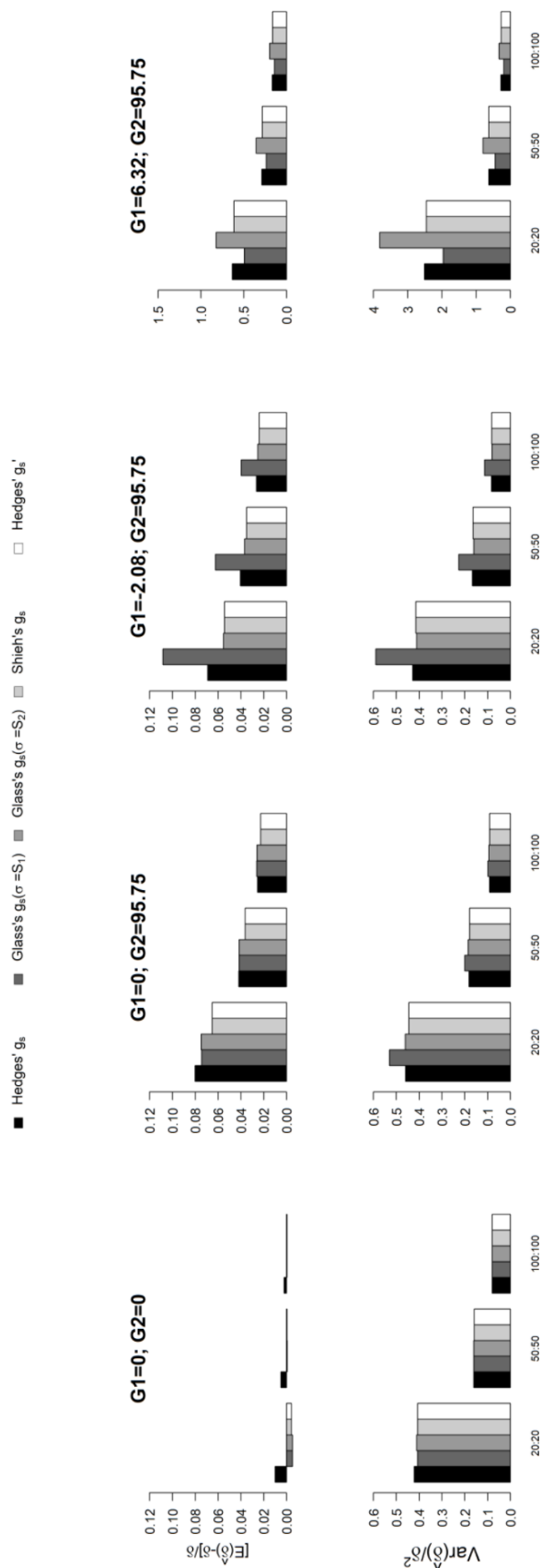
Figure 4. Bias and efficiency of estimators of standardized mean difference, when variances are unequal across groups and sample sizes are equal (condition c), as a function of n-ratio
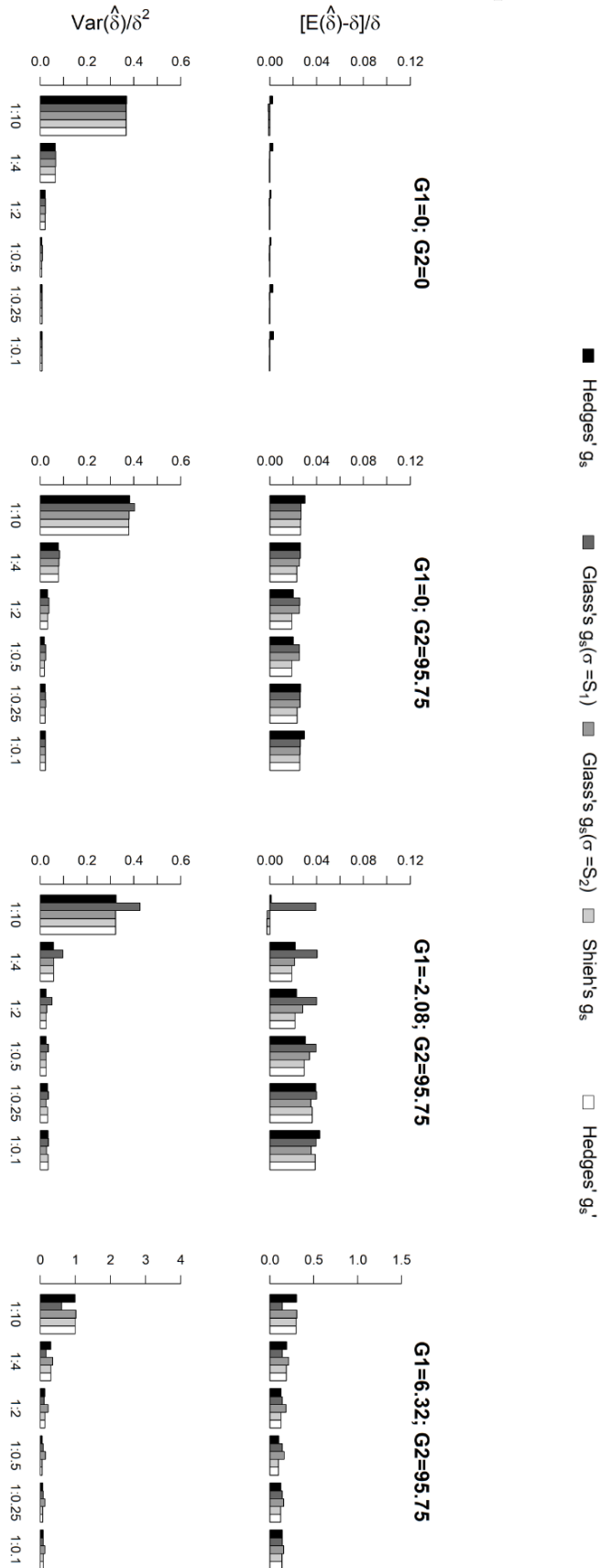
Figure 5. Bias and efficiency of estimators of standardized mean difference, when variances are unequal across groups and sample sizes are equal (condition c) as a function of the SD-ratio (when $n_1 = n_2 = 100$)

Figure 6. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 150, and $n_1 > n_2$

Figure 7. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 150, and $n_1 < n_2$
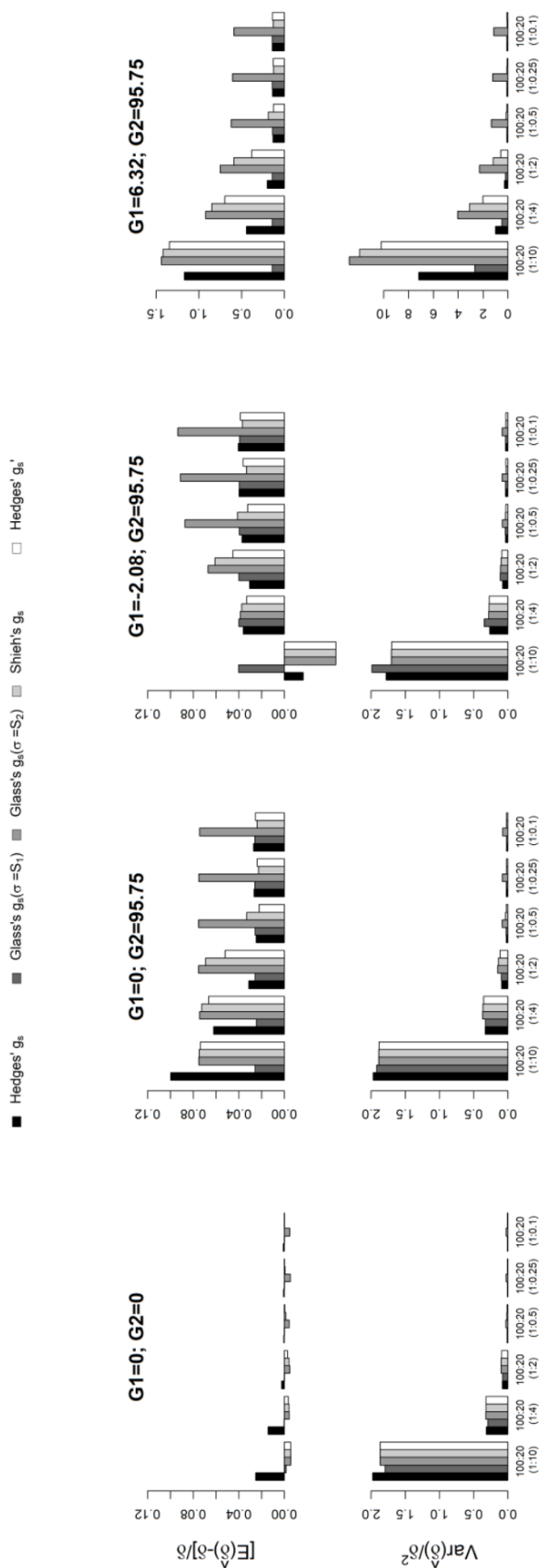
Figure 8. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 120, and $n_1 > n_2$
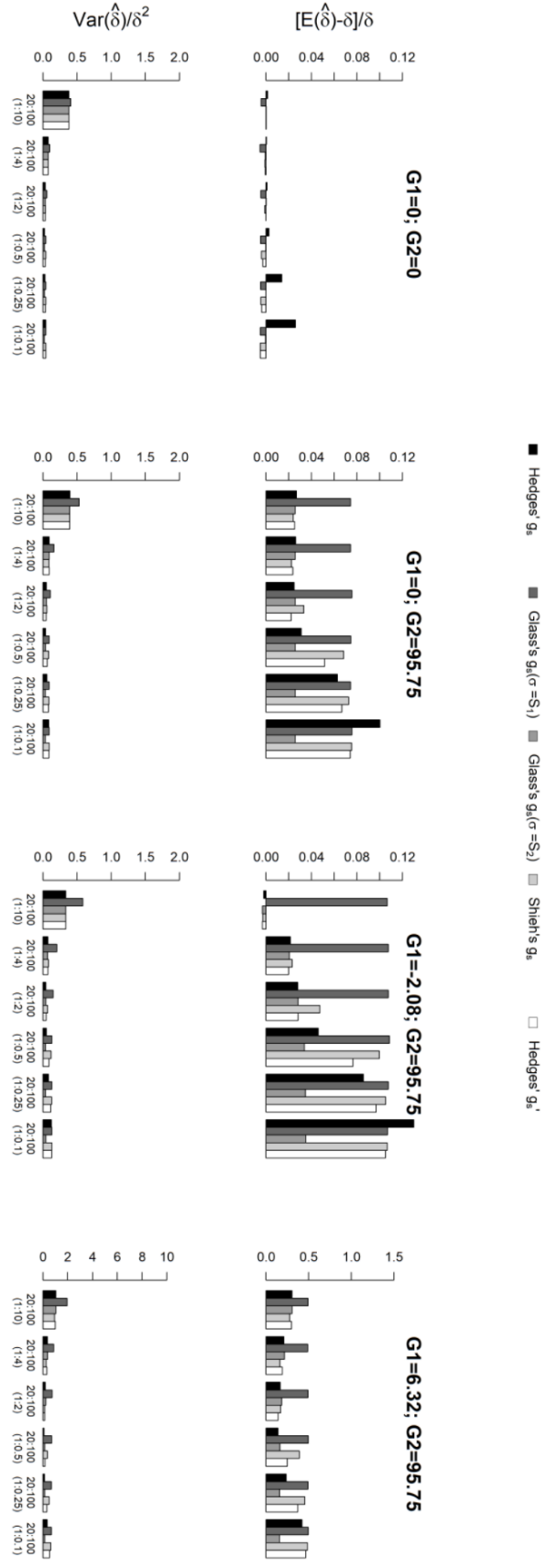
Figure 9. Bias and efficiency of estimators of standardized mean difference, when variances and sample sizes are unequal across groups (condition d), total sample (N) equals 120, and $n_1 < n_2$

Appendix

Appendix 1: The mathematical study of Shieh's δ

Paste Appendix 1 when it will be finished

Appendix 2: Confidence intervals

Paste Appendix 2 when it will be finished

Appendix 3: a priori power analyses

Paste Appendix 3 when it will be finished (Cumming & Finch, 2001)

Cumming, G., & Finch, S. (2001). A primer on the understanding, use, and calculation of confidence intervales that are based on central and noncentral distributions. *Educational and Psychological Measurement*, *61*(532), 532–574.