USAF ACADEMY
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 332
Electrical Circuits and Systems II
Summer 2016
Graded Review #1

ACADEMIC TESTING MATERIAL

ACADEMIC SECURITY: This examination is not released from academic security until 1630 on 22 July 2016. Until this time, you may not discuss the examination contents or the course material with anyone other than your instructor.

INTEGRITY: Your honor is extremely important. This academic security policy is designed to help you succeed in meeting academic requirements while practicing the honorable behavior our country rightfully demands of its military. Do not compromise your integrity by violating academic security or by taking unfair advantage of your classmates.

AUTHORIZED RESOURCES: Attached equation sheet.

- This exam is CLOSED-BOOK.
- Box your final answer where appropriate.
- Show all work to qualify for partial credit.
- Organize your work. Your instructor must be able to follow your solution process.
- Use **engineering notation** with **two significant figures**.

PROBLEM	VALUE	EARNED
1 Knowledge/Comprehension	20	
2 Comprehension/Application	20	
3 Analysis	30	
4 Analysis/Design	30	
Total	100	

NAME Solution SECTION

This page intentionally blank

1. (20 points – 4 points each)

a. What determines the **form** of the homogeneous response of 1st and 2nd order circuits?

1. input
2. output
3. characteristic equation
4. initial conditions
5. none of the above

b. Reducing C by a factor of 2 and reducing L by factor of 2 in an RLC circuit will cause the resonant frequency (undamped natural frequency) ω_o to be

1. doubled
2. halved
3. the same
4. reduced by a factor of 4
5. none of the above

c. At high frequencies, a capacitor behaves like a(n) _______ and an inductor like a(n) _______.

1. open circuit, short circuit
2. short circuit, open circuit
3. open circuit, open circuit
4. short circuit, short circuit
5. none of the above

d. The initial value of $f(t)$ given the transform $F(s) = \frac{20(s^2+10s+100)}{s(s^2+20s+100)}$ is

1. 0
2. infinite
3. 2
4. 20
5. none of the above

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s F(s) = \lim_{s \to \infty} \frac{20(s^2 + \ldots)}{(s^2 + \ldots)} = 20$$

e. The property $Af_1(t) + Bf_2(t) \xrightarrow{\mathcal{L}} AF_1(s) + BF_2(s)$ is a statement of the property of

1. Integration
2. Uniqueness
3. s-domain Translation
4. Linearity
5. none of the above

This page intentionally blank.

2a. (10 pts) Using classical techniques, find $y(t)$ that satisfies the following differential equation and initial conditions. **Be sure to solve for the constants.**

$$\frac{d^2y(t)}{dt^2} + 30\frac{dy(t)}{dt} + 450y(t) = 100u(t)$$

$$y(0) = 0 \text{ and } \frac{dy(0)}{dt} = 30$$

CE $\quad s^2 + 30s + 450 = 0 \qquad s = -15 \pm \sqrt{225 - 450}$

$$= -15 \pm j15$$

$$y_c(t) = e^{-15t}(K_1 \cos \overset{15}{\omega} t + K_2 \sin \overset{15}{\omega} t)$$

$$y_p(t) = K \qquad y_p' = y_p'' = 0$$

$$450K = 100 \qquad K = \frac{100}{450} = \frac{10}{45} = \frac{2}{9}$$

$$y(t) = \frac{2}{9} + e^{-15t}(K_1 \cos 15t + K_2 \sin 15t)$$

$$y(0) = 0 = \frac{2}{9} + K_1 \Rightarrow K_1 = -\frac{2}{9}$$

$$y'(t) = -15e^{-15t}\left(\frac{-2}{9}\cos 15t + K_2 \sin 15t\right)$$

$$+ \cancel{15}\, e^{-15t}\left(\frac{2}{9}\cdot 15 \sin 15t + 15K_2 \cos 15t\right)$$

$$y'(0) = \frac{15\cdot 2}{9} + 15K_2 = 30 \qquad K_2 = \frac{30 - \frac{30}{9}}{15} = 2 - \frac{2}{9} = \frac{16}{9}$$

$$y(t) = \left[\frac{2}{9} + e^{-15t}\left(\frac{-2}{9}\cos 15t + \frac{16}{9}\sin 15t\right)\right]u(t)$$

Same as before

2b. (10 pts) Now using Laplace techniques, find y(t) that satisfies the same differential equation and initial conditions.

$$\frac{d^2y(t)}{dt^2} + 30\frac{dy(t)}{dt} + 450y(t) = 100u(t)$$

$$y(0) = 0 \text{ and } \frac{dy(0)}{dt} = 30$$

$$\frac{100}{s} = s^2 Y(s) - s(y(0)) - y'(0) + 30s\left[Y(s) - y(0)\right] + 450\,Y(s)$$

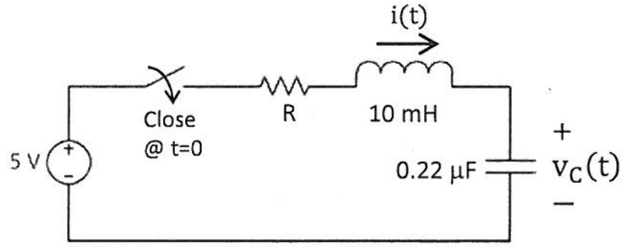
$$\left(s^2 + 30s + 450\right) Y(s) = \frac{100}{s} + 30 = \frac{100 + 30s}{s}$$

$$Y(s) = \frac{30s + 100}{s\left(s^2 + 30s + 450\right)} = \frac{K_1}{s} + \frac{K_2}{s+15-j15} + \frac{K_2^*}{s+15+j15}$$

$$K_1 = \frac{100}{450} = \frac{2}{9} \qquad K_2 = \frac{30(-15+j15) + 100}{(-15+j15)(j30)} = 0.895\angle{-97.1^\circ}$$

$$y(t) = \frac{2}{9} + 1.79\,e^{-15t}\cos\left(15t - 97.1^\circ\right)$$

3. (30 points) The switch in the figure below has been open for a long time and is closed at t = 0.



a. (10 pts) Find the value of R which falls at the boundary between an under-damped circuit and an over-damped circuit?

boundary $\Rightarrow \zeta = 1 = \frac{R}{2}\sqrt{\frac{C}{L}}$

$R = 2\sqrt{\frac{L}{C}} = 426.4\,\Omega$

b. (5 pts) You decide that you want to design the circuit to have a damping ratio of $\zeta = 0.38$ Additionally, you test the inductor and find it to have a parasitic resistance of 14.6 Ω. Find the value of the resistor R needed to achieve the desired damping

$0.38 = \frac{R_T}{2}\sqrt{\frac{C}{L}}$ $\quad R_T = (2)(0.38)\sqrt{\frac{L}{C}} = 162.0\,\Omega$

$R = R_T - R_L = 147.4\,\Omega$

c. (10 pts) After choosing the resistor R for the circuit above and re-measuring the components, you determine that the roots of the characteristic equation are

$$s_{1,2} = -8{,}000 \pm j19{,}500.$$

Write the equation that describes $v_c(t)$, the voltage across the capacitor, after the switch closes. The initial conditions are $v_c(0) = 2$ V and $i_L(0) = 0$ mA. Solve for all unknowns.

$$v_c(t) = 5 + e^{-8000t}(K_1 \cos 19500t + K_2 \sin 19500t)$$

$$v_c(0) = 2 \Rightarrow \quad K_1 = -3$$

$$v_c'(t) = -8000\, e^{-8000t}(K_1 \cos 19500t + K_2 \sin 19500t)$$

$$+ e^{-8000t}(-19500 K_1 \sin 19500t + 19500 K_2 \cos 19500t)$$

$$v_c'(0) = 0 = (-8000)(-3) + 19500 K_2 \Rightarrow K_2 = \frac{-24000}{19500} = -1.231$$

$$v_c(t) = 5 + e^{-8000t}(-3 \cos 19.5kt - 1.23 \sin 19.5kt)$$

d. (5 pts) Suppose you found the solution below. Find the current through the capacitor for $t \geq 0$.

$$v_C(t) = [5 - 3e^{-8{,}000t}\cos(19{,}500t)]u(t) \text{ V}$$

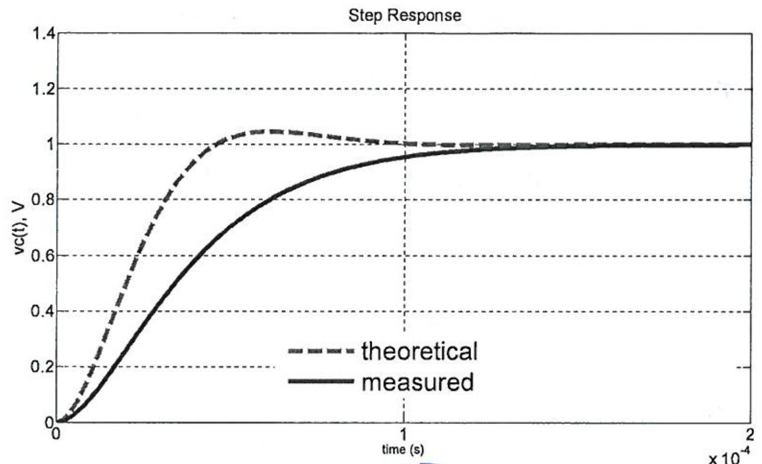
$$i_c = C\frac{dv_c}{dt} = 0.22\mu \cdot (+24k e^{-8kt} \cos 19.5kt + 58.5k e^{-8kt} \sin 19.5kt)$$

$$= 5.28 e^{-8kt} \cos 19.5kt + 12.87 e^{-8kt} \sin 19.5kt \text{ mA } u(t)$$

4. (30 points) You designed and tested a series RLC circuit and overlaid both the theoretical and measured unit step response for $v_C(t)$ below. You are unsure why the two responses didn't match using the parts below.

Parts	L=1 mH	R=100 Ω	C=0.20 μF

You do analysis and determine your measured $\zeta = 1.05$, different from your theoretical $\zeta = 0.70$.



(5 pts) Circle one. The measured response is more damped / less damped than the theoretical.

(10 pts) Explain possible reasons for the mismatch. Justify your answer using governing equations and a circuit diagram. Hint: think out non-idealities in your circuit.

Probably did not take parasitic resistance of inductor or source resistance into effect.

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$ ⇒ increased R ⇒ higher ζ more damping

other errors could be higher C or lower L

(10 pts) What capacitor C would you use instead to better align the two responses given you use the same parts for R and L?

I would choose the more damped response although the 0.7 ζ overshoot is pretty small. More damping → slightly slower but no overshoot.

(5 pts) If these two responses represented elevator deflection angle on your airplane due to a control stick input, which would you choose based on damping and overshoot? Why?

$$\zeta = 1.05 = \frac{R}{2}\sqrt{\frac{0.2\mu}{1m}} \qquad R_T = 2.10\sqrt{\frac{1m}{.2\mu}} = 148.5$$

$$\zeta = 0.7 = \frac{148.5}{2}\sqrt{\frac{C}{1m}} \qquad C = \left(\frac{(2)(0.7)}{148.5}\right)^2 1m = \boxed{0.0889\mu F}$$

Scratch Paper

Name______________ Section____