ECE 332 Lab 5s

RLC Simulation

An RLC circuit is a linear second-order system, which is described by a linear second-order differential equation. We sometimes encounter an operating circuit or physical system but we don't know values of the elements. To get them, we can observe the system's behavior on an oscilloscope and calculate their values.

For the simulation we will be doing here, we know those element values. But we are going to get data from the scope display as if we didn't and then do some reverse calculations to find out what they are. If we work carefully, the numbers should agree. In the next lab, we will build a similar circuit from real circuit elements.

Goals

The goals for this simulation are

- Build a series RLC circuit in Multisim®.
- Simulate the step response of this circuit.
- Derive from the time-domain response the values of R and L assuming we already know C.
- Observe critical damping

There is a way to get all three element values, i.e., you don't have to know one at the outset, but this is beyond what we want to do today.

Introduction

Figure 1 shows our RLC circuit. The differential equation for this series RLC circuit is

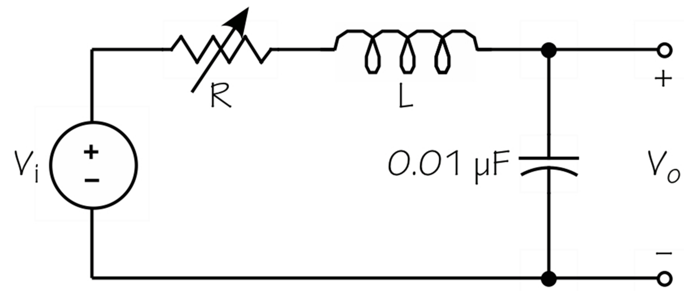

Figure 1: Series RLC Circuit

$$LC\frac{dv_o^2(t)}{dt^2} + RC\frac{dv_o(t)}{dt} + v_o(t) = v_i(t)$$

The homogeneous equation is:

$$LC\frac{dv_o^2(t)}{dt^2} + RC\frac{dv_o(t)}{dt} + v_o(t) = 0$$

There are several ways of writing a standard second-order equation; this is one of them:

$$\frac{d^2v_o(t)}{dt^2} + 2\alpha\frac{dv_o(t)}{dt} + \omega_o^2 v_o(t) = 0$$

Values of ω_o, which is the *undamped* natural freqeuncy, and α, which is the attenuation constant, are related to the circuit elements as follows:

$$\alpha = \frac{R}{2L}, \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

When we have only a time-domain display (i.e. a scope display) of the response of such a circuit, these two values are difficult to directly obtain. Instead, we will observe two others: ζ (zeta) and ω_d.

The *damping factor* ζ is related to the circuit elements by

$$\zeta = \frac{\alpha}{\omega_o} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

We can get ω_d, which is the *damped natural frequency*, from the scope display. The *undamped* natural frequency, ω_o, is not directly observable. However, it is related to the *damped* natural frequency and the damping factor by

$$\omega_d = \omega_o \sqrt{1 - \zeta^2}$$

As we will see when we start to make measurements, we can't even measure ζ and ω_d directly.

Procedure

We have four things to do in this lab:

- Build a series RLC circuit in Multisim
- Obtain a scope display of the circuit's time-domain response
- Take data from the scope display and calculate the values of R and L, knowing C = 0.01 μF, and compare the results with the actual values.
- Adjust R to get critical damping and compare this with the expected value.

A. Build the Multisim Circuit

Figure 2 is a picture of a Multisim screen for the RLC circuit. While you create a similar layout, here are a few special things you will need to do:

1. Select a 0.01 μF capacitor and use Control-R to make it vertical.
2. Select a 100 mH inductor.
3. Select a potentiometer (commonly called a pot) but before placing it, use Control-R **three times** to give it the orientation this circuit requires. Once is not enough!
4. Double click on the potentiometer symbol and change its paramters to: **Resistance = 10 kΩ** and **Increment = 1%**.

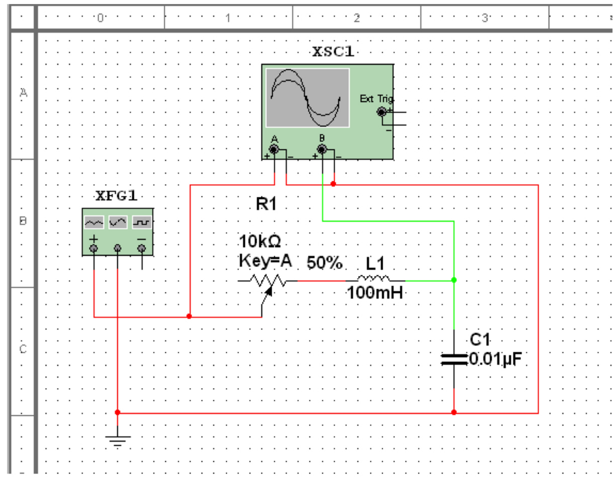

Figure 2: RLC Circuit Layout in Multisim

5. Set the pot to 10% by pressing the A key to increase the percentage, or shift-A to decrease it.
6. Instead of the source we used in Lab 2s, we will use a function generator from **Simulate⇒Instruments⇒Function Generator**.
7. Connect the elements as shown in Figure 2.
8. If you double-click on the wire connected to the B+ terminal of the scope and change its color to bright green, that signal's trace on the scope will be green, making it easier to distinguish from the input signal.

B. Simulate

1. Double-click on the Function Generator and set its parameters as in Figure 3. Note the offset is negative.
2. Double-click on the scope and set its parameters as shown in Figure 4.

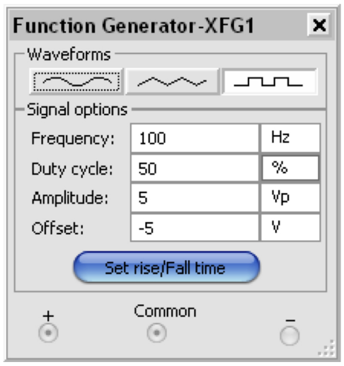

Figure 3: Function Generator Setup

3. Start the simulation via **Simulation⇒Run** or the green triangle just above your circuit drawing. You should see a response like that in Figure 4.

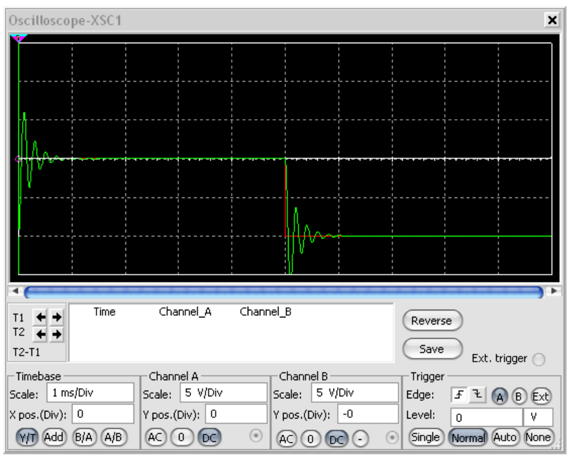

Figure 4: Initial Scope Setup

4. This scope display is not adequate for the measurements we want, so improve the display so the points of interest occupy much more of the screen as shown in Figure 5, where the final settings are 100 μs / div and 2 V / div.

5. Now that you have a proper display, freeze the screen by clicking the **Single** button and then stopping the simulation (red button above your drawing space).

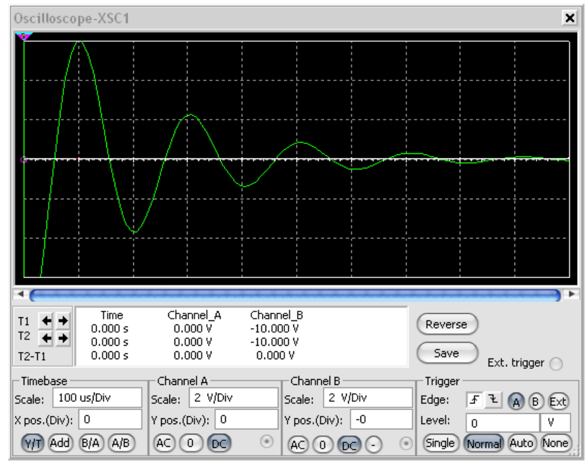

Figure 5: Improved Scope Setup

C. Collect Data and Calculate R and L

The information we get from the scope does not give us the parameters of the differential equation directly. Instead, we will measure three values and then calculate the equation parameters and the element values.

Figure 6 shows the scope with the cursors positioned at the first two peaks of the damped-sinusoidal waveform. We want from this screen the *time* between those two peaks, which is the period of the oscillation. We also want the *ratio* of the heights of these two peaks, *measured from the line onto which the oscillation is settling*.

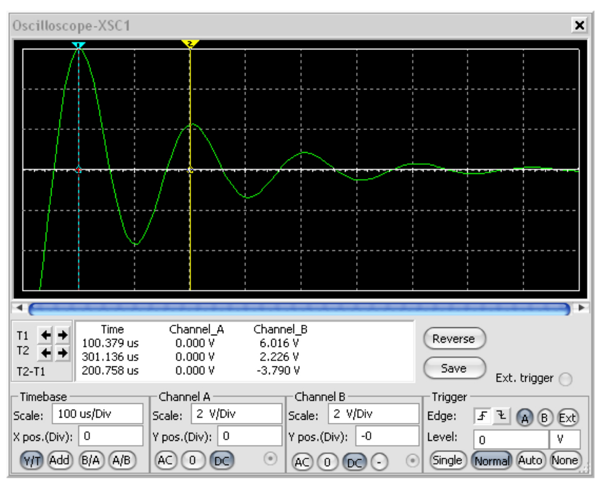

Figure 6: Cursor Measurements

1. Look at Figure 7 and use the cursors to collect and record on the hand-in page three pieces of data:

1. Distance T between the first two peaks, measured in seconds. The calibration is 100 μs / div.

2. The height, x_1, of the first peak *from the base line*, measured in volts. The calibration is 2 V / div.

3. The height, x_2, of the second peak.

2. From the period T, calculate and record the *damped* natural frequency, ω_d.

$$\omega_d = 2\pi \frac{1}{T} \quad rad/sec$$

3. Calculate and record a quantity called the *logarithmic decrement*, δ, which is related to the exponential decay of the damped sinusoid:

$$\delta = \ln \frac{x_1}{x_2}$$

4. From the logarithmic decrement, find and record the damping factor, ζ:

$$\zeta = \frac{1}{\sqrt{1 + \frac{4\pi^2}{\delta^2}}}$$

5. Finally, use the equations from the beginning of this lab to find the values of R and L and compare them with the actual circuit elements used. The resistance R is the precentage shown on the potentiometer times 10 kΩ.

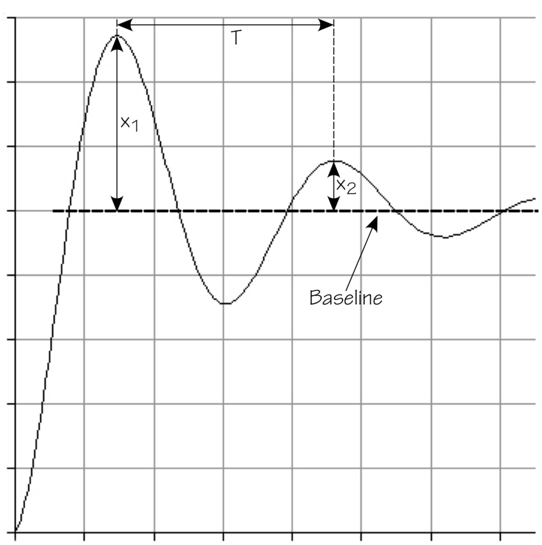

Figure 7: Reading a 2nd Order Response

D. Critical Damping

Now we will adjust the resistance to get the response to be critically damped. The point at which the response is critically damped is when the response has just barely lost its oscillatory behavior. Since R in this circuit is a potentiometer (pot for short), we can change its value and see when oscillation ceases.

1. Restart the simulation.

2. Note the pot says Key - A which means the A key on the keyboard will step the resistance. Click somewhere in the circuit window and then press A to increase the resistance by 1% of 10 kΩ. Shift-A will decrease it by 1%.

3. Continue to adjust the pot until the oscillation just barely disappears. There should be no wiggle beyond the base line.

4. Expand the vertical scale of Channel B as you get very close to critical damping by making the Scale V/Div number smaller and smaller.

5. Record the percentage reading and the actual resistance R for critical damping.

6. Calculate the damping factor ζ using this value of R with L = 100 mH and C = 0.01 μF. Ideally, $\zeta = 1$ for critical damping.

The End

Look back at the goals to see if we accomplished all of them. We started out intending to simulate the operation of a series RLC circuit, calculate from the time-domain response the values of R and L, and observe what critical damping looks like. We've done all those.

In the next lab we are going to build a physical circuit and make some of the same observations.

Report of Lab 5s Results

Name ______________________________

Show Calculations! Show Units!

Scope data:

T ________________ x_1 ________________ x_2 ________________

Calculated Values:

ω_d ________________ δ ________________ ζ ________________

Calculated element values:

C = 0.01 μF L ________________ R ________________

L_{error} ______________ R_{error} ______________

$$\%error = \frac{Measured - Theory}{Theory}(100\%)$$

Critical Damping:

R = ________________ Calculated ζ = ________________ Expected ζ = 1