

# Propositional Equivalence Rule/Identity

Prove Tautology Using Equivalence Identity

Presented by

**Md. Nazmul Huda Hridoy**

ID: 251311017

Section: A

Batch: 36th

Presented to

**Sanjoy Kumar Chakravarty**

Associate Professor,  
Dept. of CSE,  
Rajshahi University

# What is a Proposition?

A proposition is a statement that is either True (T) or False (F)



# What is a Proposition?

A proposition is a statement that is either True (T) or False (F)

Example -

- ▶ “Dhaka is the capital of India”



# What is a Proposition?

A proposition is a statement that is either True (T) or False (F)

Example -

- ▶ “Dhaka is the capital of India”

FALSE



# What is a Proposition?

A proposition is a statement that is either True (T) or False (F)

Example -

► “Dhaka is the capital of India”

FALSE

► “Are you okay?”



# What is a Proposition?

A proposition is a statement that is either True (T) or False (F)

Example -

► “Dhaka is the capital of India”

FALSE

► “Are you okay?”

Can't answer in true and false!



# Propositional Equivalence

**Two statements are equivalent if they have the same truth value in all cases.**

# Propositional Equivalence

**Two statements are equivalent if they have the same truth value in all cases.**

**Notation :**  $p \equiv q$

# Propositional Equivalence

Two statements are equivalent if they have the same truth value in all cases.

Notation :  $p \equiv q$

Example:

$$p \rightarrow q \equiv \neg p \vee q$$

# Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

# Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

# Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

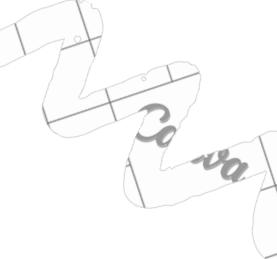
Example:

- If  $p$  = “It is raining”,
- $p \wedge T$  = “It is raining AND True” → just “It is raining.”

# Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$



# Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

Example:

- If  $p$  = “I am studying”,
- $p \vee F$  = “I am studying OR False” → just “I am studying.”



# What is a Tautology?

A tautology is a statement that is always true, no matter the truth values of its variables

# What is a Tautology?

A tautology is a statement that is always true, no matter the truth values of its variables

**Example:**

- $p \vee \neg p$  is always true.

# What is a Tautology?

A tautology is a statement that is always true, no matter the truth values of its variables

**Example:**

- $p \vee \neg p$  is always true.

Let's prove Tautology Using Equivalence Identity

# Example Problem

Prove that:

$$(p \rightarrow q) \vee (q \rightarrow p)$$

is a tautology.

# Example Problem

**Prove that:**

$$(p \rightarrow q) \vee (q \rightarrow p)$$

is a tautology.

# Example Problem

**Prove that:**

$$(p \rightarrow q) \vee (q \rightarrow p)$$

is a tautology.



# Example Problem

Prove that:

$$(p \rightarrow q) \vee (q \rightarrow p)$$

is a tautology.



## Proof Using Equivalence

1. Replace implications:

- $p \rightarrow q \equiv \neg p \vee q$
- $q \rightarrow p \equiv \neg q \vee p$

2. Expression becomes:

$$(\neg p \vee q) \vee (\neg q \vee p)$$

3. Rearrange:

$$(p \vee \neg p) \vee (q \vee \neg q)$$

4. Apply Law of Excluded Middle:

- $p \vee \neg p \equiv T$
- $q \vee \neg q \equiv T$

5. Final result:

$$T \vee T \equiv T$$

✓ Therefore, it is a tautology.

# Thank You!