

This figure belongs to Problem 14.15.

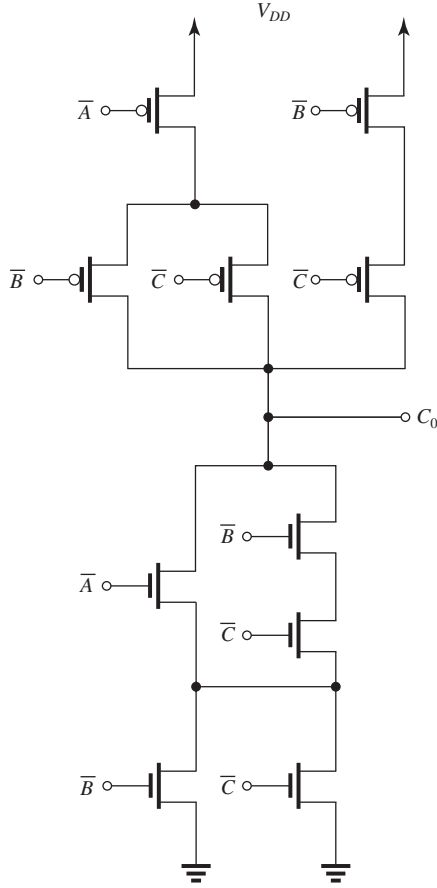


Figure 1

**14.16**  $NM_H = V_{OH} - V_{IH}$

$$= 1.8 - 1.2 = 0.6 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.9 - 0.2 = 0.7 \text{ V}$$

**14.17** (a)  $NM_H = V_{OH} - V_{IH}$

$$= 1.8 - 1.3 = 0.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 1.2 - 0.4 = 0.8 \text{ V}$$

(b)

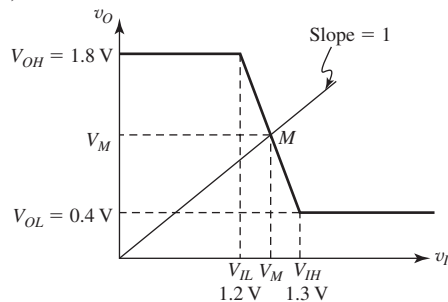


Figure 1

Refer to Fig. 1. Slope of the VTC in the transition region is:

$$\begin{aligned} \text{Slope} &= \frac{V_{OH} - V_{OL}}{V_{IL} - V_{IH}} \\ &= \frac{1.8 - 0.4}{1.2 - 1.3} = -14 \text{ V/V} \end{aligned}$$

But the slope can also be expressed as

$$\text{Slope} = \frac{V_M - V_{OH}}{V_M - V_{IH}}$$

Thus,

$$\frac{V_M - 0.4}{V_M - 1.3} = -14$$

$$\Rightarrow V_M = 1.24 \text{ V}$$

(c) The voltage gain in the transition region is equal to the slope found above, thus

$$\text{Gain} = -14 \text{ V/V}$$

**14.18**  $NM_H = V_{OH} - V_{IH}$

$$= 0.8V_{DD} - 0.6V_{DD} = 0.2V_{DD}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.4V_{DD} - 0.1V_{DD} = 0.3V_{DD}$$

$$\text{Width of transition region} = V_{IH} - V_{IL}$$

$$= 0.6V_{DD} - 0.4V_{DD} = 0.2V_{DD}$$

For a minimum noise margin of 0.4 V, we have

$$NM_H = 0.4$$

$$\Rightarrow 0.2V_{DD} = 0.4$$

$$\Rightarrow V_{DD} = 2 \text{ V}$$

**14.19**  $V_{IH} = 2 \text{ V}$

$$V_{IL} = 0.8 \text{ V}$$

$$V_{OH\min} = 2.4 \text{ V}, V_{OH\text{typ}} = 3.3 \text{ V}$$

$$V_{OL\max} = 0.4 \text{ V}, V_{OL\text{typ}} = 0.22 \text{ V}$$

(a) Worst-case  $NM_H = V_{OH\min} - V_{IH}$

$$= 2.4 - 2 = 0.4 \text{ V}$$

$$\text{Worst-case } NM_L = V_{IL} - V_{OL\max}$$

$$= 0.8 - 0.4 = 0.4 \text{ V}$$

(b) Typical average power dissipation:

$$P_D = \frac{1}{2} (5 \times 3 + 5 \times 1)$$

$$= 10 \text{ mW}$$

**14.20** (a) Refer to Fig. 14.17.

$$V_{OL} = V_{DD} \frac{R_{\text{on}}}{R + R_{\text{on}}}$$

Similarly,

$$|\text{Slope}| = \frac{V_{OH}}{V_{IH} - V_{IL}}$$

$$50 = \frac{2}{0.816 - V_{IL}}$$

$$\Rightarrow V_{IL} = 0.776 \text{ V}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= 2 - 0.816 = 1.184 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.776 - 0 = 0.776 \text{ V}$$

Since we approximated the VTC in the transition region by a straight line, the large-signal voltage gain will be equal to the small-signal voltage gain,

$$= -50 \text{ V/V}$$

**14.23** Here,  $V_{OH} = 1.2 \text{ V}$ , and  $V_{OL} = 0.0 \text{ V}$

$$\text{Also, } V_{IH} - V_{IL} \leq 1.2/3 = 0.4 \text{ V} \quad (1)$$

Now, the noise margins are “within 30% of one other.” Thus,  $NM_H = (1 \pm 0.3) NM_L$  or  $NM_L = (1 \pm 0.3) NM_H$ . Thus, they remain “within” either  $NM_H = 1.3 NM_L$  or  $NM_L = 1.3 NM_H$ , in which case either  $NM_L = 0.769 NM_H$  or  $NM_H = 0.769 NM_L$

For the former case:

$$0.769 (V_{OH} - V_{IH}) = (V_{IL} - V_{OL}) \text{ or}$$

$$0.769 (1.2 - V_{IH}) = V_{IL} - 0, \text{ whence}$$

$$V_{IL} = 0.923 - 0.769 V_{IH}$$

$$\text{Now, from (1), } V_{IH} = V_{IL} + 0.4$$

Thus,

$$V_{IL} = 0.923 - 0.769 (V_{IL} + 0.4)$$

$$= 0.615 - 0.769 V_{IL}$$

$$\text{and } V_{IL} = 0.615/1.769 = 0.349 \text{ V}$$

$$\text{whence } V_{IH} = 0.4 + 0.349 = 0.749 \text{ V}$$

Alternatively,  $NM_H = 0.769 NM_L$  and

$$(V_{OH} - V_{IH}) = 0.769 (V_{IL} - V_{OL}) \text{ or}$$

$$1.2 - V_{IH} = 0.769 V_{IL} - 0 \text{ and}$$

$$V_{IH} = 1.2 - 0.769 V_{IL}, \text{ with (1),}$$

$$V_{IL} + 0.4 = 1.2 - 0.769 V_{IL}, \text{ and}$$

$$1.769 V_{IL} = 0.8, \text{ whence } V_{IL} = 0.452 \text{ V}$$

$$\text{and } V_{IH} = 0.4 + 0.452 = 0.852$$

Thus, overall,  $V_{OH} = 1.2 \text{ V}$ ,  $V_{OL} = 0.0 \text{ V}$ ,

$V_{IH}$  ranges from 0.749 V to 0.852 V, and

$V_{IL}$  ranges from 0.349 V to 0.451 V, in

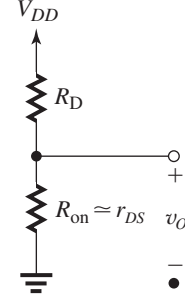
which case the margins can be as low as

$$NM_L = V_{IL} - V_{OL} = 0.349 \text{ V and}$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.852 = 0.348 \text{ V}$$

and as high as 0.451 V, and 0.451 V.

#### 14.24



*Equivalent circuit for output-low state*

The output-high level for the simple inverter circuit shown in Fig. 14.12 of the text is

$$V_{OH} = V_{DD} \Rightarrow V_{DD} = 1.2 \text{ V}.$$

When the output is low, the current drawn from the supply can be calculated as

$$I = \frac{V_{DD}}{R_D + R_{on}} = 30 \mu\text{A}$$

$$\text{Therefore: } R_D + r_{DS} = \frac{1.2}{30 \times 10^{-6}} = 40 \text{ k}\Omega$$

Also:

$$V_{OL} = 0.05 \text{ V} = \frac{r_{DS}}{R_D + r_{DS}} \times V_{DD}$$

$$\Rightarrow r_{DS} = 40 \text{ k}\Omega \times \frac{0.05}{1.2} = 1.67 \text{ k}\Omega$$

$$\text{Hence: } R_D = 40 \text{ K} - 1.67 \text{ K} = 38.3 \text{ k}\Omega$$

$$r_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)}$$

$$= \frac{1}{500 \times 10^{-6} \times \frac{W}{L} (1.2 - 0.4)}$$

$$= 1.67 \text{ k}\Omega$$

$$\Rightarrow \frac{W}{L} = 1.5$$

When the output is low:

$$P_D = V_{DD} I_{DD} = 1.2 \times 30 \mu\text{A} = 36 \mu\text{W}$$

When the output is high, the transistor is off:

$$P_D = 0 \text{ W}$$

**14.25** The output voltage swing  $= R_{C1}I = 0.5 \text{ V}$ .

with  $I_{EE} = 0.5 \text{ mA}$ ,  $R_{C1} = 1.0 \text{ k}\Omega$ , similarly,

$$R_{C2} = 1.0 \text{ k}\Omega$$

$$V_{OH} = V_{CC} = 2 \text{ V}$$

$$V_{OL} = V_{CC} - R_{C1}I_{EE} = 1.5 \text{ V}$$

**14.26** Refer to Example 14.2 on page 1107 of the text:

$$V_{OH} = V_{DD} = 1.2 \text{ V}$$

The power drawn from the supply during the low-output state is

$$P_{DD} = V_{DD}I_{DD} \Rightarrow 60 \mu\text{W} = 1.2 \times I_{DD}$$

$$\Rightarrow I_{DD} = 50 \mu\text{A}$$

In this case:

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} \Rightarrow 50 \mu\text{A} = \frac{1.2 - 0.05}{R_D}$$

$$\Rightarrow R_D = 23 \text{ k}\Omega$$

In order to determine  $\frac{W}{L}$ , we note that

$$k_n R_D = 1/V_X \text{ or } k'_n \frac{W}{L} R_D = \frac{1}{V_X}$$

Therefore, we need to first calculate  $V_X$  using Eq. (14.22) on page 1110 of the text.

$$V_{OL} = \frac{V_{DD}}{1 + \frac{V_{DD} - V_t}{V_X}} \text{ or equivalently}$$

$$0.05 \text{ V} = \frac{1.2}{1 + \frac{1.2 - 0.4}{V_X}}$$

$$\Rightarrow V_X = \frac{0.8}{23} = 0.035 \text{ V}$$

Hence,  $k'_n \frac{W}{L} R_D = \frac{1}{V_X}$  gives

$$500 \times 10^{-6} \times \frac{W}{L} \times 23 \times 10^3 = \frac{1}{0.035} \Rightarrow \frac{W}{L} = 2.5$$

Using Eq. (14.12), we obtain

$$V_{IL} = V_t + V_X = 0.4 + 0.035 = 0.435 \text{ V}$$

From Eq. (14.14) we obtain

$$\begin{aligned} V_M &= V_t + \sqrt{2(V_{DD} - V_t)V_X + V_X^2} - V_X \\ &= 0.4 + \sqrt{2(1.2 - 0.4)0.035 + 0.035^2} - 0.035 \end{aligned}$$

$$V_M = 0.6 \text{ V}$$

From Eq. (14.20) we get

$$\begin{aligned} V_{IH} &= V_t + 1.63\sqrt{V_{DD}V_X} - V_X \\ &= 0.4 + 1.63\sqrt{1.2 \times 0.035} - 0.035 = 0.7 \text{ V} \end{aligned}$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.7 = 0.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 0.435 - 0.05 = 0.385 \text{ V}$$

**14.27**  $V_t = 0.3V_{DD}$ ,  $V_M = V_{DD}/2$

From Eq. (14.13) on page 1108 of text, we obtain

$$V_X|_{V_M = \frac{V_{DD}}{2}} = \frac{\left(\frac{V_{DD}}{2} - V_t\right)^2}{V_{DD}}$$

$$= \frac{(0.5V_{DD} - 0.3V_{DD})^2}{V_{DD}}$$

$$\Rightarrow V_X = 0.04V_{DD}$$

$$V_{OH} = V_{DD}$$

From Eq. (14.12), we get

$$V_{IL} = V_t + V_X = 0.3V_{DD} + 0.04V_{DD}$$

$$= 0.34V_{DD}$$

From Eq. (14.20), we obtain

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_X} - V_X$$

$$= 0.3V_{DD} + 1.63\sqrt{V_{DD} \times 0.04V_{DD}} - 0.04V_{DD}$$

$$= 0.586V_{DD}$$

From Eq. (14.22), we get

$$V_{OL} = \frac{V_{DD}}{1 + [(V_{DD} - V_t)/V_X]}$$

$$= \frac{V_{DD}}{1 + \frac{V_{DD} - 0.3V_{DD}}{0.04V_{DD}}} = 0.054V_{DD}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= V_{DD} - 0.586V_{DD} = 0.414V_{DD}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.34V_{DD} - 0.054V_{DD} = 0.286V_{DD}$$

For  $V_{DD} = 1.2 \text{ V}$ :

$$V_X = 0.048 \text{ V}, V_{OH} = 1.2 \text{ V}, V_{IL} = 0.408 \text{ V},$$

$$V_{IH} = 0.703 \text{ V}, V_{OL} = 0.065 \text{ V},$$

$$NM_H = 0.50 \text{ V}, NM_L = 0.34 \text{ V}$$

$$P_D = V_{DD}I_D$$

$$= V_{DD} \times \frac{V_{DD} - V_{OL}}{R_D}$$

Substituting for  $R_D$  from

$$R_D = \frac{1}{k_n V_X}$$

we obtain

$$P_D = V_{DD}(V_{DD} - 0.054V_{DD}) \times k_n \times 0.04V_{DD}$$

$$P_D = 0.038V_{DD}^3 \times k'_n \left(\frac{W}{L}\right)$$

$$= 0.038 \times 1.2^3 \times 0.5 \times 10^{-3} \left(\frac{W}{L}\right)$$

$$= 0.033 \left(\frac{W}{L}\right), \text{ mW}$$

For  $V_M = \frac{V_{DD}}{2}$  we have

$$\frac{V_{DD}}{2} = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

$$\Rightarrow r = \left[ \frac{V_{DD} - V_t}{(V_{DD}/2) - V_t} \right]^2 - 1$$

For  $V_{DD} = 1.8$  V and  $V_t = 0.4$  V (from Example 14.3) we obtain

$$r = \left( \frac{1.8 - 0.4}{0.9 - 0.4} \right)^2 - 1 = 6.84$$

**14.30** Refer to Example 14.3 (page 1112 of text)

$$V_{OH} = V_{DD} = 1.2$$
 V

$$V_{OL} = (V_{DD} - V_t) \left[ 1 - \sqrt{1 - (k_p/k_n)} \right]$$

$$= (1.2 - 0.4) \left[ 1 - \sqrt{1 - \frac{1}{5}} \right]$$

$$= 0.084$$
 V

$$I_{DD} = \frac{1}{2} k_p (V_{DD} - V_t)^2$$

$$= \frac{1}{2} \times 0.1 (1.2 - 0.4)^2$$

$$= 0.032$$
 mA = 32  $\mu$ A

$$P_{av} = \frac{1}{2} V_{DD} I_{DD} = \frac{1}{2} \times 1.2 \times 32 = 19.2$$
  $\mu$ W

From the statement of Exercise 14.5, we have

$$V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r+1}}$$

where  $r = k_n/k_p = 5$ , thus

$$V_M = 0.4 + \frac{1.2 - 0.4}{\sqrt{5+1}}$$

$$= 0.73$$
 V

**14.31** (a) To obtain  $V_M = V_{DD}/2$ , the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5$$

$$\Rightarrow W_p = 2.5 W_n = 2.5 \times 1.5 \times 65 = 244$$
 nm

$$\text{Silicon area} = W_n L_n + W_p L_p$$

$$= 1.5 \times 65 \times 65 + 2.5 \times 1.5 \times 65 \times 65$$

$$= 1.5 \times 65 \times 65 (1 + 2.5)$$

$$= 22,181$$
 nm<sup>2</sup>

(b)  $V_{OH} = V_{DD} = 1$  V

$$V_{OL} = 0$$
 V

To obtain  $V_{IH}$ , we use Eq. (14.35):

$$V_{IH} = \frac{1}{8} (5 V_{DD} - 2 V_t)$$

$$= \frac{1}{8} (5 \times 1 - 2 \times 0.35)$$

$$= 0.5375$$
 V

To obtain  $V_{IL}$ , we use Eq. (14.36):

$$V_{IL} = \frac{1}{8} (3 V_{DD} + 2 V_t)$$

$$= \frac{1}{8} (3 \times 1 + 2 \times 0.35)$$

$$= 0.4625$$
 V

The noise margins can now be found as

$$NM_H = V_{OH} - V_{IH}$$

$$= 1 - 0.5375 = 0.4625$$
 V

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.4625 - 0 = 0.4625$$
 V

The noise margins are equal at approximately 0.46 V; a result of the matched design of the inverter.

(c) Since the inverter is matched, the output resistances in the two states will be equal. Thus,

$$r_{DSP} = r_{DSN} = 1 / \left[ (\mu_n C_{ox}) \left( \frac{W}{L} \right)_n (V_{DD} - V_t) \right]$$

$$= \frac{1}{0.47 \times 1.5 (1 - 0.35)} = 2.18$$
 k $\Omega$

**14.32**  $V_{OH} = 2.5$  V

$$V_{OL} = 0$$
 V

(a) For the matched case we have

$$W_p = 3.5 W_n$$

$$V_M = \frac{1}{2} V_{DD} = 1.25$$
 V

$$\text{Eq. (14.35): } V_{IH} = \frac{1}{8} (5 V_{DD} - 2 V_t)$$

$$= \frac{1}{8} (5 \times 2.5 - 2 \times 0.5)$$

$$= 1.4375$$
 V

$$\text{Eq. (14.36): } V_{IL} = \frac{1}{8} (3 V_{DD} + 2 V_t)$$

$$= \frac{1}{8} (3 \times 2.5 + 2 \times 0.5)$$

$$= 1.0625$$
 V

$$NM_H = NM_L = 1.0625$$
 V

$$\text{Silicon area} = W_n L_n + W_p L_p$$

$$= 1.5 \times 0.25 \times 0.25 + 3.5 \times 1.5 \times 0.25 \times 0.25$$

$$= 4.5 \times 1.5 \times 0.25^2 = 0.42$$
  $\mu$ m<sup>2</sup>

(b)  $W_p = W_n$  (minimum-size design):

$$\text{Eq. (14.40): } r = \sqrt{\frac{\mu_p}{\mu_n} \frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5} \times 1} = 0.53$$

$$\begin{aligned} \text{Eq. (14.39): } V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ &= \frac{0.53(2.5 - 0.5) + 0.5}{0.53 + 1} \\ &= 1.02 \text{ V} \end{aligned}$$

Thus,  $V_M$  shifts to the left by 0.23 V. Assuming  $V_{IL}$  shifts by approximately the same amount, then

$$V_{IL} \simeq 1.0625 - 0.23 \simeq 0.83 \text{ V}$$

Since  $NM_L = V_{IL}$ ,  $NM_L$  will be reduced by approximately 22% (relative to the matched case).

$$\text{Silicon area} = W_n L_n + W_p L_p$$

$$= 1.5 \times 0.25 \times 0.25 + 1.5 \times 0.25 \times 0.25$$

$$= 3 \times 0.25^2 = 0.19 \mu\text{m}^2$$

which is a reduction of 55% relative to the matched case.

(c)  $W_p = 2W_n$  (a compromise design):

$$\begin{aligned} \text{Eq. (14.40): } r &= \sqrt{\frac{\mu_p}{\mu_n} \frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5} \times \frac{2}{1}} \\ &= 0.756 \end{aligned}$$

$$\begin{aligned} \text{Eq. (14.39): } V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ &= \frac{0.756(2.5 - 0.5) + 0.5}{0.756 + 1} \\ &= 1.15 \text{ V} \end{aligned}$$

Thus, relative to the matched case the switching point ( $V_M$ ) is shifted left by  $(1.25 - 1.15) = 0.1$  V. Assuming that  $V_{IL}$  is reduced by approximately the same amount, then

$$V_{IL} = 1.0625 - 0.1 = 0.9625 \text{ V}$$

Thus,  $NM_L$  which equals  $V_{IL}$  is reduced by about 9% (relative to the matched case).

$$\text{Silicon area} = W_n L_n + W_p L_p$$

$$= 1.5 \times 0.25 \times 0.25 + 2 \times 1.5 \times 0.25 \times 0.25$$

$$= 3 \times 1.5 \times 0.25^2$$

$$= 0.28 \mu\text{m}^2$$

Compared to the matched case, the silicon area is reduced by 33%.

**14.33**  $Q_N$  will be operating in the triode region, thus

$$I_{Dn} = k'_n \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_m)V_O - \frac{1}{2} V_O^2 \right]$$

For  $V_m = 0.3V_{DD}$  and  $V_O = 0.1V_{DD}$ , we have

$$\begin{aligned} I_{Dn} &= k'_n \left( \frac{W}{L} \right)_n \\ &\left[ (V_{DD} - 0.3V_{DD}) \times 0.1V_{DD} - \frac{1}{2} \times 0.1^2 V_{DD}^2 \right] \\ &= k'_n (W/L)_n (0.07V_{DD}^2 - 0.005V_{DD}^2) \\ &= 0.065k'_n (W/L)_n V_{DD}^2 \quad \text{Q.E.D.} \end{aligned}$$

For  $V_{DD} = 1.3$  V,  $k'_n = 0.5$  mA/V<sup>2</sup> and  $I_{Dn} = 0.1$  mA, we have

$$0.1 = 0.065 \times 0.5 (W/L)_n \times 1.3^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_n = 1.82$$

**14.34** For  $v_I = +1.5$  V,  $Q_N$  will be conducting and operating in the triode region while  $Q_P$  will be off. Thus, the incremental resistance to the left of node A will be  $r_{DSN}$ ,

$$\begin{aligned} r_{DSN} &= \frac{1}{k_n(V_I - V_m)} \\ &= \frac{1}{0.2(1.5 - 0.5)} = 5 \text{ k}\Omega \end{aligned}$$

Thus,

$$\begin{aligned} v_a &= 100 \left( \frac{5}{5 + 100} \right) \\ &= 4.8 \text{ mV} \end{aligned}$$

For  $v_I = -1.5$  V,  $Q_N$  will be off but  $Q_P$  will be operating in the triode region with a resistance  $r_{DSP}$ ,

$$\begin{aligned} r_{DSP} &= \frac{1}{k_p(V_{SGP} - |V_{tp}|)} \\ &= \frac{1}{0.04(1.5 - 0.5)} = 25 \text{ k}\Omega \end{aligned}$$

Thus,

$$v_a = 100 \left( \frac{25}{25 + 100} \right) = 20 \text{ mV}$$

**14.35** From Eq. (14.39) we have

$$\begin{aligned} V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ rV_M + V_M &= r(V_{DD} - |V_{tp}|) + V_m \\ r(V_{DD} - |V_{tp}| - V_M) &= V_m - V_m \\ \Rightarrow r &= \frac{V_M - V_m}{V_{DD} - |V_{tp}| - V_M} \quad \text{Q.E.D.} \end{aligned}$$

For  $V_{DD} = 1.3$  V,  $V_m = |V_{tp}| = 0.4$  V, to obtain  $V_M = 0.6V_{DD}$ , we need

$$r = \frac{0.6 \times 1.3 - 0.4}{1.3 - 0.4 - 0.6 \times 1.3} = 3.167$$

But,

$$r = \sqrt{\frac{\mu_p}{\mu_n} \frac{W_p}{W_n}}$$

$$3.167 = \sqrt{\frac{1}{4} \times \frac{W_p}{W_n}}$$

$$\frac{W_p}{W_n} = 3.167^2 \times 4 = 40.1$$

**14.36** The current reaches its peak at  $v_I = V_M = \frac{V_{DD}}{2}$ . At this point, both  $Q_N$  and  $Q_P$  are operating in the saturation region and conducting a current

$$\begin{aligned} I_{DP} = I_{DN} &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n \left( \frac{V_{DD}}{2} - V_t \right)^2 \\ &= \frac{1}{2} \times 500 \times 1.5 \left( \frac{1.3}{2} - 0.4 \right)^2 \\ &= 23.4 \mu\text{A} \end{aligned}$$

**14.37** Refer to Example 14.4 (page 1121 of text) except here:

$V_{DD} = 1.3$  V,  $V_m = |V_{tp}| = 0.4$  V,  $\mu_n = 4 \mu_p$ , and  $\mu_n C_{ox} = 0.5$  mA/V<sup>2</sup>. Also,  $Q_N$  and  $Q_P$  have  $L = 0.13 \mu\text{m}$  and  $(W/L)_n = 1.5$ .

(a) For  $V_M = V_{DD}/2 = 0.65$  V, the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 4$$

Since  $W_n/L = 1.5$ ,  
 $W_n = 1.5 \times 0.13 = 0.195 \mu\text{m}$ . Thus,

$$W_p = 4 \times 0.195 = 0.78 \mu\text{m}$$

For this design, the silicon area is

$$\begin{aligned} A &= W_n L + W_p L = L(W_n + W_p) \\ &= 0.13(0.195 + 0.78) = 0.127 \mu\text{m}^2 \end{aligned}$$

(b)  $V_{OH} = V_{DD} = 1.3$  V

$$V_{OL} = 0 \text{ V}$$

To obtain  $V_{IH}$ , we use Eq. (14.35):

$$\begin{aligned} V_{IH} &= \frac{1}{8}(5V_{DD} - 2V_t) \\ &= \frac{1}{8}(5 \times 1.3 - 2 \times 0.4) \\ &= 0.7125 \text{ V} \end{aligned}$$

To obtain  $V_{IL}$ , we use Eq. (14.36):

$$\begin{aligned} V_{IL} &= \frac{1}{8}(3V_{DD} + 2V_t) \\ &= \frac{1}{8}(3 \times 1.3 + 2 \times 0.4) \\ &= 0.5875 \text{ V} \end{aligned}$$

We can now compute the noise margins as

$$\begin{aligned} N_{MH} &= V_{OH} - V_{IH} = 1.3 - 0.7125 \\ &= 0.5875 \text{ V} \simeq 0.59 \text{ V} \end{aligned}$$

$$\begin{aligned} N_{ML} &= V_{IL} - V_{OL} = 0.5875 - 0 \\ &= 0.5875 \text{ V} \simeq 0.59 \text{ V} \end{aligned}$$

For  $v_I = V_{IH} = 0.7125$  V, we can obtain the corresponding value of  $v_O$  by substituting in Eq. (14.34):

$$v_O = V_{IH} - \frac{V_{DD}}{2} = 0.7125 - 0.65 = 0.0625 \text{ V}$$

Thus, the worst-case value of  $V_{OL}$  is  $V_{O\max} = 0.0625 \simeq 0.06$  V, and the noise margin  $NM_L$  reduces to

$$NM_L = 0.5875 - 0.0625 = 0.5250 \text{ V}$$

or approximately 0.53 V.

From symmetry, we can obtain the value of  $v_O$  corresponding to  $v_I = V_{IL}$  as

$$\begin{aligned} v_O &= V_{DD} - 0.0625 \\ &= 1.3 - 0.0625 = 1.2375 \text{ V} \simeq 1.24 \text{ V} \end{aligned}$$

Thus, the worst-case value of  $V_{OH}$  is  $V_{OH\min} \simeq 1.24$  V, and the noise margin  $NM_H$  is reduced to

$$\begin{aligned} N_{MH} &= V_{OH\min} - V_{IH} \\ &= 1.2375 - 0.7125 = 0.5250 \text{ V} \end{aligned}$$

or approximately 0.53 V.

Note that the reduction in the noise margin (about 0.06 V) is slight.

(c) The output resistance of the inverter in the low-output state is

$$\begin{aligned} r_{DSN} &= \frac{1}{\mu_n C_{ox} (W/L)_n (V_{DD} - V_m)} \\ &= \frac{1}{0.5 \times 1.5(1.3 - 0.4)} = 1.48 \text{ k}\Omega \end{aligned}$$