as a consequence, the Student's *t*-value decreases, leading to fewer significant findings than expected with a specific alpha level. When the larger variance is associated with the *smaller* sample size, the Type 1 error rate is inflated (Nimon, 2012; Overall, Atlas, & Gibson, 1995). This inflation is caused by the under-evaluation of the error term, which increases Student's *t* value and thus leads to more significant results than are expected based on the alpha level.

As discussed earlier, Student's *t*-test is robust to unequal variances as long as the sample sizes of each group are similar (Nimon, 2012; Ruxton, 2006; Wallenstein, Zucker, & Fleiss, 1980), but, in practice, researchers often have different sample sizes in each of the independent groups (Ruxton, 2006). Unequal sample sizes are particularly common when examining measured variables, where it is not always possible to determine *a priori* how many of the collected subjects will fall in each category (e.g., sex, nationality, or marital status). However, even with complete randomized assignment to conditions, where the same number of subjects are assigned to each condition, unequal sample sizes can emerge when participants have to be removed from the data analysis due to being outliers because the experimental protocol was not followed when collecting the data (Shaw & Mitchell-Olds, 1993) or due to missing values (Wang et al., 2012).

Previous work by many researchers has shown that Student's *t*-test performs surprisingly poorly when variances are unequal and sample sizes are unequal (Glass, Peckham, & Sanders, 1972; Overall, Atlas, & Gibson, 1995; Zimmerman, 1996), especially with small sample sizes and low alpha levels (e.g., alpha = 1%; Zimmerman, 1996). The poor performance of Student's *t*-test when variances are unequal becomes visible when we look at the error rates of the test and the influence of both Type 1 errors and Type 2 errors. An increase in the Type 1 error rate leads to an inflation of the number of false positives in the literature, while an increase in the Type 2 error rate leads to a loss of statistical power (Banerjee et al., 2009).

To address these limitations of Student's *t*-test, Welch (1947) proposed a separate-variances *t*-test computed by dividing the mean difference between group $\bar{x}_1 - \bar{x}_2$ by an unpooled error term, where s_1^2 and s_2^2 are variance estimates from each independent group, and where n_1 and n_2 are the respective sample sizes for each independent group:[8]

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{3}$$

The degrees of freedom are computed as follows:

$$\mathrm{df} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \tag{4}$$

When both variances and sample sizes are the same in each independent group, the *t*-values, degrees of freedom, and the *p*-values in Student's *t*-test and Welch's *t*-test are the same (see **Table 1**). When the variance is the same in both independent groups but the sample sizes differ, the *t*-value remains identical, but the degrees of freedom differ (and, as a consequence, the *p*-value differs). Similarly, when the variances differ between independent groups but the sample sizes in each group are the same, the *t*-value is identical in both tests, but the degrees of freedom differ (and, thus, the *p*-value differs). The most important difference between Student's *t*-test and Welch's *t*-test, and indeed the main reason Welch's *t*-test was developed, is when both the variances and the sample sizes differ between groups, the *t*-value, degrees of freedom, and *p*-value all differ between Student's *t*-test and Welch's *t*-test. Note that, in practice, samples practically never show exactly the same pattern of variance as populations, especially with small sample sizes (Baguley, 2012; also see table A2 in the additional file).

Yuen's *t*-test, also called "20 percent trimmed means test", is an extension of Welch's *t*-test and is allegedly more robust in case of non-normal distributions (Wilcox & Keselman, 2003). Yuen's *t*-test consists of removing the lowest and highest 20 percent of the data and applying Welch's *t*-test on the remaining values. The procedure is explained and well-illustrated in a paper by Erceg-Hurn and Mirosevich (2008).

Simulations: Error Rates for Student's *t*-test versus Welch's *t*-test

When we are working with a balanced design, the statistical power (the probability of finding a significant effect, when there is a true effect in the population, or 1 minus the Type 2 error rate) is very similar for Student's *t*-test

	Equal variances	Unequal variances
Balanced design	$t_{Welch} = t_{Student}$ $df_{Welch} = df_{Student}$ $p_{Welch} = p_{Student}$	$t_{Welch} = t_{Student}$ $df_{Welch} \neq df_{Student}$ $p_{Welch} \neq p_{Student}$
Unbalanced design	$t_{Welch} = t_{Student}$ $df_{Welch} \neq df_{Student}$ $p_{Welch} \neq p_{Student}$	$t_{Welch} \neq t_{Student}$ $df_{Welch} \neq df_{Student}$ $p_{Welch} \neq p_{Student}$

Table 1: Comparison of *t*-value and Degrees of Freedom of Welch's and Student's *t*-test.