ECE332 Quiz 1
(10 minutes)

Name: _____

1. [60 pts] **Given**: The zero-input response of a series RLC circuit is observed to be

$$v_C(t) = 10e^{-500t} + 5e^{-2500t}\ \text{V}\ u(t)$$

$$i_L(t) = -5e^{-500t} - 12.5e^{-2500t}\ \text{mA}\ u(t)$$

a. [20 pts] **Find**: What is the circuit characteristic equation from which you find the roots?
Hint: think about which case this might be i.e. I, II, or III.

roots $s_{1,2} = -500, -2500$

$(s+500)(s+2500) = 0$

$s^2 + 3000s + 1250000 = 0$

b. [20 pts] **Find**: What are the initial values of the state variables, $v_C(t)$ and $i_L(t)$?

$v_C(0) = 10e^{-500(0)} + 5e^{-2500(0)} = 15\ \text{V}$

$i_L(0) = -5e^{-500(0)} - 12.5e^{-2500(0)} = 17.5\ \text{mA}$

c. [20 pts] **Find**: Write an expression for the voltage across the resistor R given that this is a series RLC circuit. Leave R as a variable in your expression.

$i_R = i_C \quad v_R = R \cdot i_R$

$= R(-5e^{-500t} - 12.5e^{-2500t})u(t)\ \text{mV}$

2. [40 pts] **Given**: The ODE below.

$$y''(x) - 2y'(x) - 3y(x) = \sin(4x)$$

a. [5 pts] **Find**: The complementary response $y_c(x)$ (i.e. homogeneous or natural response).

$s^2 - 2s - 3 = 0 \quad (s-3)(s+1) = 0$

$s_{1,2} = 3, -1$

$y_c(x) = c_1 e^{3x} + c_2 e^{-x}$

b. [30 pts] **Find**: The particular response $y_p(x)$ (i.e. non-homogeneous or forced response). Be sure to solve for the constants of $y_p(x)$ using undetermined coefficients.

$y_p(x) = C_3 \cos 4x + C_4 \sin 4x$

$y_p'(x) = -4C_3 \sin 4x + 4C_4 \cos 4x$

$y''(x) = -16C_3 \cos 4x - 16C_4 \sin 4x$

$\Rightarrow \quad -16C_3 - 8C_4 - 3C_3 = 0 \quad -19C_3 - 8C_4 = 0$

$-16C_4 + 8C_3 - 3C_4 = 1 \quad 8C_3 - 19C_4 = 1$

$(-19)(8)C_3 - 8(8)C_4 = 0 \quad C_4 = -\frac{19}{8^2+19^2} = -44.7m$

$(8)(19)C_3 - 19(19)C_4 = 19 \quad C_3 = \frac{(8)(8)(C_4)}{(-19)(8)} = 18.8m$

$y_p = 18.8m \cos 4x - 44.7m \sin 4x$

c. [5] **Find**: The general solution $y(x)$. Note there are no initial conditions given for $y(x)$ or $y'(x)$.

$y(x) = c_1 e^{3x} + c_2 e^{-x} + 18.8m \cos 4x - 44.7m \sin 4x$