Additional file to

Why Psychologists Should by Default Use Welch's *t*-test Instead of fStudent's *t*-test.

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Supp Mat 1 :

Type I error rate

In order to estimate the Type I error rate for Student's *t*-test and Welch's *t*-test when sample sizes are equal across groups, we simulated 1,000,000 simulations of two samples generated under 20 conditions (yielding 20,000,000 simulations in total). In each condition, the first sample was generated from a population where $\sigma_1 = 2$ and sample sizes varied from 10 to 50 in a step of 10. The standard deviation of the second sample was a function of the SDR being respectively 0.01, 0.1, 10 and 100. The set of simulations was repeated nine times, varying the distribution underlying the data. We used R commands to generate data from different distributions.

- **Two normal distributions** (See Table A1.1): in order to assess the Type I error rate of both tests, under the normality assumption, data were generated by means of the function "rnorm" (from the package "stats"; "R: The Normal Distribution," 2016).
- **Two double exponential distributions** (See Table A1.2): in order to assess the impact of high kurtosis on the Type I error rate of the three tests, data were generated by means of the function "rdoublex" (from the package "smoothmest"; "R: The double exponential (Laplace) distribution," 2012). [1]
- **One normal and one double exponential distribution** (See Table A1.3): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type I error rate, data were generated by means of the functions "rdoublex" and "rnorm".
- **Two uniform distributions** (See Table A1.4): in order to assess the impact of low kurtosis on the Type I error rate of the three tests, data were generated by means of the function "runif" (from the package "stats"; "R: The Uniform Distribution," 2016).

[1] The dispersion parameter used in the "rdoublex" function is lambda, and $\lambda = \frac{\sigma}{2}$.

- **One uniform and one normal distribution** (See Table A1.5): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type I error rate, data were generated by means of the functions “runif” and “rnorm”.
- **One uniform and one double exponential distribution** (See Table A1.6): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type I error rate, data were generated by means of the functions “runif” and “rdoublex”.
- **Two normal skewed distributions with positive skewness of 0.79** (See Table A1.7): in order to assess the impact of skewness on the Type I error rate, data were generated by means of the function “rsnorm” (from the package “fGarch”; “R: Skew Normal Distribution,” 2017). The normal skewed distribution was used because it is the only skewed distribution where the standard deviation ratio can vary without having an impact on skewness.
- **One normal skewed distribution with negative skewness (-0.79) and one normal skewed distribution with positive skewness (+0.79**; See Table A1.8): in order to assess the impact of unequal shapes, in terms of kurtosis, on the Type I error rate, when data are asymmetric, data were generated by means of the functions “rsnorm” with unequal skewness.
- **One chi(2) and one normal skewed distribution with negative skewness of -0.79** (See Table A1.9): in order to assess the impact of high skewness and kurtosis on the Type I error rate, data were generated by means of the functions “rsnorm” with different skewness and from the function “rchisq”; “R: The (non-central) Chi-Squared Distribution,” 2016).

According to the definition of Bradley (1978), one consider that the alpha risk is “sufficiently close” to the nominal alpha risk if its value falls in the interval [0.025; 0.075] (Hayes & Cai, 2007). In the tables presented below, anytime the Type I error rate is out of this interval, we have added an “*” next to the value.

Simulations show that Student’s *t*-test is robust to violations of the assumption of equal variances when sample sizes are the same between groups, even for big SDRs, as long as there are at least 10 subjects per groups and the distributions are symmetric. However, according to Gleason (2013): *“the issue of being robust to departures from normality should not be confused with being the best statistic for the situation in question”* (p.10). Even when Student’s Type I error rate is not critical, Welch’s Type I error rate is often closer of the nominal alpha risk of 5%, especially with small sample sizes (when $n_1 = n_2 = 10$), except with heavy tailed distributions, where Welch’s *t*-test tends to be more conservative (but never below .025).

With moderately skewed distributions and big SDRs (tables A1.7 and A1.8), Student’s *t*-test becomes too liberal while Welch’s *t*-test remains closer to the nominal Type I error rate when there are 10 subjects per group. With at least 20 subjects per group, both have an acceptable Type I error rate, but Welch’s *t*-test remains closer to the nominal Type I error rate.

With high levels of skewness and kurtosis (table A1.9), both tests become too liberal, when there are fewer than 30 subjects per group. With 30 subjects per group, Welch’s *t*-test have an acceptable Type I error rate while Student’s *t*-test still have a Type I error rate larger than 0.075 in some scenarios. In all cases, Welch’s *t*-test remains closer to the nominal Type I error rate.

Power

In order to estimate the power for Student's *t*-test and Welch's *t*-test when sample sizes are equal across groups, we simulated 1,000,000 simulations of two samples generated under 20 conditions (yielding 20,000,000 simulations in total). In each condition, the first sample is generated from a population where $\sigma_1 = 2$ and sample sizes vary from 10 to 50 in a step of 10. The standard deviation of the second sample is a function of the SDRs being respectively 0.01, 0.1, 10 and 100. We included a mean difference δ = 1 (giving related Cohen's effect sizes that varied from 0.29 to 0.71 which is realistic in the field of psychology). In the first step, the *p*-values of the two tests were extracted for each pair of samples and from each test, and in a second step, the percentage of *p*-value under the nominal alpha risk (5%) was computed for each test, giving the power.

In order to insure the reliability of our calculation method, we firstly used R commands to generate data from two normal distributions (See Table A1.1). Because computed power is very consistent with theoretical power, one can conclude that the method is reliable. The set of simulations was repeated nine times, varying the distribution underlying the data. We used R commands to generate data from different distributions.

- **Two normal distributions** (See Table A1.1)
- **Two double exponential distributions** (See Table A1.2)
- **One normal and one double exponential distribution** (See Table A1.3)
- **Two uniform distributions** (See Table A1.4)
- **One uniform and one normal distribution** (See Table A1.5)
- **One uniform and one double exponential distribution** (See Table A1.6)
- **Two normal skewed distributions with positive skewness of 0.79** (See Table A1.7)

- **One normal skewed distribution with negative skewness (-0.79) and one normal skewed distribution with positive skewness (+0.79**; See Table A1.8)
- **One chi(2) and one normal skewed distribution with negative skewness of -0.79** (See Table A1.9)

Simulations show that when sample sizes are equal between groups, Student's power is better than Welch's power, but very slightly. Even with extremely big SDRs (respectively 0.01, 0.1, 10 and 100) and sample sizes (10 subjects per group), the biggest advantage of Student's *t*-test over Welch's-*t* test is 5.37%, when the test is applied on two normal skewed distributions with unequal shapes. In all other cases, the difference between both tests is smaller (See Table A1.1 to A1.9).

Conclusion

Even with equal sample sizes, Welch's *t*-test is more robust than Student's *t*-test in terms of alpha risk, except with heavy tailed distributions. On the other hand, the Student's gain in power is very weak. In conclusion, we advise to always use Welch's *t*-test, instead of Student's *t*-test.

Table A1.1 : two normal distributions

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		**POWER (%)**	
$n_1 = n_2$	**SDR**	**5%**	**5%**		
10	0.01	0.050	0.065	29.34	34.07
	0.1	0.050	0.064	29.23	33.68
	10	0.051	0.065	5.25	6.73
	100	0.050	0.065	4.97	6.49
20	0.01	0.050	0.057	56.40	58.95
	0.1	0.050	0.057	56.15	58.57
	10	0.050	0.057	5.50	6.22
	100	0.050	0.057	5.00	5.72
30	0.01	0.050	0.055	75.42	76.77
	0.1	0.051	0.055	75.06	76.36
	10	0.050	0.055	5.86	6.35
	100	0.050	0.055	5.07	5.56
40	0.01	0.050	0.054	86.92	87.59
	0.1	0.050	0.053	86.66	87.31
	10	0.050	0.053	6.10	6.48
	100	0.050	0.053	5.03	5.38
50	0.01	0.050	0.053	93.40	93.71
	0.1	0.050	0.053	93.23	93.53
	10	0.050	0.053	6.34	6.66
	100	0.050	0.053	5.01	5.28

Table A1.2 : two double exponential distributions

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.042	0.057	35.41	40.11
	0.1	0.042	0.056	35.07	39.51
	10	0.042	0.057	4.58	6.06
	100	0.042	0.057	4.20	5.73
20	0.01	0.046	0.054	35.41	40.11
	0.1	0.047	0.053	35.07	39.51
	10	0.047	0.054	4.58	6.06
	100	0.046	0.054	4.20	5.73
30	0.01	0.047	0.052	35.41	40.11
	0.1	0.048	0.052	35.07	39.51
	10	0.047	0.052	4.58	6.06
	100	0.048	0.053	4.20	5.73
40	0.01	0.049	0.052	35.41	40.11
	0.1	0.048	0.052	35.07	39.51
	10	0.049	0.052	4.58	6.06
	100	0.049	0.053	4.20	5.73
50	0.01	0.049	0.051	4.86	5.14
	0.1	0.049	0.052	4.91	5.19
	10	0.049	0.051	4.85	5.13
	100	0.049	0.052	4.87	5.16

Table A1.3 : one normal and one double exponential distribution

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.042	0.057	29.35	34.06
	0.1	0.042	0.056	29.23	33.74
	10	0.042	0.057	4.58	6.05
	100	0.042	0.057	4.18	5.71
20	0.01	0.046	0.054	29.35	34.06
	0.1	0.047	0.053	29.23	33.74
	10	0.047	0.054	4.58	6.05
	100	0.046	0.054	4.18	5.71
30	0.01	0.047	0.052	29.35	34.06
	0.1	0.048	0.052	29.23	33.74
	10	0.047	0.052	4.58	6.05
	100	0.048	0.053	4.18	5.71
40	0.01	0.049	0.052	29.35	34.06
	0.1	0.048	0.052	29.23	33.74
	10	0.049	0.052	4.58	6.05
	100	0.049	0.053	4.18	5.71
50	0.01	0.049	0.051	4.86	5.14
	0.1	0.049	0.052	4.91	5.19
	10	0.049	0.051	4.85	5.13
	100	0.049	0.052	4.87	5.16

Table A1.4 : two uniform distributions

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.054	0.068	26.09	30.77
	0.1	0.054	0.068	25.96	30.45
	10	0.054	0.068	5.60	6.96
	100	0.055	0.069	5.39	6.79
20	0.01	0.051	0.058	54.78	57.45
	0.1	0.052	0.058	54.47	57.05
	10	0.052	0.059	5.61	6.33
	100	0.051	0.058	5.14	5.85
30	0.01	0.051	0.055	75.09	76.48
	0.1	0.051	0.055	74.71	76.04
	10	0.051	0.055	5.84	6.32
	100	0.051	0.055	5.08	5.55
40	0.01	0.051	0.054	87.14	87.83
	0.1	0.051	0.054	86.88	87.54
	10	0.051	0.054	6.11	6.49
	100	0.051	0.054	5.09	5.44
50	0.01	0.051	0.053	93.72	94.04
	0.1	0.051	0.053	93.50	93.80
	10	0.050	0.053	6.37	6.68
	100	0.050	0.053	5.06	5.34

Table A1.5 : one uniform and one normal distribution

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.054	0.068	26.00	30.67
	0.1	0.054	0.068	25.96	30.46
	10	0.050	0.064	5.26	6.71
	100	0.050	0.065	4.99	6.49
20	0.01	0.052	0.059	54.79	57.50
	0.1	0.052	0.058	54.50	57.09
	10	0.050	0.057	5.53	6.26
	100	0.050	0.057	5.01	5.73
30	0.01	0.051	0.056	75.01	76.40
	0.1	0.051	0.055	74.71	76.05
	10	0.050	0.055	5.82	6.32
	100	0.050	0.055	5.01	5.50
40	0.01	0.051	0.054	87.12	87.81
	0.1	0.051	0.054	86.87	87.54
	10	0.050	0.053	6.10	6.49
	100	0.050	0.053	5.06	5.42
50	0.01	0.050	0.053	93.73	94.04
	0.1	0.051	0.053	93.46	93.77
	10	0.050	0.053	6.38	6.70
	100	0.050	0.052	4.97	5.25

Table A1.6 : one uniform and one double exponential distribution

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.055	0.069	26.09	30.77
	0.1	0.054	0.068	25.97	30.43
	10	0.042	0.056	4.57	6.02
	100	0.042	0.057	4.22	5.74
20	0.01	0.051	0.059	26.09	30.77
	0.1	0.051	0.058	25.97	30.43
	10	0.046	0.053	4.57	6.02
	100	0.046	0.054	4.22	5.74
30	0.01	0.050	0.055	26.09	30.77
	0.1	0.051	0.055	25.97	30.43
	10	0.048	0.052	4.57	6.02
	100	0.048	0.053	4.22	5.74
40	0.01	0.051	0.054	26.09	30.77
	0.1	0.051	0.054	25.97	30.43
	10	0.048	0.052	4.57	6.02
	100	0.048	0.052	4.22	5.74
50	0.01	0.050	0.053	26.09	30.77
	0.1	0.051	0.053	25.97	30.43
	10	0.049	0.052	4.57	6.02
	100	0.048	0.051	4.22	5.74

Table A1.7 : two normal skewed distributions with positive skewness

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.061	0.076*	34.03	38.07
	0.1	0.062	0.075	33.89	37.68
	10	0.061	0.075	5.47	6.82
	100	0.062	0.076*	6.05	7.54
20	0.01	0.056	0.063	56.61	58.78
	0.1	0.056	0.063	56.25	58.38
	10	0.056	0.063	5.12	5.82
	100	0.057	0.064	5.57	6.28
30	0.01	0.054	0.059	73.24	74.47
	0.1	0.054	0.058	72.89	74.08
	10	0.054	0.059	5.17	5.64
	100	0.054	0.059	5.36	5.84
40	0.01	0.053	0.056	84.31	84.99
	0.1	0.053	0.057	84.05	84.72
	10	0.053	0.057	5.36	5.74
	100	0.053	0.057	5.23	5.58
50	0.01	0.053	0.056	91.14	91.51
	0.1	0.053	0.056	90.96	91.33
	10	0.053	0.055	5.58	5.88
	100	0.053	0.056	5.21	5.49

Table A1.8 : two normal skewed distributions with opposite skewness

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		**POWER (%)**	
$n_1 = n_2$	**SDR**	**5%**	**5%**		
10	0.01	0.062	0.076*	23.24	28.37
	0.1	0.062	0.076*	5.44	6.80
	10	0.061	0.075	6.08	7.56
	100	0.062	0.077*	56.44	59.47
20	0.01	0.057	0.064	55.98	58.91
	0.1	0.057	0.063	5.12	5.81
	10	0.056	0.062	5.56	6.27
	100	0.057	0.064	78.58	79.98
30	0.01	0.055	0.059	78.05	79.43
	0.1	0.054	0.059	5.18	5.66
	10	0.054	0.059	5.35	5.83
	100	0.055	0.059	90.42	91.01
40	0.01	0.053	0.057	89.98	90.56
	0.1	0.053	0.057	5.34	5.71
	10	0.053	0.057	5.26	5.61
	100	0.053	0.057	95.91	96.14
50	0.01	0.053	0.055	95.71	95.95
	0.1	0.053	0.056	5.56	5.86
	10	0.053	0.056	5.20	5.48
	100	0.052	0.055	23.23	28.60

Table A1.9 : one chi-square distribution with 2 degrees of freedom, and one normal skewed distribution with negative skewness

		Welch's *t*-test	Student's *t*-test	Welch's *t*-test	Student's *t*-test
		NOMINAL SIZE OF ALPHA RISK		POWER (%)	
$n_1 = n_2$	SDR	5%	5%		
10	0.01	0.099*	0.114*	42.76	46.01
	0.1	0.099*	0.112*	42.71	45.81
	10	0.061	0.075	7.23	8.70
	100	0.062	0.076*	6.21	7.69
20	0.01	0.081*	0.088*	59.37	61.08
	0.1	0.080*	0.087*	59.08	60.73
	10	0.057	0.064	7.12	7.87
	100	0.057	0.064	5.73	6.45
30	0.01	0.073	0.078*	72.15	73.20
	0.1	0.072	0.076*	71.91	72.92
	10	0.055	0.059	7.25	7.75
	100	0.054	0.059	5.53	6.00
40	0.01	0.068	0.071	81.59	82.24
	0.1	0.068	0.071	81.25	81.89
	10	0.053	0.057	7.44	7.83
	100	0.054	0.057	5.45	5.80
50	0.01	0.065	0.068	88.13	88.51
	0.1	0.065	0.067	87.96	88.35
	10	0.053	0.055	7.66	7.98
	100	0.053	0.056	5.44	5.72

Supp Mat 2 : Standard deviations in samples and in populations

To check if the sample SD (=S) shows the same pattern as the population SD ($= \sigma$), we simulated 1,000,000 sets of samples for each sample size from 10 to 85 under 5 population SDs: 1.84, 0.92, 1.11, 1.32 and 1.63. Because the sampling distribution of the SD was quite close to a normal distribution, the mean and SD are good estimators of the quality of the estimation of σ by S (see Table A2).

Table A2

σ:	**0.92**		**1.11**		**1.32**		**1.63**		**1.84**	
N	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
10	0.895	0.214	1.080	0.258	1.284	0.307	1.586	0.378	1.79	0.428
15	0.904	0.172	1.090	0.208	1.297	0.247	1.601	0.305	1.807	0.345
20	0.908	0.148	1.095	0.179	1.303	0.212	1.609	0.263	1.816	0.296
25	0.91	0.132	1.098	0.159	1.306	0.19	1.613	0.234	1.821	0.264
30	0.912	0.12	1.101	0.145	1.309	0.173	1.616	0.213	1.824	0.241
35	0.913	0.111	1.102	0.134	1.31	0.159	1.618	0.197	1.826	0.222
40	0.914	0.104	1.103	0.125	1.312	0.149	1.619	0.184	1.829	0.208
45	0.915	0.098	1.104	0.118	1.312	0.14	1.621	0.173	1.83	0.195
50	0.915	0.093	1.104	0.112	1.313	0.133	1.622	0.164	1.831	0.185
55	0.916	0.088	1.105	0.107	1.314	0.127	1.622	0.157	1.831	0.177
60	0.916	0.084	1.105	0.102	1.314	0.121	1.623	0.15	1.832	0.169
65	0.916	0.081	1.106	0.098	1.315	0.116	1.623	0.144	1.833	0.162
70	0.917	0.078	1.106	0.094	1.315	0.112	1.624	0.138	1.833	0.156
75	0.917	0.076	1.106	0.091	1.316	0.108	1.624	0.134	1.834	0.151
80	0.917	0.073	1.106	0.088	1.316	0.105	1.625	0.13	1.834	0.146
85	0.917	0.071	1.107	0.086	1.316	0.102	1.625	0.126	1.834	0.142

The smaller the sample size is, the further the average standard deviation is from the population standard deviation, and the bigger the dispersion around this average.

Supp Matt 3 : Type I error rate of Student's *t*-test and Welch's *t*-test when variances are unequal

Assuming a Type I error rate of 5% under the null, a test can yield either a significant result (*p*-value < 5%; or a "false positive" -FP) or a non-significant result (*p*-value > 5%; or a "true negative"-TN). The specificity is the relative frequency of effects detected as non-significant, under the null:

$$\text{Specificity} = \frac{TN}{TN+FP} = \frac{\#\ p\text{−}value|H_0 > .05)}{[\#\ p\text{−}value|H_0 > .05)] + [\#\ p\text{−}value|H_0 < .05)]}$$

The alpha risk is the complement of the specificity:

$$\frac{FP}{TN+FP} = \frac{\#\ p\text{−}value|H_0 < .05)}{[\#\ p\text{−}value|H_0 > .05)] + [\#\ p\text{−}value|H_0 < .05)]} = 1 - specificity$$

In order to estimate the Type I error rate for Student's *t*-test, Welch's *t*-test (and Yuen's *t*-test when population distributions are symmetric)[2], we simulated 1,000,000 simulations of two samples generated under 64 different conditions (yielding 64,000,000 simulations in total). In each condition, the first sample is generated from a population where σ_1=2, and its sample size varies from 10 to 40 in steps of 10. The standard deviation and the sample size of the second sample is a function of the sample sizes ratio (SSR = $\frac{n_2}{n_1}$; ranging from 0.5 to 2 in steps of 0.5) and the SDR (ranging from 0.5 to 2 in steps of .0.5). In a first step, the *p*-values of the tests were extracted for each pair of samples and for each test, and in a second step, the percent of *p*-values under the nominal alpha risk (5%) was computed for each test.

[2] The null hypothesis of Yuen's *t*-test is that the trimmed means are equal across groups. When populations are symmetric, means = trimmed means. On the other hand, when distributions are asymmetric, means ≠ trimmed means, and our simulation method is therefore not appropriate in order to assess the type I error rate of Yuen's *t*-test, because we always simulated samples extracted from populations with equal means.

The set of simulations was repeated nine times, varying the distributions underlying the data. We used R commands to generate data from different distributions.

- **Two normal distributions** (See Table A3.1): in order to assess the Type I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test under the normality assumption, data were generated by means of the function "rnorm" (from the package "stats"; "R: The Normal Distribution," 2016).
- **Two double exponential distributions** (See Table A3.2): in order to assess the impact of high kurtosis on the Type I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test, data were generated by means of the function "rdoublex" (from the package "smoothmest"; "R: The double exponential (Laplace) distribution," 2012).
- **One normal and one double exponential distribution** (See Table A3.3): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test, data were generated by means of the functions "rdoublex" and "rnorm".
- **Two uniform distributions** (See Table A3.4): in order to assess the impact of low kurtosis on the Type I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test, data were generated by means of the function "runif" (from the package "stats"; "R: The Uniform Distribution," 2016).
- **One uniform and one normal distribution** (See Table A3.5): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test, data were generated by means of the functions "runif" and "rnorm".
- **One uniform and one double exponential distribution** (See Table A3.6): in order to assess the impact of the unequal shape of distributions, in terms of kurtosis, on the Type

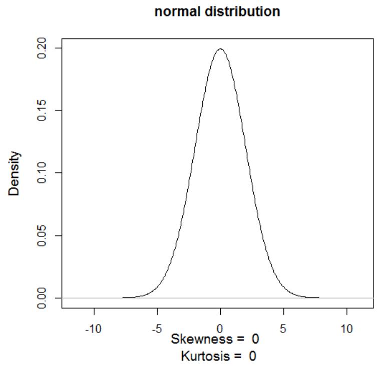
I error rate of Student's *t*-test, Welch's *t*-test and Yuen's *t*-test, data were generated by means of the functions "runif" and "rdoublex".

- **Two normal skewed distributions with positive skewness of 0.79** (See Table A3.7): in order to assess the impact of skewness on the Type I error rate of Student's *t*-test and Welch's *t*-test, data were generated by means of the function "rsnorm" (from the package "fGarch"; "R: Skew Normal Distribution," 2017). The normal skewed was used because it is the only skewed distribution where the standard deviation ratio can vary without having an impact on skewness.
- **One normal skewed distribution with negative skewness (-0.79) and one normal skewed distribution with positive skewness (+0.79**; See Table A3.8): in order to assess the impact of unequal shapes, in terms of kurtosis, on the Type I error rate of Student's *t*-test and Welch's *t*-test, when data are asymmetric, data were generated by means of the functions "rsnorm" with unequal skewness.
- **One chi(2) and one normal skewed distribution with negative skewness of -0.79** (See Table A3.9): in order to assess the impact of high skewness and kurtosis on the Type I error rate of Student's *t*-test and Welch's *t*-test, data were generated by means of the functions "rsnorm" with different skewness and from the function "rchisq"; "R: The (non-central) Chi-Squared Distribution," 2016).

Consistently with research conducted by Minitab statisticians available at http://support.minitab.com/en-us/minitab/17/Assistant_Two_Sample_t.pdf, simulations show that the Type I error rate for Student's *t*-test can differ noticeably from the nominal Type I error rate (i.e., 5%) when the groups have different variances. In light of the definition of Bradley (1978), one can consider the alpha risk "sufficiently close" to the nominal alpha risk if its value falls in the interval [0.025; 0.075] (Hayes & Cai, 2007). In the tables presented below, anytime the Type I error rate falls outside of this interval, we have added an "*" next to the value. Considering this norm, one can observe that when there is a positive correlation between sample size and standard deviation, the test is often too conservative and when there is a negative correlation between sample size and standard deviation, the test is often too liberal.

On the other hand, the Type I error rate of Welch's *t*-test remains closer of the nominal size (i.e. 5%) in all the previously contemplated cases. Yuen's *t*-test is not a good unconditional alternative, as we observe an unacceptable departure from the nominal alpha risk of 5% in several cases (See Table A3.1, A3.4). Moreover, even when Yuen's Type I error rate does not show a critical departure from the nominal alpha risk, Welch's *t*-test is better at controlling the Type I error rate at the desired alpha level (See Table A3.2, A3.3, A3.5 and A3.6).

Table A3.1

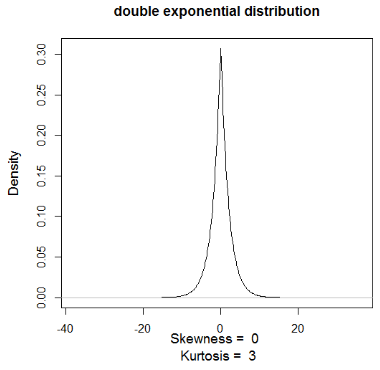


						Type I error rate		
n_2/n_1	**SDR**	**n_1**	**n_2**	σ_1	σ_2	**Welch's *t*-test**	**Student's *t*-test**	**Yuen's *t*-test**
0.5	0.5	10	5	2	1	0.047	0.021*	0.048
0.5	1	10	5	2	2	0.050	0.050	0.058
0.5	1.5	10	5	2	3	0.054	0.086*	0.071
0.5	2	10	5	2	4	0.056	0.117*	0.077*
1	0.5	10	10	2	1	0.050	0.055	0.055
1	1	10	10	2	2	0.049	0.050	0.050
1	1.5	10	10	2	3	0.049	0.052	0.052
1	2	10	10	2	4	0.050	0.054	0.055
1.5	0.5	10	15	2	1	0.051	0.087*	0.059
1.5	1	10	15	2	2	0.050	0.050	0.052
1.5	1.5	10	15	2	3	0.049	0.036	0.050
1.5	2	10	15	2	4	0.049	0.030	0.051
2	0.5	10	20	2	1	0.051	0.114*	0.061
2	1	10	20	2	2	0.050	0.050	0.054
2	1.5	10	20	2	3	0.050	0.027	0.051
2	2	10	20	2	4	0.050	0.019*	0.051

0.5	0.5	20	10	2	1	0.049	0.019*	0.050
0.5	1	20	10	2	2	0.050	0.050	0.054
0.5	1.5	20	10	2	3	0.051	0.086*	0.059
0.5	2	20	10	2	4	0.051	0.114*	0.061
1	0.5	20	20	2	1	0.050	0.053	0.053
1	1	20	20	2	2	0.049	0.050	0.051
1	1.5	20	20	2	3	0.050	0.051	0.052
1	2	20	20	2	4	0.050	0.052	0.053
1.5	0.5	20	30	2	1	0.050	0.084*	0.054
1.5	1	20	30	2	2	0.050	0.050	0.051
1.5	1.5	20	30	2	3	0.050	0.035	0.051
1.5	2	20	30	2	4	0.050	0.029	0.051
2	0.5	20	40	2	1	0.050	0.112*	0.055
2	1	20	40	2	2	0.050	0.050	0.052
2	1.5	20	40	2	3	0.050	0.027	0.051
2	2	20	40	2	4	0.050	0.018*	0.050
0.5	0.5	30	15	2	1	0.049	0.018*	0.051
0.5	1	30	15	2	2	0.050	0.050	0.053
0.5	1.5	30	15	2	3	0.050	0.085*	0.055
0.5	2	30	15	2	4	0.050	0.113*	0.056
1	0.5	30	30	2	1	0.050	0.052	0.052
1	1	30	30	2	2	0.050	0.050	0.051
1	1.5	30	30	2	3	0.050	0.050	0.051
1	2	30	30	2	4	0.050	0.051	0.051

1.5	0.5	30	45	2	1	0.050	0.083*	0.052
1.5	1	30	45	2	2	0.050	0.050	0.051
1.5	1.5	30	45	2	3	0.050	0.035	0.051
1.5	2	30	45	2	4	0.050	0.028	0.051
2	0.5	30	60	2	1	0.050	0.111*	0.052
2	1	30	60	2	2	0.050	0.050	0.051
2	1.5	30	60	2	3	0.050	0.026	0.051
2	2	30	60	2	4	0.050	0.017*	0.050
0.5	0.5	40	20	2	1	0.050	0.018*	0.051
0.5	1	40	20	2	2	0.050	0.050	0.052
0.5	1.5	40	20	2	3	0.051	0.086*	0.054
0.5	2	40	20	2	4	0.051	0.111*	0.054
1	0.5	40	40	2	1	0.050	0.052	0.051
1	1	40	40	2	2	0.050	0.050	0.051
1	1.5	40	40	2	3	0.050	0.050	0.051
1	2	40	40	2	4	0.050	0.051	0.051
1.5	0.5	40	60	2	1	0.050	0.083*	0.052
1.5	1	40	60	2	2	0.050	0.050	0.051
1.5	1.5	40	60	2	3	0.050	0.034	0.051
1.5	2	40	60	2	4	0.050	0.028	0.051
2	0.5	40	80	2	1	0.050	0.110*	0.052
2	1	40	80	2	2	0.050	0.050	0.051
2	1.5	40	80	2	3	0.050	0.026	0.050
2	2	40	80	2	4	0.050	0.017*	0.050

Table A3.2

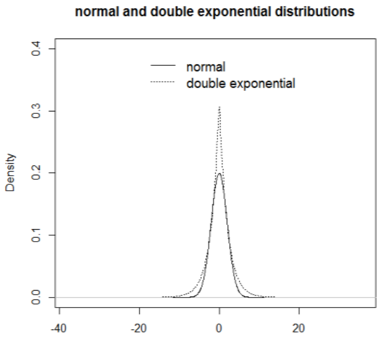


						Type I error rate		
n_2/n_1	**SDR**	**n_1**	**n_2**	σ_1	σ_2	**Welch's *t*-test**	**Student's *t*-test**	**Yuen's *t*-test**
0.5	0.5	10	5	2	1	0.041	0.019*	0.038
0.5	1	10	5	2	2	0.041	0.047	0.044
0.5	1.5	10	5	2	3	0.041	0.077*	0.050
0.5	2	10	5	2	4	0.040	0.103*	0.054
1	0.5	10	10	2	1	0.044	0.049	0.042
1	1	10	10	2	2	0.044	0.046	0.040
1	1.5	10	10	2	3	0.044	0.047	0.041
1	2	10	10	2	4	0.044	0.049	0.041
1.5	0.5	10	15	2	1	0.045	0.080*	0.044
1.5	1	10	15	2	2	0.045	0.048	0.043
1.5	1.5	10	15	2	3	0.046	0.034	0.043
1.5	2	10	15	2	4	0.046	0.028	0.043
2	0.5	10	20	2	1	0.044	0.107*	0.045
2	1	10	20	2	2	0.046	0.049	0.044
2	1.5	10	20	2	3	0.046	0.027	0.044
2	2	10	20	2	4	0.047	0.018*	0.044

0.5	0.5	20	10	2	1	0.047	0.018*	0.044
0.5	1	20	10	2	2	0.045	0.048	0.044
0.5	1.5	20	10	2	3	0.046	0.081*	0.045
0.5	2	20	10	2	4	0.045	0.108*	0.045
1	0.5	20	20	2	1	0.047	0.050	0.046
1	1	20	20	2	2	0.048	0.048	0.045
1	1.5	20	20	2	3	0.048	0.049	0.046
1	2	20	20	2	4	0.048	0.050	0.046
1.5	0.5	20	30	2	1	0.047	0.082*	0.046
1.5	1	20	30	2	2	0.048	0.049	0.047
1.5	1.5	20	30	2	3	0.049	0.035	0.047
1.5	2	20	30	2	4	0.048	0.027	0.047
2	0.5	20	40	2	1	0.047	0.109*	0.046
2	1	20	40	2	2	0.048	0.049	0.047
2	1.5	20	40	2	3	0.049	0.026	0.047
2	2	20	40	2	4	0.049	0.017*	0.048
0.5	0.5	30	15	2	1	0.048	0.017*	0.046
0.5	1	30	15	2	2	0.047	0.049	0.046
0.5	1.5	30	15	2	3	0.047	0.082*	0.045
0.5	2	30	15	2	4	0.046	0.108*	0.045
1	0.5	30	30	2	1	0.048	0.050	0.048
1	1	30	30	2	2	0.049	0.050	0.047
1	1.5	30	30	2	3	0.049	0.049	0.047
1	2	30	30	2	4	0.049	0.050	0.047

1.5	0.5	30	45	2	1	0.049	0.083*	0.048
1.5	1	30	45	2	2	0.049	0.049	0.048
1.5	1.5	30	45	2	3	0.049	0.034	0.048
1.5	2	30	45	2	4	0.049	0.028	0.048
2	0.5	30	60	2	1	0.048	0.109*	0.048
2	1	30	60	2	2	0.049	0.049	0.048
2	1.5	30	60	2	3	0.049	0.026	0.048
2	2	30	60	2	4	0.049	0.017*	0.048
0.5	0.5	40	20	2	1	0.049	0.017*	0.047
0.5	1	40	20	2	2	0.048	0.049	0.047
0.5	1.5	40	20	2	3	0.047	0.083*	0.047
0.5	2	40	20	2	4	0.047	0.109*	0.046
1	0.5	40	40	2	1	0.049	0.050	0.048
1	1	40	40	2	2	0.049	0.049	0.048
1	1.5	40	40	2	3	0.049	0.049	0.048
1	2	40	40	2	4	0.049	0.050	0.048
1.5	0.5	40	60	2	1	0.049	0.082*	0.048
1.5	1	40	60	2	2	0.049	0.049	0.048
1.5	1.5	40	60	2	3	0.049	0.034	0.048
1.5	2	40	60	2	4	0.049	0.027	0.049
2	0.5	40	80	2	1	0.049	0.109*	0.048
2	1	40	80	2	2	0.049	0.049	0.048
2	1.5	40	80	2	3	0.050	0.026	0.049
2	2	40	80	2	4	0.050	0.017*	0.049

Table A3.3

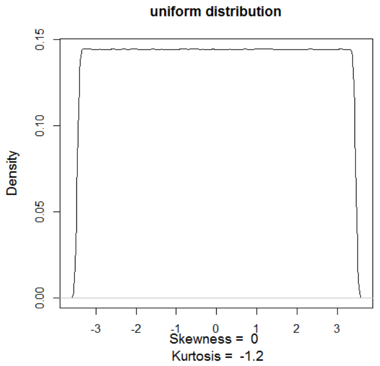


						Type I error rate		
n_2/n_1	**SDR**	**n_1**	**n_2**	σ_1	σ_2	**Welch's *t*-test**	**Student's *t*-test**	**Yuen's *t*-test**
0.5	0.5	10	5	2	1	0.046	0.020*	0.047
0.5	1	10	5	2	2	0.042	0.043	0.044
0.5	1.5	10	5	2	3	0.041	0.071	0.048
0.5	2	10	5	2	4	0.041	0.097*	0.052
1	0.5	10	10	2	1	0.050	0.055	0.057
1	1	10	10	2	2	0.047	0.049	0.048
1	1.5	10	10	2	3	0.045	0.048	0.044
1	2	10	10	2	4	0.044	0.049	0.043
1.5	0.5	10	15	2	1	0.051	0.088*	0.060
1.5	1	10	15	2	2	0.049	0.052	0.054
1.5	1.5	10	15	2	3	0.047	0.036	0.049
1.5	2	10	15	2	4	0.047	0.029	0.046
2	0.5	10	20	2	1	0.051	0.116*	0.061
2	1	10	20	2	2	0.050	0.053	0.057
2	1.5	10	20	2	3	0.048	0.028	0.052
2	2	10	20	2	4	0.048	0.019*	0.049

0.5	0.5	20	10	2	1	0.048	0.018*	0.050
0.5	1	20	10	2	2	0.046	0.046	0.046
0.5	1.5	20	10	2	3	0.045	0.078*	0.046
0.5	2	20	10	2	4	0.045	0.105*	0.045
1	0.5	20	20	2	1	0.050	0.053	0.053
1	1	20	20	2	2	0.049	0.049	0.050
1	1.5	20	20	2	3	0.048	0.049	0.048
1	2	20	20	2	4	0.048	0.050	0.047
1.5	0.5	20	30	2	1	0.050	0.085*	0.054
1.5	1	20	30	2	2	0.049	0.051	0.052
1.5	1.5	20	30	2	3	0.049	0.035	0.049
1.5	2	20	30	2	4	0.049	0.028	0.049
2	0.5	20	40	2	1	0.050	0.112*	0.054
2	1	20	40	2	2	0.050	0.052	0.053
2	1.5	20	40	2	3	0.050	0.028	0.051
2	2	20	40	2	4	0.049	0.018*	0.049
0.5	0.5	30	15	2	1	0.049	0.018*	0.051
0.5	1	30	15	2	2	0.048	0.047	0.047
0.5	1.5	30	15	2	3	0.047	0.081*	0.046
0.5	2	30	15	2	4	0.046	0.107*	0.046
1	0.5	30	30	2	1	0.050	0.051	0.052
1	1	30	30	2	2	0.049	0.049	0.050
1	1.5	30	30	2	3	0.049	0.049	0.048
1	2	30	30	2	4	0.048	0.050	0.048

1.5	0.5	30	45	2	1	0.050	0.084*	0.052
1.5	1	30	45	2	2	0.050	0.051	0.051
1.5	1.5	30	45	2	3	0.049	0.035	0.050
1.5	2	30	45	2	4	0.049	0.028	0.049
2	0.5	30	60	2	1	0.050	0.112*	0.053
2	1	30	60	2	2	0.050	0.052	0.052
2	1.5	30	60	2	3	0.050	0.027	0.050
2	2	30	60	2	4	0.049	0.017*	0.050
0.5	0.5	40	20	2	1	0.049	0.017*	0.050
0.5	1	40	20	2	2	0.048	0.047	0.048
0.5	1.5	40	20	2	3	0.048	0.081*	0.048
0.5	2	40	20	2	4	0.048	0.108*	0.047
1	0.5	40	40	2	1	0.050	0.051	0.051
1	1	40	40	2	2	0.049	0.050	0.050
1	1.5	40	40	2	3	0.049	0.050	0.049
1	2	40	40	2	4	0.049	0.050	0.049
1.5	0.5	40	60	2	1	0.050	0.084*	0.052
1.5	1	40	60	2	2	0.050	0.051	0.051
1.5	1.5	40	60	2	3	0.049	0.035	0.050
1.5	2	40	60	2	4	0.049	0.027	0.049
2	0.5	40	80	2	1	0.050	0.111*	0.052
2	1	40	80	2	2	0.050	0.051	0.051
2	1.5	40	80	2	3	0.050	0.027	0.050
2	2	40	80	2	4	0.049	0.017*	0.050

Table A3.4

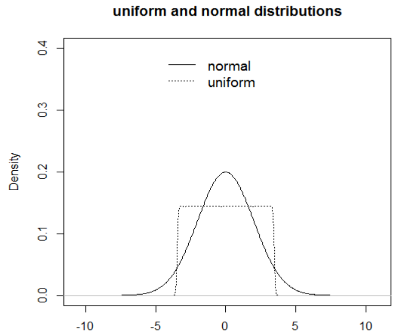


						Type I error rate		
n_2/n_1	**SDR**	**n_1**	**n_2**	**σ_1**	**σ_2**	**Welch's *t*-test**	**Student's *t*-test**	**Yuen's *t*-test**
0.5	0.5	10	5	2	1	0.050	0.024*	0.055
0.5	1	10	5	2	2	0.060	0.051	0.070
0.5	1.5	10	5	2	3	0.068	0.093*	0.091*
0.5	2	10	5	2	4	0.071	0.126*	0.103*
1	0.5	10	10	2	1	0.053	0.058	0.066
1	1	10	10	2	2	0.050	0.051	0.055
1	1.5	10	10	2	3	0.052	0.054	0.059
1	2	10	10	2	4	0.053	0.058	0.066
1.5	0.5	10	15	2	1	0.054	0.089*	0.071
1.5	1	10	15	2	2	0.051	0.051	0.058
1.5	1.5	10	15	2	3	0.050	0.037	0.055
1.5	2	10	15	2	4	0.051	0.032	0.057
2	0.5	10	20	2	1	0.055	0.116*	0.073
2	1	10	20	2	2	0.052	0.050	0.061
2	1.5	10	20	2	3	0.051	0.027	0.055
2	2	10	20	2	4	0.050	0.019*	0.054

0.5	0.5	20	10	2	1	0.051	0.020*	0.055
0.5	1	20	10	2	2	0.053	0.051	0.061
0.5	1.5	20	10	2	3	0.055	0.088*	0.070
0.5	2	20	10	2	4	0.055	0.116*	0.073
1	0.5	20	20	2	1	0.051	0.053	0.057
1	1	20	20	2	2	0.050	0.050	0.054
1	1.5	20	20	2	3	0.050	0.051	0.055
1	2	20	20	2	4	0.051	0.053	0.057
1.5	0.5	20	30	2	1	0.052	0.086*	0.059
1.5	1	20	30	2	2	0.050	0.050	0.054
1.5	1.5	20	30	2	3	0.050	0.036	0.053
1.5	2	20	30	2	4	0.051	0.029	0.054
2	0.5	20	40	2	1	0.051	0.112*	0.059
2	1	20	40	2	2	0.051	0.050	0.056
2	1.5	20	40	2	3	0.050	0.027	0.053
2	2	20	40	2	4	0.050	0.018*	0.053
0.5	0.5	30	15	2	1	0.050	0.018*	0.053
0.5	1	30	15	2	2	0.051	0.050	0.057
0.5	1.5	30	15	2	3	0.052	0.087*	0.062
0.5	2	30	15	2	4	0.053	0.114*	0.064
1	0.5	30	30	2	1	0.051	0.052	0.055
1	1	30	30	2	2	0.050	0.050	0.053
1	1.5	30	30	2	3	0.050	0.051	0.053
1	2	30	30	2	4	0.051	0.052	0.055

1.5	0.5	30	45	2	1	0.051	0.084*	0.055
1.5	1	30	45	2	2	0.050	0.050	0.053
1.5	1.5	30	45	2	3	0.050	0.035	0.052
1.5	2	30	45	2	4	0.050	0.029	0.053
2	0.5	30	60	2	1	0.051	0.111*	0.056
2	1	30	60	2	2	0.050	0.050	0.053
2	1.5	30	60	2	3	0.050	0.026	0.052
2	2	30	60	2	4	0.050	0.017*	0.052
0.5	0.5	40	20	2	1	0.050	0.018*	0.053
0.5	1	40	20	2	2	0.051	0.050	0.055
0.5	1.5	40	20	2	3	0.052	0.086*	0.058
0.5	2	40	20	2	4	0.051	0.113*	0.059
1	0.5	40	40	2	1	0.050	0.051	0.053
1	1	40	40	2	2	0.050	0.050	0.052
1	1.5	40	40	2	3	0.050	0.051	0.052
1	2	40	40	2	4	0.051	0.052	0.053
1.5	0.5	40	60	2	1	0.051	0.084*	0.054
1.5	1	40	60	2	2	0.050	0.050	0.052
1.5	1.5	40	60	2	3	0.050	0.035	0.051
1.5	2	40	60	2	4	0.050	0.028	0.052
2	0.5	40	80	2	1	0.051	0.111*	0.055
2	1	40	80	2	2	0.050	0.050	0.052
2	1.5	40	80	2	3	0.050	0.026	0.051
2	2	40	80	2	4	0.050	0.017*	0.051

Table A3.5

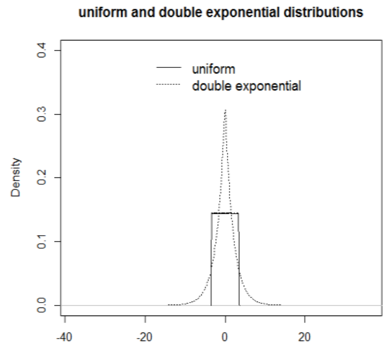


						Type I error rate		
n_2/n_1	SDR	n_1	n_2	σ_1	σ_2	Welch's *t*-test	Student's *t*-test	Yuen's *t*-test
0.5	0.5	10	5	2	1	0.049	0.023*	0.056
0.5	1	10	5	2	2	0.051	0.047	0.055
0.5	1.5	10	5	2	3	0.055	0.083*	0.065
0.5	2	10	5	2	4	0.056	0.115*	0.073
1	0.5	10	10	2	1	0.054	0.058	0.068
1	1	10	10	2	2	0.050	0.051	0.055
1	1.5	10	10	2	3	0.050	0.052	0.052
1	2	10	10	2	4	0.050	0.055	0.053
1.5	0.5	10	15	2	1	0.054	0.090*	0.073
1.5	1	10	15	2	2	0.051	0.052	0.062
1.5	1.5	10	15	2	3	0.050	0.037	0.054
1.5	2	10	15	2	4	0.050	0.031	0.052
2	0.5	10	20	2	1	0.055	0.117*	0.074
2	1	10	20	2	2	0.053	0.052	0.067
2	1.5	10	20	2	3	0.050	0.028	0.057
2	2	10	20	2	4	0.050	0.019*	0.053

0.5	0.5	20	10	2	1	0.050	0.019*	0.055
0.5	1	20	10	2	2	0.050	0.048	0.053
0.5	1.5	20	10	2	3	0.051	0.084*	0.057
0.5	2	20	10	2	4	0.052	0.113*	0.060
1	0.5	20	20	2	1	0.052	0.054	0.059
1	1	20	20	2	2	0.050	0.051	0.053
1	1.5	20	20	2	3	0.050	0.051	0.052
1	2	20	20	2	4	0.050	0.052	0.052
1.5	0.5	20	30	2	1	0.051	0.085*	0.060
1.5	1	20	30	2	2	0.051	0.051	0.056
1.5	1.5	20	30	2	3	0.050	0.036	0.053
1.5	2	20	30	2	4	0.050	0.029	0.051
2	0.5	20	40	2	1	0.051	0.113*	0.060
2	1	20	40	2	2	0.051	0.051	0.057
2	1.5	20	40	2	3	0.050	0.027	0.053
2	2	20	40	2	4	0.050	0.018*	0.052
0.5	0.5	30	15	2	1	0.051	0.018*	0.053
0.5	1	30	15	2	2	0.050	0.049	0.052
0.5	1.5	30	15	2	3	0.051	0.085*	0.054
0.5	2	30	15	2	4	0.050	0.112*	0.055
1	0.5	30	30	2	1	0.051	0.053	0.056
1	1	30	30	2	2	0.050	0.050	0.052
1	1.5	30	30	2	3	0.050	0.051	0.051
1	2	30	30	2	4	0.050	0.052	0.052

1.5	0.5	30	45	2	1	0.051	0.084*	0.056
1.5	1	30	45	2	2	0.051	0.051	0.054
1.5	1.5	30	45	2	3	0.050	0.035	0.052
1.5	2	30	45	2	4	0.050	0.028	0.051
2	0.5	30	60	2	1	0.051	0.112*	0.056
2	1	30	60	2	2	0.051	0.051	0.055
2	1.5	30	60	2	3	0.050	0.026	0.052
2	2	30	60	2	4	0.050	0.017*	0.051
0.5	0.5	40	20	2	1	0.050	0.018*	0.053
0.5	1	40	20	2	2	0.050	0.049	0.052
0.5	1.5	40	20	2	3	0.050	0.084*	0.053
0.5	2	40	20	2	4	0.050	0.111*	0.054
1	0.5	40	40	2	1	0.051	0.052	0.054
1	1	40	40	2	2	0.050	0.050	0.052
1	1.5	40	40	2	3	0.050	0.051	0.051
1	2	40	40	2	4	0.050	0.051	0.051
1.5	0.5	40	60	2	1	0.051	0.084*	0.054
1.5	1	40	60	2	2	0.050	0.051	0.053
1.5	1.5	40	60	2	3	0.050	0.035	0.051
1.5	2	40	60	2	4	0.050	0.028	0.051
2	0.5	40	80	2	1	0.051	0.111*	0.054
2	1	40	80	2	2	0.050	0.051	0.053
2	1.5	40	80	2	3	0.050	0.026	0.052
2	2	40	80	2	4	0.050	0.017*	0.051

Table A3.6

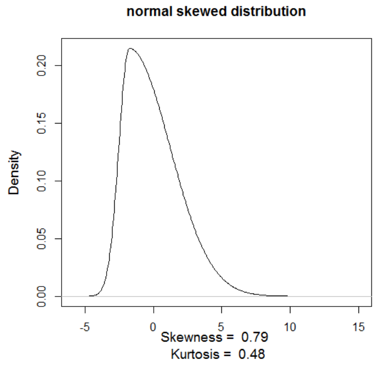


						Type I error rate		
n_2/n_1	SDR	n_1	n_2	σ_1	σ_2	Welch's *t*-test	Student's *t*-test	Yuen's *t*-test
0.5	0.5	10	5	2	1	0.048	0.023*	0.059
0.5	1	10	5	2	2	0.043	0.042	0.047
0.5	1.5	10	5	2	3	0.042	0.069	0.047
0.5	2	10	5	2	4	0.041	0.094*	0.050
1	0.5	10	10	2	1	0.053	0.058	0.071
1	1	10	10	2	2	0.048	0.050	0.059
1	1.5	10	10	2	3	0.046	0.048	0.050
1	2	10	10	2	4	0.045	0.049	0.046
1.5	0.5	10	15	2	1	0.054	0.092*	0.074
1.5	1	10	15	2	2	0.051	0.054	0.066
1.5	1.5	10	15	2	3	0.049	0.037	0.057
1.5	2	10	15	2	4	0.047	0.029	0.052
2	0.5	10	20	2	1	0.055	0.119*	0.074
2	1	10	20	2	2	0.052	0.056	0.070
2	1.5	10	20	2	3	0.050	0.030	0.062
2	2	10	20	2	4	0.049	0.019*	0.057

0.5	0.5	20	10	2	1	0.049	0.019*	0.056
0.5	1	20	10	2	2	0.046	0.044	0.049
0.5	1.5	20	10	2	3	0.046	0.076*	0.047
0.5	2	20	10	2	4	0.045	0.104*	0.046
1	0.5	20	20	2	1	0.051	0.054	0.059
1	1	20	20	2	2	0.049	0.049	0.055
1	1.5	20	20	2	3	0.048	0.049	0.051
1	2	20	20	2	4	0.048	0.050	0.048
1.5	0.5	20	30	2	1	0.052	0.087*	0.060
1.5	1	20	30	2	2	0.050	0.052	0.057
1.5	1.5	20	30	2	3	0.049	0.036	0.054
1.5	2	20	30	2	4	0.049	0.028	0.051
2	0.5	20	40	2	1	0.052	0.113*	0.061
2	1	20	40	2	2	0.051	0.053	0.058
2	1.5	20	40	2	3	0.050	0.028	0.055
2	2	20	40	2	4	0.049	0.018*	0.053
0.5	0.5	30	15	2	1	0.049	0.018*	0.054
0.5	1	30	15	2	2	0.048	0.046	0.050
0.5	1.5	30	15	2	3	0.047	0.079*	0.047
0.5	2	30	15	2	4	0.046	0.106*	0.046
1	0.5	30	30	2	1	0.051	0.053	0.056
1	1	30	30	2	2	0.050	0.050	0.053
1	1.5	30	30	2	3	0.049	0.050	0.051
1	2	30	30	2	4	0.049	0.050	0.049

1.5	0.5	30	45	2	1	0.051	0.085*	0.056
1.5	1	30	45	2	2	0.050	0.051	0.055
1.5	1.5	30	45	2	3	0.050	0.035	0.052
1.5	2	30	45	2	4	0.049	0.028	0.051
2	0.5	30	60	2	1	0.051	0.113*	0.057
2	1	30	60	2	2	0.050	0.052	0.055
2	1.5	30	60	2	3	0.050	0.027	0.053
2	2	30	60	2	4	0.050	0.017*	0.052
0.5	0.5	40	20	2	1	0.050	0.017*	0.053
0.5	1	40	20	2	2	0.048	0.047	0.050
0.5	1.5	40	20	2	3	0.048	0.081*	0.048
0.5	2	40	20	2	4	0.048	0.108*	0.048
1	0.5	40	40	2	1	0.050	0.051	0.054
1	1	40	40	2	2	0.050	0.050	0.052
1	1.5	40	40	2	3	0.049	0.050	0.051
1	2	40	40	2	4	0.049	0.050	0.050
1.5	0.5	40	60	2	1	0.050	0.084*	0.054
1.5	1	40	60	2	2	0.050	0.051	0.053
1.5	1.5	40	60	2	3	0.050	0.035	0.052
1.5	2	40	60	2	4	0.049	0.028	0.051
2	0.5	40	80	2	1	0.051	0.112*	0.055
2	1	40	80	2	2	0.050	0.052	0.054
2	1.5	40	80	2	3	0.050	0.027	0.053
2	2	40	80	2	4	0.050	0.017*	0.051

Table A3.7

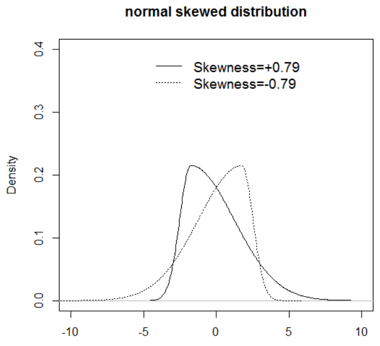


n_2/n_1	SDR	n_1	n_2	σ_1	σ_2	Type I error rate: Welch's *t*-test	Type I error rate: Student's *t*-test
0.5	0.5	10	5	2	1	0.046	0.026
0.5	1	10	5	2	2	0.053	0.049
0.5	1.5	10	5	2	3	0.065	0.087*
0.5	2	10	5	2	4	0.070	0.122*
1	0.5	10	10	2	1	0.055	0.059
1	1	10	10	2	2	0.047	0.049
1	1.5	10	10	2	3	0.050	0.053
1	2	10	10	2	4	0.055	0.059
1.5	0.5	10	15	2	1	0.058	0.090*
1.5	1	10	15	2	2	0.049	0.049
1.5	1.5	10	15	2	3	0.048	0.037
1.5	2	10	15	2	4	0.050	0.034
2	0.5	10	20	2	1	0.060	0.117*
2	1	10	20	2	2	0.052	0.049
2	1.5	10	20	2	3	0.048	0.028
2	2	10	20	2	4	0.049	0.022*

0.5	0.5	20	10	2	1	0.049	0.022*
0.5	1	20	10	2	2	0.052	0.049
0.5	1.5	20	10	2	3	0.057	0.087*
0.5	2	20	10	2	4	0.059	0.117*
1	0.5	20	20	2	1	0.053	0.055
1	1	20	20	2	2	0.049	0.050
1	1.5	20	20	2	3	0.051	0.052
1	2	20	20	2	4	0.052	0.055
1.5	0.5	20	30	2	1	0.054	0.087*
1.5	1	20	30	2	2	0.049	0.049
1.5	1.5	20	30	2	3	0.050	0.036
1.5	2	20	30	2	4	0.050	0.030
2	0.5	20	40	2	1	0.055	0.113*
2	1	20	40	2	2	0.051	0.050
2	1.5	20	40	2	3	0.050	0.027
2	2	20	40	2	4	0.050	0.019*
0.5	0.5	30	15	2	1	0.050	0.020*
0.5	1	30	15	2	2	0.052	0.050
0.5	1.5	30	15	2	3	0.054	0.086*
0.5	2	30	15	2	4	0.057	0.115*
1	0.5	30	30	2	1	0.052	0.053
1	1	30	30	2	2	0.050	0.050
1	1.5	30	30	2	3	0.050	0.051
1	2	30	30	2	4	0.051	0.053
1.5	0.5	30	45	2	1	0.053	0.085*

1.5	1	30	45	2	2	0.050	0.050
1.5	1.5	30	45	2	3	0.050	0.035
1.5	2	30	45	2	4	0.051	0.030
2	0.5	30	60	2	1	0.053	0.112*
2	1	30	60	2	2	0.051	0.050
2	1.5	30	60	2	3	0.050	0.027
2	2	30	60	2	4	0.050	0.018*
0.5	0.5	40	20	2	1	0.050	0.019*
0.5	1	40	20	2	2	0.051	0.050
0.5	1.5	40	20	2	3	0.054	0.086*
0.5	2	40	20	2	4	0.054	0.113*
1	0.5	40	40	2	1	0.051	0.053
1	1	40	40	2	2	0.050	0.050
1	1.5	40	40	2	3	0.051	0.051
1	2	40	40	2	4	0.051	0.052
1.5	0.5	40	60	2	1	0.052	0.085*
1.5	1	40	60	2	2	0.050	0.050
1.5	1.5	40	60	2	3	0.050	0.035
1.5	2	40	60	2	4	0.051	0.029
2	0.5	40	80	2	1	0.050	0.081*
2	1	40	80	2	2	0.050	0.029
2	1.5	40	80	2	3	0.050	0.016*
2	2	40	80	2	4	0.050	0.011*

Table A3.8

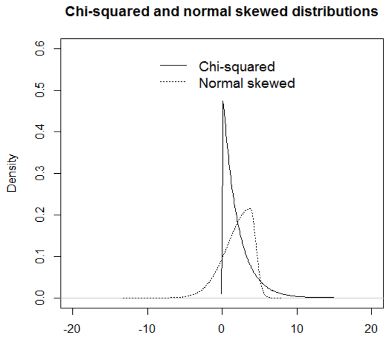


						Type I error rate	
n_2/n_1	SDR	n_1	n_2	σ_1	σ_2	Welch's *t*-test	Student's *t*-test
0.5	0.5	10	5	2	1	0.055	0.029
0.5	1	10	5	2	2	0.061	0.058
0.5	1.5	10	5	2	3	0.068	0.094*
0.5	2	10	5	2	4	0.072	0.127*
1	0.5	10	10	2	1	0.058	0.063
1	1	10	10	2	2	0.055	0.056
1	1.5	10	10	2	3	0.056	0.059
1	2	10	10	2	4	0.058	0.062
1.5	0.5	10	15	2	1	0.060	0.093*
1.5	1	10	15	2	2	0.055	0.055
1.5	1.5	10	15	2	3	0.054	0.041
1.5	2	10	15	2	4	0.054	0.036
2	0.5	10	20	2	1	0.060	0.119*
2	1	10	20	2	2	0.056	0.054
2	1.5	10	20	2	3	0.054	0.032
2	2	10	20	2	4	0.054	0.023*

0.5	0.5	20	10	2	1	0.054	0.023*
0.5	1	20	10	2	2	0.057	0.054
0.5	1.5	20	10	2	3	0.059	0.090*
0.5	2	20	10	2	4	0.061	0.119*
1	0.5	20	20	2	1	0.055	0.057
1	1	20	20	2	2	0.053	0.053
1	1.5	20	20	2	3	0.053	0.055
1	2	20	20	2	4	0.055	0.057
1.5	0.5	20	30	2	1	0.055	0.088*
1.5	1	20	30	2	2	0.053	0.053
1.5	1.5	20	30	2	3	0.052	0.038
1.5	2	20	30	2	4	0.053	0.032
2	0.5	20	40	2	1	0.056	0.114*
2	1	20	40	2	2	0.053	0.052
2	1.5	20	40	2	3	0.052	0.029
2	2	20	40	2	4	0.052	0.020*
0.5	0.5	30	15	2	1	0.053	0.021*
0.5	1	30	15	2	2	0.054	0.053
0.5	1.5	30	15	2	3	0.056	0.089*
0.5	2	30	15	2	4	0.057	0.115*
1	0.5	30	30	2	1	0.053	0.054
1	1	30	30	2	2	0.052	0.053
1	1.5	30	30	2	3	0.053	0.053
1	2	30	30	2	4	0.053	0.055
1.5	0.5	30	45	2	1	0.053	0.086*

1.5	1	30	45	2	2	0.052	0.052
1.5	1.5	30	45	2	3	0.051	0.036
1.5	2	30	45	2	4	0.052	0.030
2	0.5	30	60	2	1	0.054	0.113*
2	1	30	60	2	2	0.052	0.051
2	1.5	30	60	2	3	0.051	0.028
2	2	30	60	2	4	0.052	0.019*
0.5	0.5	40	20	2	1	0.053	0.020*
0.5	1	40	20	2	2	0.053	0.052
0.5	1.5	40	20	2	3	0.054	0.088*
0.5	2	40	20	2	4	0.055	0.114*
1	0.5	40	40	2	1	0.052	0.054
1	1	40	40	2	2	0.052	0.052
1	1.5	40	40	2	3	0.052	0.053
1	2	40	40	2	4	0.052	0.053
1.5	0.5	40	60	2	1	0.053	0.085*
1.5	1	40	60	2	2	0.052	0.051
1.5	1.5	40	60	2	3	0.052	0.036
1.5	2	40	60	2	4	0.052	0.030
2	0.5	40	80	2	1	0.050	0.081*
2	1	40	80	2	2	0.050	0.029
2	1.5	40	80	2	3	0.050	0.016*
2	2	40	80	2	4	0.050	0.011*

Table A3.9



						Type I error rate	
n_2/n_1	SDR	n_1	n_2	σ_1	σ_2	Welch's *t*-test	Student's *t*-test
0.5	0.5	10	5	2	1	0.068	0.052
0.5	1	10	5	2	2	0.066	0.077*
0.5	1.5	10	5	2	3	0.070	0.110*
0.5	2	10	5	2	4	0.071	0.137*
1	0.5	10	10	2	1	0.068	0.052
1	1	10	10	2	2	0.066	0.077*
1	1.5	10	10	2	3	0.070	0.110*
1	2	10	10	2	4	0.071	0.137*
1.5	0.5	10	15	2	1	0.084*	0.105*
1.5	1	10	15	2	2	0.065	0.058
1.5	1.5	10	15	2	3	0.059	0.043
1.5	2	10	15	2	4	0.057	0.037
2	0.5	10	20	2	1	0.088*	0.126*
2	1	10	20	2	2	0.068	0.054
2	1.5	10	20	2	3	0.060	0.032
2	2	10	20	2	4	0.057	0.024*

0.5	0.5	20	10	2	1	0.063	0.039
0.5	1	20	10	2	2	0.060	0.066
0.5	1.5	20	10	2	3	0.060	0.098*
0.5	2	20	10	2	4	0.061	0.124*
1	0.5	20	20	2	1	0.068	0.069
1	1	20	20	2	2	0.058	0.058
1	1.5	20	20	2	3	0.056	0.058
1	2	20	20	2	4	0.055	0.058
1.5	0.5	20	30	2	1	0.071	0.096*
1.5	1	20	30	2	2	0.060	0.055
1.5	1.5	20	30	2	3	0.055	0.039
1.5	2	20	30	2	4	0.055	0.033
2	0.5	20	40	2	1	0.073	0.121*
2	1	20	40	2	2	0.061	0.053
2	1.5	20	40	2	3	0.056	0.029
2	2	20	40	2	4	0.054	0.020*
0.5	0.5	30	15	2	1	0.059	0.033
0.5	1	30	15	2	2	0.057	0.062
0.5	1.5	30	15	2	3	0.057	0.094*
0.5	2	30	15	2	4	0.057	0.119*
1	0.5	30	30	2	1	0.063	0.064
1	1	30	30	2	2	0.056	0.056
1	1.5	30	30	2	3	0.054	0.055
1	2	30	30	2	4	0.054	0.056
1.5	0.5	30	45	2	1	0.066	0.093*

1.5	1	30	45	2	2	0.057	0.053
1.5	1.5	30	45	2	3	0.054	0.037
1.5	2	30	45	2	4	0.053	0.031
2	0.5	30	60	2	1	0.067	0.118*
2	1	30	60	2	2	0.058	0.052
2	1.5	30	60	2	3	0.054	0.028
2	2	30	60	2	4	0.053	0.019*
0.5	0.5	40	20	2	1	0.058	0.029
0.5	1	40	20	2	2	0.055	0.059
0.5	1.5	40	20	2	3	0.055	0.092*
0.5	2	40	20	2	4	0.055	0.116*
1	0.5	40	40	2	1	0.060	0.061
1	1	40	40	2	2	0.055	0.055
1	1.5	40	40	2	3	0.053	0.054
1	2	40	40	2	4	0.054	0.055
1.5	0.5	40	60	2	1	0.062	0.091*
1.5	1	40	60	2	2	0.055	0.052
1.5	1.5	40	60	2	3	0.053	0.037
1.5	2	40	60	2	4	0.052	0.030
2	0.5	40	80	2	1	0.063	0.116*
2	1	40	80	2	2	0.057	0.051
2	1.5	40	80	2	3	0.054	0.028
2	2	40	80	2	4	0.052	0.018*

Supp Mat 4 :

In order to estimate the Type I error rate for Welch's *t*-test when there is extreme SDR and a very unbalanced design. we simulated 1.000.000 simulations of two samples generated under 60 different conditions (yielding 60.000.000 simulations in total). In each condition. the first sample is generated from a population where σ_1=2. and its sample size varies from 10 to 30 in a step of 10. The standard deviation and the sample size of the second sample is a function of the sample sizes ratio SSR = $\frac{n_2}{n_1}$ being respectively 0.01. 0.1. 10 and 100 and SDRs ranging from 1 to 5 in steps of 1. Once again. the set of simulations was repeated nine times. varying the distributions underlying the data.

We found that even with very extreme SDRs and unbalanced designs. as long as there are at least 10 subjects per group (See Table A4; for more detail. see the appendix at https://osf.io/bver8/files/). Welch's *t*-test remains very close to the nominal alpha level (i.e.. 5%). The only exception is when at least one distribution has very high skewness and kurtosis. such as the chi-square distribution with two degrees of freedom. Under this distribution. one needs at least 30 subjects per groups to accurately control the alpha risk. With fewer than 30 subjects per groups. even Welch's *t*-test becomes too liberal when there is a negative correlation between sample size and standard deviation (See Table A3.9).

Table A4

	Welch's t-test: alpha risk	
	Minimum	**Maximum**
Two normal	0.049	0.051
Two double exponential	0.042	0.050
One normal. one double exponential	0.042	0.050
Two uniform	0.050	0.054
One uniform. one normal	0.050	0.054
One uniform. one double exponential	0.042	0.055
Two normal skewed. equal asymmetry	0.051	0.062
Two normal skewed. unequal asymmetry	0.051	0.062
One chi-squared. one normal skewed	0.050	0.100*

Supp Mat 5 : Power of Student's *t*-test and Welch's *t*-test

In order to estimate the power of Student's *t*-test and Welch's *t*-test. we performed 1.000.000 simulations of two samples generated under 64 different conditions (yielding 64.000.000 simulations in total). In each condition. the first sample was generated from a population where σ_1=2. and its sample size varied from 10 to 40 in steps of 10. The standard deviation and the sample size of the second sample is a function of the sample sizes ratio SSR $= \frac{n_2}{n_1}$; ranging from 0.5 to 2 in steps of 0.5. and SDR ranging from 0.5 to 2 in steps of .0.5. We included a mean difference $\delta = 1$ (giving related Cohen's effect sizes that vary from 0.29 to 0.71. which is realistic in the field of psychology). In a first step. the *p*-values of the two tests were extracted for each pair of samples and for each test. and in a second step. the percent of *p*-value under the nominal alpha risk (5%) was computed for each test. giving the **observed power (OP)**.

In order to insure the reliability of our calculation method. we firstly used R commands to generate data from two normal distributions (See Table A5.1). Next. the observed power computed when distributions underlying the data are normal were compared with theoretical power (TP). i.e. the power computed using the power function of Welch's *t*-test and Student's *t*-test. The computed power was very consistent with theoretical power (observed power and theoretical power were very similar). Therefore. one can conclude that the method is reliable.

The set of simulations was repeated height times. varying the distributions underlying the data. We used R commands to generate data from different distributions (for more detail. see the code at https://osf.io/bver8/files/):

- **Two double exponential distributions (See Table A5.2):** in order to investigate the impact of high kurtosis on Welch's power and Student's power. data were generated by means of the function “rdoublex” (from the package “smoothmest”; “R: The double exponential (Laplace) distribution.” 2012).

- **One normal and one double exponential distribution (See Table A5.3):** in order to investigate the impact of unequal shape. in terms of kurtosis. on Welch’s power and Student’s power. data were generated by means of the functions “rdoublex” and “rnorm”.
- **Two uniform distributions (See Table A5.4):** in order to investigate the impact of low kurtosis on Welch’s power and Student’s power. data were generated by means of the function “runif” (from the package “stats”; “R: The Uniform Distribution.” 2016).
- **One uniform and one normal distribution (See Table A5.5):** in order to investigate the impact of the unequal shape of distributions. in terms of kurtosis. on Welch’s power and Student’s power. data were generated by means of the functions “runif” and “rnorm”.
- **One uniform and one double exponential distribution (See Table A5.6):** in order to investigate the impact of the unequal shape of distributions. in terms of kurtosis. on Welch’s power and Student’s power. data were generated by means of the functions “runif” and “rdoublex”.
- **Two normal skewed distributions with positive skewness of 0.79 (See Table A5.7):** in order to investigate the impact of skewness on Welch’s power and Student’s power. data were generated by means of the function “rsnorm” (from the package “fGarch”; “R: Skew Normal Distribution.” 2017). The normal skewed was used because it is the only skewed distribution where the standard deviation ratio can vary without having an impact on skewness.
- **One normal skewed distribution with negative skewness (-0.79) and one normal skewed distribution with positive skewness (+0.79; See Table A5.8) :** in order to

investigate the impact of unequal shapes. in terms of kurtosis. on Welch's power and Student's power. when data are asymmetric. data were generated by means of the functions "rsnorm" with unequal skewness.

- **One chi(2) and one normal skewed with negative skewness of -0.79 (See Table A5.9):** in order to assess the impact of high skewness and kurtosis on Welch's power and Student's power. data were generated by means of the functions "rsnorm" with different skewness and from the function "rchisq"; "R: The (non-central) Chi-Squared Distribution." 2016).

We have not included results for Yuen's *t*-test because we saw previously than the test performed badly in terms of alpha risk in some specific conditions. Moreover. we have included results for Student's *t*-test only when the test was valid. that is. when assumptions underlying the test were met (the normality assumption and equality of variances). and/or when the designs were balanced. Indeed. when the assumption of equal variances is not met. with unequal sample sizes. the power of Student's *t*-test is underestimated when there was a positive correlation between the sample size and the standard deviation ($Power_{Student} < Power_{Welch}$). On the other hand. the power of Student's *t*-test is overestimated when there is a negative correlation between the sample size and the standard deviation ($Power_{Student} > Power_{Welch}$). The only exception is when at least one distribution has very high skewness and high kurtosis: in this case. Student's power is always slightly better than Welch's power.

Departure from the normality assumption can lead to a loss in power. for example. with one uniform and one double exponential distribution (see Table A5.6 and A6.6). However. departure from the normality assumption can also leads to a power inflation. such as with many scenarios where samples are extracted from two normal skewed distributions with positive skewness (=G1) = +.79 (See Tables A5.7 and 6.7). or from one chi-square with two degrees of freedom and one normal skewed distribution with negative skewness G1 = -.79 (See Tables

A5.9 and A6.9). We also observe a gain in power when distributions are symmetric with heavy tails. which is in contrast with previous findings (see. e.g.. Wilcox. 1998). and might seem surprising. particularly for Welch's *t*-test. considering the fact that this test was more conservative when the null hypothesis was true and samples were extracted from heavy-tailed distributions.

When comparing Student's *t*-test and Welch's *t*-test. when both tests are valid. it appears that loss in power is as true for Welch's *t*-test as it is for Student's *t*-test. For balanced designs. conclusions are given in the appendix "**Robustness of Student's *t*-test when sample sizes are equal across groups**".

Considering the cases where sample sizes are unequal and SDR = 1. Student's-*t* test is sometimes better than Welch's-*t* test. and sometimes the reverse is true. However. the difference is small (the larger observed difference is 4.29% in Table A5.7). However. because there is no correct way to insure that SDR = 1. and because variances are likely not to be equal in certain research areas. our recommendation is to always use Welch's *t*-test instead of Student's *t*-test.

Table A5.1

			Welch's *t*-test				Student's *t*-test			
		SDR	**0.5**	**1**	**1.5**	**2**	**0.5**	**1**	**1.5**	**2**
n_1	**n_2/n_1**		**Normal distributions**							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	21.61	12.96	9.5	7.9	N/A	13.54	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	25.89	18.13	12.94	10.10	27.45	18.49	13.49	10.88
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	27.18	21.29	15.86	12.30	N/A	21.70	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	27.87	23.12	17.95	14.13	N/A	23.83	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	41.97	23.13	14.51	10.74	N/A	23.85	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	48.85	33.68	22.53	16.16	49.80	33.80	22.87	16.70
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	51.26	39.35	28.34	20.70	N/A	39.64	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	52.61	42.76	32.47	24.53	N/A	43.39	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	58.79	33.22	20.05	13.94	N/A	33.92	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.74	47.69	31.96	22.39	67.32	47.75	32.21	22.84

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	69.54	55.10	40.06	29.08	N/A	55.33	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	71.01	59.34	45.84	34.61	N/A	59.87	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	71.80	42.87	25.71	17.34	N/A	43.49	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.39	59.83	40.87	28.51	79.73	59.86	41.06	28.88
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.92	67.75	50.83	37.15	N/A	67.93	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.18	72.30	57.53	44.13	N/A	72.66	N/A	N/A

Table A5.2

n₁	n₂/n₁	SDR	Welch's *t*-test 0.5	1	1.5	2	Student's *t*-test 0.5	1	1.5	2
			Double exponential distributions							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	24.59	14.52	10.03	7.76	N/A	15.31	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	29.69	20.06	14.16	10.86	25,76	18,69	14,14	11,40
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	31.59	23.46	17.16	13.24	N/A	23.85	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	32.66	25.81	19.35	15.13	N/A	25.91	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	44.54	25.69	16.39	11.82	N/A	25.82	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	51.46	35.55	24.11	17.48	52.37	35.78	24.49	18.04
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	54.05	41.23	29.64	21.89	N/A	41.33	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	55.49	44.89	33.85	25.57	N/A	44.90	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	60.22	35.76	22.22	15.51	N/A	35.74	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	67.86	49.09	33.38	23.67	68.41	49.20	33.66	24.11

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	70.56	56.37	41.23	30.18	N/A	56.47	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	71.96	60.59	46.83	35.46	N/A	60.77	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	72.39	44.94	27.78	19.04	N/A	44.93	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.57	60.54	42.11	29.76	79.90	60.60	42.31	30.14
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.97	68.28	51.63	38.09	N/A	68.37	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.17	72.63	58.06	44.87	N/A	72.89	N/A	N/A

Table A5.3

			Welch's *t*-test				Student's *t*-test			
		SDR	0.5	1	1.5	2	0.5	1	1.5	2
n1	n2/n1		One normal distribution. one double exponential distribution							
10	0.5	Theo.	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	22.33	14.05	9.91	7.81	N/A	13.46	N/A	N/A
	1	Theo.	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	26.04	18.96	13.86	10.77	27.81	19.48	14.43	11.53
	1.5	Theo.	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	27.30	21.67	16.52	12.94	N/A	22.91	N/A	N/A
	2	Theo.	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	27.79	23.33	18.48	14.73	N/A	25.04	N/A	N/A
20	0.5	Theo.	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	42.79	25.36	16.30	11.88	N/A	24.41	N/A	N/A
	1	Theo.	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	48.88	34.53	23.85	17.32	49.89	34.71	24.16	17.84
	1.5	Theo.	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	51.21	39.65	29.18	21.68	N/A	40.41	N/A	N/A
	2	Theo.	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	52.64	42.99	32.93	25.17	N/A	44.11	N/A	N/A
30	0.5	Theo.	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	59.34	35.41	22.12	15.45	N/A	34.70	N/A	N/A
	1	Theo.	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.83	48.38	33.19	23.63	67.42	48.47	33.42	24.05

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	69.61	55.37	40.75	29.98	N/A	55.80	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	70.98	59.53	46.25	35.21	N/A	60.23	N/A	N/A
40	0.5	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	72.04	44.65	27.68	18.97	N/A	44.15	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.38	60.21	41.80	29.59	79.71	60.26	41.98	29.95
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.97	67.83	51.31	38.03	N/A	68.10	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.21	72.34	57.76	44.55	N/A	72.74	N/A	N/A

Table A5.4

			Welch's *t*-test				Student's *t*-test			
		SDR	0.5	1	1.5	2	0.5	1	1.5	2
n₁	n₂/n₁		Uniform distributions							
10	0.5	Theo.	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	20.17	12.11	9.56	8.58	N/A	12.73	N/A	N/A
	1	Theo.	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	23.84	17.28	12.31	9.73	25.43	17.47	12.77	10.48
	1.5	Theo.	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	24.79	20.24	15.19	11.86	N/A	20.77	N/A	N/A
	2	Theo.	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	25.21	21.80	17.30	13.79	N/A	22.97	N/A	N/A
20	0.5	Theo.	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	40.74	21.78	13.51	10.06	N/A	22.94	N/A	N/A
	1	Theo.	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	47.51	32.81	21.82	15.56	48.49	32.86	22.12	16.10
	1.5	Theo.	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	49.87	38.48	27.57	20.17	N/A	38.88	N/A	N/A
	2	Theo.	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	51.08	41.90	31.83	24.15	N/A	42.85	N/A	N/A
30	0.5	Theo.	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	58.26	32.09	18.98	13.22	N/A	33.19	N/A	N/A
	1	Theo.	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.22	47.21	31.32	21.80	66.80	47.23	31.55	22.26

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	69.14	54.62	39.59	28.69	N/A	54.92	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	70.64	58.85	45.47	34.17	N/A	59.54	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	71.59	41.86	24.65	16.58	N/A	42.84	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.35	59.48	40.36	27.98	79.68	59.50	40.55	28.35
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.99	67.51	50.45	36.92	N/A	67.69	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.30	72.05	57.21	43.81	N/A	72.48	N/A	N/A

Table A5.5

			Welch's *t*-test				Student's *t*-test			
		SDR	**0.5**	**1**	**1.5**	**2**	**0.5**	**1**	**1.5**	**2**
n_1	**n_2/n_1**		**One uniform distribution. one normal distribution**							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	20.42	12.80	9.39	7.89	N/A	12.65	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	23.93	17.63	12.80	10.03	25.59	17.92	13.26	10.76
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	24.85	20.25	15.60	12.21	N/A	21.21	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	25.18	21.86	17.51	13.96	N/A	23.51	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	41.12	23.05	14.50	10.70	N/A	23.28	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	47.61	33.22	22.41	16.10	48.61	33.32	22.69	16.62
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	49.94	38.68	28.00	20.70	N/A	39.32	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	51.02	41.85	32.11	24.37	N/A	43.06	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	58.45	33.13	19.88	13.94	N/A	33.47	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.20	47.48	31.89	22.40	66.79	47.52	32.11	22.82

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	69.04	54.67	39.88	29.09	N/A	55.06	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	70.54	58.97	45.49	34.37	N/A	59.72	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	71.68	42.71	25.56	17.40	N/A	43.11	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.38	59.70	40.78	28.39	79.71	59.72	40.97	28.76
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.95	67.56	50.71	37.23	N/A	67.77	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.22	72.01	57.36	43.98	N/A	72.45	N/A	N/A

Table A5.6

			Welch's *t*-test				Student's *t*-test			
		SDR	**0.5**	**1**	**1.5**	**2**	**0.5**	**1**	**1.5**	**2**
n_1	n_2/n_1		**One uniform distribution. one double exponential distribution**							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	20.94	13.79	9.86	7.78	N/A	12.47	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	23.89	18.23	13.67	10.69	25.76	18.69	14.14	11.40
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	24.86	20.65	16.16	12.90	N/A	22.45	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	25.21	21.98	17.93	14.51	N/A	24.67	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	41.94	25.11	16.30	11.73	N/A	23.66	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	47.58	34.07	23.70	17.29	48.65	34.23	23.97	17.80
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	49.94	39.01	28.85	21.57	N/A	40.08	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	51.05	42.06	32.56	24.95	N/A	43.77	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	58.81	35.23	22.08	15.49	N/A	34.18	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.36	48.07	33.04	23.52	66.97	48.14	33.26	23.95

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	69.09	54.99	40.58	29.89	N/A	55.59	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	70.55	59.12	45.93	35.03	N/A	60.09	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	71.88	44.58	27.75	19.02	N/A	43.81	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	79.39	59.98	41.75	29.71	79.73	60.02	41.92	30.06
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	81.92	67.67	51.22	37.91	N/A	67.98	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	83.27	72.10	57.66	44.32	N/A	72.58	N/A	N/A

Table A5.7

			Welch's *t*-test				Student's *t*-test			
		SDR	0.5	1	1.5	2	0.5	1	1.5	2
n_1	n_2/n_1		Two normal skewed with positive skewness. equal shapes (G1 = +0.79)							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	23.33	9.68	6.37	5.84	N/A	13.97	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	29.82	18.48	10.84	7.59	31.02	18.90	11.69	8.56
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	31.54	23.42	15.27	10.56	N/A	22.23	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	32.34	26.06	18.76	13.31	N/A	24.43	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	42.95	20.54	10.81	7.55	N/A	23.99	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	49.77	34.11	20.87	13.62	50.62	34.23	21.33	14.31
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	52.06	40.39	28.05	19.26	N/A	40.17	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	53.27	44.04	32.92	23.90	N/A	44.15	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	58.95	31.61	16.58	10.63	N/A	33.89	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.00	47.97	30.78	20.10	66.58	48.02	31.09	20.63

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	68.40	55.36	40.00	28.05	N/A	55.75	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	69.53	59.38	46.17	34.16	N/A	60.38	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	71.31	41.93	22.69	14.03	N/A	43.30	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	77.87	59.89	40.22	26.70	78.23	59.92	40.44	27.15
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	80.02	67.52	50.86	36.47	N/A	68.20	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	81.17	71.52	57.65	43.85	N/A	72.83	N/A	N/A

Table A5.8

			Welch's *t*-test				Student's *t*-test			
		SDR	**0.5**	**1**	**1.5**	**2**	**0.5**	**1**	**1.5**	**2**
n_1	**n_2/n_1**		**Two normal skewed with positive skewness. equal shapes (G1 = +0.79)**							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	**N/A**	**13.53**	**N/A**	**N/A**
		Obs.	16.65	8.72	6.43	5.89	N/A	10.07	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	20.91	14.08	9.43	7.17	22.67	14.45	9.97	7.93
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	22.03	17.17	12.31	9.41	N/A	17.97	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	22.57	18.91	14.49	11.18	N/A	20.51	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	39.65	18.90	10.48	7.44	N/A	20.52	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	47.47	30.95	19.33	13.09	48.60	31.09	19.70	13.68
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	50.42	37.27	25.64	17.98	N/A	37.74	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	51.81	41.10	30.11	21.99	N/A	42.05	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	59.08	30.15	16.11	10.49	N/A	31.50	N/A	N/A
	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	68.16	46.80	29.61	19.55	68.78	46.87	29.89	20.05

	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	71.47	54.91	38.51	27.03	N/A	55.13	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	73.16	59.70	44.88	32.82	N/A	60.06	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	73.61	41.06	22.17	13.81	N/A	42.00	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	81.85	60.13	39.32	26.14	82.17	60.17	39.53	26.55
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	84.60	68.87	50.26	35.73	N/A	68.94	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	86.07	73.74	57.53	43.08	N/A	73.76	N/A	N/A

Table A5.9

n_1	n_2/n_1	SDR	Welch's *t*-test 0.5	1	1.5	2	Student's *t*-test 0.5	1	1.5	2
			One chi-square with 2 degrees of freedom. and one normal skewed with negative skewness (G1 = -0.79)							
10	**0.5**	**Theo.**	**22.34**	**12.76**	**8.81**	**7.19**	N/A	**13.53**	N/A	N/A
		Obs.	30.61	18.78	14.47	12.32	N/A	22.00	N/A	N/A
	1	**Theo.**	**25.93**	**18.51**	**13.07**	**10.07**	**26.79**	**18.51**	**13.21**	**10.28**
		Obs.	37.33	25.35	18.02	14.39	38.28	25.96	18.88	15.40
	1.5	**Theo.**	**27.09**	**21.42**	**16.01**	**12.39**	N/A	**21.68**	N/A	N/A
		Obs.	39.60	29.78	21.52	16.75	N/A	28.10	N/A	N/A
	2	**Theo.**	**27.66**	**23.13**	**18.12**	**14.27**	N/A	**23.86**	N/A	N/A
		Obs.	40.54	32.85	24.50	18.90	N/A	29.66	N/A	N/A
20	**0.5**	**Theo.**	**42.2**	**23.13**	**14.35**	**10.51**	N/A	**23.86**	N/A	N/A
		Obs.	46.56	27.92	19.24	14.95	N/A	30.17	N/A	N/A
	1	**Theo.**	**48.85**	**33.79**	**22.58**	**16.18**	**49.58**	**33.79**	**22.71**	**16.4**
		Obs.	53.04	38.27	26.53	19.84	53.68	38.46	26.96	20.45
	1.5	**Theo.**	**51.27**	**39.42**	**28.34**	**20.79**	N/A	**39.66**	N/A	N/A
		Obs.	55.27	44.14	32.44	24.37	N/A	43.32	N/A	N/A
	2	**Theo.**	**52.52**	**42.86**	**32.53**	**24.56**	N/A	**43.48**	N/A	N/A
		Obs.	56.37	47.53	36.80	28.15	N/A	46.46	N/A	N/A
30	**0.5**	**Theo.**	**58.94**	**33.28**	**19.97**	**13.89**	N/A	**33.96**	N/A	N/A
		Obs.	59.47	36.58	24.02	17.88	N/A	38.30	N/A	N/A

	1	**Theo.**	**66.79**	**47.79**	**31.96**	**22.36**	**67.32**	**47.79**	**32.09**	**22.58**
		Obs.	66.04	49.74	34.81	25.41	66.51	49.83	35.10	25.85
	1.5	**Theo.**	**69.58**	**55.12**	**40.11**	**29.16**	N/A	**55.31**	N/A	N/A
		Obs.	68.19	56.40	42.48	31.84	N/A	56.30	N/A	N/A
	2	**Theo.**	**71**	**59.45**	**45.88**	**34.62**	N/A	**59.94**	N/A	N/A
		Obs.	69.19	60.19	48.02	37.16	N/A	**60.44**	N/A	N/A
40	**0.5**	**Theo.**	**71.87**	**42.86**	**25.58**	**17.31**	N/A	**43.48**	N/A	N/A
		Obs.	69.93	44.78	29.05	20.98	N/A	**46.07**	N/A	N/A
	1	**Theo.**	**79.41**	**59.81**	**40.89**	**28.49**	**79.77**	**59.81**	**41.01**	**28.71**
		Obs.	76.01	59.90	42.88	31.07	76.35	59.95	43.09	31.44
	1.5	**Theo.**	**81.95**	**67.77**	**50.82**	**37.25**	N/A	**67.92**	N/A	N/A
		Obs.	78.02	66.79	51.79	39.16	N/A	**67.12**	N/A	N/A
	2	**Theo.**	**83.21**	**72.25**	**57.54**	**44.08**	N/A	**72.61**	N/A	N/A
		Obs.	78.94	70.42	57.92	45.46	N/A	71.41	N/A	N/A

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Table A6.1

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**Normal distributions**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	29.36	29.23	5.27	5.00
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	29.32	29.24	5.52	5.04
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	29.24	29.36	5.81	4.96
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	29.36	29.35	6.05	5.05
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	29.35	29.26	6.32	4.96
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	56.33	56.20	5.52	4.98
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	56.40	56.32	6.07	5.04
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	56.50	56.33	6.62	5.06
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	56.35	56.34	7.15	5.03
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**

		Obs.	56.45	56.41	7.74	5.05
30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	75.42	75.03	5.80	5.01
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	75.38	75.18	6.64	5.03
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	75.35	75.30	7.45	4.99
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	75.35	75.29	8.26	5.08
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	75.40	75.35	9.16	5.03

Table A6.2

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**Double exponential distributions**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	35.22	35.14	4.53	4.20
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	35.38	35.15	5.29	4.67
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	35.39	35.32	5.69	4.77
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	35.30	35.17	6.04	4.87
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	35.29	35.25	6.35	4.91
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	59.58	59.30	5.28	4.67
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	59.68	59.45	6.04	4.86
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	59.64	59.53	6.67	4.90
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	59.63	59.57	7.23	4.94
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	59.71	59.53	7.81	4.97

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	76.13	75.83	5.67	4.81
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	76.04	75.94	6.67	4.94
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	76.06	75.99	7.55	4.98
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	76.18	76.07	8.43	4.97
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	76.12	76.04	9.20	4.98

Table A6.3

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**One normal and one double exponential distributions**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	29.29	29.24	4.56	4.21
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	29.30	29.27	5.29	4.63
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	29.29	29.37	5.66	4.81
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	29.35	29.33	6.04	4.88
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	29.29	29.30	6.34	4.89
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	56.42	56.17	5.30	4.61
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	56.48	56.33	6.02	4.80
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	56.48	56.33	6.70	4.89
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	56.43	56.41	7.25	4.96
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	56.51	56.34	7.76	4.96

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	75.46	75.02	5.66	4.79
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	75.34	75.25	6.68	4.93
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	75.44	75.34	7.55	4.98
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	75.39	75.30	8.36	4.96
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	75.38	75.34	9.19	5.00

Table A6.4

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**Two uniform distributions**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	26.02	25.95	5.62	5.45
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	26.00	26.02	5.57	5.15
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	26.01	26.04	5.84	5.09
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	26.09	25.98	6.06	5.07
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	26.10	25.93	6.28	5.07
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	54.68	54.50	5.62	5.14
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	54.79	54.61	6.06	5.07
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	54.67	54.61	6.60	5.04
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	54.75	54.69	7.19	5.03
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	54.69	54.64	7.68	5.02

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	75.08	74.66	5.86	5.09
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	75.07	74.92	6.64	5.04
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	75.07	74.90	7.47	5.07
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	75.02	74.91	8.27	5.02
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	75.12	74.99	9.09	5.07

Table A6.5

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**One uniform distribution. one normal distribution**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	26.06	25.98	5.30	5.00
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	26.06	25.96	5.54	5.01
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	26.12	26.03	5.75	5.05
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	26.00	26.02	6.10	4.99
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	26.05	26.07	6.31	5.00
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	54.71	54.40	5.53	5.02
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	54.78	54.64	6.06	5.01
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	54.70	54.77	6.63	5.00
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	54.69	54.62	7.15	5.00
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	54.72	54.71	7.77	5.06

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	75.06	74.73	5.81	5.06
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	75.11	74.89	6.66	5.03
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	75.07	74.95	7.46	5.03
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	75.10	74.95	8.34	5.01
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	75.11	75.00	9.16	5.05

Table A6.6

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**One uniform. one double exponential distributions**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	25.99	26.02	4.58	4.23
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	26.06	26.02	5.27	4.63
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	26.03	25.89	5.69	4.80
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	25.98	26.05	5.99	4.86
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	25.99	26.00	6.32	4.87
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	54.74	54.43	5.27	4.63
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	54.81	54.72	6.04	4.82
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	54.78	54.66	6.67	4.93
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	54.73	54.73	7.24	4.93
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	54.84	54.69	7.81	4.96

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	75.01	74.71	5.74	4.76
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	75.08	74.82	6.68	4.91
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	75.05	74.96	7.55	4.98
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	75.06	75.01	8.38	4.98
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	75.07	74.96	9.22	4.98

Table A6.7

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**Two normal skewed with (G1 = +0.79)**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	34.03	33.92	5.47	6.03
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	34.04	34.01	5.15	5.50
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	34.06	34.03	5.19	5.38
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	33.95	33.91	5.39	5.18
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	34.01	34.03	5.54	5.16
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	56.41	56.26	5.14	5.53
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	56.48	56.43	5.31	5.26
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	56.54	56.41	5.80	5.12
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	56.59	56.50	6.32	5.06
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	56.51	56.47	6.79	5.02

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	73.21	72.94	5.15	5.40
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	73.20	72.98	5.78	5.16
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	73.28	73.16	6.61	5.07
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	73.19	73.16	7.37	5.00
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	73.18	73.19	8.31	4.99

Table A6.8

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**N skewed with G1 = +0.79. and N skewed with G1 = -0.79**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	23.34	23.22	5.44	6.04
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	23.20	23.29	5.13	5.56
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	23.31	23.28	5.16	5.37
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	23.25	23.31	5.32	5.27
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	23.34	23.29	5.53	5.18
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	56.33	56.03	5.12	5.52
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	56.34	56.18	5.36	5.22
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	56.43	56.23	5.76	5.13
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	56.42	56.27	6.31	5.10
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	56.34	56.40	6.80	5.07

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	78.54	78.06	5.15	5.35
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	78.51	78.33	5.79	5.13
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	78.55	78.48	6.58	5.08
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	78.57	78.49	7.35	5.03
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	78.51	78.43	8.19	4.98

Table A6.9

			Welch's *t*-test			
		SDR	**0.01**	**0.1**	**10**	**100**
n1	**n2/n1**		**One chi(2). one normal skewed with G1 = -0.79**			
10	**1**	**Theo.**	**29.32**	**29.19**	**5.23**	**5**
		Obs.	42.69	42.73	7.22	6.22
	2	**Theo.**	**29.32**	**29.25**	**5.51**	**5.01**
		Obs.	42.80	42.75	7.07	5.71
	3	**Theo.**	**29.32**	**29.27**	**5.79**	**5.01**
		Obs.	42.83	42.73	7.25	5.58
	4	**Theo.**	**29.32**	**29.29**	**6.06**	**5.01**
		Obs.	42.85	42.77	7.41	5.48
	5	**Theo.**	**29.32**	**29.29**	**6.33**	**5.01**
		Obs.	42.79	42.80	7.61	5.43
20	**1**	**Theo.**	**56.45**	**56.12**	**5.51**	**5.01**
		Obs.	59.37	59.07	7.11	5.76
	2	**Theo.**	**56.45**	**56.29**	**6.08**	**5.01**
		Obs.	59.32	59.17	7.43	5.47
	3	**Theo.**	**56.45**	**56.34**	**6.63**	**5.02**
		Obs.	59.31	59.24	7.86	5.36
	4	**Theo.**	**56.45**	**56.37**	**7.18**	**5.02**
		Obs.	59.31	59.25	8.35	5.28
	5	**Theo.**	**56.45**	**56.39**	**7.72**	**5.03**
		Obs.	59.32	59.30	8.81	5.31

30	**1**	**Theo.**	**75.39**	**75.04**	**5.8**	**5.01**
		Obs.	72.15	71.91	7.25	5.53
	2	**Theo.**	**75.39**	**75.22**	**6.65**	**5.02**
		Obs.	72.12	72.05	7.91	5.35
	3	**Theo.**	**75.4**	**75.28**	**7.49**	**5.03**
		Obs.	72.15	72.00	8.62	5.28
	4	**Theo.**	**75.4**	**75.31**	**8.31**	**5.03**
		Obs.	72.03	72.02	9.41	5.25
	5	**Theo.**	**75.4**	**75.32**	**9.13**	**5.04**
		Obs.	72.06	72.08	10.17	5.24

References

Gleason. J. (2013). Comparative Power Of The Anova. Randomization Anova. And Kruskal-Wallis Test. Retrieved from http://digitalcommons.wayne.edu/oa_dissertations/658/

Hayes. A. F.. & Cai. L. (2007). Further evaluating the conditional decision rule for comparing two independent means. *British Journal of Mathematical and Statistical Psychology*. *60*(2). 217–244.

R: Skew Normal Distribution. (n.d.). Retrieved January 22. 2017. from http://finzi.psych.upenn.edu/library/fGarch/html/dist-snorm.html

R: The double exponential (Laplace) distribution. (n.d.). Retrieved January 22. 2017. from https://artax.karlin.mff.cuni.cz/r-help/library/smoothmest/html/ddoublex.html

R: The (non-central) Chi-Squared Distribution. (n.d.). Retrieved January 22. 2017. from https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Chisquare.html

R: The Normal Distribution. (n.d.). Retrieved January 20. 2017. from https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Normal.html

R: The Uniform Distribution. (n.d.). Retrieved January 22. 2017. from https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html

Wilcox. R. R. (1998). How many discoveries have been lost by ignoring modern statistical methods? American Psychologist. 53(3). 300–314. DOI: https://doi.org/10.1037/0003-066X.53.3.300

Author's note: All code needed to recreate the simulations resulting in the figures and appendices are available at https://osf.io/bver8/files/. as are as the .txt files containing the results of all simulations.