Tri-Layer Setup

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1 Domain

The domain is a cartesian slab. Vertical direction is indicated by the z axis. The top of the domain is located at z_{min} while the bottom of the domain is located at z_{max}. Increasing z means going further down in the domain. Hence gravity will be positively signed. The vertical decomposition of the domain consists in three zones :

1. a stable zone extending in $z \in [z_0 = z_{min}, z_1[$ of size $\Delta z_0 = z_1 - z_{min}$,
2. a convective zone in $z \in [z_1, z_2[$ of size $\Delta z_1 = z_2 - z_1$,
3. a final stable zone spanning $z \in [z_2, z_{max}]$ of size $\Delta z_2 = z_{max} - z_2$

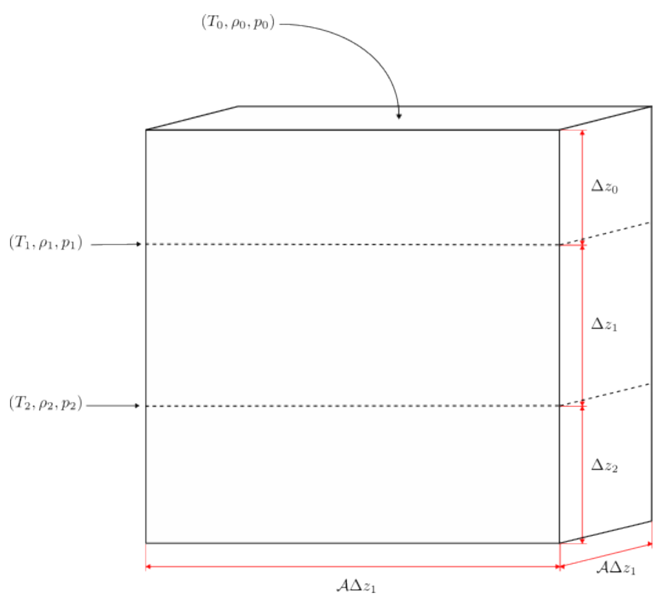

Figure 1: Description of the domain and the associated initial quantities

In the following each quantity subscripted by a number refers to the value of the quantity at the top of the corresponding layer. For instance, ρ_1 indicates the value of the density at the top of the convective zone. The localization of the quantities is described on Figure 1.

Along the horizontal axis (x, and y), the domain is periodic and spanning a certain surface which is proportional to the aspect ratio $\mathcal{A}$ between the horizontal span and the vertical span of the convective layer. Hence, if we set $x_{min} = y_{min} = 0$, we have :

$$x_{max} = y_{max} = \mathcal{A}\Delta z_1 \tag{1}$$

Free parameters :

- The size of each zone Δz_i
- The aspect ratio of the box $\mathcal{A}$

2 Hydrodynamic State

Each zone of the domain is initialized as a polytropic model. For a position in a given zone i : $z \in [z_i, z_{i+1}[$ we have the following initial conditions on the temperature T, the density ρ and the pressure P:

$$T(z) = T_i + \theta_i(z - z_i) \tag{2a}$$

$$\rho(z) = \rho_i \left(\frac{T(z)}{T_i}\right)^{m_i} \tag{2b}$$

$$P(z) = P_i \left(\frac{T(z)}{T_i}\right)^{m_i+1} \tag{2c}$$

Where θ_i is the temperature gradient in the zone and m_i the polytropic index associated with the layer. Finally, T_i, ρ_i and P_i are respectively the temperature, density and pressure at the top of the layer.

Given the value of the temperature at the top of the domain T_1 and the density at the top of the domain ρ_1 we can calculate the values of the T_i and ρ_i for $i = 1, 2$:

$$T_1 = T_0 + \theta_0 \Delta z_0 \qquad \rho_1 = \rho_0 \left(\frac{T_1}{T_0}\right)^{m_1} \tag{3}$$

$$T_2 = T_1 + \theta_1 \Delta z_1 \qquad \rho_2 = \rho_1 \left(\frac{T_2}{T_1}\right)^{m_2} \tag{4}$$

Pressure is recovered using the ideal gas equation of state with a gas coefficient $\mathcal{R} = 1$, having thus $P = \rho T$.

Gravity is kept constant in the domain. It is directed strictly towards positive z and is defined as

$$g = \theta_i(m_i + 1)\ \forall i = 0, 1, 2 \tag{5}$$

This puts a strong constraint on the values of each couple (θ_i, m_i) since as soon as g is determined by one of these couple, the values of θ_i will be automatically inferred from the values of the polytropic indices m_i. We decide to give the values of m_i to the user, and any of the θ_i. As soon as one value of θ is picked, all the other are automatically fixed.

Free parameters :

- The state at the top of the box ρ_0, T_0
- The polytropic indices m_i
- One of the temperature gradients θ_i

3 Diffusive effects

To this hydrodynamic setup should be added thermal diffusion and viscosity. The user provides the thermal dissipation coefficient C_k. This coefficient, alongside the specific heat ratio γ allows the calculation of the thermal conductivity K :

$$K = \frac{\gamma}{\gamma - 1} C_k \tag{6}$$

This thermal conductivity is then "specialized" for each zone depending on their own polytropic index : K_0, K_1, K_2. By default, we consider that $K_1 = K$. By following the litterature, we have the relations :

$$\frac{K_0}{K_1} = \frac{m_0 + 1}{m_1 + 1} \qquad\qquad \frac{K_2}{K_1} = \frac{m_2 + 1}{m_1 + 1}$$

Hence we get :

$$K_0 = K_1 \left(\frac{m_0 + 1}{m_1 + 1} \right) \tag{7a}$$

$$K_2 = K_1 \left(\frac{m_2 + 1}{m_1 + 1} \right) \tag{7b}$$

The kinematic viscosity coefficient μ is constant throughout the box and defined from the Prandtl number σ. In the normalized system, we have the following relation :

$$\mu = \sigma C_k \tag{8}$$

Free parameters :

- C_k the thermal dissipation coefficient
- σ the Prandtl number

4 Free parameters and calculations

If we sum the free parameters to define in the run we have the following set :

- The size of each zone Δz_0, Δz_1, Δz_2
- The aspect ratio of the box $\mathcal{A}$
- The state at the top of the box ρ_0, T_0
- The polytropic indices m_i
- One of the temperature gradients θ_i
- C_k the thermal dissipation coefficient
- σ the Prandtl number

Most of these parameters are left to the user. However defining T_0 and rho_0 for a given run might prove difficult. Since these points correspond to an arbitrary "top" of a solar atmosphere. ρ_0 is the density scaling parameter, and thus should be set depending on the physics/units of the run. Conveniently it can be left to 1, since it does not particularly affect the simulation.

T_0 is more difficult. One way to define it is to use the density ratio between the top and the bottom of the convection zone as a proxy. This ratio is noted $\chi_\rho = \rho_2/\rho_1$. We can find an explicit expression for χ_ρ by injecting (3) and (4) and simplifying. We get the following expression :

$$T_0 = \frac{\theta_1 \Delta z_1}{\chi_\rho^{1/m_1}} - \theta_0 \Delta z_0 \tag{9}$$