Quiz - Complexity 2020 “Do NOT turn-in”

Solutions

1. Assuming P is not equal to NP, which of the following is true ?   
   (A) NP-complete = NP  
   (B) NP-complete Intersection P = is empty   
   (C) NP-hard = NP  
   (D) P = NP-complete  
   Solution: The answer is B (no [NP-Complete](http://en.wikipedia.org/wiki/NP-complete) problem can be solved in polynomial time). Because, if one NP-Complete problem can be solved in polynomial time, then all NP problems can solved in polynomial time. If that is the case, then NP and P set become same which contradicts the given assumption.
2. Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?
3. R is NP-complete
4. R is NP-Hard
5. Q is NP-complete

(D)Q is NP-Hard  
Solution: (A) Incorrect because R is not in NP. A NP Complete problem has to be in both NP and NP-hard. (B) Correct because a NP Complete problem S is polynomial time reducible to R. (C) Incorrect because Q is not in NP. (D) Incorrect because there is no NP-complete problem that is polynomial time reducible to Q.

1. Let X be a problem that belongs to the class NP. Then which one of the following is TRUE?

(A)There is no polynomial time algorithm for X

(B) If X can be solved deterministically in polynomial time then P = NP   
(C) If X is NP-Hard, then it is NP-complete

(D)X maybe undecidable  
Solution: (A) is incorrect because set NP includes both P and NP-Complete . (B) is incorrect because X may belong to P (C) is correct because NP-Complete set is intersection of NP and NP-Hard sets. (D) is incorrect because all NP problems are decidable in finite set of operations.

1. Which of the following is true about NP-Complete and NP-Hard problems.
2. To prove that a problem X is NP-Hard, take a known NP-Hard problem and reduce Y to X
3. NP-complete is a subset of NP-Hard
4. The first problem that was proved as NP-complete was the circuit satisfiability problem
5. All of the above  
   Solution: (D) is correct since all statements are true due to the definition of NP-Completeness.
6. Every problem in NP can be solved in exponential time.
7. True
8. False  
   Solution: (A) is true since all NP problems can be solved in EXPTIME (Venn diagram).
9. If a problem X can be reduced to a known NP-hard problem, then X must be NP-hard.
10. True
11. False   
    Solution: (B) is correct. Note that if the problem statement is reversed with polynomial time reduction, then it is true.
12. Suppose you could reduce an NP complete problem to a polynomial time problem in polynomial time.   
    (A) What would be the consequence?

(B)What if the reduction required exponential time?  
Solution: (A) The consequence would be that P = NP. (B) The reduction being an exponential time reduction implies that the original NP-Complete Problem is reduced to an approximation polynomial time model.

1. **Consider a reduction of problem A to problem B. What is the most precise claim you can make about problem B for each of the following situations?**8.1 A is NP-complete and the reduction is in polynomial time. Solution: Problem B is NP-Hard; Problem B not in NP  
   1. A is in polynomial time and the reduction is also in polynomial time. Solution: the specific characteristics of problem B are unknown
   2. A is NP-complete and the reduction is in PSPACE. Solution: Problem B is in PSPACE
   3. A is in nondeterministic polynomial time and the reduction is in polynomial time. Solution: the specific characteristics of problem B are unknown
   4. A requires exponential time and the reduction is in polynomial time. Solution: Problem B is in EXPTIME
   5. A is PSPACE complete and the reduction is in PSPACE. Solution: Problem B is in PSPACE; if reduction is polynomial, problem B is in PSPACE-Complete

**9.0 What makes a NP-Complete problem strong or weak? Relate to   
 approximation possibilities?   
 Solution:** A Weak NP-Complete problem has a FPTAS approximate solution, where as a   
 Strong NP-Complete problem has only a PTAS approximate solution at best.