EQUATION SHEET

First-Order RL or RC Circuit Step Response

$v_C(t) \text{ or } i_L(t) = (IV - FV)e^{-t/\tau} + FV, t \geq 0$

Second-Order Differential Equation

Nonhomogeneous	Homogeneous
$\frac{d^2y_p(x)}{dx^2} + 2\zeta\omega_o\frac{dy_p(x)}{dx} + \omega_o^2 y_p(x) = g(x)$	$\frac{d^2y_n(x)}{dx^2} + 2\zeta\omega_o\frac{dy_n(x)}{dx} + \omega_o^2 y_n(x) = 0$

General Forms of Particular Responses

g(x)	**Form of y_P(x)**
1 (any constant)	A
5x+7	Ax+B
$3x^2$-2	Ax^2+Bx+C
x^3-x+1	Ax^3+Bx^2+Cx+D
sin(4x)	Acos(4x)+Bsin(4x)
cos(4x)	Acos(4x)+Bsin(4x)
e^{5x}	Ae^{5x}

Series RLC ODE

$\frac{d^2v_{CN}(t)}{dt^2} + \frac{R_T}{L}\frac{dv_{CN}(t)}{dt} + \frac{1}{LC}v_{CN}(t) = \frac{v_T(t)}{LC}$	$\zeta = \frac{R_T}{2}\sqrt{\frac{C}{L}} \quad \omega_o = {}^1/_{\sqrt{LC}}$

Parallel RLC ODE

$\frac{d^2i_{LN}(t)}{dt^2} + \frac{1}{R_TC}\frac{di_{LN}(t)}{dt} + \frac{1}{LC}i_{LN}(t) = \frac{i_N(t)}{LC}$	$\zeta = \frac{1}{2R_T}\sqrt{\frac{L}{C}} \quad \omega_o = {}^1/_{\sqrt{LC}}$

General Roots of Characteristic Equation and Natural Response

$s_{1,2} = \omega_o(-\zeta \pm \sqrt{\zeta^2 - 1})$	$y_N(t) = K_1e^{s_1t} + K_2e^{s_2t}, t \geq 0$

Case A

$s_{1,2} = -\alpha_1, -\alpha_2$	$y_N(t) = K_1e^{-\alpha_1t} + K_2e^{-\alpha_2t}, t \geq 0$

Case B

$s_{1,2} = -\alpha$	$y_N(t) = K_1e^{-\alpha t} + K_2te^{-\alpha t}, t \geq 0$

Case C

$s_{1,2} = -\alpha \pm j\beta$ $\alpha = \zeta\omega_o \text{ and } \beta = \omega_d = \omega_o\sqrt{1-\zeta^2}$	$y_N(t) = K_1e^{-\alpha t}\cos(\beta t) + K_2e^{-\alpha t}\sin(\beta t), t \geq 0$

Some ways to represent the characteristic equation of second-order circuits

$s^2 + 2\zeta\omega_o s + \omega_o^2 = 0$	$(s+\alpha)^2 + \beta^2 = 0$	$(s - p_1)(s - p_2) = 0$

How to determine parameters from a response plot

$\omega_d = \beta = 2\pi\frac{1}{T}$	$\delta = \ln\left(\frac{y_1 - y_\infty}{y_2 - y_\infty}\right)$	$\zeta = \frac{1}{\sqrt{1 + \frac{4\pi^2}{\delta^2}}}$

Initial and Final Value Theorems

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

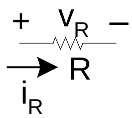
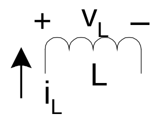
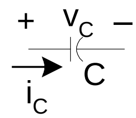
TABLE 9–2 BASIC LAPLACE TRANSFORM PAIRS

Signal	Waveform $f(t)$	Transform $F(s)$
Impulse	$\delta(t)$	1
Step function	$u(t)$	$\frac{1}{s}$
Ramp	$tu(t)$	$\frac{1}{s^2}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s+\alpha}$
Damped ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s+\alpha)^2}$
Sine	$[\sin \beta t]u(t)$	$\frac{\beta}{s^2+\beta^2}$
Cosine	$[\cos \beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped sine	$[e^{-\alpha t}\sin \beta t]u(t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
Damped cosine	$[e^{-\alpha t}\cos \beta t]u(t)$	$\frac{(s+\alpha)}{(s+\alpha)^2+\beta^2}$

TABLE 9–1 BASIC LAPLACE TRANSFORMATION PROPERTIES

Properties	Time Domain	Frequency Domain
Independent variable	t	s
Signal representation	$f(t)$	$F(s)$
Uniqueness	$\mathscr{L}^{-1}\{F(s)\}(=)[f(t)]u(t)$	$\mathscr{L}\{f(t)\} = F(s)$
Linearity	$Af_1(t) + Bf_2(t)$	$AF_1(s) + BF_2(s)$
Integration	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
Differentiation	$\frac{df(t)}{dt}$	$sF(s) - f(0-)$
	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f'(0-)$
	$\frac{d^3f(t)}{dt^3}$	$s^3F(s) - s^2f(0-) - sf'(0-) - f''(0-)$
s-Domain translation	$e^{-\alpha t}f(t)$	$F(s+\alpha)$
t-Domain translation	$f(t-a)u(t-a)$	$e^{-as}F(s)$

Form of F(s)	**Technique**	**Residues**
real distinct roots	PFE	$k_i = (s - p_i)F(s)\vert_{s=p_i}$
complex roots	determine residue k using PFE	$f(t) = 2\vert k\vert e^{-\alpha t}\cos(\omega t + \angle k)$
real repeated roots	factor repeated root then PFE	$k_i = (s - p_i)F(s)\vert_{s=p_i}$
improper function	long division then PFE	$k_i = (s - p_i)F(s)\vert_{s=p_i}$

Device/Model	Resistance – R	Inductance – L	Capacitance – C
Units	ohms, Ω	Henrys, H	Farads, F
Circuit Symbol			
Voltage Equation	$v_R = i_R R$	$v_L = L\, di_L/dt$	$v_C = v_C(0^+) + (1/C)\int i_C\, dt$
Current Equation	$i_R = v_R G = v_R/R$	$i_L = i_L(0^+) + (1/L)\int v_L\, dt$	$i_C = C\, dv_C/dt$
Power Equation	$p_R = i_R \times v_R$	$p_L = i_L \times v_L$	$p_C = i_C \times v_C$
Energy Equation	$w_R = \int p_R\, dt$	$w_L = ½\, L\, i_L^2$	$w_C = ½\, C\, v_C^2$
Energy Storage	None	Magnetic Field	Electric Field
Continuity Equation	N/A	$i_L(\tau^-) = i_L(\tau^+)$	$v_C(\tau^-) = v_C(\tau^+)$
Typical Range	1 kΩ – 10 MΩ	1 μH – 10 H	10 pF – 100 μF
Series	$R_{EQ}=R_1+R_2+\ldots$	$L_{EQ}=L_1+L_2+\ldots$	$C_{EQ} = \dfrac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\cdots}$
Parallel	$R_{EQ} = \dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$L_{EQ} = \dfrac{1}{\frac{1}{L_1}+\frac{1}{L_2}+\cdots}$	$C_{EQ}=C_1+C_2+\ldots$
Impedance	Z=R	Z=1/(jωC)	Z=jωL
Impedance @ ω=0 (dc)	R	behaves like a short	behaves like an open
Impedance @ ω=∞ (very high freq)	R	behaves like an open	behaves like a short