Exercise 2 (Some bivariate copulas)

The following exercise is an extension of Exercise 1 (introduction to copulas).

Let r_1 and r_2 denote random variables for the returns of two stocks having the following joint density:

$$\mathrm{f}(r_1, r_2) = \begin{cases} \alpha + \beta r_1 + \gamma r_2, & -1 \le r_1 \le 2,\ -2 \le r_2 \le 2.5 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha = 0.0642,\ \beta = 0.0049,\ \gamma = 0.0296$.

a) Obtain by Monte Carlo 100,000 simulated independent U(0,1) draws of (u, v). Given the following copulas, obtain as an output 100,000 dependent U(0,1) draws of (u, w). Since the first draw u is the same under the two cases, we next show how w is obtained according to different copulas:

↑θ → ↑ Dependencia

i) <u>Copula 1</u>: $w = u^{-\theta-1}\left[u^{-\theta} + v^{-\theta} - 1\right]^{-(\theta+1)/\theta}, \quad \theta > 0$ Clayton →

ii) <u>Copula 2</u>: w is from a Gaussian copula with correlation coefficient $\rho \in [-1, 1]$ Gaussian

Make a plot of points (u, v) from each copula. Consider the following parameter values:

Copula 1: $\theta = 2, 10$; Copula 2: $\rho = -0.7, 0.6$

b) Given each copula and the different parameter values, compute both the mean and standard deviation for the return portfolio A containing 25% of stock 1 and 75% of stock 2. Obtain also the simulated quantiles at 1%, 5%, 95% and 99%. Remember that the marginal distributions of the stock returns 1 and 2 were obtained in section a) from Exercise 1. Finally, obtain the correlation matrix corresponding to portfolio A under the four cases.

c) Consider the following portfolios:

P1: (50% stock 1, 50% stock 2) with copula 1 where $\theta = 2$.

P2: (20% stock 1, 80% stock 2) with copula 2 where $\rho = -0.7$.

P3: (70% stock 1, 30% stock 2) with copula 1 where $\theta = 10$.

P4: (15% stock 1, 85% stock 2) with copula 2 where $\rho = 0.6$.

<u>Note 1</u>: You do not have to generate new random numbers. They must be the ones obtained in section **a**).

<u>Note 2</u>: Use 'simuN.m' to obtain Gaussian copulas (i.e., copula 2).

Compute the correlation matrix of the four portfolios. Build the efficient frontier (allowing short selling) given the four portfolios. What is the composition of the portfolio with minimum global variance? What is the simulated VaR (quantile) of this portfolio at levels 1% and 5%.