1   Simulation checks

2   Delacre Marie[1]

3   [1] ULB

4 Simulation checks

5 **Supplemental Material 3**

6 In order to insure the reliability of our calculation method, for all scenarios where

7 $G_1 = G_2 = 0$, we compared empirical means and variances of all estimators (i.e. means and

8 variances of all estimates) with theoretical means and variances (i.e. expected means and

9 variances, computed based on equations in Tables 1, 2 and 3). Because we can draw exactly

10 the same conclusions for biased and unbiased estimators, we will simultaneously present

11 results for **biased** (Cohen's d_s, Glass's d_s using either sd_1 or sd_2 as standardizer, Shieh's d_s

12 and Cohen's d'_s) and **unbiased estimators** (Hedges' g_s, Glass's g_s using either sd_1 or sd_2

13 as standardizer, Shieh's g_s and Hedges' g'_s). Results will be subdivided into 4 conditions:

14 - When population variances and sample sizes are equal across groups (condition a; see

15 Figures A3.1 and A3.5 for respectively biased and unbiased estimators);

16 - When population variances are equal across groups and sample sizes are unequal

17 (condition b; see Figures A3.2 and A3.6 for respectively biased and unbiased estimators);

18 - When population variances are unequal across groups and sample sizes are equal

19 (condition c; see Figures A3.3 and A3.7 for respectively biased and unbiased estimators);

20 - When population variances and sample sizes are unequal across groups (condition d;

21 see Figures A3.4 and A3.8 for respectively biased and unbiased estimators).

22 Because the equations of theoretical means and variances of Cohen's d_s and Hedges' g_s

23 rely on the assumption of normality and equality of population variances, we expect

24 empirical and theoretical parameters to be very close only in conditions a and b. For all

25 other estimators, the equations of theoretical means and variances rely solely on the

26 assumption of normality and therefore, we expect empirical and theoretical parameters to be

27 very close in all conditions.

28 On average, empirical means (and variances) of all estimators are very close to

29 theoretical expectations, when population variances are equal across groups, with equal

30 sample sizes (condition a; see Table A3.1) or unequal sample sizes (condition b; see Table

31 A3.2).

32   When population variances are unequal across groups (conditions c and d; see Tables

33 A3.3 and A3.4), empirical means (and variances) of Cohen's d_s' (Cohen's g_s') and Shieh's d_s

34 (Shieh's g_s) are still very close to theoretical expectations. Regarding Glass's d_s (Glass's g_s),

35 while empirical variances remain very close to theoretical expectations, on average, one

36 observes a larger average departure between empirical and theoretical means when using

37 SD_2 as standardizer. However, when looking at simulations details (see

38 "biased_condC.xlsx", "biased_condD.xlsx", "unbiased_condC.xlsx" and

39 "unbiased_condD.xlsx" in Supplemental Material 3), one notices that the larger the

40 population effect size, the larger the departure between empirical and theoretical means, and

41 that relative to the population effect size, departures between empirical and theoretical

42 means are always very small. On the other side, both empirical bias and variance of Cohen's

43 d_s (Hedges' g_s) highly depart from theoretical expectations, even when looking and relative

44 departures to the population effect size, especially when sample sizes are unequal across

45 groups (condition d; see Table A3.4), which is not surprising, as Cohen's d_s (Hedges' g_s)

46 relies on the equality of population variances assumption.

Table A3.1

Absolute deviation between empirical and theoretical means

$|E(\hat{\delta})-\mu_{\delta}|$

Ratio between empirical and theoretical variances

$S_{\hat{\delta}}^2/\sigma_{\delta}^2$

Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Glass's $d_{s,1}$	0,022	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's $d_{s,2}$	0,023	0,000	0,005	0,007	1,005	0,889	0,966	0,035
Cohen's d'_s	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Shieh's d_s	0,006	0,000	0,001	0,002	1,006	0,910	0,976	0,028

Table A3.2

Estimator ($\hat{\delta}$)	Absolute deviation between empirical and theoretical means $\|E(\hat{\delta})-\mu_\delta\|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's $d_{s,1}$	0,019	0,000	0,004	0,006	1,006	0,891	0,966	0,037
Glass's $d_{s,2}$	0,027	0,000	0,005	0,007	1,015	0,881	0,968	0,036
Cohen's d'_s	0,010	0,000	0,003	0,002	1,007	0,902	0,965	0,034
Shieh's d_s	0,008	0,000	0,002	0,002	1,005	0,865	0,945	0,048

Table A3.3

Absolute deviation between empirical and theoretical means | Ratio between empirical and theoretical variances

$|E(\hat{\delta})-\mu_{\delta}|$ | $S_{\hat{\delta}}^2/\sigma_{\hat{\delta}}^2$

Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,080	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's $d_{s,1}$	0,037	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's $d_{s,2}$	0,230	0,000	0,012	0,033	1,008	0,883	0,974	0,032
Cohen's d'_s	0,036	0,000	0,003	0,006	1,007	0,874	0,975	0,033
Shieh's d_s	0,018	0,000	0,002	0,003	1,007	0,874	0,975	0,033

Table A3.4

Estimator ($\hat{\delta}$)	Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's $d_{s,1}$	0,026	0,000	0,005	0,006	1,009	0,881	0,972	0,033
Glass's $d_{s,2}$	0,219	0,000	0,012	0,031	1,011	0,872	0,973	0,036
Cohen's d'_s	0,030	0,000	0,003	0,006	1,011	0,860	0,974	0,034
Shieh's d_s	0,009	0,000	0,001	0,002	1,011	0,867	0,970	0,036

Table A3.5

Estimator ($\hat{\delta}$)	Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma^2_\delta$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,011	0,000	0,002	0,003	1,006	0,911	0,976	0,028
Glass's $d_{s,1}$	0,021	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's $d_{s,2}$	0,022	0,000	0,004	0,007	1,005	0,889	0,966	0,035
Cohen's d'_s	0,015	0,000	0,003	0,004	1,006	0,908	0,975	0,029
Shieh's d_s	0,008	0,000	0,002	0,002	1,006	0,908	0,975	0,029

Table A3.6

Estimator ($\widehat{\delta}$)	Absolute deviation between empirical and theoretical means $\lvert E(\widehat{\delta})-\mu_\delta\rvert$				Ratio between empirical and theoretical variances $S^2_{\widehat{\delta}}/\sigma_\delta$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's $d_{s,1}$	0,018	0,000	0,004	0,005	1,006	0,891	0,966	0,037
Glass's $d_{s,2}$	0,026	0,000	0,004	0,006	1,015	0,881	0,968	0,036
Cohen's d'_s	0,010	0,000	0,003	0,003	1,007	0,925	0,972	0,027
Shieh's d_s	0,007	0,000	0,002	0,002	1,007	0,900	0,959	0,037

Table A3.7

Estimator ($\hat{\delta}$)	Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_{\hat{\delta}}|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma^2_{\hat{\delta}}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,079	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's $d_{s,1}$	0,036	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's $d_{s,2}$	0,221	0,000	0,012	0,032	1,008	0,883	0,974	0,032
Cohen's d'_s	0,034	0,000	0,003	0,006	1,008	0,890	0,978	0,029
Shieh's d_s	0,017	0,000	0,002	0,003	1,008	0,890	0,978	0,029

Table A3.8

Estimator ($\hat{\delta}$)	Absolute deviation between empirical and theoretical means $|E(\hat{\delta})-\mu_\delta|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_\delta$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's d_s	0,250	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's $d_{s,1}$	0,025	0,000	0,004	0,006	1,009	0,881	0,972	0,033
Glass's $d_{s,2}$	0,210	0,000	0,012	0,030	1,011	0,872	0,973	0,036
Cohen's d'_s	0,029	0,000	0,003	0,005	1,011	0,882	0,977	0,029
Shieh's d_s	0,008	0,000	0,001	0,002	1,011	0,881	0,973	0,032