



Figure 10. The relationship between  $p$ -values from the TOST procedure and the SGPV for the same scenario as in Figure 9.

as the observed correlation nears -1 or 1. For example, with ten observations, an observed correlation of 0 has a symmetric 95% confidence interval ranging from -0.63 to 0.63, while an observed correlation of 0.7 has an asymmetric 95% confidence interval ranging from 0.13 to 0.92. Note that calculating confidence intervals for a correlation involves a Fisher's  $z$ -transformation, which transforms values such that they are approximately normally  $z$ -distributed, which allows one to compute symmetric confidence intervals. These confidence intervals are then retransformed into a correlation, where the confidence intervals are asymmetric if the correlation is not exactly zero.

The effect of asymmetric confidence intervals around correlations is most noticeable at smaller sample sizes. In Figure 11 we plot the  $p$ -values from equivalence tests and the SGPV (again plotted as 1-SGPV for ease of comparison) for correlations. The sample size is 30 pairs of observations, and the lower and upper equivalence bounds are set to -0.45 and 0.45, with an alpha of 0.05. As the observed correlation in the sample moves from -0.99 to 0 the  $p$ -value from the equivalence test becomes smaller, as does 1-SGPV. The pattern is quite similar to that in Figure 2. The  $p$ -value for the TOST procedure and 1-SGPV are still related as discussed above, with TOST  $p$ -values of 0.975 and 0.025 corresponding to a 1-

SGPV of 1 and 0, respectively. There are two important differences, however. First of all, the SGPV is no longer a straight line, but a curve, due to the asymmetry in the 95% CI. Second, and most importantly, the  $p$ -value for the equivalence test and the SGPV do no longer overlap at  $p = 0.5$ .

The reason that the equivalence test and SGPV no longer overlap is due to asymmetric confidence intervals. If the observed correlation falls exactly on the equivalence bound the  $p$ -value for the equivalence test is 0.5. In the equivalence test for correlations the  $p$ -value is computed based on a  $z$ -transformation which better controls error rates (Goertzen & Cribbie, 2010). This transformation is computed as follows, where  $r$  is the observed correlation and  $\rho$  is the theoretical correlation under the null:

$$z = \frac{\frac{\log(\frac{1+r}{1-r})}{2} - \frac{\log(\frac{1+\rho}{1-\rho})}{2}}{\sqrt{\frac{1}{n-3}}}$$

Because the  $z$ -distribution is symmetric, the probability of observing the observed or more extreme  $z$ -score, assuming the equivalence bound is the true effect size, is 50%. However, because the  $r$  distribution is not symmetric, this does not mean that there is always a 50% probability of observing a correlation smaller or larger