Delacre et al: Welch's t-test as Default

2007).

or when $n_1 = 3$ is sampled from a uniform distribution

and $n_3 = 3$ is sampled from a double exponential dis-

tribution). However, with extremely small sample sizes

 $(N \le 5)$, the estimate of means and standard deviations

is extremely inaccurate anyway. As we mentioned in

table A2 (see the additional file), the smaller the sam-

ple size, the further the average standard deviation is

from the population standard deviation, and the larger

² This is called the Behren-Fisher problem (Hayes & Cai,

³ In a simulation that explored Type 1 error rates, we

varied the size of the first sample from 10 to 40 in

steps of 10 and the sample sizes ratio and the stand-

ard deviation ratio from 0.5 to 2 in steps of 0.5, result-

ing in 64 simulations designs. Each design was tested

1,000,000 times. Considering these parameter values, we found that the alpha level can be inflated up to

0.11 or deflated down to 0.02 (see the additional file).

⁴ Other variants have been proposed, such as the percent

⁵ Because sample sizes are equal for each pair of sam-

ples, which sample has the bigger standard deviation

is not applicable. In this way, SDR = X will return the

same answer in terms of percent power of Levene's

test as SDR = 1/X. For example, SDR = 2 will return the

⁶ For example, many statistical users believe that the

Mann-Whitney non-parametric test can cope with both

normality and homosedasticity issues (Ruxton, 2006).

This assumption is false, since the Mann-Whitney test

remains sensitive to heterosedasticity (Grissom, 2000;

⁷ Like Bryk and Raudenbush (1988), we note that une-

qual variances between groups does not systematically

mean that population variances are different: stand-

ard deviation ratios are more or less biased estimates

of population variance (see table A2 in the additional

file). Differences can be a consequence of bias in meas-

urement, such as response styles (Baumgartner &

Steenkamp, 2001). However, there is no way to deter-

mine what part of the variability is due to error rather

Nachar, 2008; Neuhäuser & Ruxton, 2009).

the dispersion around this average.

trimmed mean (Lim & Loh, 1996).

same answer as SDR = $\frac{1}{2}$ = 0.5.

than the true population value.

right quadrant reports all p-values less than 0.05 according to Welch's t-test, but greater than 0.05 according to

Student's t-test. The larger the standard deviations ratio and the greater the sample sizes ratio, the larger the difference between p-values from Welch's t-test and Student's t-test.

When the assumption of equal variances is not met, Stu-

variance differs between groups (SDR = 2) but sample sizes

are equal $(n_1 = n_2 = 50)$. **Figure 3c** shows Student's *p*-values

plotted against Welch's p-values of Scenario 3, where both

sample sizes and variances are unequal between groups and

the larger variance is associated with the larger sample size

(SDR = 2). And, finally, **figure 3d** plots Student's *p*-values

against Welch's p-values of Scenario 4, where the greater vari-

Dots are marked on the black diagonal line when both

tests return the same p-value. The top left quadrant contains

all p-values less than 0.05 according to a Student's t-test, but

greater than 0.05 according to Welch's t-test. The bottom

ance is associated with the smaller sample size (SDR = 0.5).

dent's t-test yields unreliable results, while Welch's t-test controls Type 1 error rates as expected. The widely recom-

Conclusion

mended two-step approach, where the assumption of equal variances is tested using Levene's test and, based on the outcome of this test, a choice of Student's t-test or Welch's t-test is made, should not be used. Because the statistical power for this test is often low, researchers will inappropriately choose Student's t-test instead of more robust alternatives. Furthermore, as we have argued, it is reasonable to assume that variances are unequal in many studies in psy-

chology, either because measured variables are used (e.g.,

age, culture, gender) or because, after random assignment

to conditions, variance is increased in the experimental con-

dition compared to the control condition due to the experi-

mental manipulation. As it is explained in the additional

file, Yuen's t-test is not a better test than Welch's t-test, since it often suffers high departure from the alpha risk of 5 percent. Therefore, we argue that Welch's t-test should always be used instead of Student's *t*-test. When using Welch's t-test, a very small loss in statistical power can occur, depending on the shape of the distributions. However, the Type 1 error rate is more stable

when using Welch's t-test compared to Student's t-test, and

Welch's t-test is less dependent on assumptions that cannot

be easily tested. Welch's t-test is available in practically all

statistical software packages (and already the default in R

and Minitab) and is easy to use and report. We recommend that researchers make clear which test they use by specifying the analysis approach in the result section. Convention is a weak justification for the current practice of using Student's t-test by default. Psychologists should pay more attention to the assumptions underlying

the tests they perform. The default use of Welch's t-test is

a straightforward way to improve statistical practice.

Notes

¹ There is a Type 1 error rate inflation in a few cases where sample sizes are extremely small and SDR is big (e.g., when $n_1 = n_2 = 3$ are sampled from uniform distributions and SDR = 2, the Type 1 error rate = 0.083; Also known as the Satterwaite's test, the Smith/Welch/ Satterwaite test, the Aspin-Welch test, or the unequal

variances t-test. Competing Interests

Additional File

The additional file for this article can be found as follows:

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Author's Note

All code needed to recreate the simulations resulting in the figures and appendices is available at https://osf.io/

bver8/files/, as are as the .txt files containing the results of all simulations.

The authors have no competing interests to declare.