ECE 332 Lab 15s

Circuit for an H(s)

There isn't much excitement in what we've been doing in class: given a circuit, find the transfer function H(s). Do electrical engineers really do this? Sure, but it certainly is not their full-time job. The task is much more interesting and useful-the other way around: given the transfer function H(s), devise a circuit that implements it.

Why is this more interesting? For one thing, it's creative, and that's what engineering is all about. For another, it's because there is no single right answer. Furthermore, it is possible to start with a transfer function H(s) for which no circuit can be found.

Here's an example. Suppose we need to implement a transfer function

$$H(s) = V_{out}(s)/V_{in}(s) = 0.6.$$

Hmm, this isn't a function of s and it sure looks like d-c. Would a voltage divider do the job? Sure, so how about a 400-ohm resistor in series with the input and a 600-ohm resistor across the output? That'll yield 600/(600+400) = 0.6. But how about an 849-ohm resistor in series with the input and a 1.274-kiloohm resistor across the output? This also yields H(s) = 0.6. That's a silly combination, you might say. Maybe, but not-silly was not a design constraint.

How does one do this circuit design? That's where all the analysis you've been doing comes into play. Basically, you look at the desired H(s) and reach back into your experience to find some circuit model that looks like it might implement H(s). While there are more formal ways of approaching this design problem, experience is an important source of information.

Some Design Principles

There are some principles that, even though they are not laws written in stone, need to be considered by the designer:

- Keep basic circuits in mind as you think about the problem, circuits like the voltage divider, the inverting op-amp inverter, and the non-inverting op-amp.
- Stick to designs using resistors and capacitors; avoid inductors if at all possible. Inductors are messy circuit elements. They exhibit considerable parasitic resistance, they tend to be bulky, and they are difficult to implement on silicon.
- Watch for loading, making sure your circuit does not load the input circuit improperly. If you use a voltage divider, the circuit following it must not load the voltage divider.
- Be careful when you choose an inverting op-amp. Remember, it does draw current from the source driving it.
- Consider cascading circuits to implement portions of H(s), but not allowing one circuit to load the previous one.

Goals

This laboratory has a number of goals related to the design of circuits:

- Learn the block-diagram approach to transfer function design.
- Learn to scale circuit element values to fit available practical elements.
- Design a circuit to implement a particular H(s).

- Test the design using Matlab.
- Simulate the design using Multisim and compare with the Matlab result.
- Discuss how these compare and why.
- Combine all this in a report in Word using Matlab's publishing capabilty or Matlab's *notebook*.

Basic Circuits for H(s)

Here are four circuits that will implement several basic arrangements of H(s):

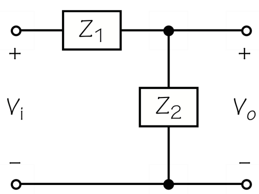

Figure 1: Voltage Divider

Voltage divider: Figure 1
Consider this H(s)

$$H_1(s) = \frac{K}{s+\alpha}$$

where K < 1. This can be implemented using the voltage divider of Figure 1, where

$$H_1(s) = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

If we arrange $H_1(s)$ a bit, we get:

$$H_1(s) = \frac{\frac{K}{s}}{1 + \frac{\alpha}{s}}$$

Remember the impedance of a capacitor is $\frac{1}{sC}$. If we match the numerator of $H_1(s)$ with the numerator of the voltage divider, then $Z_2(s) = \frac{K}{s}$, which can be implemented as a capacitor whose value is $\frac{1}{K}$ farads. Don't worry if a $\frac{1}{K}$ capacitor seems like it could be an impractical value; we will deal with that later.

Now match the denominators:

$$Z_1(s) + Z_2(s) = 1 + \frac{\alpha}{s}$$

If we substitute our newly found value of $Z_2(s)$ into this, we can solve for $Z_1(s)$:

$$Z_1(s) = 1 + \frac{\alpha}{s} - \frac{K}{s} = 1 + \frac{\alpha - K}{s}$$

This is a series combination of two circuit elements: a resistor of 1 Ω and a capacitor of $\frac{1}{K\alpha}$ farads. Figure 2 shows the circuit we have just designed. We will leave the quest of practical values for later.

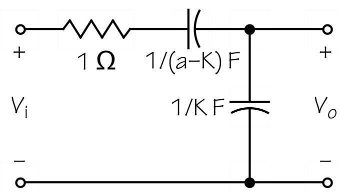

Figure 2: Voltage Divider for $\frac{K}{s+\alpha}$

Voltage Divider with Gain, Figure 3

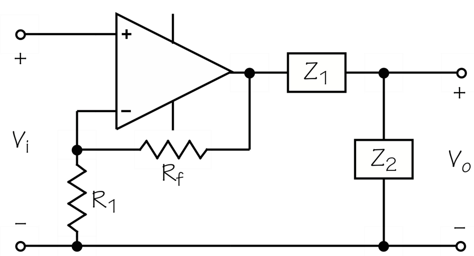

Figure 3: Voltage Divider with Gain

Suppose $H_1(s)$ in the previous design example requires a gain of 1 or greater. We now segregate the gain K from the voltage divider as shown in Figure 3 and we separate K from the rest of the H(s) in the process:

$$H_2(s) = \frac{K}{s+\alpha} = \frac{K}{\alpha}\frac{\alpha}{s+\alpha} = \frac{K}{\alpha}\frac{\frac{\alpha}{s}}{1+\frac{\alpha}{s}}$$

Matching as before:

$$Z_2(s) = \frac{\alpha}{s},\ Z_1(s) = 1$$

This can be implemented by a $\frac{1}{\alpha}$ farad capacitor across the output and a 1-ohm resistor in series with the input, as shown in Figure 4. Note the resistor ratio for the op-amp yields a gain of K.

Figure 5 shows two voltage dividers cascaded. This circuit could implement:

$$H_3(s) = \frac{K}{(s+\alpha)(s+\beta)} = \left[\frac{\frac{K_1}{s}}{1+\frac{\alpha}{s}}\right]\left[\frac{K_2}{\beta}\frac{\frac{\beta}{s}}{1+\frac{\beta}{s}}\right]$$

The first term is a voltage divider; the second is a voltage divider with gain.

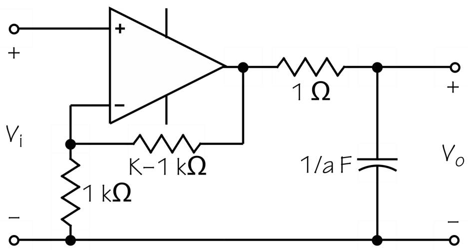

Figure 4: Voltage Divider for $\frac{K}{s+\alpha}$

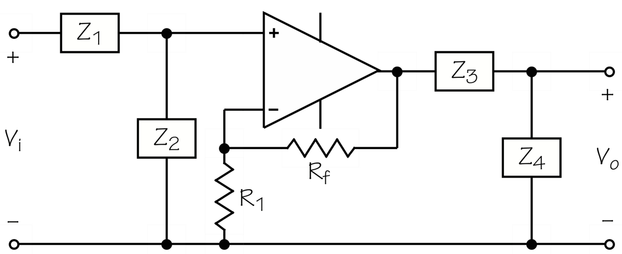

Figure 5: Two Voltage Dividers with Gain and Buffer

Inverting op-amp in Figure 6

Figure 6 is an inverter with a parallel RC circuit in both the input and the feedback paths. The transfer function for this circuit is:

$$H(s) = -\frac{Z_2(s)}{Z_1(s)}$$

If we remember admittance is one over impedance, we get a very nice result involving the resistors and capacitors:

$$H(s) = -\frac{Y_1(s)}{Y_2(s)} = -\frac{C_1 s + \frac{1}{R_1}}{C_2 s + \frac{1}{R_2}}$$

Now consider implementing

$$H_4(s) = -K\frac{s+\beta}{s+\alpha} = -\frac{Ks+K\beta}{s+\alpha}$$

Matching terms givers $C_1 = K$ farads, $R_1 = \frac{1}{K\beta}$ ohms, $C_2 = 1$ farad, and $R_2 = \frac{1}{\alpha}$ ohms. We will handle the strange element values in the next section.

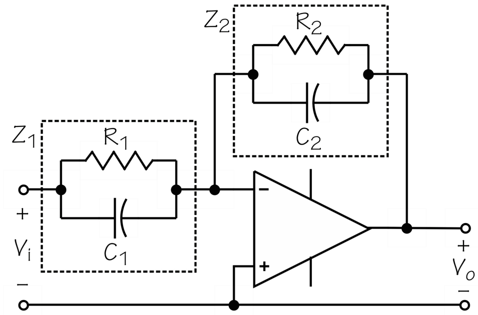

Figure 6: Inverter for $-\frac{K(s+\beta)}{s+\alpha}$

Scaling

Scaling takes the strange element values we have just created and converts them into practical values. Scaling relies on the fact that, if we multiply all of the impedances in a circuit by a certain factor, overall circuit performance will be unchanged. This is because our transfer functions are all ratios of impedances. Multiply the numerator and the denominator by the same factor and H(s) is unchanged.

The tricky part of scaling is remembering the impedance of a resistor depends on the value of the resistance, while the impedance of a capacitor depends on the reciprocal of the capacitance.

Suppose we have a circuit that contains a resistor that is designed to be 1/10 ohms and a capacitor that is designed to be 1/25 farads. We decide to scale the capacitor so its value becomes 0.1 μF. This means multiplying the capacitor's value by:

$$\frac{0.1\times 10^{-6}}{1/25} = 2.5\times 10^{-6}$$

This action has *decreased* the capacitor's value by a factor of 2.5×10^{-6}. Hence, we must *increase* the value of the resistor by that factor, dividing by the scale factor, thereby making the resistor value

$$\frac{1/10}{2.5\times 10^{-6}} = 40\ k\Omega$$

Our 1/10-Ω resistor is now 40 $k\Omega$ and our 1/25-F capacitor is not 0.1 μF, both practical values.

Remember, if we decrease capacitors in our circuit by a certain factor, we must increase all the resistors by the same factor.

Design Example

To bring all this together, let's design a circuit for:

$$H(s) = \frac{5000}{s+200}$$

We will use a voltage divider with gain, as shown in Figure 3. We rearrange H(s) into the right form for that circuit:

$$H(s) = \frac{5000}{200}\frac{200}{s+200} = 25\frac{200/s}{1+200/s}$$

The resultant circuit, before scaling, is in Figure 7 where the non-inverter has a gain of 25 using practical resistors.

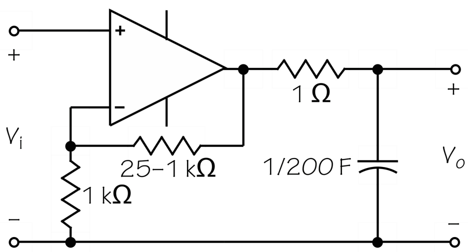

Figure 7: Example Before Scaling

Now we need to scale to practical values. Suppose we choose to use a 0.22 μF capacitor. The scaling factor is:

$$\frac{0.22\times10^{-6}}{1/200} = 4.4\times10^{-5}$$

This makes the resistor

$$\frac{1}{4.4\times10^{-5}} = 22.7\ k\Omega$$

Figure 8 shows the resulting circuit.

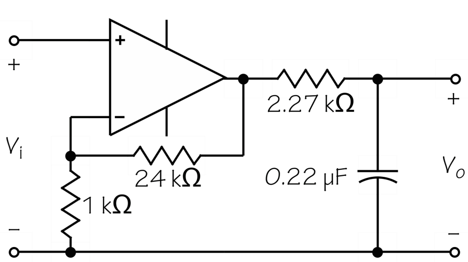

Figure 8: Example with reasonable values

Specifications

Design a network that meets the following specifications. Then, predict its response using Matlab, simulate its operation using MultiSim, compare the results, and discuss how well your design does or does not meet specifications, including possible causes of errors, how you dealt with them, and any changes you would make for a new design.

- Voltage transfer function is:

$$H(s) = \pm\frac{2\times10^4(s+100)}{(s+500)(s+10000)}$$

- All capacitors $< 1\ \mu$F; all resistors $\geq 10\ k\Omega$.
- No more than 3 op-amps.
- Source resistance = 5 kΩ.
- Output loading = 500 Ω.

Procedure

There are several steps needed to accomplish the requirements of this laboratory. All of the results should be combined into a single Word or pdf document, neatly formatted, with help from Matlab's publishing capability or Matlab's notebook.

1. Prediction

a. Use Matlab to create for the given H(s) its pole-zero plot, its step response, its Bode plot of frequency response, and its inverse Laplace transform

b. Create a table like the one below and fill in all the values.

	Parameter	Matlab	Simulate	% Error
Step	t_s (ms)			
Response	Final Value			
Freq	$\omega_{c1}(rad/s)$			
Response	$\omega_{c2}(rad/s)$			
	K(V/V)			

Note t_s is the 2% settling time.

2. Design

a. Design a prototype network that satisfies the H(s) specification using design techniques stated here and in your text and in class.
b. Scale your circuit to use reasonable values and standard parts.
c. Account for both source and load resistances.
d. Create a properly labeled drawing of your circuit. You may use Multisim for this requirement.

3. Simulate
a. Simulate your design using Multisim.
b. Create appropriately labeled drawings of the network's step and frequency responses.
c. Copy and paste from Multisim to Word document the circuit and the results of the simulations.
d. From the data provided by your simulation, fill in the rest of the table that was started in Step 1b.

4. Comparison

For each of the following, you are to compare data from Matlab and Multisim. If you wish to combine the graphical data on one plot of each type, instructions on how to export data from Multisim and import it into Matlab are in the lab directory of the course website.

5. Conclusions
Discuss briefly in your report whether your design met specs, focusing on where it differs from those specs. If there are differences, discuss why. Note errors that you made. Tell how you did troubleshooting if your network did not work properly the first time. Indicate how you could accomplish this design in better fashion.

Reporting

Print out a copy of your report and submit to your instructor.

The End

Your work on this exercise is finished when you have accomplished all the assigned tasks and submitted your report.