1   Theoretical variance, as a function of population parameters

2   Theoretical variance, as a function of population parameters

3   **The variance**

4   Note: while we focus on the theoretical variance of biased estimators (Cohen's d,

5   Glass's d, Shieh's d and Cohen's d^*) when the normality assumption is met, it is interesting

6   to notice that our main conclusions seem to generalize to biased estimators when samples

7   are extracted from symmetric distributions. Moreover, unbiased estimators depend on the

8   same factors as biased estimators, so our conclusions remain similar for unbiased

9   estimators when samples are extracted from heavy-tailed symmetric distributions.

10   **Cohen's d**

11   **When variances are equal across populations.**

12   ***When $\delta_{Cohen} = 0$.***

13   When the population effect size is zero, the variance of Cohen's d can be simplified as

14   follows:

$$Var_{Cohen's\ d} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

15   The **variance** of Cohen's d is a function of total sample size (N) and the sample sizes

16   allocation ratio $\left(\frac{n_2}{n_1}\right)$:

17   • The larger the total sample size, the lower the variance. The variance tends to zero

18   when the total sample size tends to infinity (see Figure 1);

19   • The further the sample sizes allocation ratio is from 1, the larger the variance (see

20   Figure 2).

21 ***When $\delta_{Cohen} \neq 0$.***

22   While the variance of Cohen's d still depends on the total sample size and the sample

23 sizes allocation ratio, it also depends on the population effect size (δ_{Cohen}). The larger the

24 population effect size, the larger the variance. Note that the effect of the population effect

25 size decreases when sample sizes increase since

$$\lim_{n_1 \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

26   This is illustrated in Figure 3.

27   **In summary.** The variance of Cohen's d is a function of the population effect size

28 (δ_{Cohen}), the total sample size (N) and the sample sizes ratio ($\frac{n_2}{n_1}$):

29   • The variance decreases when the total sample size increases;

30   • The variance also decreases when the sample sizes ratio gets closer to 1;

31   • Finally, the variance increases when δ_{Cohen} increases. Note that the effect of δ_{Cohen} is

32   moderated by the total sample size (the larger N, the smaller the effect of δ_{Cohen} on

33   the variance).

34 **Glass's d**

35   **When variances are equal across populations.**

When $\delta_{Glass} = 0$.

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{1}{n_e}\right)$$

In this configuration, the **variance** of Glass's d is a function of the sample sizes of both control (n_c) and experimental (n_e) groups as well as of the sample sizes allocation ratio $\left(\frac{n_c}{n_e}\right)$:

- The larger the sample sizes, the lower the variance (Figure 4);

The sample sizes ratio associated with the lowest variance is not exactly 1 (because of the term $\frac{df}{df-2}$, df depending only on n_c), but is very close to 1 (and the larger the total sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance). The further from this sample size ratio, the larger the variance (see Figure 5).

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases. On the other hand, the effect of the population effect size does *not* depend on the size of the experimental group since

$$\lim_{n_c \to \infty}\left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right] = 0$$

$$\lim_{n_e \to \infty}\left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right] \neq 0$$

These limits are illustrated in Figure 6.

55   Note: while the sample sizes ratio associated with the lowest variance was very close

56   to 1 with a null population effect size, this is not true anymore when the population effect

57   size is not zero. Indeed, because of the second term in the addition, when computing the

58   variance, one gives much more weight to the effect size of the control group (see Figure 7),

59   especially when the effect size gets larger. For example, when $\delta_{Glass} = 4$, the lowest variance

60   will occur when n_c is approximately 3 times larger than n_e. When $\delta_{Glass} = 7$, the lowest

61   variance will occur when n_c is approximately 5 times larger than n_e, etc.

62   **When variances are unequal across populations, with equal sample sizes.**

63   ***When $\delta_{Glass} = 0$.***

64   When the population effect size is zero, the variance of Glass's d can be simplified as

65   follows:

$$Var_{Glass's\ d} = \frac{n-1}{n(n-3)}\left(1 + \frac{\sigma_e^2}{\sigma_c^2}\right)$$

66   where $n = N/2 =$ sample size of each group. The variance is therefore a function of the

67   total sample size and the SD-ratio $\left(\frac{\sigma_c}{\sigma_e}\right)$:

68   • The larger the total sample size, the lower the variance (See Figure 8);

69   • The larger the SD-ratio (i.e. the larger is σ_c in comparison with σ_e), the lower the

70   variance (see Figure 9). However, the effect of the SD-ratio decreases when sample

71   sizes increase, because $\lim_{n(=n_c=n_e)\to\infty}\left[\frac{df}{n(df-2)}\right] = 0$.

72   ***When $\delta_{Glass} \neq 0$.***

73   While the variance of Glass's d still depends on the total sample size and the

74   SD-ratio, it also depends on the population effect size (δ_{Glass}). The larger the population

75   effect size, the larger the variance. However, the effect of the population effect size

76   decreases when the control group increases, as previously explained and illustrated in

77   Figure 6.

78   **When variances are unequal across populations, with unequal sample sizes.**

79   **When $\delta_{Glass} = 0$.**

80   When the population effect size is zero, the variance of Glass's d can be simplified as

81 follows:

$$Var_{Glass's\,d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e\sigma_c^2}\right)$$

82   The variance of Glass's d is therefore a function of the total sample size (N), the

83 SD-ratio and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

84 • For any SD and sample sizes pairing, increasing n_c and/or n_e will decrease the

85   variance (see Figure 10);

86   – The effect of the sample sizes ratio depends on the SD-ratio:

87   * We previously mentioned that when $\sigma_c = \sigma_e$, the variance is minimized

88   when sample sizes of both groups are almost identical (see Figure 5),

89   meaning that it is more efficient, in order to reduce variance, to add

90   subjects uniformly in both groups;

91   * When $\sigma_e > \sigma_c$, more weight is given to n_e, meaning that it is more efficient,

92   in order to reduce variance, to add subjects in the experimental group (n_e;

93   see bottom plots in Figure 10);

94   * When $\sigma_c > \sigma_e$, less weight is given to n_e, meaning that it is more efficient, in

95   order to reduce variance, to add sujects in the control group (n_c; see top

96   plots in Figure 10).

97 • Finally, there is also a main effect of the SD-ratio: the larger is σ_c in comparison

98   with σ_e, the lower the variance, as we can observe in Figure 11. We can also notice

99   that in Figure 10, the maximum variance is much larger in the two bottom plots

100   (where $\sigma_c < \sigma_e$) than in the two top plots (where $\sigma_c > \sigma_e$).

101   Note that the effect of the SD-ratio, and the interaction effect between SD-ratio and

102   sample sizes ratio decreases when the sample size of the control group increases (because

103   $\frac{n_c - 1}{n_c - 3}$ gets closer to 1).

104   **_When $\delta_{Glass} \neq 0$._**

105   While the variance of Glass's d still depends on the total sample size, the SD-ratio

106   and the interaction between the SD-ratio and the sample sizes ratio, it also depends on the

107   population effect size (δ_{Glass}): the larger the population effect size, the larger the variance.

108   However, the effect of the population effect size decreases when the sample size of the

109   control group increases, as previously explained and illustrated in Figure 6.

110   Note: when the population effect size was null, when $\sigma_c < \sigma_e$, it was much more

111   efficient to add subjects in the experimental group in order to reduce the variance (because

112   much more weight was given to n_e). When $\delta_{Glass} \neq 0$, it is important to add subjects in

113   both groups in order to reduce the variance (because $\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2$ is only a

114   function of the sample size of the control group). With huge population effect size, it is

115   even always more important to add subjects in the control group (e.g. when $\delta_{Glass} = 30$).

116   **In summary.**   The variance of Glass's d is a function of the population effect size

117   (δ_{Glass}), the SD-ratio, the total sample size and the interaction between sample sizes ratio

118   and SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e} \right)$:

119   • The variance decreases when the SD-ratio increases (i.e. when $\sigma_e >> \sigma_c$);

120   • The variance always decreases when the control and/or the experimental group

121   increases. The benefit of adding subjects rather in the control, in the experimental,

122   or in both groups, in order to reduce the variance, varies as a function of the

123   SD-ratio and the population effect size. The only situation where it is optimal to

maximize the experimental group is when $\sigma_e > \sigma_c$ and $\delta_{Glass} \approx 0$. Most of the time, it

is more efficient to maximize the control groups (e.g. anytime $\sigma_e < \sigma_c$, and when

δ_{Glass} is very large) or to uniformly add subjects in both groups (e.g. when $\sigma_e > \sigma_c$

and δ_{Glass} is neither null nor huge);

- The variance increases when δ_{Glass} increases. Note that the effect of δ_{Glass} is

  moderated by the control group size (the larger n_e, the smaller the effect of δ_{Glass} on

  the variance).

Cohen's d^*

When variances are equal across populations.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* is computed as

follows :

$$Var_{Cohen's\ d*} = \frac{df}{df - 2} \times \frac{N}{n_1 n_2}$$

with

$$df = \frac{4(n_1 - 1)(n_2 - 1)}{n_1 + n_2 - 2}$$

In this configuration, the degrees of freedom as well as the variance of Cohen's d^* depend

on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see

  Figure 12);

- The larger the total sample size, the lower the bias (see Figure 13).

142   ***When $\delta^*_{Cohen} \neq 0$.***

143   While the variance of Cohen's d^* still depends on the total sample size and the

144   sample sizes ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the

145   population effect size, the larger the variance. However, the effect of the population effect

146   size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or

147   the sample sizes ratio get closer to 1), as illustrated in Figure 14 since

$$\lim_{df \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

148   **When variances are unequal across populations, with equal sample sizes.**

149   ***When $\delta^*_{Cohen} = 0$.***

150   When the population effect size is zero, the variance of Cohen's d^* can be simplified

151   as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df - 2} \times \frac{2}{n}$$

152   where $n = N/2$=sample size of each group, and $df = \frac{(n-1)(\sigma_1^4 + \sigma_2^4 + 2\sigma_1^2\sigma_2^2)}{\sigma_1^4 + \sigma_2^4}$. In this

153   configuration, the degrees of freedom as well as the variance of Cohen's d^* depend on the

154   total sample size (N) and the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$:

155   • The further the SD-ratio is from 1, the larger the variance (see Figure 15);

156   • The larger the total sample size, the lower the variance (see Figure 16).

157   Note: for a constant SD-ratio, the size of the variance does not matter (see Figure

158   17).

159   ***When $\delta^*_{Cohen} \neq 0$.***

160   While the variance of Cohen's d^* still depends on the total sample size and the

161   SD-ratio, it also depends on the population effect size (δ^*_{Cohen}): the larger the population

162   effect size, the larger the variance. However, the effect of the population effect size

163   decreases when the degrees of freedom increase (i.e. when the total sample size increases

164   and/or the SD-ratio get closer to 1), as previously illustrated in Figure 14.

165   **When variances are unequal across populations, with unequal sample sizes.**

166   ***When $\delta^*_{Cohen} = 0$.***

167   When the population effect size is zero, the variance of Cohen's d^* can be simplified

168   as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df - 2} \times \frac{2\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2 + \sigma_2^2}$$

169   with $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$. In this configuration, the degrees of freedom are a function of

170   the total sample size (N) and the interaction between sample sizes ratio and the SD-ratio

171    $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

172   • The larger the total sample size, the lower the variance (illustration in Figure 18);

173   • The smallest variance always occurs when there is a positive pairing between

174   variances and sample sizes, because one gives more weight to the smallest variance in

175   the denominator of the df computation and in the numerator of the variance

176   computation. Moreover, the further the SD-ratio is from 1, the further from 1 will

177   also be the sample sizes ratio associated with the smallest variance (see Figure 19).

178   This can be explained by splitting the numerator and the denominator of the df

179   computation (see the file "Theoretical Bias, as a function of population parameters").

180   Note: for a constant SD-ratio, the variance does not matter. (See Figure 20).

181   ***When $\delta^*_{Cohen} \neq 0$.***

182   While the variance of Cohen's d^* still depends on the total sample size, the SD-ratio

183   and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the

184   population effect size (δ^*_{Cohen}): the larger the population effect size, the larger the variance.

185   However, the effect of the population effect size decreases when the degrees of freedom

186   increase (i.e. when the total sample size increases and/or when there is a positive pairing

187   between the sample sizes ratio and the SD-ratio), as previously illustrated in Figure 14.

188   **In summary.**   The variance of Cohen's d^* is a function of the population effect size

189   (δ^*_{Cohen}), the total sample size (N) and the interaction between sample sizes ratio and

190   SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1} \right)$:

191   • The variance always decreases when the control and/or the experimental group

192   increases. The benefit of adding subjects rather in the control or in the experimental

193   group depends on the SD-ratio. Indeed, the smallest variance always occurs when

194   there is a positive pairing between variances and sample sizes. Moreover, the further

195   the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio

196   associated with the smallest variance;

197   • The variance increases when δ^*_{Cohen} increases. Note that the effect of δ^*_{Cohen} is

198   moderated by the total sample size and the interaction between sample sizes ratio

199   and SD-ratio.

Shieh's d

201   **When variances are equal across populations.**

202   ***When $\delta_{Shieh} = 0$.***

203   When the population effect size is zero, the variance of Shieh's d can be simplified as

204   follows:

$$Var_{Shieh's\,d} = \frac{df}{(df - 2)N}$$

with $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the sample sizes allocation ratio $\left(\frac{n_2}{n_1}\right)$:

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure 21);

- The larger the total sample size, the lower the variance. It does not matter whether the sample sizes ratio is constant (see Figure 22) or not (see Figure 23).

Note: in "Theoretical Bias, as a function of population parameters," we noticed that moving the sample sizes ratio away from 1 when adding subjects in only one group could decrease the degrees of freedom. However, due to the total sample size term (N) in the denominator of the variance computation, even when degrees of freedom decrease due to the fact that one adds subjects only in one group, the variance still decreases (because the denominator of the variance computation increases; see Figure 23).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase, without increasing the sample sizes ratio, and/or the sample sizes ratio gets closer to 1), as previously illustrated in Figure 14.

Note: we previously noticed that when the effet size is zero, the variance of Shieh's d decreases, even when the sample sizes ratio increases. It is no longer true when there is a non-null effect size because the larger the sample sizes ratio, the more the variance will increase with increasing effect size.

229   **When variances are unequal across populations, with equal sample sizes.**

230   ***When $\delta_{Shieh} = 0$.***

231   When the population effect size is zero, the variance of Shieh's d can be simplified as

232 follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

233 with $df = \frac{(\sigma_1^2+\sigma_2^2)^2 \times (n-1)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the

234 variance of Shieh's d depend on the total sample size (N) and the SD-ratio ($\frac{\sigma_2}{\sigma_1}$).

235   • The further the SD-ratio is from 1, the larger the variance (see Figure 24);

236   • The larger the total sample size, the lower the variance (see Figure 25).

237   Note: for a constant SD-ratio, the size of the variance does not matter (see Figure

238 26).

239   ***When $\delta_{Shieh} \neq 0$.***

240   While the variance of Shieh's d still depends on the total sample size and the

241 SD-ratio, it also depends on the population effect size (δ_{Shieh}): the larger the population

242 effect size, the larger the variance. However, the effect of the population effect size

243 decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the

244 SD-ratio gets closer to 1), as previously illustrated in Figure 14.

245   **When variances are unequal across populations, with unequal sample sizes.**

246   ***When $\delta_{Shieh} = 0$.***

247   When the population effect size is zero, the variance of Shieh's d can be simplified as

248 follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1} + \frac{(\sigma_2^2/n_2)^2}{n_2-1}}$. In this configuration, the degrees of freedom as well as the

variance of Shieh's d depend on the total sample size (N) and the interaction between the

sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the total sample size, the lower the variance. It remains true when the

  sample sizes ratio is constant (see Figure 27) and when it is not (see Figure 28).

  Note: When variances were equal across populations, adding subjects only in the first

group had the same impact on the variance as adding subjects only in the second group

(see Figure 23). When variances are unequal across groups, this is not true anymore (see

Figure 28).

- The smallest variance always occurs when there is a positive pairing between

  variances and sample size. Moreover, the further the SD-ratio is from 1, the further

  from 1 will also be the sample sizes ratio associated with the smallest variance (see

  Figure 29).

  Moreover, for a constant SD-ratio, the variances do not matter (See Figure 30).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the

interaction between the sample sizes ratio and the SD-ratio, it also depends on the

population effect size (δ_{Shieh}): the larger the population effect size, the larger the variance.

However, the effect of the population effect size decreases when the degrees of freedom

increase, as previously illustrated in Figure 14.

In summary.   The variance of Shieh's d is a function of the population effect size

(δ_{Shieh}), the total sample size (N) and the interaction between sample sizes ratio and

SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;

- The variance increases when δ_{Shieh} increases. Note that the effect of δ_{Shieh} is moderated by the total sample size and the interaction between the sample sizes ratio and the SD-ratio.
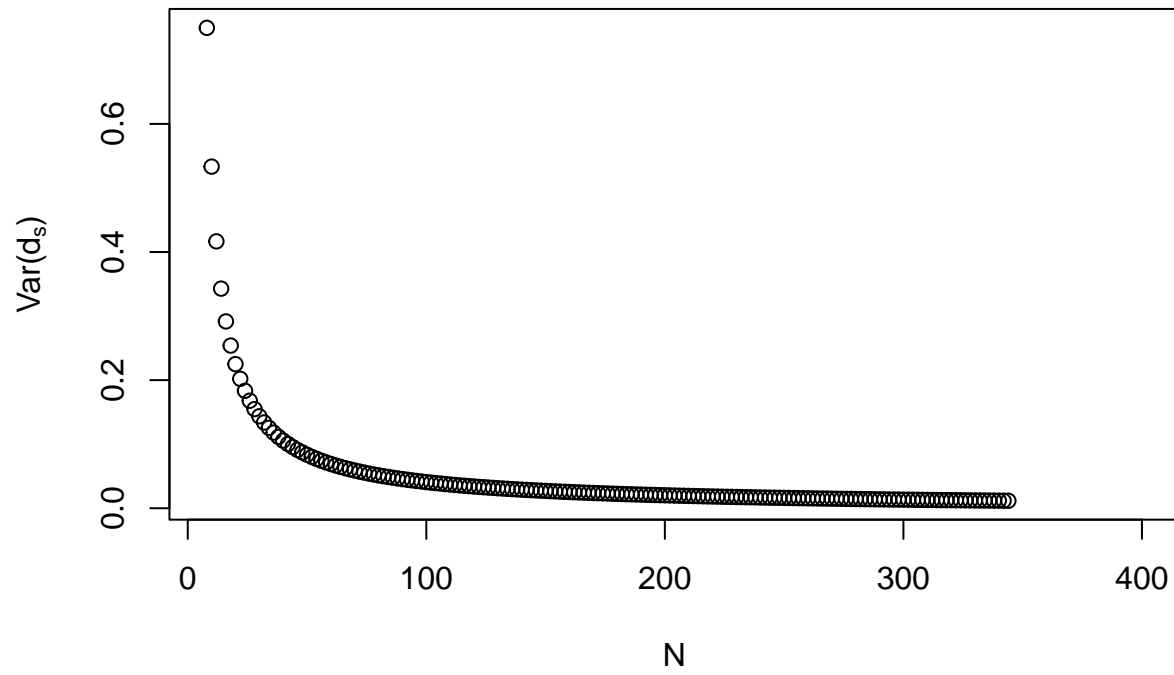
Figure 1. Variance of Cohen's d, when variances are equal across groups, as a function of the total sample size (N).
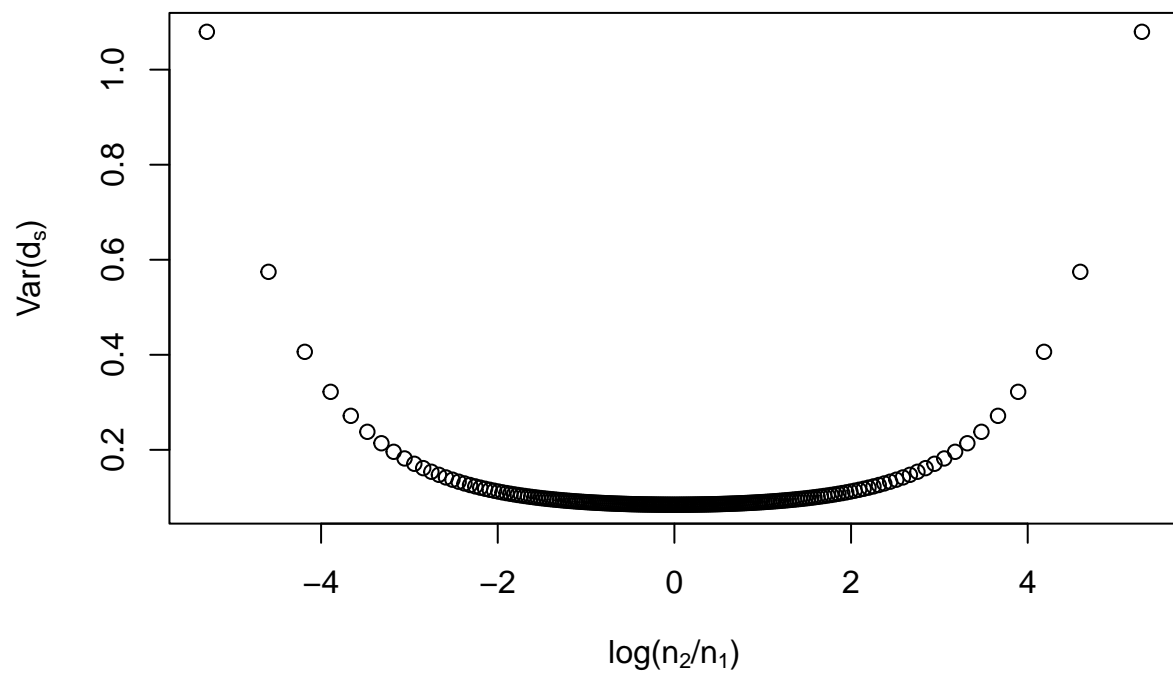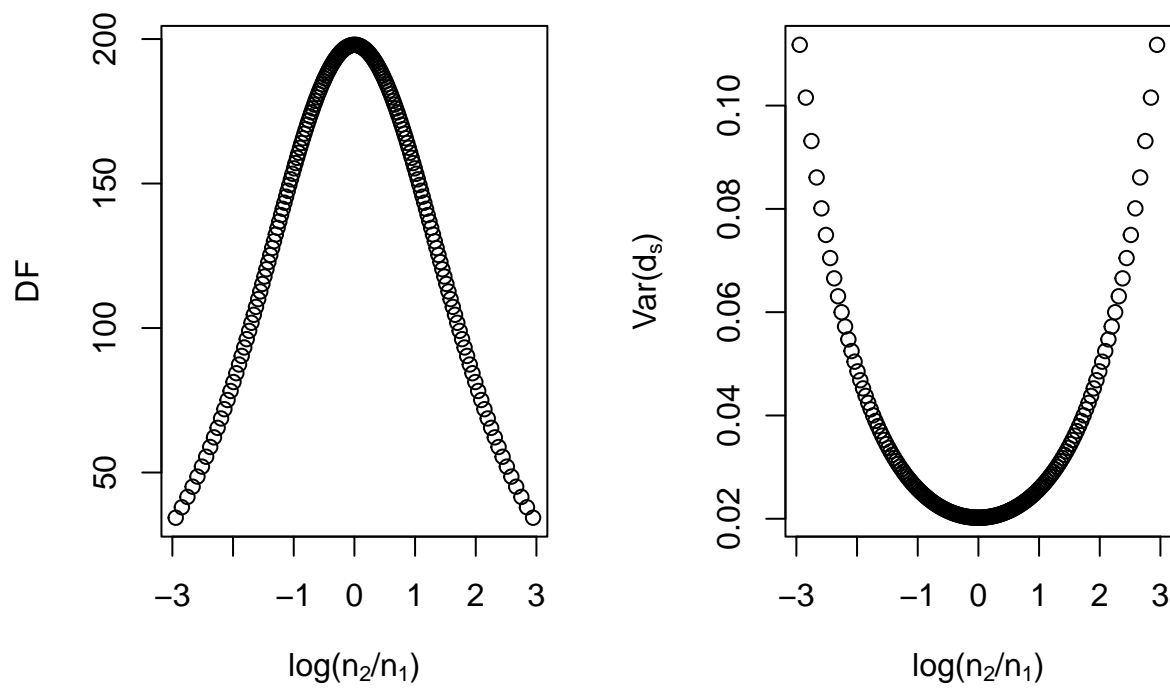
Figure 2. Variance of Cohen's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_2}{n_1}\right)\right)$.
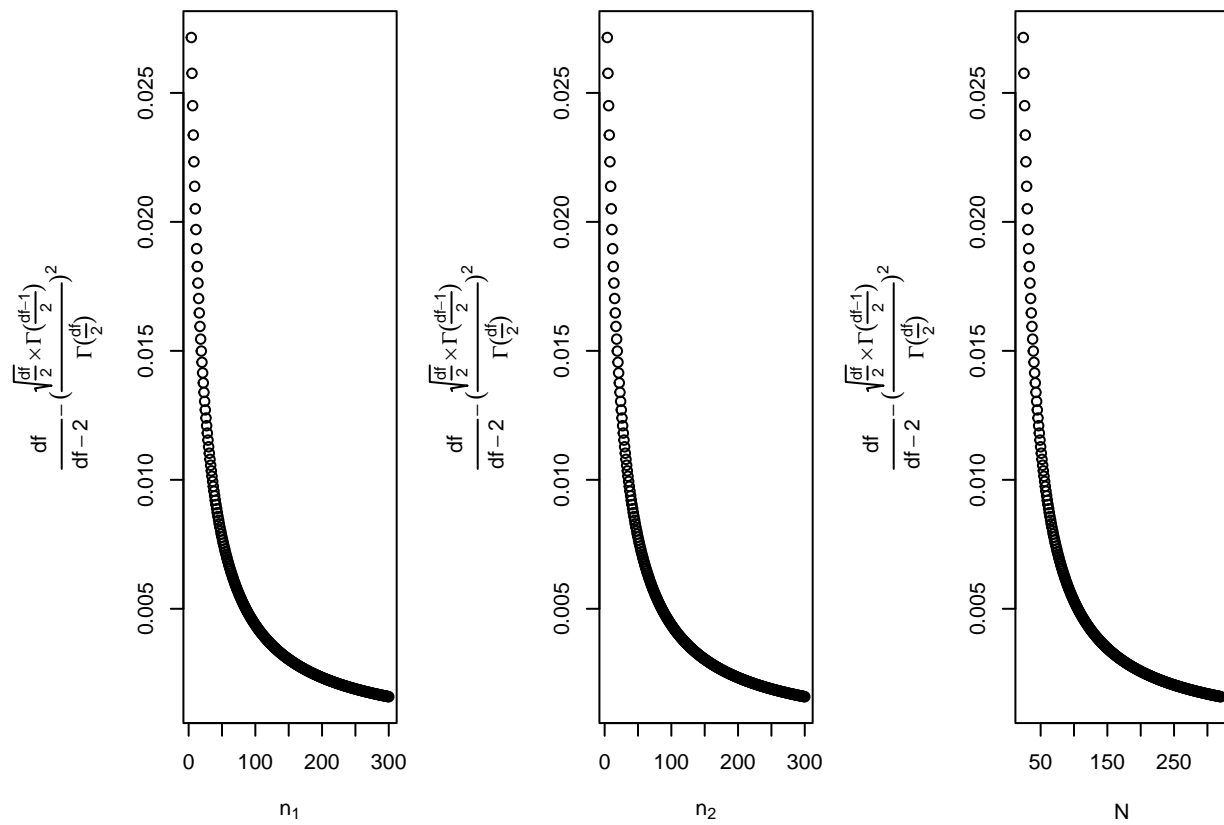
Figure 3. Effect size moderator, when computing the variance of Cohen's d, as a function of n_1 (left), n_2 (center) and $N = n_1 + n_2$ (right).
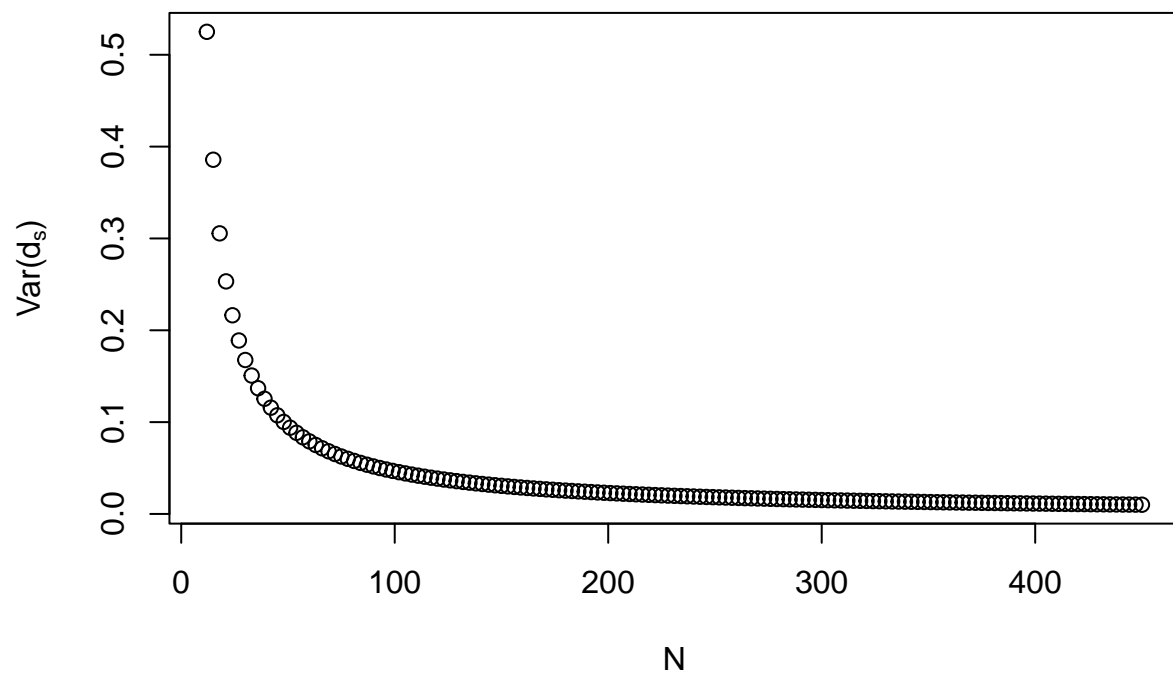
Figure 4. Variance of Glass's *d*, when variances are equal across groups, as a function of the total sample size (N).
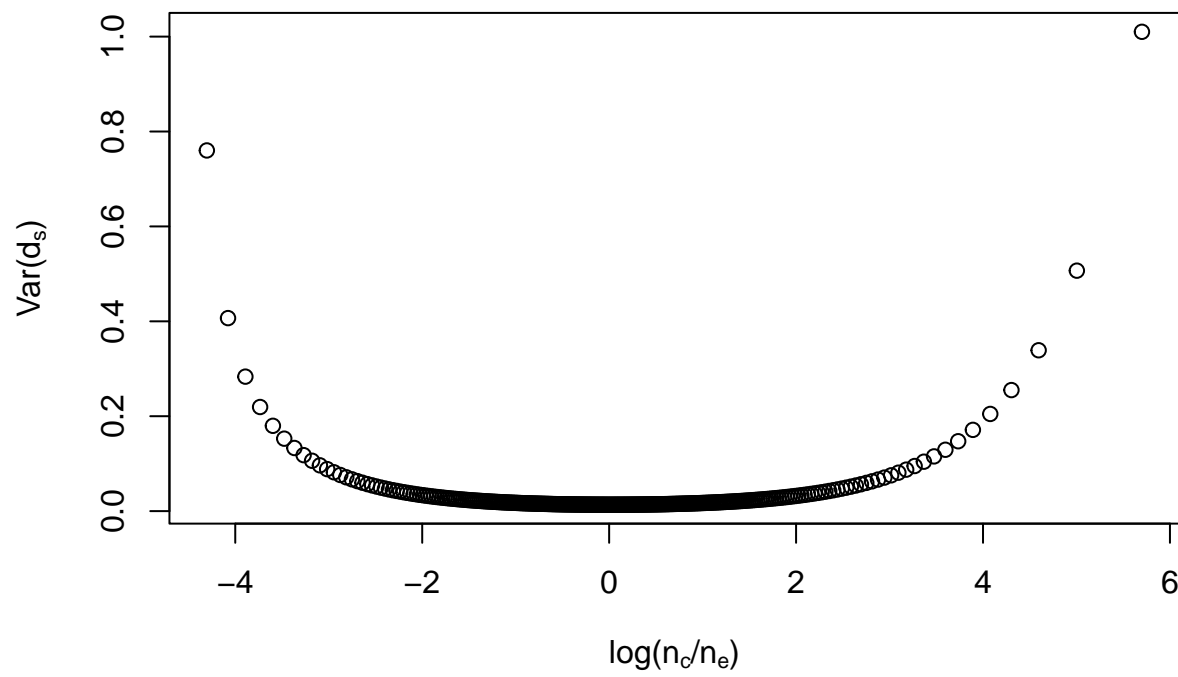
Figure 5. Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_c}{n_e}\right)\right)$.
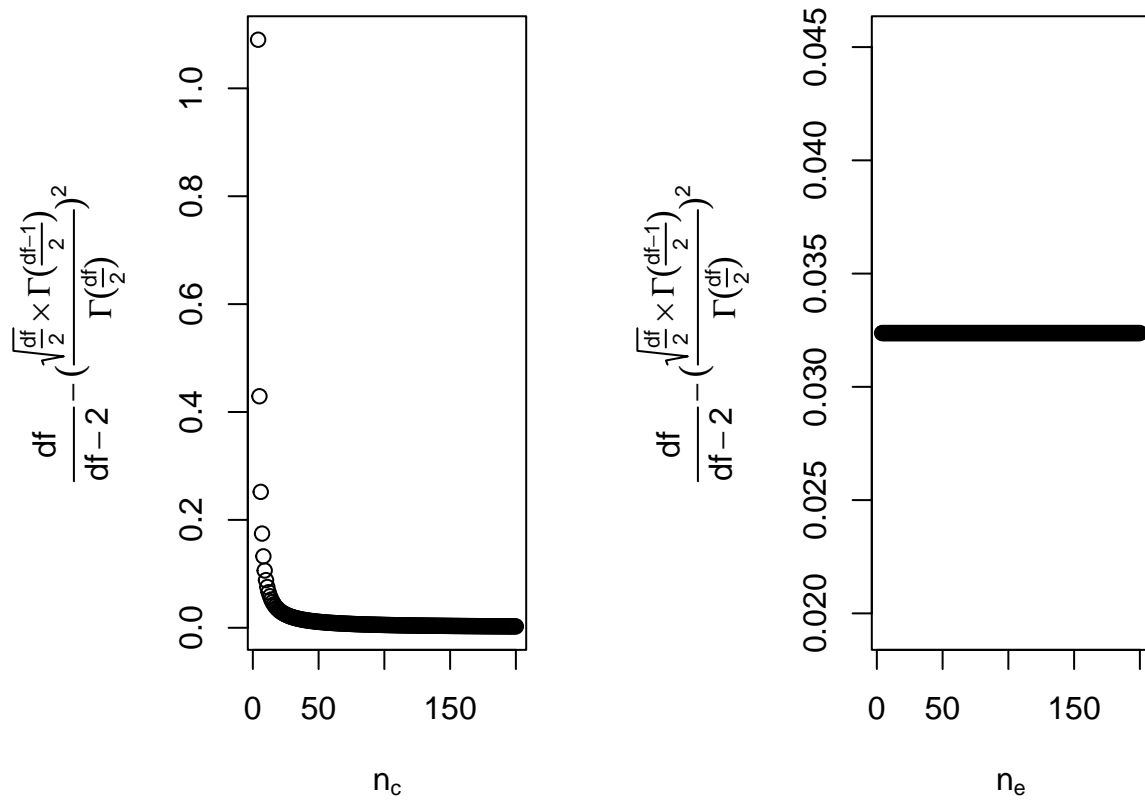
Figure 6. Effect size moderator, when computing the variance of Glass's *d*, as a function of the size of the control group (left) and experimental group (right).
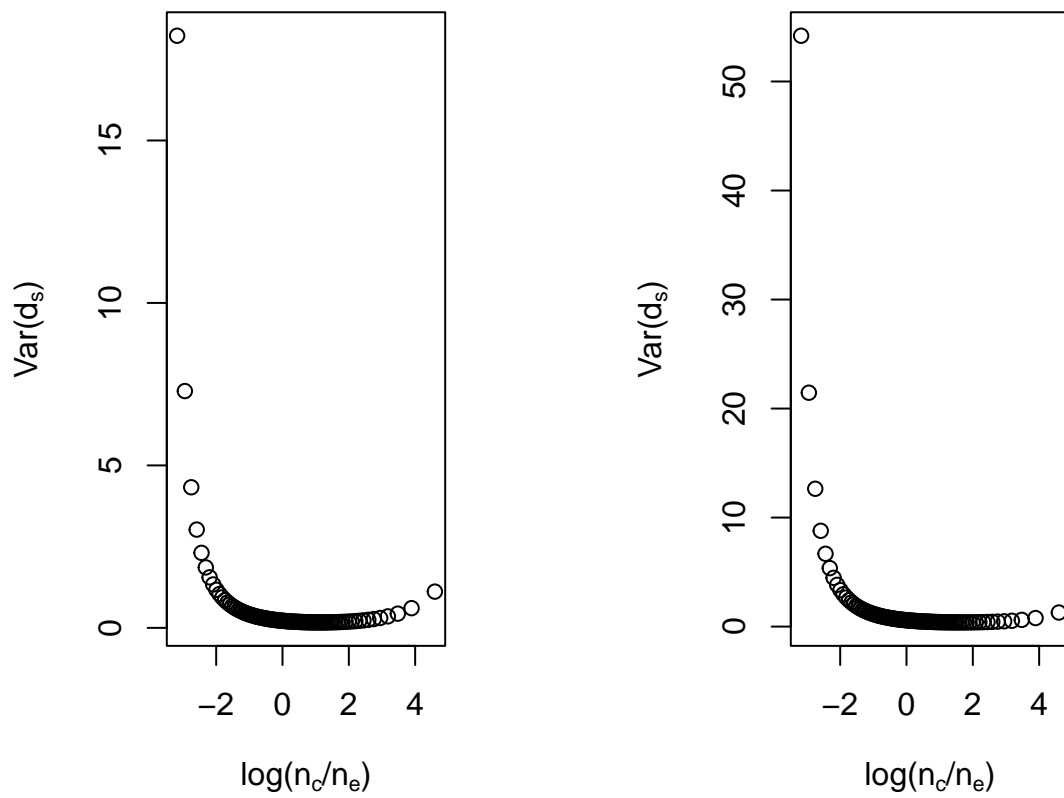
Figure 7. Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_c}{n_e}\right)$) when δ_{Glass} equals 4 (left) or 7 (right).
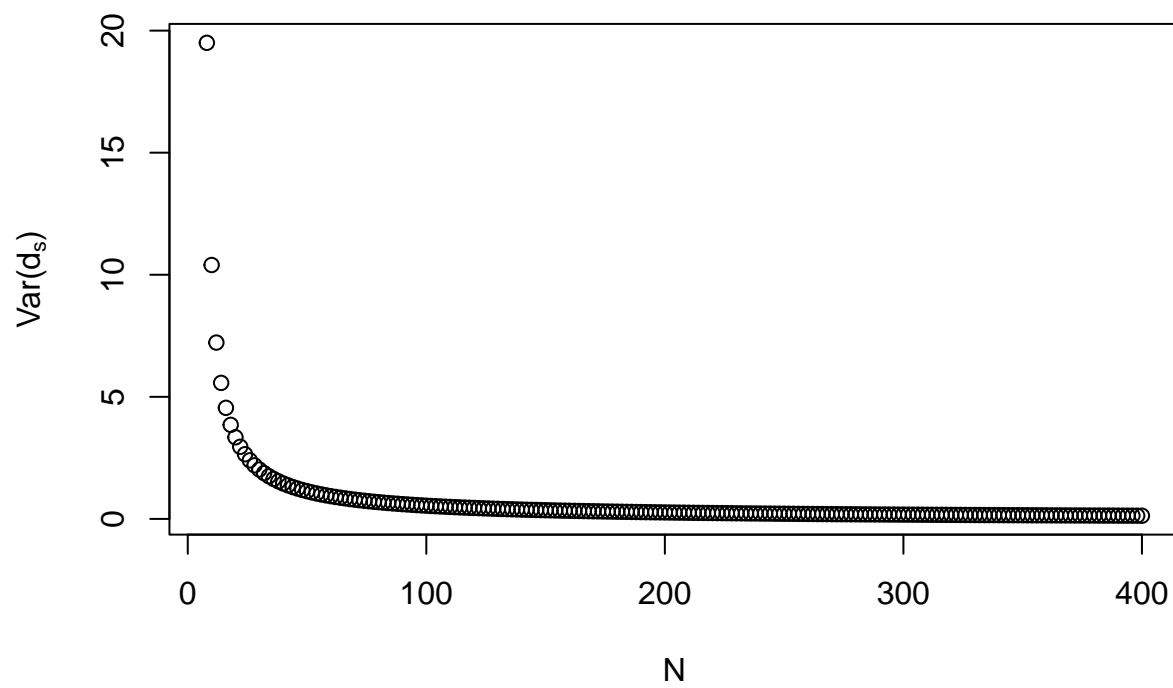
Figure 8. Variance of Glass's *d*, when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes (N).
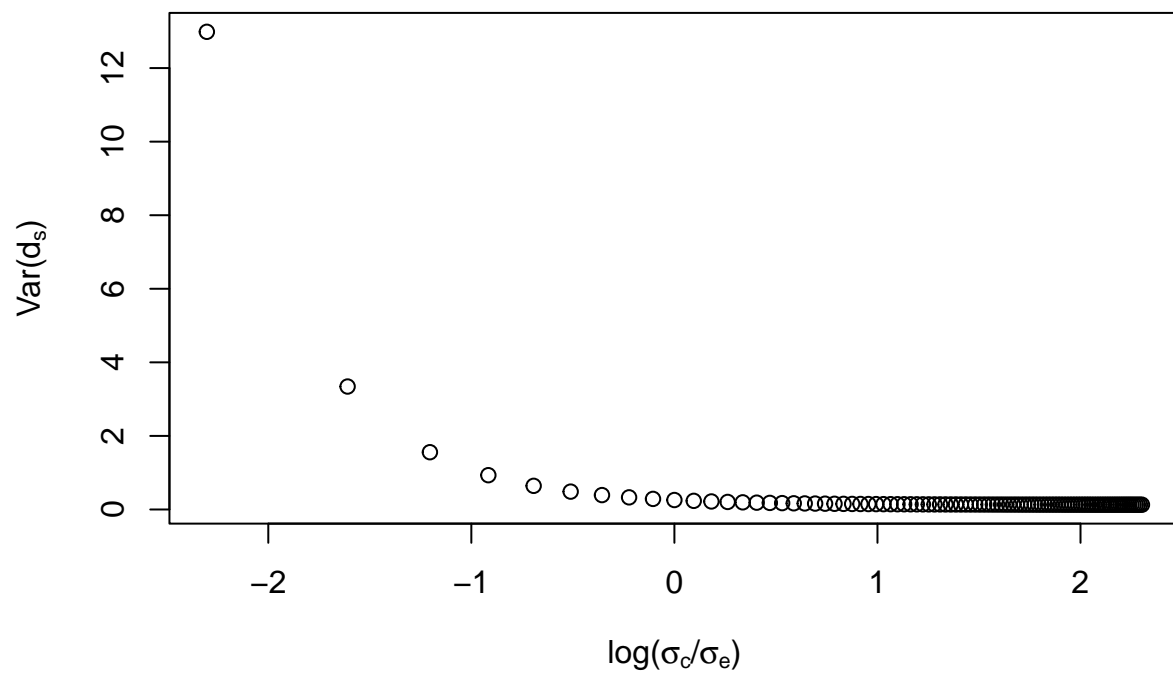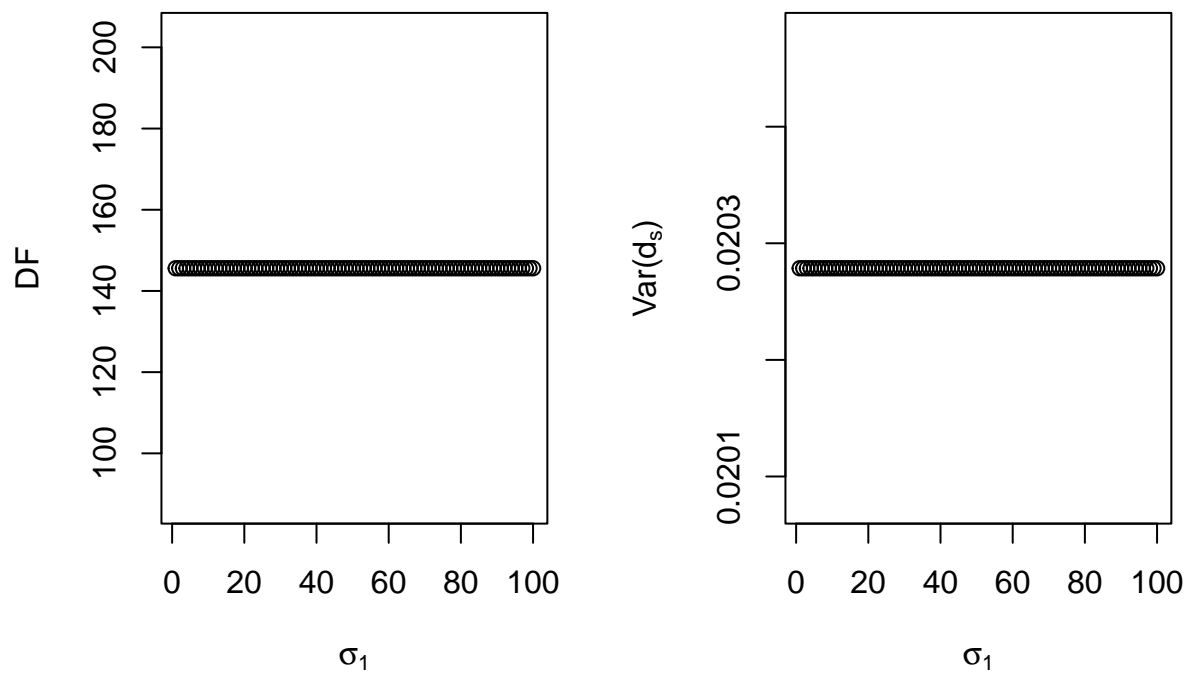
Figure 9. Variance of Glass's d, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
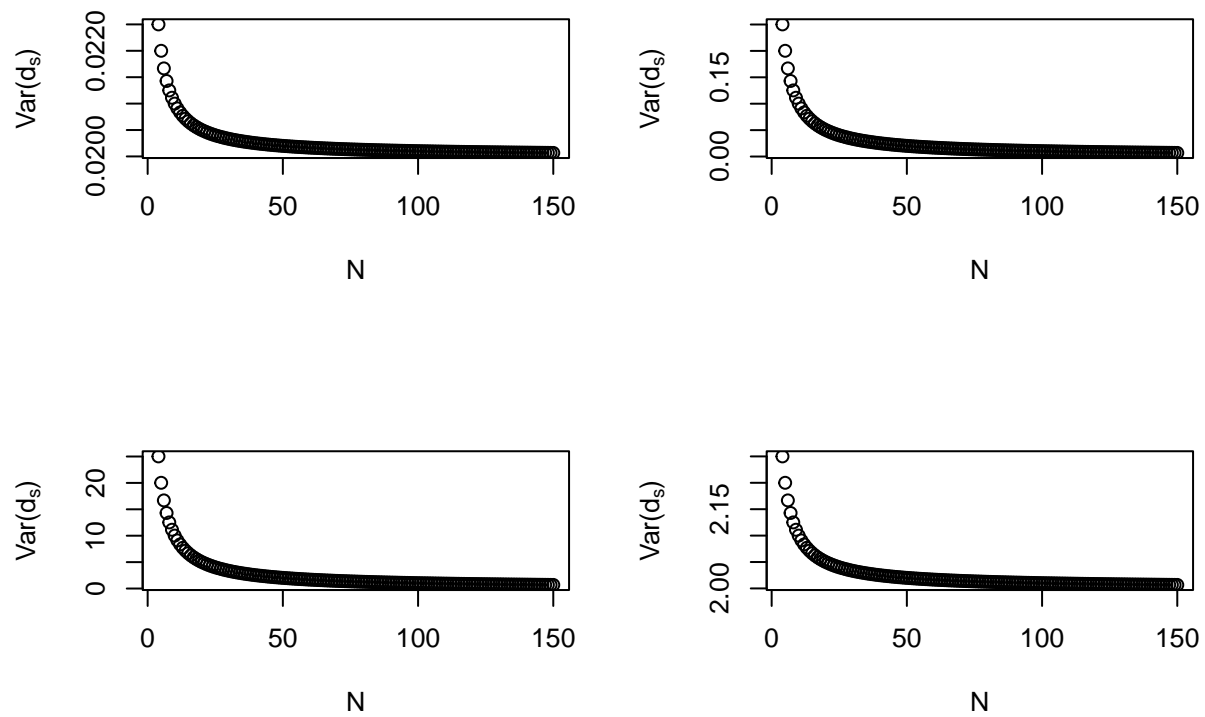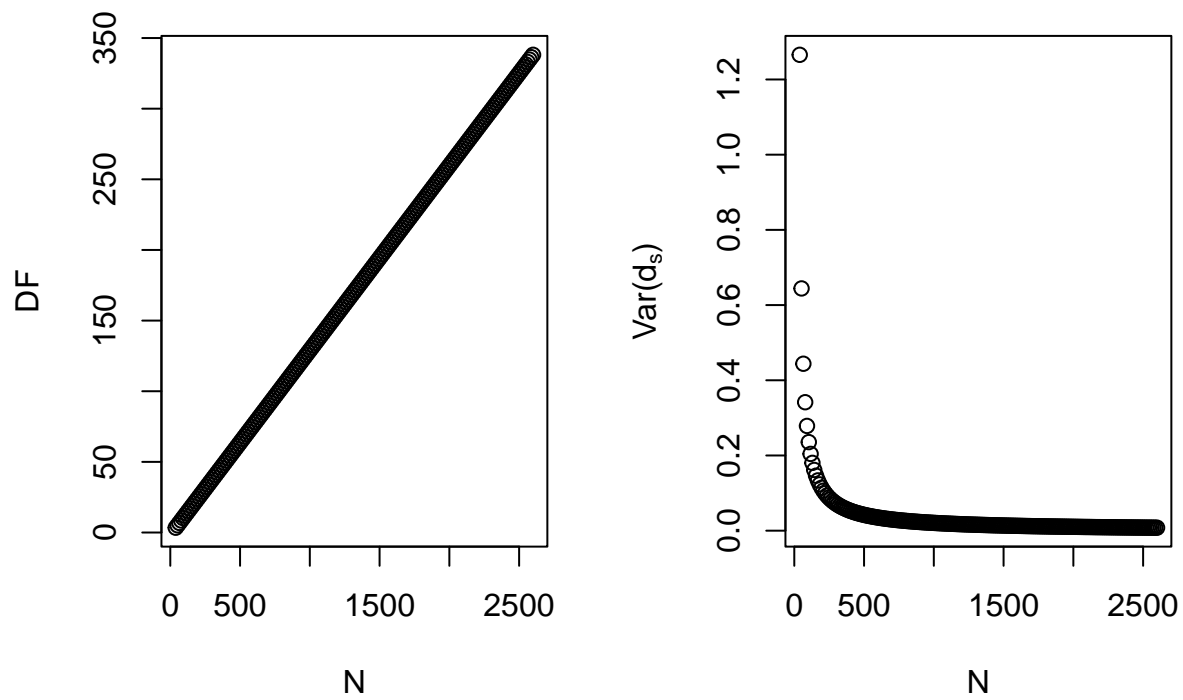
Figure 10. Variance of Glass's *d*, when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when $\sigma_c > \sigma_e$ (top plots) or $\sigma_c < \sigma_e$ (bottom plots).
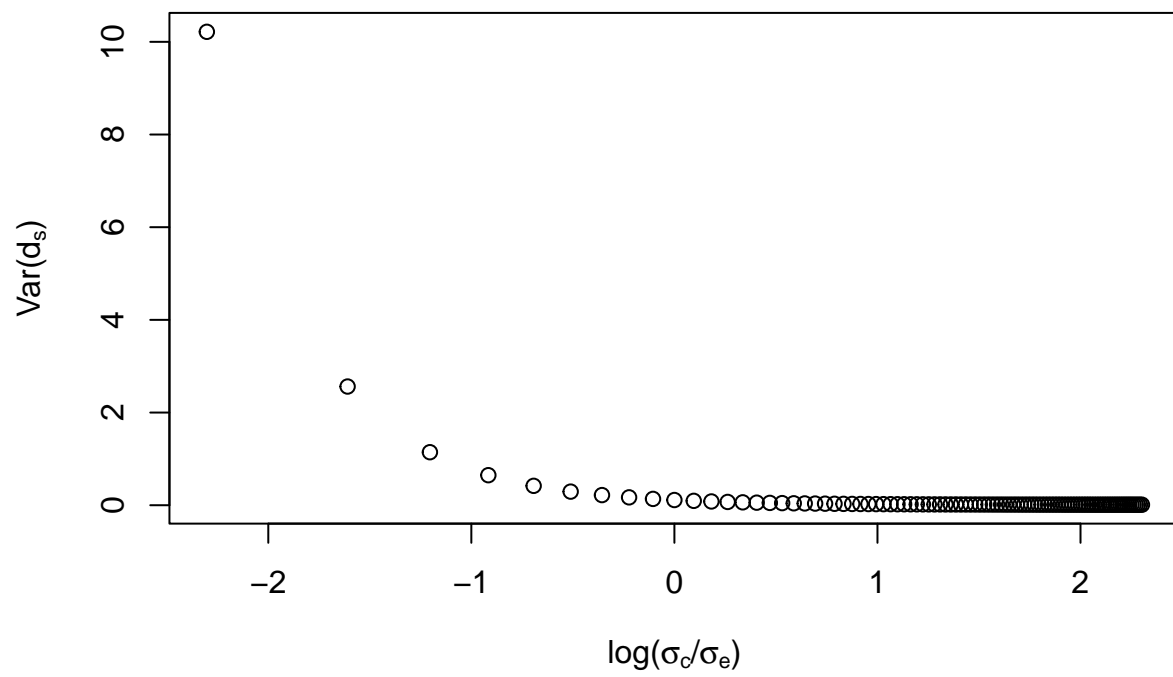
Figure 11. Variance of Glass's d, when sample sizes and variances are unequal across groups, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_c}{\sigma_e}\right)\right)$.
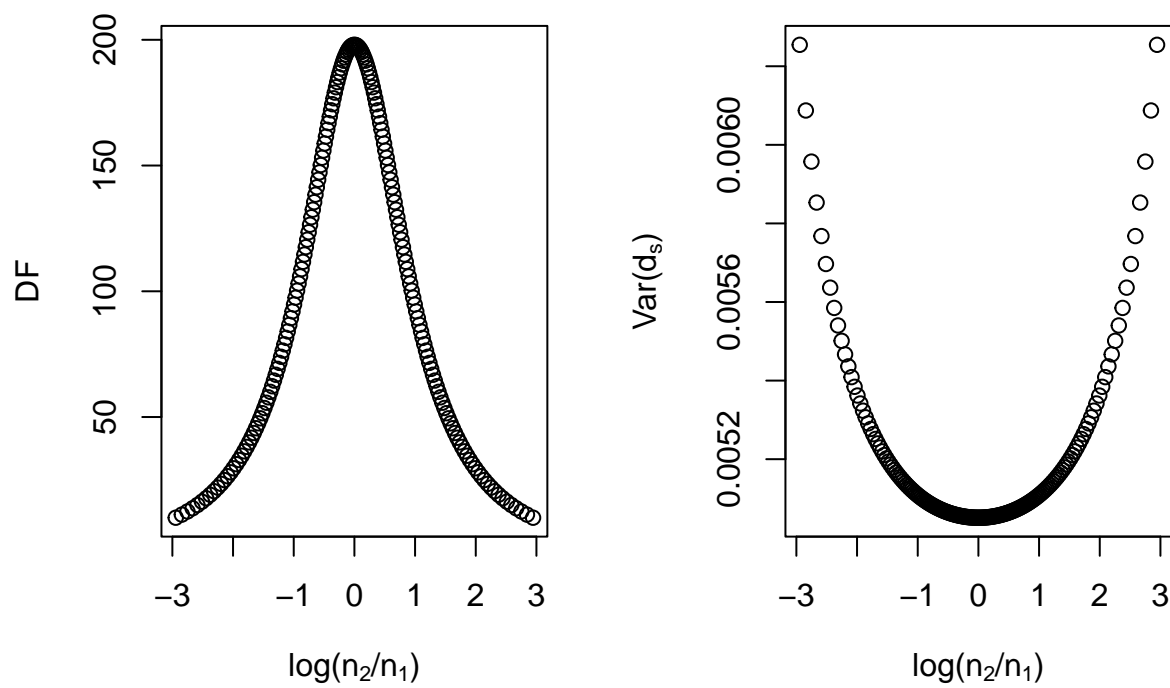
Figure 12. Variance of Cohen's d^* when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_2}{n_1}\right)\right)$.
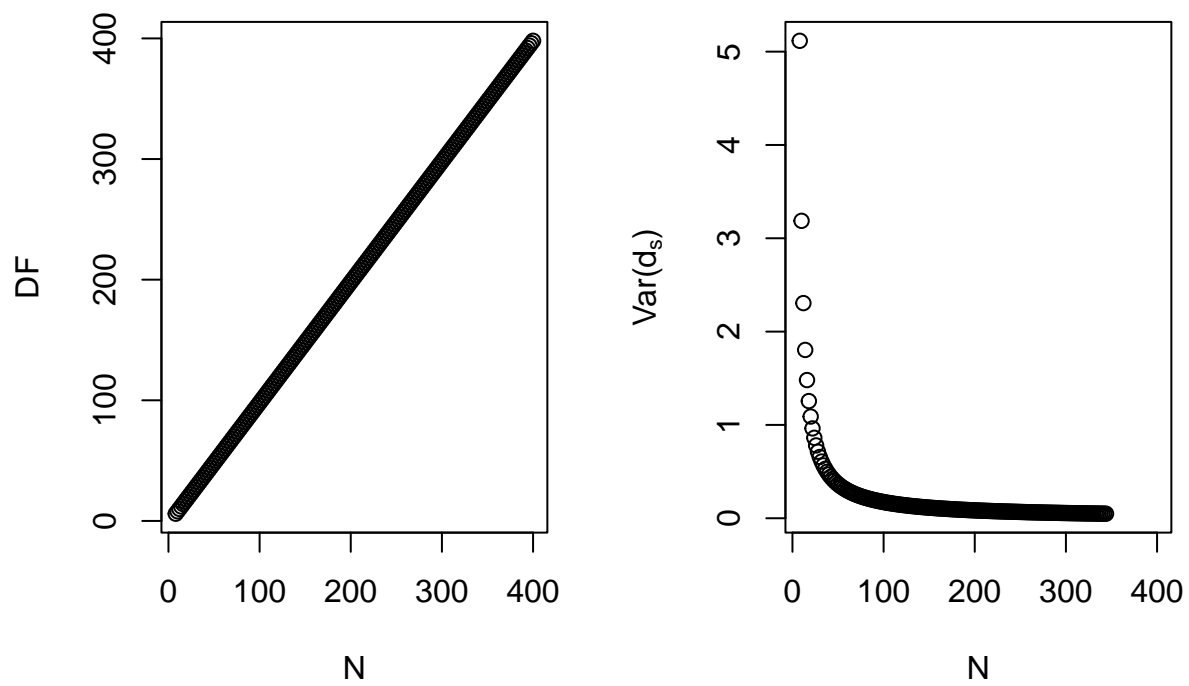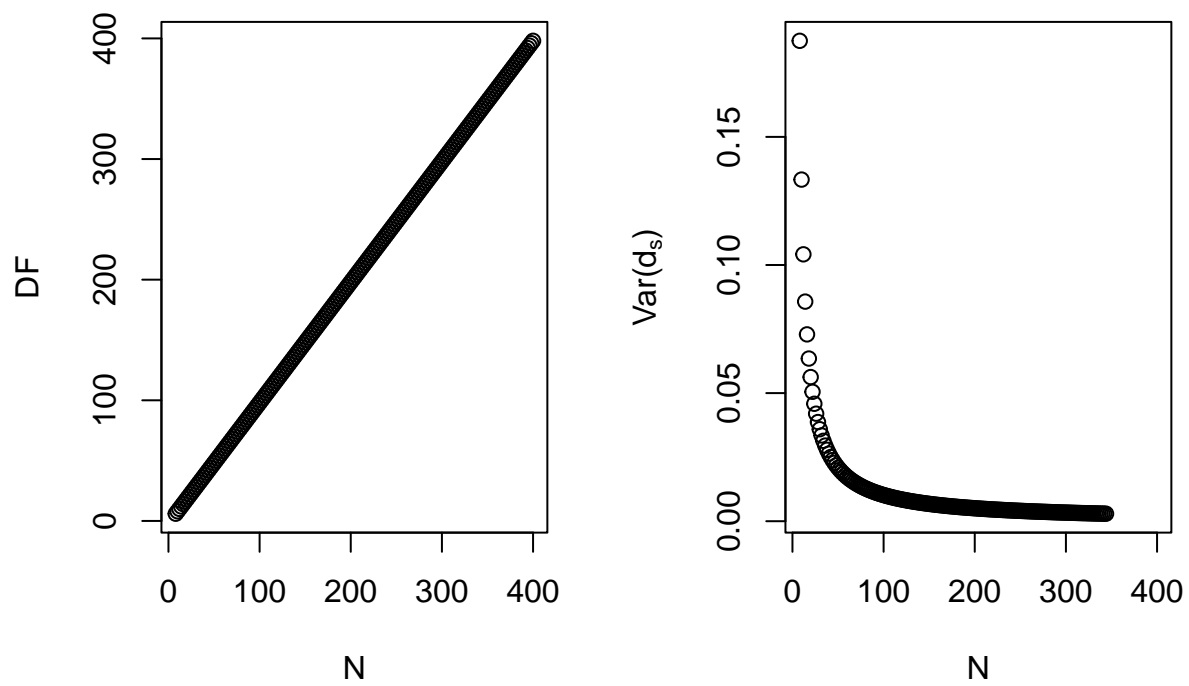
Figure 13. Variance of Cohen's d^* when variances are equal across groups, as a function of the total sample size (N).
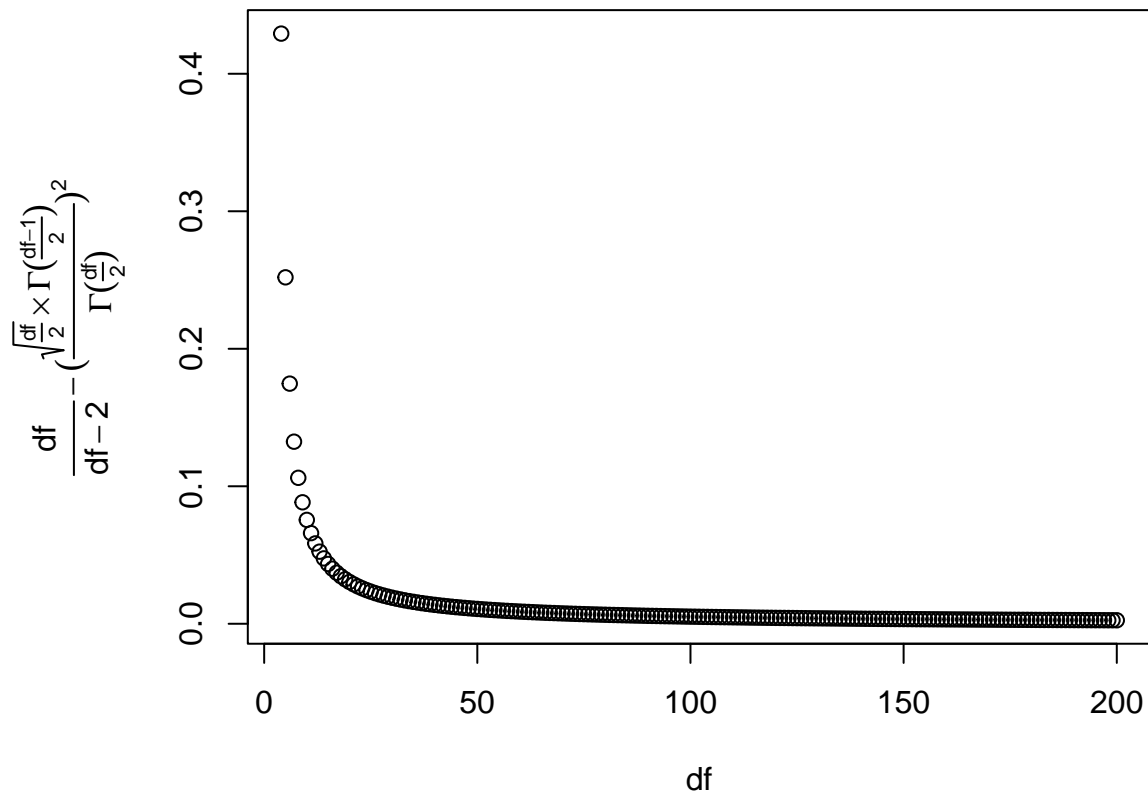
Figure 14. Effect size moderator (for all estimators), as a function of the degrees of freedom.
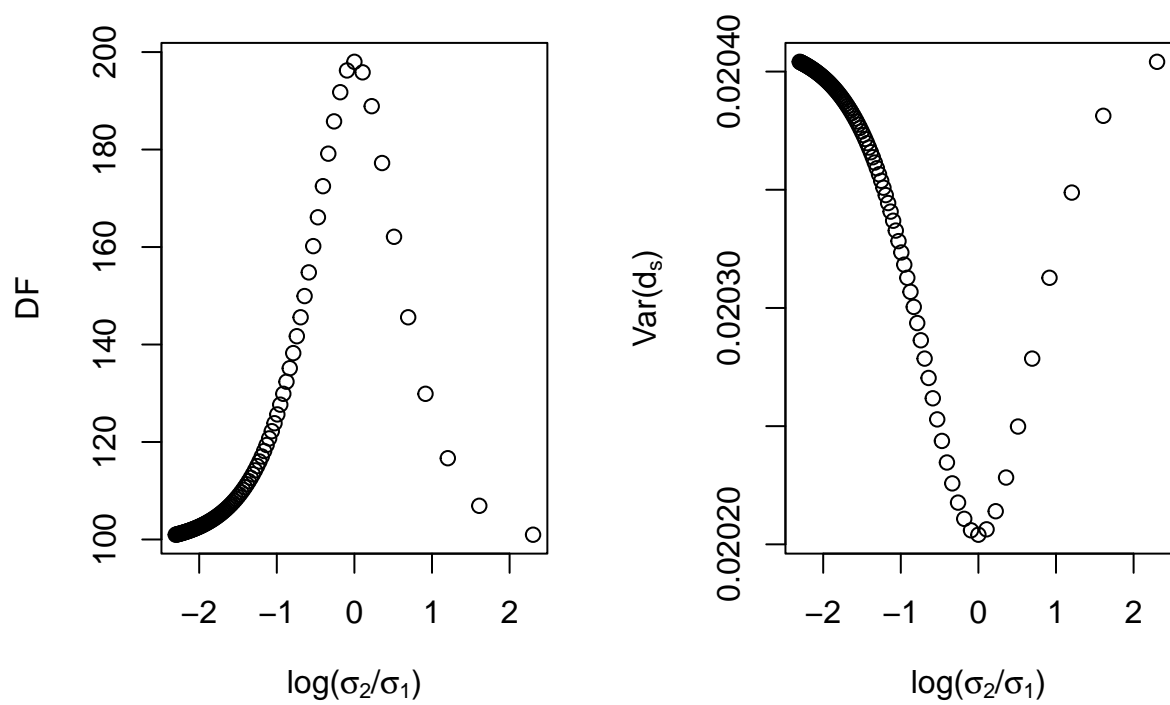
Figure 15. Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_2}{\sigma_1}\right)\right)$.
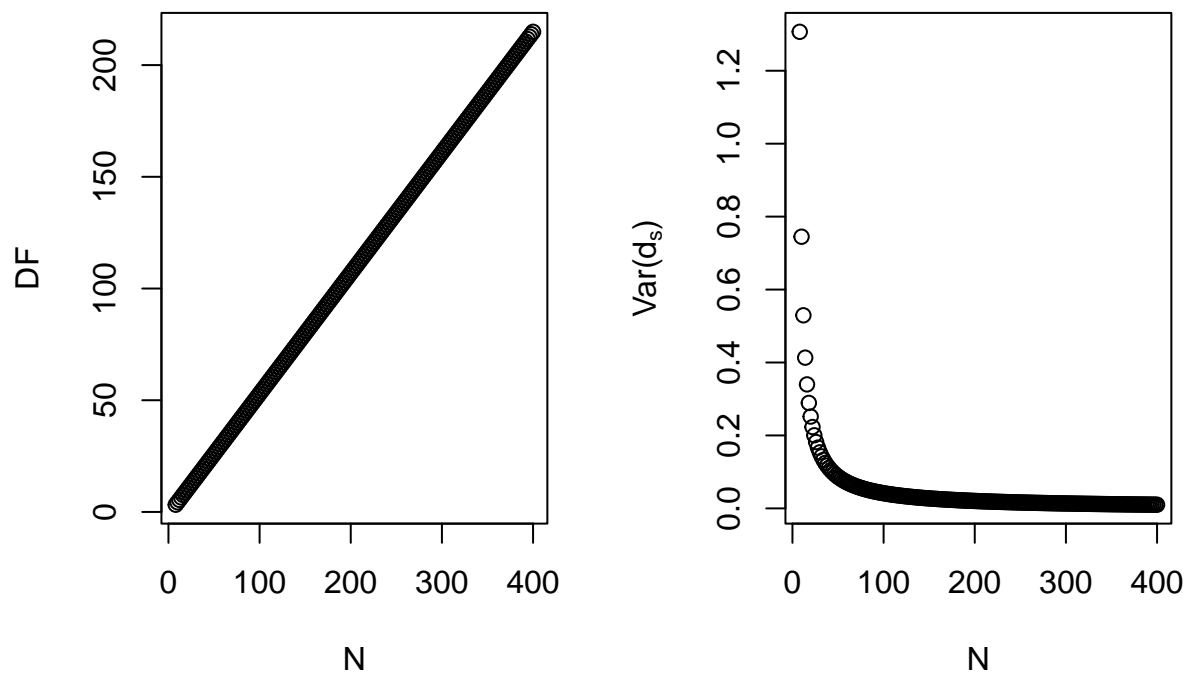
Figure 16. Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 17. Variance of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

Figure 18. Variance of Cohen's d^* when variances and sample sizes are unequal across groups, as a function of the total sample size (N).
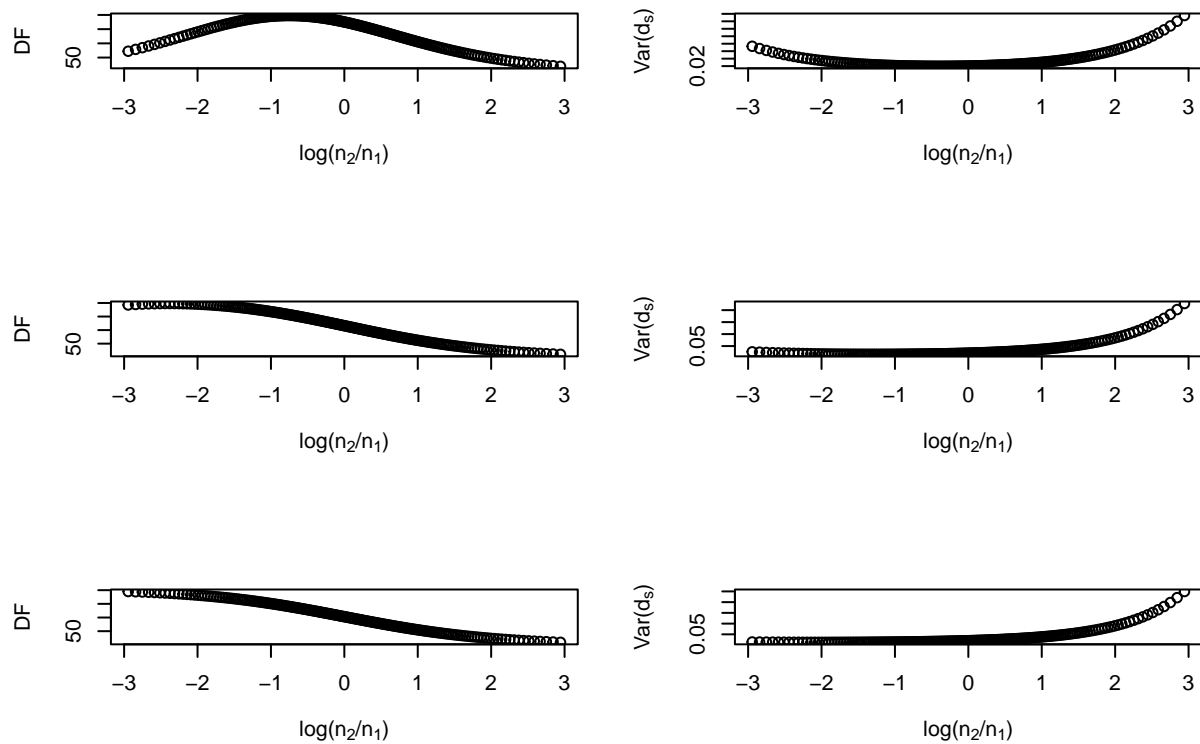
Figure 19. The variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_2}{n_1}\right))$, when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).
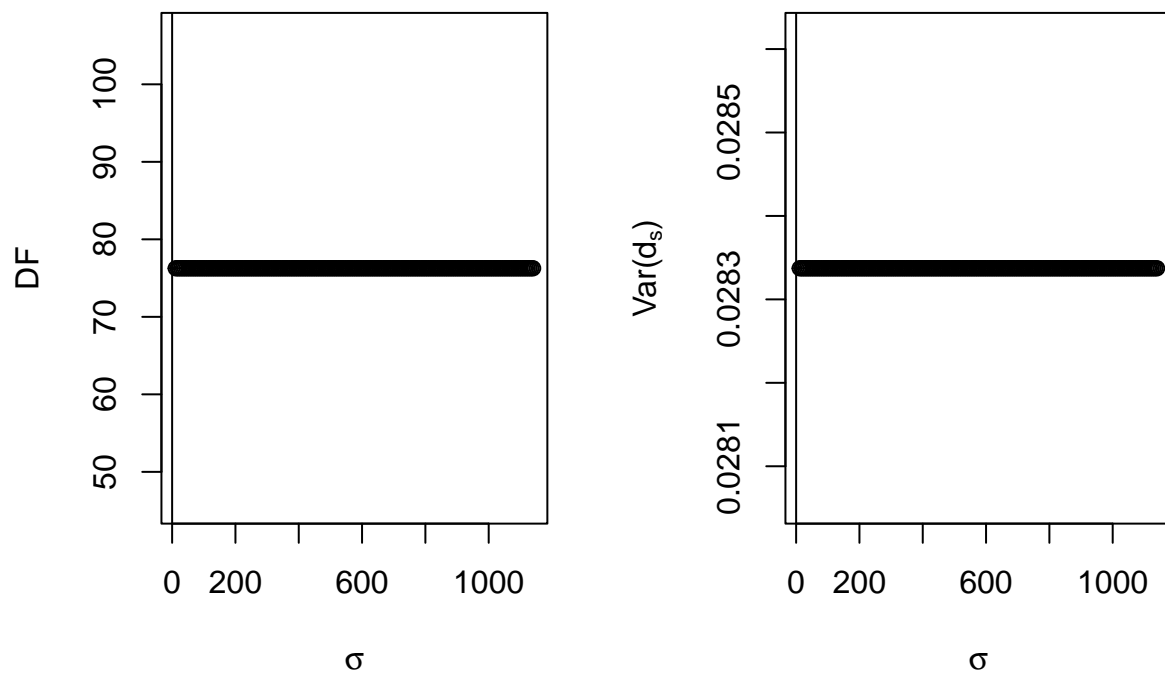
Figure 20. Variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

Figure 21. Variance of Shieh's d when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $\left(log\left(\frac{n_2}{n_1}\right)\right)$.

Figure 22. Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_2}{n_1}\right)\right)$.
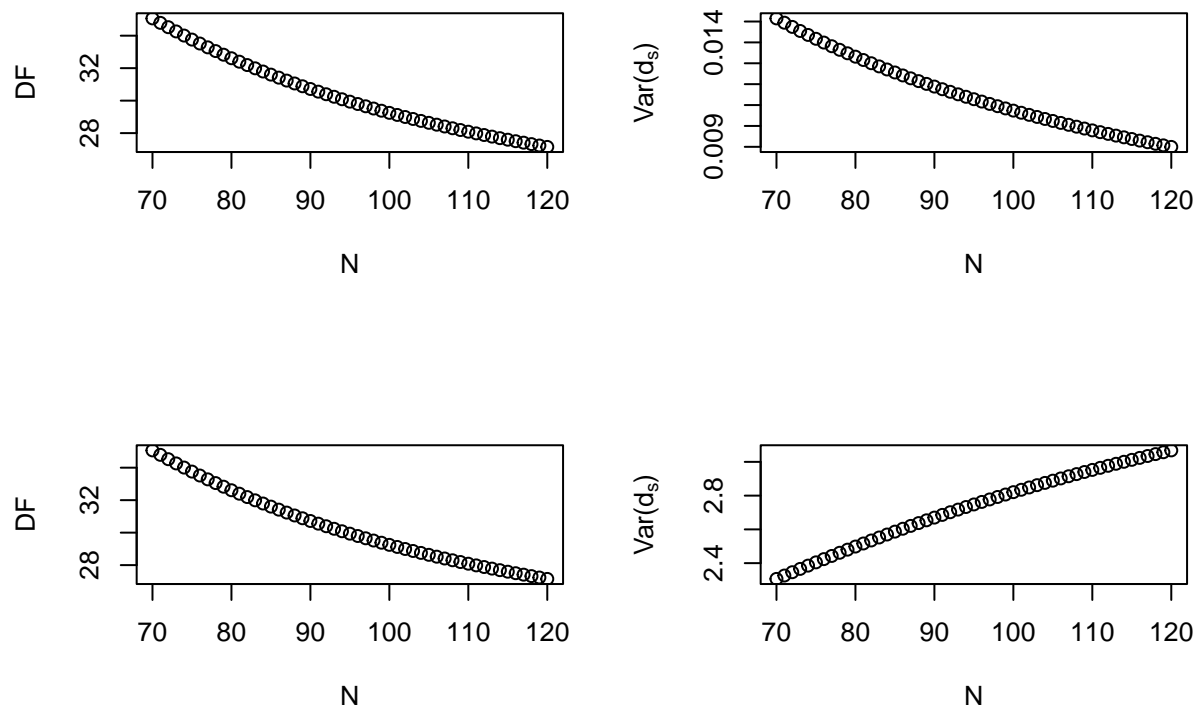
Figure 23. Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).
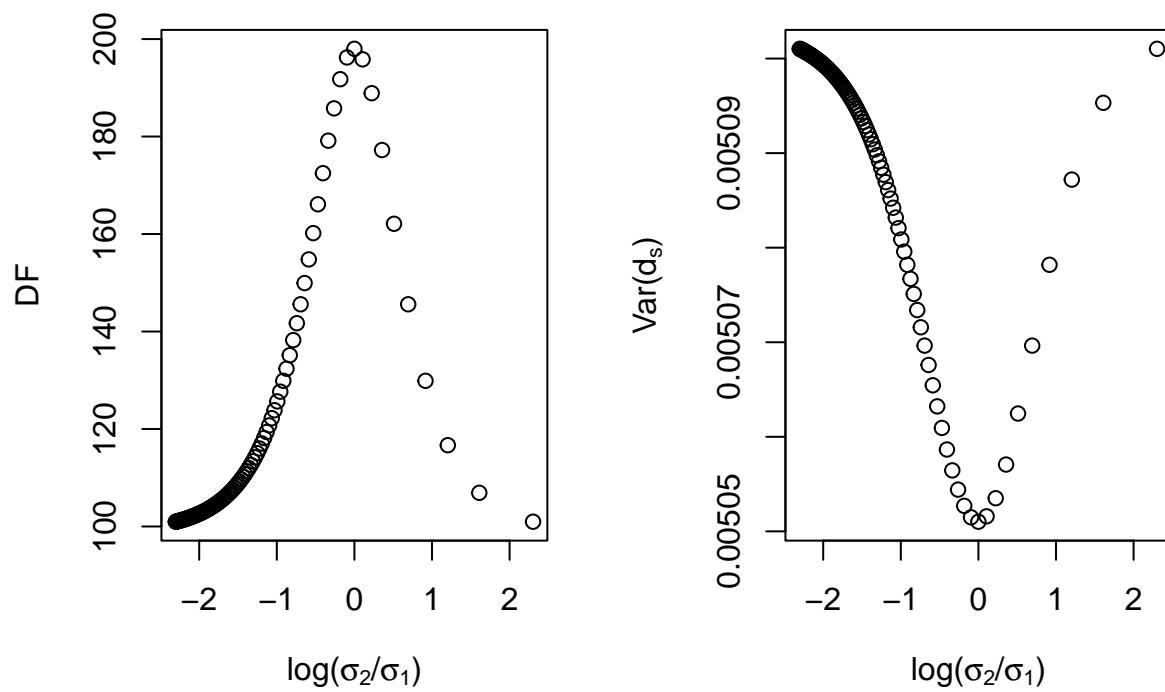
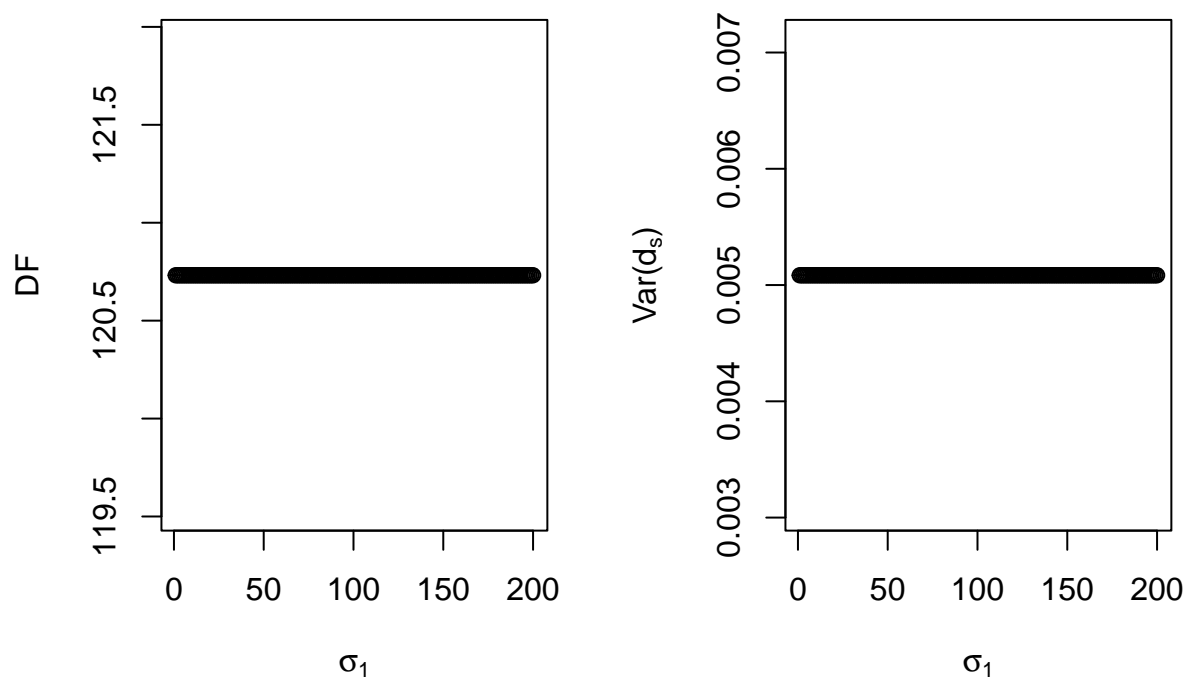*Figure 24*. Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio $\left(log\left(\frac{\sigma_2}{\sigma_1}\right)\right)$.
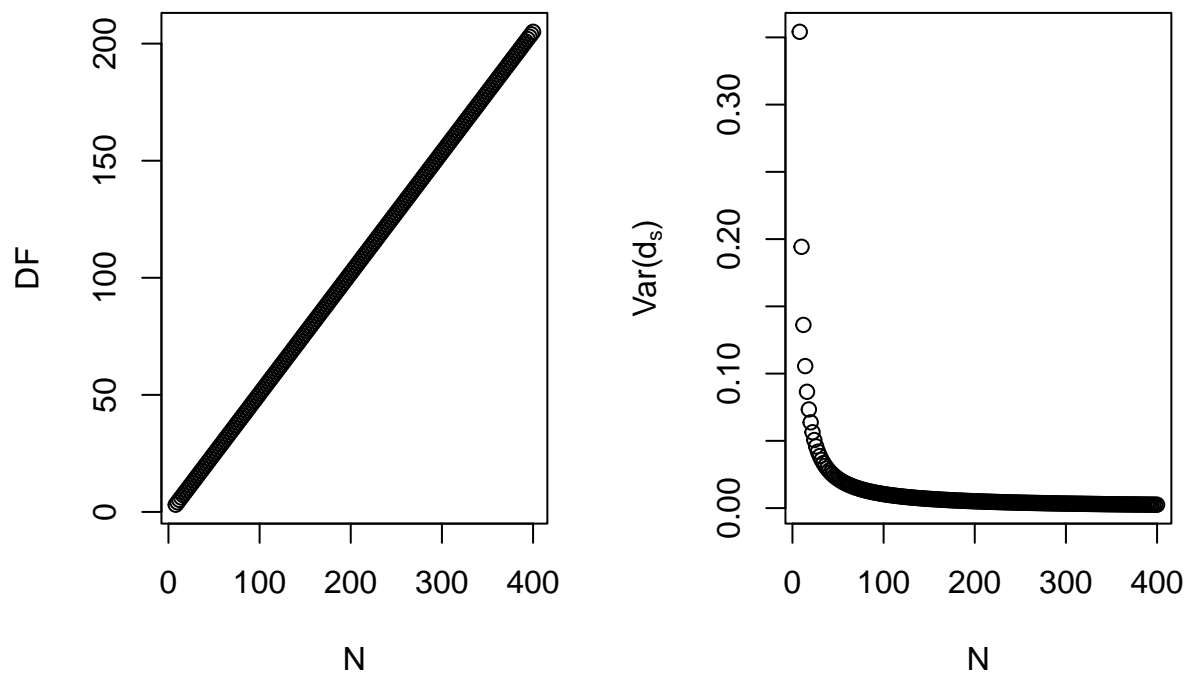
Figure 25. Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Figure 26. Variance of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.
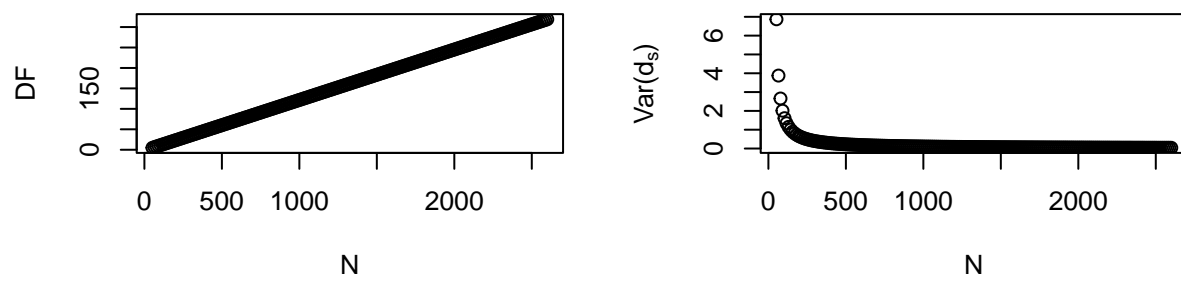
Figure 27. Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), for a constant sample sizes ratio $\left(log\left(\frac{n_2}{n_1}\right)\right)$.
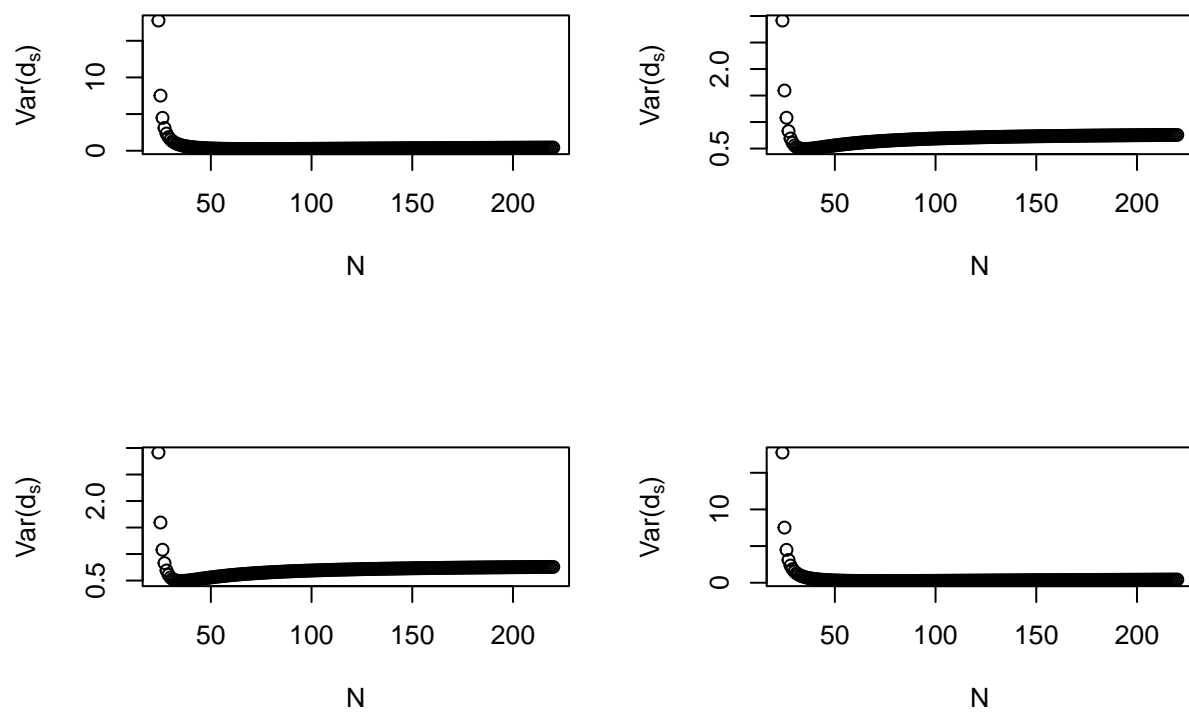
Figure 28. Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and $\sigma_1 > \sigma_2$ (top plots) or $\sigma_1 < \sigma_2$ (bottom plots).
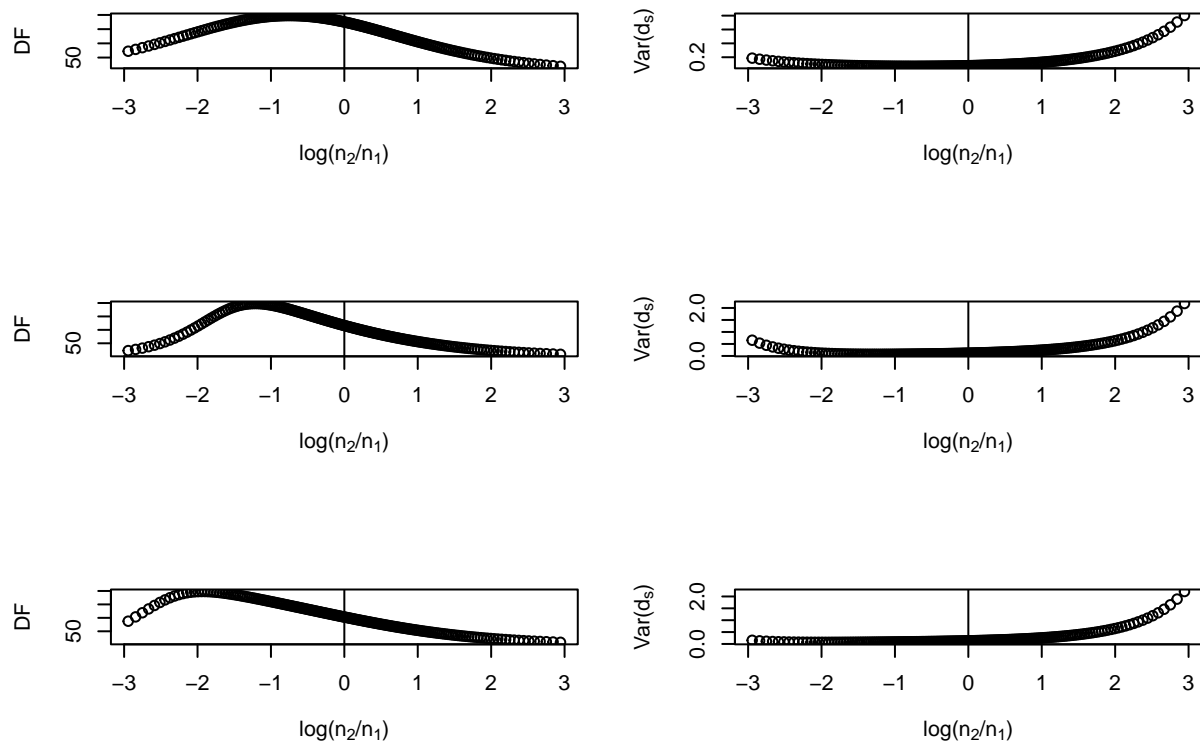
Figure 29. The variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio $(log\left(\frac{n_2}{n_1}\right))$, when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).
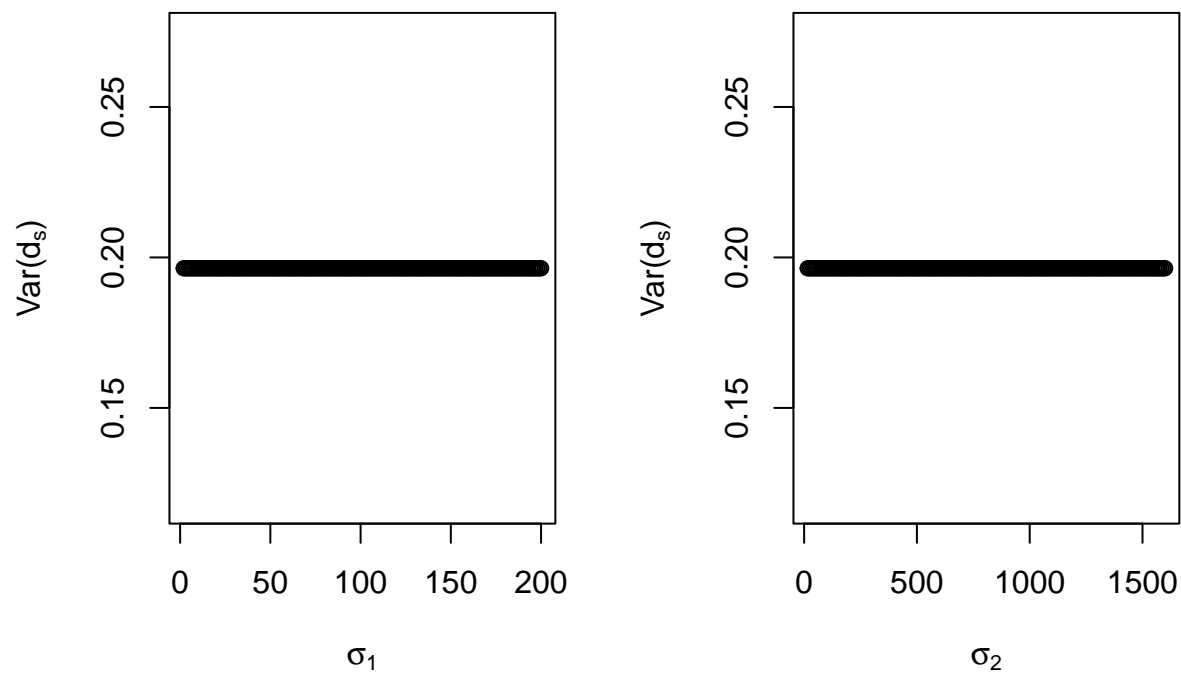
Figure 30. Variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of σ_1 (left) or σ_2 (right), for a constant SD-ratio.