

## Supplemental Material

## Supplemental Material

### Supplemental Material 1 :

#### Theoretical bias

For all “biased” estimators, when the population effect size is null so is the bias. We will therefore focus on configurations where there is a non-null population effect size. The sampling distribution of Cohen’s  $d$  (and therefore its bias) is only known under the assumptions of normality and homoscedasticity. On the other hand, the biases of Glass’s  $d$ , Cohen’s  $d^*$  and Shieh’s  $d$  are theoretically known for all configurations where the normality assumption is met. In order to simplify the analysis of their bias, it is convenient to subdivide all configurations into 3 conditions :

- when population variances are equal across groups;
- when population variances are unequal across groups, with equal sample sizes;
- when population variances are unequal across groups, with unequal sample sizes.

**Preliminary note.** For all previously mentioned estimators (Cohen’s  $d$ , Glass’s  $d$ , Cohen’s  $d^*$  and Shieh’s  $d$ ), the theoretical expectation is computed by multiplying the population effect size (respectively  $\delta_{Cohen}$ ,  $\delta_{Glass}$ ,  $\delta_{Cohen}^*$  and  $\delta_{Shieh}$ ) by the following multiplier coefficient:

$$\gamma = \frac{\sqrt{\frac{df}{2}} \times \Gamma_{\frac{df-1}{2}}}{\Gamma_{\frac{df}{2}}} \quad (1)$$

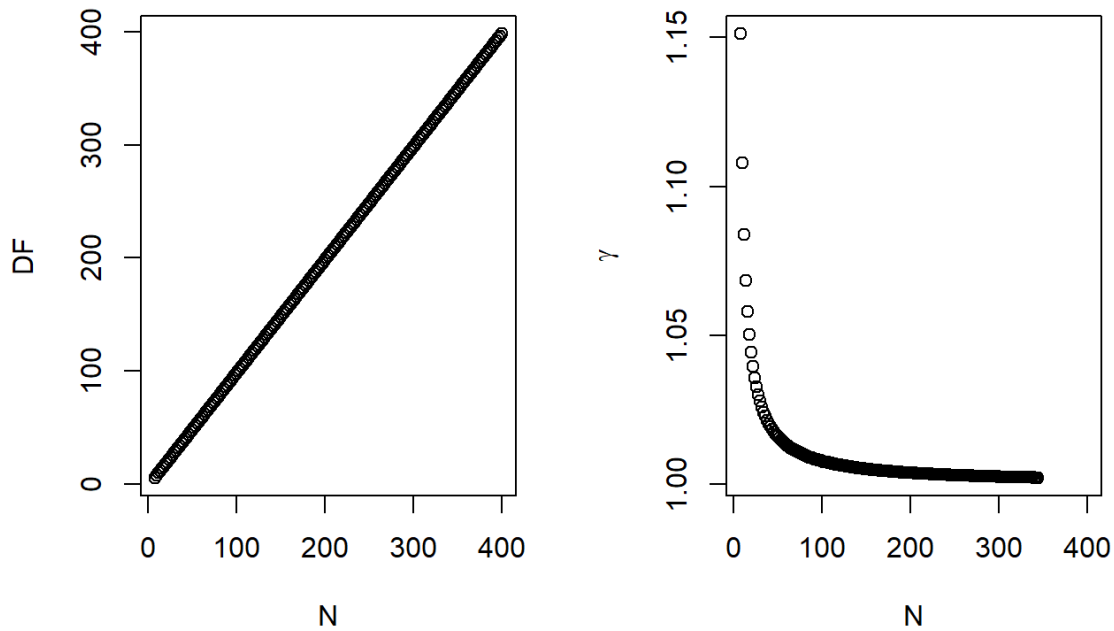
where  $df$  are the degrees of freedom (see the main article).  $\gamma$  is *always* positive, meaning that when the population effect size is not zero, all estimators will overestimate the population effect size. Moreover, its limit tends to 1 when the degrees of freedom ( $df$ ) tend to infinity, meaning that the larger the degrees of freedom, the lower the bias.

While we focus on the theoretical bias of biased estimators when the normality assumption is met, it is interesting to notice that our main conclusions seem to generalize to :

- biased estimators when samples are extracted from symmetric distributions;
- unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

**Cohen's  $d$  (see Table 2).** Under the assumptions that independant residuals are normally distributed with equal variances, the **bias** of Cohen's  $d$  is a function of total sample size ( $N$ ) and the population effect size ( $\delta_{Cohen}$ ) :

- The larger the population effect size, the more Cohen's  $d$  will overestimate  $\delta_{Cohen}$ ;
- The larger the total sample size, the lower the bias (see Figure SM1.1);



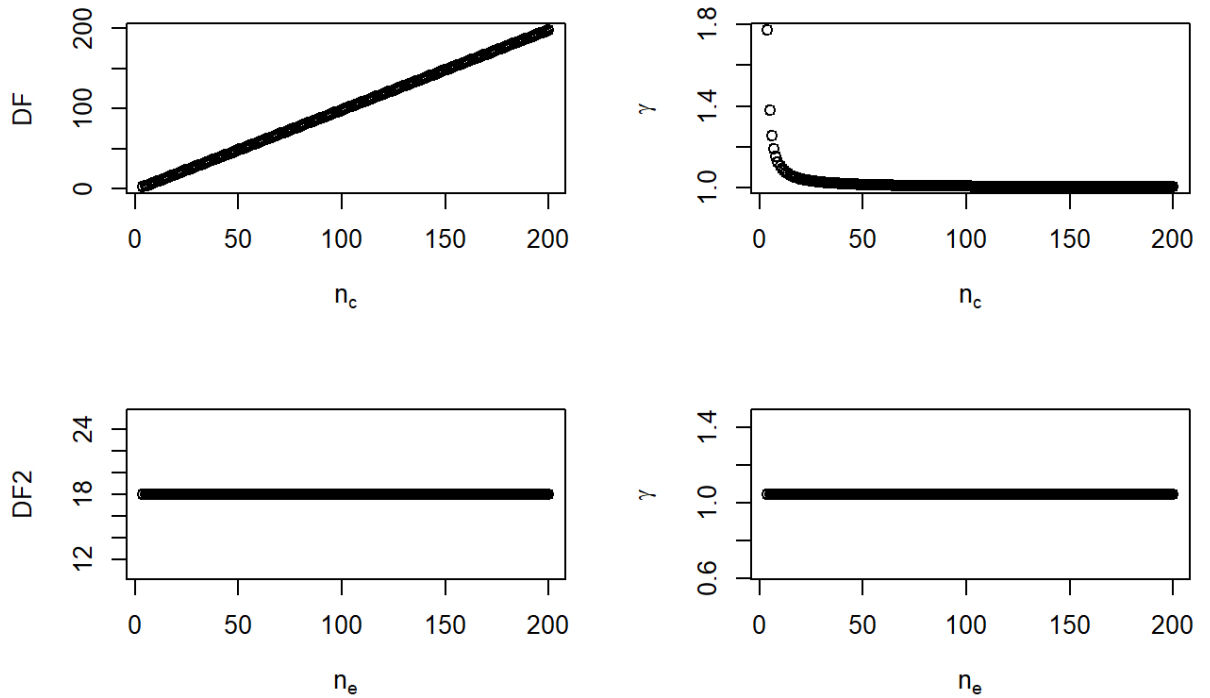
*Figure SM1.1* : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d$ , when variances are equal across groups, as a function of the total sample size ( $N$ ).

- Of course, considering the degrees of freedom, the sample size ratio does not matter (i.e. the bias will decrease when increasing  $n_1$ ,  $n_2$  or both sample sizes).

**Glass's  $d$**  (see **Table 3**). Because degrees of freedom depend only on the control group size (neither on  $\sigma_1$  nor on  $\sigma_2$ ), there is no need to distinguish between cases where there is homoscedasticity or heteroscedasticity!

The **bias** of Glass's  $d$  is a function of the control group size ( $n_c$ ) and the population effect size ( $\delta_{Glass}$ ) :

- The larger the population effect size, the more Glass's  $d$  will overestimate  $\delta_{Glass}$ ;
- The larger the size of the control group, the lower the bias (see the two top plots in Figure SM1.2). On the other hand, increasing the experimental group size does not impact the bias (see the two bottom plots in Figure SM1.2).



*Figure SM1.2* : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Glass's  $d$ , when variances are equal across groups, as a function of  $n_c$  (top) and  $n_e$  (bottom).

Cohen's  $d^*$  (see Table 3).

*When variances are equal across populations.*

When  $\sigma_1 = \sigma_2 = \sigma$  :

$$df_{Cohen's d^*} = \frac{(n_1 - 1)(n_2 - 1)(2\sigma^2)^2}{(n_2 - 1)\sigma^4 + (n_1 - 1)\sigma^4} = \frac{(n_1 - 1)(n_2 - 1) \times 4\sigma^4}{\sigma^4(n_1 + n_2 - 2)} = \frac{4(n_1 - 1)(n_2 - 1)}{n_1 + n_2 - 2}$$

One can see that degrees of freedom depend only on the total sample size ( $N$ ) and the sample size allocation ratio  $\left(\frac{n_2}{n_1}\right)$ . As a consequence, the **bias** of Cohen's  $d^*$  is a function of the population effect size ( $\delta_{Cohen}^*$ ), the sample size allocation ratio  $\left(\frac{n_2}{n_1}\right)$  and the total sample size ( $N$ ).

- The further the sample size allocation ratio is from 1, the larger the bias (see Figure SM1.3);

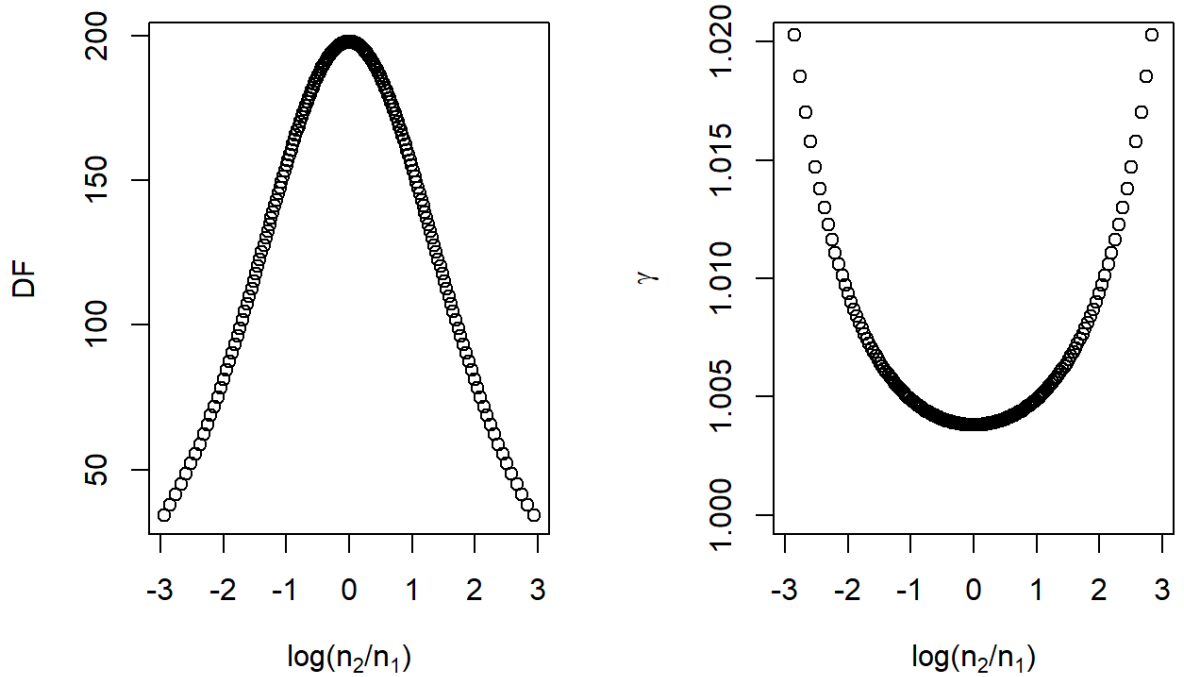


Figure SM1.3 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances are equal across groups, as a function of the logarithm of the sample sizes ratio  $\log\left(\frac{n_2}{n_1}\right)$ .

- The larger the population effect size, the more Cohen's  $d^*$  will overestimate  $\delta_{Cohen}^*$ ;
- The larger the total sample size, the lower the bias (see Figure SM1.4).

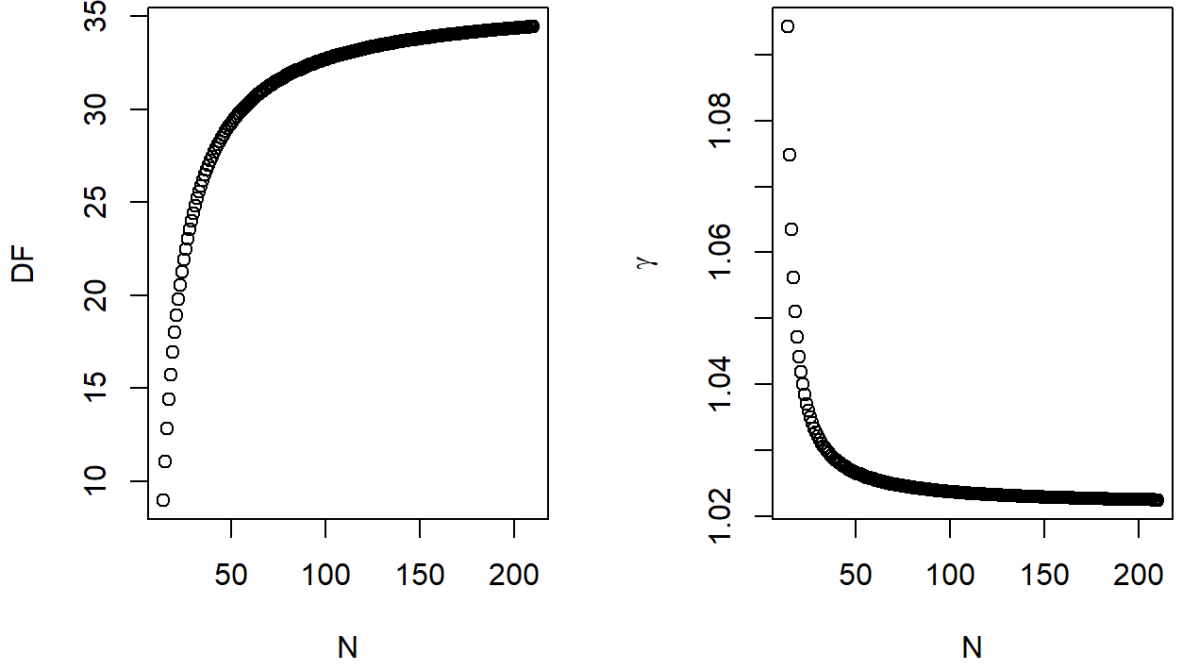


Figure SM1.4 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances are equal across groups, as a function of the total sample size ( $N$ ).

***When variances are unequal across populations, with equal sample sizes.***

When  $n_1 = n_2 = n$ :

$$df_{Cohen's\ d^*} = \frac{(n-1)^2(\sigma_1^2 + \sigma_2^2)^2}{(n-1)(\sigma_1^4 + \sigma_2^4)} = \frac{(n-1)(\sigma_1^4 + \sigma_2^4 + 2\sigma_1^2\sigma_2^2)}{\sigma_1^4 + \sigma_2^4}$$

One can see that degrees of freedom depend only on the total sample size ( $N$ ) and the  $SD$ -ratio  $\left(\frac{\sigma_2}{\sigma_1}\right)$ . As a consequence, the **bias** of Cohen's  $d^*$  is a function of the population effect size ( $\delta_{Cohen}^*$ ), the  $SD$ -ratio  $\left(\frac{\sigma_2}{\sigma_1}\right)$  and the total sample size ( $N$ ) :

- The larger the population effect size, the more Cohen's  $d^*$  will overestimate  $\delta_{Cohen}^*$ ;
- The further the  $SD$ -ratio is from 1, the larger the bias (see Figure SM1.5);

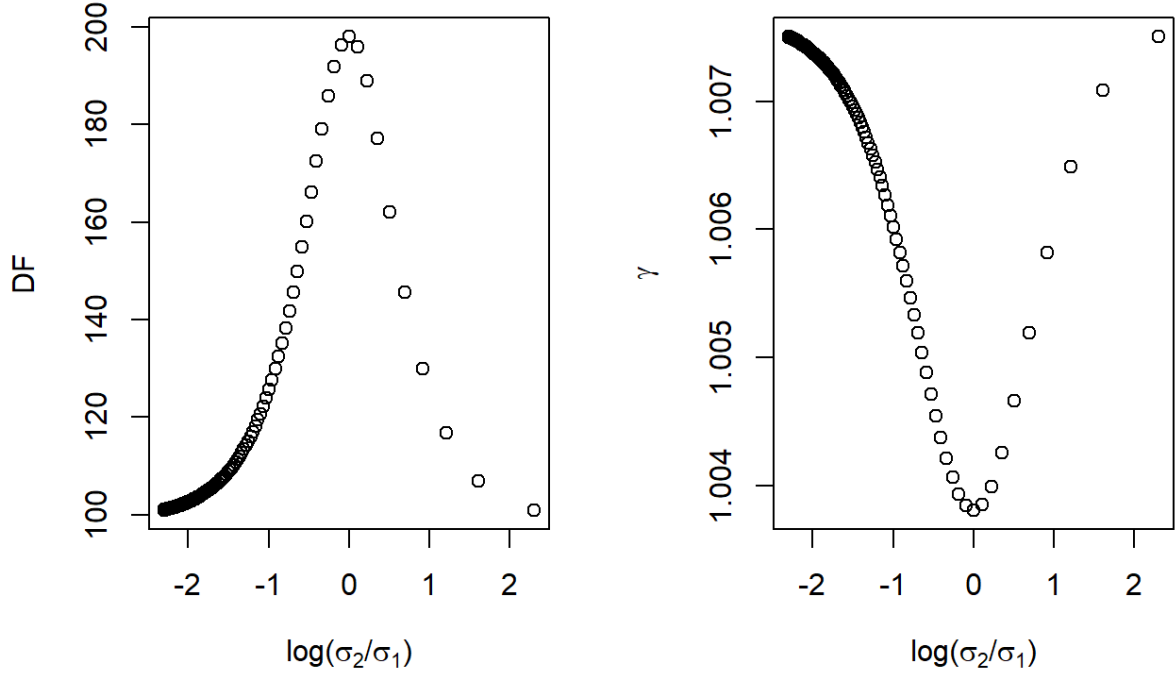


Figure SM1.5 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the  $SD$ -ratio ( $\log\left(\frac{\sigma_2}{\sigma_1}\right)$ ).

- The larger the total sample size, the lower the bias (see Figure SM1.6).

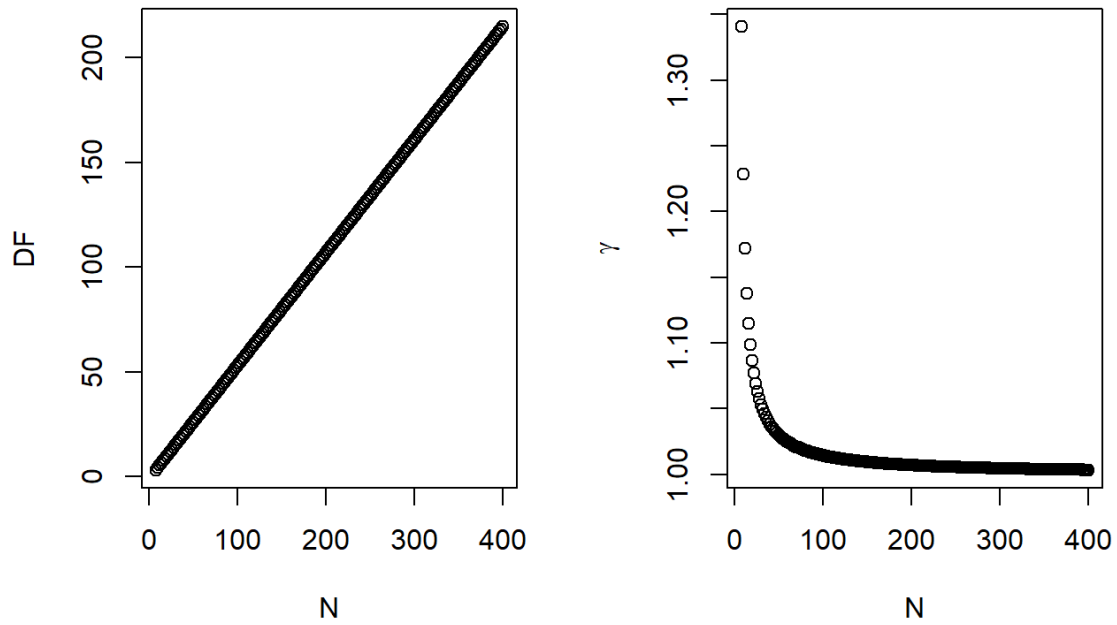


Figure SM1.6 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances are unequal across groups and sample sizes are equal, as a function of the total sample size ( $N$ ).



Note : for a constant  $SD$ -ratio,  $\sigma_1$  and  $\sigma_2$  do not matter (see Figure SM1.7).

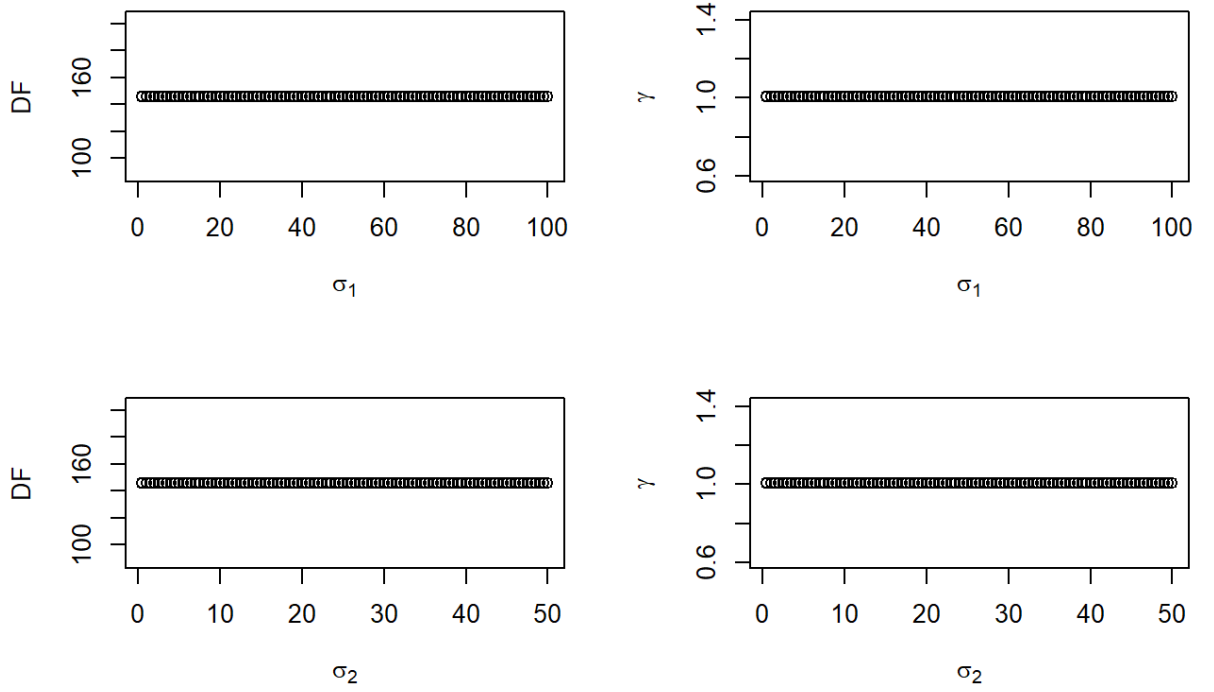


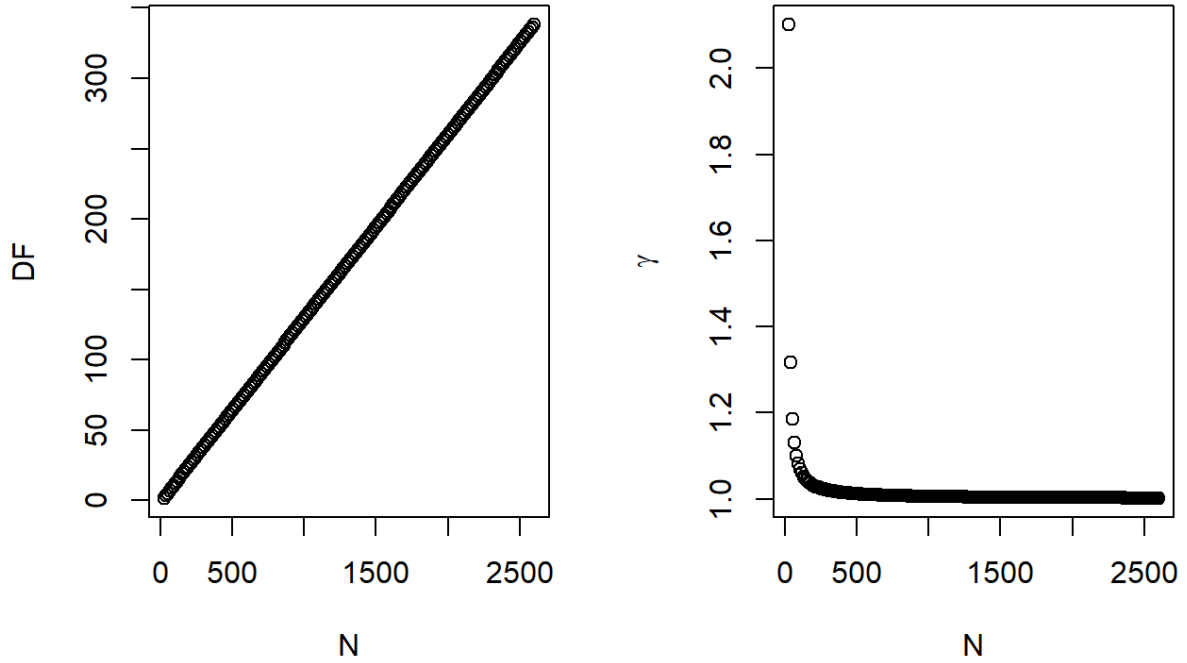
Figure SM1.7 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances are unequal across groups and sample sizes are equal, as a function of  $\sigma_1$  (top plots) and  $\sigma_2$  (bottom plots), for a constant  $SD$ -ratio.

***When variances are unequal across populations, with unequal sample sizes.***

The **bias** of Cohen's  $d^*$  is a function of the population effect size ( $\delta_{Cohen}^*$ ), the total sample size ( $N$ ), and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$  :

- The larger the population effect size, the more Cohen's  $d^*$  will overestimate  $\delta_{Cohen}^*$ ;

- The larger the total sample size, the lower the bias (see Figure SM1.8);



*Figure SM1.8* : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances and sample sizes are unequal across groups, as a function of the total sample size ( $N$ ).

- The smallest bias always occurs when there is a positive pairing between variances and sample sizes, because one gives more weight to the smallest variance, in the denominator of the  $df$  computation. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest bias (see Figure SM1.9). This can be explained by splitting the numerator and the denominator in the  $df$  computation.

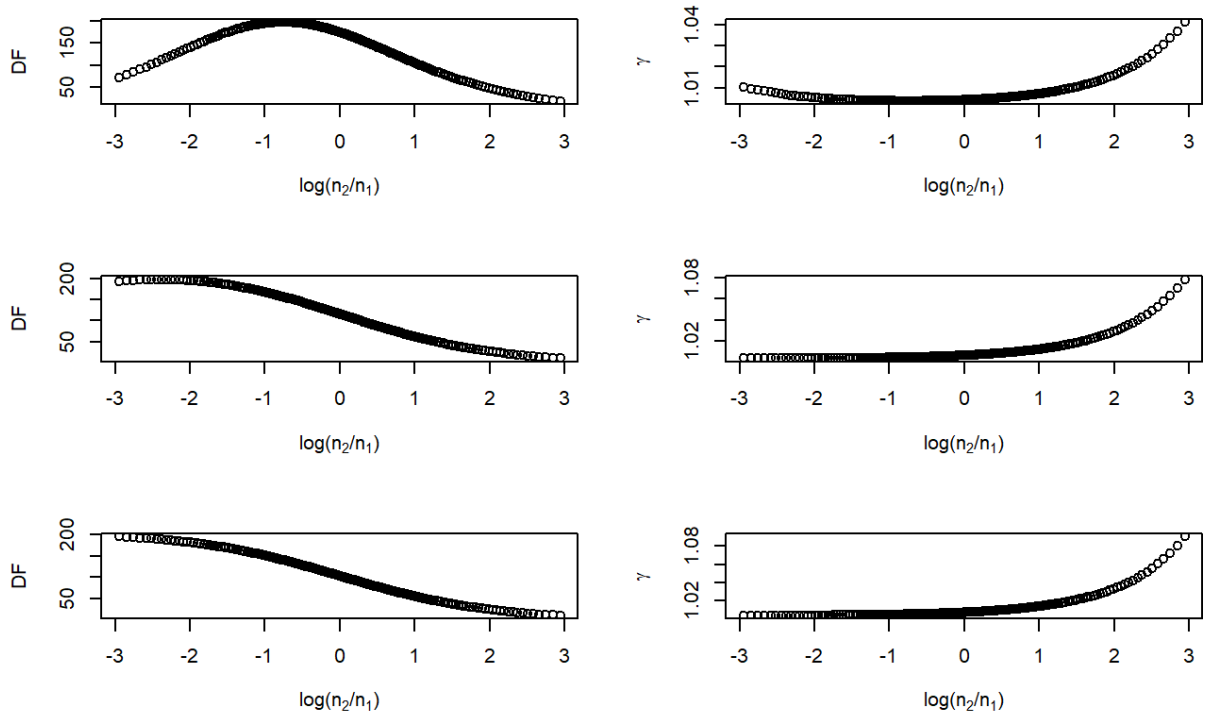
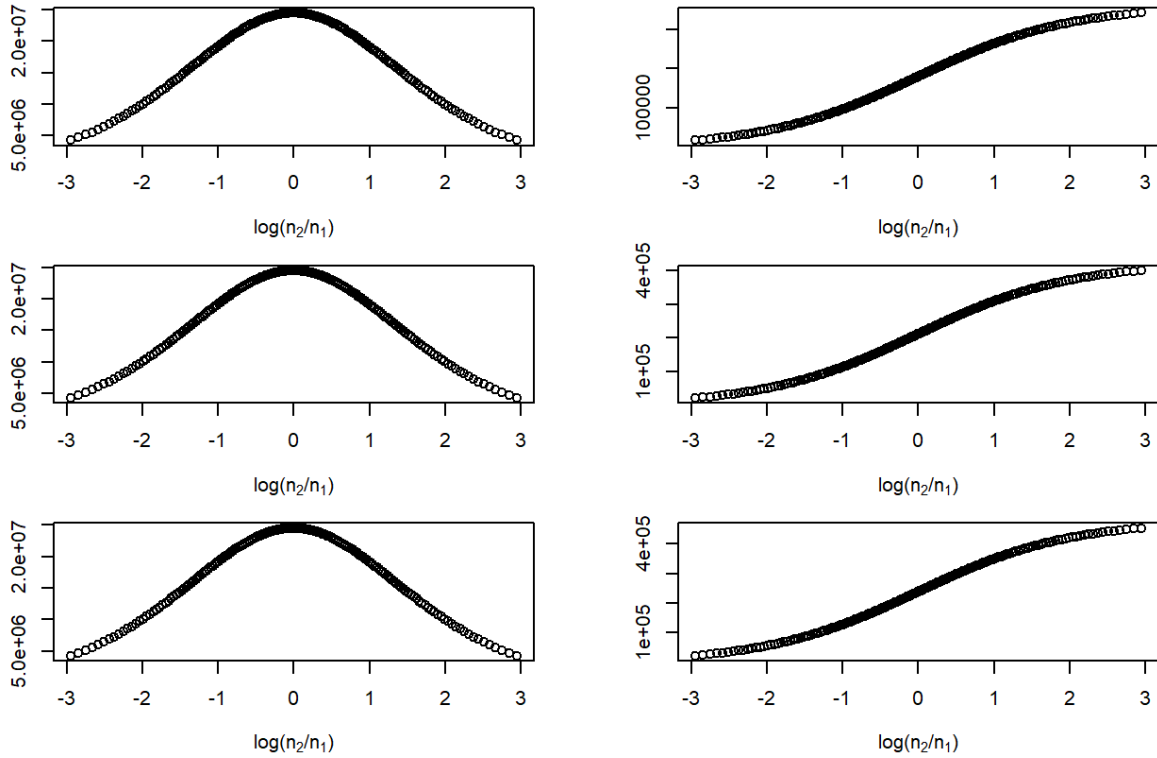


Figure SM1.9 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$  when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ), when  $SD$ -ratio equals .68 (first row), .29 (second row) or .14 (third row).

As illustrated in Figure SM1.10, for any  $SD$ -ratio, the numerator of the degrees of freedom will be maximized when sample sizes are equal across groups (and is not impacted by the  $SD$ -ratio). On the other hand, the denominator will be minimized when there is a positive pairing between variances and sample sizes. For example, when  $\sigma_1 > \sigma_2$ , the smallest denominator occurs when  $\frac{n_2}{n_1}$  reaches its minimum value and the further from 1 the  $SD$ -ratio, the larger the impact of the sample sizes ratio on the denominator.



*Figure SM1.10* : Numerator and denominator of the degrees of freedom ( $DF$ ) computation, when computing the bias of Cohen's  $d^*$  when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ), when  $SD$ -ratio equals .68 (first row), .29 (second row) or .14 (third row).

Note : for a constant  $SD$ -ratio, the variance does not matter. (See Figure SM1.11).

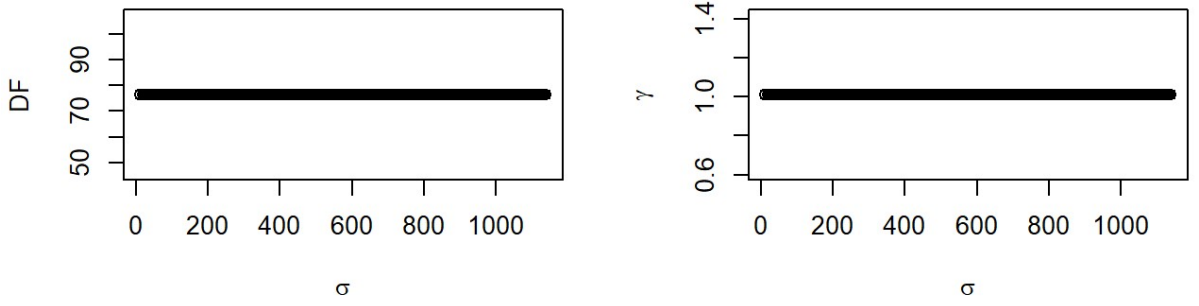


Figure SM1.11 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Cohen's  $d^*$ , when variances and sample sizes are unequal across groups, as a function of  $\sigma = \frac{(\sigma_1^2 + \sigma_2^2)}{2}$ , for a constant  $SD$ -ratio.

Shieh's  $d$  (see Table 3).

**When variances are equal across populations.**

When  $\sigma_1 = \sigma_2 = \sigma$ :

$$\begin{aligned}
 df_{Shieh's\ d} &= \frac{\left(\frac{n_2\sigma^2 + n_1\sigma^2}{n_1n_2}\right)^2}{\frac{(n_2-1)\left(\frac{\sigma^2}{n_1}\right)^2 + (n_1-1)\left(\frac{\sigma^2}{n_2}\right)^2}{(n_1-1)(n_2-1)}} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{[\sigma^2(n_1 + n_2)]^2}{n_1^2n_2^2} \times \frac{(n_1 - 1)(n_2 - 1)}{(n_2 - 1) \times \frac{\sigma^4}{n_1^2} + (n_1 - 1) \times \frac{\sigma^4}{n_2^2}} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{\sigma^4 N^2}{n_1^2n_2^2} \times \frac{(n_1 - 1)(n_2 - 1)}{\sigma^4 \left(\frac{n_2-1}{n_1^2} + \frac{n_1-1}{n_2^2}\right)} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{N^2(n_1 - 1)(n_2 - 1)}{n_1^2n_2^2 \left(\frac{n_2^2(n_2-1) + n_1^2(n_1-1)}{n_1^2n_2^2}\right)} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{N^2(n_1 - 1)(n_2 - 1)}{n_2^2(n_2 - 1) + n_1^2(n_1 - 1)}
 \end{aligned}$$

One can see that degrees of freedom depend only on the total sample size ( $N$ ) and the sample size allocation ratio  $\left(\frac{n_2}{n_1}\right)$ . As a consequence, the **bias** of Shieh's  $d$  is a function of the population effect size ( $\delta_{Shieh}$ ), the sample size allocation ratio  $\left(\frac{n_2}{n_1}\right)$  and the total sample size ( $N$ ).

- The larger the population effect size, the more Shieh's  $d$  will overestimate  $\delta_{Shieh}$ ;
- The further the sample size allocation ratio is from 1, the larger the bias (see Figure SM1.12);

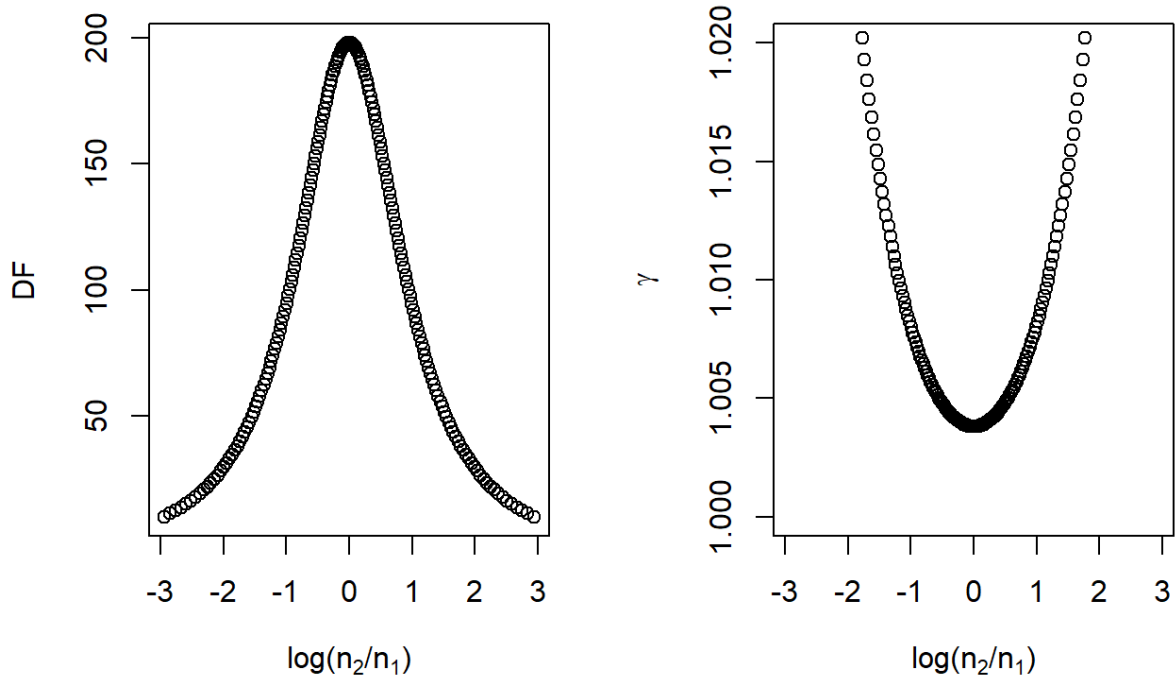


Figure SM1.12 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ).

- For a constant sample sizes ratio, the larger the total sample size, the lower the bias (see Figure SM1.13).

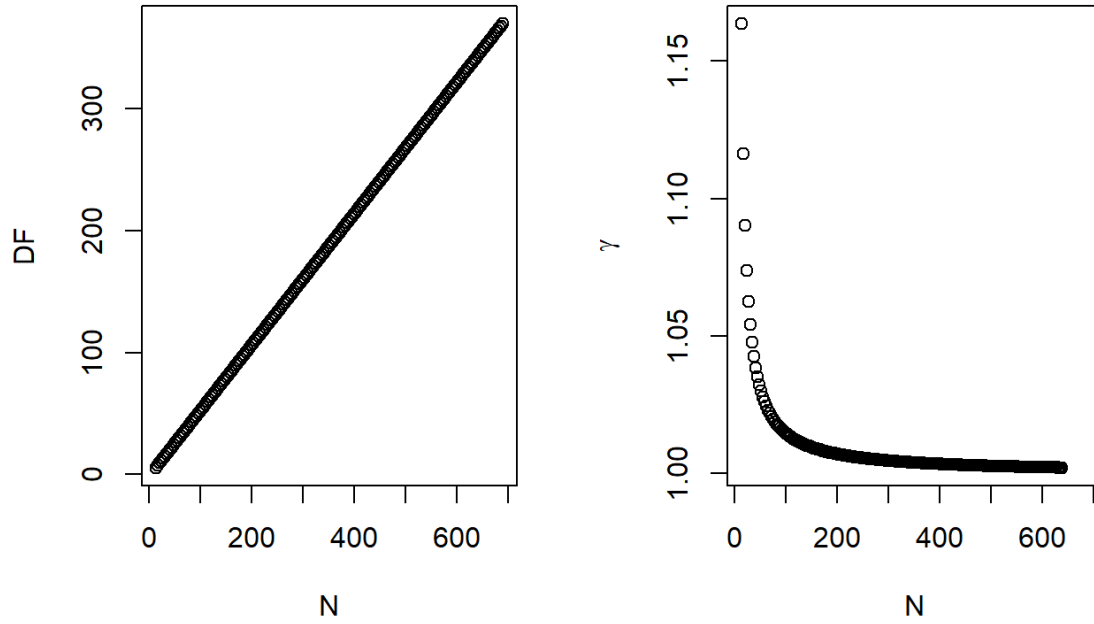
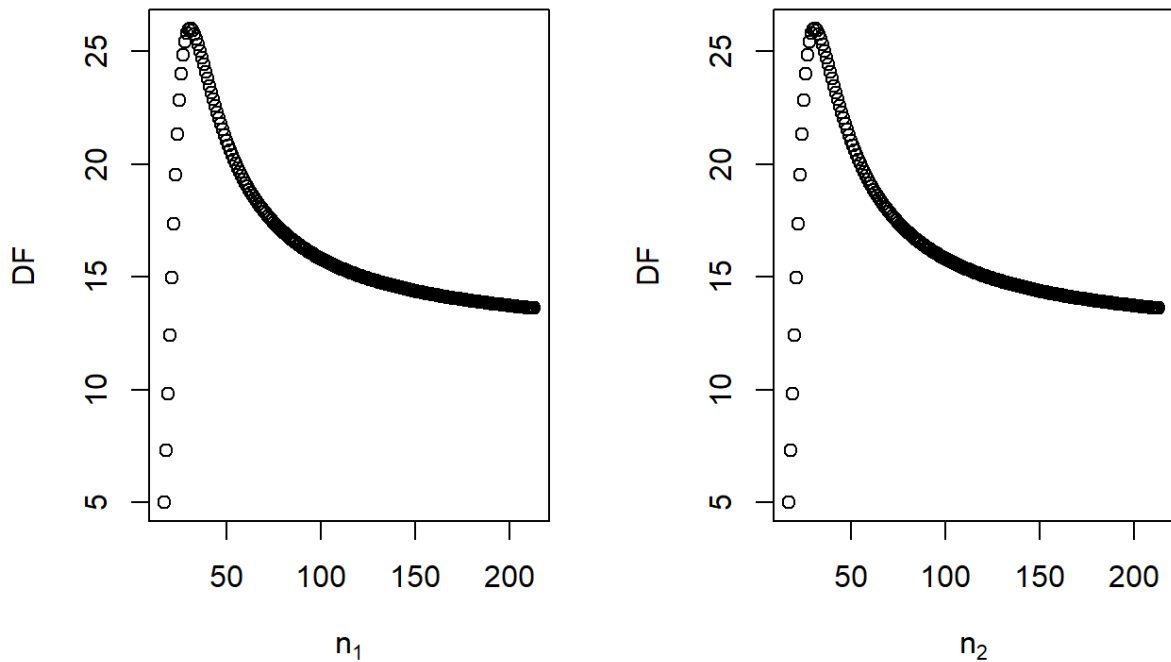


Figure SM1.13 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances are equal across groups, as a function of the total sample size ( $N$ ).

Note : when computing Cohen's  $d^*$ , degrees of freedom increased when adding subjects in either one or both groups, even when the sample size ratio increased. When computing Shieh's  $d$ , this is not true anymore : there is a larger impact of the sample sizes ratio such that moving the sample sizes ratio away from 1 when adding subjects in only one group can decrease the degrees of freedom and therefore, increase the bias (See Figure SM1.14).



*Figure SM1.14* : Degrees of freedom ( $DF$ ), when computing the bias of Shieh's  $d$ , when variances are equal across groups, when adding subjects only in the first group (left) or in the second group (right).



*When variances are unequal across populations, with equal sample sizes.*

When  $n_1 = n_2 = n$ :

$$\begin{aligned}
 df_{Shieh's\ d} &= \frac{\left(\frac{\sigma_1^2 + \sigma_2^2}{n}\right)^2}{\frac{(\sigma_1^2/n)^2 + (\sigma_2^2/n)^2}{n-1}} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{(\sigma_1^2 + \sigma_2^2)^2}{n^2} \times \frac{n-1}{\frac{\sigma_1^4 + \sigma_2^4}{n^2}} \\
 \Leftrightarrow df_{Shieh's\ d} &= \frac{(\sigma_1^2 + \sigma_2^2)^2 \times (n-1)}{\sigma_1^4 + \sigma_2^4}
 \end{aligned}$$

One can see that degrees of freedom depend on the total sample size ( $N$ ) and the  $SD$ -ratio  $\left(\frac{\sigma_2}{\sigma_1}\right)$ . As a consequence, the bias depends on the population effect size ( $\delta_{Shieh}$ ), the  $SD$ -ratio  $\left(\frac{\sigma_2}{\sigma_1}\right)$  and the total sample size ( $N$ ).

- The larger the population effect size, the more Shieh's  $d$  will overestimate  $\delta_{Shieh}$ ;

- The further the  $SD$ -ratio is from 1, the larger the bias (see Figure SM1.15);

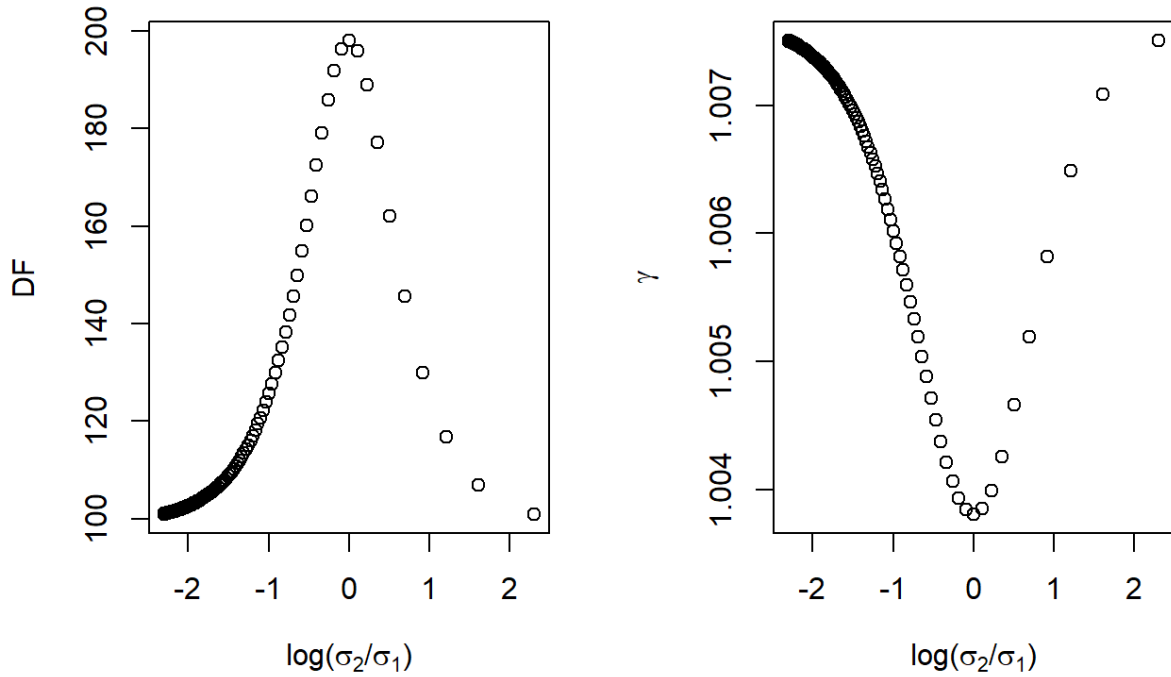
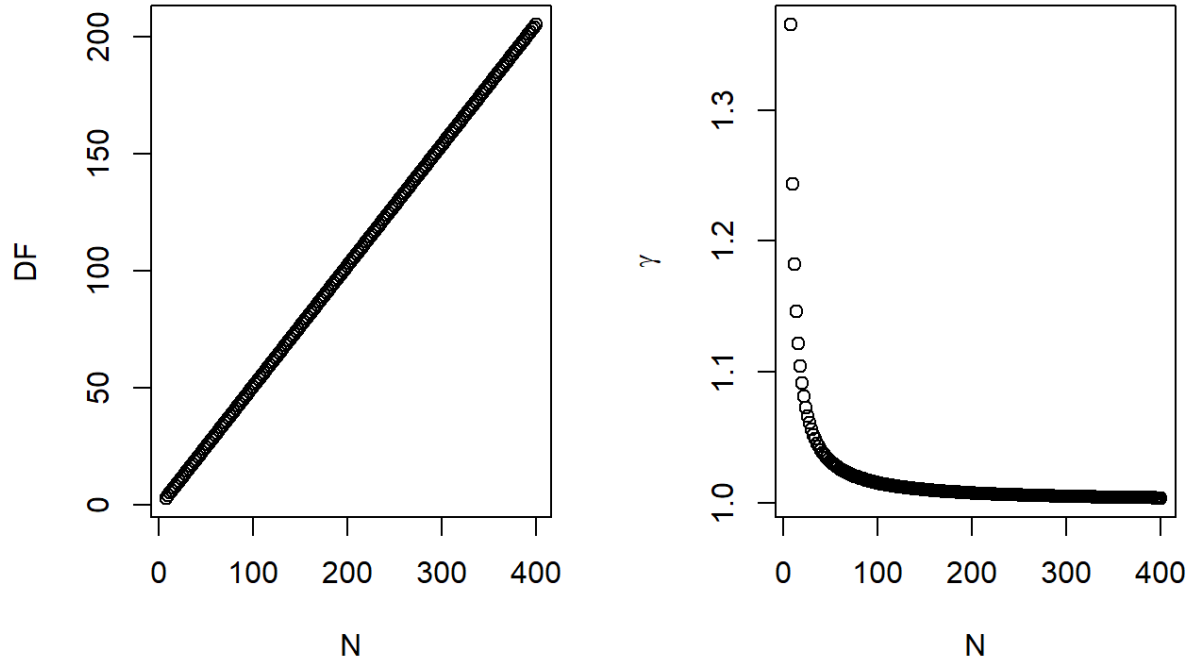


Figure SM1.15 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the  $SD$ -ratio ( $\log\left(\frac{\sigma_2}{\sigma_1}\right)$ ).

- The larger the total sample size, the lower the bias (see Figure SM1.16);



*Figure SM1.16* : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of the total sample size ( $N$ ).

Note : for a constant  $SD$ -ratio,  $\sigma_1$  and  $\sigma_2$  do not matter (see Figure SM1.17).

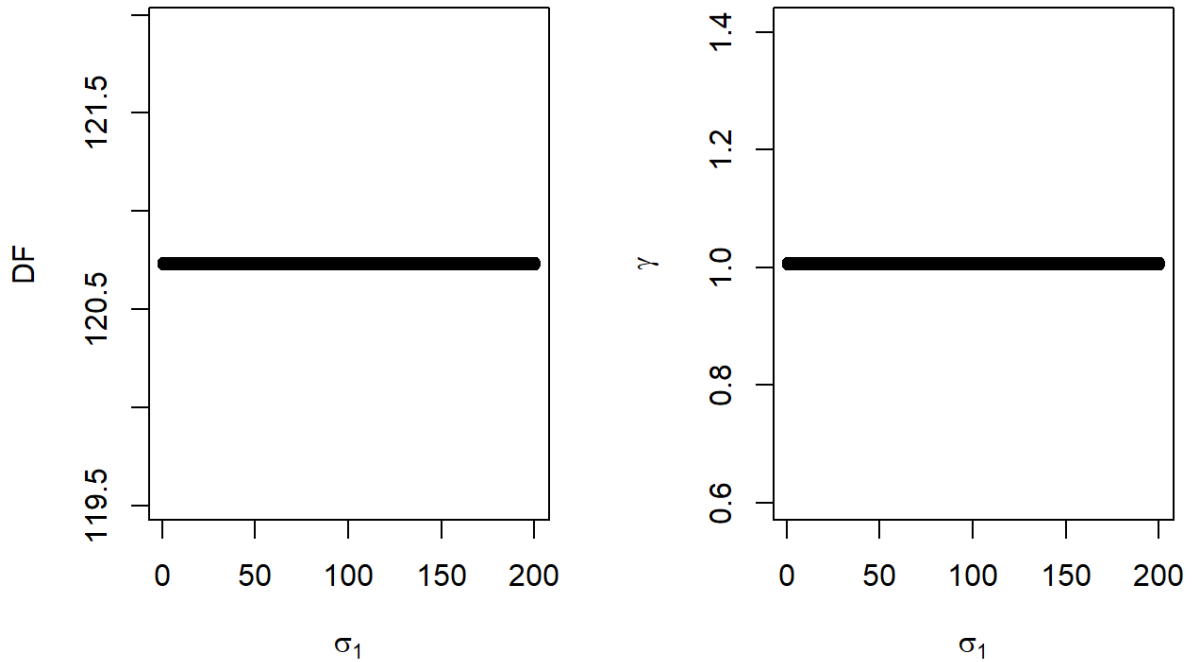


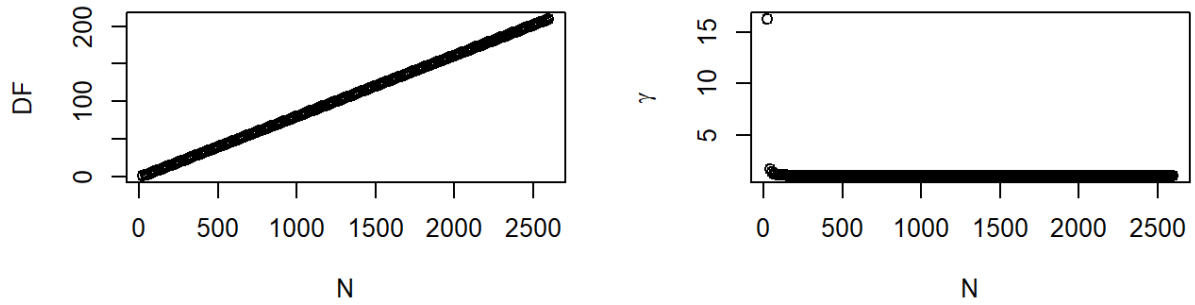
Figure SM1.17 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of  $\sigma_1$ , for a constant  $SD$ -ratio.

***When variances are unequal across populations, with unequal sample sizes.***

The **bias** of Shieh's  $d$  is a function of the population effect size ( $\delta_{Shieh}$ ), the sample sizes ( $n_1$  and  $n_2$ ), and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$  :

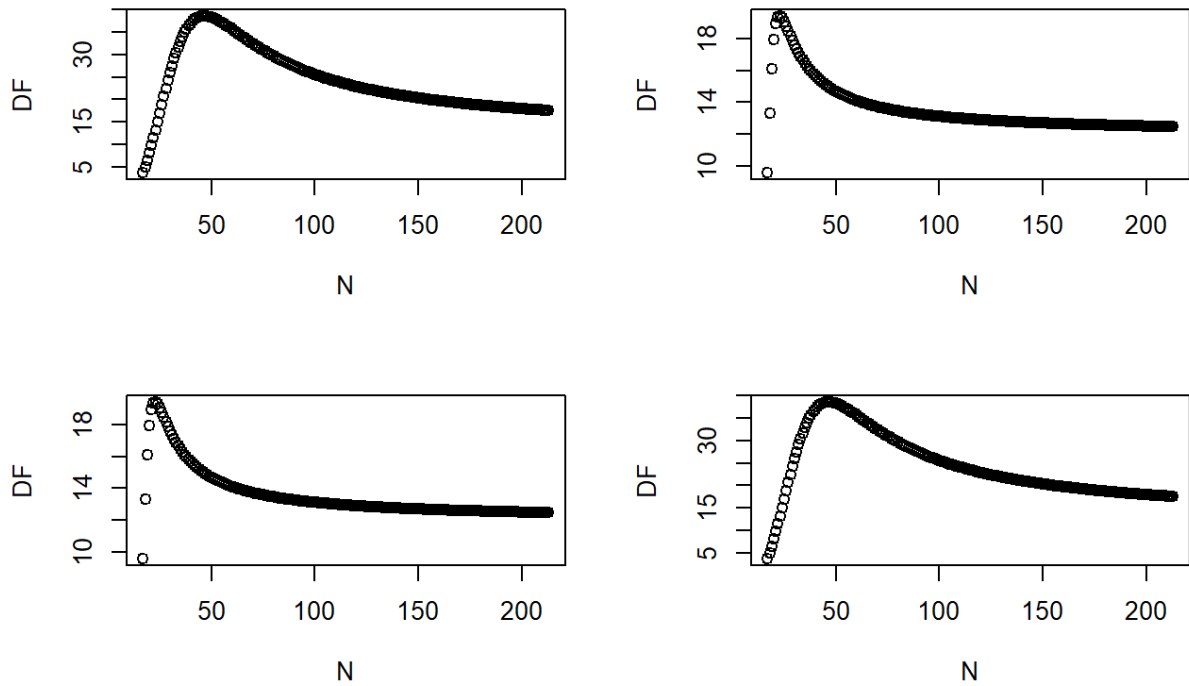
- The larger the population effect size, the more Shieh's  $d$  will overestimate  $\delta_{Shieh}$ ;

- For a constant sample sizes ratio, the larger the sample sizes, the lower the bias (See Figure SM1.18);



*Figure SM1.18* : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of the total sample size ( $N$ ).

Note : When variances were equal across populations, adding subjects only in the first group had the same impact on degrees of freedom (and therefore on bias) than adding subjects only in the second group (see Figure SM1.14). When variances are unequal across groups, this is not true anymore (see Figure SM1.19).



*Figure SM1.19* : Degrees of freedom ( $DF$ ), when computing the bias of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of the total sample size, when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots), and  $\sigma_1 > \sigma_2$  (left plots) or  $\sigma_1 < \sigma_2$  (right plots).

- The smallest bias always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest bias (See Figure SM1.20);

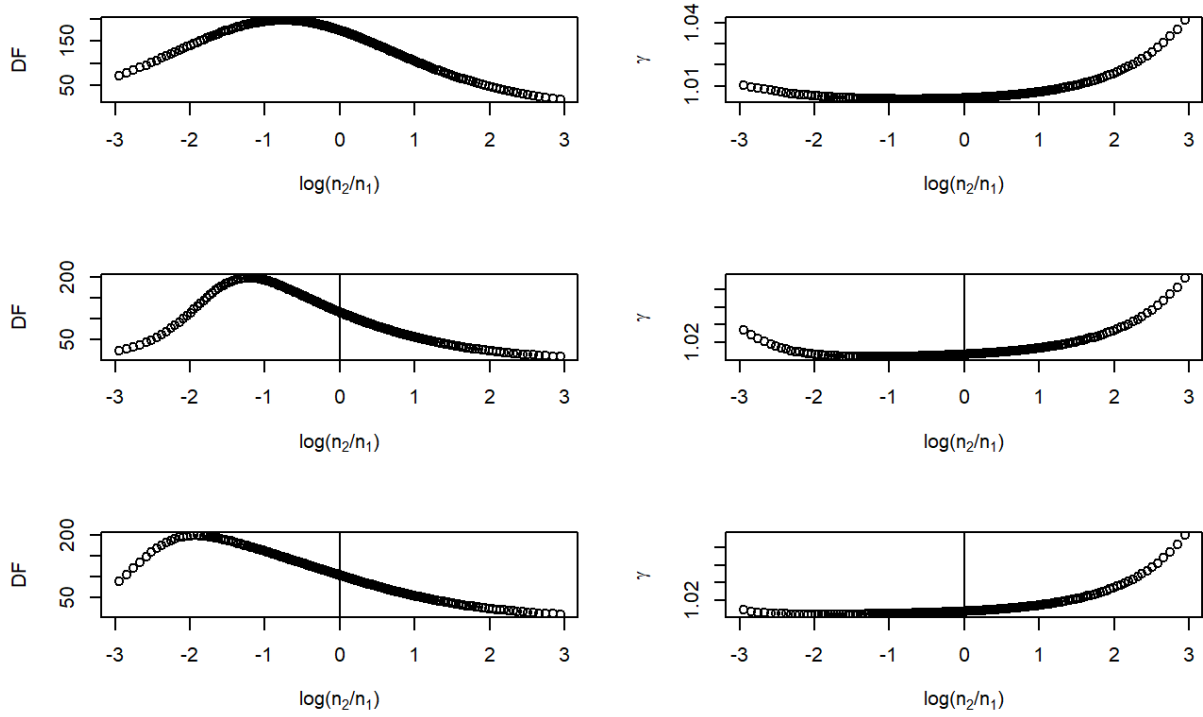


Figure SM1.20 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ), when  $SD$ -ratio equals .68 (first row), .29 (second row) or .14 (third row).

Moreover, for a constant  $SD$ -ratio, the variances do not matter (See Figure SM1.21).

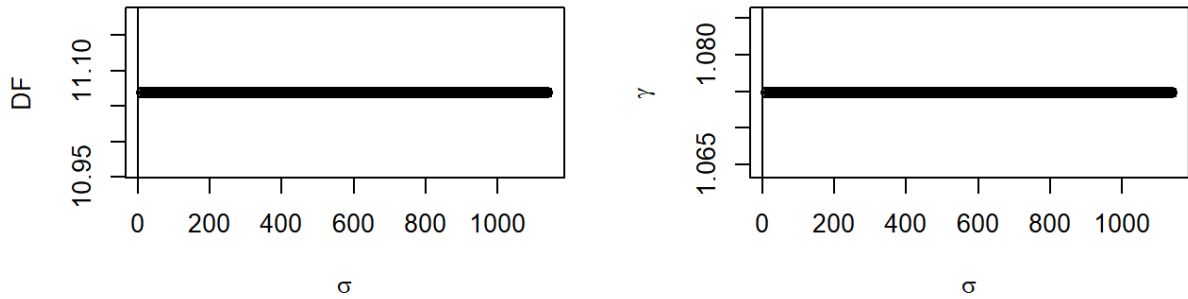


Figure SM1.21 : Degrees of freedom ( $DF$ ) and  $\gamma$ , when computing the bias of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of  $\sigma_1$  and  $\sigma_2$ , for a constant  $SD$ -ratio.

**In summary.** The **bias** of Cohen's  $d$  is a function of the population effect size  $\delta_{Cohen}$  and the total sample size ( $N$ ) :

- When  $\delta_{Cohen}$  is null, the bias is null. In all other configurations, the larger  $\delta_{Cohen}$ , the more Cohen's  $d$  will overestimate  $\delta_{Cohen}$ ;
- The bias decreases when the total sample size increases (it does not matter whether one adds subjects in only one group or in both).

The **bias** of Glass's  $d$  is a function of the population effect size ( $\delta_{Glass}$ ) and the size of the control group ( $n_e$ ) :

- When  $\delta_{Glass}$  is null, the bias is null. In all other configurations, the larger  $\delta_{Glass}$ , the more Glass's  $d$  will overestimate  $\delta_{Glass}$ ;
- The bias decreases when the size of the control group increases. On the other hand, increasing the size of the experimental group does not impact the bias.

The **bias** of Cohen's  $d^*$  is a function of the population effect size ( $\delta_{Cohen}^*$ ), the total sample size, and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$  :



- When  $\delta_{Cohen}^*$  is null, the bias is null. In all other configurations, the larger  $\delta_{Cohen}^*$ , the more Cohen's  $d^*$  will overestimate  $\delta_{Cohen}^*$ ;
- The bias decreases when the total sample size increases (it does not matter whether one adds subjects in only one group or in both);
- The smallest bias always occurs when there is a positive pairing between  $\frac{\sigma_2}{\sigma_1}$  and  $\frac{n_2}{n_1}$ . Moreover, the larger the  $SD$ -ratio, the further from 1 is the sample sizes ratio associated with the smallest bias.

The **bias** of Shieh's  $d$  is a function of the population effect size ( $\delta_{Shieh}$ ), the total sample size, and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$ :

- When  $\delta_{Shieh}$  is null, the bias is null. In all other configurations, the larger  $\delta_{Shieh}$ , the more Shieh's  $d$  will overestimate  $\delta_{Shieh}$ ;
- For a constant sample sizes ratio, the bias decreases when the total sample size increases;
- The smallest bias always occurs when there is a positive pairing between  $\frac{\sigma_2}{\sigma_1}$  and  $\frac{n_2}{n_1}$ . Moreover, the larger the  $SD$ -ratio, the further from 1 is the sample sizes ratio associated with the smallest bias (for more details, see “Theoretical Bias, as a function of population parameters”).

## Theoretical variance

Note : while we focus on the theoretical variance of biased estimators (Cohen's  $d$ , Glass's  $d$ , Shieh's  $d$  and Cohen's  $d^*$ ) when the normality assumption is met, it is interesting to notice that our main conclusions seem to generalize to biased estimators when samples are extracted from symmetric distributions. Moreover, unbiased estimators depend on the same factors as biased estimators, so our conclusions remain similar for unbiased estimators when samples are extracted from heavy-tailed symmetric distributions.

### Cohen's $d$ .

#### *When variances are equal across populations.*

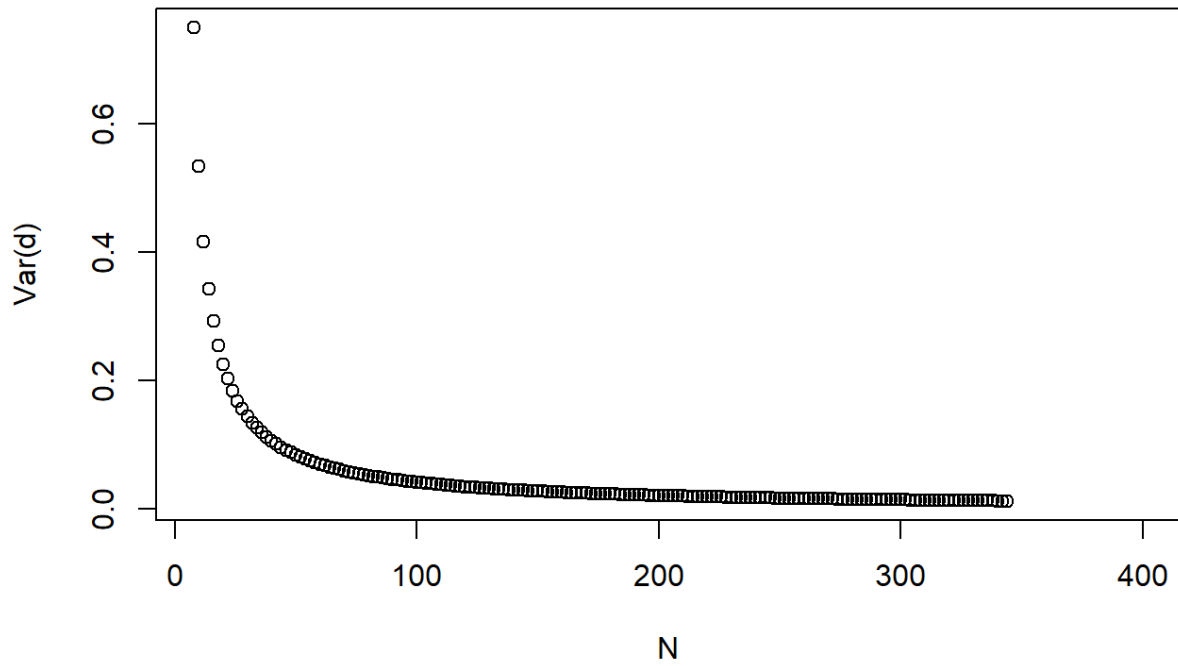
When  $\delta_{Cohen} = 0$ .

When the population effect size is zero, the variance of Cohen's  $d$  can be simplified as follows:

$$Var_{Cohen's\ d} = \frac{N(N-2)}{n_1 n_2 (N-4)}$$

The **variance** of Cohen's  $d$  is a function of total sample size ( $N$ ) and the sample sizes allocation ratio  $\left(\frac{n_2}{n_1}\right)$  :

- The larger the total sample size, the lower the variance. The variance tends to zero when the total sample size tends to infinity (see Figure SM1.22);



*Figure SM1.22* : Variance of Cohen's  $d$ , when variances are equal across groups, as a function of the total sample size ( $N$ ).

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.23).

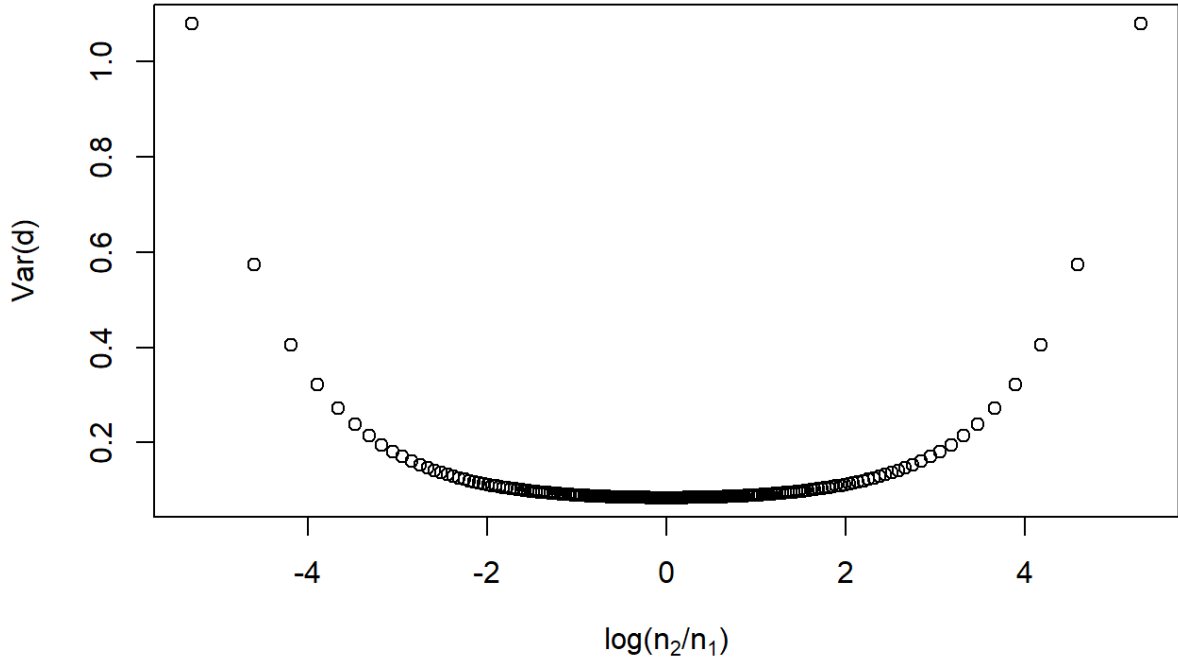


Figure SM1.23 : Variance of Cohen's  $d$ , when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ).

When  $\delta_{Cohen} \neq 0$ .

While the variance of Cohen's  $d$  still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size ( $\delta_{Cohen}$ ). The larger the population effect size, the larger the variance. Note that the effect of the population effect size decreases when sample sizes increase since

$$\lim_{n_1 \rightarrow \infty} \left[ \frac{df}{df-2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_2 \rightarrow \infty} \left[ \frac{df}{df-2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{N \rightarrow \infty} \left[ \frac{df}{df - 2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

This is illustrated in Figure SM1.24.

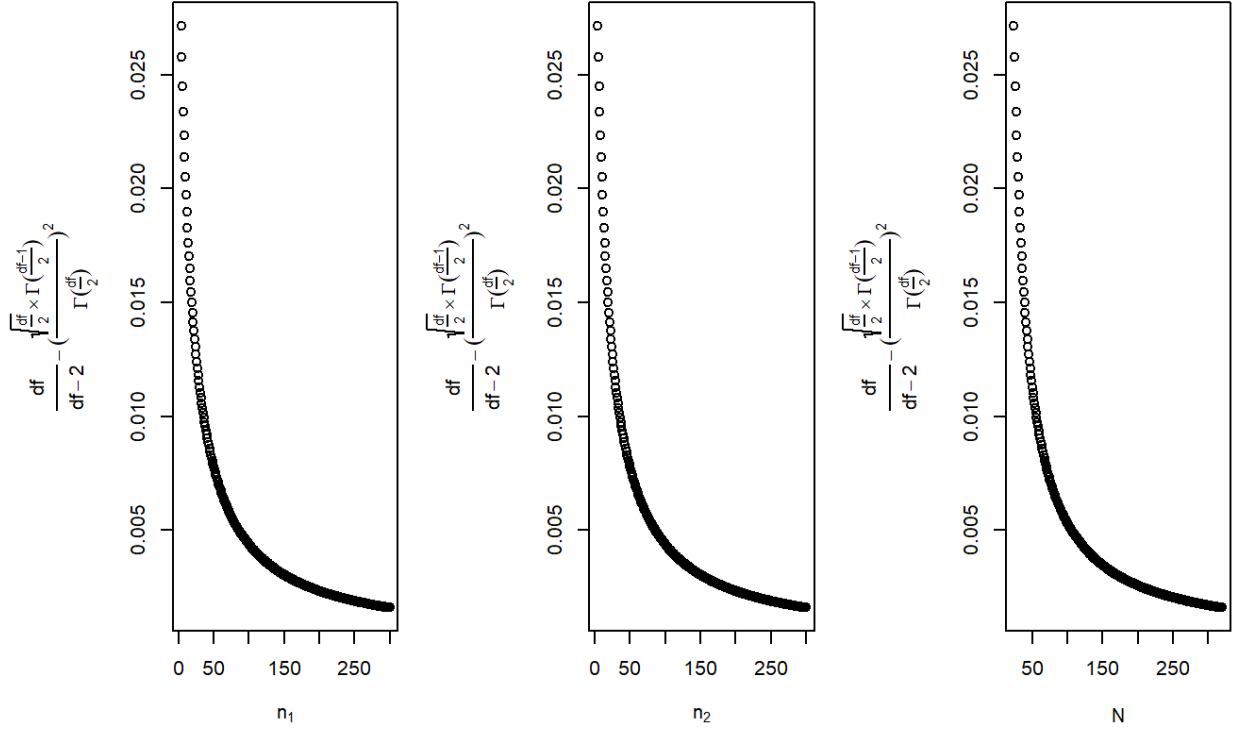


Figure SM1.24 : Effect size moderator, when computing the variance of Cohen's  $d$ , as a function of  $n_1$  (left),  $n_2$  (center) and  $N = n_1 + n_2$  (right).

***In summary.***

The variance of Cohen's  $d$  is a function of the population effect size ( $\delta_{Cohen}$ ), the total sample size ( $N$ ) and the sample sizes ratio ( $\frac{n_2}{n_1}$ ) :

- The variance decreases when the total sample size increases;
- The variance also decreases when the sample sizes ratio gets closer to 1;
- Finally, the variance increases when  $\delta_{Cohen}$  increases. Note that the effect of  $\delta_{Cohen}$  is moderated by the total sample size (the larger  $N$ , the smaller the effect of  $\delta_{Cohen}$  on the variance).

Glass's  $d$ .

*When variances are equal across populations.*

When  $\delta_{Glass} = 0$ .

When the population effect size is zero, the variance of Glass's  $d$  can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3} \left( \frac{1}{n_c} + \frac{1}{n_e} \right)$$

In this configuration, the **variance** of Glass's  $d$  is a function of the sample sizes of both control ( $n_c$ ) and experimental ( $n_e$ ) groups as well as of the sample sizes allocation ratio  $\left( \frac{n_c}{n_e} \right)$  :

- The larger the sample sizes, the lower the variance (Figure SM1.25);

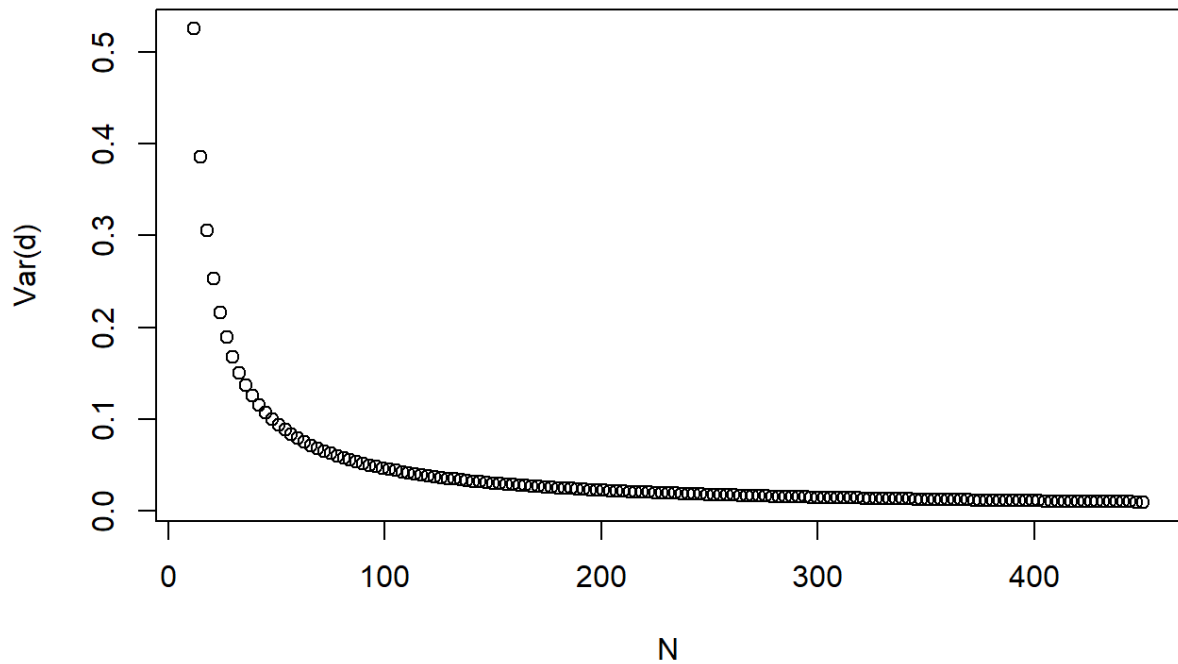


Figure SM1.25 : Variance of Glass's  $d$ , when variances are equal across groups, as a function of the total sample size ( $N$ ).

The sample sizes ratio associated with the lowest variance is not exactly 1 (because of the term  $\frac{df}{df-2}$ ,  $df$  depending only on  $n_c$ ), but is very close to 1 (and the larger the total sample size, the closer to 1 is the sample sizes ratio associated with the lowest variance). The further from this sample size ratio, the larger the variance (see Figure SM1.26).

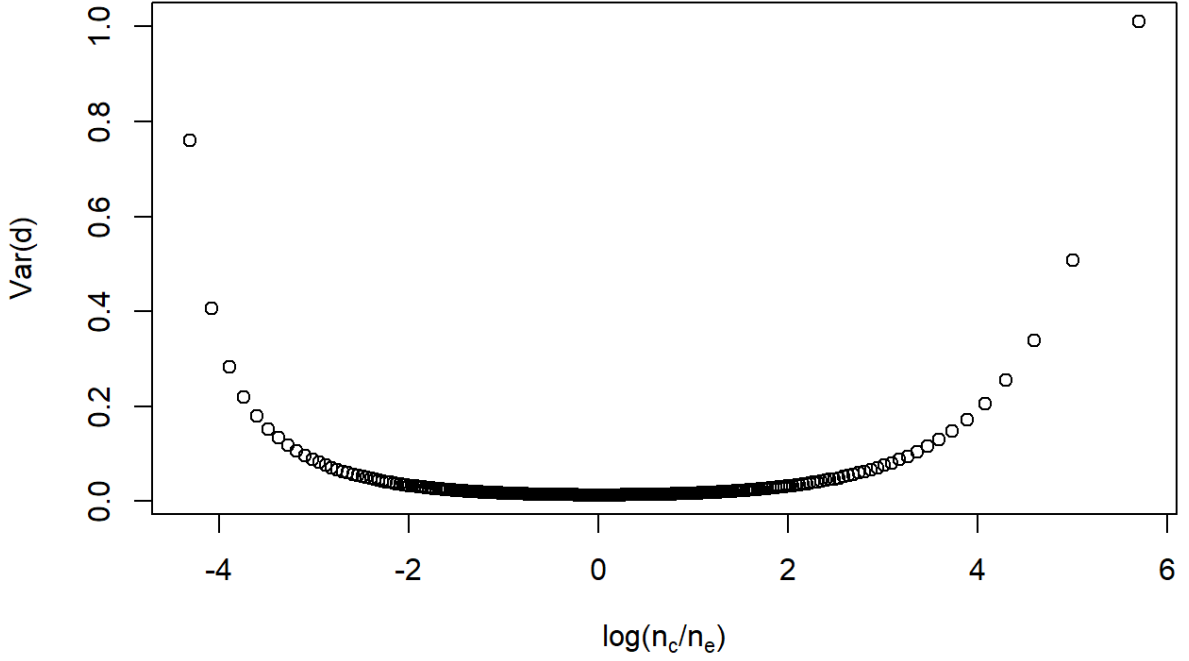


Figure SM1.26 : Variance of Glass's  $d$ , when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_c}{n_e}\right)$ ).

When  $\delta_{Glass} \neq 0$ .

While the variance of Glass's  $d$  still depends on the total sample size and the sample sizes allocation ratio, it also depends on the population effect size ( $\delta_{Glass}$ ). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases. On the other hand, the effect of the population effect size does *not* depend on the size of the experimental group since

$$\lim_{n_c \rightarrow \infty} \left[ \frac{df}{df-2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

$$\lim_{n_e \rightarrow \infty} \left[ \frac{df}{df-2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] \neq 0$$

These limits are illustrated in Figure SM1.27.

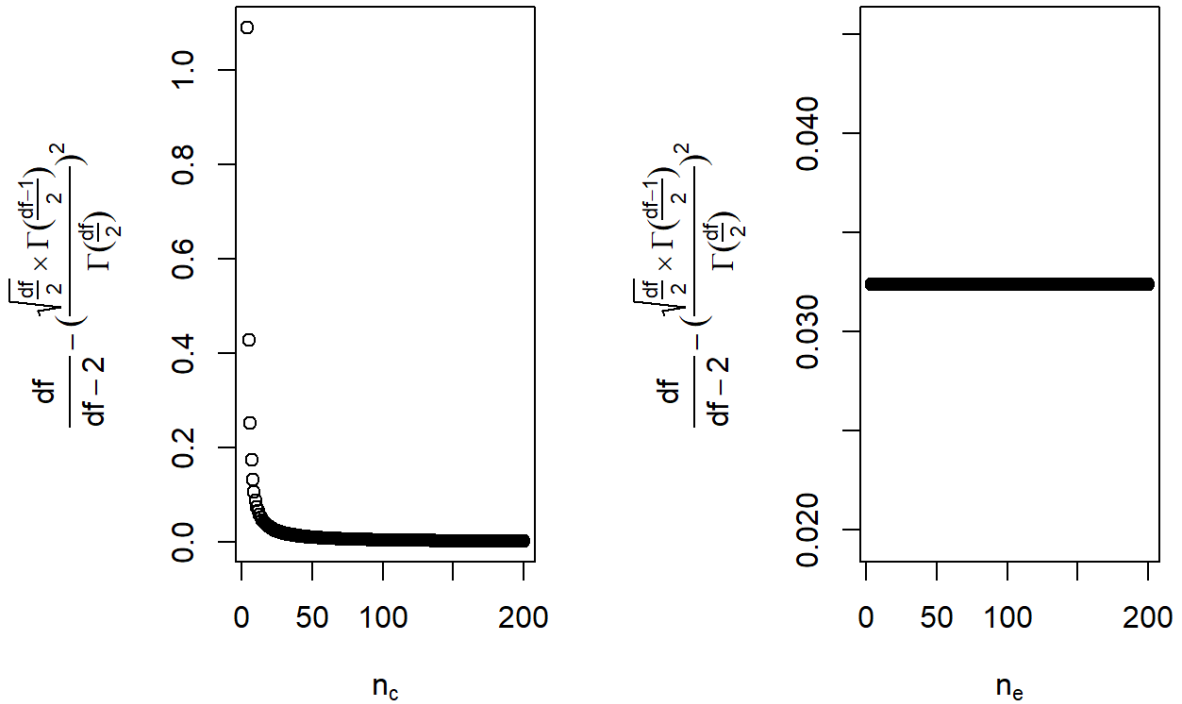


Figure SM1.27 : Effect size moderator, when computing the variance of Glass's  $d$ , as a function of the size of the control group (left) and experimental group (right).

Note : while the sample sizes ratio associated with the lowest variance was very close to 1 with a null population effect size, this is not true anymore when the population effect size is not zero. Indeed, because of the second term in the addition, when computing the variance, one gives much more weight to the effect size of the control group (see Figure SM1.28), especially when the effect size gets larger. For example, when  $\delta_{Glass} = 4$ , the lowest variance will occur when  $n_c$  is approximately 3 times larger than  $n_e$ . When  $\delta_{Glass} = 7$ , the lowest variance will occur when  $n_c$  is approximately 5 times larger than  $n_e$ , etc.



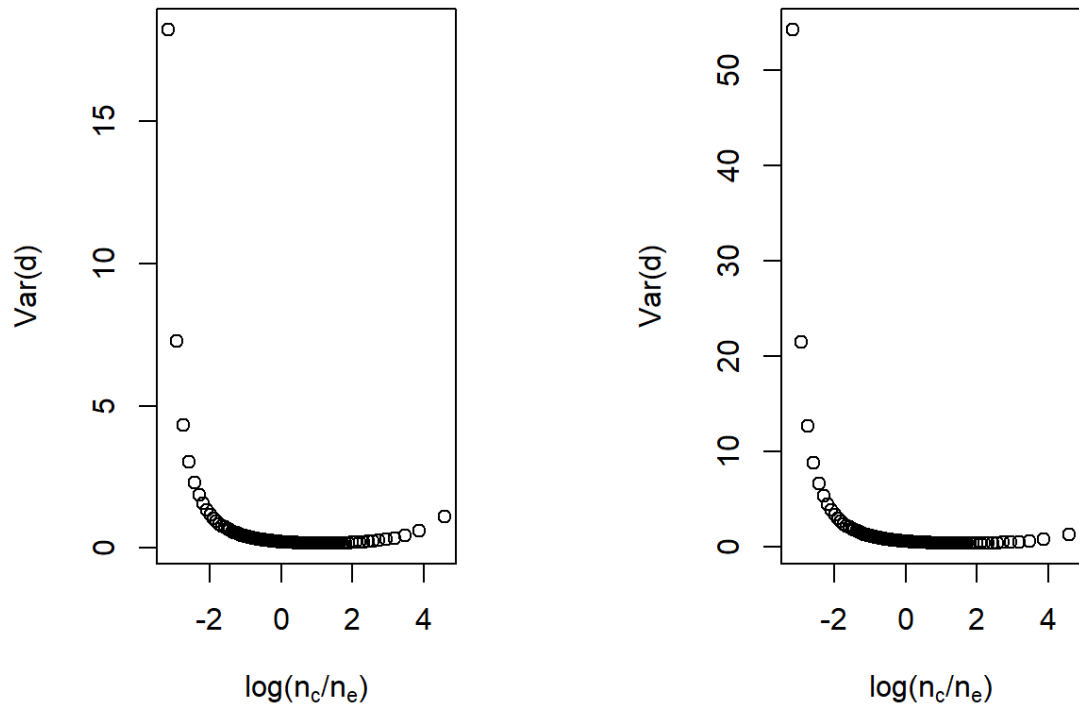


Figure SM1.28 : Variance of Glass's  $d$ , when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_c}{n_e}\right)$ ) when  $\delta_{Glass}$  equals 4 (left) or 7 (right).

***When variances are unequal across populations, with equal sample sizes.***

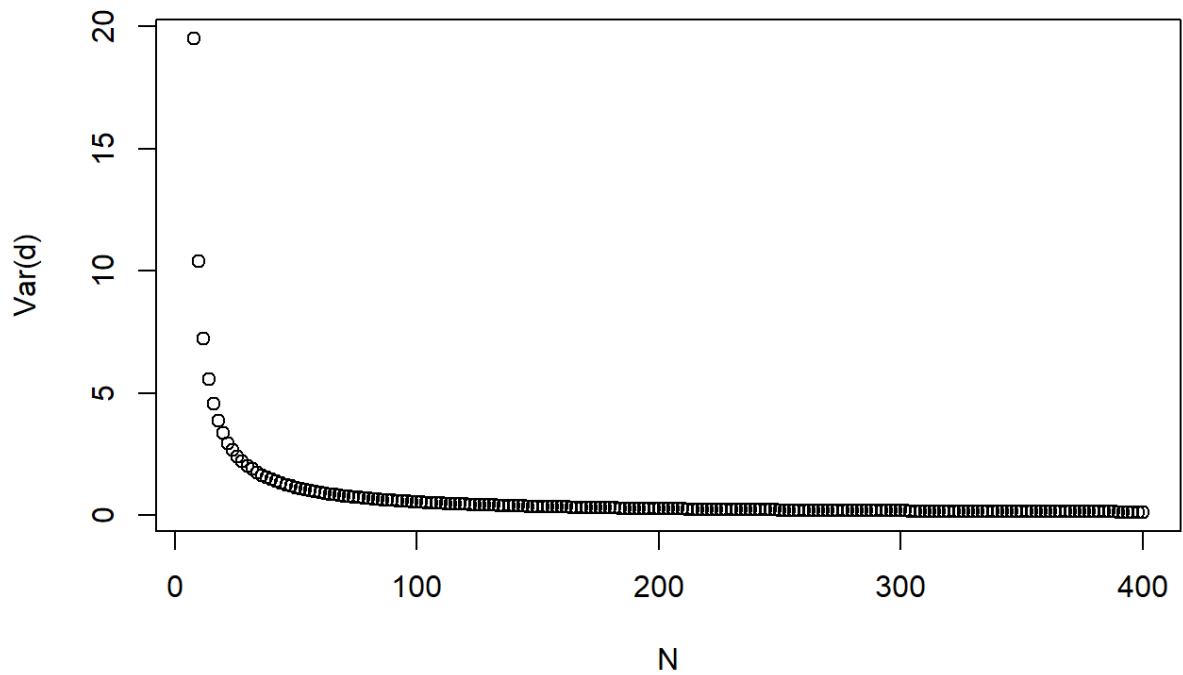
When  $\delta_{Glass} = 0$ .

When the population effect size is zero, the variance of Glass's  $d$  can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n-1}{n(n-3)} \left( 1 + \frac{\sigma_e^2}{\sigma_c^2} \right)$$

where  $n = N/2$  = sample size of each group. The variance is therefore a function of the total sample size and the  $SD$ -ratio ( $\frac{\sigma_c}{\sigma_e}$ ) :

- The larger the total sample size, the lower the variance (See Figure SM1.29);



*Figure SM1.29* : Variance of Glass's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of the total sample sizes ( $N$ ).

- The larger the  $SD$ -ratio (i.e. the larger is  $\sigma_c$  in comparison with  $\sigma_e$ ), the lower the variance (see Figure SM1.30). However, the effect of the  $SD$ -ratio decreases when sample sizes increase, because  $\lim_{n(=n_c=n_e) \rightarrow \infty} \left[ \frac{df}{n(df-2)} \right] = 0$ .

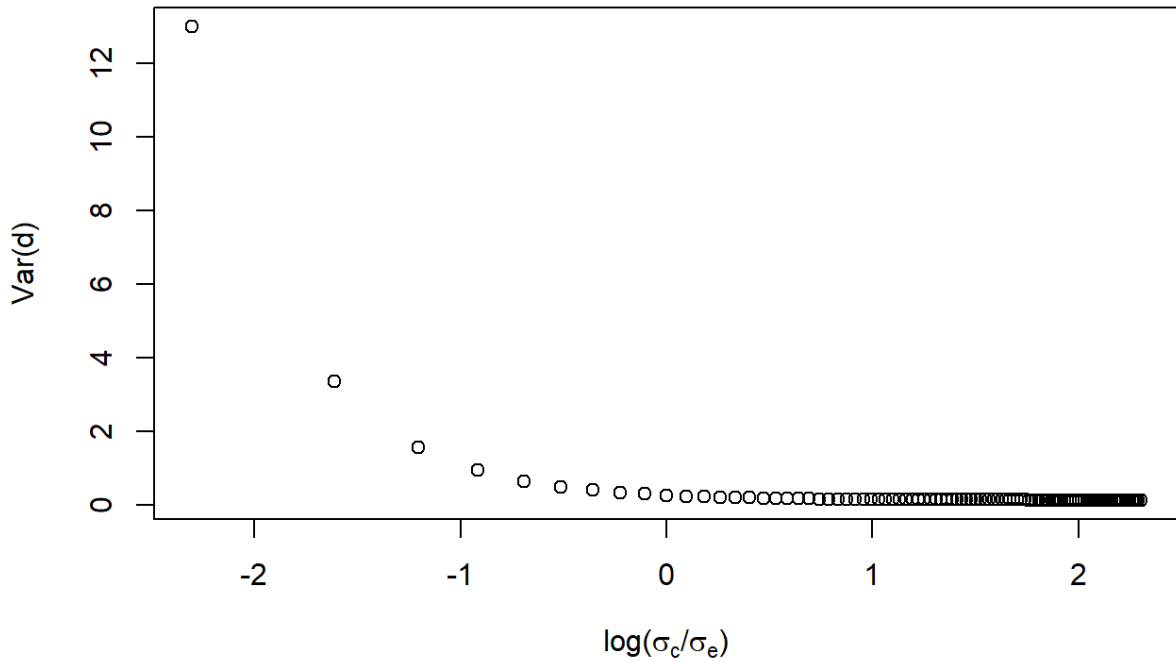


Figure SM1.30 : Variance of Glass's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the  $SD$ -ratio ( $\log\left(\frac{\sigma_c}{\sigma_e}\right)$ ).

When  $\delta_{Glass} \neq 0$ .

While the variance of Glass's  $d$  still depends on the total sample size and the  $SD$ -ratio, it also depends on the population effect size ( $\delta_{Glass}$ ). The larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the control group increases, as previously explained and illustrated in Figure SM1.27.

*When variances are unequal across populations, with unequal sample sizes.*

When  $\delta_{Glass} = 0$ .

When the population effect size is zero, the variance of Glass's  $d$  can be simplified as follows:

$$Var_{Glass's\ d} = \frac{n_c - 1}{n_c - 3} \left( \frac{1}{n_c} + \frac{\sigma_e^2}{n_e \sigma_c^2} \right)$$

The variance of Glass's  $d$  is therefore a function of the total sample size ( $N$ ), the  $SD$ -ratio and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left( \frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e} \right)$ :

- For any  $SD$  and sample sizes pairing, increasing  $n_c$  and/or  $n_e$  will decrease the variance (see Figure SM1.31);

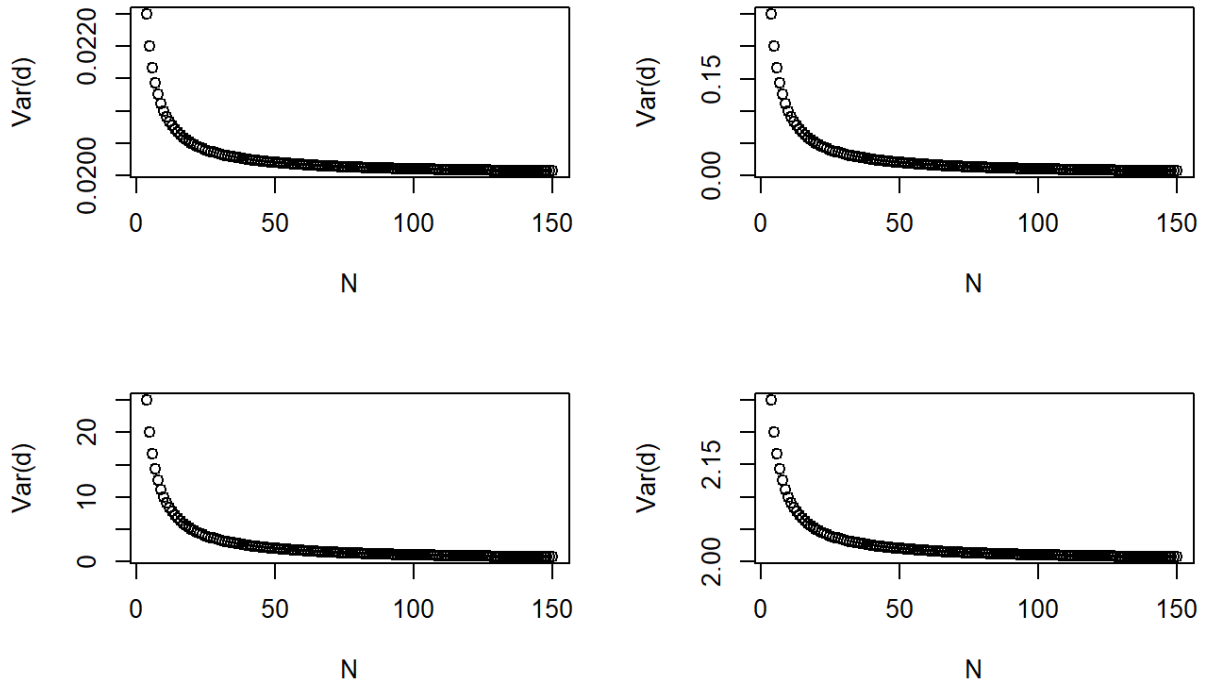


Figure SM1.31 : Variance of Glass's  $d$ , when variances and sample sizes are unequal across groups, as a function of the total sample sizes, when increasing only the control (right) or the experimental (left) group, when  $\sigma_c > \sigma_e$  (top plots) or  $\sigma_c < \sigma_e$  (bottom plots).

- The effect of the sample sizes ratio depends on the  $SD$ -ratio :
  - We previously mentioned that when  $\sigma_c = \sigma_e$ , the variance is minimized when sample sizes of both groups are almost identical (see Figure SM1.26), meaning that it is more efficient, in order to reduce variance, to add subjects uniformly in both groups;
  - When  $\sigma_e > \sigma_c$ , more weight is given to  $n_e$ , meaning that it is more efficient, in order to reduce variance, to add subjects in the experimental group ( $n_e$ ; see bottom plots in Figure M1.31);
  - When  $\sigma_c > \sigma_e$ , less weight is given to  $n_e$ , meaning that it is more efficient, in order to reduce variance, to add subjects in the control group ( $n_c$ ; see top plots in Figure SM1.31).

- Finally, there is also a main effect of the  $SD$ -ratio : the larger is  $\sigma_c$  in comparison with  $\sigma_e$ , the lower the variance, as we can observe in Figure SM1.32. We can also notice that in Figure SM1.31, the maximum variance is much larger in the two bottom plots (where  $\sigma_c < \sigma_e$ ) than in the two top plots (where  $\sigma_c > \sigma_e$ ).

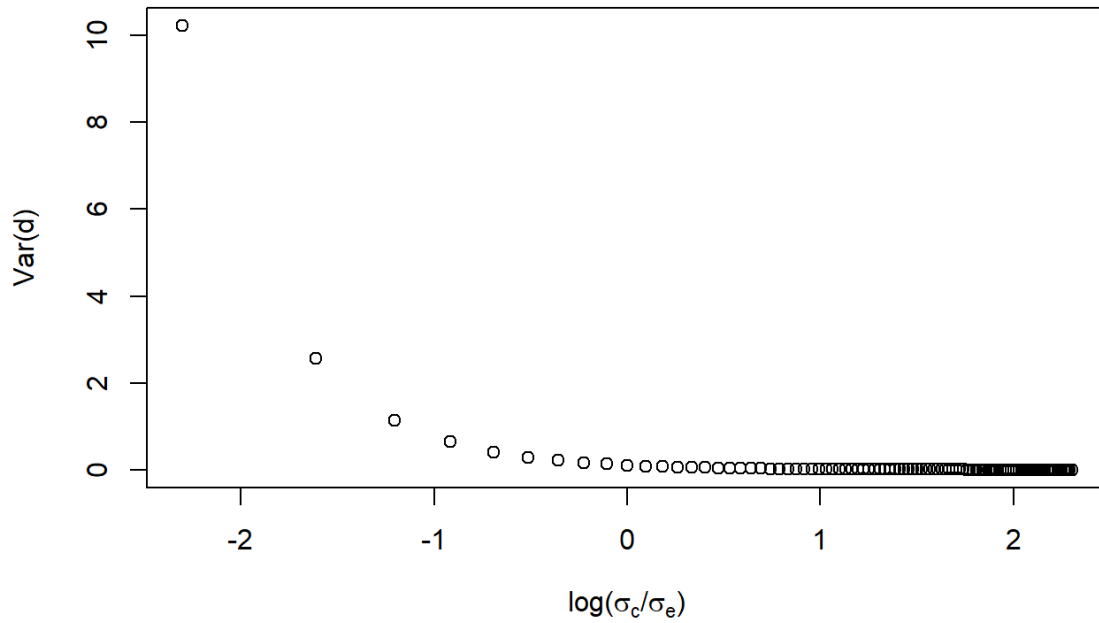


Figure SM1.32 : Variance of Glass's  $d$ , when sample sizes and variances are unequal across groups, as a function of the logarithm of the  $SD$ -ratio ( $\log\left(\frac{\sigma_c}{\sigma_e}\right)$ ).

Note that the effect of the  $SD$ -ratio, and the interaction effect between  $SD$ -ratio and sample sizes ratio decreases when the sample size of the control group increases (because  $\frac{n_c-1}{n_c-3}$  gets closer to 1).

When  $\delta_{Glass} \neq 0$ .

While the variance of Glass's  $d$  still depends on the total sample size, the  $SD$ -ratio and the interaction between the  $SD$ -ratio and the sample sizes ratio, it also depends on the population effect size ( $\delta_{Glass}$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the sample size of the control group increases, as previously explained and illustrated in Figure SM1.27.

Note : when the population effect size was null, when  $\sigma_c < \sigma_e$ , it was much more efficient to add subjects in the experimental group in order to reduce the variance (because much more weight was given to  $n_e$ ). When  $\delta_{Glass} \neq 0$ , it is important to add subjects in both groups in order to reduce the variance (because  $\frac{df}{df-2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma(\frac{df-1}{2})}{\Gamma(\frac{df}{2})} \right)^2$  is only a function of the sample size of the control group). With huge population effect size, it is even always more important to add subjects in the control group (e.g. when  $\delta_{Glass} = 30$ ).

***In summary.***

The variance of Glass's  $d$  is a function of the population effect size ( $\delta_{Glass}$ ), the  $SD$ -ratio, the total sample size and the interaction between sample sizes ratio and  $SD$ -ratio  $\left( \frac{n_c}{n_e} \times \frac{\sigma_c}{\sigma_e} \right)$  :

- The variance decreases when the  $SD$ -ratio increases (i.e. when  $\sigma_e >> \sigma_c$ );
- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control, in the experimental, or in both groups, in order to reduce the variance, varies as a function of the  $SD$ -ratio and the population effect size. The only situation where it is optimal to maximize the experimental group is when  $\sigma_e > \sigma_c$  and  $\delta_{Glass} \approx 0$ . Most of the time, it is more efficient to maximize the control groups (e.g. anytime  $\sigma_e < \sigma_c$ , and when  $\delta_{Glass}$  is very large) or to uniformly add subjects in both groups (e.g. when  $\sigma_e > \sigma_c$  and  $\delta_{Glass}$  is neither null nor huge);
- The variance increases when  $\delta_{Glass}$  increases. Note that the effect of  $\delta_{Glass}$  is moderated by the control group size (the larger  $n_e$ , the smaller the effect of  $\delta_{Glass}$  on the variance).

Cohen's  $d^*$ .

*When variances are equal across populations.*

When  $\delta_{Cohen}^* = 0$ .

When the population effect size is zero, the variance of Cohen's  $d^*$  is computed as follows :

$$Var_{Cohen's d^*} = \frac{df}{df - 2} \times \frac{N}{n_1 n_2}$$

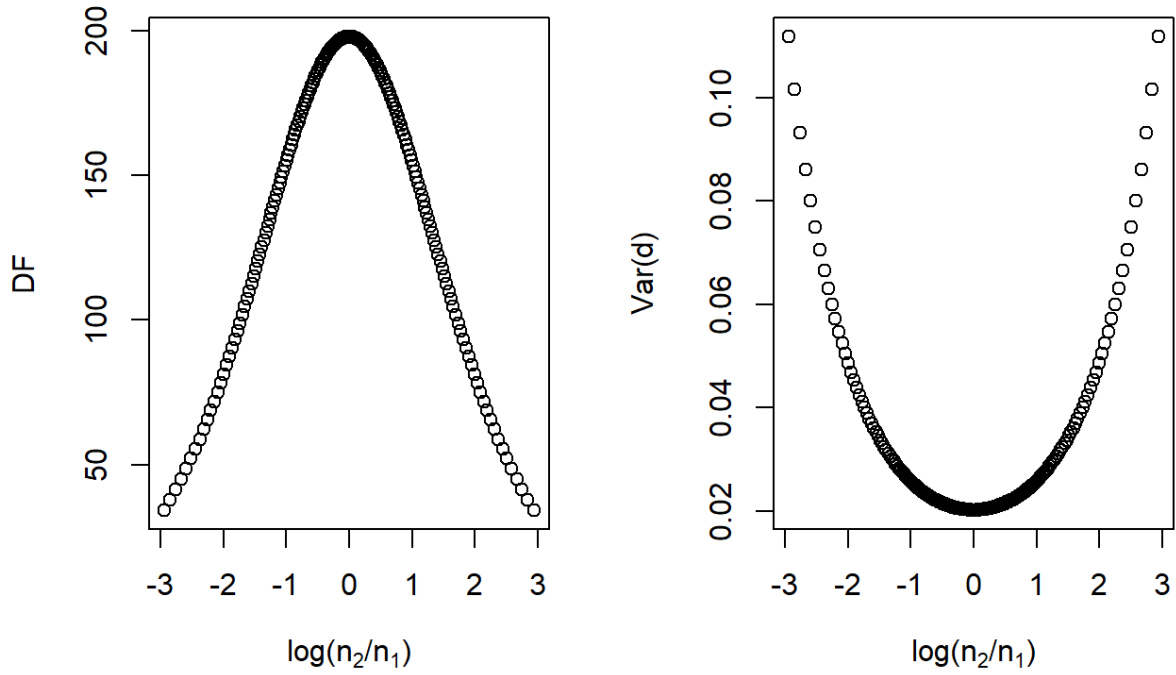
with

$$df = \frac{4(n_1 - 1)(n_2 - 1)}{n_1 + n_2 - 2}$$

In this configuration, the degrees of freedom as well as the variance of Cohen's  $d^*$  depend on the total sample size ( $N$ ) and the sample sizes allocation ratio ( $\frac{n_2}{n_1}$ ):

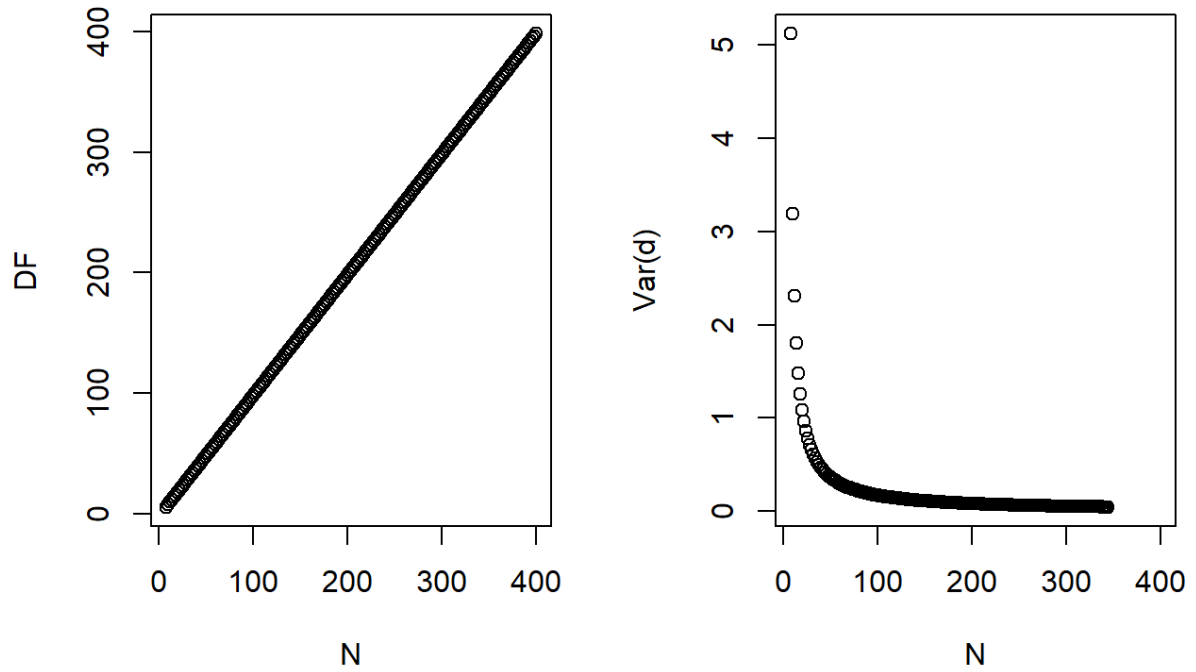


- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.33);



*Figure SM1.33* : Variance of Cohen's  $d^*$  when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ).

- The larger the total sample size, the lower the bias (see Figure SM1.34).



*Figure SM1.34* : Variance of Cohen's  $d^*$  when variances are equal across groups, as a function of the total sample size ( $N$ ).

When  $\delta_{Cohen}^* \neq 0$ .

While the variance of Cohen's  $d^*$  still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size ( $\delta_{Cohen}^*$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the sample sizes ratio get closer to 1), as illustrated in Figure SM1.35 since

$$\lim_{df \rightarrow \infty} \left[ \frac{df}{df - 2} - \left( \frac{\sqrt{\frac{df}{2}} \times \Gamma\left(\frac{df-1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)} \right)^2 \right] = 0$$

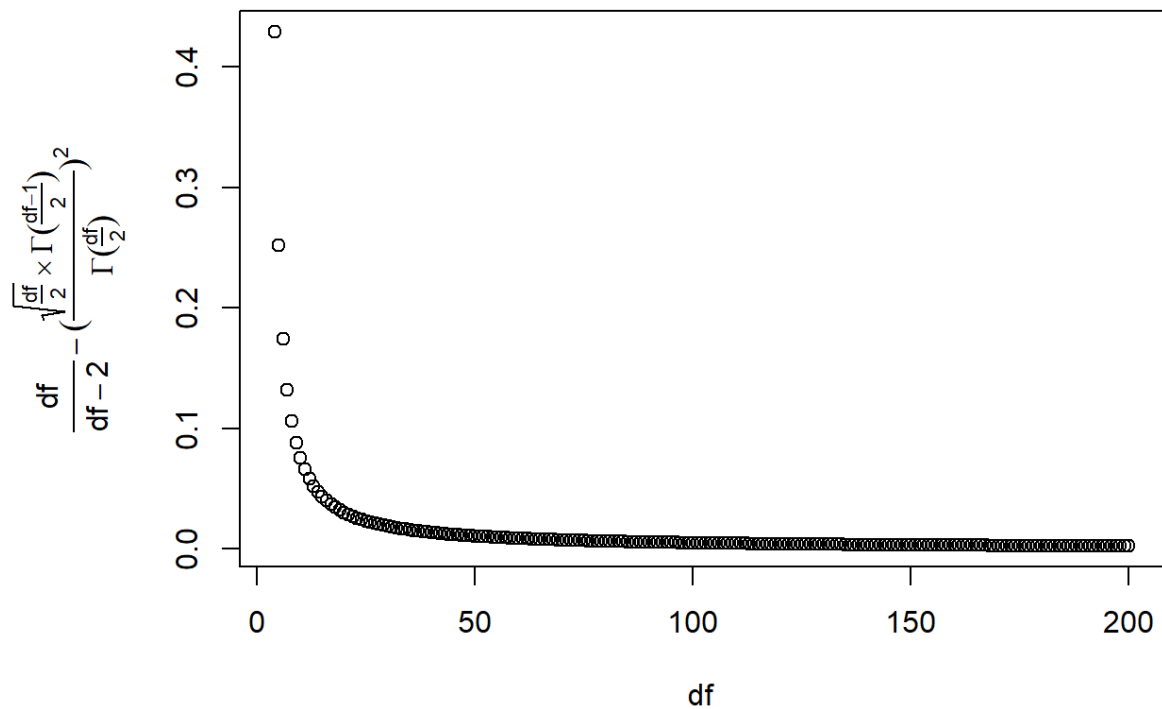


Figure SM1.35 : Effect size moderator (for all estimators), as a function of the degrees of freedom.

*When variances are unequal across populations, with equal sample sizes.*

When  $\delta_{Cohen}^* = 0$ .

When the population effect size is zero, the variance of Cohen's  $d^*$  can be simplified as follows:

$$Var_{Cohen's d^*} = \frac{df}{df - 2} \times \frac{2}{n}$$

where  $n = N/2$ =sample size of each group, and  $df = \frac{(n-1)(\sigma_1^4 + \sigma_2^4 + 2\sigma_1^2\sigma_2^2)}{\sigma_1^4 + \sigma_2^4}$ . In this configuration, the degrees of freedom as well as the variance of Cohen's  $d^*$  depend on the total sample size ( $N$ ) and the  $SD$ -ratio  $\left(\frac{\sigma_2}{\sigma_1}\right)$ :

- The further the  $SD$ -ratio is from 1, the larger the variance (see Figure SM1.36);

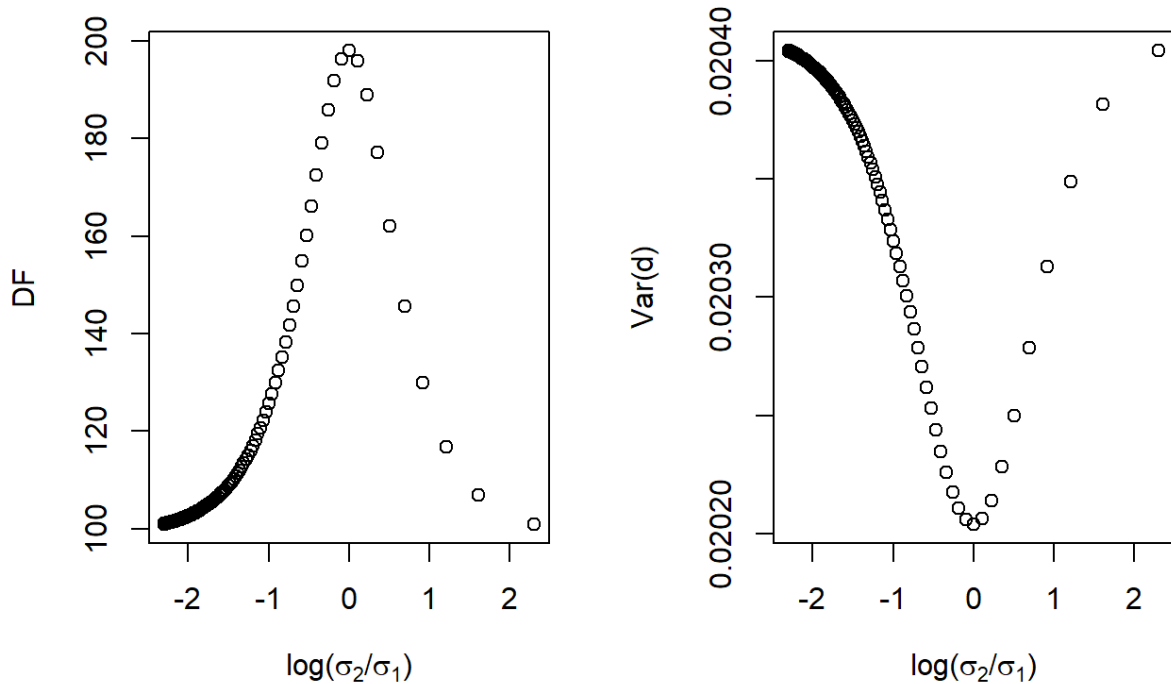
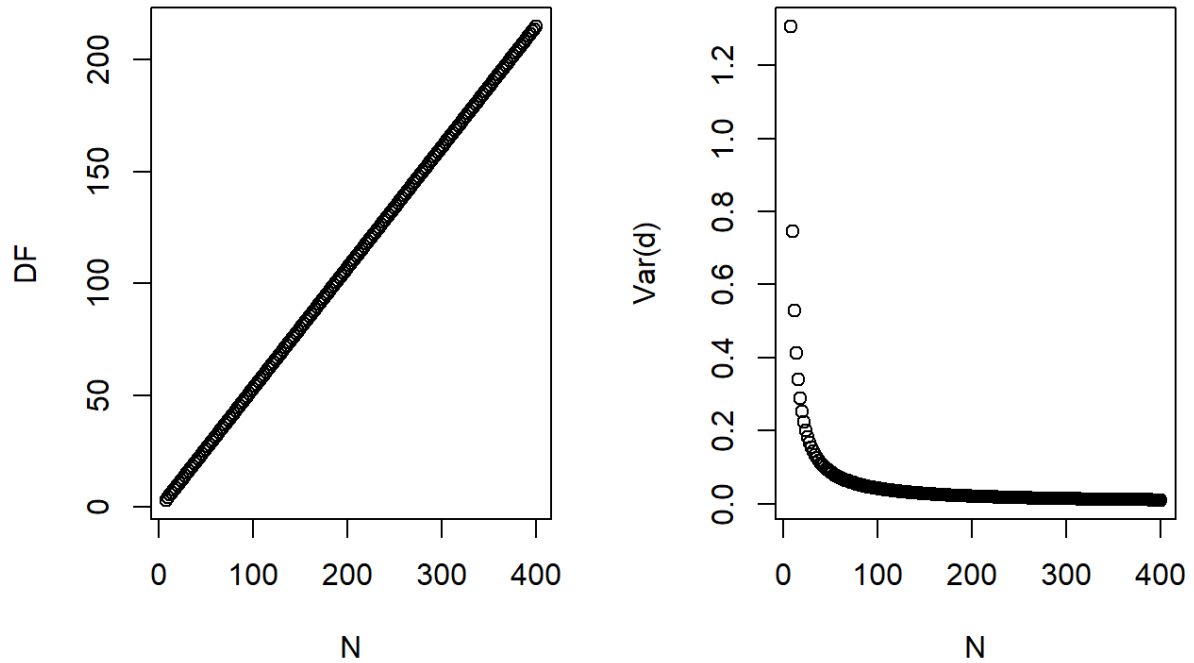


Figure SM1.36 : Variance of Cohen's  $d^*$  when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the  $SD$ -ratio  $\left(\log\left(\frac{\sigma_2}{\sigma_1}\right)\right)$ .

- The larger the total sample size, the lower the variance (see Figure SM1.37).



*Figure SM1.37* : Variance of Cohen's  $d^*$  when variances are unequal across groups and sample sizes are equal, as a function of the total sample size ( $N$ ).

Note : for a constant  $SD$ -ratio, the size of the variance does not matter (see Figure SM1.38).

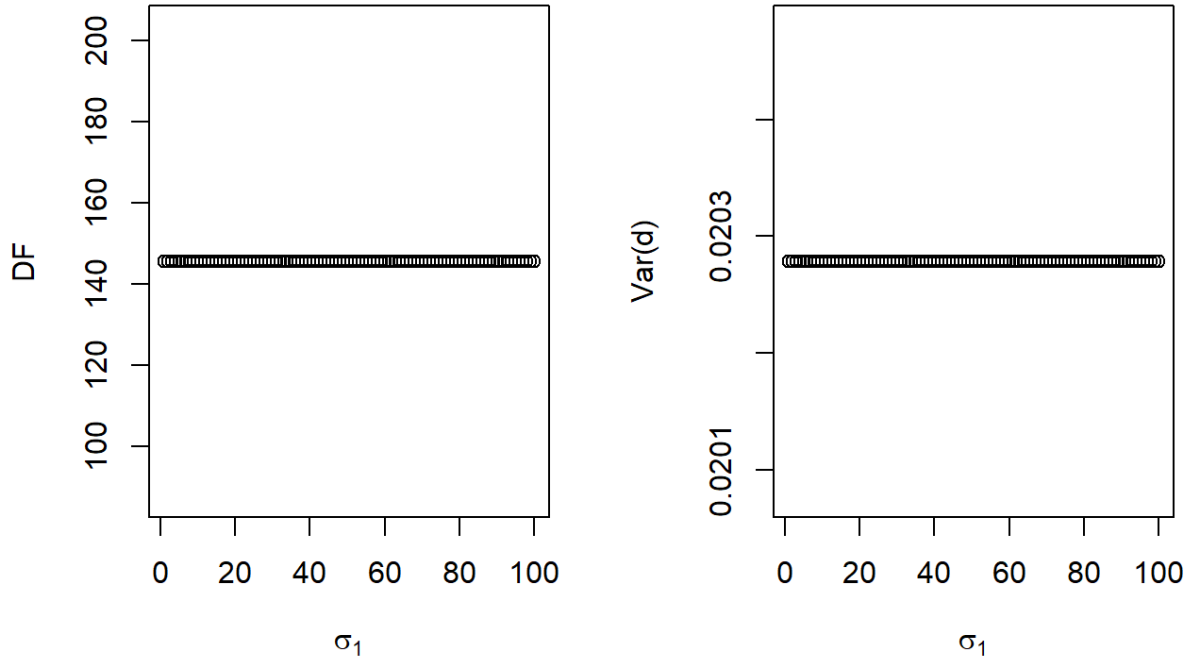


Figure SM1.38 : Variance of Cohen's  $d^*$ , when variances are unequal across groups and sample sizes are equal, as a function of  $\sigma_1$  and  $\sigma_2$ , for a constant  $SD$ -ratio.

When  $\delta_{Cohen}^* \neq 0$ .

While the variance of Cohen's  $d^*$  still depends on the total sample size and the  $SD$ -ratio, it also depends on the population effect size ( $\delta_{Cohen}^*$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or the  $SD$ -ratio get closer to 1), as previously illustrated in Figure SM1.35.

*When variances are unequal across populations, with unequal sample sizes.*

When  $\delta_{Cohen}^* = 0$ .

When the population effect size is zero, the variance of Cohen's  $d^*$  can be simplified as follows:

$$Var_{Cohen's d^*} = \frac{df}{df - 2} \times \frac{2 \left( \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \right)}{\sigma_1^2 + \sigma_2^2}$$

with  $df = \frac{(n_1-1)(n_2-1)(\sigma_1^2+\sigma_2^2)^2}{(n_2-1)\sigma_1^4+(n_1-1)\sigma_2^4}$ . In this configuration, the degrees of freedom are a function of the total sample size ( $N$ ) and the interaction between sample sizes ratio and the  $SD$ -ratio  $\left( \frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1} \right)$ :

- The larger the total sample size, the lower the variance (illustration in Figure SM1.39);

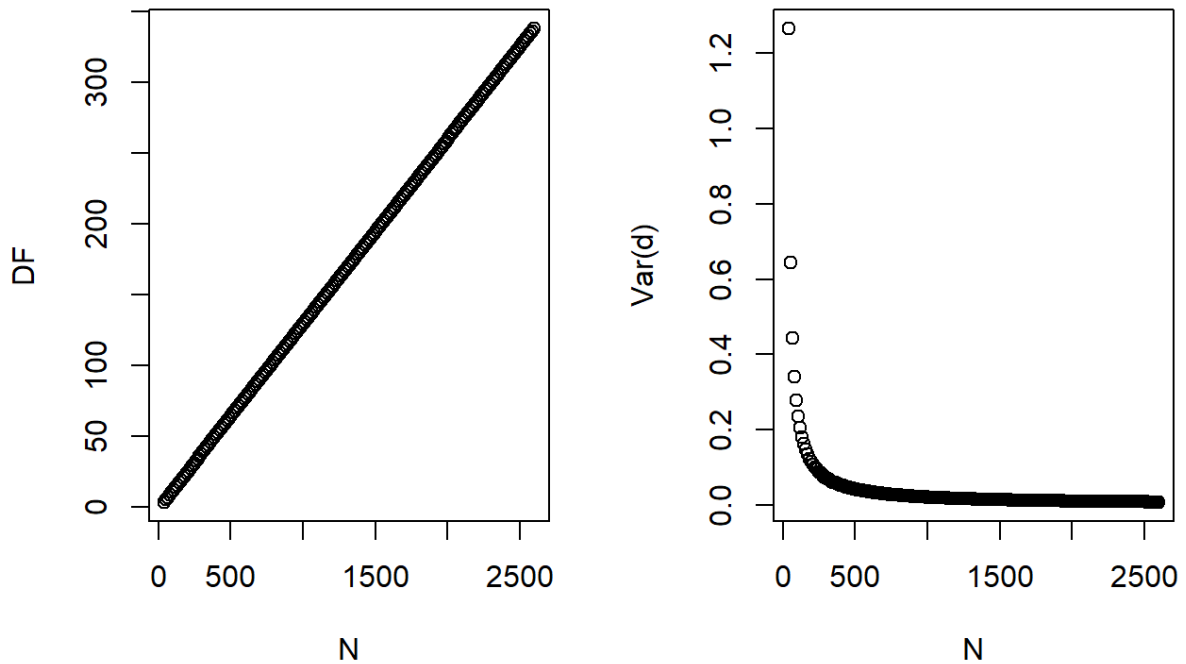


Figure SM1.39 : Variance of Cohen's  $d^*$  when variances and sample sizes are unequal across groups, as a function of the total sample size ( $N$ ).

- The smallest variance always occurs when there is a positive pairing between variances and sample sizes, because one gives more weight to the smallest variance in the denominator of the  $df$  computation and in the numerator of the variance computation. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure SM1.40). This can be explained by splitting the numerator and the denominator of the  $df$  computation (see the file “Theoretical Bias, as a function of population parameters”).

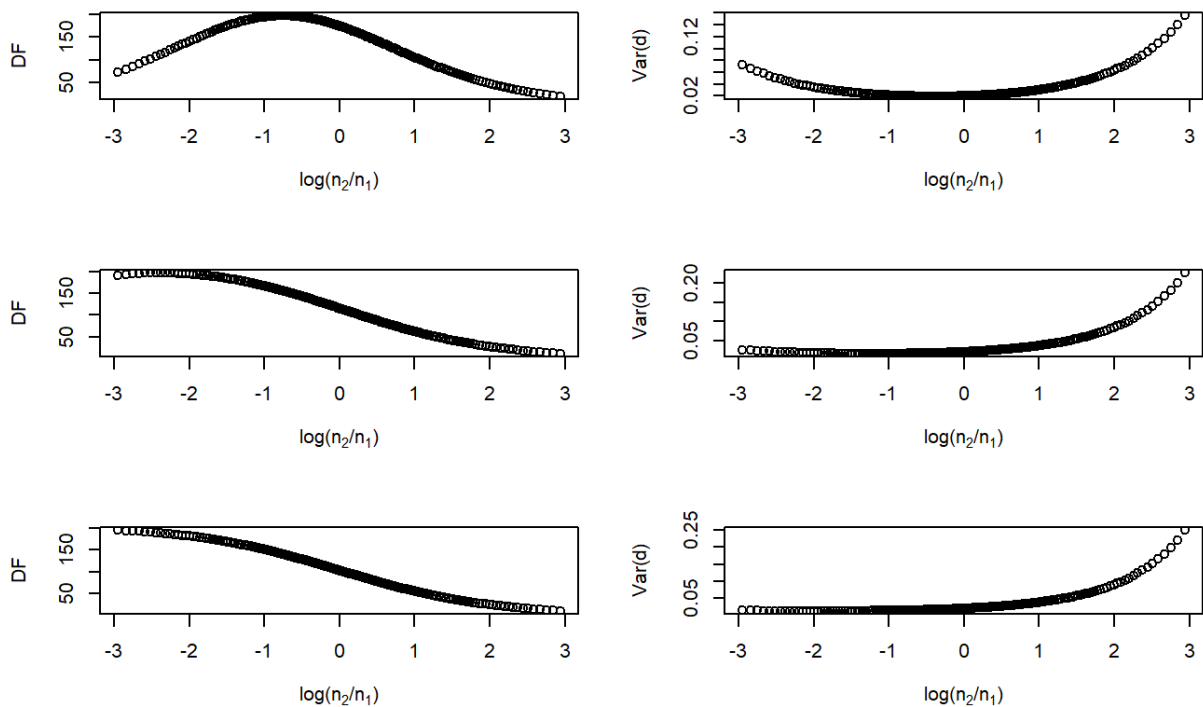


Figure SM1.40 : The variance of Cohen's  $d^*$ , when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ), when  $SD$ -ratio equals .68 (first row), .29 (second row) or .14 (third row).



Note : for a constant  $SD$ -ratio, the variance does not matter. (See Figure SM1.41).

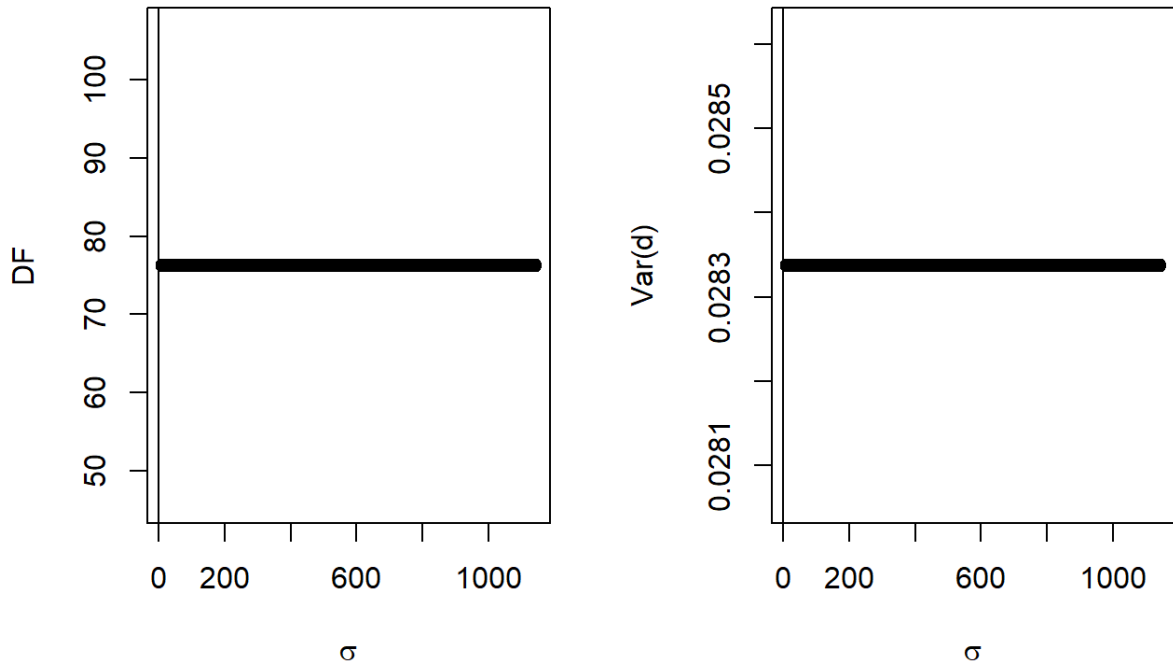


Figure SM1.41 : Variance of Cohen's  $d^*$ , when variances and sample sizes are unequal across groups, as a function of  $\sigma_1$  and  $\sigma_2$ , for a constant  $SD$ -ratio.

When  $\delta_{Cohen}^* \neq 0$ .

While the variance of Cohen's  $d^*$  still depends on the total sample size, the  $SD$ -ratio and the interaction between the sample sizes ratio and the  $SD$ -ratio, it also depends on the population effect size ( $\delta_{Cohen}^*$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when the total sample size increases and/or when there is a positive pairing between the sample sizes ratio and the  $SD$ -ratio), as previously illustrated in Figure SM1.35.

***In summary.***

The variance of Cohen's  $d^*$  is a function of the population effect size ( $\delta_{Cohen}^*$ ), the total sample size ( $N$ ) and the interaction between sample sizes ratio and  $SD$ -ratio ( $\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}$ ) :

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the  $SD$ -ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;
- The variance increases when  $\delta_{Cohen}^*$  increases. Note that the effect of  $\delta_{Cohen}^*$  is moderated by the total sample size and the interaction between sample sizes ratio and  $SD$ -ratio.

**Shieh's  $d$ .**

***When variances are equal across populations.***

When  $\delta_{Shieh} = 0$ .

When the population effect size is zero, the variance of Shieh's  $d$  can be simplified as follows:

$$Var_{Shieh's d} = \frac{df}{(df - 2)N}$$

with  $df = \frac{N^2(n_1-1)(n_2-1)}{n_2^2(n_2-1)+n_1^2(n_1-1)}$ . In this configuration, the degrees of freedom as well as the variance of Shieh's  $d$  depend on the total sample size ( $N$ ) and the sample sizes allocation ratio ( $\frac{n_2}{n_1}$ ):

- The further the sample sizes allocation ratio is from 1, the larger the variance (see Figure SM1.42);

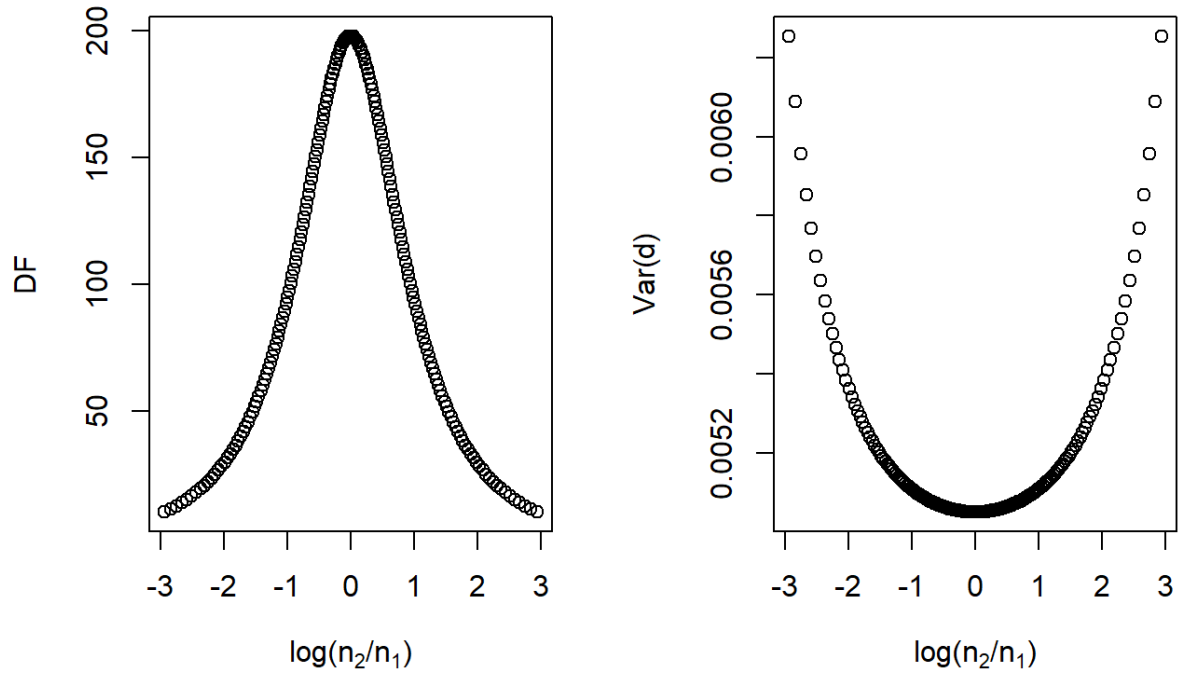


Figure SM1.42 : Variance of Shieh's  $d$  when variances are equal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ).

- The larger the total sample size, the lower the variance. It does not matter whether the sample sizes ratio is constant (see Figure SM1.43) or not (see Figure SM1.44).

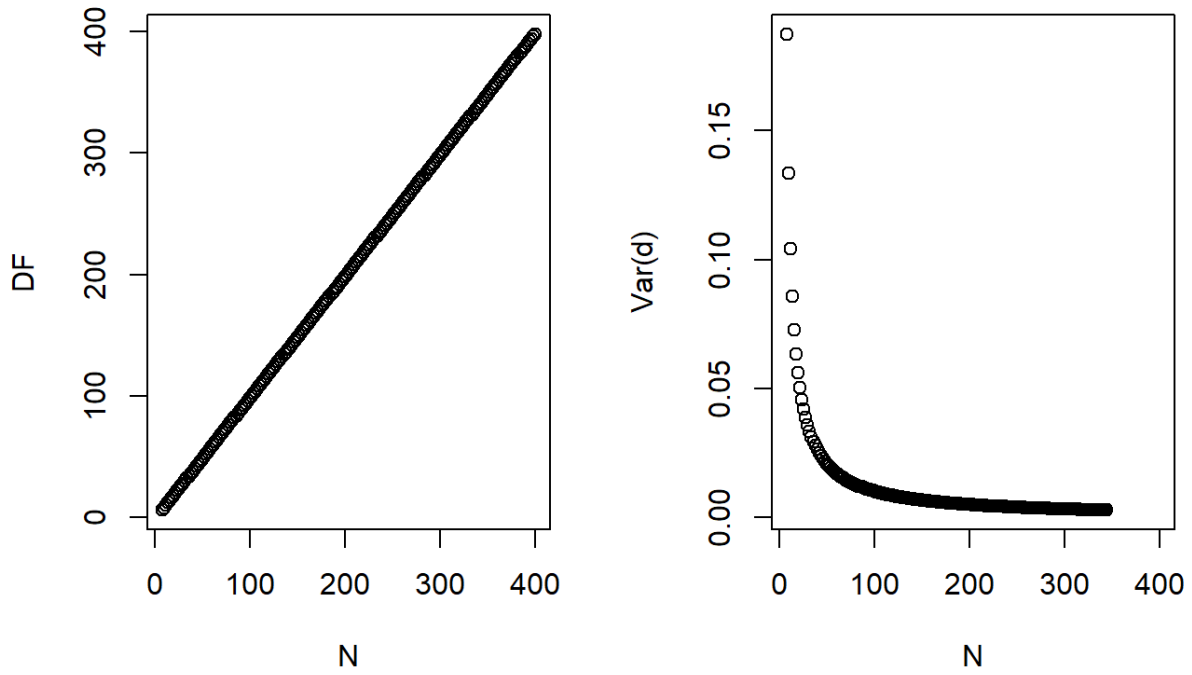
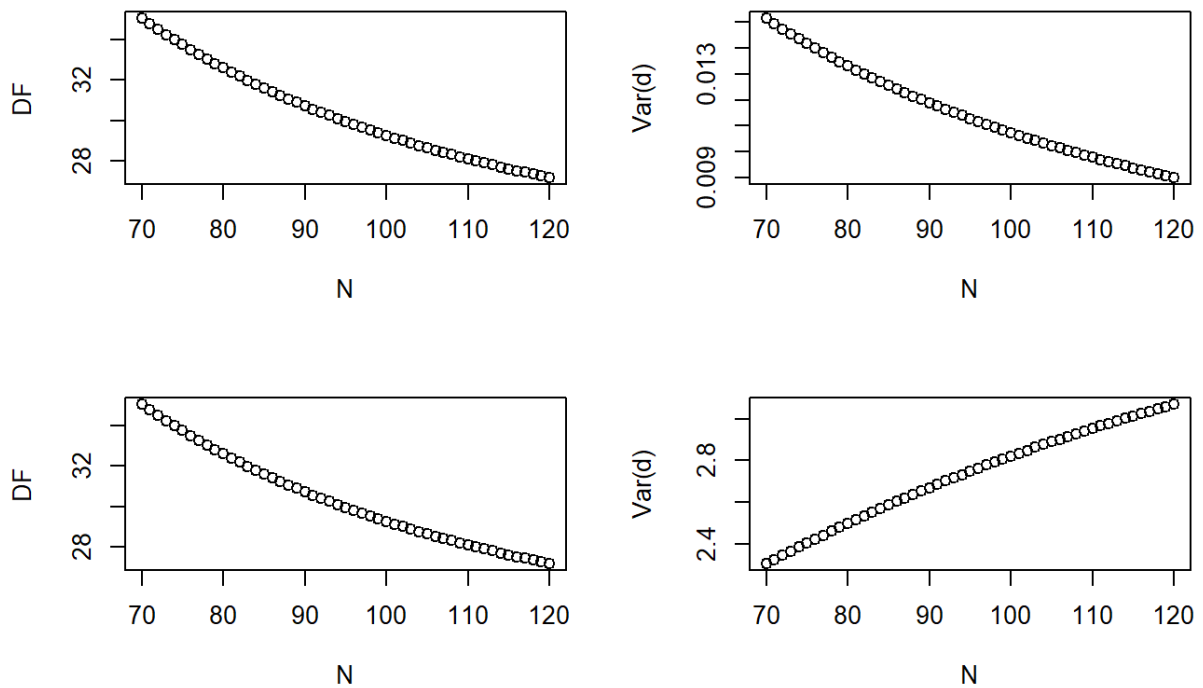


Figure SM1.43 : Variance of Shieh's  $d$  when variances are equal across groups, as a function of the total sample size ( $N$ ), for a constant sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ).



*Figure SM1.44* : Variance of Shieh's  $d$  when variances are equal across groups, as a function of the total sample size ( $N$ ), when adding subjects only in one group (either in the first group; see top plots; or in the second group; see bottom plots).

Note : in “Theoretical Bias,” we noticed that moving the sample sizes ratio away from 1 when adding subjects in only one group could decrease the degrees of freedom. However, due to the total sample size term ( $N$ ) in the denominator of the variance computation, even when degrees of freedom decrease due to the fact that one adds subjects only in one group, the variance still decreases (because the denominator of the variance computation increases; see Figure SM1.44).

When  $\delta_{Shieh} \neq 0$ .

While the variance of Shieh's  $d$  still depends on the total sample size and the sample sizes ratio, it also depends on the population effect size ( $\delta_{Shieh}$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase, without

increasing the sample sizes ratio, and/or the sample sizes ratio gets closer to 1), as previously illustrated in Figure SM1.35.

Note : we previously noticed that when the effect size is zero, the variance of Shieh's  $d$  decreases, even when the sample sizes ratio increases. It is no longer true when there is a non-null effect size because the larger the sample sizes ratio, the more the variance will increase with increasing effect size.

*When variances are unequal across populations, with equal sample sizes.*

When  $\delta_{Shieh} = 0$ .

When the population effect size is zero, the variance of Shieh's  $d$  can be simplified as follows:

$$Var_{Shieh's d} = \frac{df}{(df - 2)N}$$

with  $df = \frac{(\sigma_1^2 + \sigma_2^2)^2 \times (n-1)}{\sigma_1^4 + \sigma_2^4}$ . In this configuration, the degrees of freedom as well as the variance of Shieh's  $d$  depend on the total sample size ( $N$ ) and the  $SD$ -ratio ( $\frac{\sigma_2}{\sigma_1}$ ).

- The further the  $SD$ -ratio is from 1, the larger the variance (see Figure SM1.45);

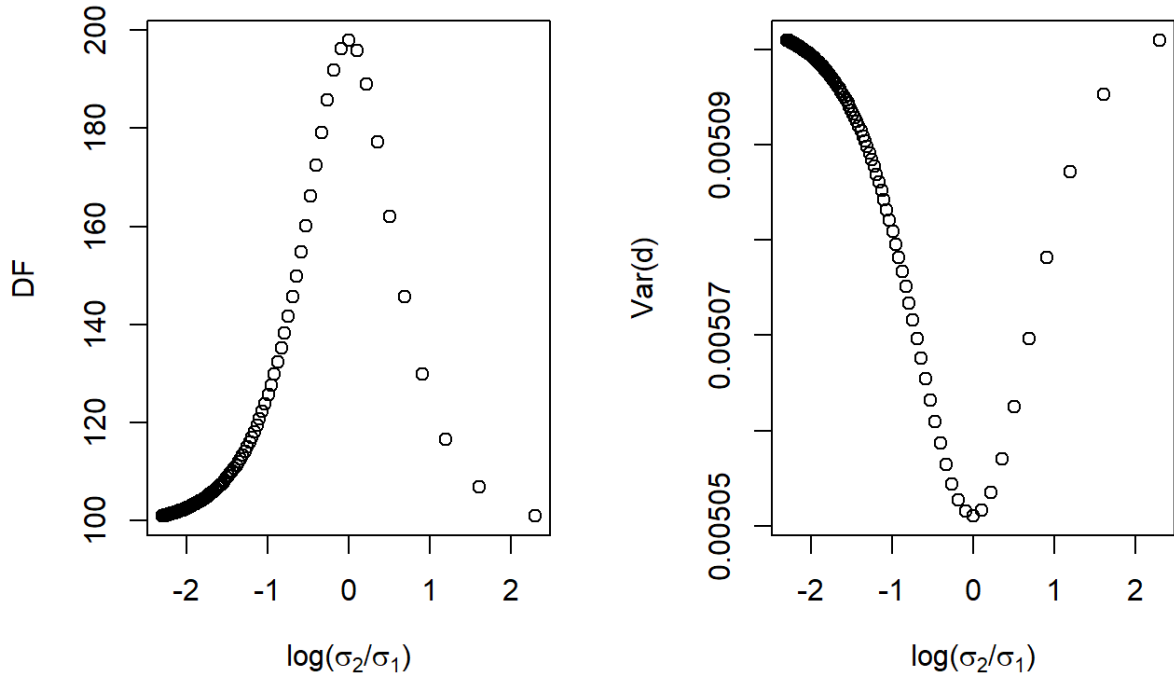
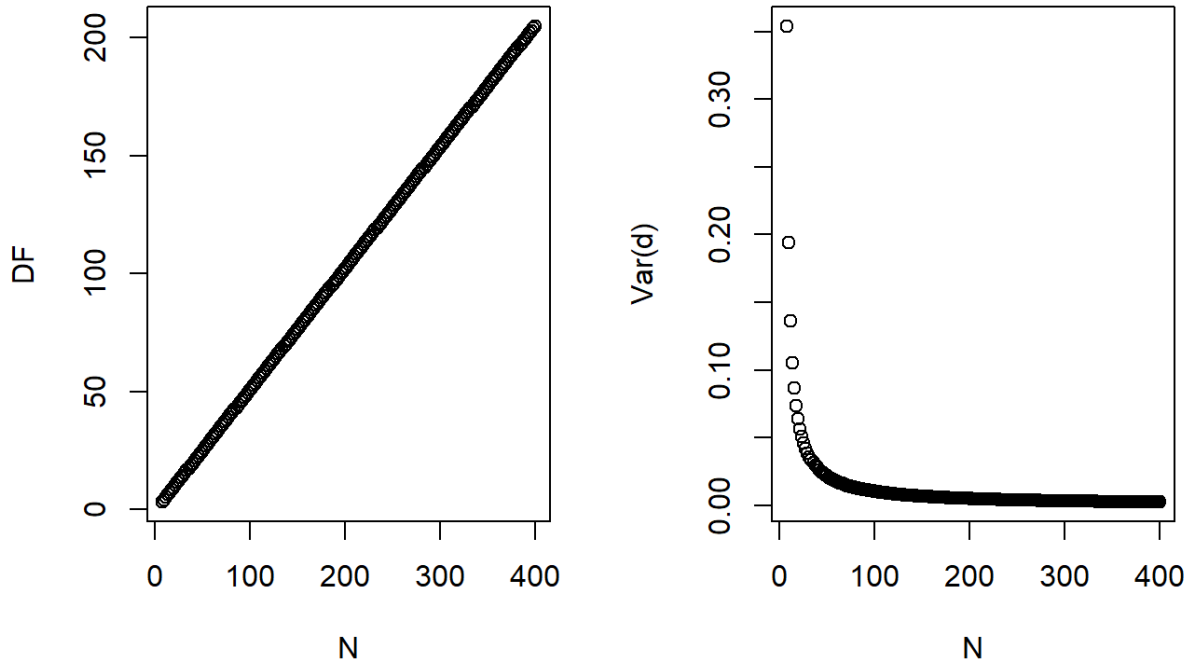


Figure SM1.45 : Variance of Shieh's  $d$  when variances are unequal across groups and sample sizes are equal, as a function of the logarithm of the  $SD$ -ratio ( $\log\left(\frac{\sigma_2}{\sigma_1}\right)$ ).

- The larger the total sample size, the lower the variance (see Figure SM1.46).



*Figure SM1.46* : Variance of Shieh's  $d$  when variances are unequal across groups and sample sizes are equal, as a function of the total sample size ( $N$ ).



Note : for a constant  $SD$ -ratio, the size of the variance does not matter (see Figure SM1.47).

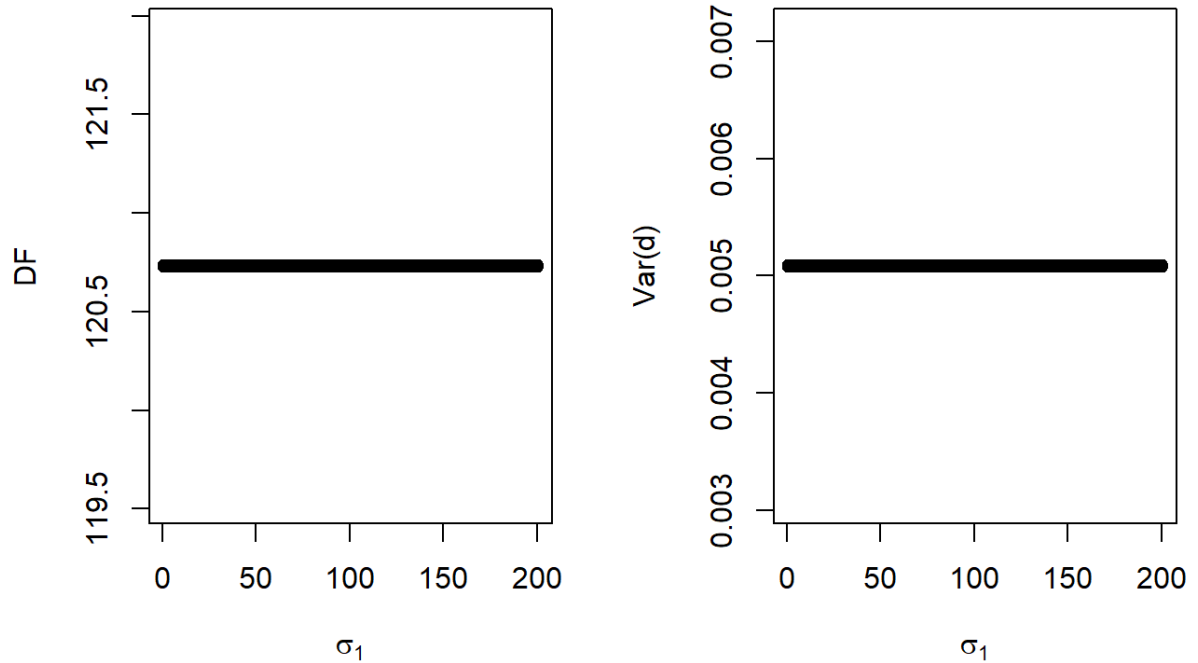


Figure SM1.47 : Variance of Shieh's  $d$ , when variances are unequal across groups and sample sizes are equal, as a function of  $\sigma_1$  and  $\sigma_2$ , for a constant  $SD$ -ratio.

When  $\delta_{Shieh} \neq 0$ .

While the variance of Shieh's  $d$  still depends on the total sample size and the  $SD$ -ratio, it also depends on the population effect size ( $\delta_{Shieh}$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase (i.e. when sample sizes increase and/or the  $SD$ -ratio gets closer to 1), as previously illustrated in Figure SM1.35.

*When variances are unequal across populations, with unequal sample sizes.*

When  $\delta_{Shieh} = 0$ .

When the population effect size is zero, the variance of Shieh's  $d$  can be simplified as follows:

$$Var_{Shieh's d} = \frac{df}{(df - 2)N}$$

with  $df = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{(\sigma_1^2/n_1)^2}{n_1-1} + \frac{(\sigma_2^2/n_2)^2}{n_2-1}}$ . In this configuration, the degrees of freedom as well as the variance of Shieh's  $d$  depend on the total sample size ( $N$ ) and the interaction between the sample sizes ratio and the  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$ :

- The larger the total sample size, the lower the variance. It remains true when the sample sizes ratio is constant (see Figure SM1.48) and when it is not (see Figure SM1.49).

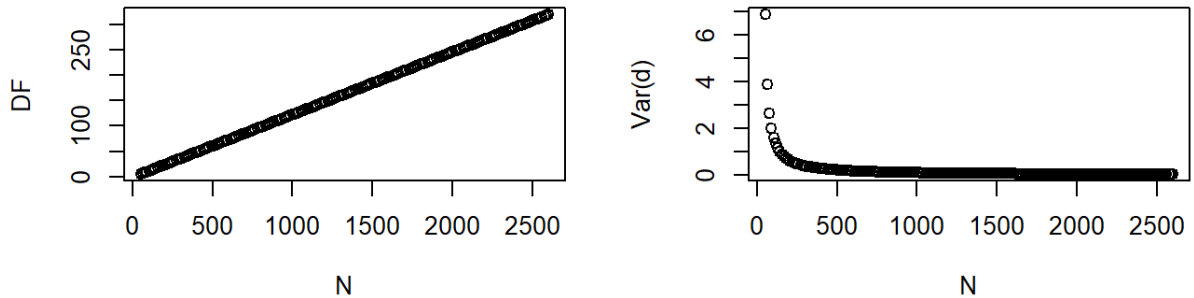


Figure SM1.48 : Variance of Shieh's  $d$  when variances and sample sizes are unequal across groups, as a function of the total sample size ( $N$ ), for a constant sample sizes ratio  $\left(\log\left(\frac{n_2}{n_1}\right)\right)$ .

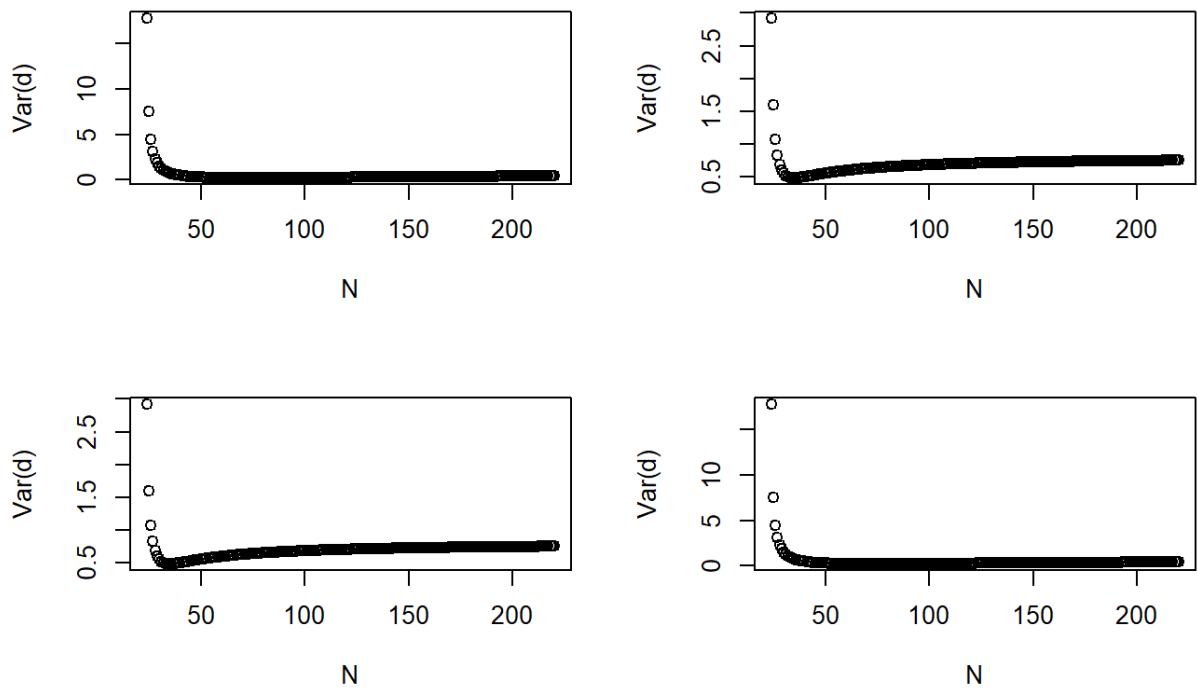


Figure SM1.49 : Variance of Shieh's  $d$  when variances and sample sizes are unequal across groups, as a function of the total sample size ( $N$ ), when adding subjects only in one group (either in the first group; see left plots; or in the second group; see right plots), and  $\sigma_1 > \sigma_2$  (top plots) or  $\sigma_1 < \sigma_2$  (bottom plots).

Note : When variances were equal across populations, adding subjects only in the first group had the same impact on the variance as adding subjects only in the second group (see Figure SM1.34). When variances are unequal across groups, this is not true anymore (see Figure SM1.49).

- The smallest variance always occurs when there is a positive pairing between variances and sample size. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance (see Figure SM1.50).

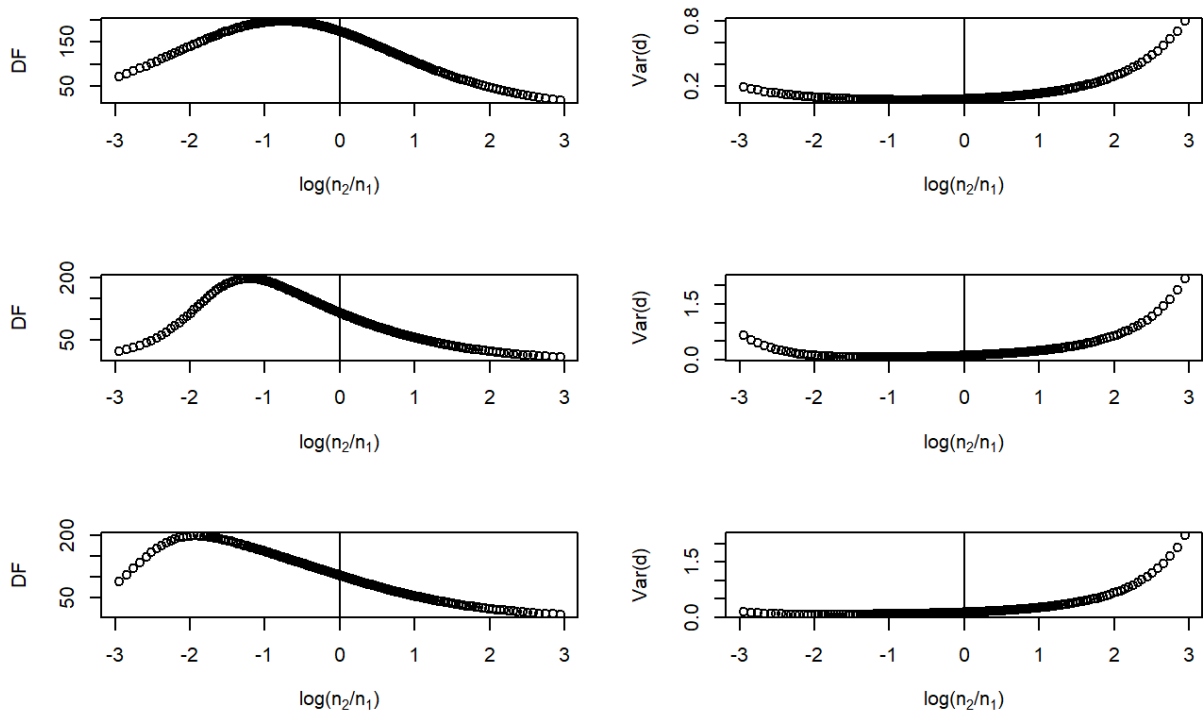


Figure SM1.50 : The variance of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of the logarithm of the sample sizes ratio ( $\log\left(\frac{n_2}{n_1}\right)$ ), when  $SD$ -ratio equals .68 (first row), .29 (second row) or .14 (third row).

Moreover, for a constant  $SD$ -ratio, the variances do not matter (See Figure SM1.51).

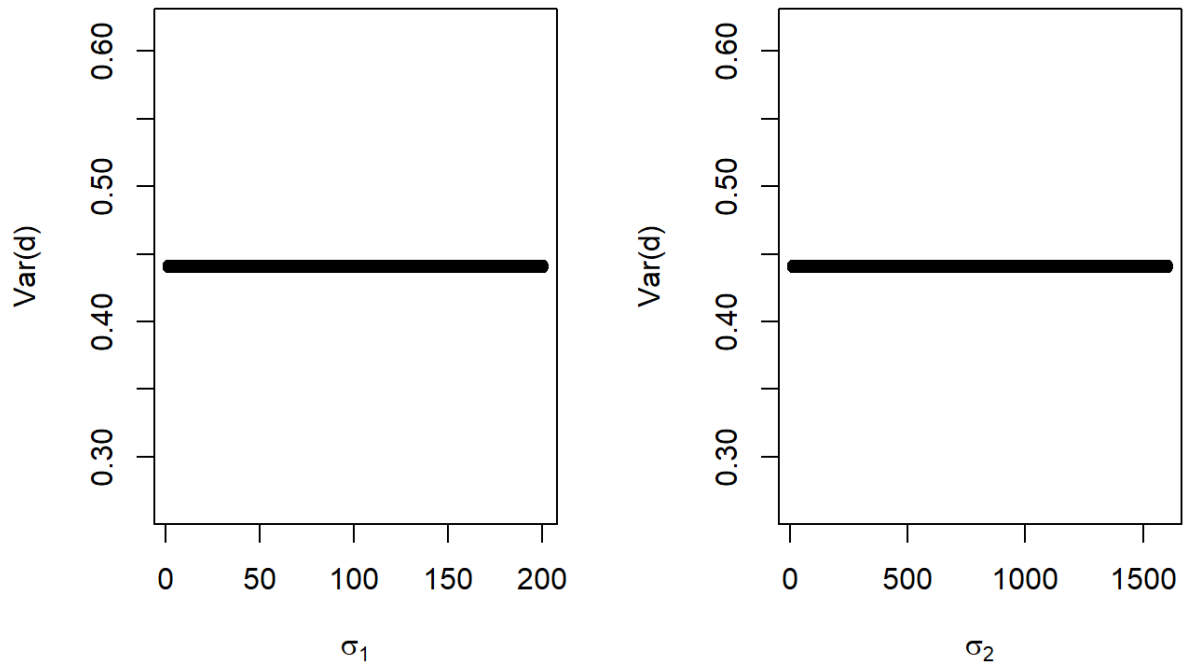


Figure SM1.51 : Variance of Shieh's  $d$ , when variances and sample sizes are unequal across groups, as a function of  $\sigma_1$  (left) or  $\sigma_2$  (right), for a constant  $SD$ -ratio.

When  $\delta_{Shieh} \neq 0$ .

While the variance of Shieh's  $d$  still depends on the total sample size and the interaction between the sample sizes ratio and the  $SD$ -ratio, it also depends on the population effect size ( $\delta_{Shieh}$ ) : the larger the population effect size, the larger the variance. However, the effect of the population effect size decreases when the degrees of freedom increase, as previously illustrated in Figure SM1.35.

***In summary.***

The variance of Shieh's  $d$  is a function of the population effect size ( $\delta_{Shieh}$ ), the total sample size ( $N$ ) and the interaction between sample sizes ratio and  $SD$ -ratio  $\left(\frac{n_2}{n_1} \times \frac{\sigma_2}{\sigma_1}\right)$  :

- The variance always decreases when the control and/or the experimental group increases. The benefit of adding subjects rather in the control or in the experimental group depends on the  $SD$ -ratio. Indeed, the smallest variance always occurs when there is a positive pairing between variances and sample sizes. Moreover, the further the  $SD$ -ratio is from 1, the further from 1 will also be the sample sizes ratio associated with the smallest variance;
- The variance increases when  $\delta_{Shieh}$  increases. Note that the effect of  $\delta_{Shieh}$  is moderated by the total sample size and the interaction between the sample sizes ratio and the  $SD$ -ratio.

## Supplemental Material 2 :

In order to insure the reliability of our calculation method, for all scenarios where  $G_1 = G_2 = 0$ , we compared empirical means and variances of all estimators (i.e. means and variances of all estimates) with theoretical means and variances (i.e. expected means and variances, computed based on equations in Tables 2, 3 and 4 in the main article). Because we can draw exactly the same conclusions for **biased** (Cohen's  $d$ , Glass's  $d$  using either  $S_1$  or  $S_2$  as standardizer, Shieh's  $d$  and Cohen's  $d^*$ ) and **unbiased** (Hedges'  $g$ , Glass's  $g$  using either  $S_1$  or  $S_2$  as standardizer, Shieh's  $g$  and Hedges'  $g^*$ ) estimators, we will simultaneously present results for both categories of estimators. Results will be subdivided into 4 conditions:

- When population variances and sample sizes are equal across groups (condition a; see Tables SM2.1 and SM2.5 for respectively biased and unbiased estimators);
- When population variances are equal across groups and sample sizes are unequal (condition b; see Tables SM2.2 and SM2.6 for respectively biased and unbiased estimators);
- When population variances are unequal across groups and sample sizes are equal (condition c; see Tables SM2.3 and SM2.7 for respectively biased and unbiased estimators);
- When population variances and sample sizes are unequal across groups (condition d; see Tables SM2.4 and SM2.8 for respectively biased and unbiased estimators).

Because the equations of theoretical means and variances of Cohen's  $d$  and Hedges'  $g$  rely on the assumption of normality and equality of population variances, we expect empirical and theoretical parameters to be very close only in conditions a and b. For all other estimators, the equations of theoretical means and variances rely solely on the assumption of normality and therefore, we expect empirical and theoretical parameters to be very close in all conditions.

On average, empirical means (and variances) of all estimators are very close to theoretical expectations when population variances are equal across groups, with equal

sample sizes (condition a; see Tables SM2.1 and SM2.5) or unequal sample sizes (condition b; see Tables SM2.2 and SM2.6).

When population variances are unequal across groups (conditions c and d; see Tables SM2.3, SM2.4, SM2.7 and SM2.8), empirical means (and variances) of Cohen's  $d^*$  (Hedges'  $g^*$ ) and Shieh's  $d$  (Shieh's  $g$ ) are still very close to theoretical expectations. Regarding Glass's  $d$  (Glass's  $g$ ), on average, while empirical variances remain very close to theoretical expectations, one observes larger departures between empirical and theoretical means when using  $S_2$  as standardizer. However, when looking at details in results for each scenario (see “biased\_estimator\_condition C.xlsx,” “biased\_estimator\_condition D.xlsx,” “unbiased\_estimator\_condition C.xlsx” and “unbiased\_estimator\_condition C.xlsx” in Supplemental Material 2), one notices that the larger the population effect size, the larger the departure between empirical and theoretical means, and that relative to the population effect size, departures between empirical and theoretical means are always very small. On the other hand, both empirical bias and variance of Cohen's  $d$  (Hedges'  $g$ ) highly depart from theoretical expectations, even when looking at relative departures to the population effect size, especially when sample sizes are unequal across groups (condition d; see Tables SM2.4 and SM2.8), which is not surprising, as Cohen's  $d$  (Hedges'  $g$ ) relies on the equality of population variances assumption.



Table SM2.1.

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are equal across groups (condition a).*

Estimator ( $\widehat{\delta}$ )	Absolute deviation between empirical and theoretical means $ E(\widehat{\delta}) - \mu_{\delta} $				Ratio between empirical and theoretical variances $S^2_{\widehat{\delta}} / \sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $d$	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Glass's $d_1$	0,022	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's $d_2$	0,023	0,000	0,005	0,007	1,005	0,889	0,966	0,035
Cohen's $d^*$	0,012	0,000	0,002	0,003	1,006	0,910	0,976	0,028
Shieh's $d$	0,006	0,000	0,001	0,002	1,006	0,910	0,976	0,028

Table SM2.2

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).*

Estimator ( $\hat{d}$ )	Absolute deviation between empirical and theoretical means $ E(\hat{d}) - \mu_{\delta} $				Ratio between empirical and theoretical variances $S^2_{\hat{d}}/\sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $d$	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's $d_1$	0,019	0,000	0,004	0,006	1,006	0,891	0,966	0,037
Glass's $d_2$	0,027	0,000	0,005	0,007	1,015	0,881	0,968	0,036
Cohen's $d^*$	0,010	0,000	0,003	0,002	1,007	0,902	0,965	0,034
Shieh's $d$	0,008	0,000	0,002	0,002	1,005	0,865	0,945	0,048

Table SM2.3

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).*

Estimator ( $\hat{\delta}$ )	Absolute deviation between empirical and theoretical means $ E(\hat{\delta}) - \mu_{\delta} $				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}} / \sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $d$	0,080	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's $d_1$	0,037	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's $d_2$	0,230	0,000	0,012	0,033	1,008	0,883	0,974	0,032
Cohen's $d^*$	0,036	0,000	0,003	0,006	1,007	0,874	0,975	0,033
Shieh's $d$	0,018	0,000	0,002	0,003	1,007	0,874	0,975	0,033

Table SM2.4

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for biased estimators, when population variances and sample sizes are unequal across groups (condition d).*

Estimator ( $\hat{\delta}$ )	Absolute deviation between empirical and theoretical means $ \mathbb{E}(\hat{\delta})-\mu_{\delta} $				Ratio between empirical and theoretical variances $S^2_{\hat{\delta}}/\sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $d$	0,252	0,000	0,015	0,034	5,624	0,208	1,638	1,357
Glass's $d_1$	0,026	0,000	0,005	0,006	1,009	0,881	0,972	0,033
Glass's $d_2$	0,219	0,000	0,012	0,031	1,011	0,872	0,973	0,036
Cohen's $d^*$	0,030	0,000	0,003	0,006	1,011	0,860	0,974	0,034
Shieh's $d$	0,009	0,000	0,001	0,002	1,011	0,867	0,970	0,036

Table SM2.5

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are equal across groups (condition a).*

Estimator ( $\hat{\delta}$ )	Absolute deviation between empirical and theoretical means $ \mathbb{E}(\hat{\delta}) - \mu_{\delta} $				Ratio between empirical and theoretical variances $S_{\hat{\delta}}^2 / \sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $g$	0,011	0,000	0,002	0,003	1,006	0,911	0,976	0,028
Glass's $g_1$	0,021	0,000	0,004	0,006	1,006	0,897	0,966	0,033
Glass's $g_2$	0,022	0,000	0,004	0,007	1,005	0,889	0,966	0,035
Cohen's $g^*$	0,015	0,000	0,003	0,004	1,006	0,908	0,975	0,029
Shieh's $g$	0,008	0,000	0,002	0,002	1,006	0,908	0,975	0,029

Table SM2.6

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are equal across groups and sample sizes are unequal (condition b).*

Estimator ( $\widehat{\delta}$ )	Absolute deviation between empirical and theoretical means $ \widehat{E(\delta)} - \mu_{\delta} $				Ratio between empirical and theoretical variances $S_{\widehat{\delta}}^2/\sigma_{\delta}^2$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $g$	0,005	0,000	0,001	0,001	1,017	0,951	0,985	0,017
Glass's $g_1$	0,018	0,000	0,004	0,005	1,006	0,891	0,966	0,037
Glass's $g_2$	0,026	0,000	0,004	0,006	1,015	0,881	0,968	0,036
Cohen's $g^*$	0,010	0,000	0,003	0,003	1,007	0,925	0,972	0,027
Shieh's $g$	0,007	0,000	0,002	0,002	1,007	0,900	0,959	0,037

Table SM2.7

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances are unequal across groups and sample sizes are equal (condition c).*

Estimator ( $\hat{\delta}$ )	Absolute deviation between empirical and theoretical means $ \mathbb{E}(\hat{\delta}) - \mu_{\delta} $				Ratio between empirical and theoretical variances $S_{\hat{\delta}}^2/\sigma_{\delta}$			
	Max	Min	Mean	Standard deviation	Max	Min	Mean	Standard deviation
Cohen's $g$	0,079	0,000	0,010	0,015	1,753	1,005	1,175	0,208
Glass's $g_1$	0,036	0,000	0,005	0,007	1,004	0,888	0,973	0,030
Glass's $g_2$	0,221	0,000	0,012	0,032	1,008	0,883	0,974	0,032
Cohen's $g^*$	0,034	0,000	0,003	0,006	1,008	0,890	0,978	0,029
Shieh's $g$	0,017	0,000	0,002	0,003	1,008	0,890	0,978	0,029

Table SM2.8

*Absolute deviation between empirical and theoretical means as well as ratio between empirical and theoretical variances for unbiased estimators, when population variances and sample sizes are unequal across groups (condition d).*

Estimator ( $\hat{\delta}$ )	Absolute deviation between empirical and theoretical means $ \mathbb{E}(\hat{\delta})-\mu_{\delta} $					Ratio between empirical and theoretical variances $S_{\hat{\delta}}^2/\sigma_{\delta}$				
	Max	Min	Mean	Standard deviation		Max	Min	Mean	Standard deviation	
Cohen's $g$	0,250	0,000	0,015	0,034		5,624	0,208	1,638	1,357	
Glass's $g_1$	0,025	0,000	0,004	0,006		1,009	0,881	0,972	0,033	
Glass's $g_2$	0,210	0,000	0,012	0,030		1,011	0,872	0,973	0,036	
Cohen's $g^*$	0,029	0,000	0,003	0,005		1,011	0,882	0,977	0,029	
Shieh's $g$	0,008	0,000	0,001	0,002		1,011	0,881	0,973	0,032	