Table A1. *Summary of the data of the fictive case*

	A1	A2	A3
n_i	41.00	21.00	31.00
$\bar{X}$	24	23	27
s^2	81.75	10.075	38.40

Appendix 1: The Mathematical Development of the *F*-test, *W*-test, and *F**-test: Numerical Example

A summary is presented in Table A1. The complete example is available on Github. The DV is a score that can vary from 0 to 40. The IV is a three-level factor A (levels = A_1, A_2 and A_3).

The global mean (i.e. the mean of the global dataset) is a weighted mean of the group means:

$$\frac{(41*24)+(21*23)+(31*27)}{41+21+31} = \frac{2304}{93} \approx 24.77$$

The F-test statistic and degrees of freedom are computed by applying formulas (1), (2) and (3):

$$F = \frac{\frac{1}{3-1}[41*(24-\frac{2304}{93})^2+21*(23-\frac{2304}{93})^2+31*(27-\frac{2304}{93})^2]}{\frac{1}{93-3}[(41-1)*81.75+(21-1)*10.075+(31-1)*38.4]} \approx 2.377$$

$$df_n = 3-1 = 2$$

$$df_d = 93-3 = 90$$

The F*-test and his degrees of freedom are computed by applying formulas 4, 5 and 6:

$$F^* = \frac{41*(24-\frac{2304}{93})^2+21*(23-\frac{2304}{93})^2+31*(27-\frac{2304}{93})^2}{(1-\frac{41}{93})*81.75+(1-\frac{21}{93})*10.075+(1-\frac{31}{93})*38.4} \approx 3.088$$

$$df_n = 3-1 = 2$$

$$df_d = \frac{1}{\frac{(\frac{(1-\frac{41}{93})*81.75}{\sum_{j=1}^{k}(1-\frac{n_j}{N})s_j^2})^2}{41-1}+\frac{(\frac{(1-\frac{21}{93})*10.075}{\sum_{j=1}^{k}(1-\frac{n_j}{N})s_j^2})^2}{21-1}+\frac{(\frac{(1-\frac{31}{93})*38.4}{\sum_{j=1}^{k}(1-\frac{n_j}{N})s_j^2})^2}{31-1}} \approx 81.149$$

$$Where \sum_{j=1}^{k}(1-\frac{n_j}{N})*s_j^2 \approx 79.11$$

Finally, the W-test and his degrees of freedom are computed in applying formulas 7, 8 and 9:

$$W = \frac{\frac{1}{3-1}[\frac{41}{81.75}(24-\bar{X}')^2 + \frac{21}{10.075}(23-\bar{X}')^2 + \frac{31}{38.4}(27-\bar{X}')^2]}{\frac{2(3-2)}{3^2-1}[(\frac{1}{41-1})(1-\frac{\frac{41}{81.75}}{w})^2 + (\frac{1}{21-1})(1-\frac{\frac{21}{10.075}}{w})^2 + (\frac{1}{31-1})(1-\frac{\frac{31}{38.4}}{w})^2] + 1} \approx 4.606$$

Where:

$w = \sum_{j=1}^{k} w_j \approx 3.39$

$\bar{X}' = \frac{\sum_{j=1}^{k}(w_j \bar{x_j})}{w} \approx 24.1$

$$df_n = 3 - 1$$

$$df_d = \frac{3^2-1}{3[\frac{(1-\frac{w_j}{w})^2}{41-1} + \frac{(1-\frac{w_j}{w})^2}{21-1} + \frac{(1-\frac{w_j}{w})^2}{31-1}]} \approx 59.32$$

One should notice that in this example, the biggest sample size has the biggest variance. As previously mentioned, it means that the F-test will be too conservative, because the F value decreases. The F^*-test will also be a little too conservative, even if the test is less affected than the F-test. As a consequence: $W > F^* > F$.

Appendix 2: Justification for the choice of distributions in simulations

The set of simulations described in the article was repeated for 7 distributions. We used R commands to generate data from different distributions:

- k normal distributions (Figure A2.1): in order to assess the Type I error rate and power of the different tests under the assumption of normality, data were generated by means of the function "rnorm" (from the package "stats"; "R: The Normal Distribution," 2016).
- k double exponential distributions (Figure A2.2): In order to assess the impact of high kurtosis on the Type I error rate and power of all tests, data were generated by means of the function "rdoublex" (from the package "smoothmest"; "R: The double exponential (Laplace) distribution," 2012).
- k mixed normal distributions (Figure A2.3): In order to assess the impact of extremely high kurtosis on the Type I error rate and power of all tests, regardless of variance, data were generated by means of the function "rmixnorm" (from the package "bda"; Wang & Wang, 2015).
- k normal right skewed distributions (Figure A2.4): In order to assess the impact of moderate skewness on the Type I error rate and power, data were generated by means of the function "rsnorm" (from the package "fGarch"; "R: Skew Normal Distribution," 2017). The normal skewed distribution was chosen because it is the only skewed distribution where the standard deviation ratio can vary without having an impact on skewness.
- k-1 normal left skewed distributions (Figure A2.5) and 1 normal right skewed distribution (Figure A2.5): In order to assess the impact of unequal shapes, in terms of skewness, on the Type I error rate and power, when data have moderate skewness, data were generated by means of the functions "rsnorm" (from the package "fGarch"; "R: Skew Normal Distribution," 2017).
- k-1 chi-squared distributions with two degrees of freedom (See Figure A2.7), and one normal rigt skewed distribution (Figure A2.5): In order to assess the impact of high asymetry on the Type I error rate an power, k-1 distributions were generated by means of the functions "rchisq" ("R: The (non-central) Chi-squared Distribution," 2016). The last distribution was generated by means of "rsnorm" in order to follow a normal right skewed distribution with a mean of 2 (from the package "fGarch"; "R: Skew Normal Distribution," 2017).

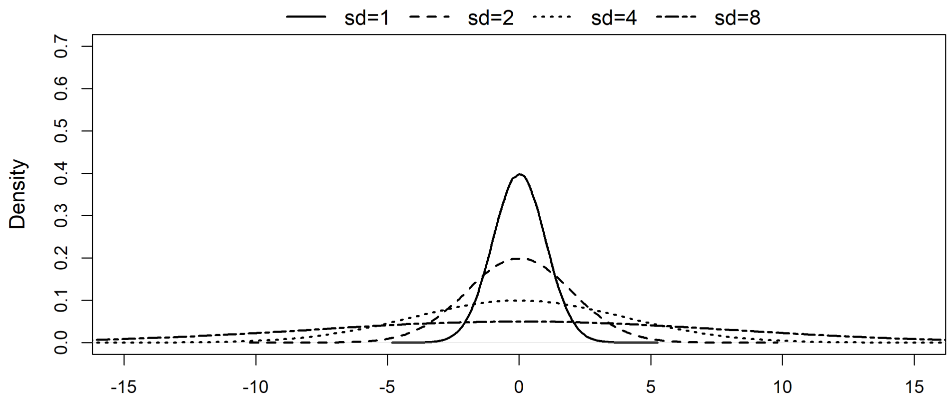

Figure 1: centered normal probability density function, as a function of the population SD

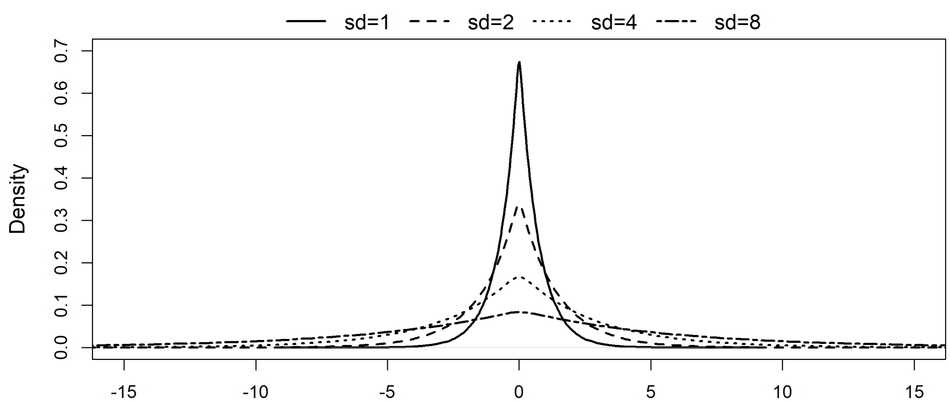

Figure 2: centered double exponential probability density function, as a function of the population SD

- *k*-1 chi-squared distributions with two degrees of freedom (See Figure A2.7), and one normal left skewed distribution (Figure A2.5): In order to assess the impact of unequal shapes, in terms of skewness, on Type I error rate and power when distributions have extreme skewness, k-1 distributions were generated by means of the functions “rchisq” (“R: The (non-central) Chi-squared Distribution,” 2016). The last distribution was generated by means of “rsnorm” in order to follow a normal right skewed distribution with a mean of 2 (from the package “fGarch”; “R: Skew Normal Distribution,” 2017)

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normal right #skewed probability density function, as a function of the population SD"}
#knitr::include_graphics("Rmarkdown folder/Rmarkdown inputs/") #
```

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#{r "", echo=FALSE, fig.width = 15,fig.height=8,out.width = '400px',fig.cap = "centered
normal left #skewed probability density function, as a function of the population SD"}
#knitr::include_graphics("Rmarkdown folder/Rmarkdown inputs/") #
```

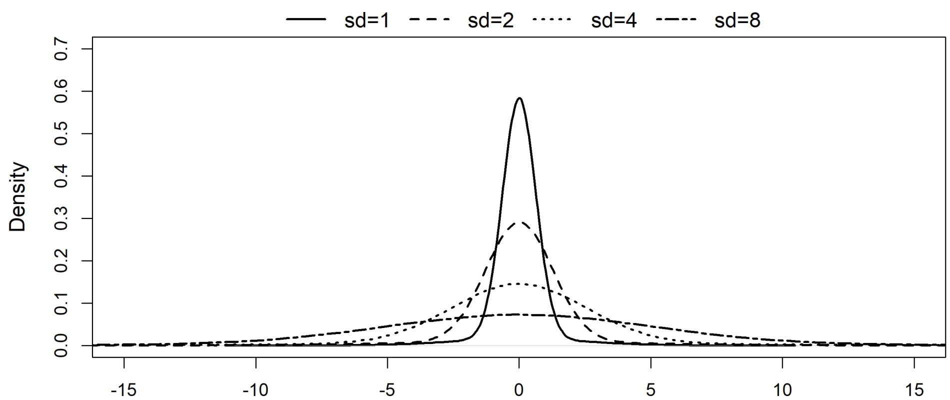

Figure 3: centered mixed normal probability density function, as a function of the population SD

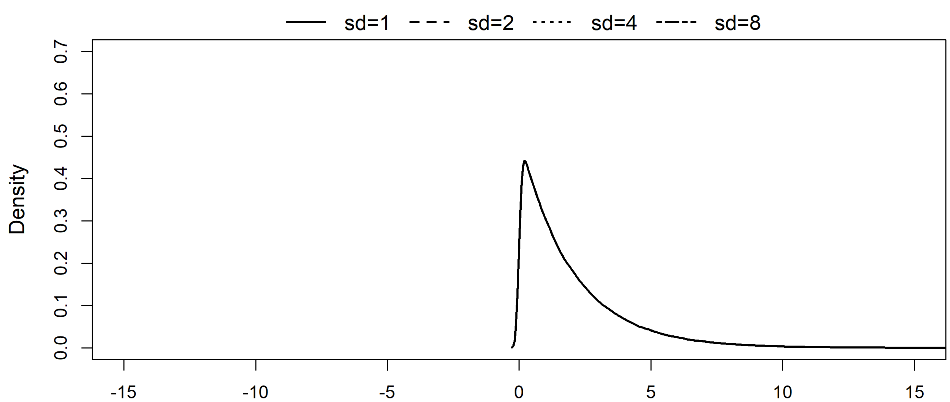

Figure 4: chi-squared with 2 degrees of freedom probability density function, as a function of the population SD