1) Sea la función:

$$Z(s) = \frac{(s^2+3)(s^2+1)}{s(s^2+2)}$$

Se pide hallar la topología circuital y los valores de los componentes para:

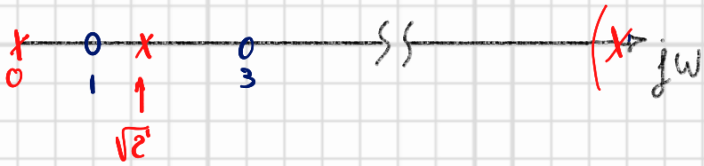
a) Síntesis de $Z(s)$ mediante el método de Foster en su versión "paralelo" o "derivación".

b) Idem **a)** mediante Cauer 1 y 2.

Foster

$$Z(s) = \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$$

$F(j\omega)$



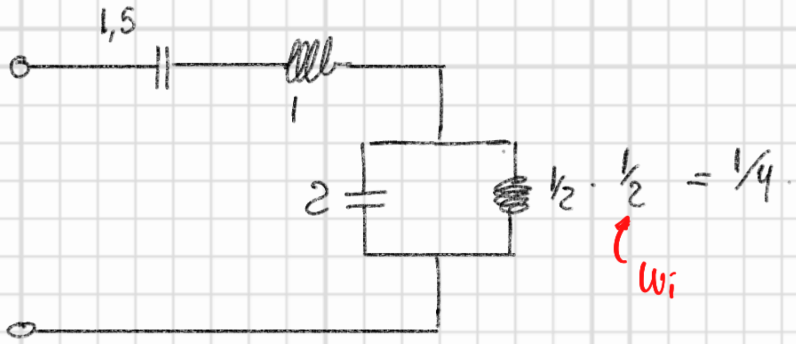
$$F(s) = \frac{k_0}{s} + k_\infty s + \frac{2k_i s}{s^2 + \omega_i^2}$$

$$k_0 = \lim_{s \to 0} s F(s) = \frac{\cancel{s}(s^2+3)(s^2+1)}{\cancel{s}(s^2+2)} = \frac{3}{2} = \boxed{3/2 = k_0}$$

$$k_\infty = \lim_{s \to \infty} \frac{F(s)}{s} = \frac{(s^2+3)(s^2+1)}{s^2(s^2+2)} = \frac{s^4+4s^2+3}{s^4+2s^2} = \boxed{1 = k_\infty}$$

$$2k_i = \lim_{s^2 \to -2} \frac{\cancel{s^2+2}}{s} \frac{(s^2+3)(s^2+1)}{s\,\cancel{(s^2+2)}} = \frac{s^4+4s^2+3}{s^2} = \boxed{1/2 = 2k_i}$$

Si $F(s) = Z(s)$



Foster en paralelo

Para poder realizar esto es necesario tener una Y, por lo tanto invertimos Z.

$$Z(\$) = \frac{\$^4 + 4\$^2 + 3}{\$^3 + 2\$} \Rightarrow Y(\$) = \frac{\$^3 + 2\$}{\$^4 + 4\$^2 + 3} = \frac{\$(\$^2+2)}{(\$^2+3)(\$^2+1)} = \frac{2K_3\,\$}{\$^2+3} + \frac{2K_1\$}{\$^2+1}$$

$$2K_3 = \lim_{\$^2 \to -3} \frac{(\$^2+3)}{\$} \cdot Y(\$) = \frac{\$^2+2}{\$^2+1} = \frac{-1}{-2} = \boxed{\frac{1}{2} = 2K_3}$$

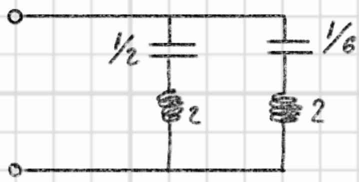
$$2K_1 = \lim_{\$^2 \to -1} \frac{(\$^2+1)}{\$} \cdot Y(\$) = \frac{\$^2+2}{\$^2+3} = \boxed{\frac{1}{2} = 2K_1}$$

$$\left.\frac{2K_i\,\$}{\$^2 + \omega_i^2}\right|_{\omega_i^2=3} = \frac{1}{2}\,\frac{\$}{\$^2+3} = \frac{1}{2\$ + \frac{6}{\$}}$$

L C

$$\frac{2K_i\$}{\$^2+\omega_i^2} = \frac{1}{2}\,\frac{\$}{\$^2+1} = \frac{1}{2\$ + \frac{2}{\$}}$$

L C



Cauer

Retiro polos en infinito

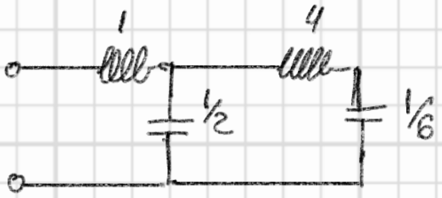
$$Z(s) = \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$$

$$\begin{array}{l|l} s^4 + 4s^2 + 3 & s^3 + 2s \\ -\,s^4 + 2s^2 & \boxed{s} \;\; Z_1 \\ \hline \quad +2s^2 + 3 & \end{array}$$

$$\begin{array}{l|l} s^3 + 2s & 2s^2 + 3 \\ s^3 + \frac{3}{2}s & \boxed{s/2} \;\; Y_3 \\ \hline \quad \frac{1}{2}s & \end{array}$$

$$\begin{array}{l|l} 2s^2 + 3 & \frac{1}{2}s \\ 2s^2 & \boxed{4s} \;\; Z_5 \\ \hline \quad 3 & \end{array}$$

$$\begin{array}{l|l} \frac{1}{2}s & 3 \\ \frac{1}{2}s & \boxed{\frac{s}{6}} \;\; Z_7 \\ \hline \quad 0 & \end{array}$$



Retiro polos en cero

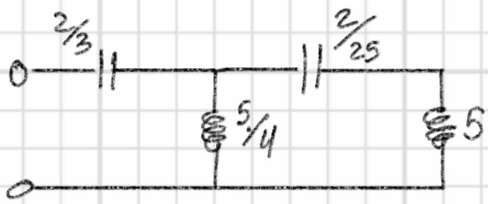
$$Z = \frac{P}{Q} = C + \frac{R}{Q} = C + \frac{1}{\frac{Q}{R}}$$

$$\begin{array}{l|l} 3 + 4s^2 + s^4 & 2s + s^3 \\ 3 + \frac{3}{2}s^2 & \boxed{\frac{3}{2s}} \;\; Z_1 \\ \hline 0 + \frac{5}{2}s^2 + s^4 & \end{array}$$

$$\begin{array}{l|l} 2s + s^3 & \frac{5}{2}s^2 + s^4 \\ 2s + \frac{4}{5}s^3 & \boxed{\frac{4}{5s}} \;\; Y_3 \\ \hline 0 \quad \frac{1}{5}s^3 & \end{array}$$

$$\begin{array}{l|l} \frac{5}{2}s^2 + s^4 & \frac{1}{5}s^3 \\ \frac{5}{2}s^2 & \boxed{\frac{25}{2s}} \;\; Z_5 \\ \hline 0 \quad s^4 & \end{array}$$

$$\begin{array}{l|l} \frac{1}{5}s^3 & s^4 \\ \frac{1}{5}s^3 & \boxed{\frac{1}{5s}} \;\; Y_7 \\ \hline \quad 0 & \end{array}$$

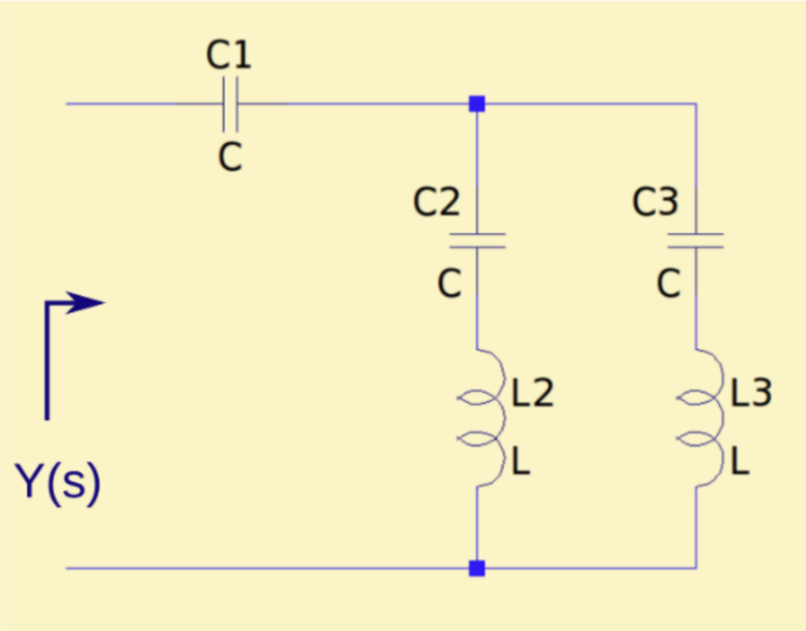


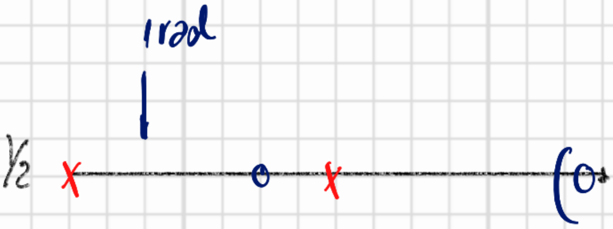
2) Sea

$$Y(s) = \frac{3s(s^2 + 7/3)}{(s^2+2)(s^2+5)}$$

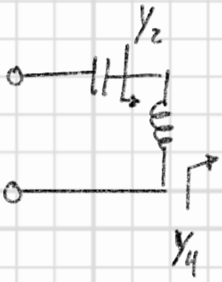
Obtenga los valores de los componentes de la siguiente red sabiendo que L2 y C2 resuenan a 1 r/s.

L_2 y C_2 resuenen a $1 \frac{rad}{seg}$





Retiro un poco del polo en cero para mover el cero a un rad.



No se puede ☹

Rompemos la estructura de la red.

Sacamos un poco de el polo en cero para mover el cero

$$Z(s) = \frac{1}{Y(s)} = \frac{(s^2+2)(s^2+5)}{3s\,(s^2+7/3)} \qquad Z_2(s) = Z(s) - \frac{k_1'}{s} \Rightarrow Z_2(s)\Big|_{s=1} = 0$$

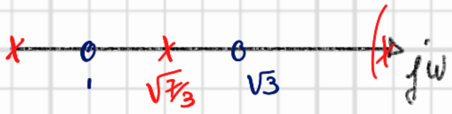
$$k_0' = Z(s)\cdot s\Big|_{s^2=-1}^{=0} = \frac{(s^2+2)(s^2+5)}{3\,(s^2+7/3)} = \frac{1\cdot 4}{4} = 1$$

$$\boxed{k_0' = 1}$$

$$Z_2(s) = Z(s) - 1/s$$

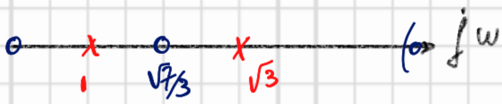
$$Z_2(s) = \frac{s^4 + \overset{4}{\cancel{7}}s^2 + \overset{3}{\cancel{10}} - \cancel{3s^2} - \cancel{7}}{3s\,(s^2+7/3)}$$

$$\boxed{Z_2(s) = \frac{s^4+4s^2+3}{3s\,(s^2+7/3)}}$$



Pasamos a Y para fijar el polo/cero en 1

$$Y_2(s) = \frac{1}{Z_2(s)} = \frac{3s\,(s^2+7/3)}{(s^2+1)(s^2+3)}$$



$$Y_3(s) = Y_2(s) - \frac{2k_1 s}{s^2+1} \Rightarrow 2k_1 = \lim_{s^2\to -1} \frac{Y_2\cdot(s^2+1)}{s} = \boxed{2 = 2k_1}$$

$$Y_3(s) = Y_2(s) - \frac{2s}{s^2+1} = \frac{3s\,(s^2+7/3) - 2s(s^2+3)}{(s^2+1)(s^2+3)} = \boxed{\frac{s^3+s}{(s^2+1)(s^2+3)} = Y_3(s)} = \frac{s}{s^2+3}$$

$$Y(s) = \frac{k_0'}{s} + \frac{1}{\frac{2k_1 s}{s^2+1} + Y_3(s)} = \frac{1}{s} + \frac{1}{\frac{2s}{s^2+1} + \frac{s}{s^2+3}}$$

$$Y(s) = \underbrace{\frac{1}{s}}_{C} + \frac{1}{\frac{1}{\underset{L}{\frac{s}{2}} + \underset{C}{\frac{1}{2s}}} + \frac{1}{\underset{L}{s} + \underset{C}{\frac{3}{s}}}}$$