Supplemental Material

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Supplemental Material 1 : theoretical bias and variance of estimators

Theoretical bias

For all “biased” estimators, when the population effect size is null so is the bias. We will therefore focus on configurations where there is a non-null population effect size. The sampling distribution of Cohen’s *d* (and therefore its bias) is only known under the assumptions of normality and homoscedasticity. On the other hand, the biases of Glass’s *d*, Cohen’s d^* and Shieh’s *d* are theoretically known for all configurations where the normality assumption is met. In order to simplify the analysis of their bias, it is convenient to subdivide all configurations into 3 conditions :

- when population variances are equal across groups;
- when population variances are unequal across groups, with equal sample sizes;
- when population variances are unequal across groups, with unequal sample sizes.

Preliminary note. For all previously mentioned estimators (Cohen’s *d*, Glass’s *d*, Cohen’s d^* and Shieh’s *d*), the theoretical expectation is computed by multiplying the population effect size (respectively δ_{Cohen}, δ_{Glass}, δ^*_{Cohen} and δ_{Shieh}) by the following multiplier coefficient:

$$\gamma = \frac{\sqrt{\frac{df}{2}} \times \Gamma\frac{df-1}{2}}{\Gamma\frac{df}{2}} \tag{1}$$

where *df* are the degrees of freedom (see the main article). γ is *always* positive, meaning that when the population effect size is not zero, all estimators will overestimate the population effect size. Moreover, its limit tends to 1 when the degrees of freedom (*df*) tend to infinity, meaning that the larger the degrees of freedom, the lower the bias.

While we focus on the theoretical bias of biased estimators when the normality assumption is met, it is interesting to notice that our main conclusions seem to generalize to :

- biased estimators when samples are extracted from symmetric distributions;

- unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

Cohen's *d* (see Table 2). Under the assumptions that independant residuals are normally distributed with equal variances, the **bias** of Cohen's d is a function of total sample size (N) and the population effect size (δ_{Cohen}) :

- The larger the population effect size, the more Cohen's d will overestimate δ_{Cohen};
- The larger the total sample size, the lower the bias (see Figure SM1.1);

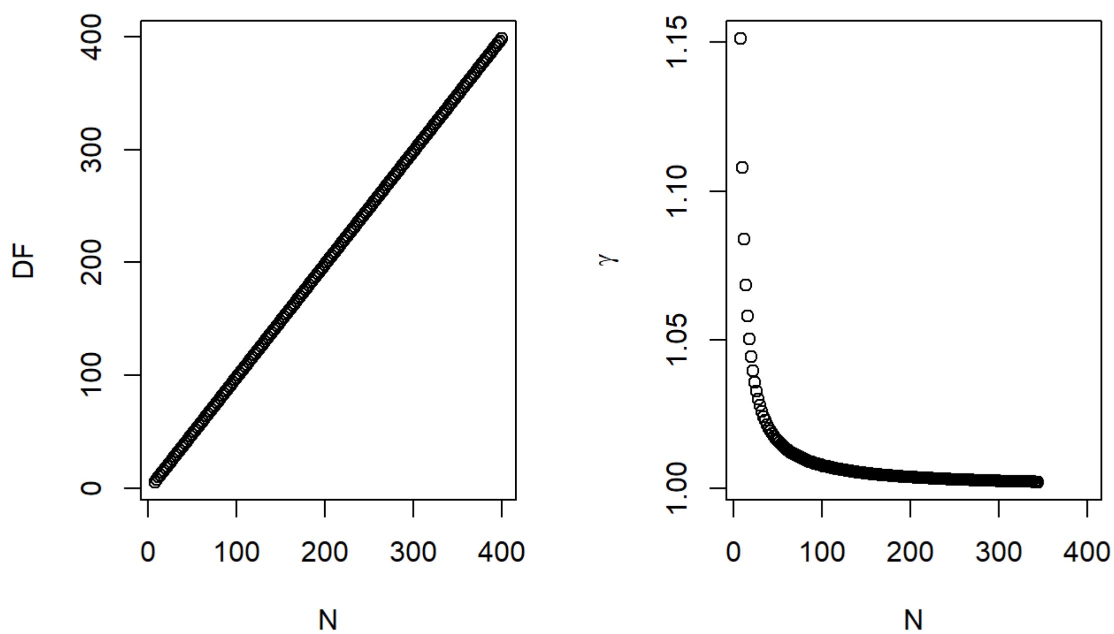

Figure SM1.1 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d, when variances are equal across groups, as a function of the total sample size (N).

- Of course, considering the degrees of freedom, the sample size ratio does not matter (i.e. the bias will decrease when increasing n_1, n_2 or both sample sizes).

Glass's *d* (see Table 3). Because degrees of freedom depend only on the control group size (neither on σ_1 nor on σ_2), there is no need to distinguish between cases where there is homoscedasticity or heteroscedasticity!

The **bias** of Glass's d is a function of the control group size (n_c) and the population effect size (δ_{Glass}) :

- The larger the population effect size, the more Glass's d will overestimate δ_{Glass};
- The larger the size of the control group, the lower the bias (see the two top plots in Figure SM1.2). On the other hand, increasing the experimental group size does not impact the bias (see the two bottom plots in Figure SM1.2).

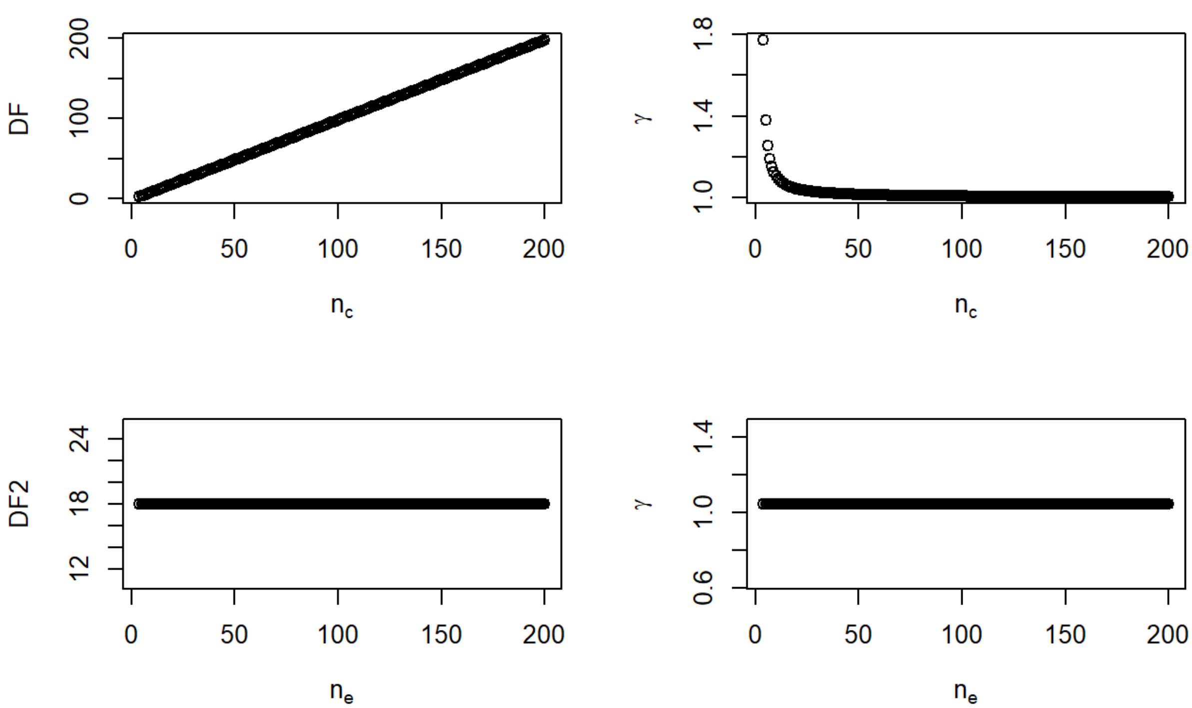

Figure SM1.2 : Degrees of freedom (DF) and γ, when computing the bias of Glass's d, when variances are equal across groups, as a function of n_c (top) and n_e (bottom).

Cohen's d^* (see Table 3).

When variances are equal across populations.

When $\sigma_1 = \sigma_2 = \sigma$:

$$df_{Cohen's\ d^*} = \frac{(n_1-1)(n_2-1)(2\sigma^2)^2}{(n_2-1)\sigma^4 + (n_1-1)\sigma^4} = \frac{(n_1-1)(n_2-1)\times 4\sigma^4}{\sigma^4(n_1+n_2-2)} = \frac{4(n_1-1)(n_2-1)}{n_1+n_2-2}$$

One can see that degrees of freedom depend only on the total sample size (N) and the sample size allocation ratio $\left(\frac{n_2}{n_1}\right)$. As a consequence, the **bias** of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the sample size allocation ratio $\left(\frac{n_2}{n_1}\right)$ and the total sample size (N).

- The further the sample size allocation ratio is from 1, the larger the bias (see Figure SM1.3);

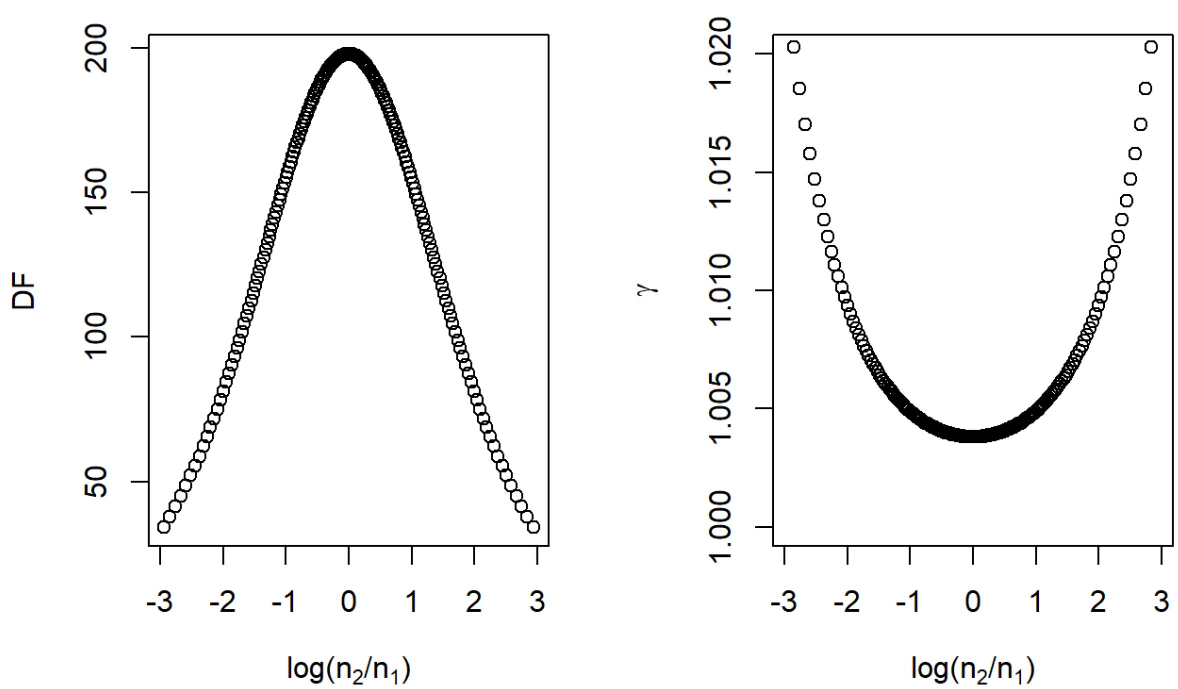

Figure SM1.3 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio $log\left(\frac{n_2}{n_1}\right)$.

- The larger the population effect size, the more Cohen's d^* will overestimate δ^*_{Cohen};
- The larger the total sample size, the lower the bias (see Figure SM1.4).

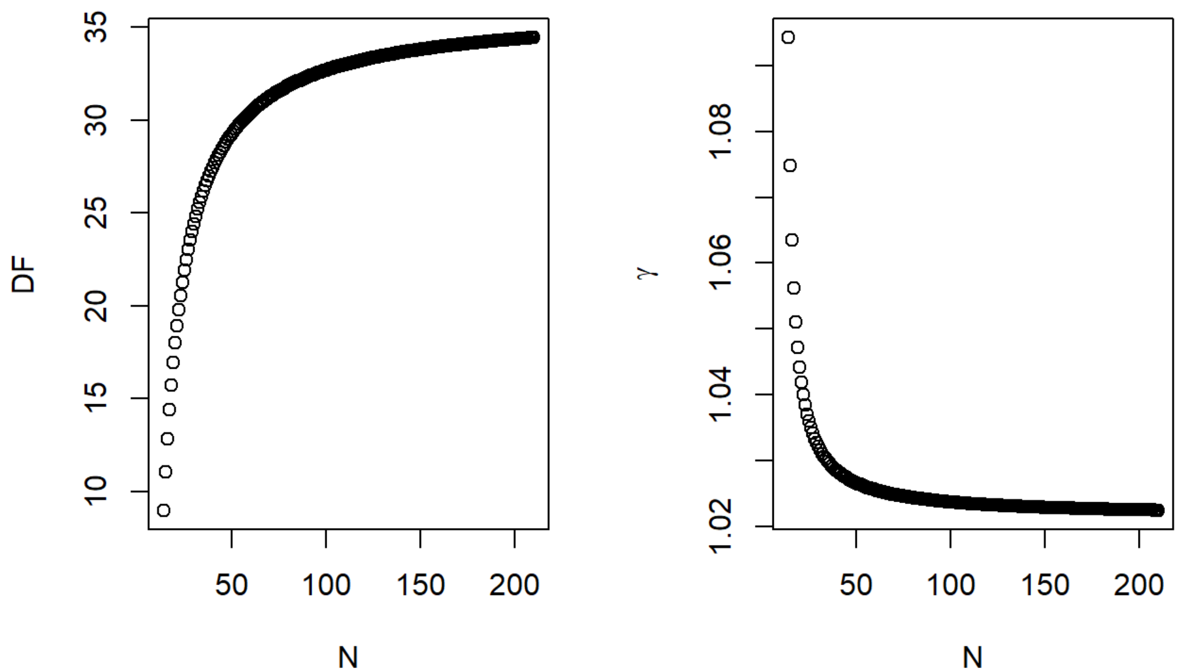

Figure SM1.4 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances are equal across groups, as a function of the total sample size (N).

When variances are unequal across populations, with equal sample sizes.

When $n_1 = n_2 = n$:

$$df_{Cohen's\ d^*} = \frac{(n-1)^2(\sigma_1^2+\sigma_2^2)^2}{(n-1)(\sigma_1^4+\sigma_2^4)} = \frac{(n-1)(\sigma_1^4+\sigma_2^4+2\sigma_1^2\sigma_2^2)}{\sigma_1^4+\sigma_2^4}$$

One can see that degrees of freedom depend only on the total sample size (N) and the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$. As a consequence, the **bias** of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$ and the total sample size (N) :

- The larger the population effect size, the more Cohen's d^* will overestimate δ^*_{Cohen};
- The further the SD-ratio is from 1, the larger the bias (see Figure SM1.5);

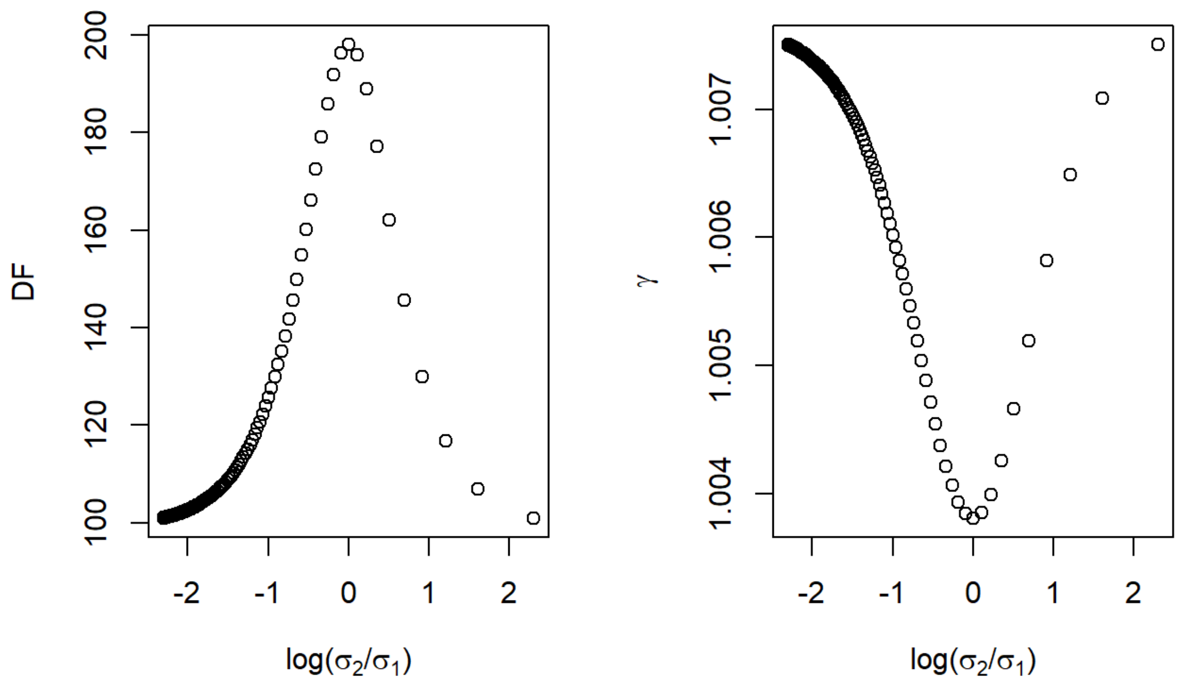

Figure SM1.5 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

- The larger the total sample size, the lower the bias (see Figure SM1.6).

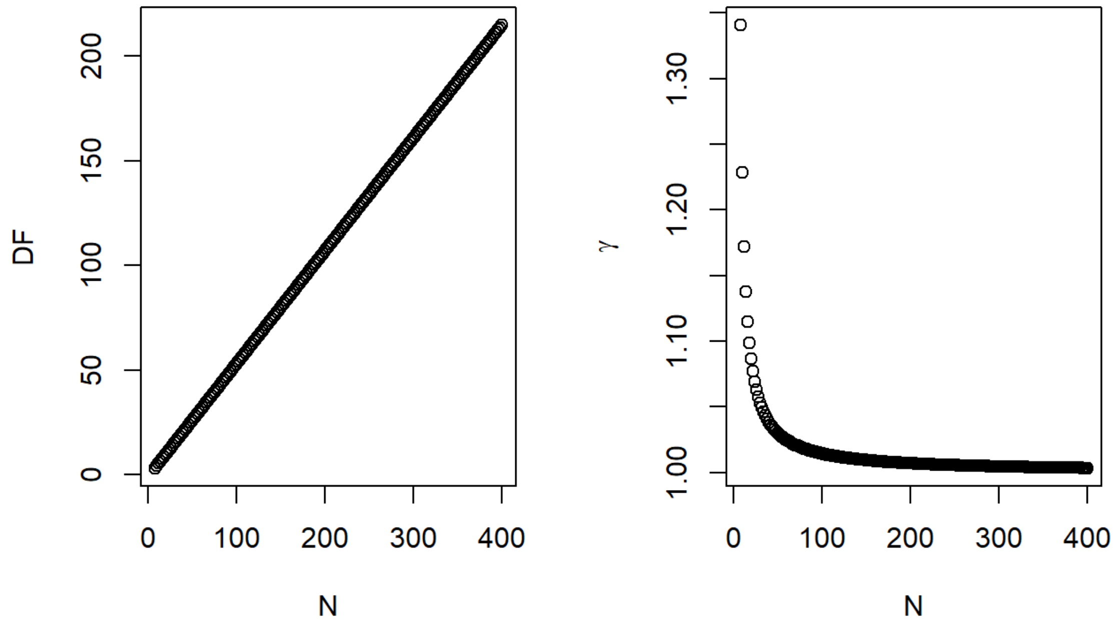

Figure SM1.6 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Note : for a constant SD-ratio, σ_1 and σ_2 do not matter (see Figure SM1.7).

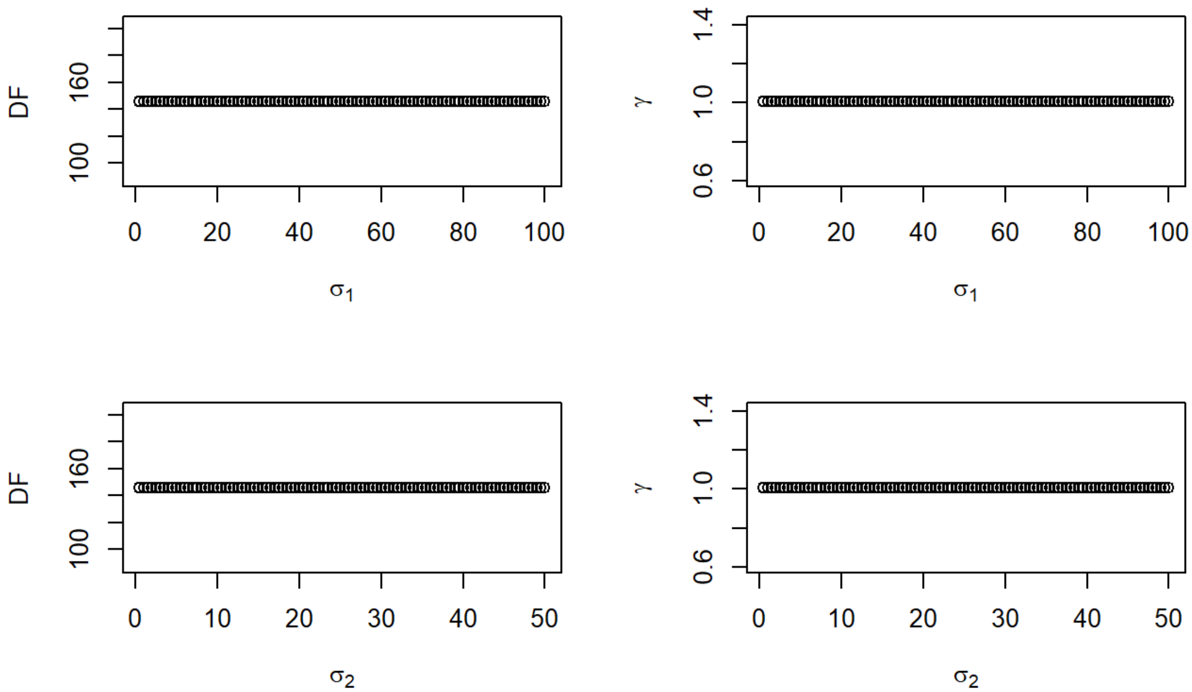

Figure SM1.7 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 (top plots) and σ_2 (bottom plots), for a constant SD-ratio.

When variances are unequal across populations, with unequal sample sizes.

The **bias** of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the total sample size (N), and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the population effect size, the more Cohen's d^* will overestimate δ^*_{Cohen};

- The larger the total sample size, the lower the bias (see Figure SM1.8);

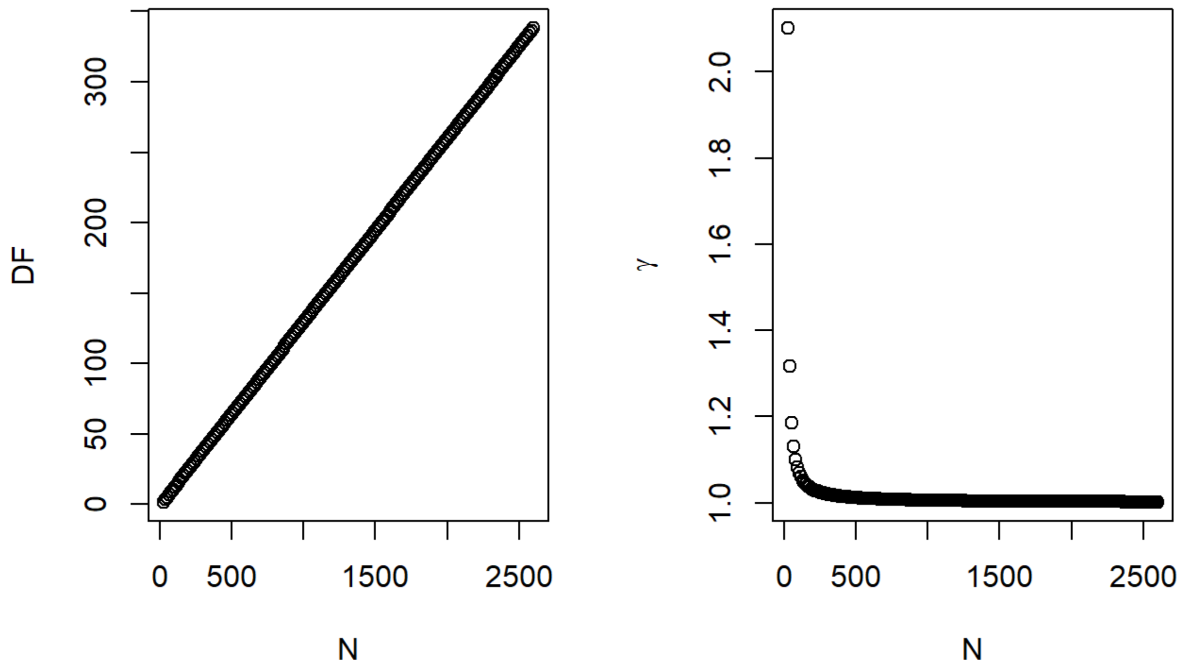

Figure SM1.8 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

- The smallest bias always occurs when there is a positive pairing between variances and sample sizes, because one gives more weight to the smallest variance, in the denominator of the *df* computation. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest bias (see Figure SM1.9). This can be explained by splitting the numerator and the denominator in the *df* computation.

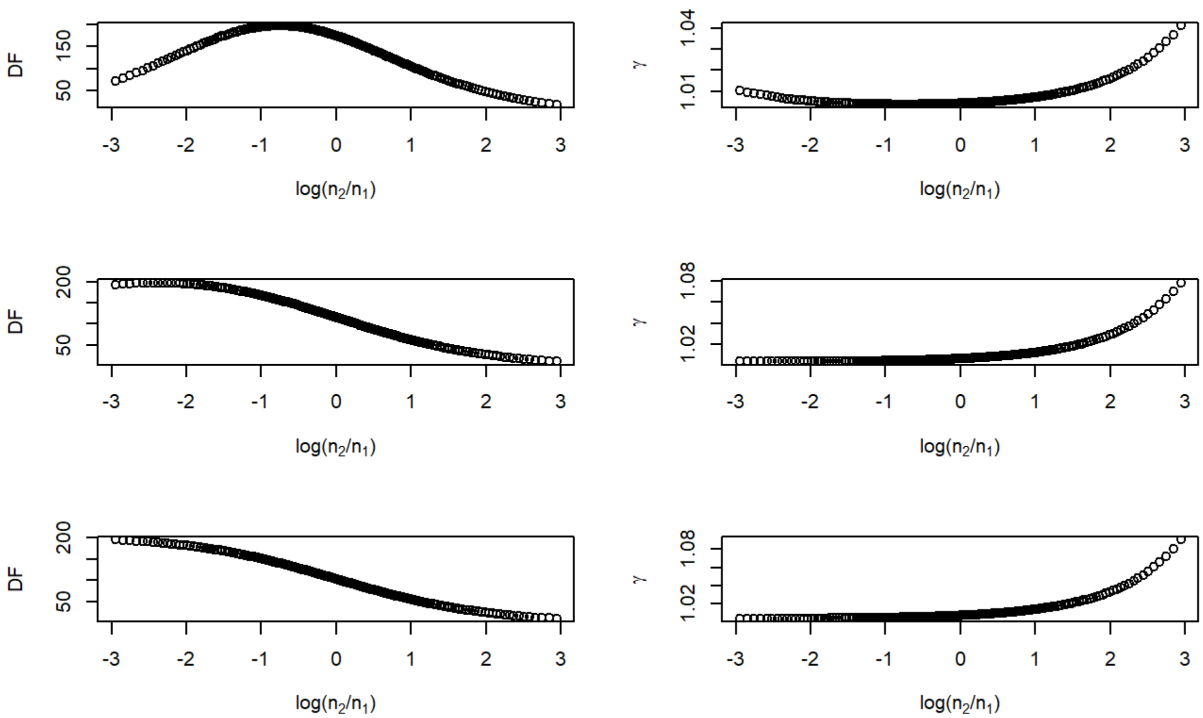

Figure SM1.9 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^* when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

As illustrated in Figure SM1.10, for any SD-ratio, the numerator of the degrees of freedom will be maximized when sample sizes are equal across groups (and is not impacted by the SD-ratio). On the other hand, the denominator will be minimized when there is a positive pairing between variances and sample sizes. For example, when $\sigma_1 > \sigma_2$, the smallest denominator occurs when $\frac{n_2}{n_1}$ reaches its minimum value and the further from 1 the SD-ratio, the larger the impact of the sample sizes ratio on the denominator.

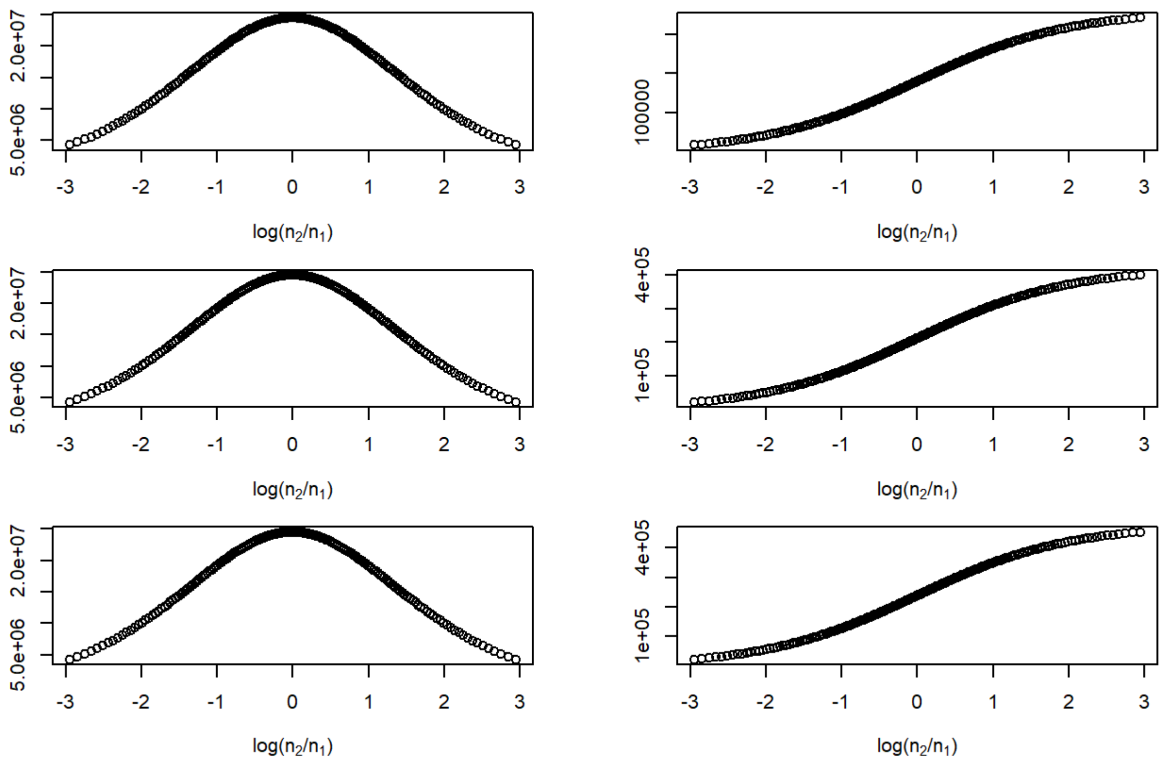

Figure SM1.10 : Numerator and denominator of the degrees of freedom (DF) computation, when computing the bias of Cohen's d^* when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

Note : for a constant SD-ratio, the variance does not matter. (See Figure SM1.11).

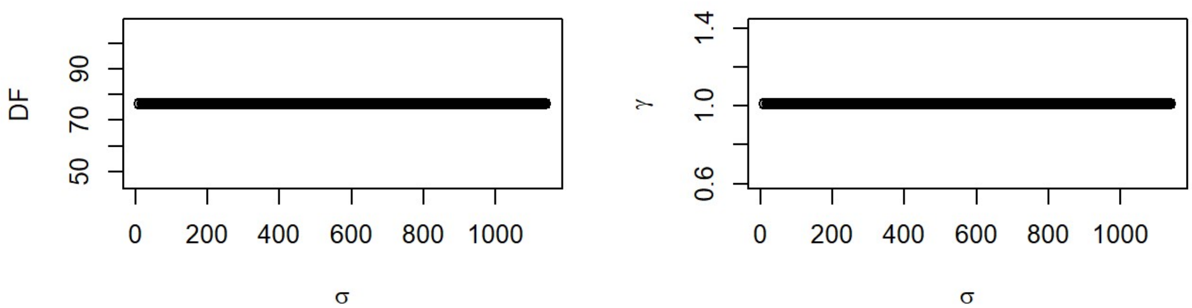

Figure SM1.11 : Degrees of freedom (DF) and γ, when computing the bias of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of $\sigma = \frac{(\sigma_1^2+\sigma_2^2)}{2}$, for a constant SD-ratio.

Shieh's *d* (see Table 3).

When variances are equal across populations.

When $\sigma_1 = \sigma_2 = \sigma$:

$$df_{Shieh's\ d} = \frac{\left(\frac{n_2\sigma^2+n_1\sigma^2}{n_1 n_2}\right)^2}{\frac{(n_2-1)\left(\frac{\sigma^2}{n_1}\right)^2+(n_1-1)\left(\frac{\sigma^2}{n_2}\right)^2}{(n_1-1)(n_2-1)}}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{[\sigma^2(n_1+n_2)]^2}{n_1^2 n_2^2} \times \frac{(n_1-1)(n_2-1)}{(n_2-1)\times\frac{\sigma^4}{n_1^2}+(n_1-1)\times\frac{\sigma^4}{n_2^2}}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{\sigma^4 N^2}{n_1^2 n_2^2} \times \frac{(n_1-1)(n_2-1)}{\sigma^4\left(\frac{n_2-1}{n_1^2}+\frac{n_1-1}{n_2^2}\right)}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{N^2(n_1-1)(n_2-1)}{n_1^2 n_2^2\left(\frac{n_2^2(n_2-1)+n_1^2(n_1-1)}{n_1^2 n_2^2}\right)}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$$

One can see that degrees of freedom depend only on the total sample size (N) and the sample size allocation ratio $\left(\frac{n_2}{n_1}\right)$. As a consequence, the **bias** of Shieh's d is a function of the population effect size (δ_{Shieh}), the sample size allocation ratio $\left(\frac{n_2}{n_1}\right)$ and the total sample size (N).

- The larger the population effect size, the more Shieh's d will overestimate δ_{Shieh};
- The further the sample size allocation ratio is from 1, the larger the bias (see Figure SM1.12);

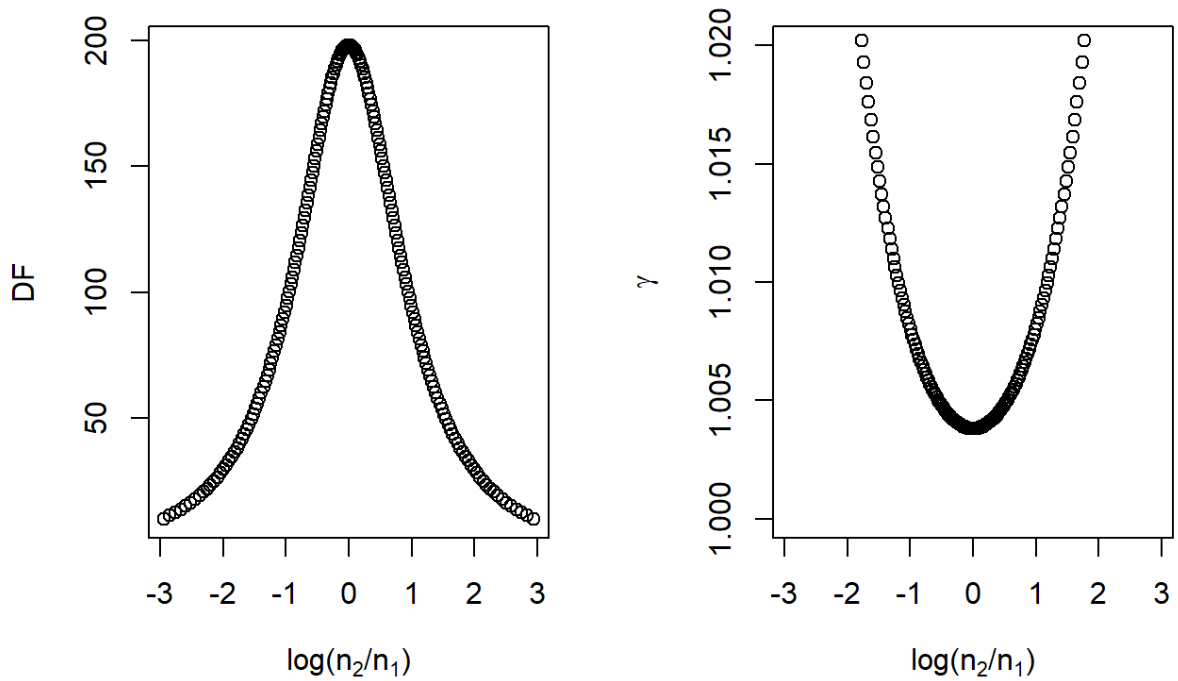

Figure SM1.12 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

- For a constant sample sizes ratio, the larger the total sample size, the lower the bias (see Figure SM1.13).

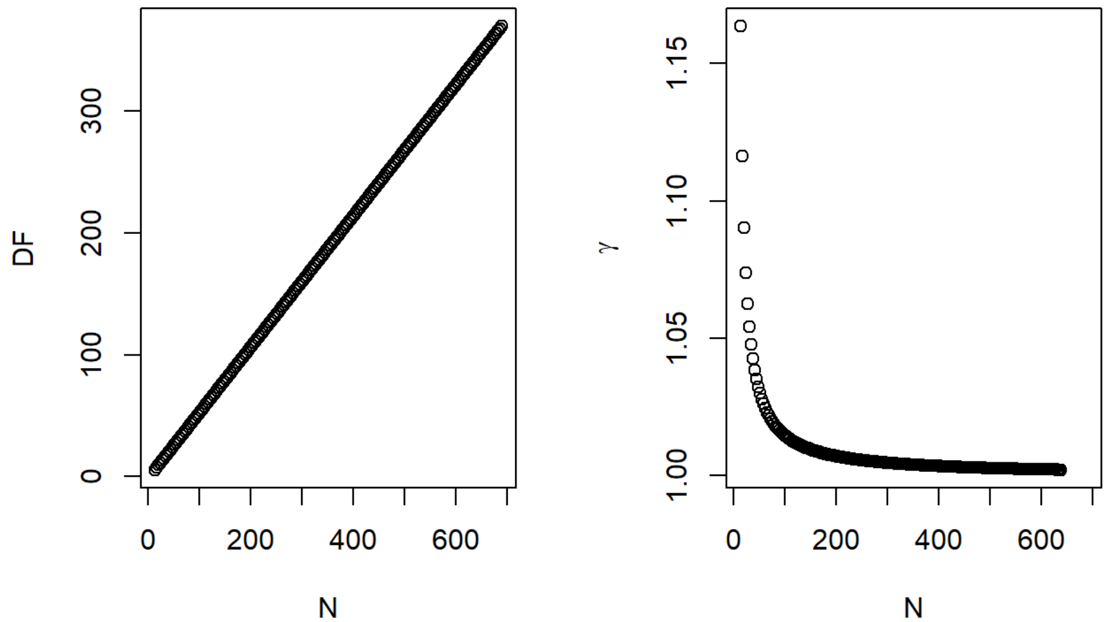

Figure SM1.13 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances are equal across groups, as a function of the total sample size (N).

Note : when computing Cohen's d^*, degrees of freedom increased when adding subjects in either one or both groups, even when the sample size ratio increased. When computing Shieh's d, this is not true anymore : there is a larger impact of the sample sizes ratio such that moving the sample sizes ratio away from 1 when adding subjects in only one group can decrease the degrees of freedom and therefore, increase the bias (See Figure SM1.14).

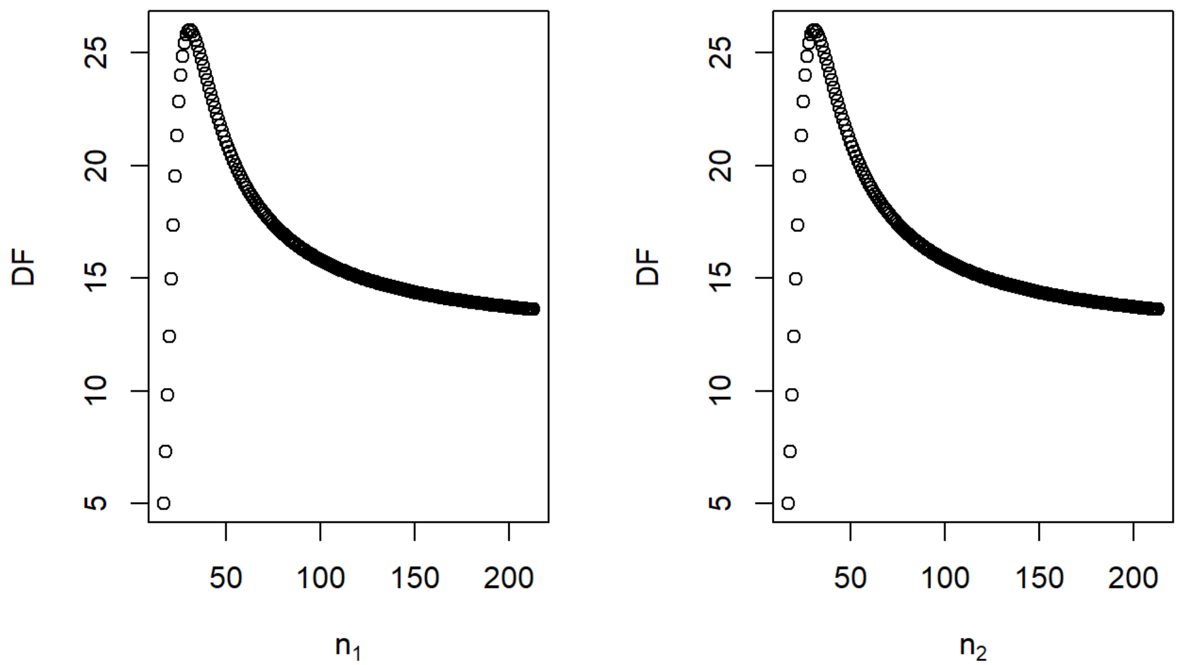

Figure SM1.14 : Degrees of freedom (DF), when computing the bias of Shieh's d, when variances are equal across groups, when adding subjects only in the first group (left) or in the second group (right).

When variances are unequal across populations, with equal sample sizes.

When $n_1 = n_2 = n$:

$$df_{Shieh's\ d} = \frac{\left(\frac{\sigma_1^2+\sigma_2^2}{n}\right)^2}{\frac{(\sigma_1^2/n)^2+(\sigma_2^2/n)^2}{n-1}}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{(\sigma_1^2+\sigma_2^2)^2}{n^2} \times \frac{n-1}{\frac{\sigma_1^4+\sigma_2^4}{n^2}}$$

$$\leftrightarrow df_{Shieh's\ d} = \frac{(\sigma_1^2+\sigma_2^2)^2 \times (n-1)}{\sigma_1^4+\sigma_2^4}$$

One can see that degrees of freedom depend on the total sample size (N) and the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$. As a consequence, the bias depends on the population effect size (δ_{Shieh}), the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$ and the total sample size (N).

- The larger the population effect size, the more Shieh's d will overestimate δ_{Shieh};

- The further the SD-ratio is from 1, the larger the bias (see Figure SM1.15);

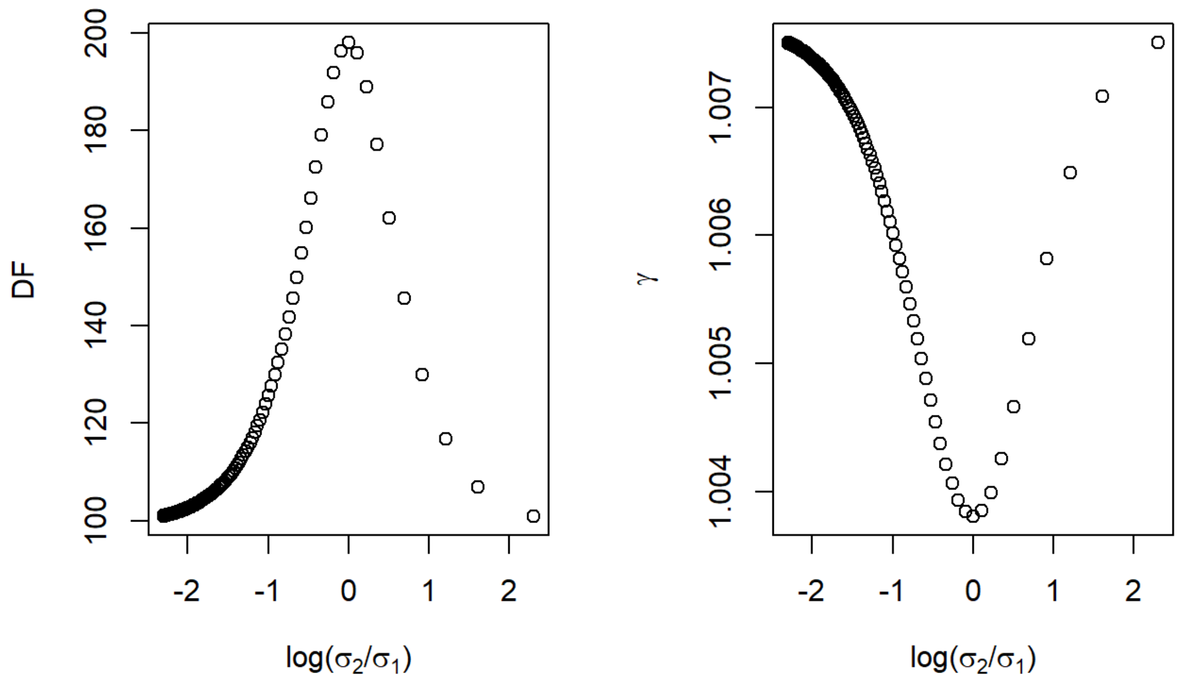

Figure SM1.15 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

- The larger the total sample size, the lower the bias (see Figure SM1.16);

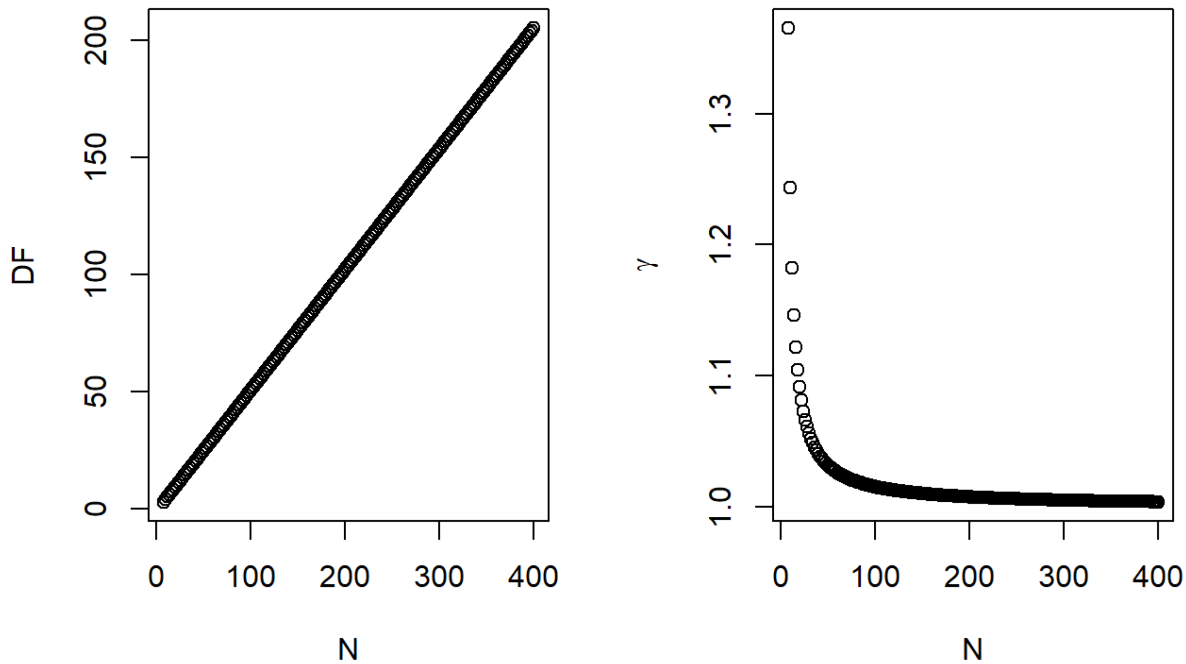

Figure SM1.16 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Note : for a constant SD-ratio, σ_1 and σ_2 do not matter (see Figure SM1.17).

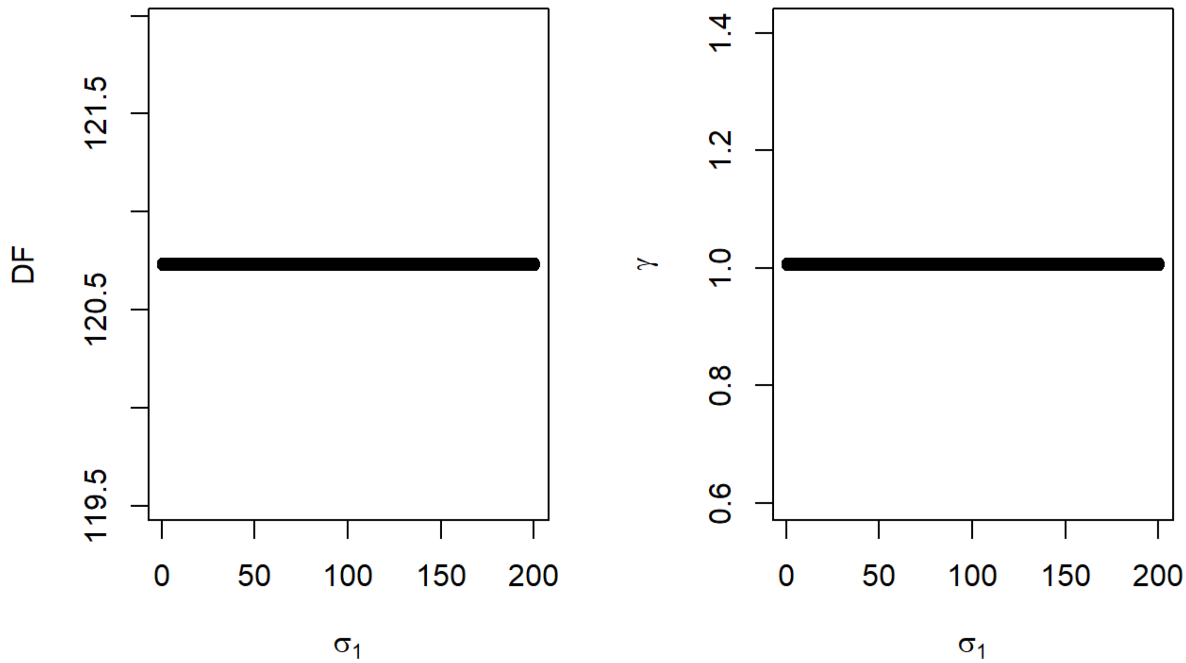

Figure SM1.17 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of σ_1, for a constant SD-ratio.

When variances are unequal across populations, with unequal sample sizes.

The **bias** of Shieh's d is a function of the population effect size (δ_{Shieh}), the sample sizes (n_1 and n_2), and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the population effect size, the more Shieh's d will overestimate δ_{Shieh};

- For a constant sample sizes ratio, the larger the sample sizes, the lower the bias (See Figure SM1.18);

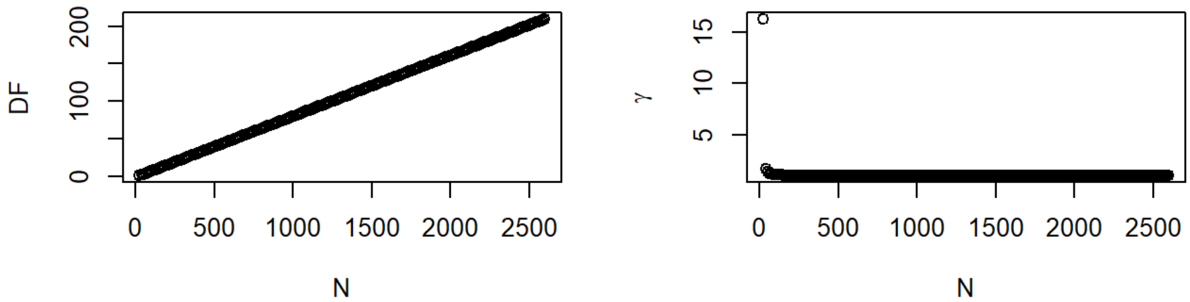

Figure SM1.18 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

Note : When variances were equal across populations, adding subjects only in the first group had the same impact on degrees of freedom (and therefore on bias) than adding subjects only in the second group (see Figure SM1.14). When variances are unequal across groups, this is not true anymore (see Figure SM1.19).

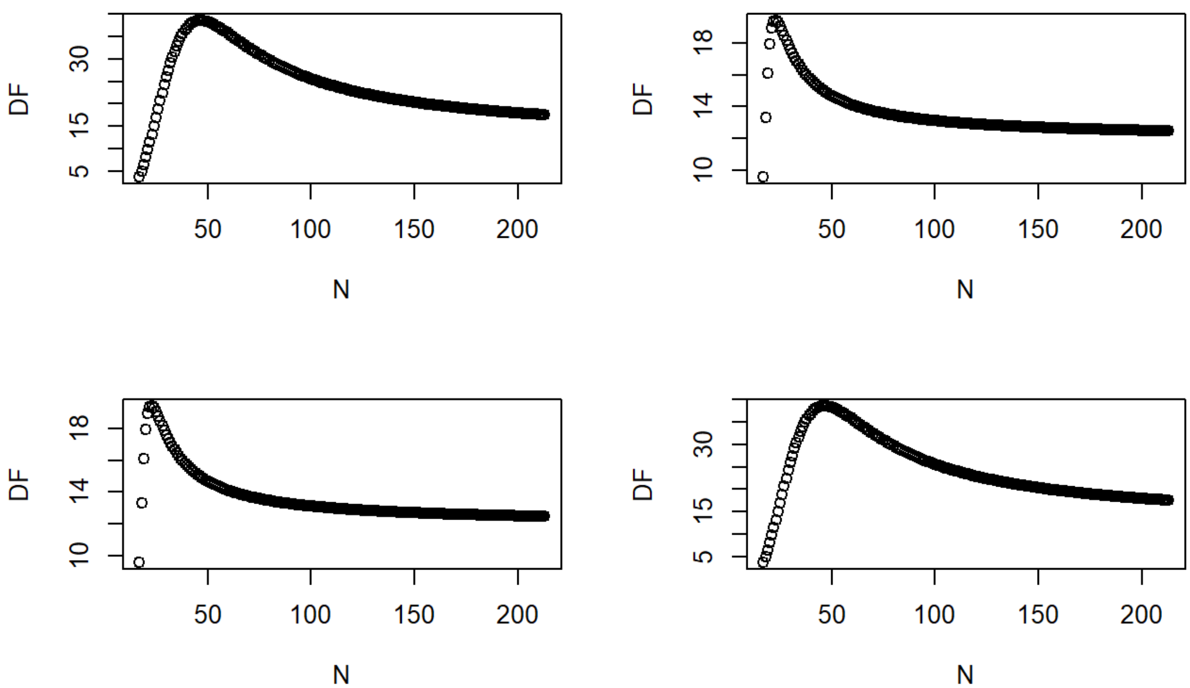

Figure SM1.19 : Degrees of freedom (DF), when computing the bias of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the total sample size, when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots), and $\sigma_1 > \sigma_2$ (left plots) or $\sigma_1 < \sigma_2$ (right plots).

- The smallest bias always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest bias (See Figure SM1.20);

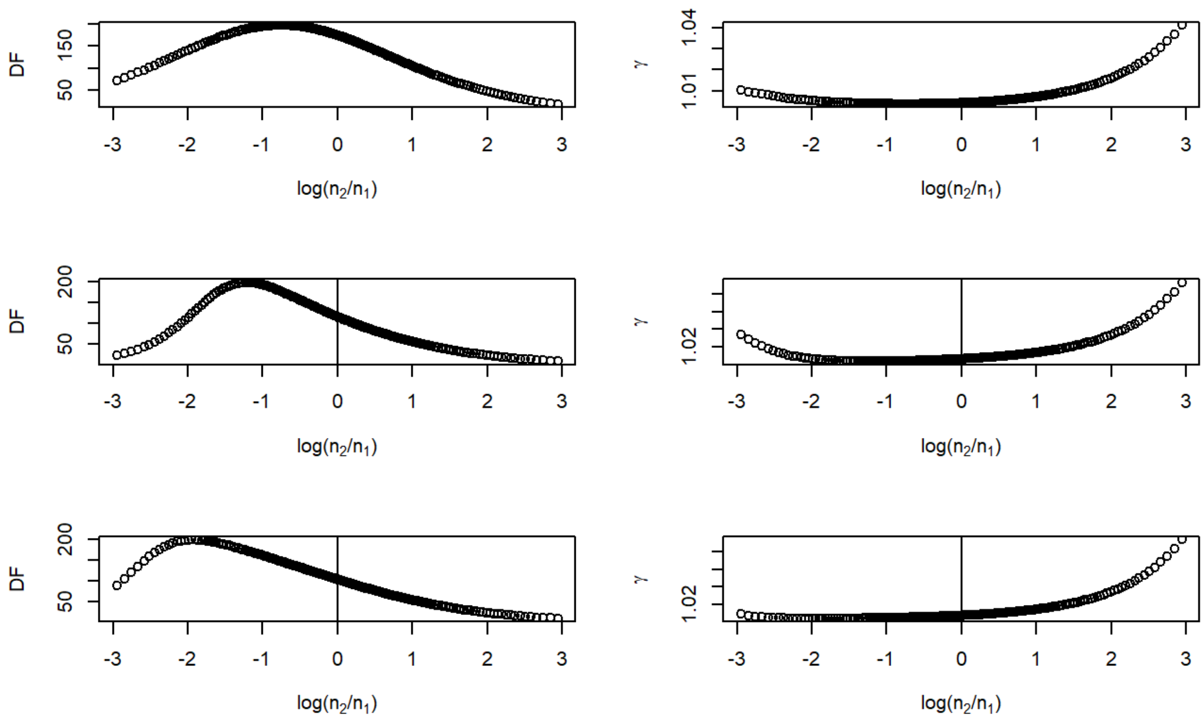

Figure SM1.20 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

Moreover, for a constant SD-ratio, the variances do not matter (See Figure SM1.21).

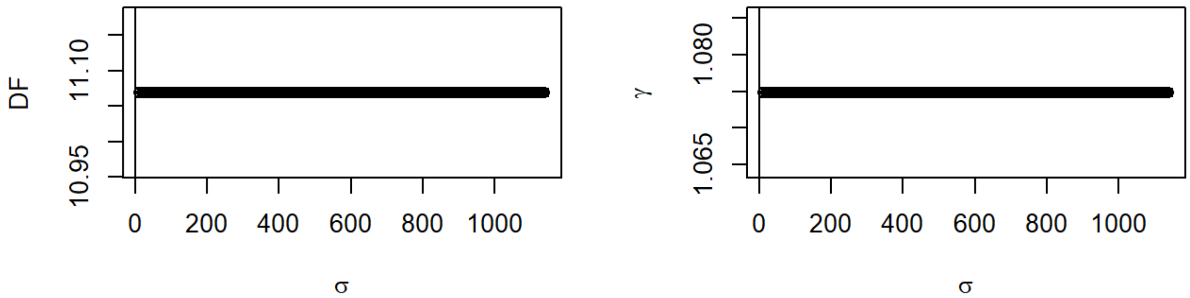

Figure SM1.21 : Degrees of freedom (DF) and γ, when computing the bias of Shieh's d, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

In summary. The **bias** of Cohen's d is a function of the population effect size δ_{Cohen} and the total sample size (N) :

- When δ_{Cohen} is null, the bias is null. In all other configurations, the larger δ_{Cohen}, the more Cohen's d will overestimate δ_{Cohen};
- The bias decreases when the total sample size increases (it does not matter whether one adds subjects in only one group or in both).

The **bias** of Glass's d is a function of the population effect size (δ_{Glass}) and the size of the control group (n_e) :

- When δ_{Glass} is null, the bias is null. In all other configurations, the larger δ_{Glass}, the more Glass's d will overestimate δ_{Glass};
- The bias decreases when the size of the control group increases. On the other hand, increasing the size of the experimental group does not impact the bias.

The **bias** of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the total sample size, and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- When δ^*_{Cohen} is null, the bias is null. In all other configurations, the larger δ^*_{Cohen}, the more Cohen's d^* will overestimate δ^*_{Cohen};
- The bias decreases when the total sample size increases (it does not matter whether one adds subjects in only one group or in both);
- The smallest bias always occurs when there is a positive pairing between $\frac{\sigma_2}{\sigma_1}$ and $\frac{n_2}{n_1}$. Moreover, the larger the SD-ratio, the further from 1 is the sample sizes ratio associated with the smallest bias.

The **bias** of Shieh's d is a function of the population effect size (δ_{Shieh}), the total sample size, and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- When δ_{Shieh} is null, the bias is null. In all other configurations, the larger δ_{Shieh}, the more Shieh's d will overestimate δ_{Shieh};
- For a constant sample sizes ratio, the bias decreases when the total sample size increases;
- The smallest bias always occurs when there is a positive pairing between $\frac{\sigma_2}{\sigma_1}$ and $\frac{n_2}{n_1}$. Moreover, the larger the SD-ratio, the further from 1 is the sample sizes ratio associated with the smallest bias (for more details, see "Theoretical Bias, as a function of population parameters").

Theoretical variance

Note : while we focus on the theoretical variance of biased estimators (Cohen's d, Glass's d, Shieh's d and Cohen's d^*) when the normality assumption is met, it is interesting to notice that our main conclusions seem to generalize to biased estimators when samples are extracted from symmetric distributions. Moreover, unbiased estimators depend on the same factors as biased estimators, so our conclusions remain similar for unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

Cohen's *d*.

When variances are equal across populations.

When $\delta_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d can be simplified as follows:

$$Var_{Cohen's\ d} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

The **variance** of Cohen's d is a function of total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$) :

- The larger the total sample size, the lower the variance. The variance tends to zero when the total sample size tends to infinity (see Figure SM1.22);

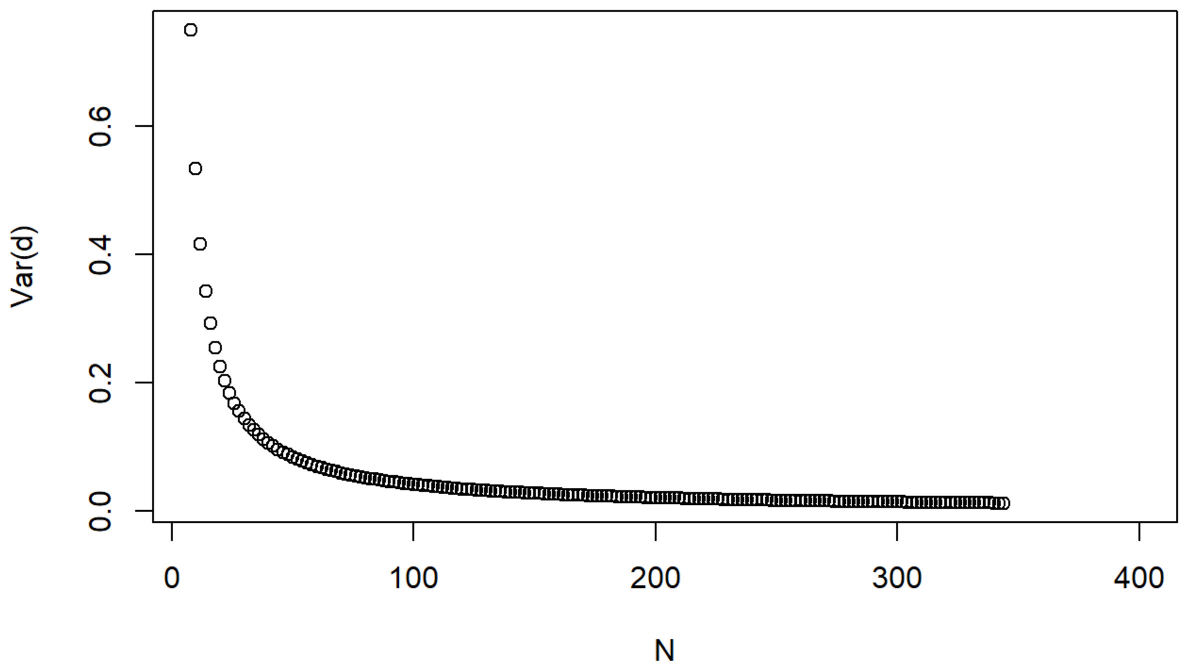

Figure SM1.22 : Variance of Cohen's d, when variances are equal across groups, as a function of the total sample size (N).

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.23).

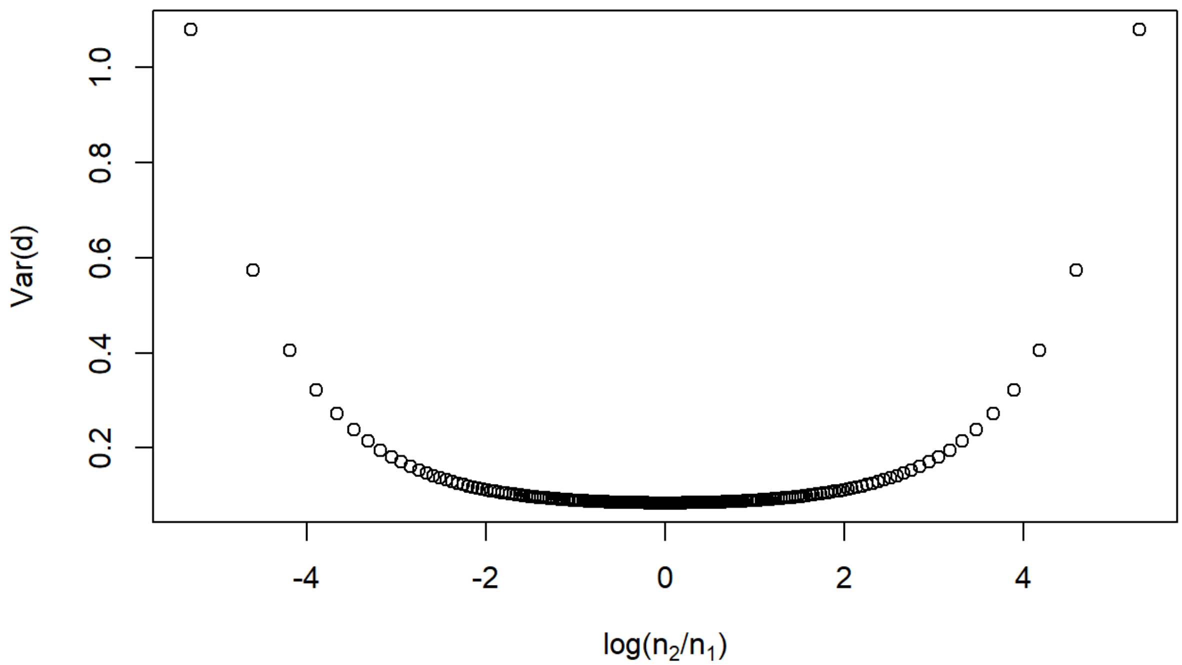

Figure SM1.23 : Variance of Cohen's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

When $\delta_{Cohen} \neq 0$.

While the variance of Cohen's d still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size (δ_{Cohen}). The larger the population effect size, the larger the variance. Note that the effect of the population effect size decreases when sample sizes increase since

$$\lim_{n_1 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N\to\infty}\left[\frac{df}{df-2}-\left(\frac{\sqrt{\frac{df}{2}}\times\Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)}\right)^2\right]=0$$

This is illustrated in Figure SM1.24.

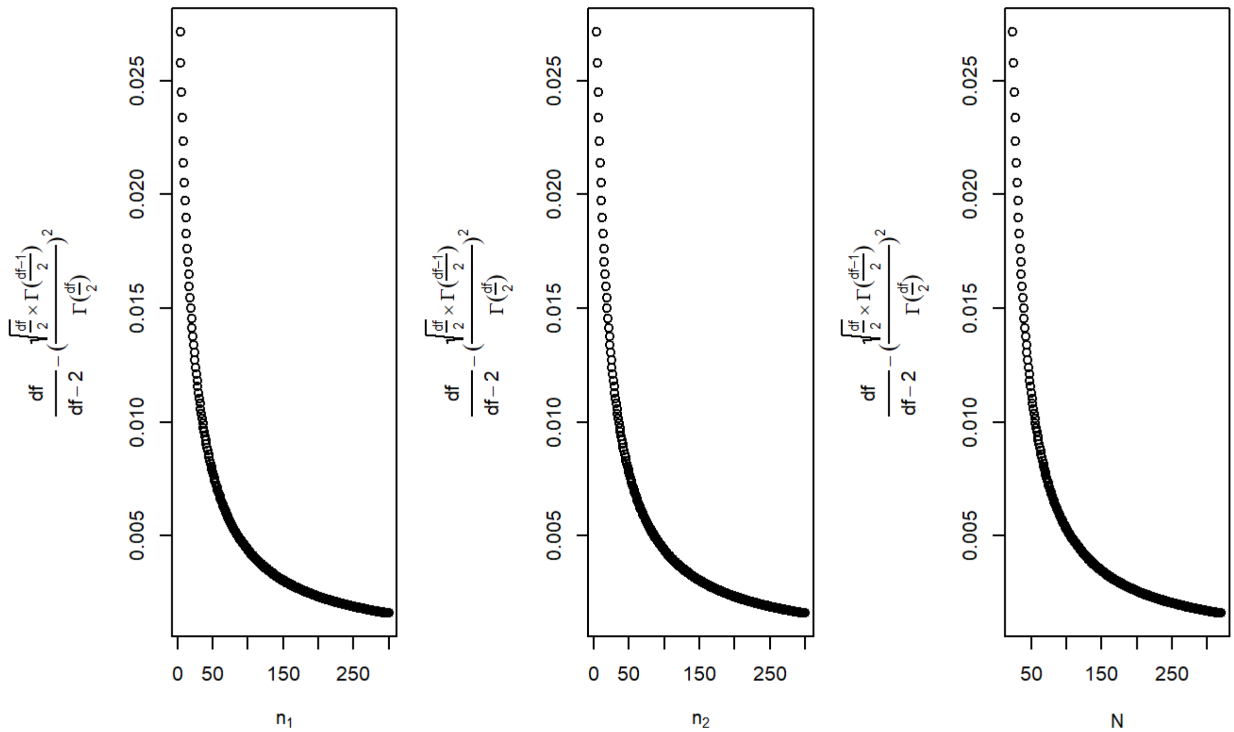

Figure SM1.24 : Effect size moderator, when computing the variance of Cohen's d, as a function of n_1 (left), n_2 (center) and $N = n_1 + n_2$ (right).

In summary.

The variance of Cohen's d is a function of the population effect size (δ_{Cohen}), the total sample size (N) and the sample sizes ratio ($\frac{n_2}{n_1}$) :

- The variance decreases when the total sample size increases;
- The variance also decreases when the sample sizes ratio gets closer to 1;
- Finally, the variance increases when δ_{Cohen} increases. Note that the effect of δ_{Cohen} is moderated by the total sample size (the larger N, the smaller the effect of δ_{Cohen} on the variance).

Glass's *d*.

When variances are equal across populations.

When $\delta_{Glass} = 0$.

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{1}{n_e}\right)$$

In this configuration, the **variance** of Glass's d is a function of the sample sizes of both control (n_c) and experimental (n_e) groups as well as of the sample sizes allocation ratio $\left(\frac{n_c}{n_e}\right)$:

- The larger the sample sizes, the lower the variance (Figure SM1.25);

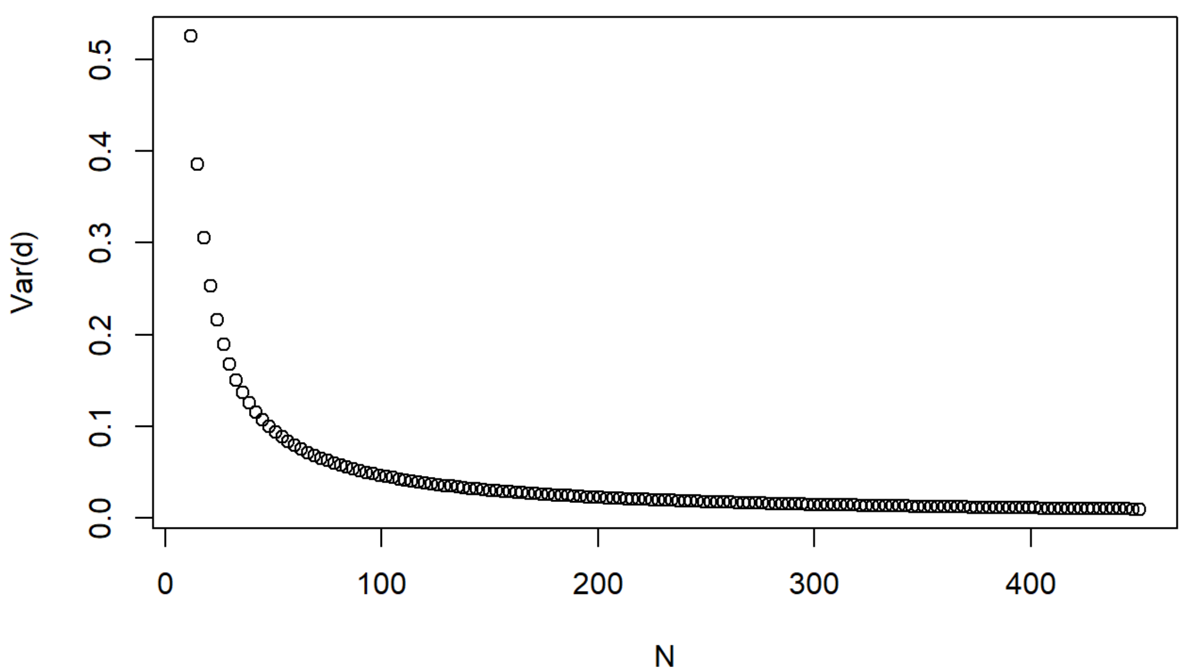

Figure SM1.25 : Variance of Glass's d, when variances are equal across groups, as a function of the total sample size (N).

The sample sizes ratio associated with the lowest variance is not exactly 1 (because of the term $\frac{df}{df-2}$, df depending only on n_c), but is very close to 1 (and the larger the total sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance). The further from this sample size ratio, the larger the variance (see Figure SM1.26).

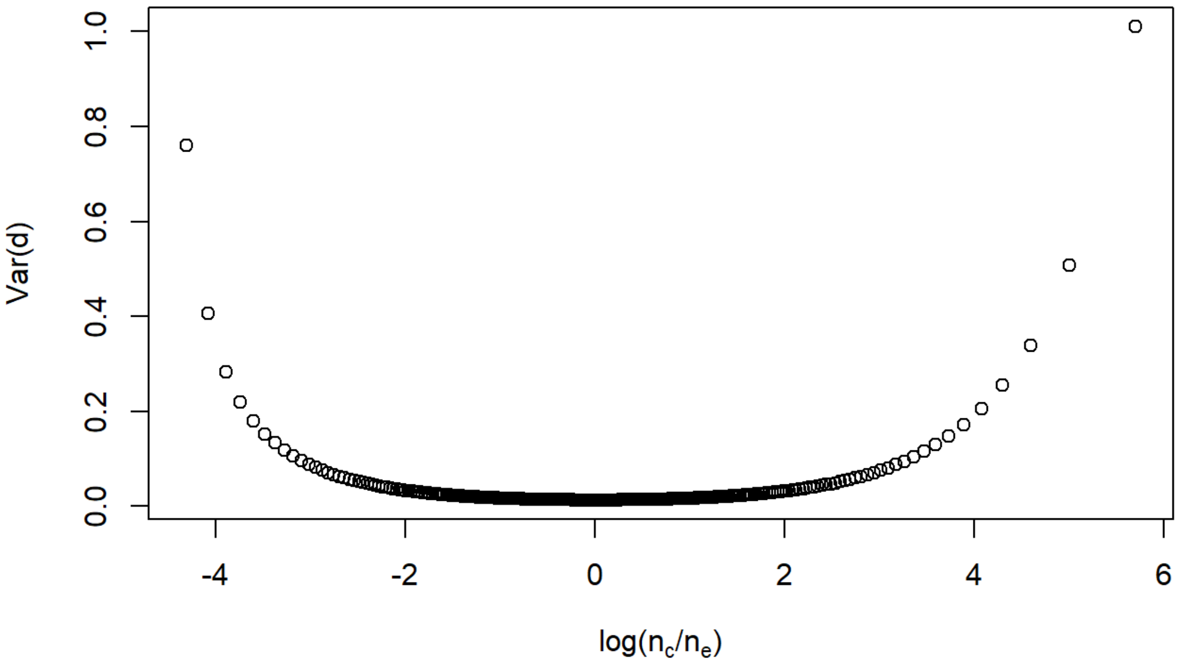

Figure SM1.26 : Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_c}{n_e}\right)$).

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases. On the other hand, the effect of the population effect size does *not* depend on the size of the experimental group since

$$\lim_{n_c \to \infty} \left[\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_e \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] \neq 0$$

These limits are illustrated in Figure SM1.27.

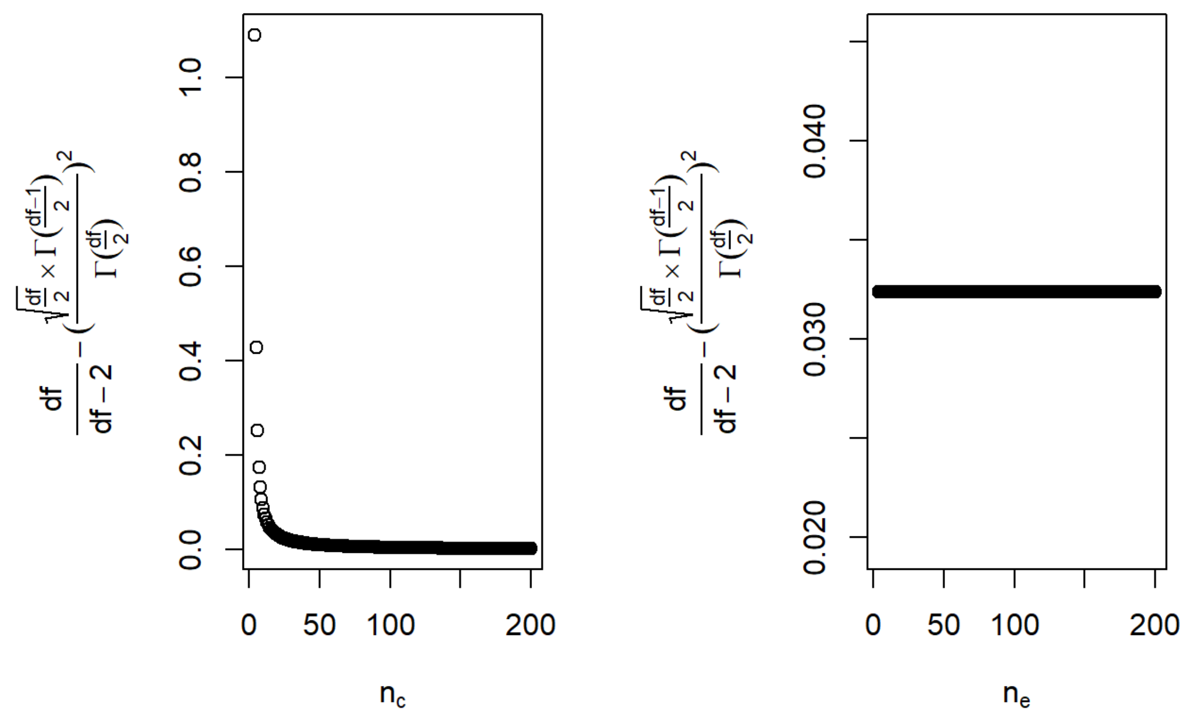

Figure SM1.27 : Effect size moderator, when computing the variance of Glass's d, as a function of the size of the control group (left) and experimental group (right).

Note : while the sample sizes ratio associated with the lowest variance was very close to 1 with a null population effect size, this is not true anymore when the population effect size is not zero. Indeed, because of the second term in the addition, when computing the variance, one gives much more weight to the effect size of the control group (see Figure SM1.28), especially when the effect size gets larger. For example, when $\delta_{Glass}= 4$, the lowest variance will occur when n_c is approximately 3 times larger than n_e. When $\delta_{Glass}=$ 7, the lowest variance will occur when n_c is approximately 5 times larger than n_e, etc.

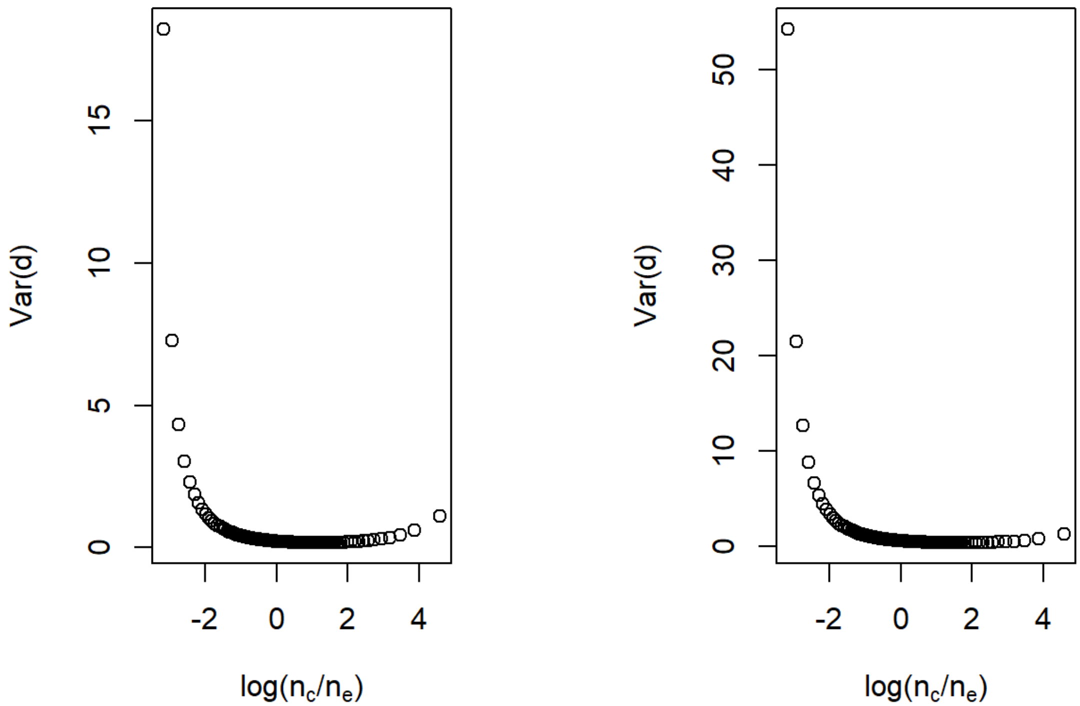

Figure SM1.28 : Variance of Glass's d, when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_c}{n_e}\right)$) when δ_{Glass} equals 4 (left) or 7 (right).

When variances are unequal across populations, with equal sample sizes.

When $\delta_{Glass} = 0$*.*

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n-1}{n(n-3)}\left(1+\frac{\sigma_e^2}{\sigma_c^2}\right)$$

where $n = N/2 =$ sample size of each group. The variance is therefore a function of the total sample size and the SD-ratio $(\frac{\sigma_c}{\sigma_e})$:

- The larger the total sample size, the lower the variance (See Figure SM1.29);

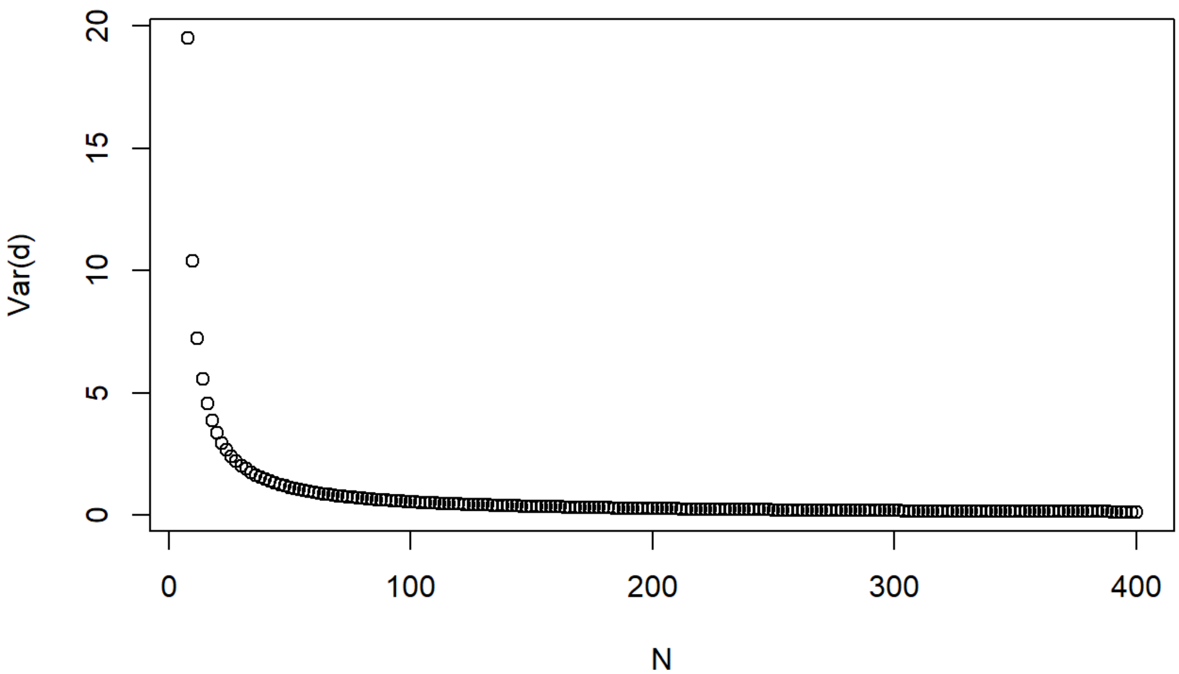

Figure SM1.29 : Variance of Glass's d, when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes (N).

- The larger the SD-ratio (i.e. the larger is σ_c in comparison with σ_e), the lower the variance (see Figure SM1.30). However, the effect of the SD-ratio decreases when sample sizes increase, because $\lim_{n(=n_c=n_e)\to\infty}\left[\frac{df}{n(df-2)}\right]=0$.

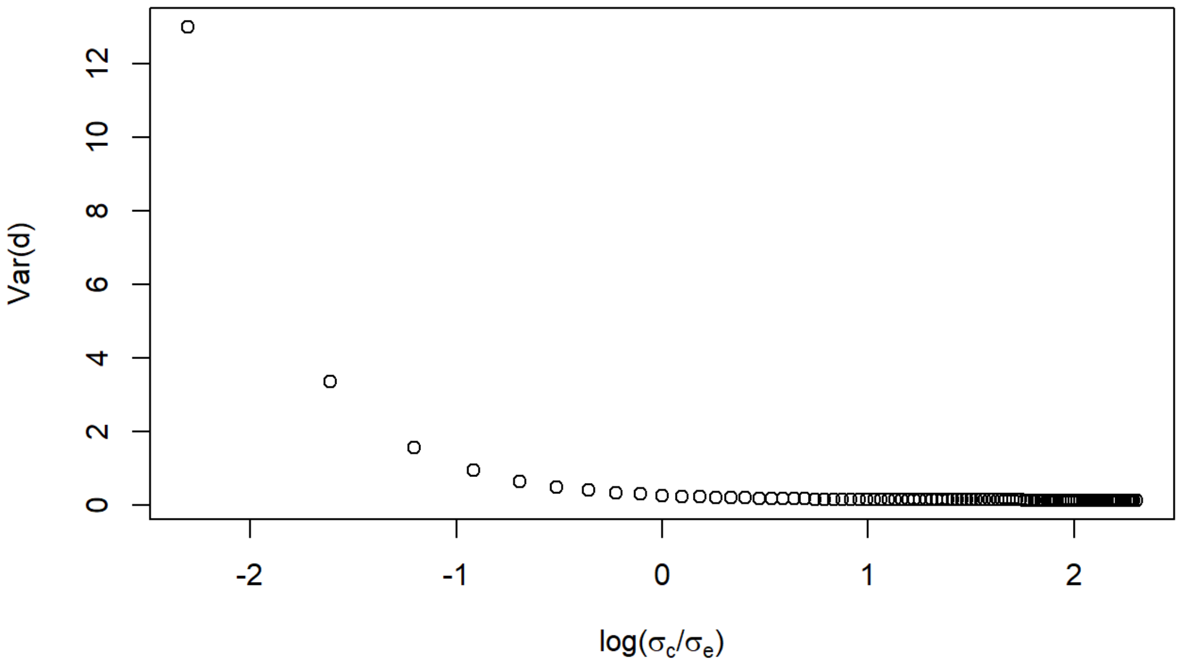

Figure SM1.30 : Variance of Glass's d, when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_c}{\sigma_e}\right)$).

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ_{Glass}). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases, as previously explained and illustrated in Figure SM1.27.

When variances are unequal across populations, with unequal sample sizes.

When $\delta_{Glass} = 0$*.*

When the population effect size is zero, the variance of Glass's d can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3}\left(\frac{1}{n_c} + \frac{\sigma_e^2}{n_e \sigma_c^2}\right)$$

The variance of Glass's d is therefore a function of the total sample size (N), the SD-ratio and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

- For any SD and sample sizes pairing, increasing n_c and/or n_e will decrease the variance (see Figure SM1.31);

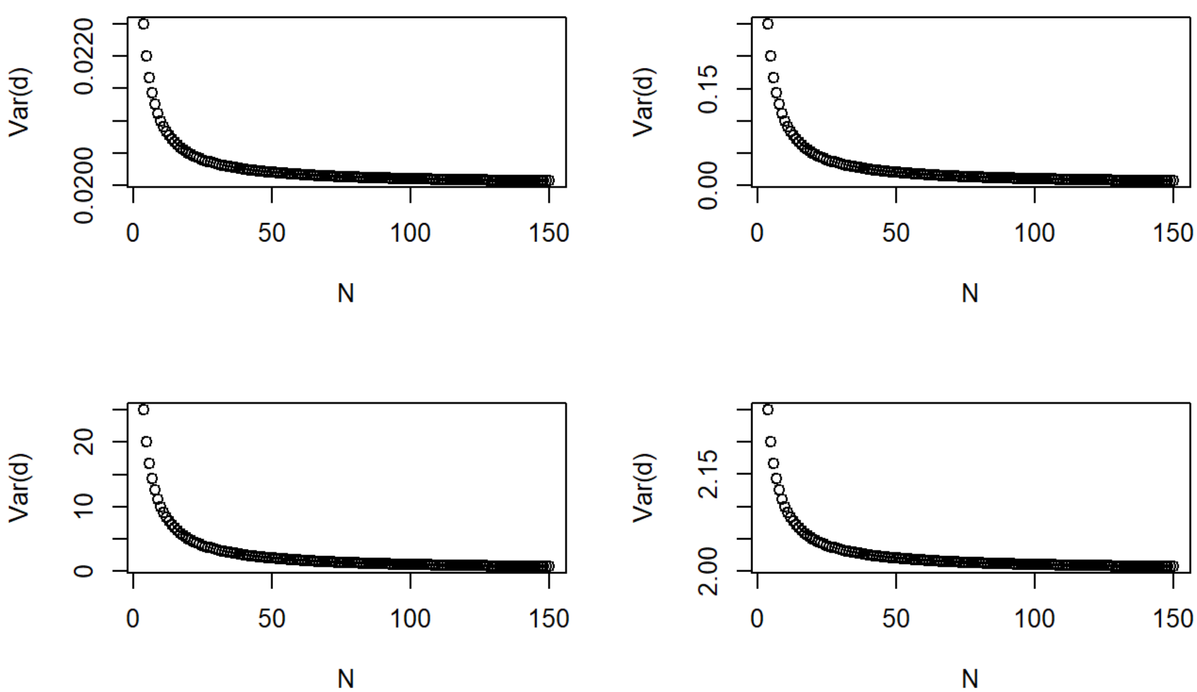

Figure SM1.31 : Variance of Glass's d, when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when $\sigma_c > \sigma_e$ (top plots) or $\sigma_c < \sigma_e$ (bottom plots).

- The effect of the sample sizes ratio depends on the SD-ratio :
  - We previously mentioned that when $\sigma_c = \sigma_e$, the variance is minimized when sample sizes of both groups are almost identical (see Figure SM1.26), meaning that it is more efficient, in order to reduce variance, to add subjects uniformly in both groups;
  - When $\sigma_e > \sigma_c$, more weight is given to n_e, meaning that it is more efficient, in order to reduce variance, to add subjects in the experimental group (n_e; see bottom plots in Figure M1.31);
  - When $\sigma_c > \sigma_e$, less weight is given to n_e, meaning that it is more efficient, in order to reduce variance, to add sujects in the control group (n_c; see top plots in Figure SM1.31).

- Finally, there is also a main effect of the SD-ratio : the larger is σ_c in comparison with σ_e, the lower the variance, as we can observe in Figure SM1.32. We can also notice that in Figure SM1.31, the maximum variance is much larger in the two bottom plots (where $\sigma_c < \sigma_e$) than in the two top plots (where $\sigma_c > \sigma_e$).

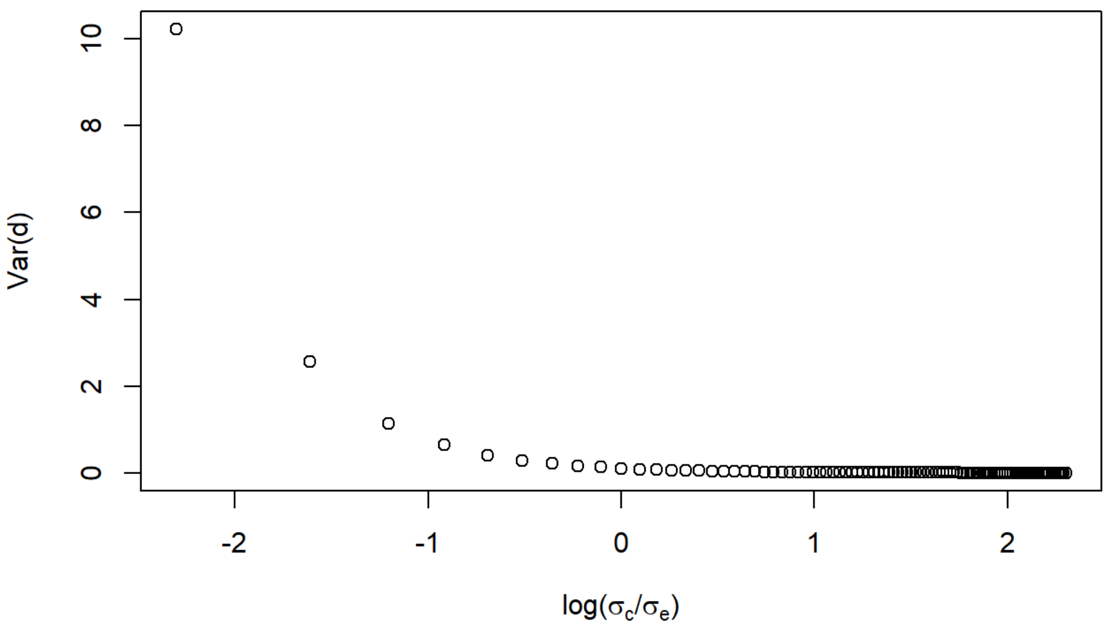

Figure SM1.32 : Variance of Glass's d, when sample sizes and variances are unequal across groups, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_c}{\sigma_e}\right)$).

Note that the effect of the SD-ratio, and the interaction effect between SD-ratio and sample sizes ratio decreases when the sample size of the control group increases (because $\frac{n_c-1}{n_c-3}$ gets closer to 1).

When $\delta_{Glass} \neq 0$.

While the variance of Glass's d still depends on the total sample size, the SD-ratio and the interaction between the SD-ratio and the sample sizes ratio, it also depends on the population effect size (δ_{Glass}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the sample size of the control group increases, as previously explained and illustrated in Figure SM1.27.

Note : when the population effect size was null, when $\sigma_c < \sigma_e$, it was much more efficient to add subjects in the experimental group in order to reduce the variance (because much more weight was given to n_e). When $\delta_{Glass} \neq 0$, it is important to add subjects in both groups in order to reduce the variance (because $\frac{df}{df-2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2$ is only a function of the sample size of the control group). With huge population effect size, it is even always more important to add subjects in the control group (e.g. when $\delta_{Glass} = 30$).

In summary.

The variance of Glass's d is a function of the population effect size (δ_{Glass}), the SD-ratio, the total sample size and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e}\right)$:

- The variance decreases when the SD-ratio increases (i.e. when $\sigma_e >> \sigma_c$);
- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control, in the experimental, or in both groups, in order to reduce the variance, varies as a function of the SD-ratio and the population effect size. The only situation where it is optimal to maximize the experimental group is when $\sigma_e > \sigma_c$ and $\delta_{Glass} \approx 0$. Most of the time, it is more efficient to maximize the control groups (e.g. anytime $\sigma_e < \sigma_c$, and when δ_{Glass} is very large) or to uniformly add subjects in both groups (e.g. when $\sigma_e > \sigma_c$ and δ_{Glass} is neither null nor huge);
- The variance increases when δ_{Glass} increases. Note that the effect of δ_{Glass} is moderated by the control group size (the larger n_e, the smaller the effect of δ_{Glass} on the variance).

Cohen's d^*.

When variances are equal across populations.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* is computed as follows :

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{N}{n_1 n_2}$$

with

$$df = \frac{4(n_1-1)(n_2-1)}{n_1+n_2-2}$$

In this configuration, the degrees of freedom as well as the variance of Cohen's d^* depend on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.33);

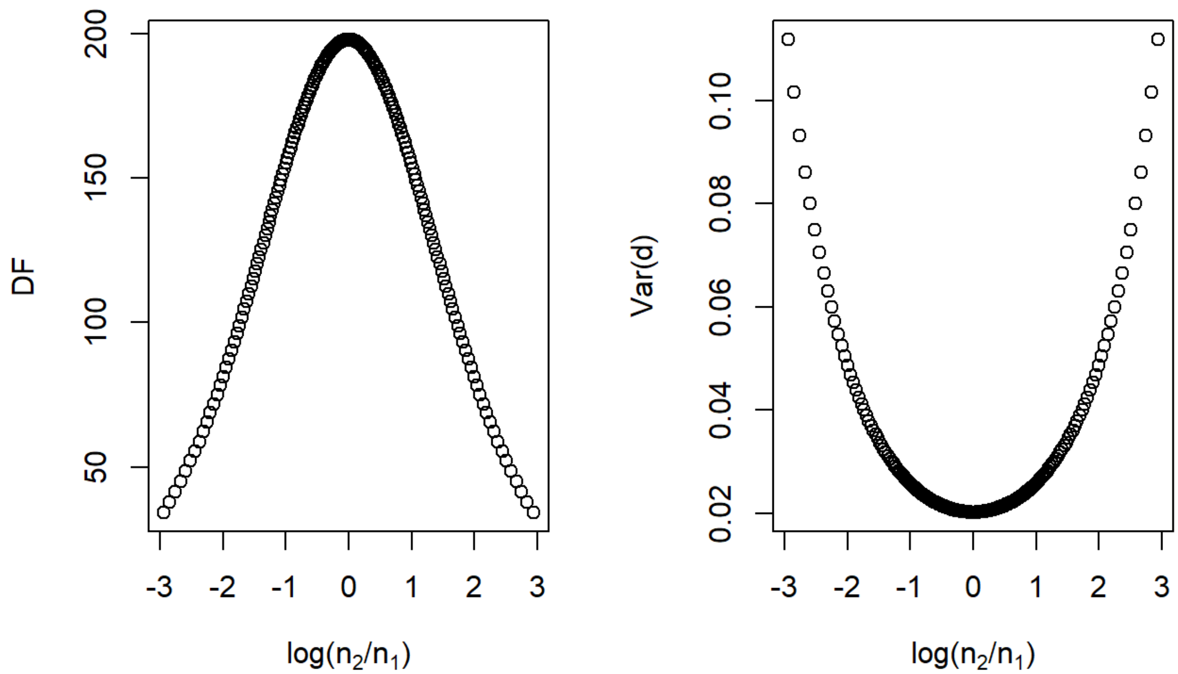

Figure SM1.33 : Variance of Cohen's d^* when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

- The larger the total sample size, the lower the bias (see Figure SM1.34).

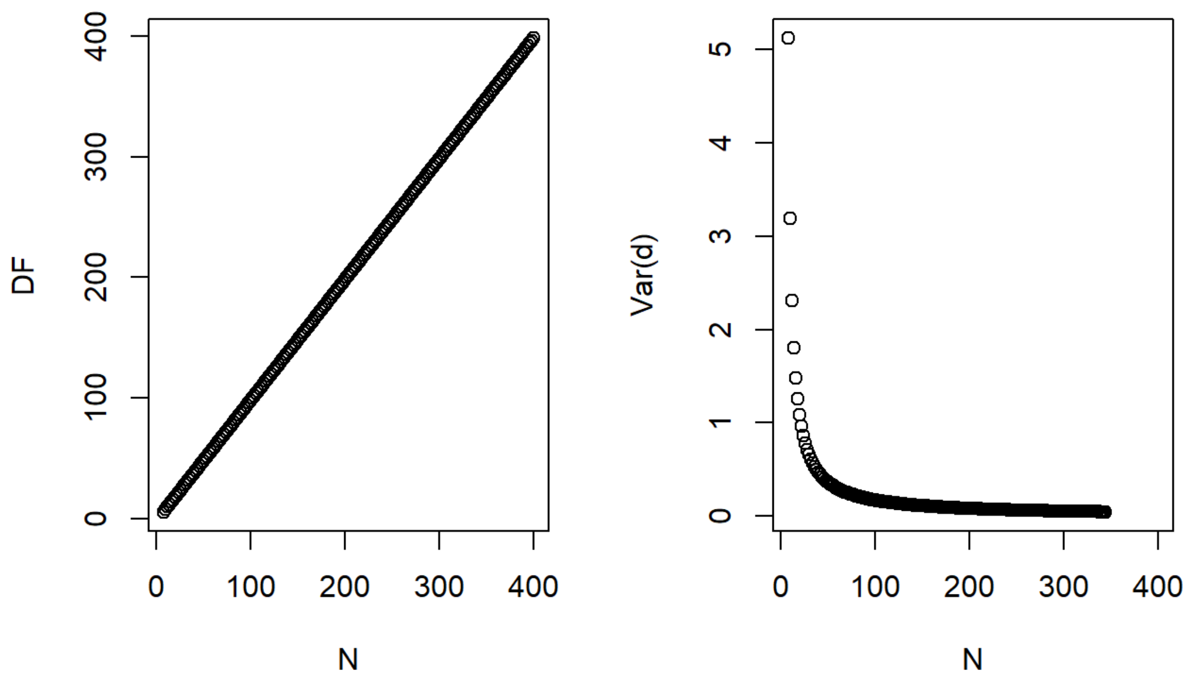

Figure SM1.34 : Variance of Cohen's d^* when variances are equal across groups, as a function of the total sample size (N).

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size (δ^*_{Cohen}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the sample sizes ratio get closer to 1), as illustrated in Figure SM1.35 since

$$\lim_{df \to \infty} \left[\frac{df}{df - 2} - \left(\frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

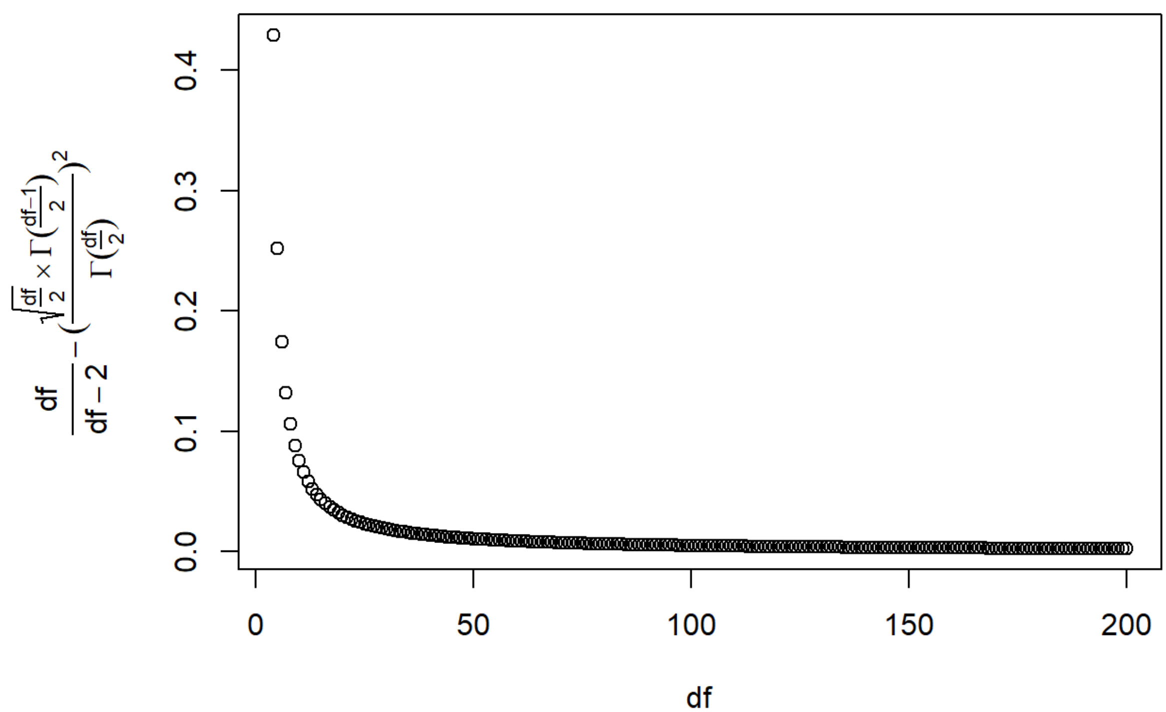

Figure SM1.35 : Effect size moderator (for all estimators), as a function of the degrees of freedom.

When variances are unequal across populations, with equal sample sizes.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* can be simplified as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{2}{n}$$

where $n = N/2$=sample size of each group, and $df = \frac{(n-1)(\sigma_1^4+\sigma_2^4+2\sigma_1^2\sigma_2^2)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the variance of Cohen's d^* depend on the total sample size (N) and the SD-ratio $\left(\frac{\sigma_2}{\sigma_1}\right)$:

- The further the SD-ratio is from 1, the larger the variance (see Figure SM1.36);

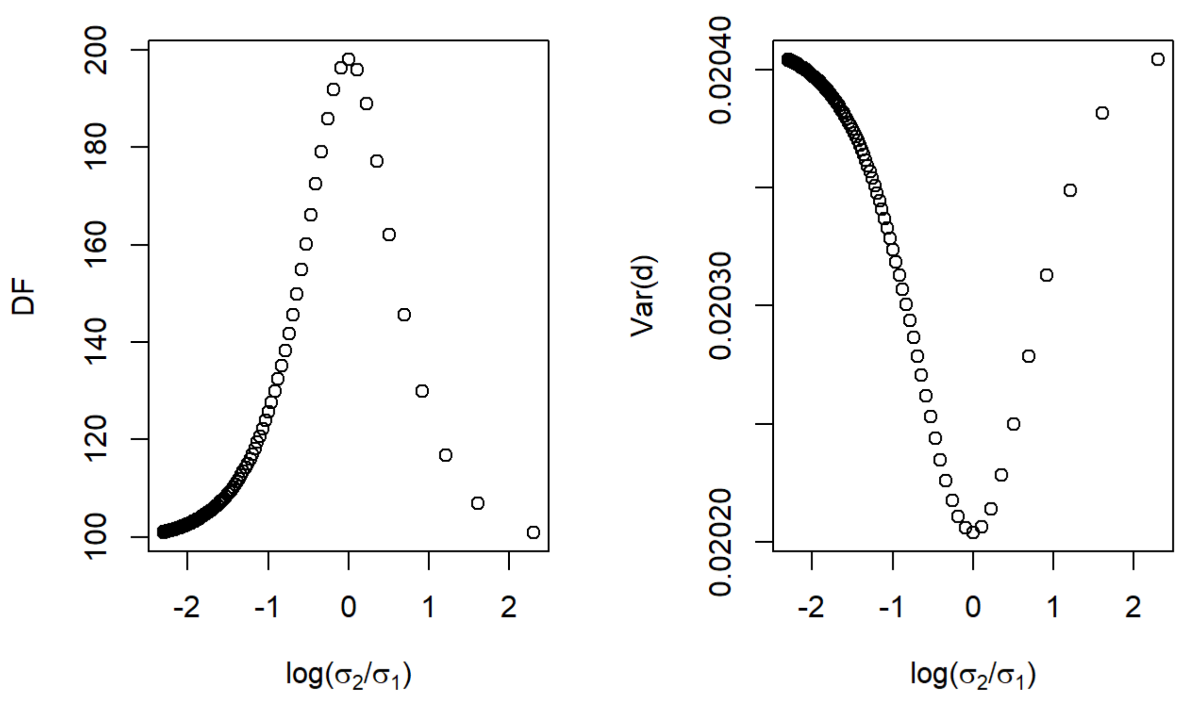

Figure SM1.36 : Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

- The larger the total sample size, the lower the variance (see Figure SM1.37).

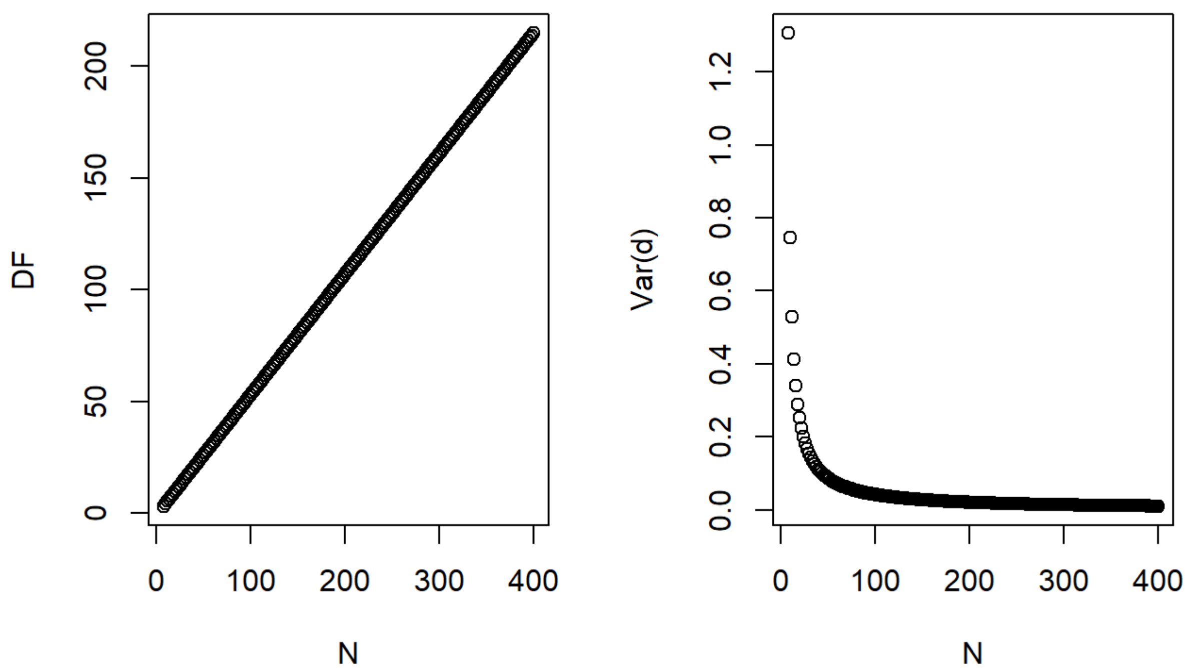

Figure SM1.37 : Variance of Cohen's d^* when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Note : for a constant SD-ratio, the size of the variance does not matter (see Figure SM1.38).

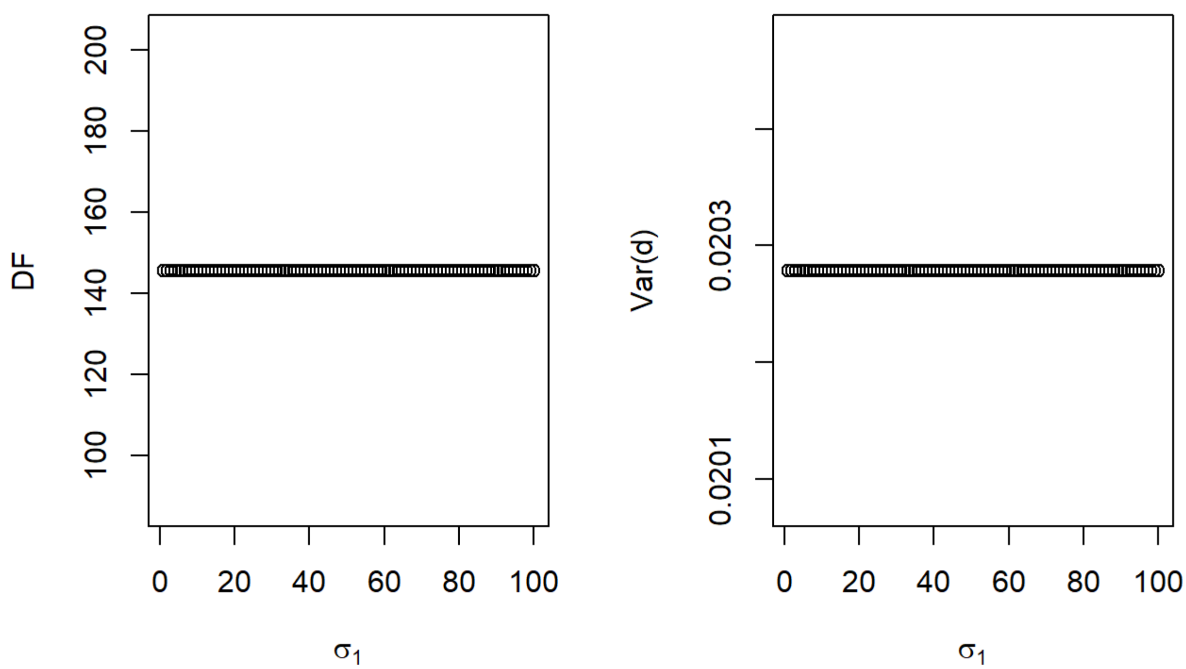

Figure SM1.38 : Variance of Cohen's d^*, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ^*_{Cohen}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or the SD-ratio get closer to 1), as previously illustrated in Figure SM1.35.

When variances are unequal across populations, with unequal sample sizes.

When $\delta^*_{Cohen} = 0$.

When the population effect size is zero, the variance of Cohen's d^* can be simplified as follows:

$$Var_{Cohen's\ d^*} = \frac{df}{df-2} \times \frac{2\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)}{\sigma_1^2+\sigma_2^2}$$

with $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$. In this configuration, the degrees of freedom are a function of the total sample size (N) and the interaction between sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the total sample size, the lower the variance (illustration in Figure SM1.39);

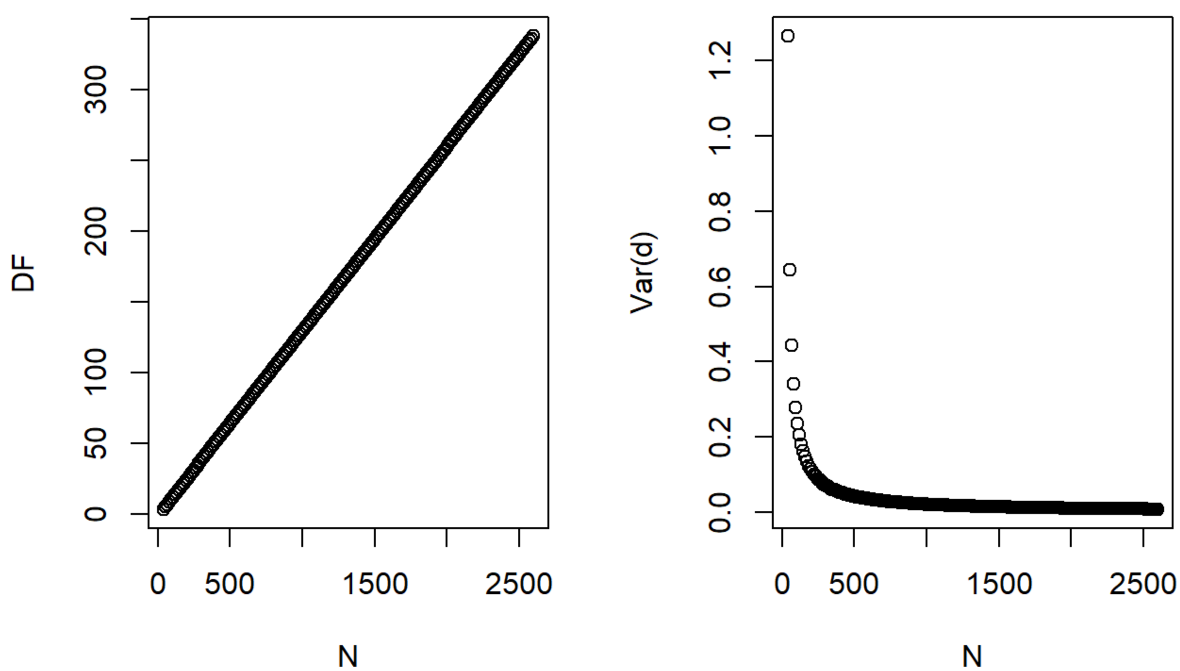

Figure SM1.39 : Variance of Cohen's d^* when variances and sample sizes are unequal across groups, as a function of the total sample size (N).

- The smallest variance always occurs when there is a positive pairing between variances and sample sizes, because one gives more weight to the smallest variance in the denominator of the *df* computation and in the numerator of the variance computation. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure SM1.40). This can be explained by splitting the numerator and the denominator of the *df* computation (see the file "Theoretical Bias, as a function of population parameters").

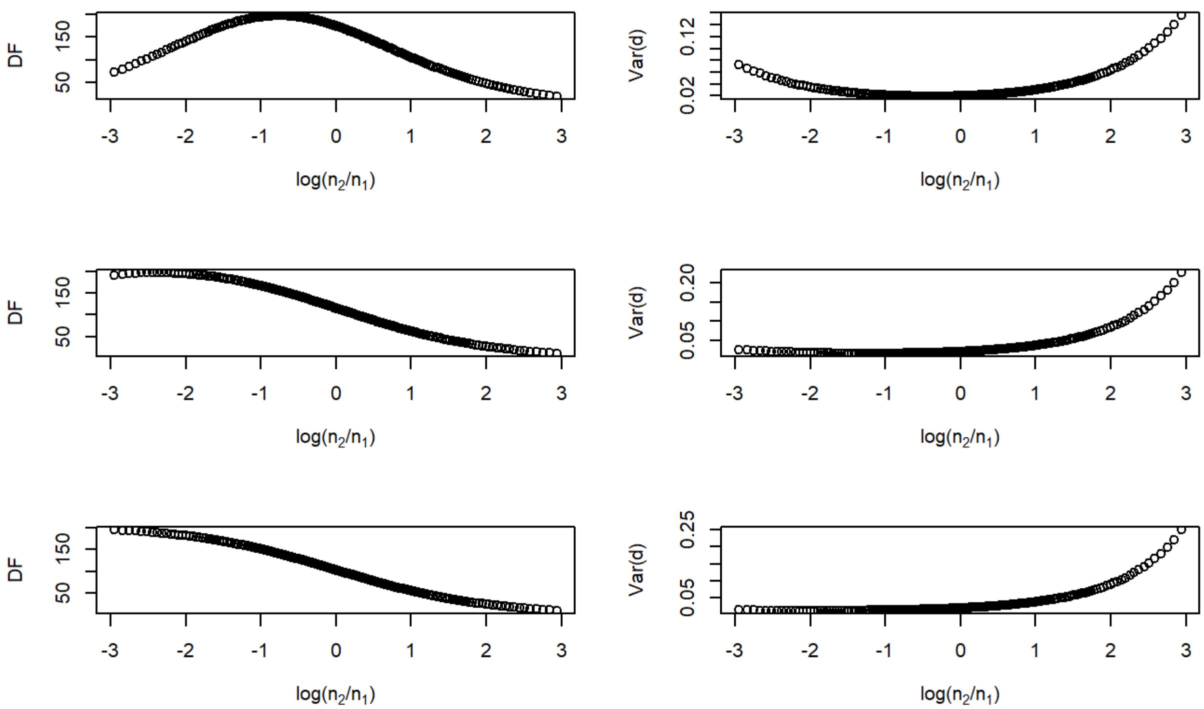

Figure SM1.40 : The variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

Note : for a constant SD-ratio, the variance does not matter. (See Figure SM1.41).

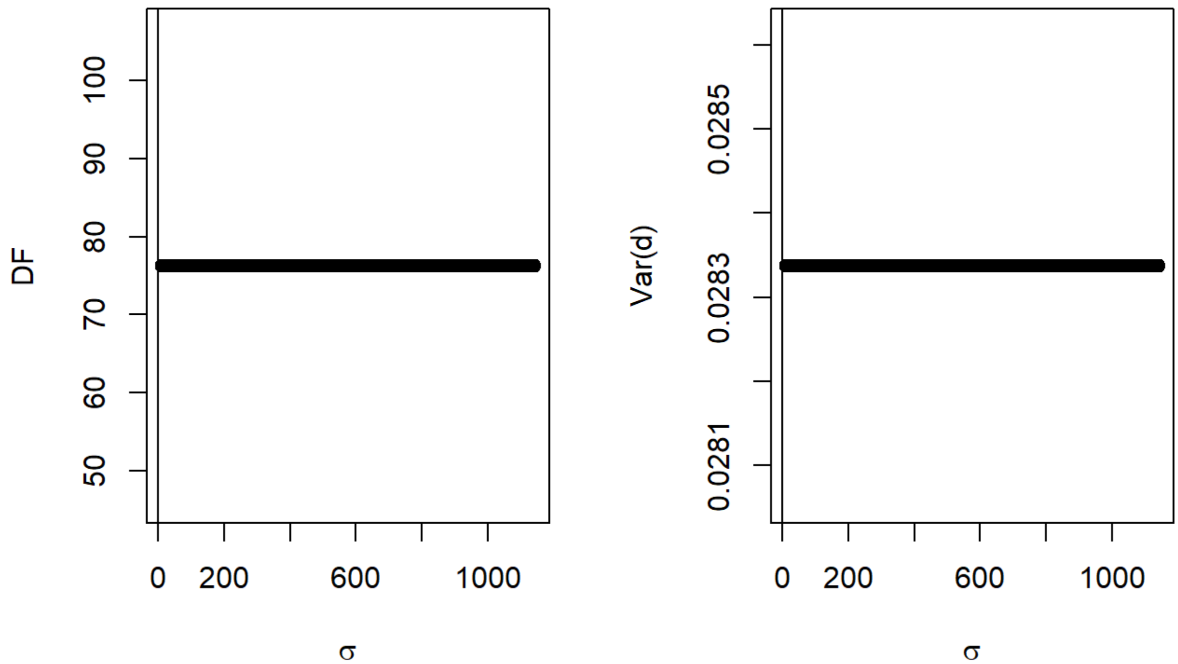

Figure SM1.41 : Variance of Cohen's d^*, when variances and sample sizes are unequal across groups, as a function of σ_1 and σ_2, for a constant SD-ratio.

When $\delta^*_{Cohen} \neq 0$.

While the variance of Cohen's d^* still depends on the total sample size, the SD-ratio and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the population effect size (δ^*_{Cohen}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or when there is a positive pairing between the sample sizes ratio and the SD-ratio), as previously illustrated in Figure SM1.35.

In summary.

The variance of Cohen's d^* is a function of the population effect size (δ^*_{Cohen}), the total sample size (N) and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;
- The variance increases when δ^*_{Cohen} increases. Note that the effect of δ^*_{Cohen} is moderated by the total sample size and the interaction between sample sizes ratio and SD-ratio.

Shieh's *d*.

When variances are equal across populations.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the sample sizes allocation ratio ($\frac{n_2}{n_1}$):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.42);

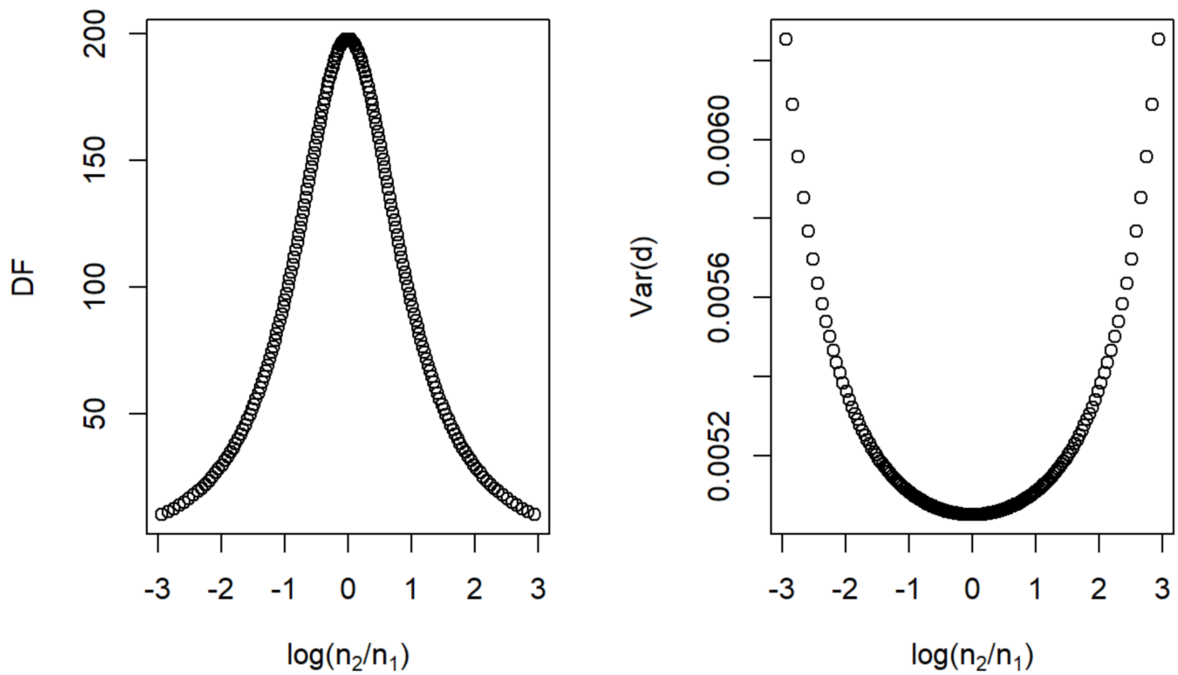

Figure SM1.42 : Variance of Shieh's d when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

- The larger the total sample size, the lower the variance. It does not matter whether the sample sizes ratio is constant (see Figure SM1.43) or not (see Figure SM1.44).

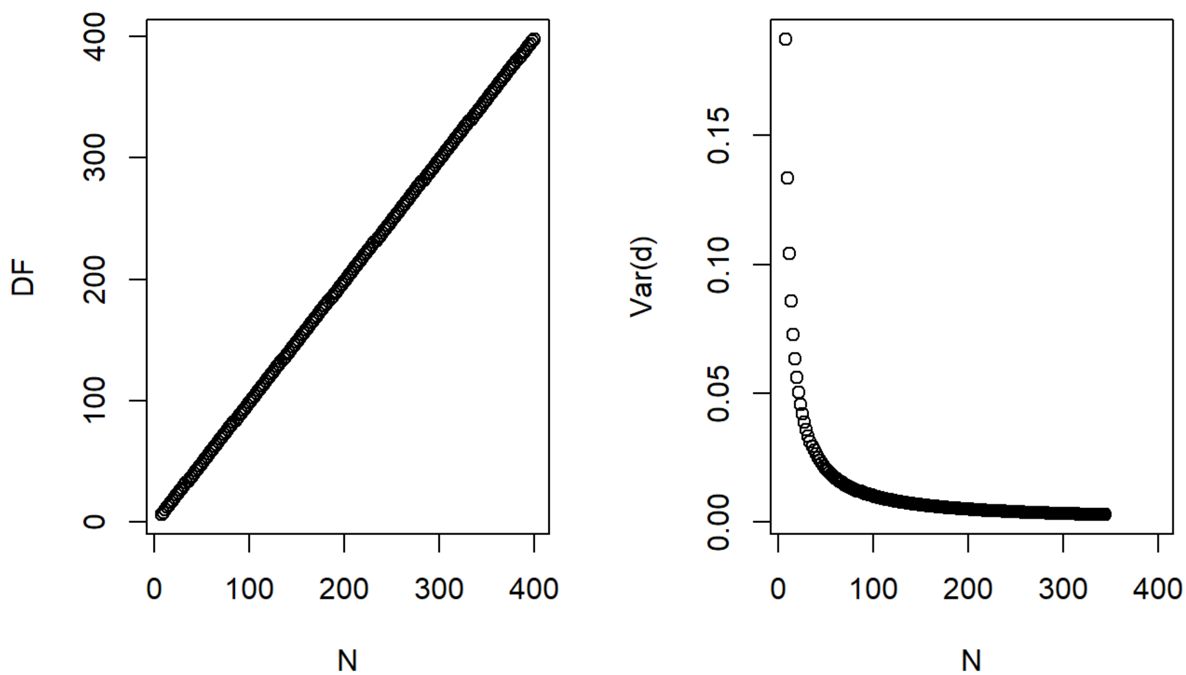

Figure SM1.43 : Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), for a constant sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

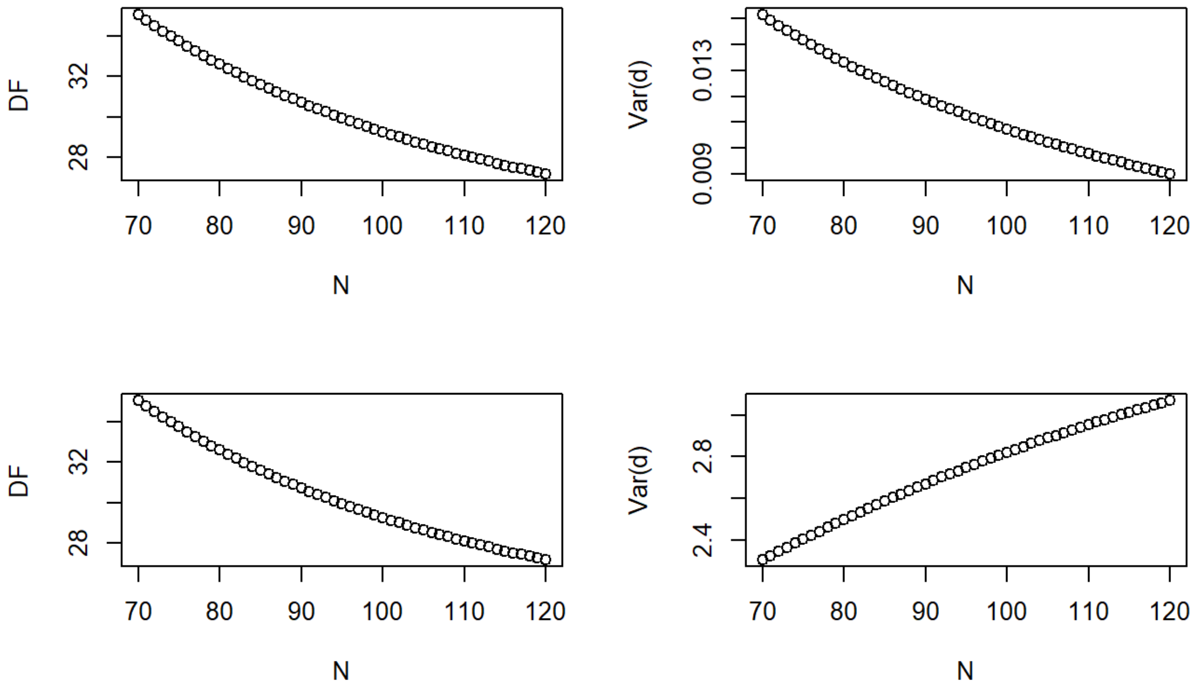

Figure SM1.44 : Variance of Shieh's d when variances are equal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).

Note : in "Theoretical Bias," we noticed that moving the sample sizes ratio away from 1 when adding subjects in only one group could decrease the degrees of freedom. However, due to the total sample size term (N) in the denominator of the variance computation, even when degrees of freedom decrease due to the fact that one adds subjects only in one group, the variance still decreases (because the denominator of the variance computation increases; see Figure SM1.44).

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size (δ_{Shieh}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase, without increasing the sample sizes ratio, and/or the sample sizes ratio gets closer to 1), as previously illustrated in Figure SM1.35.

Note : we previously noticed that when the effet size is zero, the variance of Shieh's d decreases, even when the sample sizes ratio increases. It is no longer true when there is a non-null effect size because the larger the sample sizes ratio, the more the variance will increase with increasing effect size.

When variances are unequal across populations, with equal sample sizes.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\ d} = \frac{df}{(df-2)N}$$

with $df = \frac{(\sigma_1^2+\sigma_2^2)^2 \times (n-1)}{\sigma_1^4+\sigma_2^4}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the SD-ratio ($\frac{\sigma_2}{\sigma_1}$).

- The further the SD-ratio is from 1, the larger the variance (see Figure SM1.45);

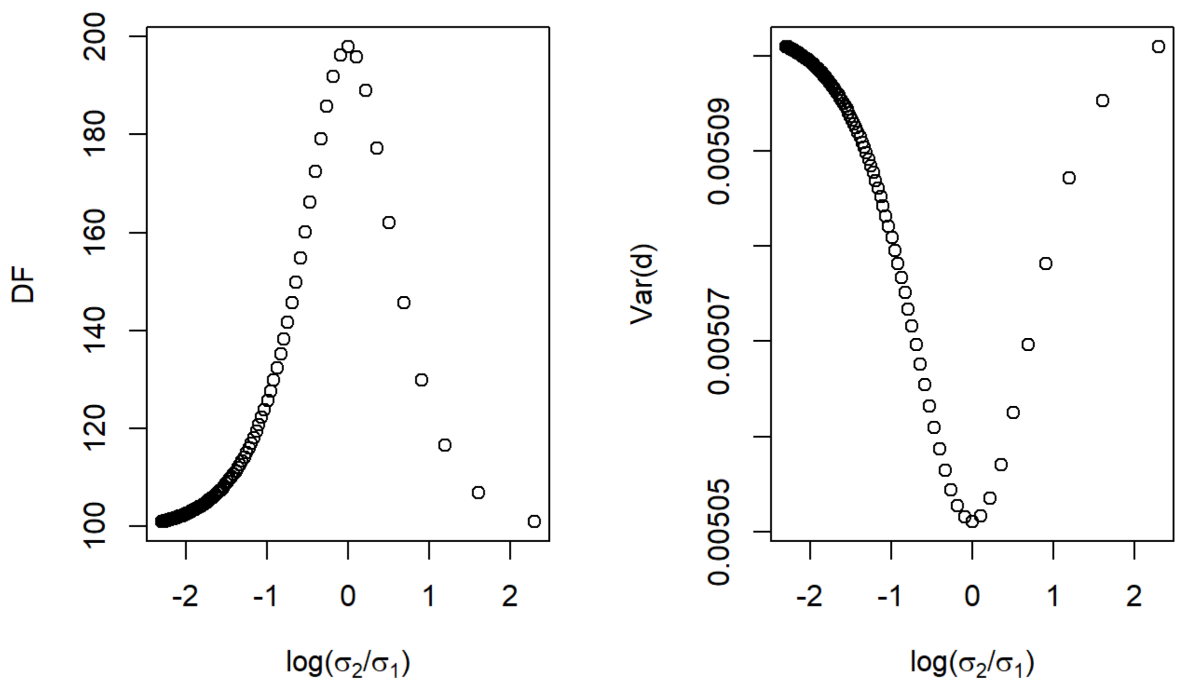

Figure SM1.45 : Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the SD-ratio ($log\left(\frac{\sigma_2}{\sigma_1}\right)$).

- The larger the total sample size, the lower the variance (see Figure SM1.46).

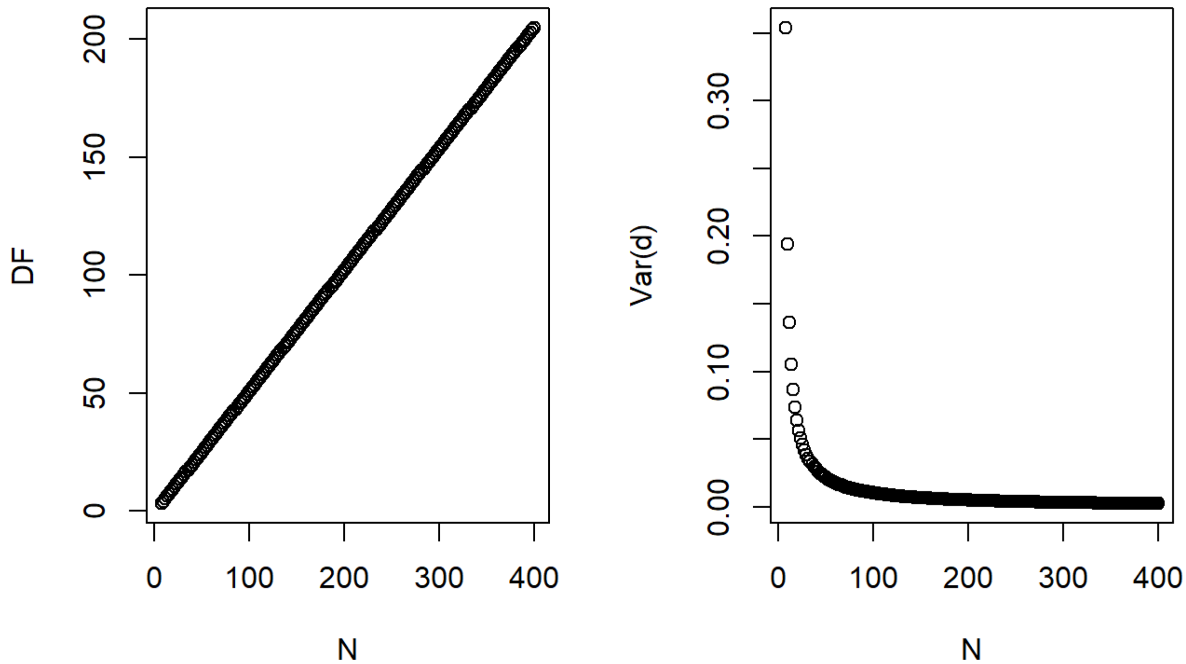

Figure SM1.46 : Variance of Shieh's d when variances are unequal across groups and sample sizes are equal, as a function of the total sample size (N).

Note : for a constant SD-ratio, the size of the variance does not matter (see Figure SM1.47).

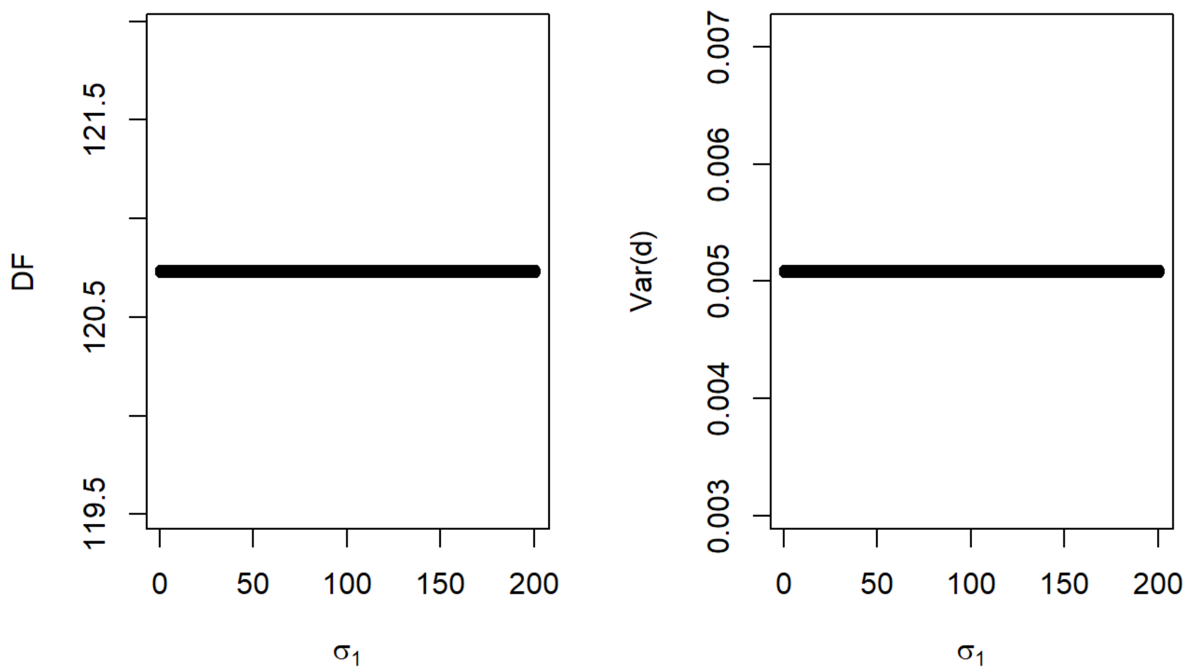

Figure SM1.47 : Variance of Shieh's d, when variances are unequal across groups and sample sizes are equal, as a function of σ_1 and σ_2, for a constant SD-ratio.

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the SD-ratio, it also depends on the population effect size (δ_{Shieh}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the SD-ratio gets closer to 1), as previously illustrated in Figure SM1.35.

When variances are unequal across populations, with unequal sample sizes.

When $\delta_{Shieh} = 0$.

When the population effect size is zero, the variance of Shieh's d can be simplified as follows:

$$Var_{Shieh's\, d} = \frac{df}{(df-2)N}$$

with $df = \frac{\left(\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1}+\frac{(\sigma_2^2/n_2)^2}{n_2-1}}$. In this configuration, the degrees of freedom as well as the variance of Shieh's d depend on the total sample size (N) and the interaction between the sample sizes ratio and the SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The larger the total sample size, the lower the variance. It remains true when the sample sizes ratio is constant (see Figure SM1.48) and when it is not (see Figure SM1.49).

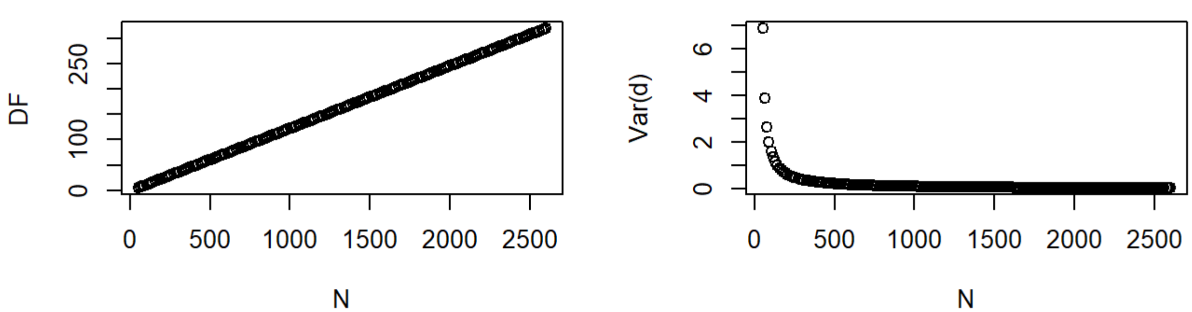

Figure SM1.48 : Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), for a constant sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$).

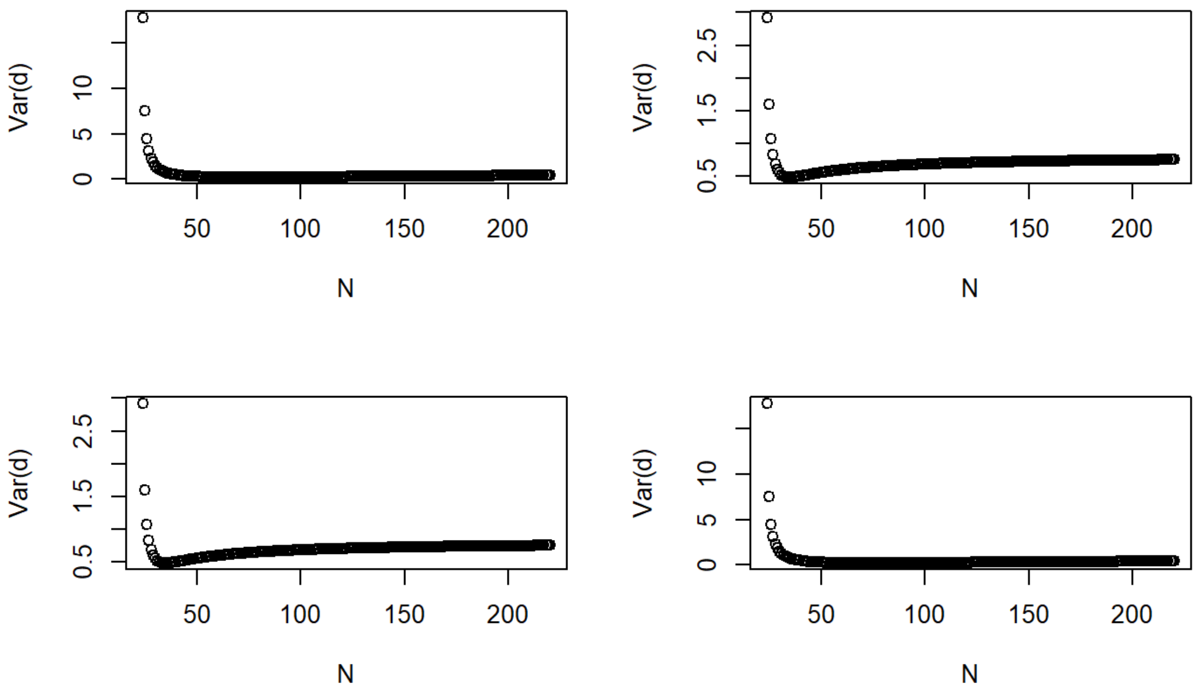

Figure SM1.49 : Variance of Shieh's d when variances and sample sizes are unequal across groups, as a function of the total sample size (N), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and $\sigma_1 > \sigma_2$ (top plots) or $\sigma_1 < \sigma_2$ (bottom plots).

Note : When variances were equal across populations, adding subjects only in the first group had the same impact on the variance as adding subjects only in the second group (see Figure SM1.34). When variances are unequal across groups, this is not true anymore (see Figure SM1.49).

- The smallest variance always occurs when there is a positive pairing between variances and sample size. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure SM1.50).

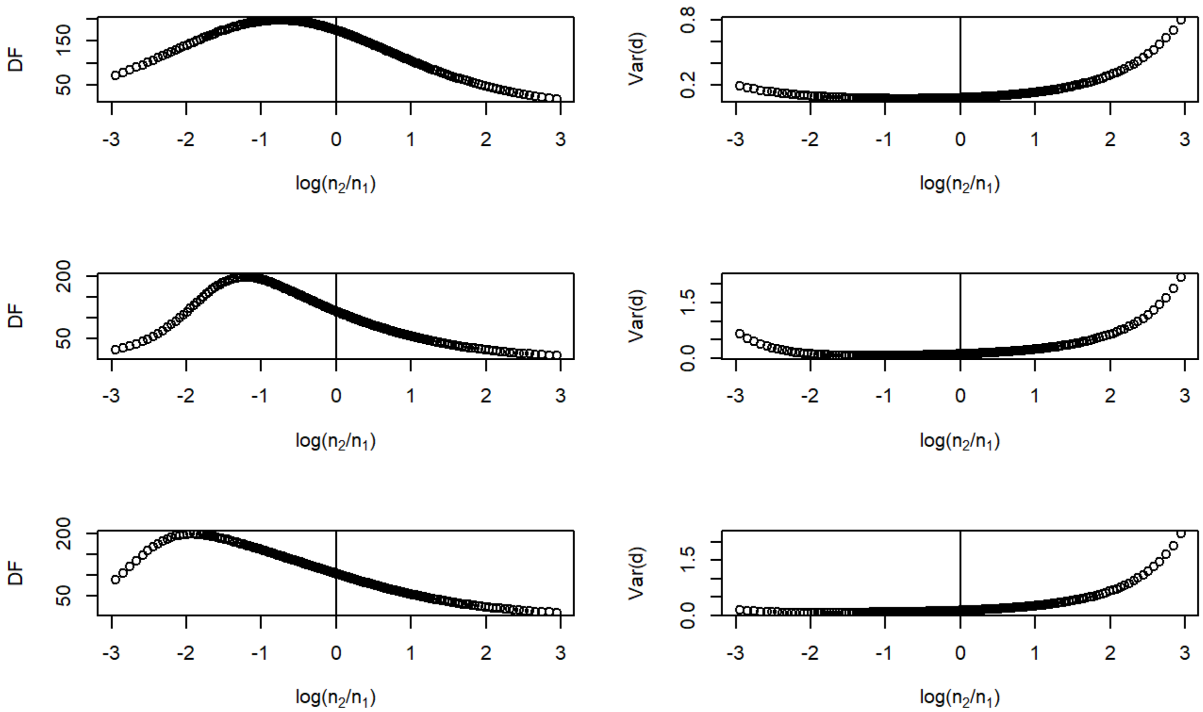

Figure SM1.50 : The variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ($log\left(\frac{n_2}{n_1}\right)$), when SD-ratio equals .68 (first row), .29 (second row) or .14 (third row).

Moreover, for a constant SD-ratio, the variances do not matter (See Figure SM1.51).

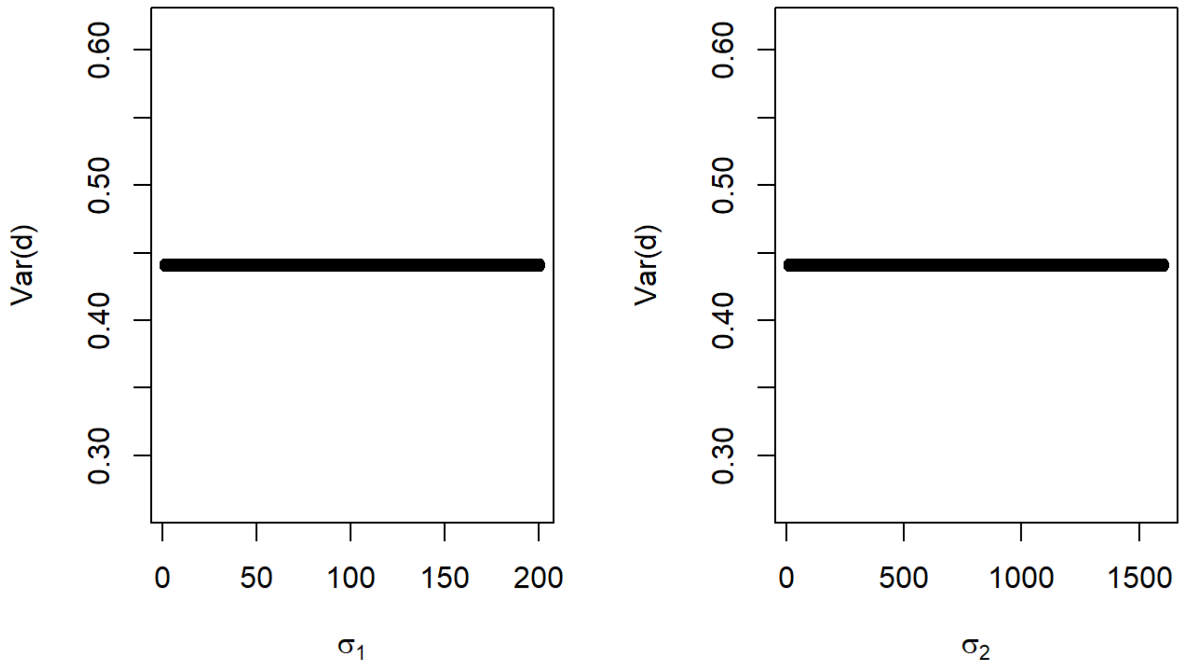

Figure SM1.51 : Variance of Shieh's d, when variances and sample sizes are unequal across groups, as a function of σ_1 (left) or σ_2 (right), for a constant SD-ratio.

When $\delta_{Shieh} \neq 0$.

While the variance of Shieh's d still depends on the total sample size and the interaction between the sample sizes ratio and the SD-ratio, it also depends on the population effect size (δ_{Shieh}) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase, as previously illustrated in Figure SM1.35.

In summary.

The variance of Shieh's d is a function of the population effect size (δ_{Shieh}), the total sample size (N) and the interaction between sample sizes ratio and SD-ratio $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$:

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the SD-ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the SD-ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;

- The variance increases when δ_{Shieh} increases. Note that the effect of δ_{Shieh} is moderated by the total sample size and the interaction between the sample sizes ratio and the SD-ratio.

Supplemental Material 2 : Simulation checks

In order to insure the reliability of our calculation method, for all scenarios where $G_1 = G_2 = 0$, we compared empirical means and variances of all estimators (i.e. means and variances of all estimates) with theoretical means and variances (i.e. expected means and variances, computed based on equations in Tables 2, 3 and 4 in the main article). Because we can draw exactly the same conclusions for **biased** (Cohen's d, Glass's d using either S_1 or S_2 as standardizer, Shieh's d and Cohen's d^*) and **unbiased** (Hedges' g, Glass's g using either S_1 or S_2 as standardizer, Shieh's g and Hedges' g^*) estimators, we will simultaneously present results for both categories of estimators. Results will be subdivided into 4 conditions :

- When population variances and sample sizes are equal across groups (condition a; see Tables SM2.1 and SM2.5 for respectively biased and unbiased estimators);

- When population variances are equal across groups and sample sizes are unequal (condition b; see Tables SM2.2 and SM2.6 for respectively biased and unbiased estimators);

- When population variances are unequal across groups and sample sizes are equal (condition c; see Tables SM2.3 and SM2.7 for respectively biased and unbiased estimators);

- When population variances and sample sizes are unequal across groups (condition d; see Tables SM2.4 and SM2.8 for respectively biased and unbiased estimators).

Because the equations of theoretical means and variances of Cohen's d and Hedges' g rely on the assumption of normality and equality of population variances, we expect empirical and theoretical parameters to be very close only in conditions a and b. For all other estimators, the equations of theoretical means and variances rely solely on the assumption of normality and therefore, we expect empirical and theoretical parameters to be very close in all conditions.

On average, empirical means (and variances) of all estimators are very close to theoretical expectations when population variances are equal across groups, with equal sample sizes (condition a; see Tables SM2.1 and SM2.5) or unequal sample sizes (condition b; see Tables SM2.2 and SM2.6).

When population variances are unequal across groups (conditions c and d; see Tables SM2.3, SM2.4, SM2.7 and SM2.8), empirical means (and variances) of Cohen's d^* (Hedges' g^*) and Shieh's d (Shieh's g) are still very close to theoretical expectations. Regarding Glass's d (Glass's g), on average, while empirical variances remain very close to theoretical expectations, one observes larger departures between empirical and theoretical means when using S_2 as standardizer. However, when looking at details in results for each scenario (see "biased estimator_condition C.xlsx," "biased estimator_condition D.xlsx," "unbiased estimator_condition C.xlsx" and "unbiased estimator_condition C.xlsx" in Supplemental Material 2), one notices that the larger the population effect size, the larger the departure between empirical and theoretical means, and that relative to the population effect size, departures between empirical and theoretical means are always very small. On the other hand, both empirical bias and variance of Cohen's d (Hedges' g) highly depart from theoretical expectations, even when looking at relative departures to the population effect size, especially when sample sizes are unequal across groups (condition d; see Tables SM2.4 and SM2.8), which is not surprising, as Cohen's d (Hedges' g) relies on the equality of population variances assumption.

Table SM2.1.

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are equal across groups (condition a).

	Absolute deviation between empirical and theoretical means $\|E(\hat{\delta})-\mu_{\delta}\|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's *d*	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Glass's d_1	0,022	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's d_2	0,023	0,000	0,005	0,007	1,005	0,889	0,966	0,035
Cohen's *d**	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Shieh's *d*	0,006	0,000	0,001	0,002	1,006	0,910	0,976	0,028

Table SM2.2

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's d	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's d_1	0,019	0,000	0,004	0,006	1,006	0,891	0,966	0,037
Glass's d_2	0,027	0,000	0,005	0,007	1,015	0,881	0,968	0,036
Cohen's d^*	0,010	0,000	0,003	0,002	1,007	0,902	0,965	0,034
Shieh's d	0,008	0,000	0,002	0,002	1,005	0,865	0,945	0,048

Table SM2.3

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's d	0,080	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's d_1	0,037	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's d_2	0,230	0,000	0,012	0,033	1,008	0,883	0,974	0,032
Cohen's d^*	0,036	0,000	0,003	0,006	1,007	0,874	0,975	0,033
Shieh's d	0,018	0,000	0,002	0,003	1,007	0,874	0,975	0,033

Table SM2.4

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are unequal across groups (condition d).

	Absolute deviation between empirical and theoretical means $\|E(\hat{\delta})-\mu_{\delta}\|$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's d	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's d_1	0,026	0,000	0,005	0,006	1,009	0,881	0,972	0,033
Glass's d_2	0,219	0,000	0,012	0,031	1,011	0,872	0,973	0,036
Cohen's d^*	0,030	0,000	0,003	0,006	1,011	0,860	0,974	0,034
Shieh's d	0,009	0,000	0,001	0,002	1,011	0,867	0,970	0,036

Table SM2.5

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are equal across groups (condition a).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	**Max**	**Min**	**Mean**	**Standard deviation**	**Max**	**Min**	**Mean**	**Standard deviation**
Cohen's g	0,011	0,000	0,002	0,003	1,006	0,911	0,976	0,028
Glass's g_1	0,021	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's g_2	0,022	0,000	0,004	0,007	1,005	0,889	0,966	0,035
Cohen's g^*	0,015	0,000	0,003	0,004	1,006	0,908	0,975	0,029
Shieh's g	0,008	0,000	0,002	0,002	1,006	0,908	0,975	0,029

Table SM2.6

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})\text{-}\mu_{\delta}\rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's g	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's g_1	0,018	0,000	0,004	0,005	1,006	0,891	0,966	0,037
Glass's g_2	0,026	0,000	0,004	0,006	1,015	0,881	0,968	0,036
Cohen's g^*	0,010	0,000	0,003	0,003	1,007	0,925	0,972	0,027
Shieh's g	0,007	0,000	0,002	0,002	1,007	0,900	0,959	0,037

Table SM2.7

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).

	Absolute deviation between empirical and theoretical means $\lvert E(\hat{\delta})-\mu_{\delta} \rvert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's g	0,079	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's g_1	0,036	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's g_2	0,221	0,000	0,012	0,032	1,008	0,883	0,974	0,032
Cohen's g^*	0,034	0,000	0,003	0,006	1,008	0,890	0,978	0,029
Shieh's g	0,017	0,000	0,002	0,003	1,008	0,890	0,978	0,029

Table SM2.8

Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are unequal across groups (condition d).

	Absolute deviation between empirical and theoretical means $\vert E(\hat{\delta})-\mu_{\hat{\delta}}\vert$				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\hat{\delta}}$			
Estimator ($\hat{\delta}$)	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's g	0,250	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's g_1	0,025	0,000	0,004	0,006	1,009	0,881	0,972	0,033
Glass's g_2	0,210	0,000	0,012	0,030	1,011	0,872	0,973	0,036
Cohen's g^*	0,029	0,000	0,003	0,005	1,011	0,882	0,977	0,029
Shieh's g	0,008	0,000	0,001	0,002	1,011	0,881	0,973	0,032

Supplemental Material 3 : correlation between sample means and *SD*

Introduction

The d-family effect sizes are commonly used with between-subject designs where individuals are randomly assigned into one of two independent groups and group means are compared. The population effect size is defined as

$$\delta = \frac{\mu_1 - \mu_2}{\sigma}$$

where both populations follow a normal distribution with mean μ_j in the j^{th} population (j=1,2) and common standard deviation σ. There exist different estimators of this population effect size, varying as a function of the chosen standardizer. When the equality of variances assumption is met, σ is estimated by pooling both sample standard deviations (S_1 and S_2) :

$$S_{Cohen's\ d} = \sqrt{\frac{(n_1 - 1) \times S_1^2 + (n_2 - 1) \times S_2^2}{n_1 + n_2 - 2}}$$

When the equality of variances assumption is not met, we are considering three alternative estimates :

- Using the standard deviation of the control group (S_c) as standardizer :

$$S_{Glass's\ d} = S_c$$

- Using a standardizer that takes the sample sizes allocation ratio $\left(\frac{n_2}{n_1}\right)$ into account :

$$S_{Shieh's\ d} = \sqrt{S_1^2/q_1 + S_2^2/q_2}; \quad q_j = \frac{n_j}{N}(j = 1, 2)$$

- Or using the square root of the non pooled average of both variance estimates (S_1^2 and S_2^2) as standardizer :

$$S_{Cohen's\ d^*} = \sqrt{\frac{(S_1^2 + S_2^2)}{2}}$$

As we previously mentioned, the use of these formulas requires to meet the assumption of normality. Using them when distributions are not normal will have consequences on both bias and variance of all estimators. More specifically, when samples are extracted from skewed distributions, correlations might occur between the sample mean difference ($\bar{X}_1 - \bar{X}_2$) and standardizers (S). Throughout this Supplemental Material, we will study when these correlations occur. To this end, we will distinguish 3 situations :

- when $\sigma_1 = \sigma_2$ and $n_1 = n_2$ (condition a);
- when $\sigma_1 = \sigma_2$ and $n_1 \neq n_2$ (condition b);
- when $\sigma_1 \neq \sigma_2$ and $n_1 = n_2$ (condition c).

Before studying conditions a, b and c, we will briefly introduce the impact of these correlations on the bias. Note that we will compute correlations using the coefficient of Spearman's ρ. We decided to use Spearman's ρ instead of Pearson's ρ because some plots revealed non-perfectly linear relations.

How correlations between the mean difference ($\bar{X}_1 - \bar{X}_2$) and standardizers affect the bias of estimators.

When population distributions are right-skewed, there is a positive (negative) correlation between S_1 (S_2) and ($\bar{X}_1 - \bar{X}_2$). When distributions are left-skewed, there is a negative (positive) correlation between S_1 (S_2) and ($\bar{X}_1 - \bar{X}_2$). When the population mean difference ($\mu_1 - \mu_2$) is positive (like in our simulations), all other parameters being equal, an estimator is always less biased and variable when choosing a standardizer that is positively correlated with $\bar{X}_1 - \bar{X}_2$ than when choosing an estimator that is negatively correlated with $\bar{X}_1 - \bar{X}_2$. When the population mean difference is negative, the reverse is true.

"All other parameters being equal" is mentioned because it is always possible that other factors in action have an opposite effect on bias and variance in order that increasing the magnitude of the correlation between S_j and $\bar{X}_1 - \bar{X}_2$ does not necessarily reduce the bias and the variance. For example, when population variances are equal across groups and sample sizes are unequal, we will see below that the lower n_j, the larger the magnitude of the correlation between S_j and $\bar{X}_1 - \bar{X}_2$. When the correlation between S_j and $\bar{X}_1 - \bar{X}_2$ is positive, the smaller the sample size, the larger the positive correlation. At the same time, we know that increasing the sample size decreases the bias. This is a nice example of situations where two factors might have an opposite action on bias.

Correlations between the mean difference ($\bar{X}_1 - \bar{X}_2$) and all standardizers

When equal population variances are estimated based on equal sample sizes (condition a). While $\bar{X}_j$ and S_j (j=1,2) are uncorrelated when samples are extracted from symmetric distributions (see Figure SM3.1), there is a non-null correlation between $\bar{X}_j$ and S_j when distributions are skewed (Zhang, 2007).

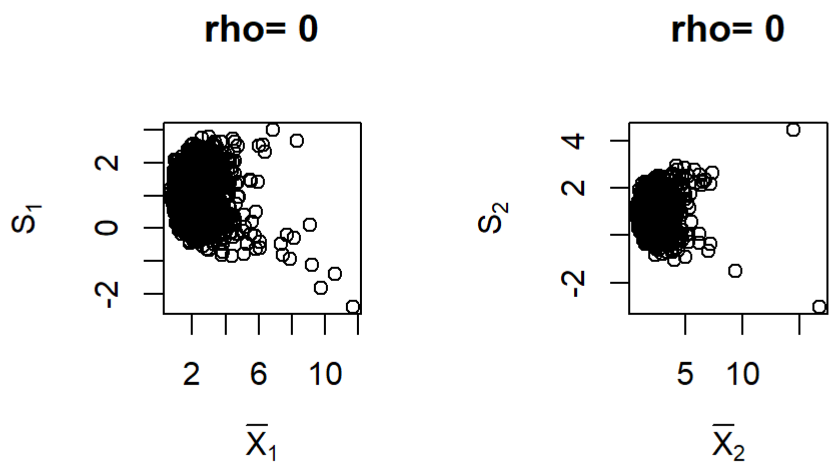

Figure SM3.1 : S_j as a function of $\bar{X}_j$ (j=1,2), when samples are extracted from symmetric distributions ($\gamma_1 = 0$).

More specifically, when distributions are right-skewed, there is a **positive** correlation between $\bar{X}_j$ and S_j (see the two top plots in Figure **??**), resulting in a *positive* correlation between S_1 and $\bar{X}_1 - \bar{X}_2$ and in a *negative* correlation between S_2 and $\bar{X}_1 - \bar{X}_2$ (see the two bottom plots in Figure SM3.2). This can be explained by the fact that $\bar{X}_1$ and $\bar{X}_1 - \bar{X}_2$ are positively correlated while $\bar{X}_2$ and $\bar{X}_1 - \bar{X}_2$ are negatively correlated (of course, correlations would be trivially reversed if we computed $\bar{X}_2 - \bar{X}_1$ instead of $\bar{X}_1 - \bar{X}_2$).

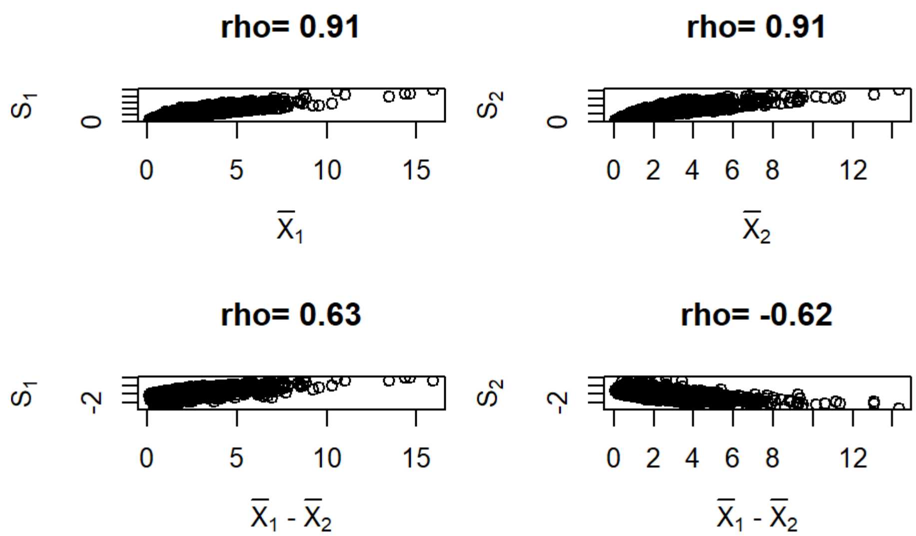

Figure SM3.2 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$).

One should also notice that both correlations between S_j and $\bar{X}_1 - \bar{X}_2$ are equal, in absolute terms (possible tiny differences might be observed due to sampling error in our simulations). As a consequence, when computing a standardizer taking both S_1 and S_2 into account, it results in a standardizer that is uncorrelated with $\bar{X}_1 - \bar{X}_2$ (see Figure SM3.3).

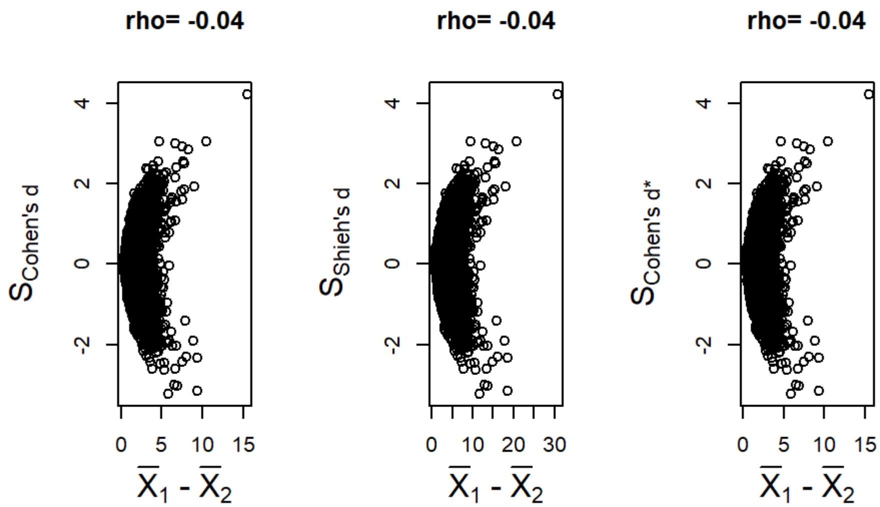

Figure SM3.3 : $S_{Cohen's\ d}$, $S_{Shieh's\ d}$ and $S_{Cohen's\ d^*}$ as a function of the mean difference ($\bar{X}_1 - \bar{X}_2$), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$).

On the other hand, when distributions are left-skewed, there is a **negative** correlation between $\bar{X}_j$ and S_j (see the two top plots in Figure SM3.4), resulting in a *negative* correlation between S_1 and $\bar{X}_1 - \bar{X}_2$ and in a *positive* correlation between S_2 and $\bar{X}_1 - \bar{X}_2$ (see the two bottom plots in Figure SM3.4).

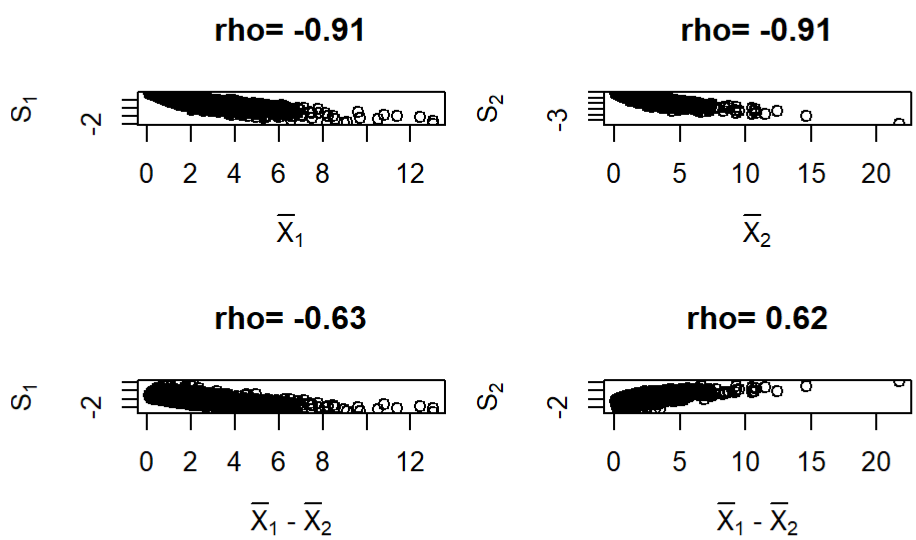

Figure SM3.4 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$).

Again, because correlations between S_j and $\bar{X}_1 - \bar{X}_2$ are similar in absolute terms, any standardizers taking both S_1 and S_2 into account will be uncorrelated with $\bar{X}_1 - \bar{X}_2$ (see Figure SM3.5).

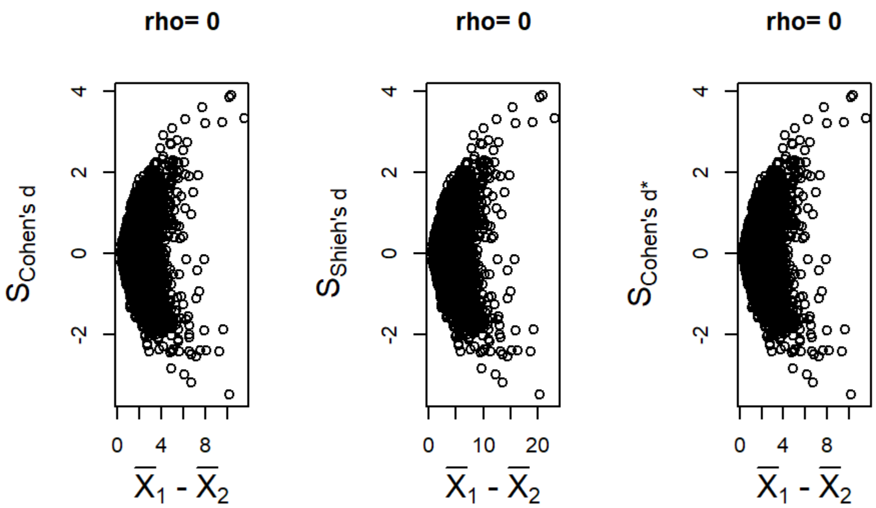

Figure SM3.5 : $S_{Cohen's\ d}$, $S_{Shieh's\ d}$ and $S_{Cohen's\ d^*}$ as a function of the mean difference ($\bar{X}_1 - \bar{X}_2$), when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$).

When equal population variances are estimated based on unequal sample sizes (condition b). When distributions are skewed, there are again non-null correlations between $\bar{X}_j$ and S_j, however $cor(S_1, \bar{X}_1) \neq cor(S_2, \bar{X}_2)$, because of the different sample sizes.

When distributions are skewed, one observes that the larger the sample size, the lower the correlation between S_j and $\bar{X}_j$ (See Figures SM3.6 and SM3.7).

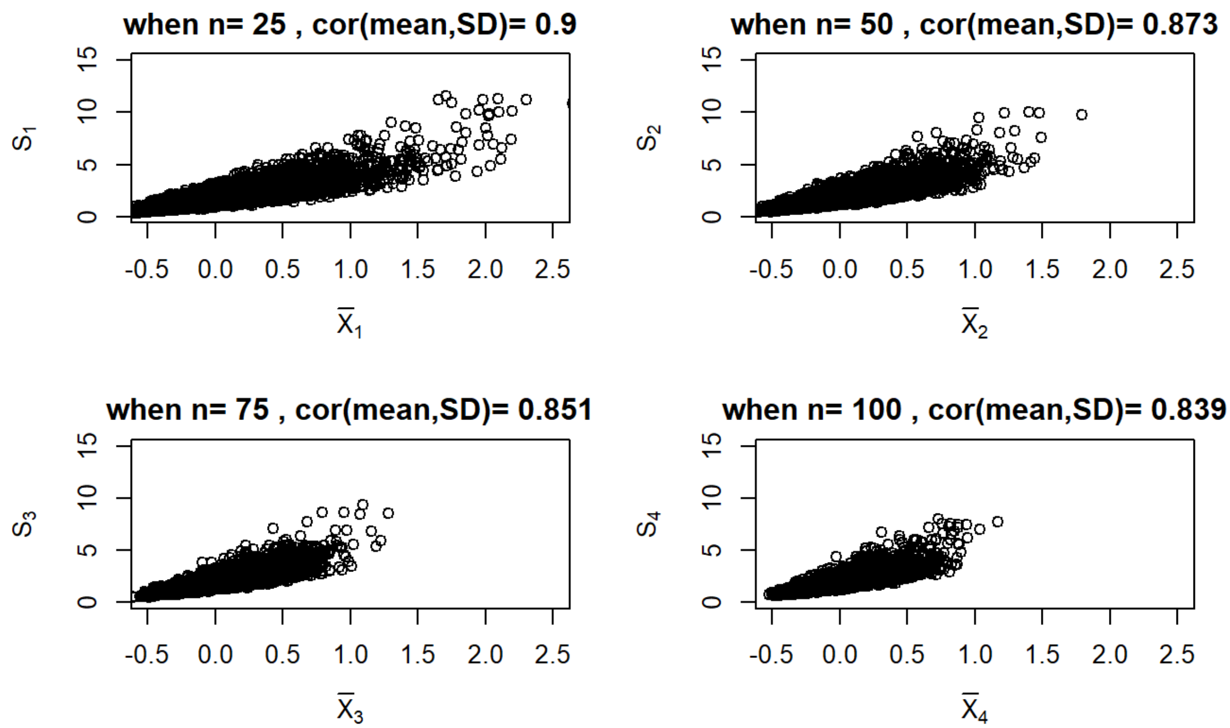

Figure SM3.6 : Correlation between S_j and $\bar{X}_j$ when $n = 25$, 50, 75 or 100 and samples are extracted from right skewed distributions ($\gamma_1 = 6.32$).

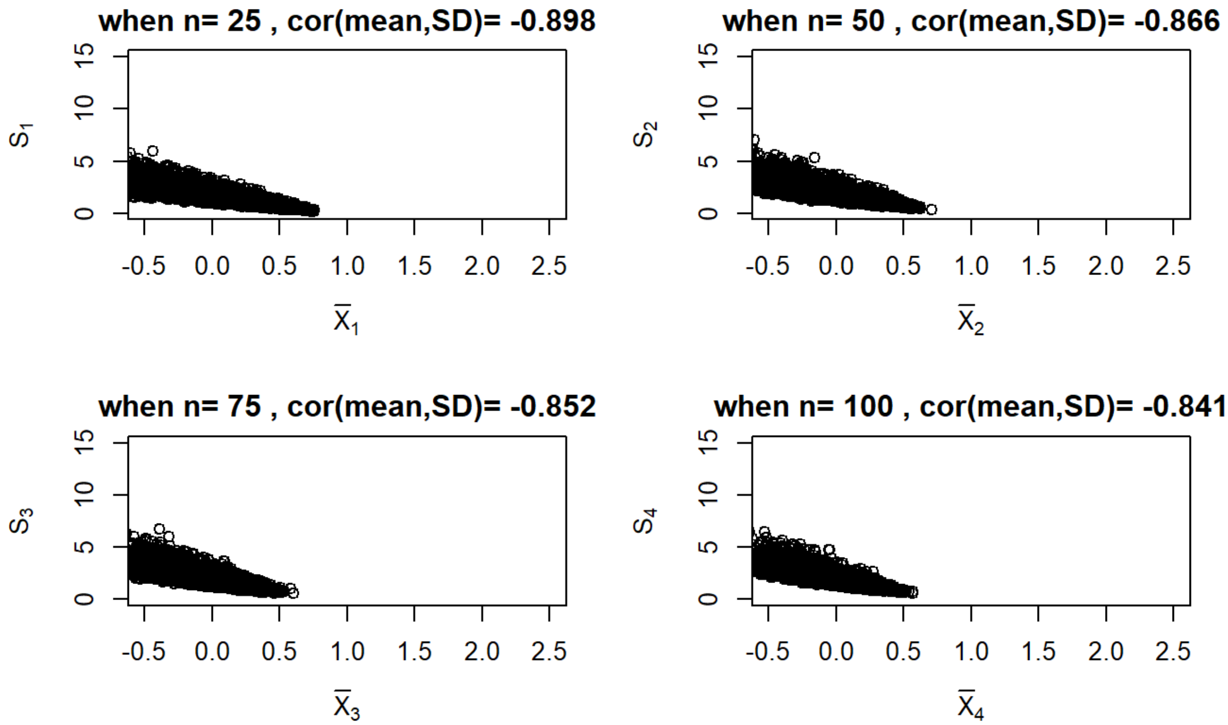

Figure SM3.7 : Correlation between S_j and $\bar{X}_j$ when $n = 25$, 50, 75 or 100 and samples are extracted from left skewed distributions ($\gamma_1 = -6.32$).

This might explain that the magnitude of the correlation between S_j and $\bar{X}_1 - \bar{X}_2$ is lower in the larger sample (see bottom plots in Figures SM3.8 and SM3.9). With no surprise, there is a positive (negative) correlation between S_1 and $\bar{X}_1 - \bar{X}_2$ and a negative (positive) correlation between S_2 and $\bar{X}_1 - \bar{X}_2$ when distributions are right-skewed (left-skewed), as illustrated in the two bottom plots of Figures SM3.8 and SM3.9.

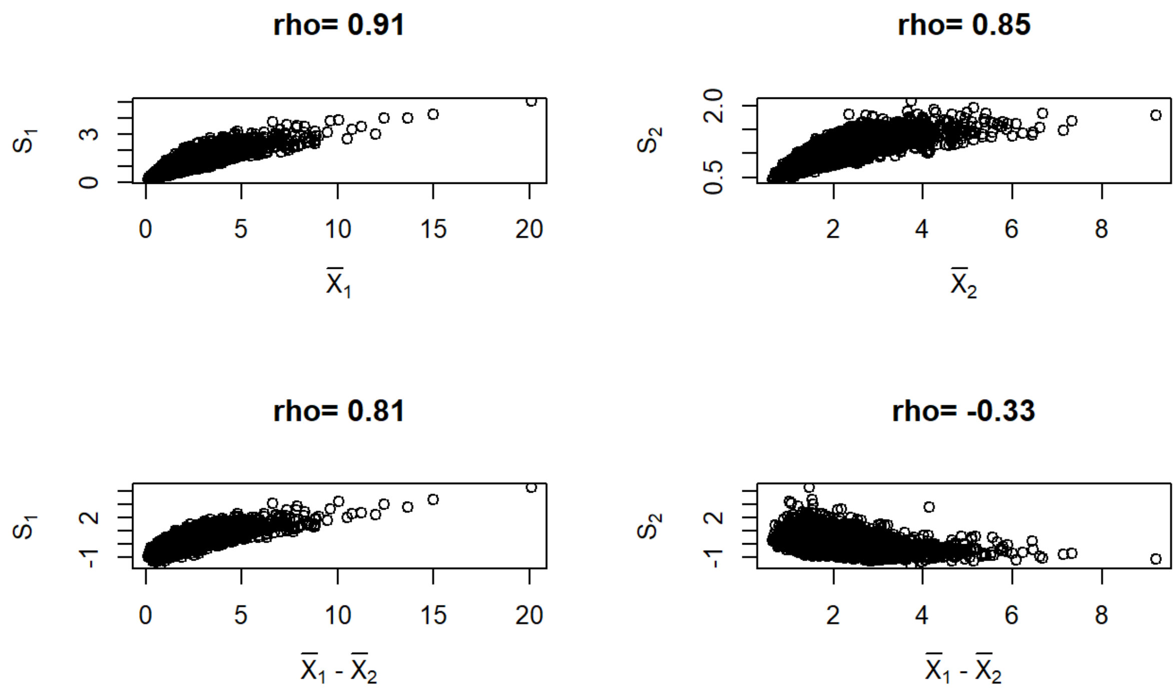

Figure SM3.8 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$), with n_1=20 and n_2=100.

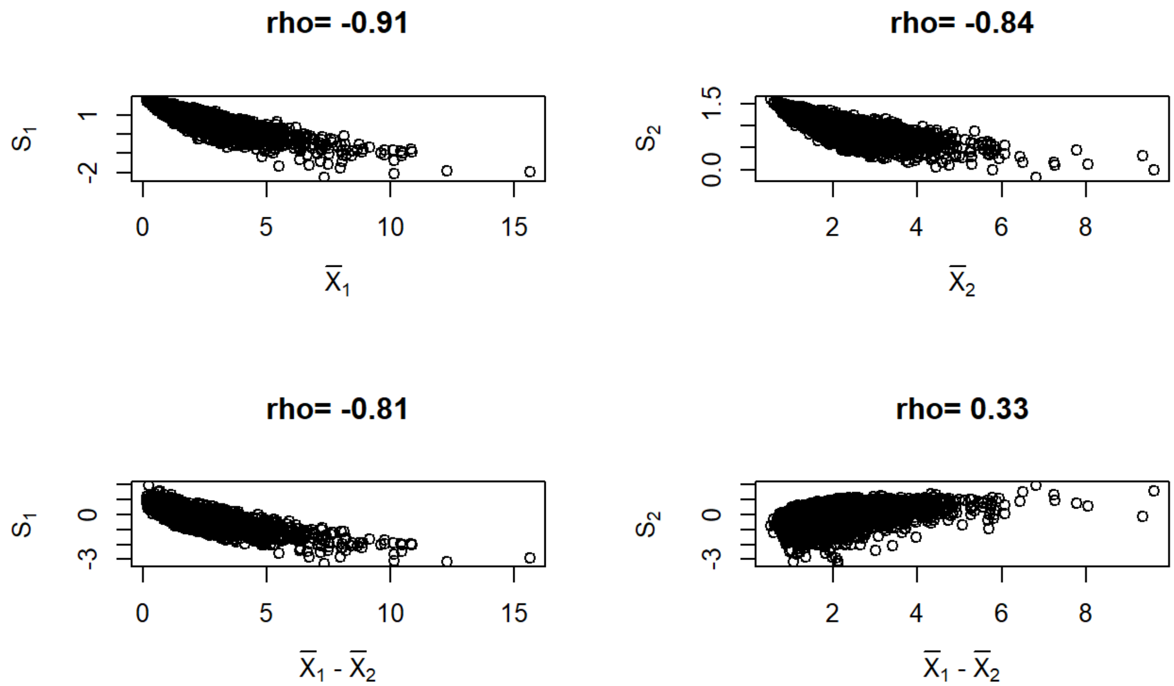

Figure SM3.9 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$), with n_1=20 and n_2=100.

This might also explain that standardizers of Shieh's d and Cohen's d^* are **correlated** with $\bar{X}_1 - \bar{X}_2$ (see Figures SM3.10 and SM3.11) :

- When computing $S_{Cohen's\ d^*}$, the same weight is given to both S_1 and S_2. Therefore, it does not seem surprising that the sign of the correlation between $S_{Cohen's\ d^*}$ and $\bar{X}_1 - \bar{X}_2$ is the same as the size of the correlation between $\bar{X}_1 - \bar{X}_2$ and the SD of the smallest sample;

- When computing $S_{Shieh's\ d}$, more weight is given to the SD of the smallest sample, it is therefore not really surprising to observe that the correlation between $S_{Shieh's\ d}$ and $\bar{X}_1 - \bar{X}_2$ is closer of the correlation between the SD of the smallest group and $\bar{X}_1 - \bar{X}_2$ (i.e. $|cor(S_{Shieh's\ d}, \bar{X}_1 - \bar{X}_2)| > |cor(S_{Cohen's\ d^*}, \bar{X}_1 - \bar{X}_2)|$);

- When computing $S_{Cohen's\ d}$, more weight is given to the SD of the largest sample, which by compensation effect brings the correlation very close to 0.

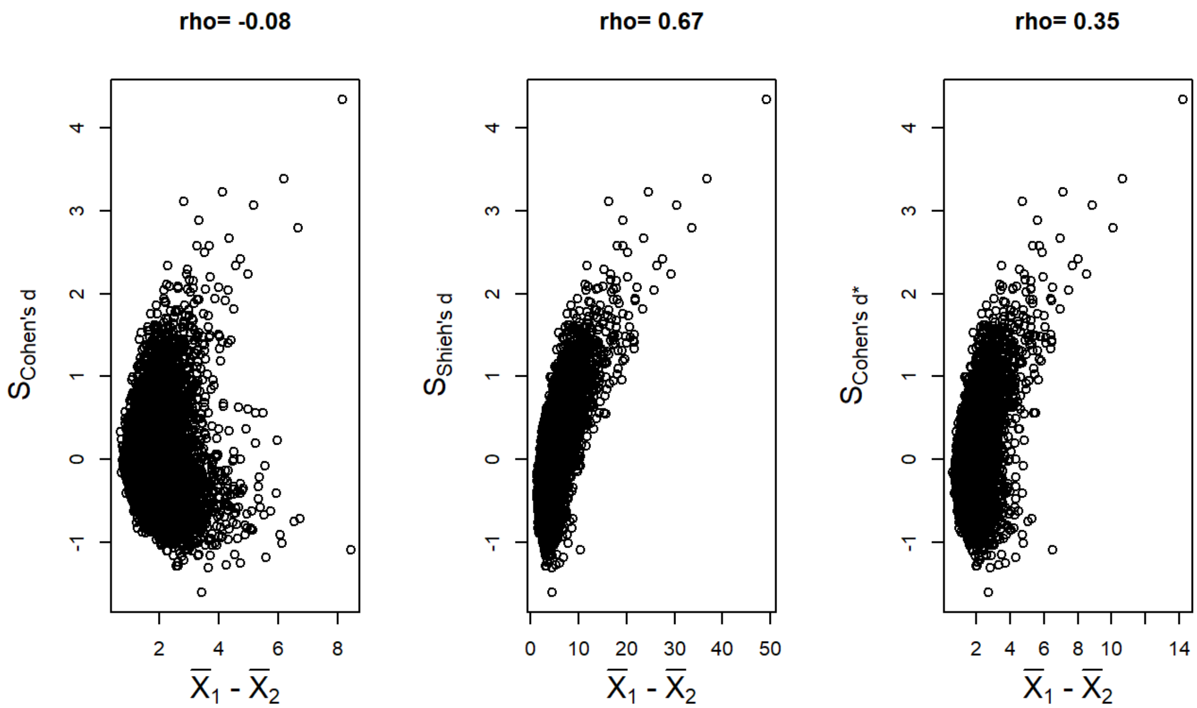

Figure SM3.10 : $S_{Cohen's\,d}$, $S_{Shieh's\,d}$ and $S_{Cohen's\,d^*}$ as a function of the mean difference ($\bar{X}_1 - \bar{X}_2$), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$, with n_1=20 and n_2=100).

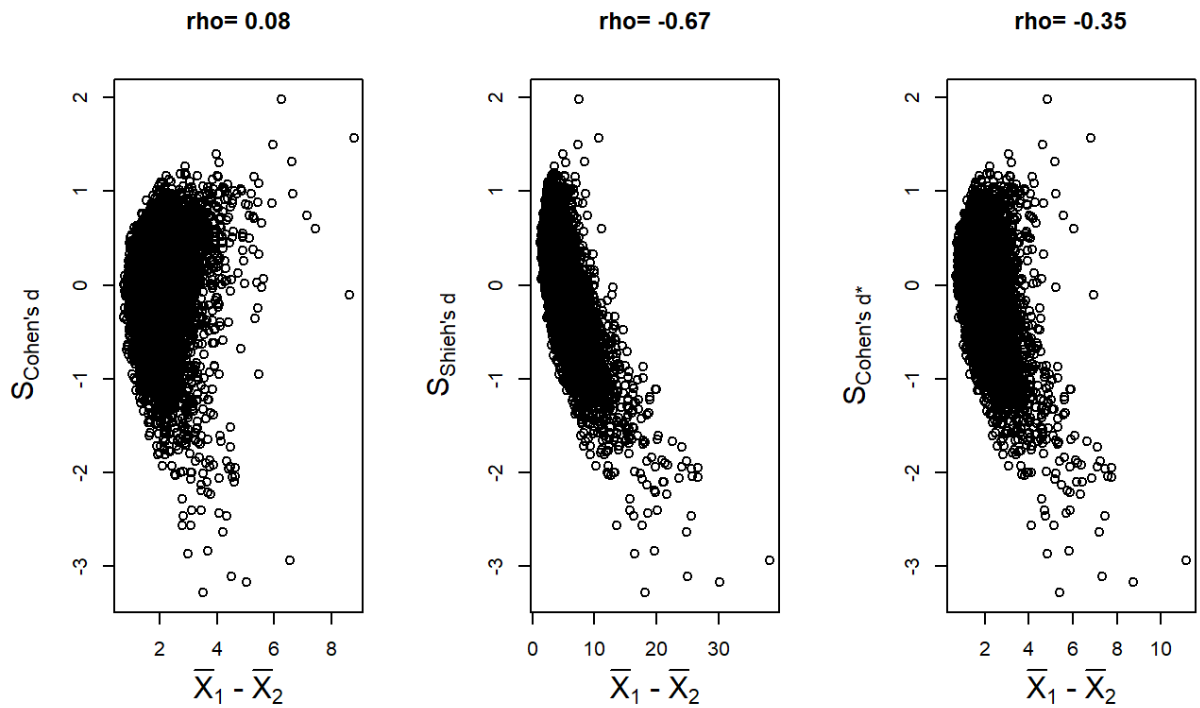

Figure SM3.11 : $S_{Cohen's\,d}$, $S_{Shieh's\,d}$ and $S_{Cohen's\,d^*}$ as a function of the mean difference ($\bar{X}_1 - \bar{X}_2$), when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$), with n_1=20 and n_2=100.

The correlation between $\bar{X}_1 - \bar{X}_2$ and respectively S_1, S_2, the standardizer of Cohen's d^*, the standardizer of Shieh's d and the standardizer of Cohen's d are summarized in Table 1.

Table 1

Correlation between standardizers (S_1, S_2, $S_{Cohen's\ d}$, $S_{Shieh's\ d}$ and $S_{Cohen's\ d^*}$) and $\bar{X}_1 - \bar{X}_2$, when samples are extracted from skewed distributions with equal variances, and $n_1 = n_2$ (condition a) or $n_1 \neq n_2$ (condition b)

	population distribution	
	right-skewed	*left-skewed*
When $n_1 = n_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	$S_{Cohen's\ d}$: *null*	$S_{Cohen's\ d}$: *null*
	$S_{Shieh's\ d}$: *null*	$S_{Shieh's\ d}$: *null*
	$S_{Cohen's\ d^*}$: *null*	$S_{Cohen's\ d^*}$: *null*
When $n_1 > n_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	$S_{Cohen's\ d}$: *null*	$S_{Cohen's\ d}$: *null*
	$S_{Shieh's\ d}$: *negative*	$S_{Shieh's\ d}$: *positive*
	$S_{Cohen's\ d^*}$: *positive (but very small)*	$S_{Cohen's\ d^*}$: *negative (but very small)*
When $n_1 < n_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	$S_{Cohen's\ d}$: *negative (but very small)*	$S_{Cohen's\ d}$: *positive (but very small)*
	$S_{Shieh's\ d}$: *positive*	$S_{Shieh's\ d}$: *negative*
	$S_{Cohen's\ d^*}$: *positive*	$S_{Cohen's\ d^*}$: *negative*

When unequal population variances are estimated based on equal sample sizes (condition c). When distributions are skewed, there are again non-null correlations between $\bar{X}_j$ and S_j. As illustrated in Figures SM3.12 and SM3.13, the correlation remains the same for any population SD (σ). However, the magnitude of the correlation between S_j and $\bar{X}_1 - \bar{X}_2$ differs : it is stronger in the sample extracted from the larger population variance (see Figures SM3.14 and SM3.15).

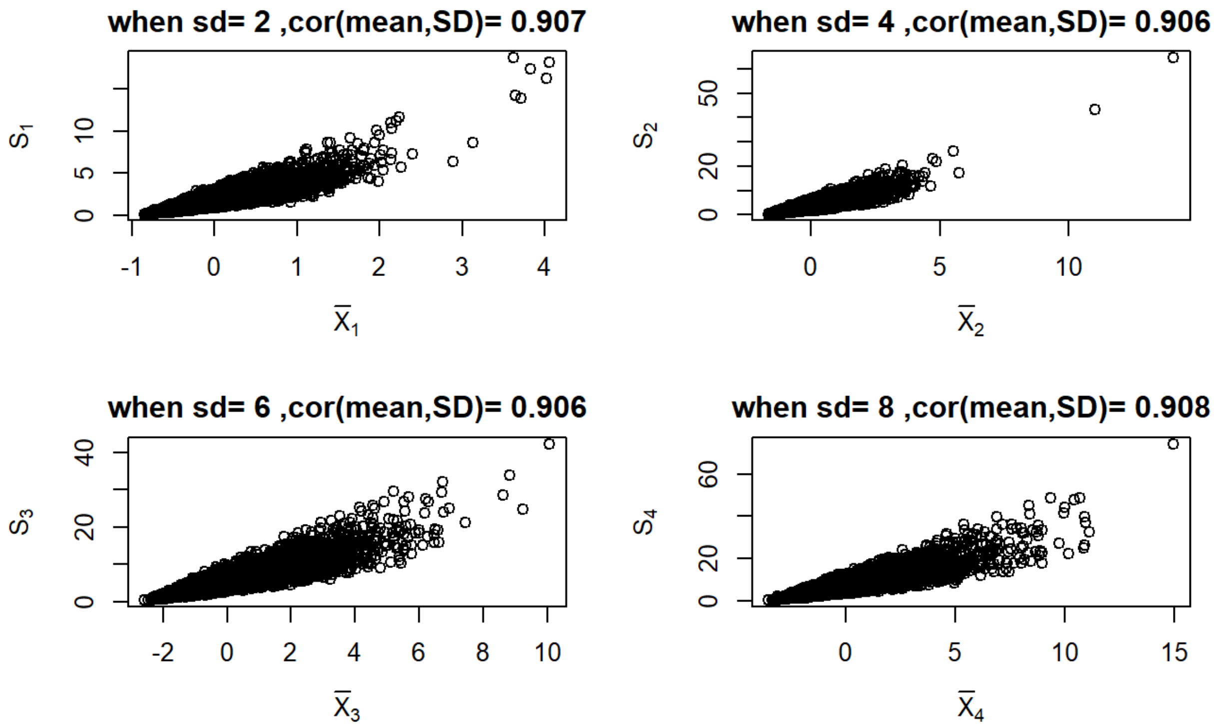

Figure SM3.12 : Correlation between S_j and $\bar{X}_j$ when SD = 2, 4, 6 or 8 and samples are extracted from right skewed distributions (γ_1 = 6.32).

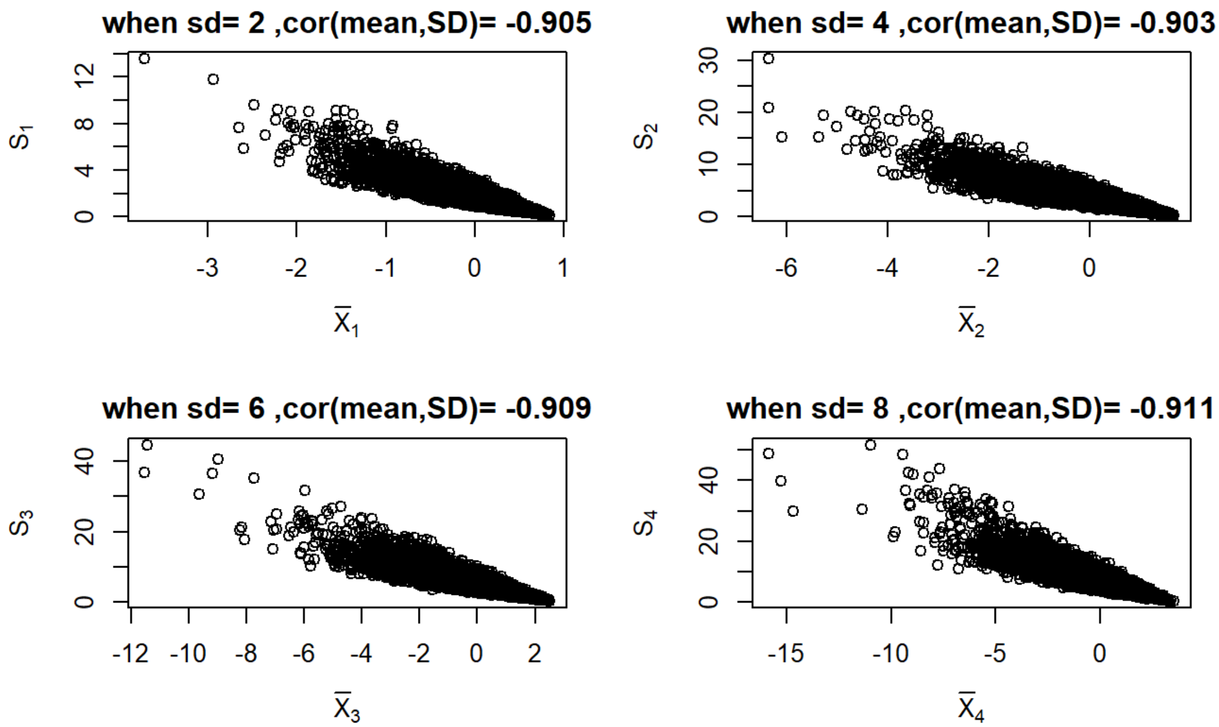

Figure SM3.13 : Correlation between S_j and $\bar{X}_j$ when $SD = 2$, 4, 6 or 8 and samples are extracted from left skewed distributions ($\gamma_1 = -6.32$).

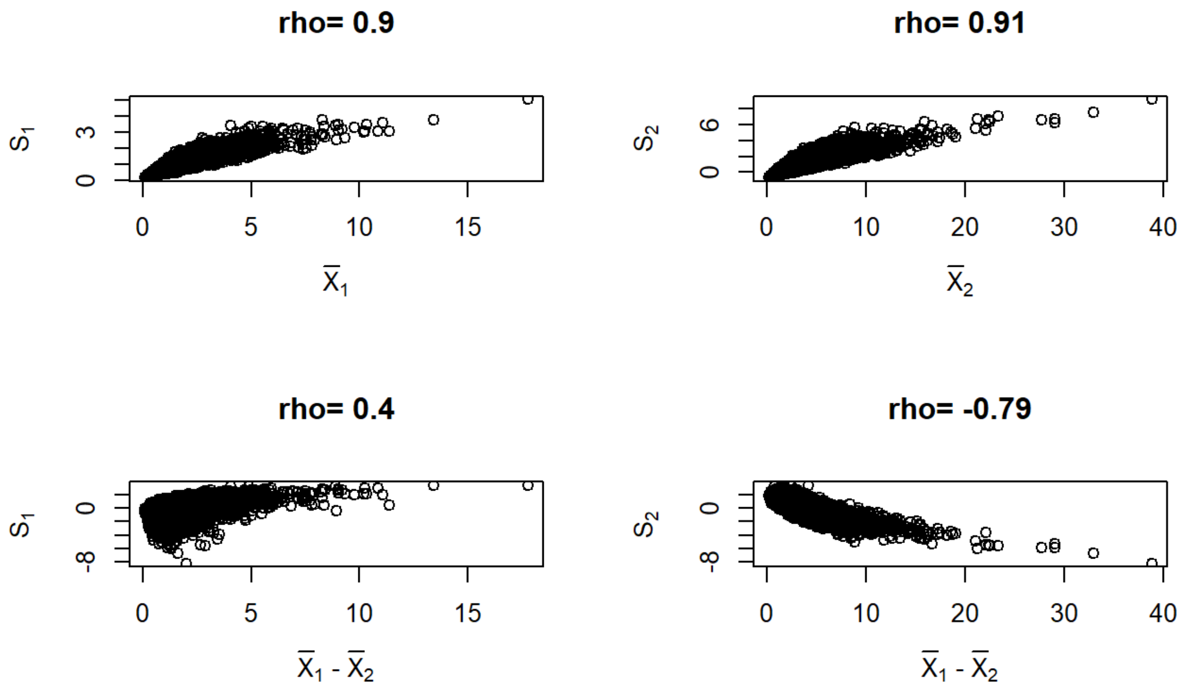

Figure SM3.14 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$), with S_1=2 and S_2=4.

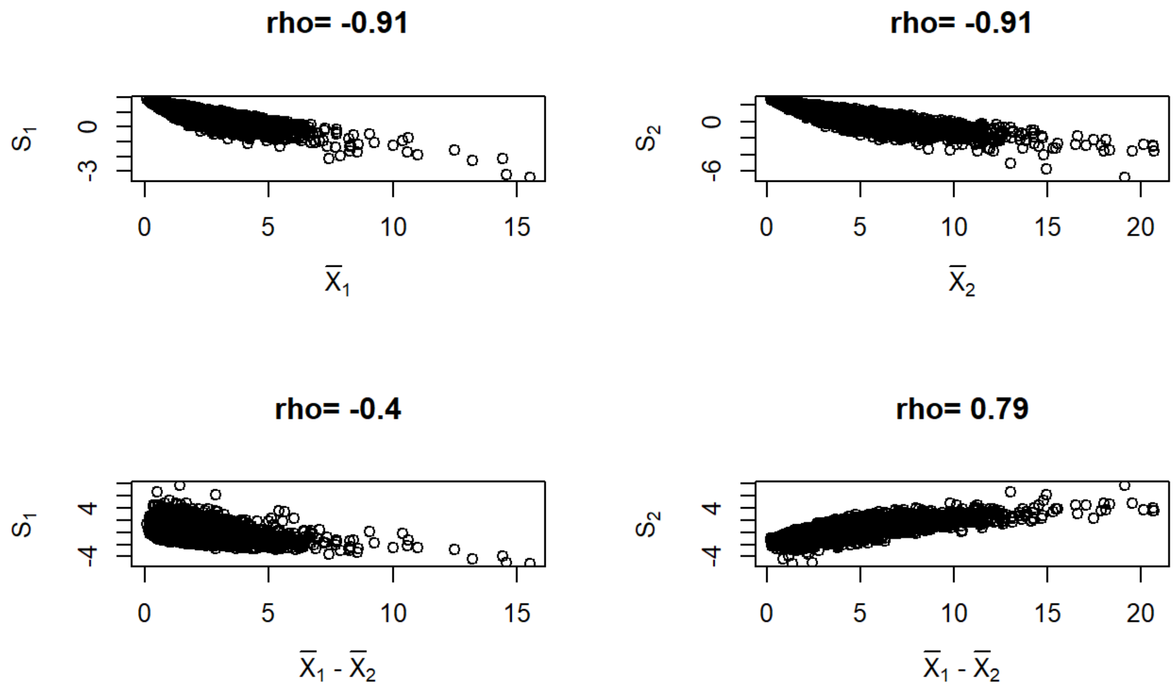

Figure SM3.15 : S_j (j=1,2) as a function of $\bar{X}_j$ (top plots) or $\bar{X}_1 - \bar{X}_2$ (bottom plots), when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$), with S_1=2 and S_2=4.

This also explains that when computing a standardizer that takes both S_1 and S_2 into account, it results in a standardizer that is correlated with $\bar{X}_1 - \bar{X}_2$ (see Figures SM3.16 and SM3.17). The correlation between the mean difference ($\bar{X}_1 - \bar{X}_2$) and respectively the standardizer of Shieh's d, Cohen's d^* and Cohen's d will have the same sign as the correlation between ($\bar{X}_1 - \bar{X}_2$) and the larger SD. Table 2 summarizes the sign of the correlation between $\bar{X}_1 - \bar{X}_2$ and respectively S_1, S_2 and the three standardizers taking both S_1 and S_2 into account (see "Others" in the Table).

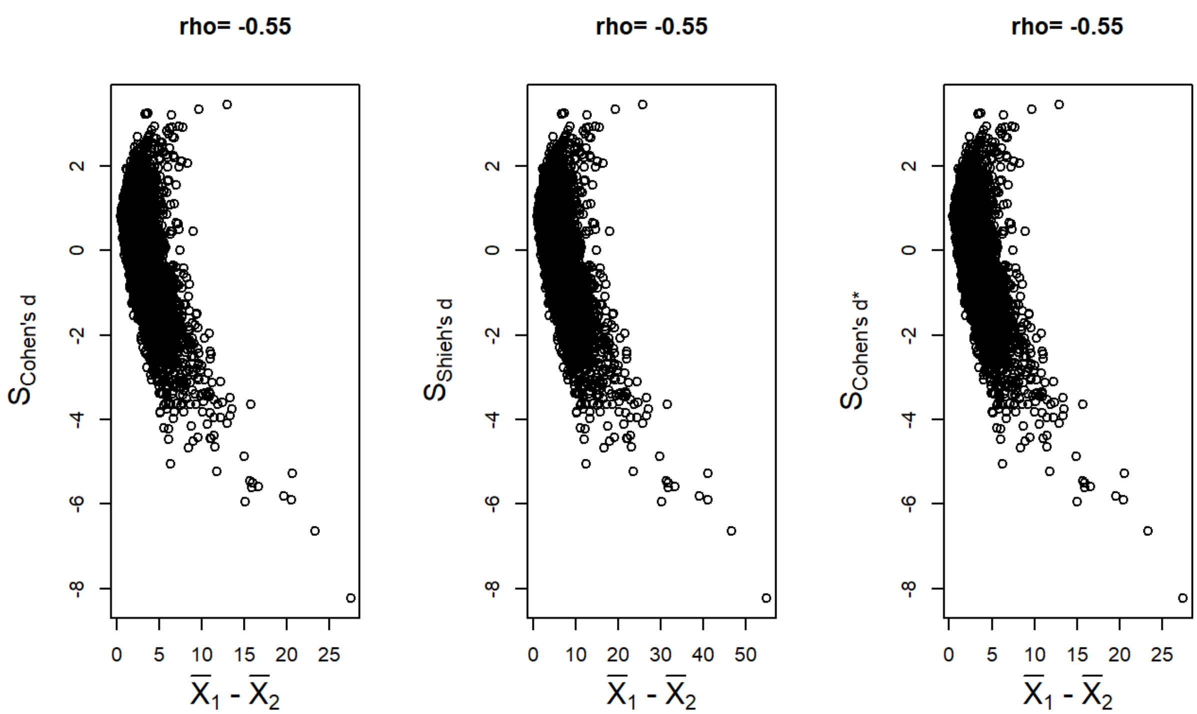

Figure SM3.16 : $S_{Cohen's\, d}$, $S_{Shieh's\, d}$ and $S_{Cohen's\, d^*}$ as a function of the mean difference ($\bar{X}_1 - \bar{X}_2$), when samples are extracted from right skewed distributions ($\gamma_1 = 6.32$), with S_1=2 and S_2=4.

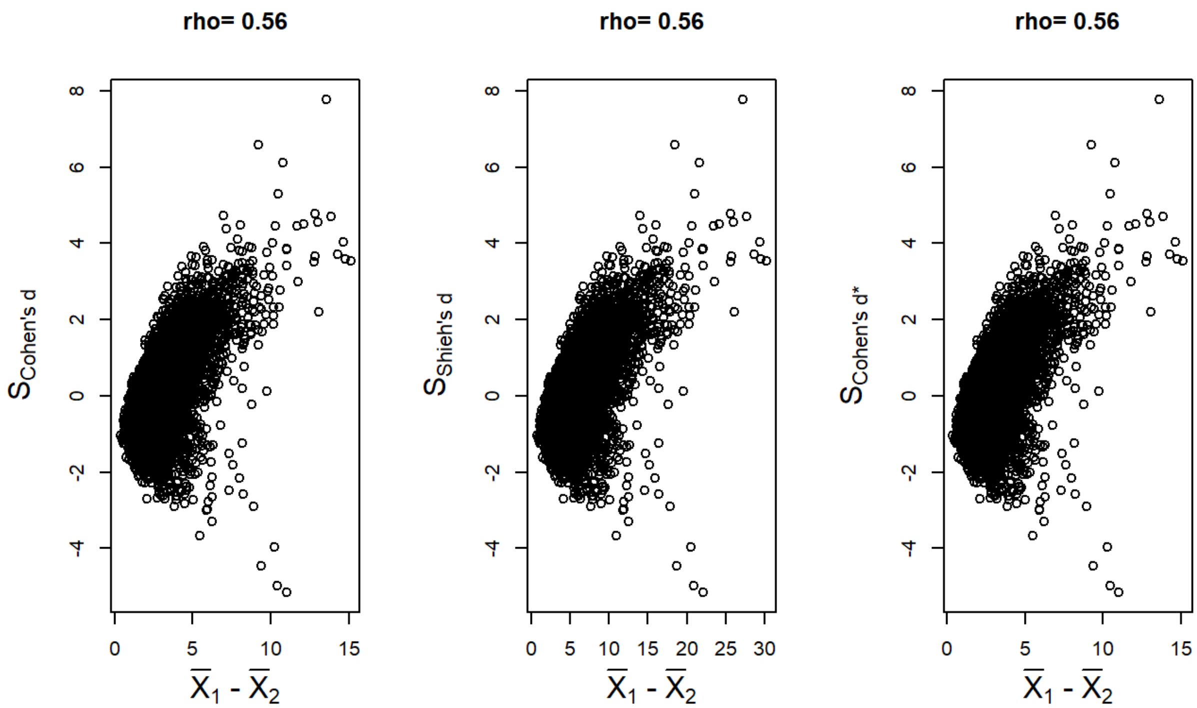

Figure SM3.17 : $S_{Cohen's\,d}$, $S_{Shieh's\,d}$ and $S_{Cohen's\,d^*}$ as a function of the mean difference $(\bar{X}_1 - \bar{X}_2)$, when samples are extracted from left skewed distributions ($\gamma_1 = -6.32$), with S_1=2 and S_2=4.

Table 2

Correlation between standardizers (S_1, S_2 and others) and $\bar{X}_1 - \bar{X}_2$, when samples are extracted from skewed distributions with equal sample sizes, as a function of the SD-ratio.

	population distribution	
	right-skewed	*left-skewed*
When $\sigma_1 = \sigma_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	Others : *null*	Others : *null*
When $\sigma_1 > \sigma_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	Others : *positive*	Others : *negative*
When $\sigma_1 < \sigma_2$	S_1 : *positive*	S_1 : *negative*
	S_2 : *negative*	S_2 : *positive*
	Others : *negative*	Others : *positive*