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# Seminario Finanzas

## *Asset Allocation*

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*Alumno*

Manuel DE LA LLAVE\*

*Profesor*

Ángel LEÓN

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\*manudela@ucm.es

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Se pueden encontrar los scripts empleados [en este repositorio de GitHub](#) y también en el [anexo correspondiente](#) para su consulta y réplica. Para correr el Problem Set 1 es necesaria la función naiveMV.m y para el Problem Set 2 se necesita, además de la anterior función, las funciones clayton.m y simuN.m. Estas dos últimas han sido modificadas respecto a las originales para hacerlo con 2 variables en lugar de 3.

Todos los cálculos los he realizado con Matlab y se pueden comprobar en la sección correspondiente de cada script.

## 1 Mean Variance Investing

Let be the following sample expected return vector,  $\mu$ , and the sample covariance matrix,  $\Omega$ , obtained from 3 time series of yearly stock returns (denoted as 1, 2, and 3):

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.09 \\ 0.05 \end{pmatrix}; \quad \Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} = \begin{pmatrix} 0.7 & 0.1 & -0.06 \\ 0.1 & 0.5 & -0.1 \\ -0.06 & -0.1 & 0.1 \end{pmatrix}$$

answer the following questions:

- a) Compute the efficient frontier (EF) by allowing short-selling. Which stocks in the EF do increase when increasing  $\mu_p$ ? Make a graphic for each single stock weight in the EF by using a grid of values for  $\mu_p$  (just start with the expected return of gmv).

A continuación presento un gráfico donde están representadas las fronteras eficientes tanto con las ventas a corto permitidas (línea roja) como no permitidas (línea azul). Aunque en el gráfico no se distingan, la EF que permite short-selling está ligeramente por encima de la que no, algo que esperaríamos, pues las ventas a corto nos amplían la frontera de posibilidades de inversión y nos facilita la diversificación, reduciendo el riesgo por rentabilidad. Los tres puntos marcados son cada uno de los stocks que componen el portfolio eficiente.

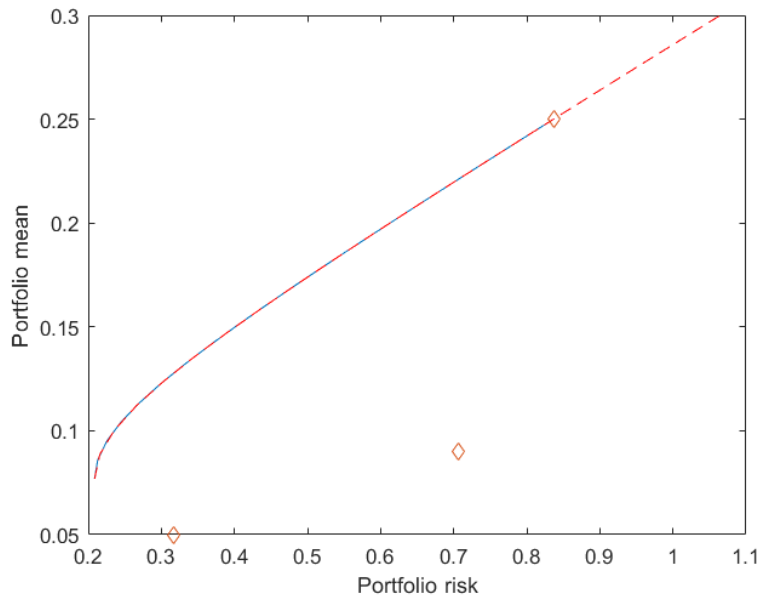


Figura 1: Frontera eficiente con y sin short-selling

En la siguiente imagen podemos observar que sólo el precio del primer activo es creciente con  $\mu_p$ :

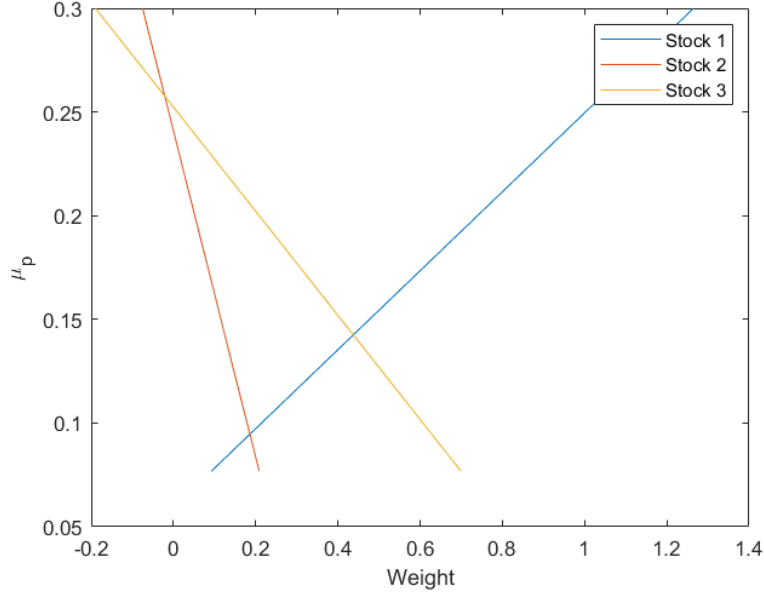


Figura 2: Ponderaciones de los activos según  $\mu_p$

- b) Consider an investor with a quadratic expected utility function. Assume that you invest all your wealth into three different portfolios such that each follows a certain target. One portfolio is for ‘retirement’ with  $b = 0.5$ , another is for ‘education’ of children with  $b = 0.3$  and finally, the last portfolio is for ‘bequest’ with  $b = 0.1$ . The total investor’s wealth is shared across the different portfolios in the following way: 60% in the portfolio for retirement, 25% in the portfolio for education, and finally the remaining 15% in the portfolio for bequest. Compute the optimal portfolio composition, mean and risk for each target portfolio and the mixed portfolio (i.e., the one containing the three target portfolios). Are the three target portfolios efficient? Does the ‘mixed portfolio’ belong to the PF? In case of being the ‘mixed portfolio’ in the PF, then try to obtain this portfolio as a combination of portfolios gmv and d.

Los cálculos se pueden encontrar en la sección *h* del script, donde  $Wstar$  son los pesos de cada portfolio óptimo según el grado de aversión siendo  $muP$  y  $sigP$  la media y desviación típica de cada uno mientras que  $aux, muMixed$  y  $sigMixed$  hacen referencia a la mezcla de portfolios según indica el enunciado.

	$b = 0.5$	$b = 0.3$	$b = 0.1$	Mixed
$S_1$	57.26%	89.27%	249.34%	94.07%
$S_2$	9.23%	1.50%	-37.20%	0.33%
$S_3$	33.51%	9.23%	-112.14%	5.59%
$\mu$	0.1682	0.2291	0.5338	0.2383
$\sigma$	0.4757	0.7425	2.1479	0.7836

Cuadro 1: Resultados

Podemos comprobar que cuanto más disminuye la aversión al riesgo, más se invierte en  $S_1$ , que recordemos que era el único activo creciente con  $\mu_p$ . Dado que los portfolios calculados son óptimos, también serán eficientes y como cualquier combinación lineal de portfolios eficientes es un portfolio eficiente, podemos decir el portfolio mixto también es eficiente, es decir, estos cuatro portfolios son eficientes. Gráficamente comprobamos que todos los

portfolios están sobre la EF:

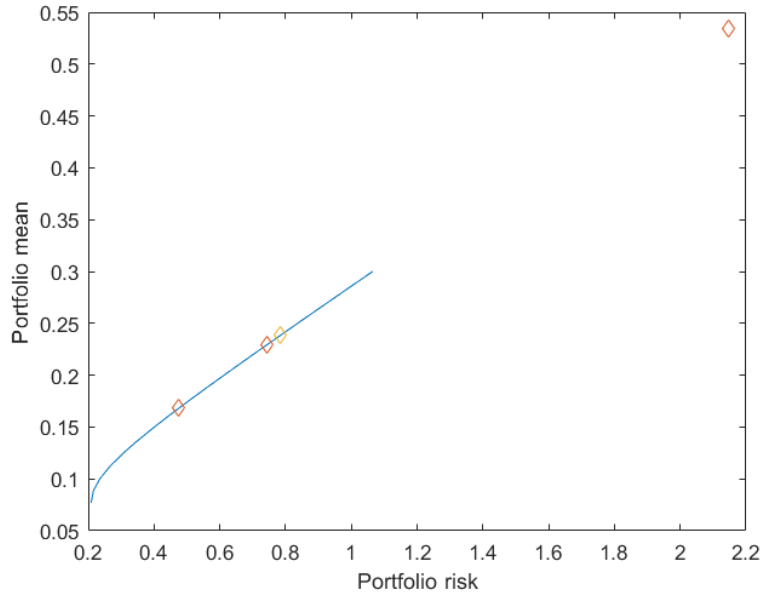


Figura 3: Frontera eficiente y portfolios

Para comprobar que nuestro portfolio mixto se puede expresar como combinación lineal del portfolio gmv y de máximo ratio de Sharpe, aplicamos la siguiente fórmula para que nos de cada peso:

$$w_{p,i} = w_{gmv,i} + \frac{A(C\mu_p - A)(w_{d,i} - w_{gmv,i})}{D}$$

Ahora si multiplicamos los pesos obtenidos por el vector  $\mu$  de rendimientos iniciales, obtendríamos que (en nuestro script)  $mu_{pi} = muMixed$  y  $W_{pi} = Wmixed$ , es decir, que hemos replicado el portfolio mixto como una combinación del portfolio de mínima varianza global y el de máximo ratio de Sharpe.

- c) Suppose that the return for the risk-free asset with return  $r=4\%$ . Obtain the tangency portfolio, denoted as e, in the PF (corresponding to the three risky assets allowing short-selling), its mean and risk. Finally, obtain and represent the capital allocation line (CAL). Does a portfolio containing 50% in the risk-free asset, 15% in stock 1, 20% in stock 2 and 15% in stock 3 belong to the CAL?

A continuación de los resultados y, más abajo, la CAL representada gráficamente.

	$S_1$	$S_2$	$S_3$	$\mu$	$\sigma$
Portfolio tangente (e)	37.63%	13.98%	45.39%	0.1309	0.3278

Cuadro 2: Portfolio tangente con un activo libre de riesgo

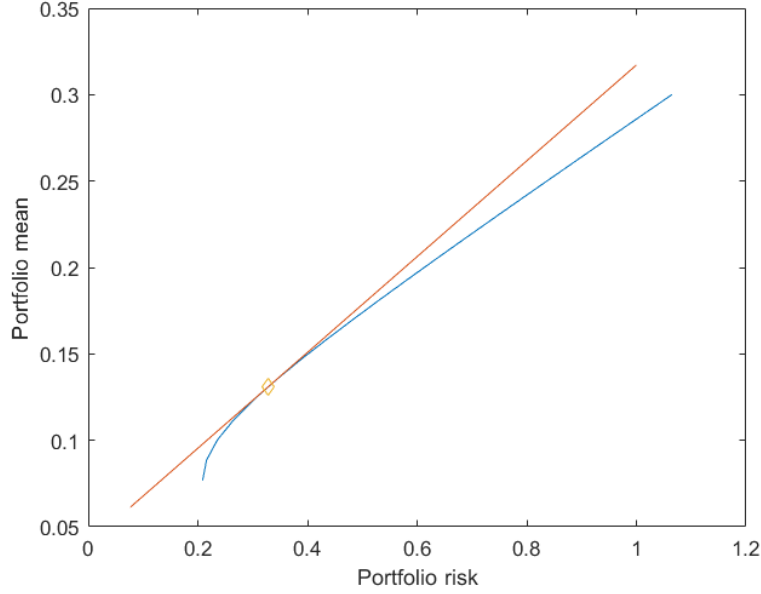


Figura 4: CAL y portfolio tangente

Para comprobar que dicho portfolio se encuentra en la CAL, primero hallamos el rendimiento de dicho portfolio ( $\mu_{\text{test}} = 0.083$ ) y calculamos alfa tal que  $\alpha = \frac{\mu_{\text{test}} - r_f}{\mu_e - r_f} = 0.4733$ . Multiplicando alfa por los pesos del portfolio tangente comprobamos que los pesos requeridos no coinciden con los del enunciado ( $S_1 = 17.81\%$ ,  $S_2 = 6.62\%$  y  $S_3 = 22.9\%$ ) y por tanto el portfolio propuesto no pertenece a la CAL.

- d) By using Matlab, try to compute the EF already obtained in section a but now only long positions in stocks are allowed.

Hecho en el apartado a).

## 2 Copulas<sup>1</sup>

Let  $r_1$  and  $r_2$  denote random variables for the returns of two stocks having the following joint density:

$$f(r_1, r_2) = \begin{cases} \alpha + \beta r_1 + \gamma r_2, & -1 \leq r_1 \leq 2, \quad -2 \leq r_2 \leq 2.5 \\ 0, & \text{otherwise} \end{cases}$$

where  $\alpha = 0.0642$ ,  $\beta = 0.0049$ ,  $\gamma = 0.0296$ .

- a) Obtain by Monte Carlo 100,000 simulated independent  $U(0, 1)$  draws of  $(u, v)$ . Given the following copulas, obtain as an output 100,000 dependent  $U(0, 1)$  draws of  $(u, w)$ . Since the first draw  $u$  is the same under the two cases, we next show how  $w$  is obtained according to different copulas:

i) Copula 1:  $w = u^{-\theta-1} [u^{-\theta} + v^{-\theta} - 1]^{-(\theta+1)/\theta}$ ,  $\theta > 0$

ii) Copula 2:  $w$  is from a Gaussian copula with correlation coefficient  $\rho \in [-1, 1]$

Make a plot of points  $(u, v)$  from each copula. Consider the following parameter values:

---

<sup>1</sup>Como trabajamos con variables aleatorias hemos establecido la semilla `rng(1585)` para su replicación.

Copula 1:  $\theta = 2, 10$ ; Copula 2:  $\rho = -0.7, 0.6$

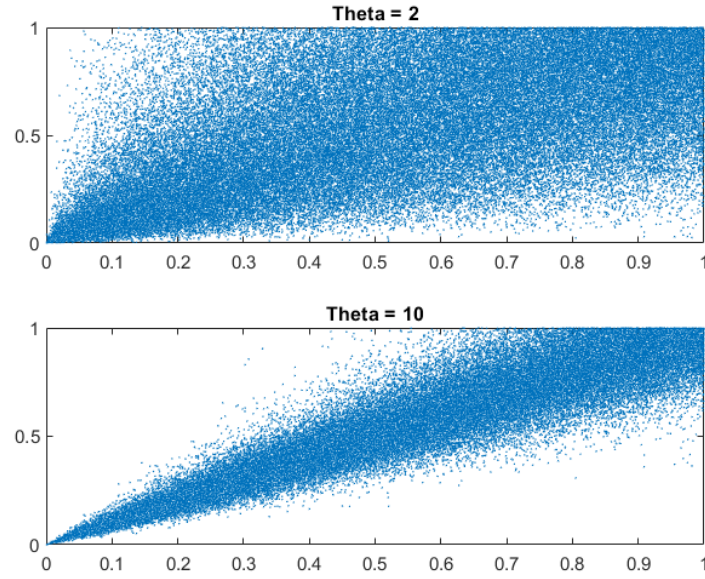


Figura 5: Cópula de Clayton

Vemos que cuanto mayor dependencia (mayor  $\theta$ ), menor dispersión encontramos.

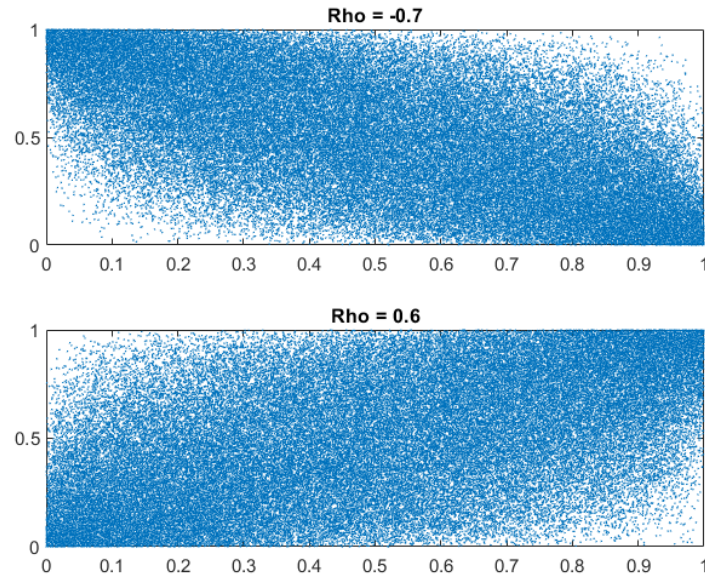


Figura 6: Cópula gaussiana

- b) Given each copula and the different parameter values, compute both the mean and standard deviation for the return portfolio A containing 25% of stock 1 and 75% of stock 2. Obtain also the simulated quantiles at 1%, 5%, 95% and 99%. Finally, obtain the correlation matrix corresponding to portfolio A under the four cases.

	$\mu$	$\sigma$	$q_{1\%}$	$q_{5\%}$	$q_{95\%}$	$q_{99\%}$
Clayton ( $\theta = 2$ )	0.8255	0.9939	-1.5162	-1.0445	2.1346	2.2755
Clayton ( $\theta = 10$ )	0.8264	1.0374	-1.5212	-1.0706	2.2168	2.3179
Gauss( $\rho = -0.7$ )	0.8266	0.7009	-0.8549	-0.5040	1.7148	1.8854
Gauss( $\rho = 0.6$ )	0.8258	0.9690	-1.4217	-0.9557	2.1734	2.3166

Cuadro 3: Performance del portfolio según la cópula empleada

Y la matriz de correlaciones:

$$\Gamma = \begin{pmatrix} 1.0000 & 0.9147 & 0.0431 & 0.9605 \\ 0.9147 & 1.0000 & -0.3178 & 0.8189 \\ 0.0431 & -0.3178 & 1.0000 & 0.1843 \\ 0.9605 & 0.8189 & 0.1843 & 1.0000 \end{pmatrix}$$

c) Consider the following portfolios:

- P1: (50% stock 1, 50% stock 2) with copula 1 where  $\theta = 2$ .
- P2: (20% stock 1, 80% stock 2) with copula 2 where  $\rho = -0.7$ .
- P3: (70% stock 1, 30% stock 2) with copula 1 where  $\theta = 10$ .
- P4: (15% stock 1, 85% stock 2) with copula 2 where  $\rho = 0.6$ .

Compute the correlation matrix of the four portfolios. Build the efficient frontier (allowing short selling) given the four portfolios. What is the composition of the portfolio with minimum global variance? What is the simulated VaR (quantile) of this portfolio at levels 1% and 5% .

La matriz de correlaciones es la siguiente:

$$\Gamma = \begin{pmatrix} 1.0000 & 0.9414 & -0.1889 & 0.8925 \\ 0.9414 & 1.0000 & -0.4762 & 0.7134 \\ -0.1889 & -0.4762 & 1.0000 & 0.1971 \\ 0.8925 & 0.7134 & 0.1971 & 1.0000 \end{pmatrix}$$

En esta matriz el orden del P2 y el P3 se ha invertido, es decir,  $a_{21} = 0.9414$  es la correlación entre P1 y P3 mientras que  $a_{31} = -0.1889$  es la correlación entre P1 y P2, ya que en Matlab he trabajado con ese orden de cópulas. Por otro lado, la representación de las fronteras eficientes, junto con la localización de cada portfolio estudiado, se muestra a continuación:



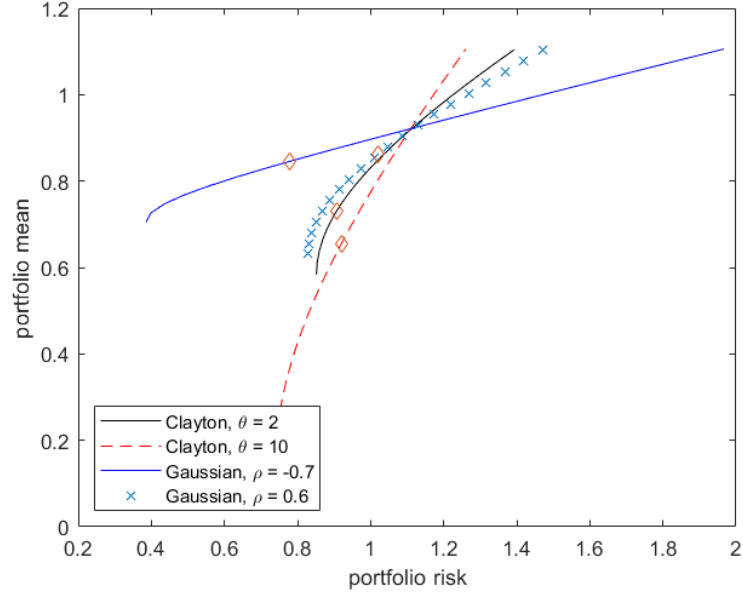


Figura 7: Fronteras eficientes

y los resultados de computar los diferentes portfolios de mínima varianza global:

	$\mu$	$\sigma$	$q_{1\%}$	$q_{5\%}$	$S_1$	$S_2$
P1 ( $\theta = 2$ )	0.5838	0.8518	-1.0216	-0.8267	89.09%	10.91%
P2 ( $\rho = -0.7$ )	0.7051	0.3857	-0.2013	0.0959	57.08%	42.92%
P3 ( $\theta = 10$ )	0.1333	0.7386	-1.0374	-0.7775	207.70%	-107.70%
P4 ( $\rho = 0.6$ )	0.6310	0.8287	-1.0175	-0.7352	77.45%	22.55%

Cuadro 4: Portfolio de mínima varianza global según la cópula empleada

Vemos que los valores obtenidos se corresponden con la gráfica representada.

# A Código de Matlab

## A.1 Problem Set 1

```
1 %% Cleaning
2 clear('all')
3 clc
4 close('all')
5
6 %% Initial parameters
7 muR = [0.25;
8        0.09;
9        0.05];
10 covR = [ 0.7, 0.1, -0.06;
11          0.1, 0.5, -0.1;
12          -0.06, -0.1, 0.1];
13 l = ones(length(muR),1);
14 MuMax = max(muR);
15 Nw = 20;
16 sigR = diag(diag(sqrt(covR)));
17 corrR = sigR \ covR / sigR; % corrR = corrcov(covR)
18
19 % %% PCTR
20 % eq_w = 1/length(muR); % Equally weighted
21 % alt_w = [0.2;
22 %          0.6;
23 %          0.2]; % alternative pportfolio
24 %
25 % eqw_sdev = sqrt(eq_w' * covR * eq_w);
26 % eqw_MCTR = (covR * eq_w) / eqw_sdev;
27 % eqw_PCTR = eq_w .* eqw_MCTR / eqw_sdev;
28 %
29 % alt_sdev = sqrt(alt_w' * covR * alt_w);
30 % alt_MCTR = (covR * alt_w) / alt_sdev;
31 % alt_PCTR = eq_w .* alt_MCTR / alt_sdev;
32
33 %% Section e) and m): EF with and without short-selling
34 invcov = inv(covR);
35
36 m1 = invcov*l; %vector of order nx1
37 m2 = invcov*muR; %vector of order nx1
38
39 A = l'*m2; %real number
40 B = muR'*m2; %real number (positive)
41 C = l'*m1; %real number (positive)
42 D = B*C-A^2; %real number (positive)
43
44 g = (1/D)*(B*m1-A*m2); %vector of order nx1
45 h = (1/D)*(C*m2-A*m1); %vector of order nx1
46
47 %Min variance EF portfolio (portfolio mvp)
48
49 Wmvp = (1/C)*m1; %weight of mvp
50 Mumvp = A/C; %mean of mvp
51 Sigmvp = sqrt(1/C); %sig of mvp
52
53 %Build EF under short position
54
```

```

55 q = 1.2;
56 MuMax1 = q*MuMax;
57 mup = linspace(Mumvp,MuMax1,Nw);
58
59 Wshort = zeros(Nw,length(muR));
60 muShort = zeros(Nw,1);
61 sigShort = zeros(Nw,1);
62
63 for i=1:Nw
64     w = g + h*mup(i); %column vector of order length(muR)
65     Wshort(i,:) = w';
66     muShort(i) = w'*muR;
67     sigShort(i) = sqrt(w'*covR*w);
68 end
69
70 [PRisk, PRoR, PWts] = NaiveMV(muR, covR, Nw);
71
72 WmvpLo = PWts(1,:)' ;      % Weight of mvp (Long)
73 MumvpLo = PRoR(1);        % Mean of mvp (Long)
74 SigmvpLo = PRisk(1);      % Sig of mvp (Long)
75
76 % Portfolio comparison: Long vs Short
77
78 compaP = [WmvpLo, Wmvp]
79 compaMV = [MumvpLo, Mumvp; SigmvpLo, SigmvpLo]
80
81 % Plotting
82
83 figure(1);
84 plot(PRisk,PRoR) % Long
85 hold on
86 plot(sigShort, muShort, 'r--') % Short selling is allowed
87 hold on
88 plot(diag(sigR),muR,'d')
89 xlabel('Portfolio risk')
90 ylabel('Portfolio mean')
91
92 figure(2);
93 plot(Wshort(:,1), muShort, Wshort(:,2), muShort, Wshort(:,3), muShort)
94 xlabel('Weight')
95 ylabel('\mu_p')
96 legend('Stock 1', 'Stock 2', 'Stock 3')
97
98 %% Section h): quadratic expected utility function
99
100 Wd = m2/A;
101 b = [0.5; 0.3; 0.1];
102 Wstar = zeros(3);
103 muP = zeros(1,3);
104 sigP = zeros(1,3);
105
106 for i = 1:3
107     Wstar(:,i) = Wmvp + A * (Wd - Wmvp) / b(i);
108     muP(i) = Wstar(:,i)'*muR;
109     sigP(i) = sqrt(Wstar(:,i)' * covR * Wstar(:,i));
110 end
111
112 Wmixed = [0.6, 0.25, 0.15];

```

```

113 muMixed = Wmixed * muP';
114 aux = sum(Wmixed .* Wstar, 2);
115 sigMixed = sqrt(aux' * covR * aux);
116
117 % Proof that all of them are efficient (in the PF):
118 figure(3);
119 plot(sigShort, muShort) % Short selling is allowed
120 hold on
121 plot(sigP,muP,'d', sigMixed,muMixed,'d')
122 xlabel('Portfolio risk')
123 ylabel('Portfolio mean')
124
125 for i = 1:3
126     Wpi(i) = Wmvp(i) + A * (C * muMixed - A) * (Wd(i) - Wmvp(i)) / D;
127 end
128
129 mupi = Wpi * muR; % mupi is equal to muMixed.
130
131 %% Section i): risk-free asset:
132
133 rf = 0.04;
134 We = invcov * (muR - rf) / (1'*invcov * (muR - rf));
135 muE = We' * muR;
136 sigE = sqrt(We' * covR * We);
137
138 sigCAL = linspace(Mumvp,1,Nw);
139 muCAL = rf + (muE - rf) * sigCAL/sigE ;
140
141 figure(4);
142 plot(sigShort, muShort) % Short selling is allowed
143 hold on
144 plot(sigCAL, muCAL)
145 hold on
146 plot(sigE,muE,'d')
147 xlabel('Portfolio risk')
148 ylabel('Portfolio mean')
149
150 WtestCAL = [0.15, 0.2, 0.15];
151 muTestCAL = WtestCAL * muR + (1 - sum(WtestCAL)) * rf;
152 alpha = (muTestCAL - rf)/(muE - rf);
153 test = alpha * We;

```

## A.2 Problem Set 2

```
1 %% Cleaning
2 clear('all')
3 clc
4 close('all')
5
6 %% Inputs
7 rng(1585); % For replication
8 N = 1e5; % Number of simulations
9 U = rand(N,2); % Indep. U(0,1) random matrix of order (N,2)
10
11 %% Section a)
12 % Clayton
13 theta1 = 2; % Small dependence relationship
14 theta2 = 10; % Greater dependence relationship
15
16 w1_clay = clayton(theta1,U);
17 w2_clay = clayton(theta2,U);
18
19 % Gaussian
20 rho1 = -0.7;
21 corr1 = [1, rho1;
22          rho1, 1];
23 rho2 = 0.6;
24 corr2 = [1, rho2;
25          rho2, 1];
26 w1_gauss = simuN(corr1,U);
27 w2_gauss = simuN(corr2,U);
28
29 % Plotting
30
31 figure(1);
32 subplot(2,1,1)
33 plot(w1_clay(:,1), w1_clay(:,2), 'o','MarkerSize',0.5)
34 title('Theta = 2')
35
36 subplot(2,1,2)
37 plot(w2_clay(:,1), w2_clay(:,2), 'o','MarkerSize',0.5)
38 title('Theta = 10')
39
40 figure(2);
41 subplot(2,1,1)
42 plot(w1_gauss(:,1), w1_gauss(:,2), 'o','MarkerSize',0.5)
43 title('Rho = -0.7')
44
45 subplot(2,1,2)
46 plot(w2_gauss(:,1), w2_gauss(:,2), 'o','MarkerSize',0.5)
47 title('Rho = 0.6')
48
49 %% Section b)
50 %Parameters of the bivariate pdf given by (I)
51 %f(r1,r2) = alfa + beta*r1 + gamma*r2
52
53 alfa = 0.0642;
54 beta = 0.0049;
55 gamma = 0.0296;
56
```

```

57 %Conditional sampling
58 %Conditional distribution from bivar. distrib. = F(r1|r2)
59
60 % Marginal cumulative distribution of r2: G(r2)
61 % G(r2) = G0 + G1*r2 + G2*r2^2
62
63 G0 = 0.2222;
64 G1 = 0.2;
65 G2 = 0.0444;
66
67 % Marginal pdf of r2 = g(r2)
68 % g(r2) = g00 + g11*r2
69 % r2 belongs to [-2,2.5]
70
71 g00 = 0.2;
72 g11 = 0.0889;
73
74 r2 = zeros(N,4); % draws from r2
75 r1 = zeros(N,4); % draws from r1
76
77 d = alfa -0.5*beta;
78
79 copu(:, :, 1) = w1_clay;
80 copu(:, :, 2) = w2_clay;
81 copu(:, :, 3) = w1_gauss;
82 copu(:, :, 4) = w2_gauss;
83
84 for i=1:N
85     for j = 1:4
86         G0_aux = G0 - copu(i,2,j);
87         A = [G2 G1 G0_aux];
88         R = roots(A);
89
90         if R(1) < -2
91             r2(i,j) = R(2);
92         elseif R(1) > 2.5
93             r2(i,j) = R(2);
94         else
95             r2(i,j) = R(1);
96         end
97     end
98 end
99
100 % r1 by conditional sampling
101 % r1 belongs to [-1,2]
102
103
104 for i=1:N
105     for j = 1:4
106         c1 = 1/(g00 + g11*r2(i,j));
107         c2 = d + gamma*r2(i,j);
108         c3 = alfa + gamma*r2(i,j);
109         c4 = 0.5*beta;
110         c5 = c2 - U(i,1)/c1;
111         B = [c4 c3 c5];
112         R = roots(B);
113         if R(1) < -1
114             r1(i,j) = R(2);

```

```

115         elseif R(1) > 2
116             r1(i,j) = R(2);
117         else
118             r1(i,j) = R(1);
119         end
120     end
121 end
122
123 wa = 0.25; % Weight stock 1
124
125 RpA = wa*r1 + (1 - wa)*r2;
126 %correA = corrcoef(r1, r2)
127 correA = corrcoef(RpA)
128 mRpMCA = mean(RpA) %mean
129 stdRpMCA = std(RpA) %std
130
131 Var1A = quantile(RpA,0.01) % quantile(1%)
132 Var5A = quantile(RpA,0.05) % quantile(5%)
133 Var95A = quantile(RpA,0.95) % quantile(95%)
134 Var99A = quantile(RpA,0.99) % quantile(99%)
135
136 %% Section c)
137 wpx = [0.5, 0.7, 0.2, 0.15]; % Weights for each portfolio
138 Rpx = zeros(N, length(wpx));
139 for i = 1:length(wpx)
140     Rpx(:,i) = wpx(i) * r1(:,i) + (1 - wpx(i)) * r2(:,i);
141 end
142
143 correx = corrcoef(Rpx)
144
145 % Build Efficient Frontiers (EFs) (short positions allowed)
146
147 Nw = 20;
148
149 muR = zeros(2, length(Rpx(1,:)));
150 covR = zeros(2,2,length(Rpx(1,:)));
151 invcov = zeros(2,2,length(Rpx(1,:)));
152 l = ones(2,1);
153 m1 = zeros(2, length(Rpx(1,:)));
154 m2 = zeros(2, length(Rpx(1,:)));
155 A = zeros(1, length(Rpx(1,:)));
156 B = zeros(1, length(Rpx(1,:)));
157 C = zeros(1, length(Rpx(1,:)));
158 D = zeros(1, length(Rpx(1,:)));
159 g = zeros(2, length(Rpx(1,:)));
160 h = zeros(2, length(Rpx(1,:)));
161 Wmvp = zeros(2, length(Rpx(1,:)));
162 Mumvp = zeros(1, length(Rpx(1,:)));
163 Sigmvp = zeros(1, length(Rpx(1,:)));
164 Wshort = zeros(Nw,2,length(Rpx(1,:)));
165 muShort = zeros(Nw,length(Rpx(1,:)));
166 sigShort = zeros(Nw,length(Rpx(1,:)));
167
168 for i = 1:length(Rpx(1,:))
169     muR(:,i) = [mean(r1(:,i)); mean(r2(:,i))];
170     covR(:, :, i) = cov(r1(:,i), r2(:,i));
171     invcov(:, :, i) = inv(covR(:, :, i));
172     m1(:,i) = invcov(:, :, i)*l; %vector of order nx1

```

```

173     m2(:,i) = invcov(:, :, i)*muR(:,i); %vector of order nx1
174
175     A(i) = 1'*m2(:,i); %real number
176     B(i) = muR(:,i)'*m2(:,i); %real number (positive)
177     C(i) = 1'*m1(:,i); %real number (positive)
178     D(i) = B(i)*C(i)-A(i)^2; %real number (positive)
179
180     g(:,i) = (1/D(i))*(B(i)*m1(:,i)-A(i)*m2(:,i)); %vector of order nx1
181     h(:,i) = (1/D(i))*(C(i)*m2(:,i)-A(i)*m1(:,i)); %vector of order nx1
182
183     %Min variance EF portfolio (portfolio mvp)
184
185     Wmvp(:,i) = (1/C(i))*m1(:,i); %weight of mvp
186     Mumvp(i) = A(i)/C(i); %mean of mvp
187     Sigmvp(i) = sqrt(1/C(i)); %sig of mvp
188
189     %Build EF under short position
190
191     q = 1.2;
192     MuMax1(i) = q*max(muR(:,i));
193     mup(i,:) = linspace(Mumvp(i), MuMax1(i), Nw);
194
195     for j=1:Nw
196         w = g(:,i) + h(:,i)*mup(i,j); %column vector of order length(muR)
197         Wshort(j,:,i) = w';
198         muShort(j,i) = w'*muR(:,i);
199         sigShort(j,i) = sqrt(w'*covR(:, :, i)*w);
200     end
201 end
202
203 %Plotting
204
205 figure(3);
206 plot(sigShort(:,1), muShort(:,1),'k') %Clayton copula
207 %title('Clayton, \theta = 2')
208 hold on
209 plot(sigShort(:,2), muShort(:,2),'r--') %Clayton copula
210 %title('Clayton, \theta = 10')
211 hold on
212 plot(sigShort(:,3), muShort(:,3),'b') %Gaussian copula
213 %title ('Gaussian, \rho = -0.7')
214 hold on
215 plot(sigShort(:,4), muShort(:,4),'x') %Gaussian copula
216 %title ('Gaussian, \rho = 0.6')
217 hold on
218 plot(std(Rpx),mean(Rpx),'d')
219 legend('Clayton, \theta = 2', 'Clayton, \theta = 10', ...
220 'Gaussian, \rho = -0.7', 'Gaussian, \rho = 0.6','Location','southwest')
221 xlabel('portfolio risk')
222 ylabel('portfolio mean')
223
224 newR = zeros(N,4);
225 for i = 1:length(wpx)
226     newR(:,i) = Wmvp(1,i) * r1(:,i) + Wmvp(2,i) * r2(:,i);
227 end
228
229 VaR1 = quantile(newR,0.01) % quantile(1%)
230 VaR5 = quantile(newR,0.05) % quantile(5%)

```