Homework 3, due on 11/01

Problem 1. If $\mathbb{E}\,X = -2$ and $Var(X) = 3$, find
(a) $\mathbb{E}(-1+2X)^2$
(b) $Var(5-3X)$

Problem 2. A system consisting of one original unit plus a spare can function for a random amount of time X. If the density of X is given (in units of months) by

$$f(x) = \begin{cases} Cx^2e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

What is the probability that the system functions for at least 10 months?

Problem 3. If X is uniformly distributed over $(-1, 3)$, find
(a) $\mathbb{P}(|X| > 2)$
(b) The density function of the random variable $|X|$

Problem 4. If X is an exponential random variable with parameter $\lambda = 2$, compute the probability density function of the random variable Y defined by $Y = \log X + 1$.

Problem 5. The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 50$ and $\sigma = 6$. What is the probability that starting with this year, it will take more than 10 years before a year occurs having a rainfall of less than 30 inches? What assumptions are you making?

Problem 6. Urn A contains three balls labeled from 1 to 3 and urn B contains 2 balls labeled from 1 to 2. A ball is randomly selected from urn A and another ball is randomly selected from urn B. Find the joint probability mass function of X and Y when
(a) X is the largest value obtained on the selected balls and Y is the sum of the values;
(b) X is the value on the ball from urn A and Y is the larger of the two values;
(c) X is the smallest and Y is the largest value obtained on the balls.

Problem 7. The joint probability density function of X and Y is given by

$$f(x,y) = C(x^2 + xy), \quad x \in (0,1), \quad y \in (0,2).$$

(a) Find C.
(b) Compute the density of X.

(c) Find $\mathbb{P}(X > Y)$.
(d) Find $\mathbb{E}\,X$ and $\mathbb{E}\,Y$.

Problem 8. A man and a woman agree to meet at a certain location about 10:00 a.m. If the man arrives at a time uniformly distributed between 9:30 and 10:30 a.m., and if the woman independently arrives at a time uniformly distributed between 9:45 and 11:00 a.m., find the probability that the first to arrive waits no longer than 10 minutes. What is the probability that the woman arrives first?