**Problem 10–10.** **(D)** For the circuit of Figure P10–10:

(a) Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeroes.

Combine the inductor and capacitor in parallel and then combine the result in series with the resistor.

$$Z_{\text{EQ}}(s) = R + \left(Ls \parallel \frac{1}{Cs}\right) = R + \frac{(Ls)(1/Cs)}{Ls + 1/Cs}$$

$$= R + \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}} = \frac{Rs^2 + \frac{1}{C}s + \frac{R}{LC}}{s^2 + \frac{1}{LC}}$$

$$= \frac{R\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}{s^2 + \frac{1}{LC}}$$

The poles are located at $s = \pm j1/\sqrt{LC}$ and the zeroes are located at

$$s = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

(b) If $R = 15$ kΩ, select values of $L$ and $C$ to locate poles at $\pm j200$ krad/s. Where are the resulting zeroes?

Pick $C = 0.01\ \mu$F. Solve for $L$.

$$(200\,\text{krad/s})^2 = \frac{1}{LC}$$

$$L = \frac{1}{(200\,\text{krad/s})^2(0.01\ \mu\text{F})} = 2.5\ \text{mH}$$

The zeroes are located at $s = -3.333 \pm j199.972$ krad/s.

**Problem 10–19.** For the circuit of Figure P10–19:

(a) Use current division to find $I_2(s)$.

Apply two-path current division.

$$I_2(s) = \frac{R_1 + Ls}{R_1 + R_2 + Ls} I_1(s)$$

(b) Use the look-back method to find $Z_N(s)$.

We have the following results:

$$Z_N(s) = (R_1 + Ls) \parallel R_2 = \frac{(R_1 + Ls)(R_2)}{R_1 + R_2 + Ls}$$

(c) If $I_1(s)$ is equal to $I_A/s$, find the poles and zeroes of $I_2(s)$ and identify the natural and the forced poles.

We have the following results:

$$I_2(s) = \frac{R_1 + Ls}{R_1 + R_2 + Ls}\left(\frac{I_A}{s}\right) = \frac{s + \dfrac{R_1}{L}}{s\left(s + \dfrac{R_1 + R_2}{L}\right)} I_A$$

The zero is located at $s = -R_1/L$ and the poles are located at zero and $s = -(R_1 + R_2)/L$. The natural pole is at $s = -(R_1 + R_2)/L$ and the forced pole is at zero.

**Problem 10–26.** The switch in Figure P10–25 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the $s$ domain and solve for $I_{\rm L}(s)$ and $i_{\rm L}(t)$ in symbolic form.

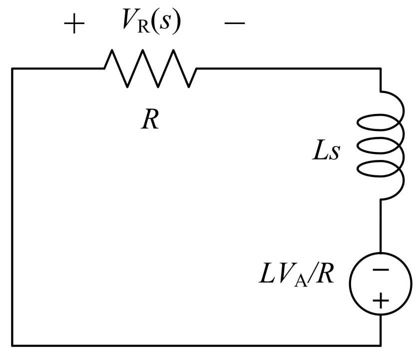
The initial inductor current is zero, so the transformed circuit after the switch moves is the source in series with the inductor and the $2R$ resistor. The voltage source is $V_{\rm S}(s) = V_{\rm A}/s$. Apply Ohm's law to find the current.

$$I_{\rm L}(s) = \frac{\dfrac{V_{\rm A}}{s}}{Ls + 2R} = \frac{\dfrac{V_{\rm A}}{L}}{s\left(s + \dfrac{2R}{L}\right)} = \frac{\dfrac{V_{\rm A}}{2R}}{s} - \frac{\dfrac{V_{\rm A}}{2R}}{s + \dfrac{2R}{L}}$$

$$i_{\rm L}(t) = \frac{V_{\rm A}}{2R}\left(1 - e^{-2Rt/L}\right)u(t)$$

**Exercise 10–2.** The source of the $t$-domain circuit of Figure 10–8(a) is suddenly turned off. Use Laplace techniques to solve for the voltage across the resistor $v_R(t)$.

In the steady state, the inductor current is $V_A/R$, so this is the initial inductor current when the source is turned off. In the $s$ domain, we have resistor $R$ in series with $Ls$ and voltage source $Li_L(0) = LV_A/R$. The resulting $s$-domain circuit is shown below.




Apply voltage division to find the voltage across the resistor.

$$V_R(s) = \frac{R}{R + Ls}\left(\frac{LV_A}{R}\right) = \frac{V_A}{s + \frac{R}{L}}$$

$$v_R(t) = \left[V_A e^{-Rt/L}\right] u(t)\ \text{V}$$

**Problem 10–13.** For the circuit of Figure P10–13:

(a) Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeroes.

Working from right to left, combine the resistor and capacitor in series. Combine that result in parallel with the other resistor. Combine that result in series with the other capacitor.

$$Z_{\text{EQ}}(s) = \left[\left(2R + \frac{1}{Cs}\right) \parallel 2R\right] + \frac{1}{2Cs} = \frac{4R^2 + 2R/Cs}{4R + 1/Cs} + \frac{1}{2Cs}$$

$$= \frac{4R^2Cs + 2R}{4RCs + 1} + \frac{1}{2Cs} = \frac{4RCs + 1 + 8R^2C^2s^2 + 4RCs}{2Cs(4RCs + 1)}$$

$$= \frac{8R^2C^2s^2 + 8RCs + 1}{2Cs(4RCs + 1)} = \frac{2RC\left(s^2 + \frac{1}{RC}s + \frac{1}{8R^2C^2}\right)}{2Cs\left(s + \frac{1}{4RC}\right)}$$

The poles are located at zero and $s = -1/4RC$. The zeroes are located at

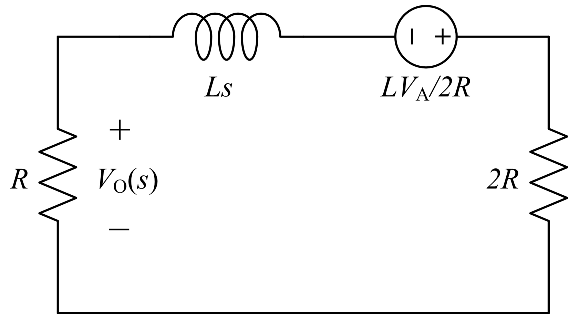
$$s = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{1}{2R^2C^2}}}{2} = \frac{1}{RC}\left(-\frac{1}{2} \pm \sqrt{\frac{1}{8}}\right)$$

(b) If its poles were located at $-20$ krad/s and zero, where would the poles move to if the value of $R$ was reduced to half of its current value?

If $R$ is reduced by a factor of two, the poles would increase in magnitude by a factor of two to be located at zero and $s = -40$ krad/s.

**Problem 10–25.** The switch in Figure P10–25 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the $s$ domain and solve for $I_L(s)$, $i_L(t)$, $V_O(s)$, and $v_O(t)$ in symbolic form.

The initial current through the inductor is $i_L(0) = V_A/2R$. The transformed circuit is shown below.




Apply Ohm's law to find the current and the output voltage.

$$I_L(s) = \frac{Li_L(0)}{2R + R + Ls} = \frac{\frac{LV_A}{2R}}{Ls + 3R} = \frac{\frac{V_A}{2R}}{s + \frac{3R}{L}}$$

$$i_L(t) = \frac{V_A}{2R} e^{-3Rt/L} u(t)$$

$$V_O(s) = -RI_L(s) = \frac{-\frac{V_A}{2}}{s + \frac{3R}{L}}$$

$$v_O(t) = -\frac{V_A}{2} e^{-3Rt/L} u(t)$$

**Problem 10–49.** **(D)** There is no initial energy stored in the circuit in Figure P10–49. Use circuit reduction to find the output network function $V_2(s)/V_1(s)$. Then select values of $R$ and $C$ so that the poles of the network function are approximately −2618 and −382 rad/s.

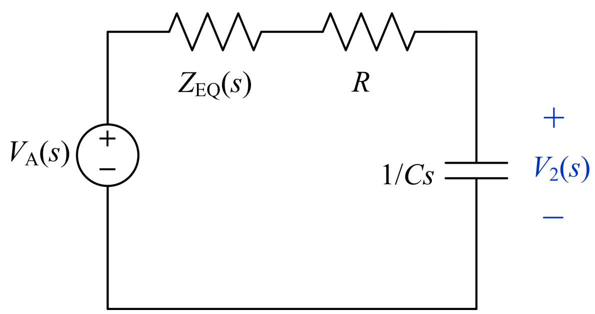
Perform a source transformation to get a current source $CsV_1(s)$ in parallel with the left capacitor. Combine the capacitor in parallel with the left resistor to get $Z_{EQ}(s)$.

$$Z_{EQ}(s) = \frac{R/Cs}{R + 1/Cs} = \frac{R}{RCs + 1}$$

Perform another source transformation to get a new voltage source $V_A(s)$.

$$V_A(s) = Z_{EQ}(s)CsV_1(s) = \left[\frac{R}{RCs+1}\right]\left[CsV_1(s)\right] = \frac{RCsV_1(s)}{RCs+1}$$

The resulting circuit is shown below.




Apply voltage division

$$V_2(s) = \left(\frac{\frac{1}{Cs}}{\frac{R}{RCs+1} + R + \frac{1}{Cs}}\right)\left(\frac{RCsV_1(s)}{RCs+1}\right)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R}{R + R^2Cs + R + R + \frac{1}{Cs}} = \frac{\frac{1}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}$$

For the required poles, we have:

$$(s+2618)(s+382) = s^2 + 3000s + 10^6$$

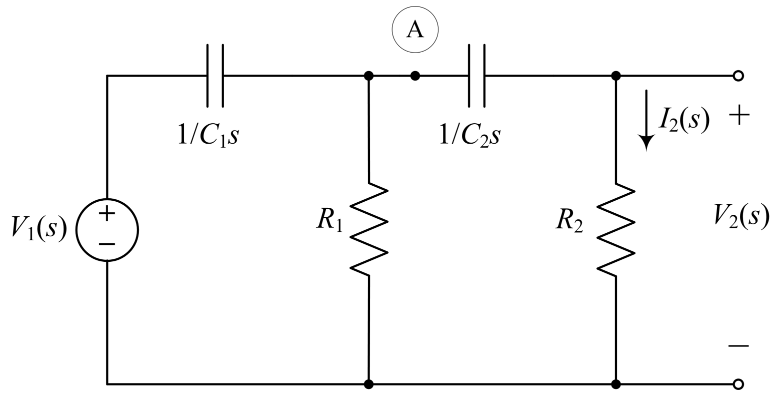
$$\frac{1}{RC} = 1000$$

Choose $R = 1\ \text{k}\Omega$ and $C = 1\ \mu\text{F}$ to meet the specifications. There are other valid solutions.

**Problem 10–53. (E)** There is no initial energy stored in the circuit in Figure P10–53.

(a) Transform the circuit into the $s$ domain and formulate node-voltage equations.

The $s$ domain circuit is shown below.




Write the node-voltage equations by inspection.

$$\left(C_1s + \frac{1}{R_1} + C_2s\right)V_A(s) - C_2sV_B(s) = C_1sV_1(s)$$

$$-C_2sV_A(s) + \left(C_2s + \frac{1}{R_2}\right)V_B(s) = 0$$

(b) Solve these equations for $V_2(s)$ in symbolic form.

Solve the second equation for $V_A(s)$ and substitute into the first equation.

$$C_2sV_A(s) = \left(C_2s + \frac{1}{R_2}\right)V_B(s) = \left(\frac{R_2C_2s+1}{R_2}\right)V_B(s)$$

$$V_A(s) = \left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_B(s)$$

$$C_1sV_1(s) = \left(C_1s + \frac{1}{R_1} + C_2s\right)\left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_B(s) - C_2sV_B(s)$$

$$R_1C_1sV_1(s) = \left(R_1C_1s + 1 + R_1C_2s\right)\left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_B(s) - R_1C_2sV_B(s)$$

$$R_1R_2C_1C_2s^2V_1(s) = (R_1R_2C_1C_2s^2 + R_1C_1s + R_2C_2s + 1 + R_1C_2s)V_B(s)$$

$$V_2(s) = V_B(s) = \frac{R_1R_2C_1C_2s^2V_1(s)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

(c) Insert an OP AMP buffer at point A and solve for $V_2(s)$ in symbolic form. How did inserting the buffer change the denominator, and therefore, the location of the poles?

We can find the output by applying voltage division twice.

$$V_2(s) = \left(\frac{R_1}{R_1 + 1/C_1 s}\right)\left(\frac{R_2}{R_2 + 1/C_2 s}\right)V_1(s)$$

$$= \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} = \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1}$$

The buffer reduced the coefficient of the $s$ term in the denominator. In general, this change reduces the damping coefficient.

(d) Using both separate circuits (with and without the OP AMP), select values of $R_1$, $R_2$, $C_1$, and $C_2$ to locate a pole at $-10$ krad/s and a second pole at $-100$ krad/s. Evaluate the two approaches and give pros and cons for each design.

Without the OP AMP, we need to determine the location of the poles in symbolic form. We have the following results:

$$s = \frac{-(R_1 C_1 + R_2 C_2 + R_1 C_2) \pm \sqrt{(R_1 C_1 + R_2 C_2 + R_1 C_2)^2 - 4R_1 R_2 C_1 C_2}}{2R_1 R_2 C_1 C_2}$$

Pick $R_1 = R_2 = 1$ kΩ and solve for $C_1$ and $C_2$ using MATLAB. The MATLAB code and results are shown below.

```
syms C1 C2 positive
R1 = 1000;
R2 = 1000;
Eqn1 = (R1*C1+R2*C2+R1*C2)/R1/R2/C1/C2 - 110e3;
Eqn2 = 1/R1/R2/C1/C2 - 1e9;
Soln = solve(Eqn1,Eqn2,C1,C2);
C1num = double(Soln.C1)
C2num = double(Soln.C2)
```

```
C1num =
    22.9844e-009
    87.0156e-009
C2num =
    43.5078e-009
    11.4922e-009
```

There are two solutions. Using the first solution, we have $C_1 = 0.02298$ $\mu$F and $C_2 = 0.04351$ $\mu$F.

In the OP AMP design, the poles are visible in the development of the output voltage, with $s = -1/R_1C_1 = -10$ krad/s and $s = -1/R_2C_2 = -100$ krad/s. Choose $R_1 = R_2 = 1$ kΩ, $C_1 = 0.1$ $\mu$F, and $C_2 = 0.01$ $\mu$F to place the poles as specified.

Both approaches can achieve the same results, but the design with the OP AMP is much easier to analyze and much easier to work with in selecting components to meet a design specification. We will use the OP AMP approach in future circuit design problems.