Homework 4, due on 11/08

Problem 1. Let X and Y be random variables with joint density

$$f(x,y) = Cxy, \quad 0 \le x \le 1, \quad 0 \le y \le 2, 0 \le 2x + y \le 2$$

and $f(x,y) = 0$ otherwise.

(a) Find C

(b) Find $\mathbb{E}[1/(1+X)]$

(c) Find $\mathbb{P}(2X + Y < 1)$

Problem 2. Let X, Y, Z be independent uniform random variables on $[0,1]$. What is the probability that Y lies between X and Z.

Problem 3. The joint probability mass function of X and Y is given by

$$p(1,1) = \frac{1}{8}, \quad p(1,2) = \frac{1}{4}, \quad p(2,1) = \frac{1}{8}, \quad p(2,2) = \frac{1}{2}$$

(a) Compute the conditional mass function of Y given $X = i$, $i = 1, 2$.

(b) Are X and Y independent?

(c) Compute $\mathbb{P}(XY \le 5/2)$, $\mathbb{P}(X + Y \ge 7/3)$ and $\mathbb{P}(X/Y > 3/2)$.

Problem 4. The joint density of X and Y is given by

$$f(x,y) = Cye^{-y(2+x)}, \quad x, y > 0.$$

(a) Find C.

(b) Find the conditional density of X given $Y = y$.

(c) Find the density of $Z = XY$.

Problem 5. X and Y have joint density function

$$f(x,y) = C\frac{1}{x^3y^2}, \quad x, y \ge 1.$$

(a) Find C

(b) Find the joint density of $U = X/Y$ and $V = XY$.

(c) What are their marginal densities? Are they independent?

Note: The second midterm is on 11/15.