

(10 minutes)

[40 pts] **Find:** Develop and express the center frequency gain $|T(j\omega_0)|$ exclusively in terms of R_1 , R_2 , and/or C . Show your work!

$$T(s) = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_2 C} s + \frac{1}{C^2 R_1 R_2}}$$

a $s = j\omega = j\omega_0$ $T(j\omega_0) = \frac{\frac{1}{R_1 C} j\omega_0}{\frac{2}{R_2 C} j\omega_0}$

$$|T(j\omega_0)| = \frac{1}{R_1 C} \cdot \frac{R_2 C}{2} = \frac{R_2}{2R_1} = 10 \quad R_2 = 20 R_1$$

[10 pts] **Find:** Determine R_2 if $R_1 = 1 \text{ k}\Omega$. Assume you found $|T(j\omega_0)| = \frac{1}{2} \frac{R_2}{R_1} = \frac{1}{2\zeta^2}$ from above.

$$\frac{R_2}{R_1} = \frac{1}{\zeta^2} \quad R_1 = 1 \text{ k} \quad R_2 = \frac{1 \text{ k}}{\zeta^2}$$

$$\zeta = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{R_1}{20R_1}} = \sqrt{0.05} = 0.223$$

[10 pts] **Find:** Determine C using R_1 and R_2 you found above. Assume you found $\omega_0 = \frac{1}{C\sqrt{R_1 R_2}}$.

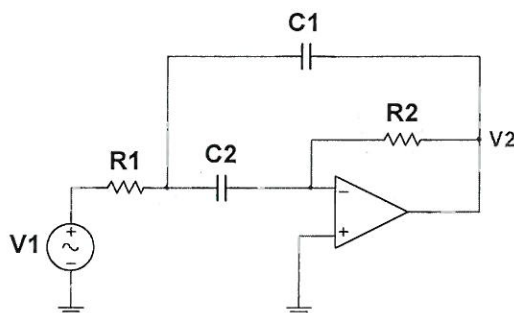
$$C = \frac{1}{\omega_0 \sqrt{R_1 R_2}} = \frac{1}{\omega_0 \sqrt{\frac{1 \text{ M}}{\zeta^2}}} = \frac{\zeta}{1 \text{ k} \cdot \omega_0}$$

Name: Solution

1. [100 pts] **Given:** The Bandpass Sallen-Key topology below with the following requirements.

$$\omega_0 = 10,000 \text{ rad}\cdot\text{s}^{-1} \quad |T(j\omega_0)| = 10 \text{ V/V}$$

Using Equal Capacitor method $T(s) = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$



$$\frac{\frac{1}{R_1 C}}{\frac{2}{R_2 C}} = 10$$

$$\frac{R_2}{R_1} = 20$$

- [20 pts] **Find:** Develop and express ω_0 exclusively in terms of R_1 , R_2 , and/or C using $T(j\omega)$ above. Show your work!

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}$$

$$\Rightarrow \omega_0^2 = \frac{1}{R_1 R_2 C^2} \quad \omega_0 = \frac{1}{\sqrt{R_1 R_2} \cdot C}$$

$$-\omega_0^2 + \frac{1}{R_2 R_1 C^2} = 0 \quad \omega_0 = \frac{1}{\sqrt{R_1 R_2} \cdot C}$$

$$|T(j\omega_0)| =$$

$$\frac{\frac{1}{R_1 C} j\omega_0}{(j\omega_0)^2 + \frac{2}{R_2 C} j\omega_0 + \frac{1}{R_1 R_2 C^2}}$$

- [20 pts] **Find:** Develop and express ζ exclusively in terms of R_1 , R_2 , and/or C using $T(j\omega)$ above. Show your work!

$$2\zeta\omega_0 = \frac{2}{R_2 C}$$

$$\zeta \frac{1}{\sqrt{R_1 R_2} \cdot C} = \frac{1}{R_2 C}$$

$$\zeta = \frac{\sqrt{R_1 R_2}}{R_2} = \sqrt{\frac{R_1}{R_2}}$$

$$\zeta^2 = \frac{R_1}{R_2}$$