

# Efficient Deep Learning for Massive MIMO Channel State Estimation



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## Background

Role of CSI in MIMO

## CSI Estimation

Convolutional Neural Networks

## Spherical Normalization

Spherical Normalization

## MarkovNet

Differential Encoding

## SphNet-Quant

Results

# Background

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Massive MIMO is a key enabling technology for future wireless communications networks.

- ▶ 5G, Ultra-Dense Networks, IoT

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S. Marek, "Sprint Spent \$1B on Massive MIMO for Its 5G Network in Q2," *SDxCentral*, <https://www.sdxcentral.com/articles/news/sprint-spent-1b-on-massive-mimo-for-its-5g-network-in-q2/2018/06/>. Accessed: Feb 22, 2020.

Massive MIMO is a key enabling technology for future wireless communications networks.

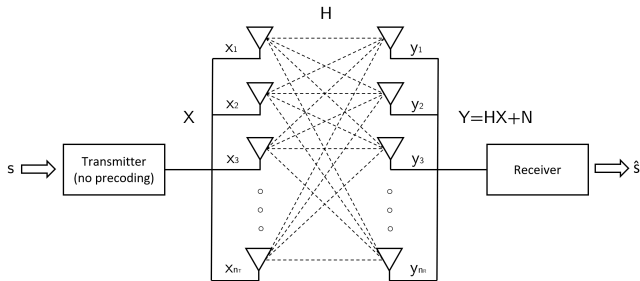
- 5G, Ultra-Dense Networks, IoT

The efficacy of MIMO depends on accurate *Channel State Information (CSI)*.

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S. Marek, "Sprint Spent \$1B on Massive MIMO for Its 5G Network in Q2," *SDxCentral*, <https://www.sdxcentral.com/articles/news/sprint-spent-1b-on-massive-mimo-for-its-5g-network-in-q2/2018/06/>. Accessed: Feb 22, 2020.

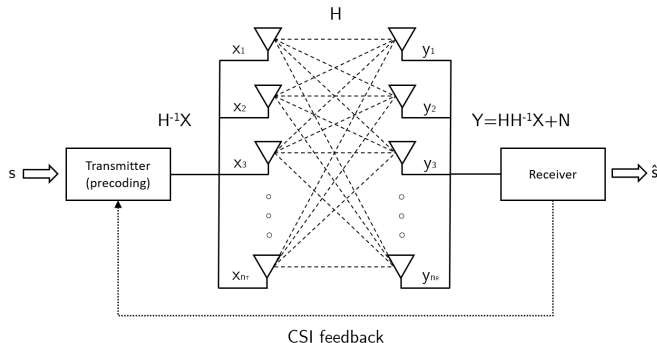
Massive MIMO uses numerous antennas to endow transceivers with spatial diversity.



The fading coefficients between each set of Tx/Rx antennas constitute **Channel State Information (CSI)**,  $\mathbf{H}$ . For  $n_T$ ,  $n_R$  antennas,

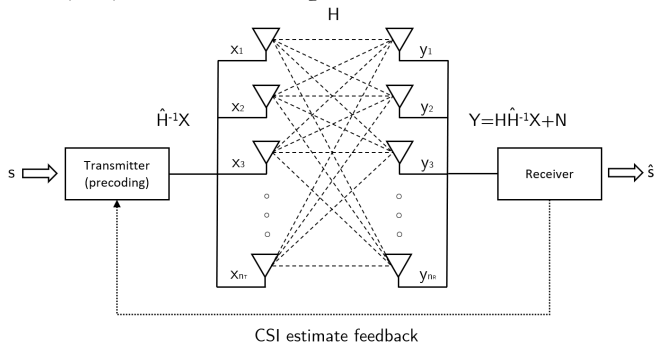
$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,n_T} \\ h_{2,1} & h_{2,2} & \dots & h_{2,n_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n_R,1} & h_{n_R,2} & \dots & h_{n_R,n_T} \end{bmatrix}$$

**Perfect CSI** (i.e., exact knowledge of the channel,  $\mathbf{H}$ ) allows us to maximize the power of the received symbol by precoding.



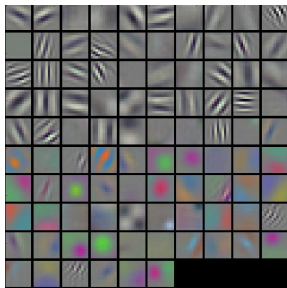


However, transmitting  $\mathbf{H}$  is costly. Instead, generate **CSI Estimates**,  $\hat{\mathbf{H}}$ , based on **compressed feedback**.

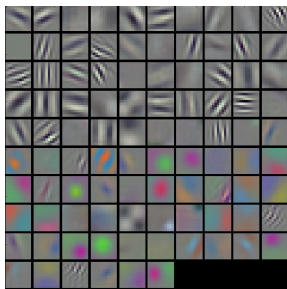


**Goal:** Find low-dimensional representation, feed back to transmitter for recovery of  $\hat{\mathbf{H}}$  which is an accurate approximation of  $\mathbf{H}$  in MSE sense.

- ▶ CNNs = state-of-the art performance in image processing applications
- ▶ Capable of extracting features from 2D, grid-like data

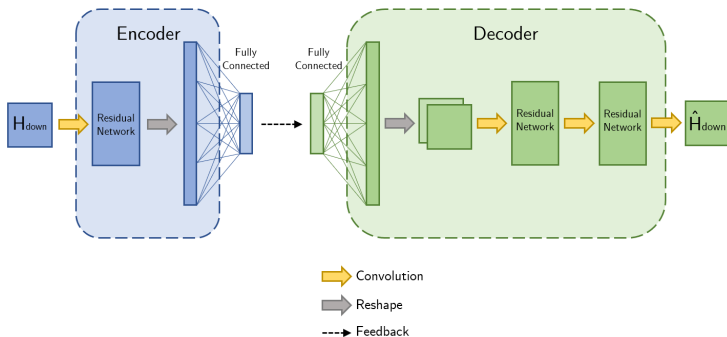


- ▶ CNNs = state-of-the art performance in image processing applications
- ▶ Capable of extracting features from 2D, grid-like data



- ▶ **Recently, CNNs applied to CSI estimation**

- CNN-based autoencoder for learned CSI compression and feedback [1]



# Spherical Normalization

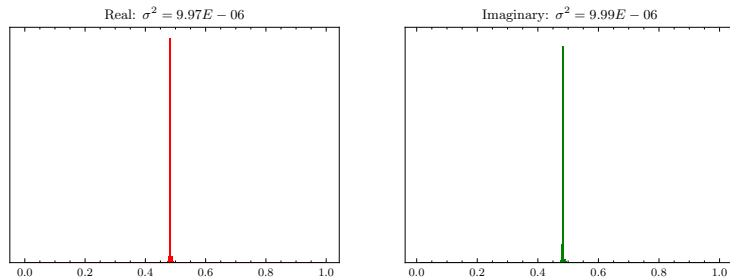
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Power-based normalization for improved CSI reconstruction accuracy.

Most works perform **minmax scaling** – Take the extrema ( $\mathbf{H}_{\min}, \mathbf{H}_{\max}$ ) of the real and imaginary channels,

$$\mathbf{H}_{n,\text{minmax}}(i, j) = \frac{\mathbf{H}_n(i, j) - \mathbf{H}_{\min}}{\mathbf{H}_{\max} - \mathbf{H}_{\min}} \in [0, 1],$$

for  $n \in [1, \dots, N]$  given  $N$  samples and  $i, j$  indexing rows/columns of CSI matrices.



**Figure:** Distribution and variance of minmax-normalized COST2100 real/imaginary channels ( $N = 99000$ ) images.

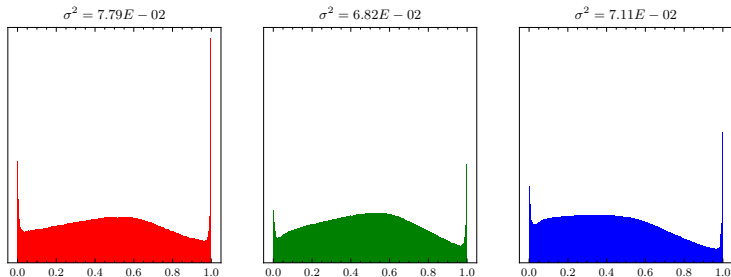


Figure: Distribution and variance of minmax-normalized ImageNet color channels ( $N = 50000$ ) images.

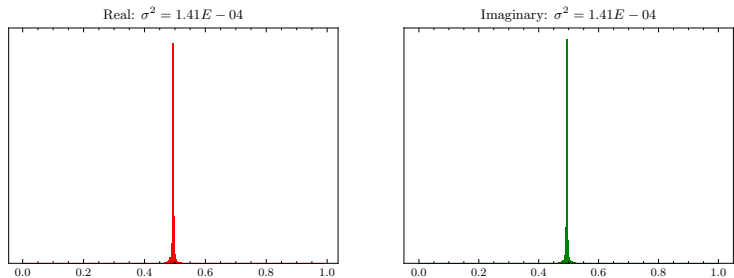


**Spherical normalization** – scale each channel sample by its power,

$$\check{\mathbf{H}}_d^n = \frac{\mathbf{H}_d^n}{\|\mathbf{H}_d^n\|_2}. \quad (1)$$

After applying (1) to each sample, minmax scaling is applied to the entire dataset.

The resulting dataset under spherical normalization can exhibit a larger variance than the same dataset under minmax scaling.



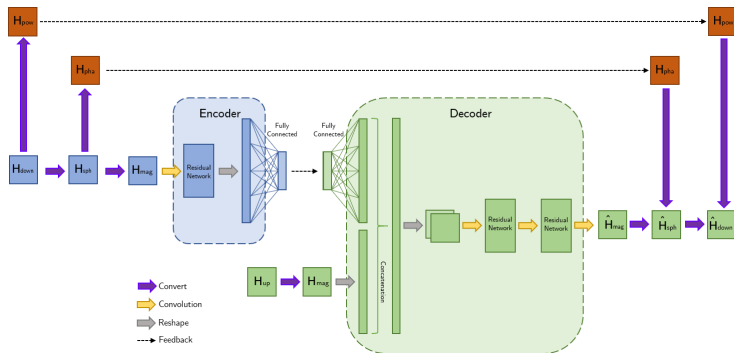
**Figure:** Distribution and variance of COST2100 real/imaginary channels under spherical normalization ( $N = 99000$ ) images.

Under spherical normalization, MSE loss becomes equivalent to NMSE.  
Recall the definitions,

$$\text{MSE} = \frac{1}{N} \sum_{k=1}^N \|\mathbf{H}_k - \hat{\mathbf{H}}_k\|^2, \quad \text{NMSE} = \frac{1}{N} \sum_{k=1}^N \frac{\|\mathbf{H}_k - \hat{\mathbf{H}}_k\|^2}{\|\mathbf{H}_k\|^2}$$

The MSE of the spherically normalized estimator is equivalent to the NMSE of the regular estimator, i.e.

$$\begin{aligned} \text{MSE}_{\text{Sph}} &= \frac{1}{N} \sum_{k=1}^N \|\check{\mathbf{H}}_k - \hat{\mathbf{H}}_k\|^2 \\ &= \frac{1}{N} \sum_{k=1}^N \left\| \frac{\mathbf{H}_k}{\|\mathbf{H}_k\|^2} - \frac{\hat{\mathbf{H}}_k}{\|\mathbf{H}_k\|^2} \right\|^2 \\ &= \frac{1}{N} \sum_{k=1}^N \frac{\|\mathbf{H}_k - \hat{\mathbf{H}}_k\|^2}{\|\mathbf{H}_k\|^2} \quad \square. \end{aligned}$$



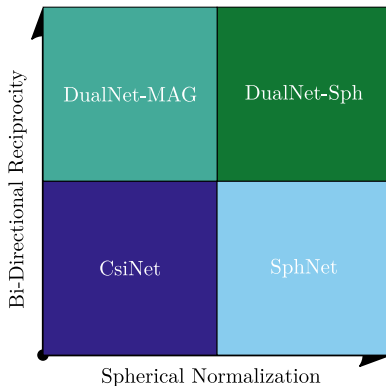


Figure: Illustration of techniques used in different models.

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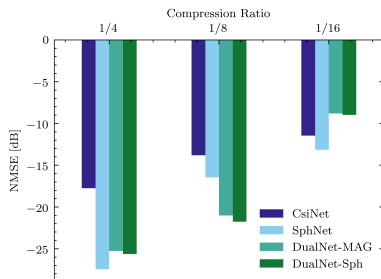
Z. Liu, **M. del Rosario**, X. Liang, L. Zhang, and Z. Ding, “Spherical normalization for learned compressive feedback in massive mimo csi acquisition,” in *2020 IEEE International Conference on Communications Workshops (ICC Workshops)*, pp. 1–6, 2020

Two MIMO scenarios using COST 2100 model with 32 antennas at gNB and single UE (single antenna), 1024 subcarriers.

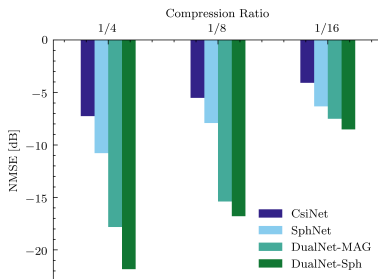
1. **Indoor** environment using 5.3GHz, 0.1 m/s UE mobility, square area of length 20m
2. **Outdoor** environment using 300MHz, 1 m/s UE mobility, square area of length 400m

**Dataset:**  $10^5$  channel samples – 70%/30% training/test split.

**Hyperparameters:** Adam optimizer with learning rate  $10^{-3}$ , batch size 200, 1000 epochs, MSE loss



(a) Indoor



(b) Outdoor

**Figure:** NMSE (lower is better) comparison of bidirectional reciprocity and spherical normalization against CsiNet for increasing compression ratio [2]

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Z. Liu, M. del Rosario, X. Liang, L. Zhang, and Z. Ding, "Spherical normalization for learned compressive feedback in massive mimo csi acquisition," in *2020 IEEE International Conference on Communications Workshops (ICC Workshops)*, pp. 1–6, 2020

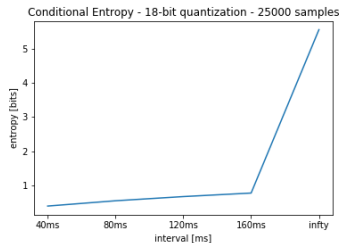
# MarkovNet

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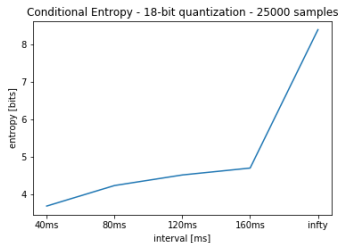
A deep differential autoencoder for efficient temporal learning.



CSI estimation techniques benefit from temporal information.



(a) Indoor



(b) Outdoor

**Figure:** Conditional entropy between CSI matrices for different feedback intervals. COST2100 model used for (a) Indoor and (b) Outdoor network

Recurrent neural networks (RNNs) contain trainable long short-term memory (LSTM) cells which learn temporal relationships.

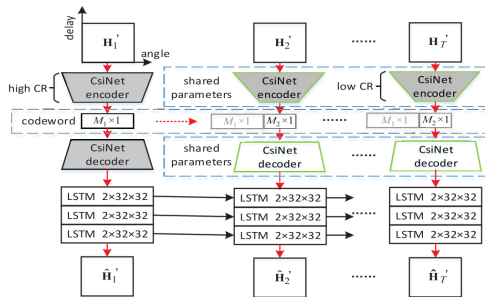


Figure: CsiNet-LSTM network architecture [3].

The number of parameters/FLOPs for RNNs is large.

**Table:** Model size and computational complexity of CsiNet-LSTM and CsiNet. M: million.

	Parameters		FLOPs	
CR	CsiNet-LSTM	CsiNet	CsiNet-LSTM	CsiNet
1/4	132.7 M	2.1 M	412.9 M	7.8 M
1/8	123.2 M	1.1 M	410.8 M	5.7 M
1/16	118.5 M	0.5 M	409.8 M	4.7 M
1/32	116.1 M	0.3 M	409.2 M	4.1 M
1/64	115.0 M	0.1 M	409.0 M	3.9 M

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T. Wang, C. Wen, S. Jin, and G. Y. Li, “Deep Learning-Based CSI Feedback Approach for Time-Varying Massive MIMO Channels,” *IEEE Wireless Comm. Letters*, vol. 8, pp. 416–419, April 2019

Instead of learning a temporal dependency across multiple timeslots, we proposed a one-step differential encoder.

For short enough time intervals between  $t$  and  $t - 1$ , we view CSI data as a Markov chain, i.e.

$$\mathbf{H}_t = \gamma \mathbf{H}_{t-1} + \mathbf{V}_t,$$

with  $\gamma \in \mathbb{R}^+$  and i.i.d  $\mathbf{V}_t$  such that  $\mathbf{V}_t \sim \mathcal{CN}(\mathbf{0}, \Sigma_V)$ .

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Z. Liu †, **M. del Rosario** †, and Z. Ding, “A Markovian Model-Driven Deep Learning Framework for Massive MIMO CSI Feedback,” *arXiv e-prints*, Sept.

The ordinary least-squares solution,  $\gamma$ , is given as

$$\gamma = \frac{\text{Trace}(\mathbb{E} [\mathbf{H}_{t-1}^H \mathbf{H}_t])}{\mathbb{E} \|\mathbf{H}_t^H \mathbf{H}_t\|^2}.$$

We utilize the estimator,  $\hat{\gamma}$ , based on the second-order statistics of the CSI matrices,

$$\hat{\gamma} = \frac{\sum_{i=1}^N \text{Trace}([\mathbf{H}_{t-1}^H(i) \mathbf{H}_t(i)])}{\sum_{i=1}^N \|\mathbf{H}_t^H(i) \mathbf{H}_t(i)\|^2},$$

for training set of size  $N$ .

With the one-step estimator  $\hat{\gamma}$ , we propose train an encoder for the estimation error as

$$\mathbf{s}_t = f_{e,t}(\mathbf{H}_t - \hat{\gamma}\hat{\mathbf{H}}_{t-1}),$$

and we jointly train a decoder,

$$\hat{\mathbf{H}}_t = f_{d,t}(\mathbf{s}_t) + \hat{\gamma}\hat{\mathbf{H}}_{t-1}$$

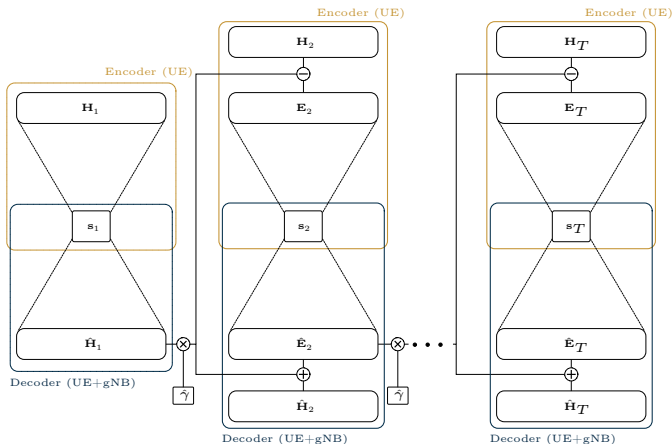
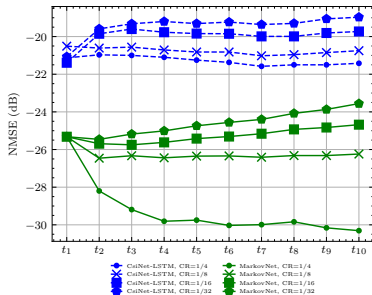
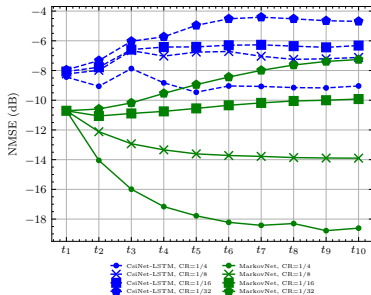


Figure: Abstract architecture for MarkovNet. Networks at  $t \geq 2$  are trained to predict the estimation error,  $E_t$ .



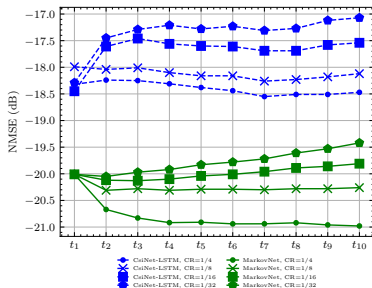
(a) Indoor



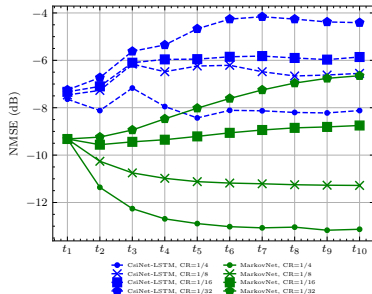
(b) Outdoor

Figure:  $\text{NMSE}_{\text{truncated}}$  comparison of MarkovNet and CsiNet-LSTM at various compression ratios (CR).



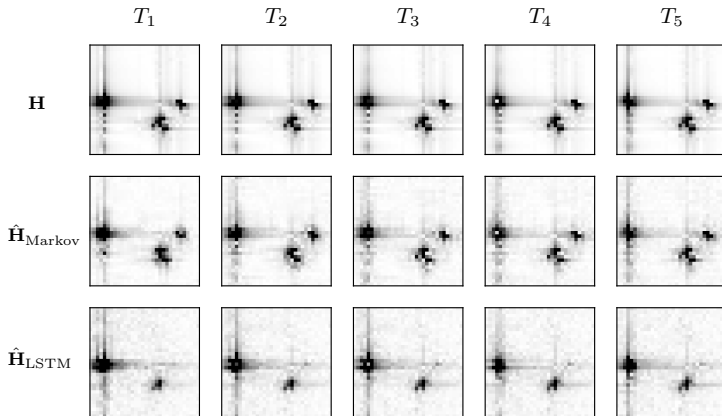


(a) Indoor



(b) Outdoor

Figure: NMSE<sub>all</sub> comparison of MarkovNet and CsiNet-LSTM at various compression ratios (CR).

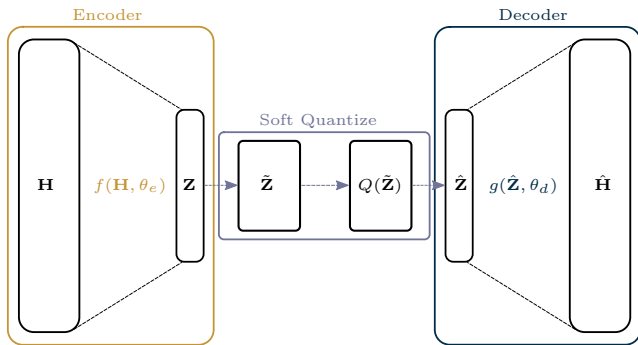


**Figure:** Ground truth CSI ( $\mathbf{H}$ ), MarkovNet estimates ( $\hat{\mathbf{H}}_{\text{Markov}}$ ), and CsiNet-LSTM estimates ( $\hat{\mathbf{H}}_{\text{LSTM}}$ ) for five timeslots ( $T_1$  through  $T_5$ ) on one outdoor sample from the test set (both networks at  $\text{CR} = \frac{1}{4}$ ).

# SphNet-Quant

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An end-to-end trained autoencoder with learned feedback quantization.



**Figure:** Abstract architecture for CsiNet-Quant. SoftQuantize layer ( $Q(\tilde{\mathbf{Z}})$ ) is a continuous, softmax-based relaxation of a  $d$ -dimensional quantization of the latent layer  $\mathbf{Z}$ .

Define the  $m$ -dimensional codebook of size  $L$  as  $\mathbf{C} \in \mathbb{R}^{m \times L}$ . The soft assignments of the  $j$ -th latent vector  $\tilde{\mathbf{z}}_j$  can be written as,

$$\phi(\tilde{\mathbf{z}}_j) = \left[ \frac{\exp(-\sigma \|\tilde{\mathbf{z}}_j - \mathbf{c}_\ell\|^2)}{\sum_{i=1}^L \exp(-\sigma \|\tilde{\mathbf{z}}_j - \mathbf{c}_i\|^2)} \right]_{\ell \in [L]} \in \mathbb{R}^L, \quad (2)$$

which is referred to as the ‘softmax’ function.  $\sigma$  is a *temperature* or *annealing* parameter which controls the degree of quantization,

$$\lim_{\sigma \rightarrow \infty} \phi(\tilde{\mathbf{z}}_j) = \text{onehot}(\tilde{\mathbf{z}}_j) = \begin{cases} 1 & \ell = \underset{\ell}{\operatorname{argmax}} \phi(\tilde{\mathbf{z}}_j)[\ell] \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The soft assignments  $\phi$  admit probability masses over the codewords,

$$q_j = \phi(\tilde{\mathbf{z}}).$$

Based on finite samples, we define the histogram probability estimates  $p_j$

$$p_j = \frac{|\{e_l(\mathbf{z}_i) | l \in [m], i \in [N], e_l(\mathbf{z}_i) = j\}|}{mN}.$$

Our target for the rate loss is the crossentropy between  $p_j$  and  $q_j$  term,

$$H(\phi) := H(p, q) = - \sum_{j=1}^L p_j \log q_j = H(p) + D_{\text{KL}}(p \| q).$$

Loss function for soft quantization = regularized rate-distortion function.

$$\operatorname{argmin}_{\theta_e, \theta_d, \mathbf{C}} L_d(\mathbf{H}, \hat{\mathbf{H}}) + \lambda L_{\ell^2}(\theta_e, \theta_d, \mathbf{C}) + \beta L_r(\theta_e, \mathbf{C}) \quad (4)$$

Where the different loss terms are

Term	Definition	Description
$L_d(\mathbf{H}, \hat{\mathbf{H}})$	$\frac{1}{N} \sum_{i=1}^N \ \mathbf{H}_i - g(Q(f(\mathbf{H}_i, \theta_e), \mathbf{C}), \theta_d)\ ^2$	distortion loss
$L_{\ell^2}(\theta_e, \theta_d, \mathbf{C})$	$\ \theta_e\ ^2 + \ \theta_d\ ^2 + \ \mathbf{C}\ ^2$	$\ell^2$ penalty
$L_r(\theta_e, \mathbf{C})$	$m\beta H(\phi)$	rate loss

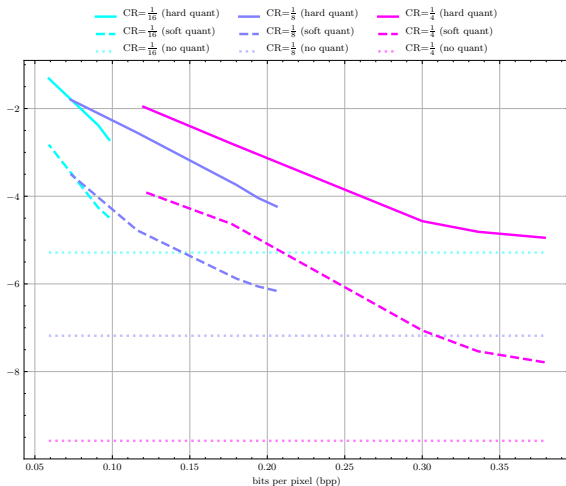


Figure: Rate distortion of CsiNet-Quant under minmax normalization using:  $L = 1024$  centers,  $d = 4$ .



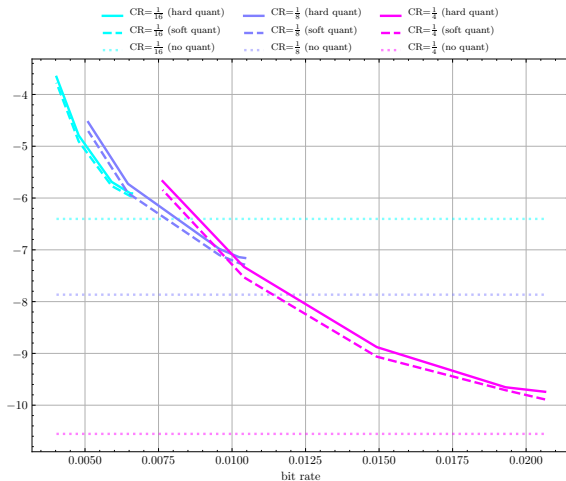


Figure: Rate distortion of SphNet-Quant using:  $L = 1024$  centers,  $CR = \frac{1}{4}$ ,  $d = 4$ . Bit rates are realized under arithmetic coding of quantized features.

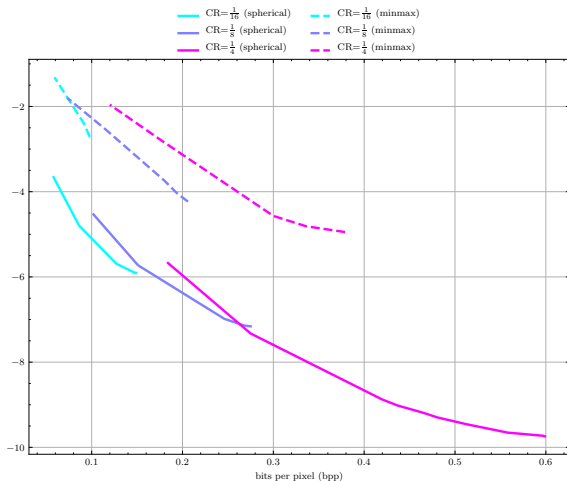


Figure: Rate distortion of CsiNet-Quant under both minmax (dotted line) and spherical (solid line) normalization using:  $L = 1024$  centers,  $d = 4$ . Hard quantization performance shown for each CR.

Questions?

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- [1] C. Wen, W. Shih, and S. Jin, “Deep learning for massive mimo csi feedback,” *IEEE Wireless Communications Letters*, vol. 7, pp. 748–751, Oct 2018.
- [2] Z. Liu, **M. del Rosario**, X. Liang, L. Zhang, and Z. Ding, “Spherical normalization for learned compressive feedback in massive mimo csi acquisition,” in *2020 IEEE International Conference on Communications Workshops (ICC Workshops)*, pp. 1–6, 2020.
- [3] T. Wang, C. Wen, S. Jin, and G. Y. Li, “Deep Learning-Based CSI Feedback Approach for Time-Varying Massive MIMO Channels,” *IEEE Wireless Comm. Letters*, vol. 8, pp. 416–419, April 2019.
- [4] Z. Liu †, **M. del Rosario** †, and Z. Ding, “A Markovian Model-Driven Deep Learning Framework for Massive MIMO CSI Feedback,” *arXiv e-prints*, Sept. 2020.  
  
Submitted to IEEE Transactions on Wireless Communications.
- [5] L. Kozachenko and N. N. Leonenko, “Sample estimate of the entropy of a random vector,” *Problemy Peredachi Informatsii*, vol. 23, no. 2, pp. 9–16, 1987.
- [6] G. Ver Steeg, G. Ballabio, E. Sennesh, M. Rebo, and D. Ulianych, “Non-parametric entropy estimation (npeet),” October 2019.

† → equal contribution

# Appendix

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Rather than scalar  $\hat{\gamma} \in \mathbb{R}^+$ , we can derive a multivariate  $p$ -step predictor,  $\mathbf{W}_1, \dots, \mathbf{W}_p$ . Given  $p$  prior CSI samples, the mean-square optimal predictor  $\hat{H}_t$  is a linear combination of these the prior CSI samples,

$$\hat{\mathbf{H}}_t = \mathbf{H}_{t-1} \mathbf{W}_1 + \dots + \mathbf{H}_{t-p} \mathbf{W}_p + \mathbf{E}_t. \quad (5)$$

Error terms are uncorrelated with the CSI samples (i.e.  $\mathbf{H}_{t-i}^H \mathbf{E}_t = 0$  for all  $i \in [0, \dots, p]$ ), and we pre-multiply by  $\mathbf{H}_{t-i}^H$ ,

$$\begin{aligned}\mathbf{H}_{t-i}^H \hat{\mathbf{H}}_t &= \mathbf{H}_{t-i}^H \mathbf{H}_{t-1} \mathbf{W}_1 + \dots + \mathbf{H}_{t-i}^H \mathbf{H}_{t-p} \mathbf{W}_p + \mathbf{H}_{t-i}^H \mathbf{E}_t \\ &= \mathbf{H}_{t-i}^H \mathbf{H}_{t-1} \mathbf{W}_1 + \dots + \mathbf{H}_{t-i}^H \mathbf{H}_{t-p} \mathbf{W}_p.\end{aligned}\tag{6}$$

Denote the correlation matrix  $\mathbf{R}_i = \mathbb{E}[\mathbf{H}_{t-i}^H \mathbf{H}_t]$ . We presume the CSI matrices are generated by a stationary process, and consequently, they have the following properties:

1.  $\mathbf{R}_i = \mathbb{E}[\mathbf{H}_{t-i}^H \mathbf{H}_t] = \mathbb{E}[\mathbf{H}_t^H \mathbf{H}_{t+i}]$
2.  $\mathbf{R}_i = \mathbf{R}_{-i}^H$



Taking the expectation, we can write (6) as a linear combination of correlation matrices,

$$\mathbf{R}_{i+1} = \mathbf{R}_i \mathbf{W}_1 + \cdots + \mathbf{R}_{i-p+1} \mathbf{W}_p.$$

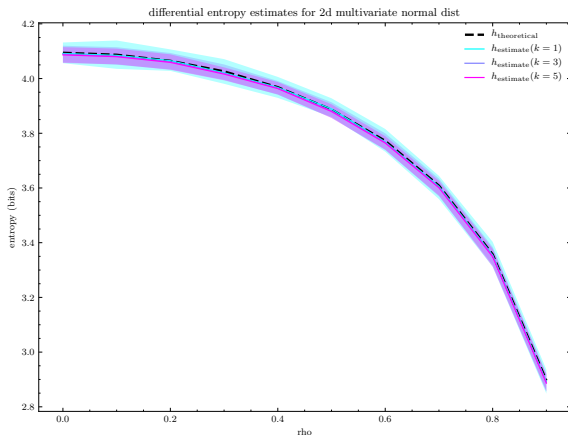
For  $p$  CSI samples, we can write a system of  $p$  equations, which admits the following,

$$\begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \vdots \\ \mathbf{R}_p \end{bmatrix} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{R}_1^H & \cdots & \mathbf{R}_{p-1}^H \\ \mathbf{R}_1 & \mathbf{R}_0 & \cdots & \mathbf{R}_{p-2}^H \\ \vdots & & \ddots & \vdots \\ \mathbf{R}_{p-1} & \mathbf{R}_{p-2} & \cdots & \mathbf{R}_0 \end{bmatrix} \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \vdots \\ \mathbf{W}_p \end{bmatrix}.$$

Solving for the coefficient matrices admits the solution

$$\begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \dots \\ \mathbf{W}_p \end{bmatrix} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{R}_1^H & \dots & \mathbf{R}_{p-1}^H \\ \mathbf{R}_1 & \mathbf{R}_0 & \dots & \mathbf{R}_{p-2}^H \\ \vdots & & \ddots & \vdots \\ \mathbf{R}_{p-1} & \mathbf{R}_{p-2} & \dots & \mathbf{R}_0 \end{bmatrix}^+ \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \dots \\ \mathbf{R}_p \end{bmatrix}, \quad (7)$$

where  $[\cdot]^+$  denotes the Moore-Penrose pseudoinverse.



**Figure:** Differential entropy and estimates for 2d multivariate normal distribution. Estimates are based on the KL estimator [5] using the NPEET library [6].