5.1 $t_{ox} = 2 \sim 10$ nm

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$\epsilon_{ox} = 34.5 \text{ pF/m}$$

$$C_{ox}^{-1} = 58 \sim 290 \text{ m}^2/\text{F} \left(\frac{\mu\text{m}^2}{\text{pF}}\right)$$

For 10 pF:

Area $= 580 \sim 2900 \ (\mu\text{m}^2)$

so

$d = 24 \sim 54\ \mu\text{m}$

5.2 $C_{ox} = 9 \text{ fF}/\mu\text{m}^2,\ V_{OV} = 0.2$ V

$L = 0.36\ \mu\text{m},\ V_{DS} = 0$ V

$W = 3.6\ \mu\text{m}$

$Q = C_{ox}.W.L.V_{OV} = 2.33$ fC

5.3 $k'_n = \mu_n C_{ox}$

$$= \frac{\text{m}^2}{\text{V}\cdot\text{s}}\ \frac{\text{F}}{\text{m}^2} = \frac{\text{F}}{\text{V}\cdot\text{s}} = \frac{\text{C/V}}{\text{V}\cdot\text{s}} = \frac{\text{C}}{\text{s}}\ \frac{1}{\text{V}^2}$$

$$= \frac{\text{A}}{\text{V}^2}$$

Since $k_n = k'_n W/L$ and W/L is dimensionless, k_n has the same dimensions as k'_n; that is, A/V^2.

5.4 With v_{DS} small, compared to V_{OV}, Eq. (5.13a) applies:

$$r_{DS} = \frac{1}{(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{OV})}$$

(a) V_{OV} is doubled $\rightarrow r_{DS}$ is halved. factor $= 0.5$

(b) W is doubled $\rightarrow r_{DS}$ is halved. factor $= 0.5$

(c) W and L are doubled $\rightarrow r_{DS}$ is unchanged. factor $= 1.0$

(d) If oxide thickness t_{ox} is halved, and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

then C_{ox} is doubled. If W and L are also halved, r_{DS} is halved, factor $= 0.5$.

5.5 The transistor size will be minimized if W/L is minimized. To start with, we minimize L by using the smallest feature size,

$L = 0.18\ \mu\text{m}$

$$r_{DS} = \frac{1}{k'_n\,(W/L)\,(v_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k'_n\,(W/L)\,v_{OV}}$$

Two conditions need to met for v_{OV} and r_{DS}

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6}\,(W/L) v_{OV,1}}$$

$= 250 \Rightarrow (W/L)\ v_{OV,1} = 10$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6}\,(W/L)\,v_{OV,2}}$$

$= 1000 \Rightarrow (W/L)\ v_{OV,2} = 2.5$

If condition 1 is met, condition 2 will be met since the over-drive voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease *W/L* as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints. This requires our using the largest possible $v_{GS,1}$ voltage.
$v_{GS,1} = 1.8$ V so $v_{OV,1} = 1.8 - 0.5 = 1.3$ V, and

$$W/L = \frac{10}{v_{OV,1}} = \frac{10}{1.3} = 7.69$$

Condition 2 now can be used to find $v_{GS,2}$

$$v_{OV,2} = \frac{2.5}{W/L} = \frac{2.5}{7.69} = 0.325$$

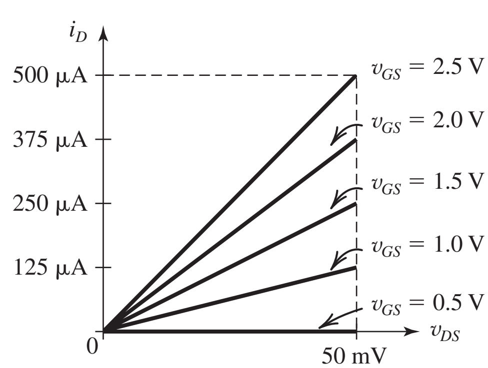
$\Rightarrow v_{GS,2} = 0.825 \text{ V} \Rightarrow 0.825 \text{ V} \le v_{GS} \le 1.8$ V

5.6 $k_n = 5 \text{ mA/V}^2,\ V_{tn} = 0.5$ V,

small v_{DS}

$i_D = k_n(v_{GS} - V_t)\,v_{DS} = k_n v_{OV} v_{DS}$

$$g_{DS} = \frac{1}{r_{DS}} = k_n v_{OV}$$



This table belongs to Exercise 5.6.

V_{GS} (V)	V_{OV} (V)	g_{DS} (mA/V)	r_{DS} (Ω)
0.5	0	0	∞
1.0	0.5	2.5	400
1.5	1.0	5.0	200
2.0	1.5	7.5	133
2.5	2.0	10	100

5.7 $t_{ox} = 4$ nm, $V_t = 0.5$ V

$L_{\min} = 0.18\ \mu$m, small v_{DS},

$k'_n = 400\ \mu\text{A/V}^2$, $0 < v_{GS} < 1.8$ V.

$$r_{DS}^{-1} = k'_n\ W/L\ (v_{GS} - V_t) \le 1\ \text{mA/V} = \frac{1}{1\ \text{k}\Omega}$$

$W/L \le 1.92$

$W \le 0.35\ \mu$m

5.8 $r_{ds} = 1/\left.\dfrac{\partial i_D}{\partial v_{DS}}\right|_{v_{DS} = V_{DS}}$

$$= \left[\frac{\partial}{\partial v_{DS}}\left(k_n\left(V_{OV}v_{DS} - \frac{1}{2}v_{DS}^2\right)\right)\right]^{-1}$$

$$= \left[k_n\left(\frac{\partial}{\partial v_{DS}}\right)(v_{OV}v_{DS}) - 1/2\frac{\partial}{\partial v_{DS}}\left(v_{DS}^2\right)\right]^{-1}$$

$$= \left[k_n\left(V_{OV} - \frac{1}{2}\cdot 2V_{DS}\right)\right]^{-1}$$

$$= \frac{1}{k_n(V_{OV} - V_{DS})}$$

If $V_{DS} = 0 \Rightarrow r_{ds} = \dfrac{1}{k_nV_{OV}}$

If $V_{DS} = 0.2V_{OV} \Rightarrow r_{ds} = \dfrac{1.25}{V_{OV}}$

If $V_{DS} = 0.5V_{OV} \Rightarrow r_{ds} = \dfrac{1}{k_n(V_{OV} - 0.5V_{OV})}$

$$= 1/k_n(0.5V_{OV}) = \frac{2}{k_nV_{OV}}$$

If $V_{DS} = 0.8V_{OV} \Rightarrow r_{ds} = \dfrac{1}{k_n(V_{OV} - 0.8V_{OV})}$

$$= 1/k_n(0.2V_{OV}) = \frac{5}{k_nV_{OV}}$$

If $V_{DS} = V_{OV}$,

$$r_{ds} = \frac{1}{0} \Rightarrow \infty$$

5.9 $V_{DS\ \text{sat}} = V_{OV}$

$V_{OV} = V_{GS} - V_t = 1 - 0.5 = 0.5$ V

$\Rightarrow V_{DS\ \text{sat}} = 0.5$ V

In saturation:

$$i_D = \frac{1}{2}k'_n\left(\frac{W}{L}\right)V_{OV}^2 = \frac{1}{2}k_nV_{OV}^2$$

$$i_D = \frac{1}{2} \times \frac{4\ \text{mA}}{\text{V}^2} \times (0.5\ \text{V})^2$$

$i_D = 0.5$ mA

5.10 $L_{\min} = 0.25\ \mu$m

$t_{ox} = 6$ nm

$$\mu_n = 460\ \frac{\text{cm}^2}{\text{V}\cdot\text{s}} = 460 \times 10^{-4}\ \frac{\text{m}^2}{\text{V}\cdot\text{s}}$$

(a) $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{34.5\ \text{pF/m}}{6\ \text{nm}}$

$$= 5.75 \times 10^{-3}\ \frac{\text{F}}{\text{m}^2}\left(\frac{\text{pF}}{\mu\text{m}^2}\right)$$

$k'_n = \mu_nC_{ox} = 265\ \mu\text{A/V}^2$

(b) For $\dfrac{W}{L} = \dfrac{20}{0.25}$, $k_n = 21.2\ \text{mA/V}^2$

$\therefore 0.5\ \text{mA} = I_D = \dfrac{1}{2}k_nV_{OV}^2$

$V_{OV} = 0.22$ V

$V_{GS} = 0.72$ V

$V_{DS} \ge 0.22$ V

(c) $g_{DS} = \dfrac{1}{100\ \Omega} = k_nV_{OV}$

$\therefore V_{OV} = 0.47$ V.

$V_{GS} = 0.97$ V.

5.11 $V_{tp} = -0.7$ V

(a) $|V_{SG}| = |V_{tp}| + |V_{OV}|$

$= 0.7 + 0.4 = 1.1$ V

$\Rightarrow V_G = -1.1$ V

(b) For the p-channel transistor to operate in saturation, the drain voltage must not exceed the gate voltage by more than $|V_{tp}|$. Thus

$v_{D\max} = -1.1 + 0.7 = -0.4$ V

Put differently, V_{SD} must be at least equal to $|V_{OV}|$, which in this case is 0.4 V. Thus $v_{D\max} = -0.4$ V.

(c) In (b), the transistor is operating in saturation, thus

$$I_D = \frac{1}{2}k_p|V_{OV}|^2$$

$$0.5 = \frac{1}{2} \times k_p \times 0.4^2$$

$$\Rightarrow k_p = 6.25 \text{ mA/V}^2$$

For $V_D = -20$ mV, the transistor will be operating in the triode region. Thus

$$I_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 6.25\left[0.02 \times 0.4 - \frac{1}{2}(0.02)^2\right]$$

$$= 0.05 \text{ mA}$$

For $V_D = -2$ V, the transistor will be operating in saturation, thus

$$I_D = \frac{1}{2}k_p|V_{OV}|^2 = \frac{1}{2} \times 6.25 \times 0.4^2 = 0.5 \text{ mA}$$

5.12 $i_D = \frac{1}{2}k_n'\frac{W}{L}|V_{OV}|^2 \qquad k_n' = \mu_n C_{ox}$

For equal drain currents:

$$\mu_n C_{ox}\frac{W_n}{L} = \mu_p C_{ox}\frac{W_p}{L}$$

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = 2.5$$

5.13 For small $v_{DS}, i_D \simeq k_n'\frac{W}{L_1}(V_{GS} - V_t)V_{DS}$,

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k_n'\frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{100 \times 10^{-6} \times 20 \times (5 - 0.7)}$$

$$r_{DS} = 116.3\ \Omega \qquad V_{DS} = r_{DS} \times i_D = 116.3 \text{ mV}$$

For the same performance of a p-channel device:

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L} \times 2.5$$

$$= 20 \times 2.5 \Rightarrow \frac{W_p}{L} = 50$$

5.14 $t_{ox} = 6$ nm, $\mu_n = 460$ cm^2/V·s, $V_t = 0.5$ V, and $W/L = 10$.

$$k_n = \mu_n C_{ox}\frac{W}{L} = 460 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{6 \times 10^{-9}} \times 10$$

$$= 2.645 \text{ mA/V}^2$$

(a) $v_{GS} = 2.5$ V and $v_{DS} = 1$ V

$$v_{OV} = v_{GS} - V_t = 2 \text{ V}$$

Thus $v_{DS} < v_{OV} \Rightarrow$ triode region,

$$I_D = k_n\left[v_{DS}v_{OV} - \frac{1}{2}v_{DS}^2\right]$$

$$= 2.645\left[1 \times 2 - \frac{1}{2} \times 1\right] = 4 \text{ mA}$$

(b) $v_{GS} = 2$ V and $v_{DS} = 1.5$ V

$$v_{OV} = v_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

Thus, $v_{DS} = v_{OV} \Rightarrow$ saturation region,

$$i_D = \frac{1}{2}k_n v_{OV}^2 = \frac{1}{2} \times 2.645 \times 1.5^2$$

$$= 3 \text{ mA}$$

(c) $v_{GS} = 2.5$ V and $v_{DS} = 0.2$ V

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus, $v_{DS} < v_{OV} \Rightarrow$ triode region,

$$i_D = k_n\left[v_{DS}v_{OV} - \frac{1}{2}v_{DS}^2\right]$$

$$= 2.645[0.2 \times 2 - \frac{1}{2}0.2^2] = 1 \text{ mA}$$

(d) $v_{GS} = v_{DS} = 2.5$ V

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus, $v_{DS} > v_{OV} \Rightarrow$ saturation region,

$$i_D = \frac{1}{2}k_n v_{OV}^2$$

$$= \frac{1}{2} \times 2.645 \times 2^2 = 5.3 \text{ mA}$$

5.15 See Table on next page.

5.16 $i_D = k_n\left[v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2\right]$

$$\frac{i_D}{k_n} = v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2 \qquad (1)$$

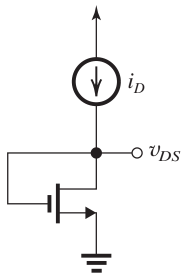
Figure 1 shows graphs for i_D/k_n versus v_{DS} for various values of v_{OV}. Since the right-hand side of Eq. (1) does not have any MOSFET parameters, these graphs apply for any n-channel MOSFET with the assumption that $\lambda = 0$. They also apply to p-channel devices with v_{DS} replaced by v_{SD}, k_n by k_p, and v_{OV} with $|v_{OV}|$. The slope of each graph at $v_{DS} = 0$ is found by differentiating Eq. (1) relative to v_{DS} with $v_{OV} = V_{OV}$ and then substituting $v_{DS} = 0$. The result is

$$\left.\frac{d(i_D/k_n)}{dv_{DS}}\right|_{v_{DS}=0,\ v_{OV}=V_{OV}} = V_{OV}$$

Figure 1 shows the tangent at $v_{DS} = 0$ for the graph corresponding to $v_{OV} = V_{OV3}$. Observe that it intersects the horizontal line $i_D/k_n = \frac{1}{2}V_{OV3}^2$ at $v_{DS} = \frac{1}{2}V_{OV3}$. Finally, observe that the curve representing the boundary between the triode region and the saturation region has the equation

$$i_D/k_n = \frac{1}{2}v_{DS}^2$$

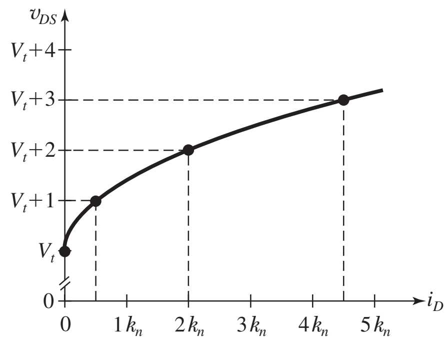
5.26



$v_{DS} = v_{GS}$

$i_D = \frac{1}{2}k_n(v_{DS} - V_t)^2$

$\therefore v_{DS} = \sqrt{\frac{2i_D}{k_n}} + V_t$



5.27 $V_{DS} = V_D - V_S \quad V_{GS} = V_G - V_S$

$V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$

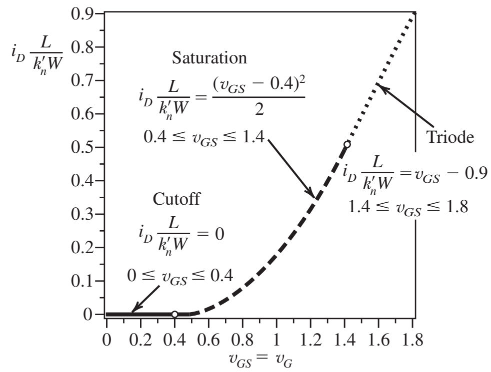
According to Table 5.1, three regions are possible.

Case	V_S	V_G	V_D	V_{GS}	V_{OV}	V_{DS}	Region of operation
a	+1.0	+1.0	+2.0	0	–1.0	+1.0	Cutoff
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	Sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	Sat.
d	+1.0	+1.5	0	+0.5	–0.5	–1.0	Sat.*
e	0	+2.5	1.0	+2.5	+1.5	+1.0	Triode
f	+1.0	+1.0	+1.0	0	–1.0	0	Cutoff
g	–1.0	0	0	+1.0	0	+1.0	Sat.
h	–1.5	0	0	+1.5	+0.5	+1.5	Sat.
i	–1.0	0	+1.0	+1.0	0	+2.0	Sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	Triode

* With the source and drain interchanged.

5.28 The cutoff–saturation boundary is determined by $v_{GS} = V_t$, thus $v_{GS} = 0.4$ V at the boundary.

The saturation–triode boundary is determined by $v_{GD} = V_t$, and $v_{DS} = V_{DD} = 1$ V, and since $v_{GS} = v_{GD} + v_{DS}$, one has $v_{GS} = 0.4 + 1.0 = 1.4$ V at the boundary.



5.29 (a) Let Q_1 have a ratio (W/L) and Q_2 have a ratio $1.03\,(W/L)$. Thus

$$I_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - V_t)^2$$

$$I_{D2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right) \times 1.03 \times (1 - V_t)^2$$

Thus,

$$\frac{I_{D2}}{I_{D1}} = 1.03$$

That is, a 3% mismatch in the W/L ratios results in a 3% mismatch in the drain currents.

(b) Let Q_1 have a threshold voltage $V_t = 0.6$ V and Q_2 have a threshold voltage $V_t + \triangle V_t = 0.6 + 0.01 = 0.61$ V.

Thus

$$I_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - 0.6)^2$$

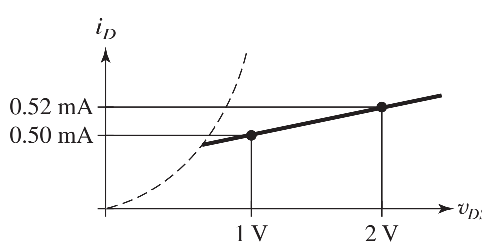
$$I_{D2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(1 - 0.61)^2$$

and

$$\frac{I_{D2}}{I_{D1}} = \frac{(1 - 0.61)^2}{(1 - 0.6)^2} = 0.95$$

That is, a 10-mV mismatch in the threshold voltage results in a 5% mismatch in drain currents.

5.30



5.38

$$k_p = k_p'\left(\frac{W}{L}\right) = 100\ \mu\text{A/V}^2$$

$V_{tp} = -1\text{ V}\quad \lambda = -0.02\text{ V}^{-1}$

$V_G = 0,\quad V_S = +5\text{ V} \Rightarrow V_{SG} = 5\text{ V}$

$|V_{OV}| = V_{SG} - |V_{tp}| = 5 - 1 = 4$

• For $v_D = +4$ V, $v_{SD} = 1\text{ V} < |V_{OV}| \Rightarrow$ triode-region operation,

$$i_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 100\left(1 \times 4 - \frac{1}{2} \times 1\right) = 350\ \mu\text{A}$$

• For $v_D = +2$ V, $v_{SD} = 3\text{ V} < |V_{OV}| \Rightarrow$ triode-region operation,

$$i_D = k_p\left[v_{SD}|V_{OV}| - \frac{1}{2}v_{SD}^2\right]$$

$$= 100\left(3 \times 4 - \frac{1}{2} \times 9\right) = 750\ \mu\text{A}$$

• For $v_D = +1$ V, $v_{SD} = 4\text{ V} = |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2}k_p|V_{OV}|^2(1 + |\lambda|v_{SD})$$

$$= \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 4) = 864\ \mu\text{A}$$

• For $v_D = 0$ V, $v_{SD} = 5\text{ V} > |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 5) = 880\ \mu\text{A}$$

• For $v_D = -5$ V, $v_{SD} = 10\text{ V} > |V_{OV}| \Rightarrow$ saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 16(1 + 0.02 \times 10) = 960\ \mu\text{A}$$

5.39 $V_{tp} = 0.8\text{ V},\quad |V_A| = 40\text{ V}$

$|v_{GS}| = 3\text{ V},\quad |v_{DS}| = 4\text{ V}$

$i_D = 3\text{ mA}$

$|V_{OV}| = |v_{GS}| - |V_{tp}| = 2.2\text{ V}$

$|v_{DS}| > |V_{OV}| \Rightarrow$ saturation mode

$v_{GS} = -3\text{ V}$

$v_{SG} = +3\text{ V}$

$v_{DS} = -4\text{ V}$

$v_{SD} = 4\text{ V}$

$V_{tp} = -0.8\text{ V}$

$V_A = -40\text{ V}$

$\lambda = -0.025\text{ V}^{-1}$

$$i_D = \frac{1}{2}k_p(v_{GS} - V_{tp})^2(1 + \lambda\ v_{DS})$$

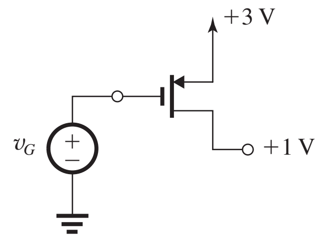
$$3 = \frac{1}{2}k_p[-3 - (-0.8)]^2(1 - 0.025 \times -4)$$

$\Rightarrow k_p = 1.137\text{ mA/V}^2$

5.40 PMOS with $V_{tp} = -1$ V

Case	V_S	V_G	V_D	V_{SG}	$\|V_{OV}\|$	V_{SD}	Region of operation
a	+2	+2	0	0	0	2	Cutoff
b	+2	+1	0	+1	0	2	Cutoff–Sat.
c	+2	0	0	+2	1	2	Sat.
d	+2	0	+1	+2	1	1	Sat–Triode
e	+2	0	+1.5	+2	1	0.5	Triode
f	+2	0	+2	+2	1	0	Triode

5.41



$V_{tp} = -0.5\text{ V}$

$v_G = +3\text{ V} \rightarrow 0\text{ V}$

As v_G reaches +2.5 V, the transistor begins to conduct and enters the saturation region, since v_{DG} will be negative. The transistor continues to operate in the saturation region until v_G reaches 0.5 V, at which point v_{DG} will be 0.5 V, which is equal to $|V_{tp}|$, and the transistor enters the triode region. As v_G goes below 0.5 V, the transistor continues to operate in the triode region.

5.42 Case a, assume, sat,

$$\frac{(1 - V_t)^2}{(1.5 - V_t)^2} = \frac{100}{400} \Rightarrow V_t = 0.5,$$

$V_{GD} \leq V_t$

$\therefore$ sat;

Case b — same procedure, except use V_{SG} and V_{SD}.