Mark Demore II, 2d Lt

CSCE686 - Dr. Lamont

Spr 2020 - Homework 1

**Exercise 1.1 Related problems to maximum clique.** Given an undirected graph

G = (V,E). A clique Q of the graph G is a subset of V where any two vertices in Q

are adjacent:

∀i, j ∈ Q × Q, (i, j) ∈ E

A maximum clique is a clique with the largest cardinality. The problem of finding the

maximum clique is NP-hard. The clique number is the cardinality of the maximum

clique. Given the following problems:

• The subset I ⊆ V of maximum cardinality such as the set of edges of the subgraph

induced by I is empty.

• Graph coloring.

Find the relationships between the formulated problems and the maximum clique

problem. How these problems are identified in the literature?

Exercise 1.1:

The maximum clique problem attempts to find the largest set of adjacent vertices in a graph. Graph coloring assigns colors to the vertices of a graph so that no adjacent vertices share the same color. The maximum independent set problem involves finding the set of vertices in a graph that are not adjacent and therefore would result in an empty subgraph.

The maximum clique problem and the maximum independent set problem are complementary. As such, the solution of one can be used to find the other for the same graph. Both problems have a complexity of NP-hard. A clique is one graph is an independent set in the graph’s complement.

The maximum clique problem is related to graph coloring by the number of colors needed to color the graph. All members of a clique are adjacent to each other, and therefore will all require a different color. As such, the cardinality of the maximum clique is equal to the number of colors needed to color the graph. So, determining the number of colors needed for a graph is also the maximum clique problem, and therefore NP-hard.

**Exercise 1.2 Easy versus hard optimization problem.** Let us consider the set

bipartitioning problem. Given a set X of n positive integers e1, e2, . . . , en where n is

an even value. The problem consists in partitioning the set X into two subsets Y and

Z of equal size. How many possible partitions of the set X exist?

Two optimization problems may be defined:

• Maximum set bipartitioning that consists in maximizing the difference between

the sums of the two subsets Y and Z.

• Minimum set bipartitioning that consists in minimizing the difference between

the sums of the two subsets Y and Z.

To which complexity class the two optimization problems belong? Let us consider

the minimum set bipartitioning problem. Given the following greedy heuristic: sort

the set X in decreasing order. For each element of X[i] with i = 1 to n, assign it to the

set with the smallest current sum. What is the time complexity of this heuristic?

Exercise 1.2:

The maximum and minimum set bi-partitioning problems are both a complexity of NP-hard.

The greedy heuristic for the minimum set bi-partitioning problem results in an algorithm with time complexity O(n log n).