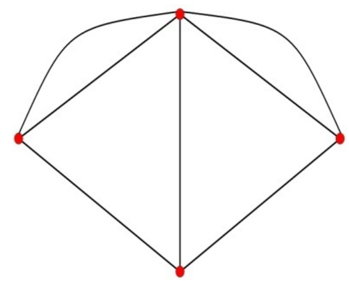
Social and Economic Networks: Models and Analysis



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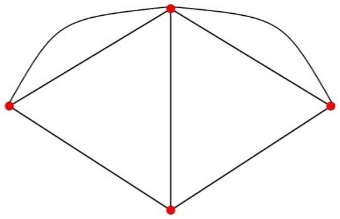
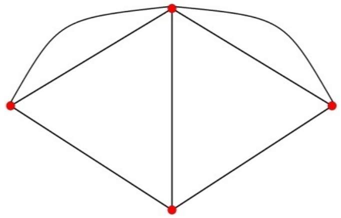
4.1: Strategic Network Formation

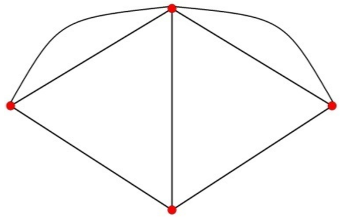


Outline



- Part I: Background and Fundamentals
  - Definitions and Characteristics of Networks (1,2)
  - Empirical Background (3)
- Part II: Network Formation
  - Random Network Models (4,5)
  - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
  - Diffusion and Learning (7,8)
  - Games on Networks (9)

Economic Game Theoretic Models of Network Formation

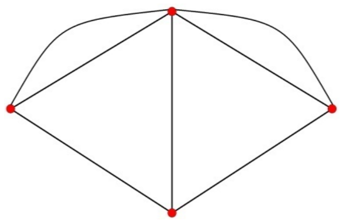


- Costs and benefits for each agent associated with each network

- Agents choose links

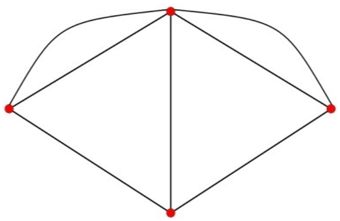
- Contrast incentives and social efficiency

Modeling Choices



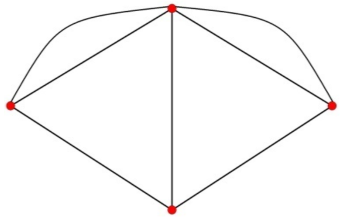
- How should we model incentives to form and sever links?
  - is consensus needed (undirected/directed)?
  - can they coordinate changes in the network?
  - is the process dynamic or static?
  - how sophisticated are agents?
  - what do they know when making a decision?
  - do they make errors?
  - what happens on the network?
  - can they compensate each other for relationship?
  - are links ajustable in intensity?

Some Questions



- Which networks are likely to form?
- Are some more stable than others to various perturbations?
- Are the networks that form efficient?
- How inefficient are they if they are not efficient?
- Can intervention help improve efficiency?
- Can such models provide insight into observed characteristics of networks?

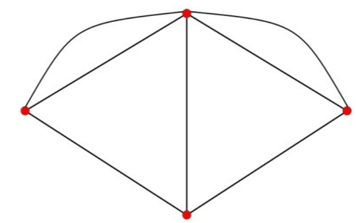
An Economic Analysis: Jackson Wolinsky (96)



- u_i (g) - payoff to i if the network is g

- undirected network formation

Connections Model JW96

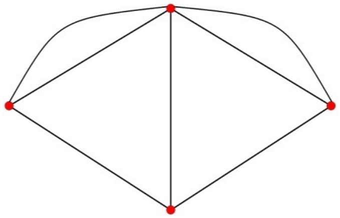


- $0 \leq \delta \leq 1$ a benefit parameter for i from connection between i and j

- $0 \leq c_{ij}$ cost to i of link to j

- $\ell(i,j)$ shortest path length between i,j

$$u_i(g) = \sum_j \delta^{\ell(i,j)} - \sum_{j \text{ in } N_i(g)} c_{ij}$$

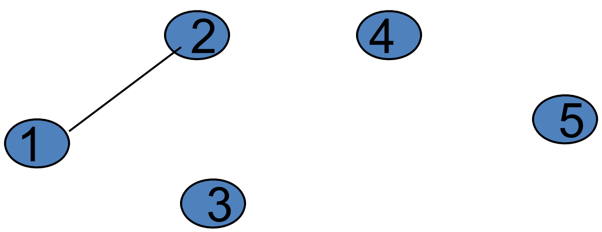


Symmetric Version:

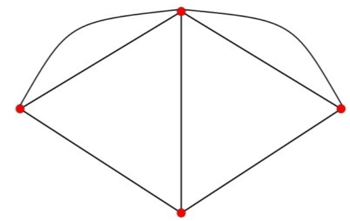
- benefit from a friend is $\delta<1$
- benefit from a friend of a friend is δ^2,...
- cost of a link is $c>0$

$u_2 = \delta\text{-}c$

$u_1 = \delta\text{-}c$



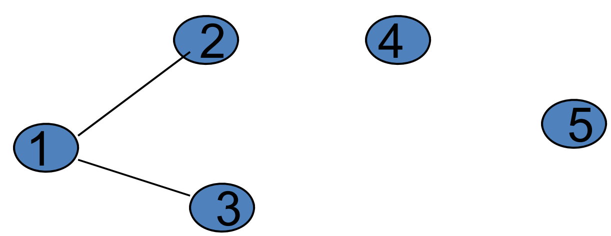
Symmetric Version:



- benefit from a friend is $\delta<1$
- benefit from a friend of a friend is δ^2,...
- cost of a link is $c>0$

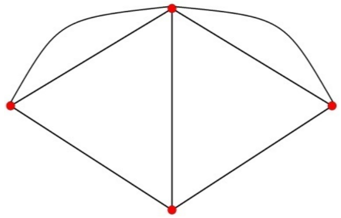
$u_2 = \delta + \delta^2 - c$

$u_1 = 2\delta - 2c$



$u_3 = \delta + \delta^2 - c$

Symmetric Version:



- benefit from a friend is δ<1
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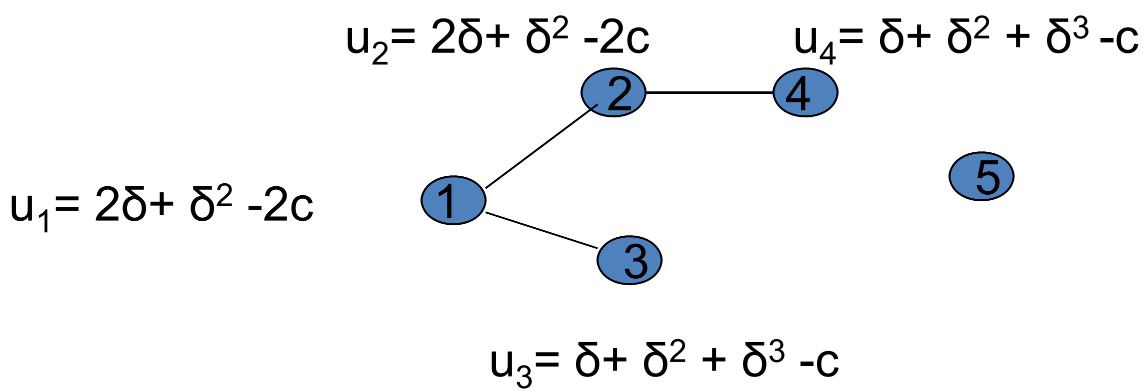
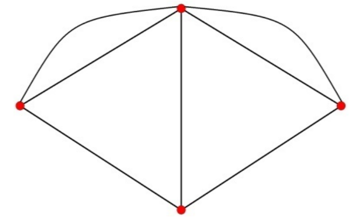
- cost of a link is c>0

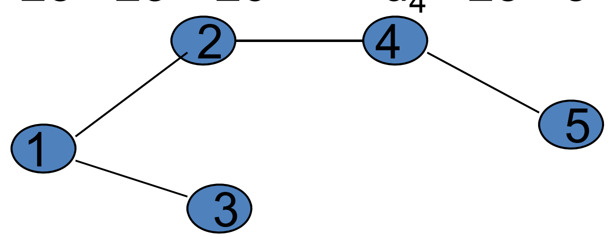


Symmetric Version:



- benefit from a friend is $\delta<1$
- benefit from a friend of a friend is δ^2,...
- cost of a link is $c>0$

$u_2 = 2\delta + 2\delta^2 - 2c$ $\quad$ $u_4 = 2\delta + \delta^2 + \delta^3 - 2c$

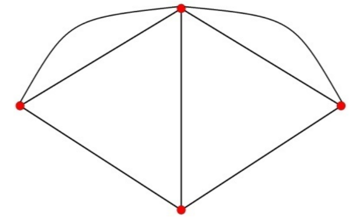


$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$

$u_5 = \delta + \delta^2 + \delta^3 + \delta^4 - c$

$u_3 = \delta + \delta^2 + \delta^3 + \delta^4 - c$

Symmetric Version:

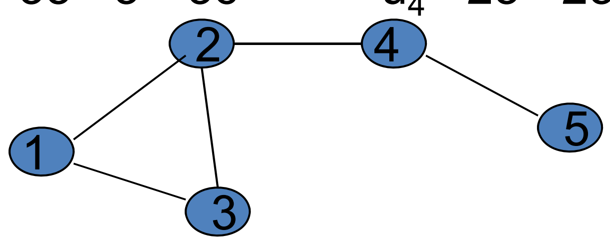


- benefit from a friend is $\delta<1$
- benefit from a friend of a friend is δ^2,...
- cost of a link is $c>0$

$u_2 = 3\delta + \delta^2 - 3c$

$u_4 = 2\delta + 2\delta^2 - 2c$

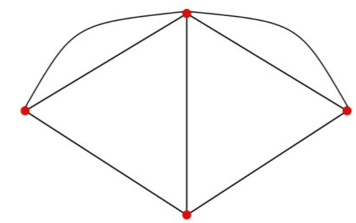
$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$



$u_5 = \delta + \delta^2 + 2\delta^3 - c$

$u_3 = 2\delta + \delta^2 + \delta^3 - 2c$

Symmetric Version:



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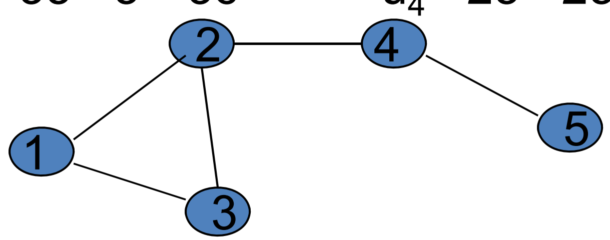
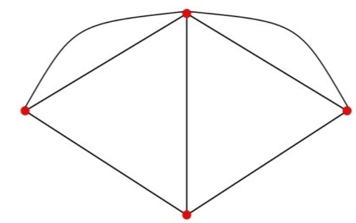


$u_5 = \delta + \delta^2 + 2\delta^3 - c$

$u_3 = 2\delta + \delta^2 + \delta^3 - 2c$

Questions:



- Which networks are best for society?
- Which networks are formed by the agents?

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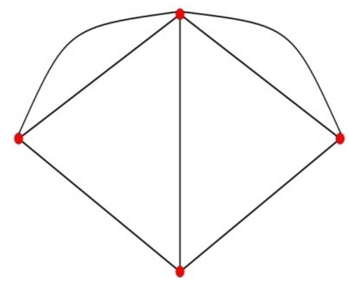

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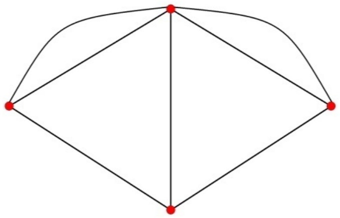
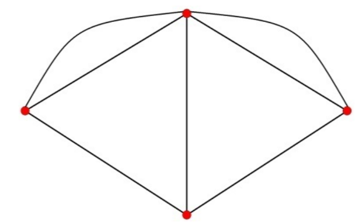
4.2: Pairwise Stability and Efficiency



Modeling Incentives/Equilibrium

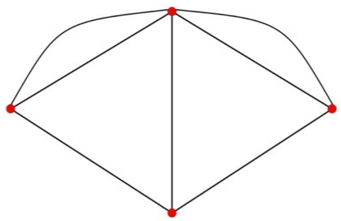


- What if model as a game where each agent announces who they wish to link to and a link forms if and only if both agents name each other?

- Nash equilibrium: no agent can gain from changing his/her action

Both are Nash equilibria: both announce each other is an equilibrium
neither announces the other is an equilibrium...

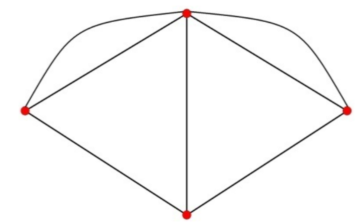
Modeling Incentives: Pairwise Stability



- no agent gains from severing a link – relationships must be beneficial to be maintained

- no two agents both gain from adding a link (at least one strictly) – beneficial relationships are pursued when available

Pairwise Stability



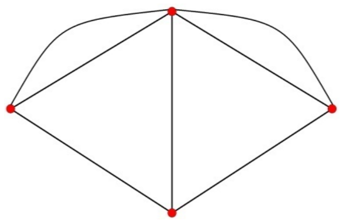
- $u_i(g) \geq u_i(g\text{-}ij)$ for i and ij in g
  - no agent gains from severing a link

- $u_i(g+ij) > u_i(g)$ implies $u_j(g+ij) < u_j(g)$ for ij ij not in g
  - no two agents both gain from adding a link (at least one strictly)

- a weak concept, but often narrows things down

Both are Nash equilibria, but only the dyad is pairwise stable

Pairwise Stability



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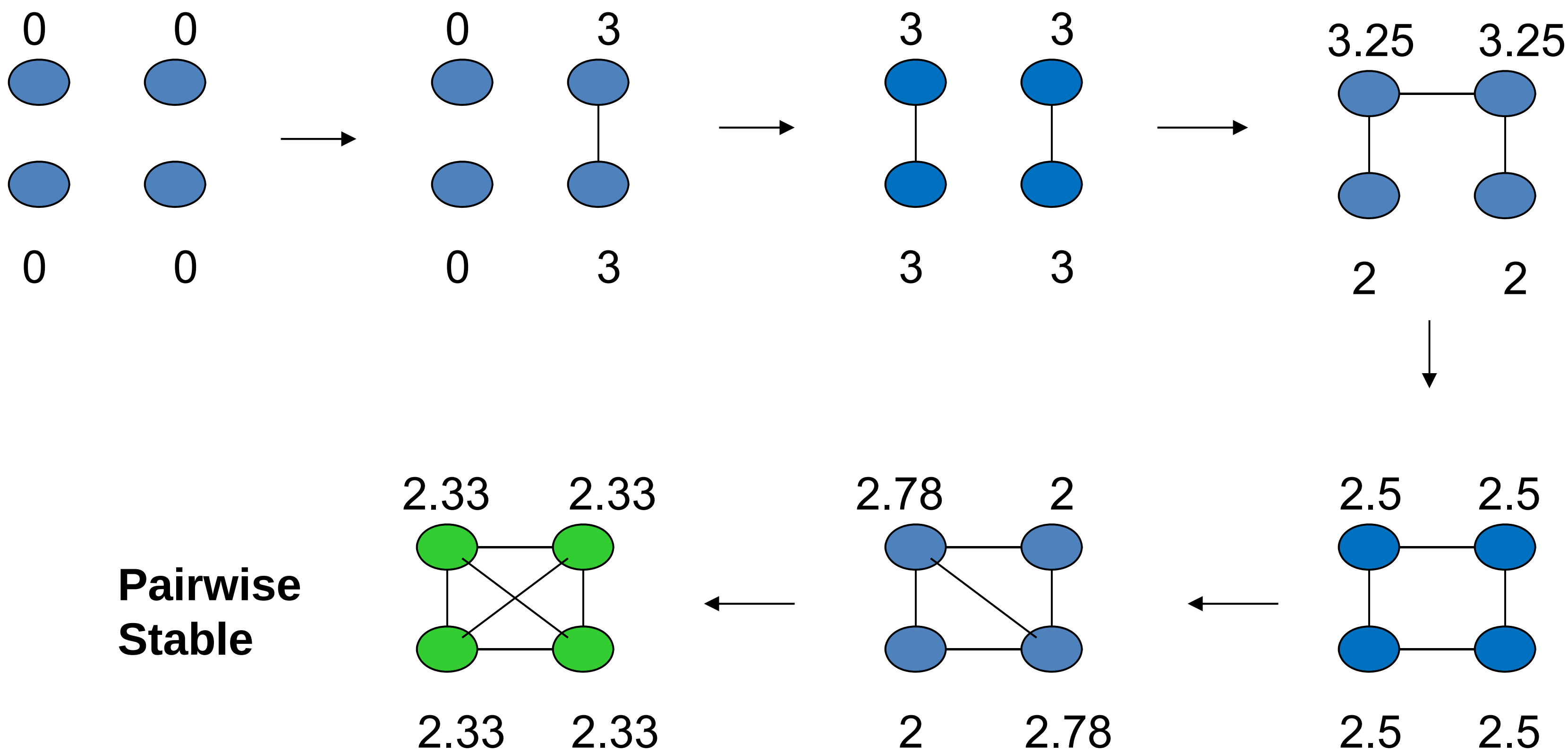
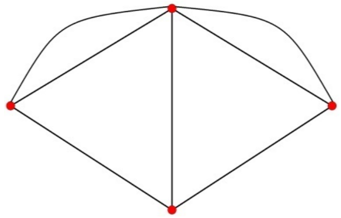
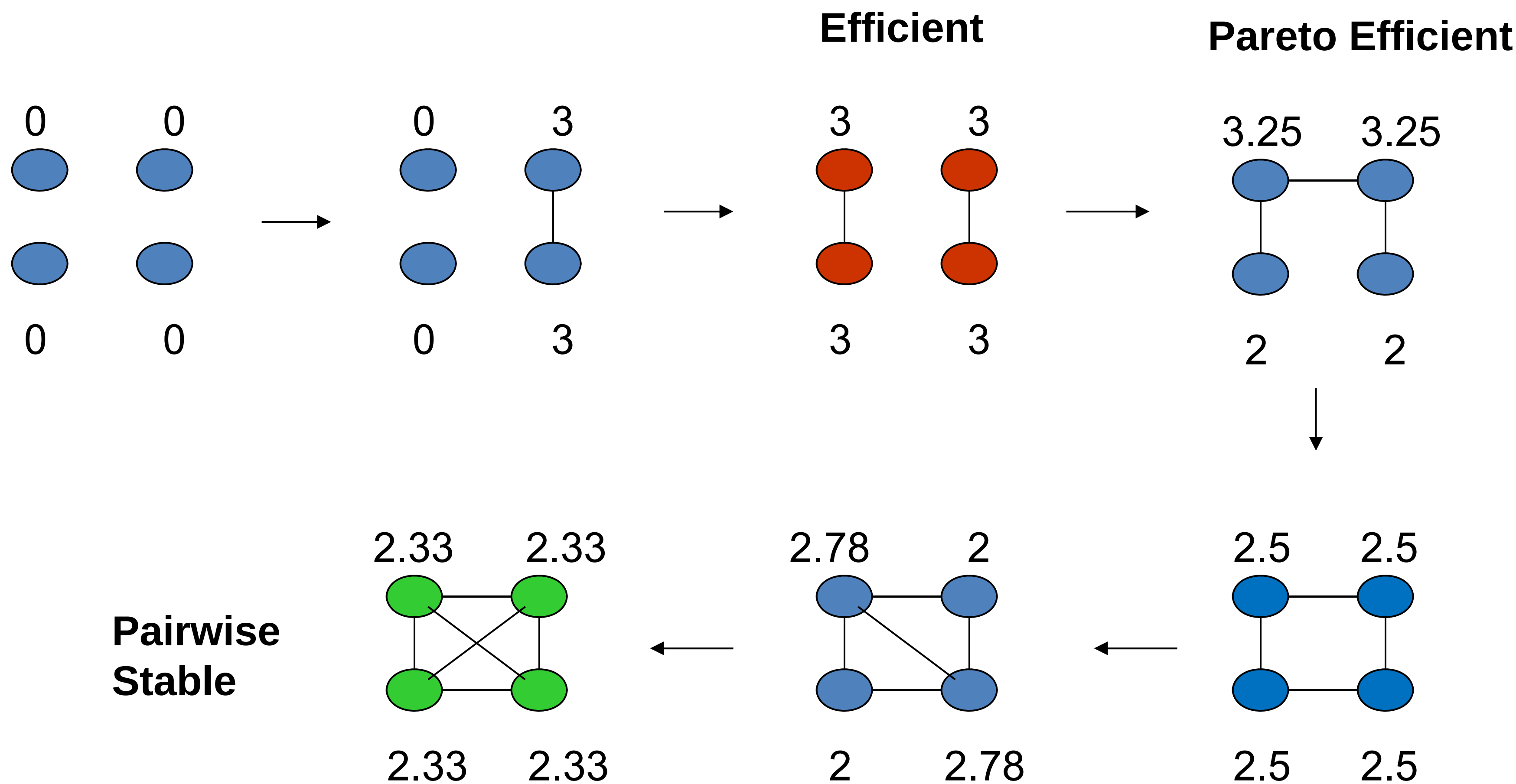

0
0
0
0
0
3
0
3
3
3
3
3
3.25
3.25
2
2
2.5
2.5
2.5
2.5
2.78
2
2
2.78
2.33
2.33
2.33
2.33
Pairwise
Stable

Efficiency



- **Pareto efficient** g: there does not exist g' s.t.
  - $u_i(g') \geq u_i(g)$ for all i, strict for some

- **Efficient** g (Pareto if transfers):
  - g maximizes $\sum u_i(g')$



Efficient
Pareto Efficient
0 0
0 0
0 3
0 3
3 3
3 3
3.25 3.25
2 2
2.5 2.5
2.5 2.5
2.78 2
2 2.78
2.33 2.33
2.33 2.33
Pairwise Stable

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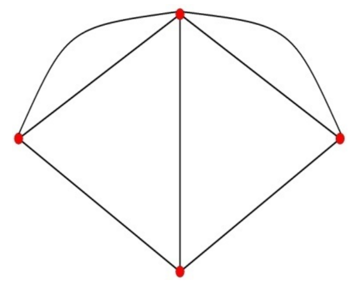

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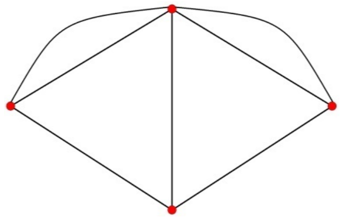
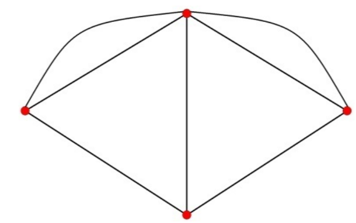
4.3: Connections Model



Connections Model JW96



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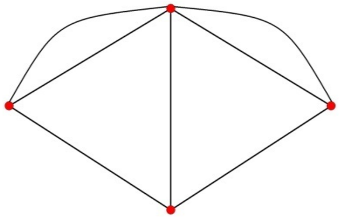
- $\ell(i,j)$ shortest path length between i,j

$$u_i(g) = \sum_j \delta_{ij}^{\ \ell(i,j)} - \sum_{j \text{ in } N_i(g)} c_{ij}$$

Symmetric Version:



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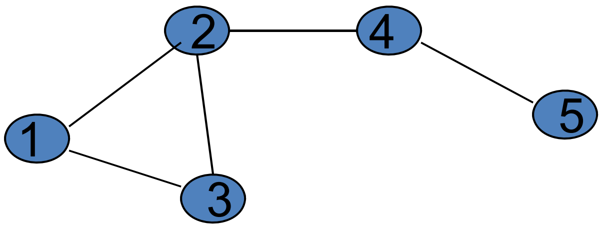
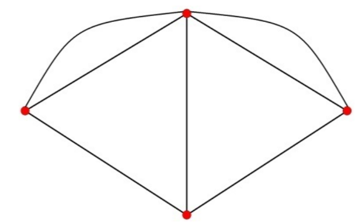
$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$



$u_5 = \delta + \delta^2 + 2\delta^3 - c$

Efficient Networks in the Symmetric Connections Model

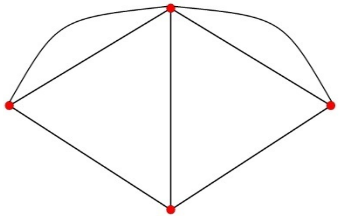


- low cost: $c< \delta-\delta^2$
  - complete network is uniquely efficient

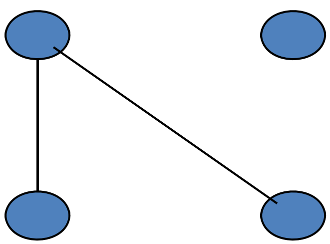
- medium cost: $\delta-\delta^2 < c < \delta+(n-2)\delta^2/2$
  - star networks with all agents are uniquely efficient

- high cost: $\delta+(n-2)\delta^2/2 < c$
  - empty network is uniquely efficient

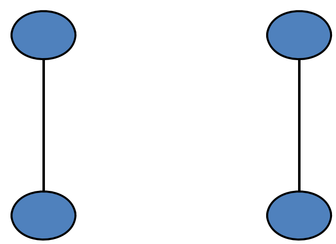
Why Stars?



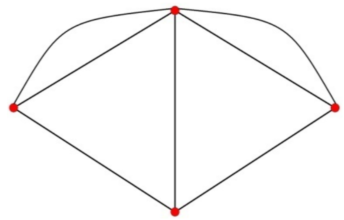
$4\delta + 2\delta^2 - 4c$



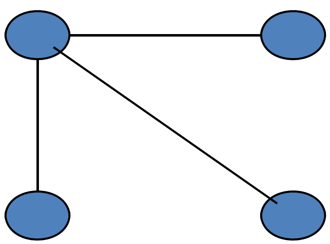
$4\delta - 4c$



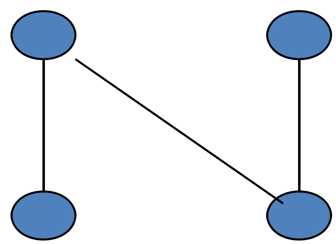
Why Stars?



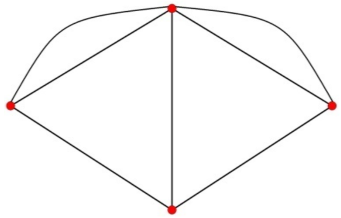
$6\delta + 6\delta^2 - 6c$



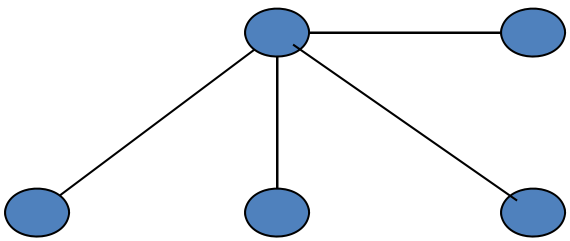
$6\delta + 4\delta^2 + 2\delta^3 - 6c$



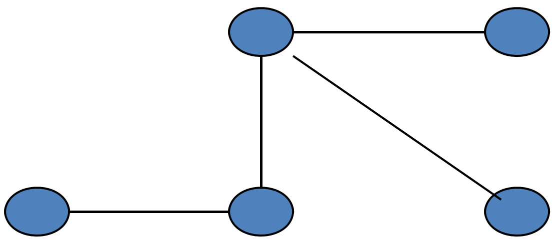
Why Stars?



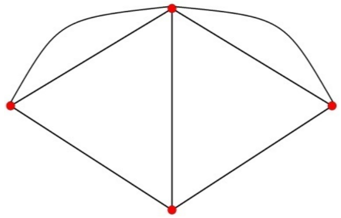
$8\delta + 12\delta^2 - 8c$



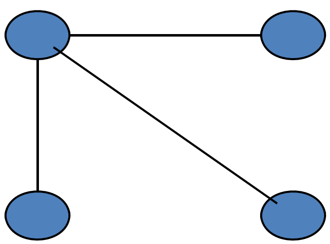
$8\delta + 8\delta^2 + 4\delta^3 - 8c$



Star versus Complete:



$6\delta + 6\delta^2 - 6c$



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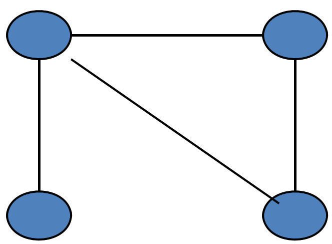

$2\delta - 2\delta^2 - 2c$

better if $\delta - \delta^2 > c$

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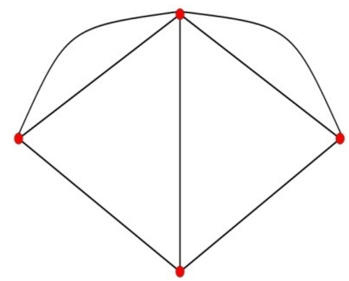

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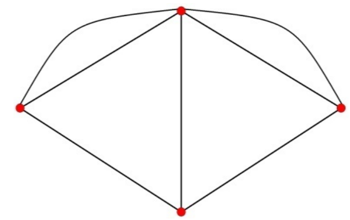
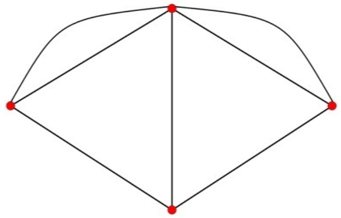
4.4: Efficiency in the Connections Model



Efficient Networks in the Symmetric Connections Model



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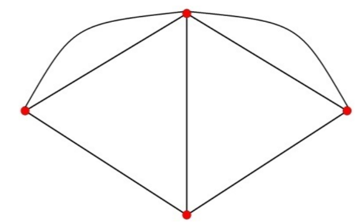
  - complete network is uniquely efficient

- medium cost: $\delta - \delta^2 < c < \delta + (n-2)\delta^2/2$
  - star networks with all agents are uniquely efficient

- high cost: $\delta + (n-2)\delta^2/2 < c$
  - empty network is uniquely efficient

Proof

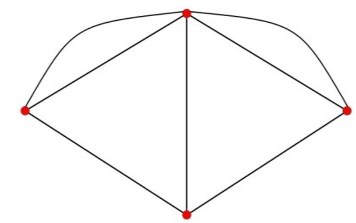


- $c < \delta - \delta^2$ then $u_i(g+ij) > u_i(g)$ if ij not in g

Also $u_k(g+ij) \geq u_k(g)$ if ij not in g for every k, thus

$\Sigma_k u_k(g+ij) > \Sigma_k u_k(g)$

- $c > \delta - \delta^2$ first, show that the value of a component is highest when the component is a star



- value of a star with k players is

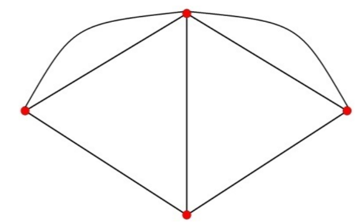
  $2(k-1)\ [\ \delta - c] + (k-1)(k-2)\delta^2$

- value of a network with k players and m links ($m \geq k-1$) is at most

  $2m\ [\ \delta - c]\ + [k(k-1)-2m]\delta^2$

- difference is

  $2(m-(k-1))\ [\delta^2 - (\delta - c)]\ > 0$ if $m > k-1$

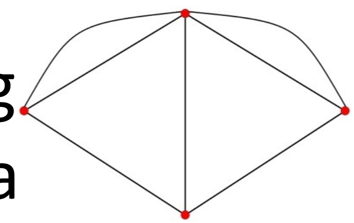


- If m = k-1 and not a star, then some pair is at a distance of more than 2, so less value than a star:
- value of a star with k players is

  $2(k-1)\ [\ \delta - c] + (k-1)(k-2)\delta^2$
- value of a component with k players and k-1 links that is not a star is at most

  $2(k-1)\ [\ \delta - c] + [(k-1)(k-2)-1]\delta^2 + \delta^3$
- Star is better

Check that if two separate star components each g nonnegative utility, then one star with all those pla generates higher utility

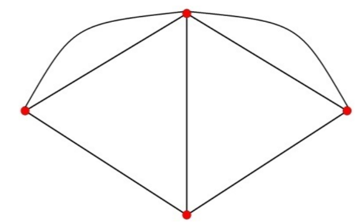


- Separate: $2(k-1)\,[\,\delta - c] + (k-1)(k-2)\delta^2 + 2(k'-1)\,[\,\delta - c] + (k'-1)(k'-2)\delta^2$

$= 2(k+k'-2)\,[\,\delta - c] + [(k-1)(k-2)+ (k'-1)(k'-2)]\delta^2$

- As one star: $2(k+k'-1)\,[\,\delta - c] + (k+k'-1)(k+k'-2)\delta^2$

- second expression is greater...

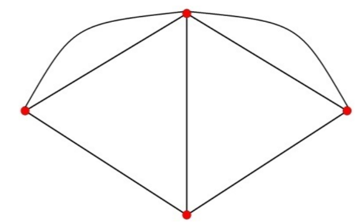


- So efficient networks are collections of stars or empty networks
- So, either a star with all players or empty:

- Want a star if its value is >0, so when

$$2(n-1)\ [\ \delta - c] + (n-1)(n-2)\delta^2 > 0$$

Efficient Networks in the Symmetric Connections Model

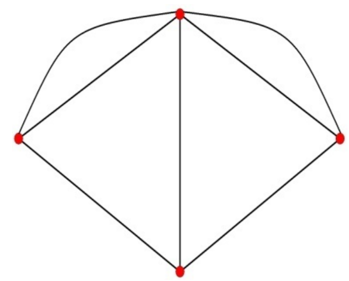


- low cost: $c< \delta-\delta^2$
  - complete network is uniquely efficient

- medium cost: $\delta-\delta^2 < c < \delta+(n-2)\delta^2/2$
  - star networks with all agents are uniquely efficient

- high cost: $\delta+(n-2)\delta^2/2 < c$
  - empty network is uniquely efficient

Social and Economic Networks: Models and Analysis



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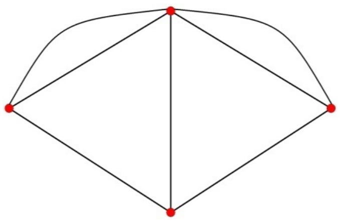
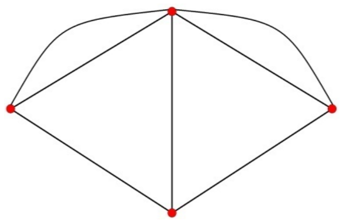
4.5: Pairwise Stability in the Connections Model



Pairwise Stability



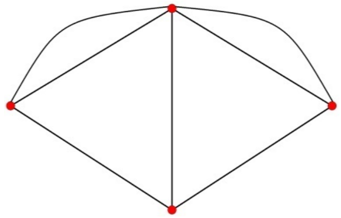
- low cost: $c < \delta - \delta^2$
  - complete network is pairwise stable

- medium/low cost: $\delta - \delta^2 < c < \delta$
  - star network is pairwise stable
  - others are also pairwise stable

- medium/high cost: $\delta < c < \delta + (n-2)\delta^2/2$
  - star network is not pairwise stable (no loose ends)
  - nonempty pairwise stable networks are over-connected and may include too few agents

- high cost: $\delta + (n-2)\delta^2/2 < c$
  - empty network is pairwise stable

Inefficiency:

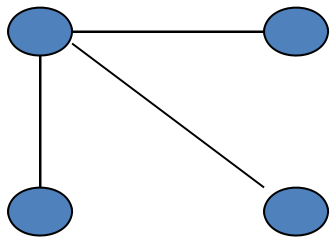


payoff to center:

$3\delta - 3c$

not pairwise stable if

$\delta < c$

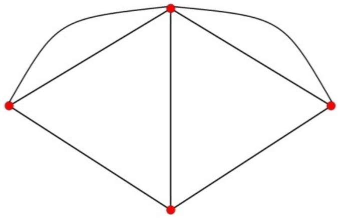


Overall payoff: $6\delta + 6\delta^2 - 6c$

Peripheral players gain indirect benefits

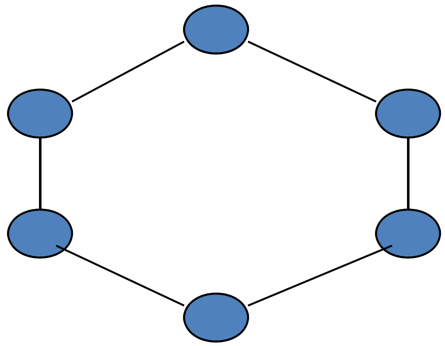
Center player does not account for them

Example: Pairwise stable and inefficient

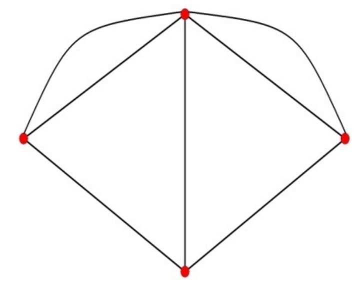


unique nonempty pairwise stable network architecture if

$\color{red}{\delta < c} < (\delta+\delta^2+\delta^3)(1-\delta^2)$, n=6



Social and Economic Networks: Models and Analysis



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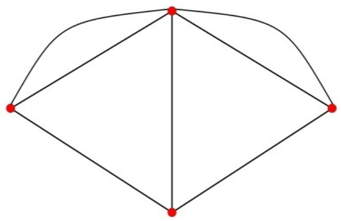
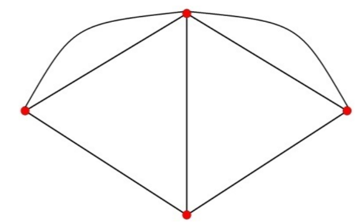
4.6: Externalities and the Coauthor Model



Externalities



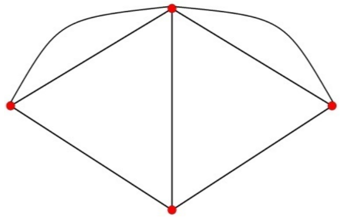
- Positive:

  $u_k(g+ij) \geq u_k(g)$ if ij not ing for every $k \neq i,j$

- Negative:

  $u_k(g+ij) \leq u_k(g)$ if ij not in g for every $k \neq i,j$

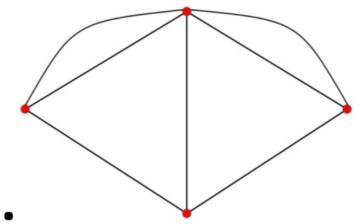
Externalities



- Inefficiency in connections model due to positive externalities - ``no loose ends’’

- What about models with negative externalities?

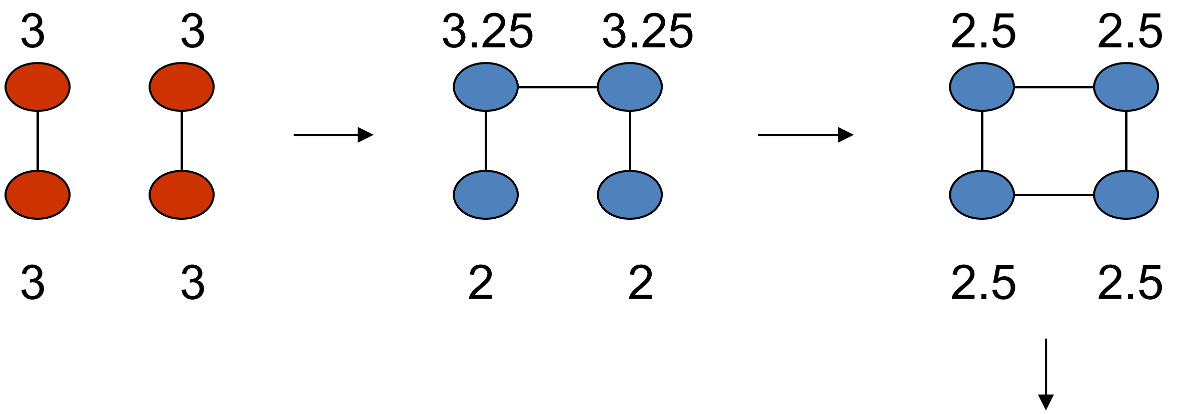
Example: ``Coauthor'' JW96



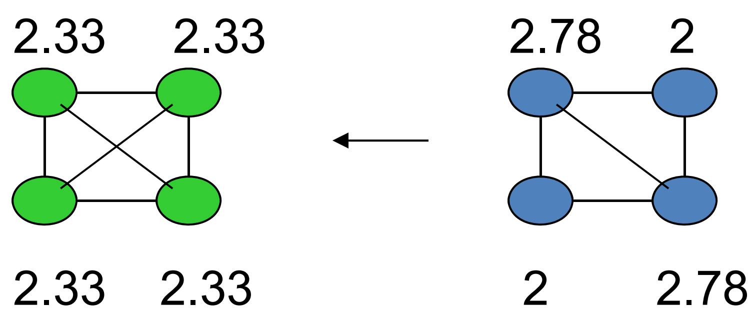
- Agents get value from research collaboration
  - value for each relationship depends on time each puts into it
  - plus an interaction term, which is product of the times spent

$$u_i(g) = \sum_{j:\ ij \text{ in } g} [1/d_i + 1/d_j + 1/(d_i d_j)]$$
$$= 1 + \sum_{j:\ ij \text{ in } g} [1/d_j + 1/(d_i d_j)]$$

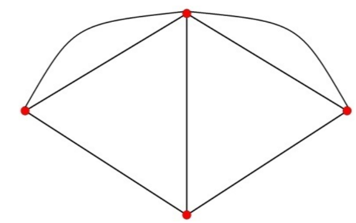
Efficient:



Pairwise Stable:

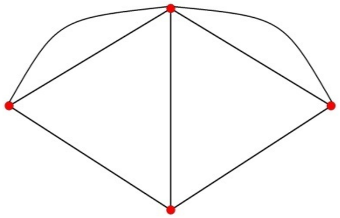


- no direct costs to links



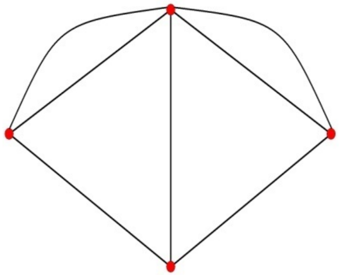
- n is even:
  - efficient networks: pairs
  - pairwise stable networks consist of completely connected components, each of a different size, one has more than the square of the number of nodes in the other
  - by adding a link, dilute existing synergies, only add if new coauthor brings comparable worth

Stable and Efficient only coincide in special cases



- Can transfers help in other cases?
- What can we say about when conflict exists?
- What can we say about when transfers improve efficiency?
- Are transfers in players' interests?

Social and Economic Networks: Models and Analysis



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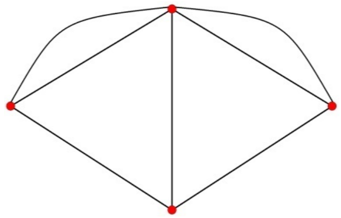
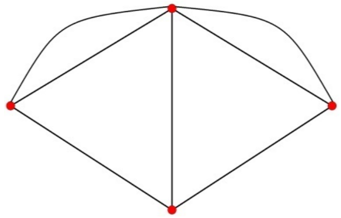
4.7: Network Formation and Transfers

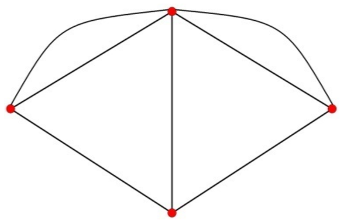


Outline



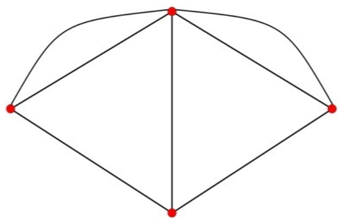
- Part I: Background and Fundamentals
  - Definitions and Characteristics of Networks (1,2)
  - Empirical Background (3)
- Part II: Network Formation
  - Random Network Models (4,5)
  - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
  - Diffusion and Learning (7,8)
  - Games on Networks (9)

Strategic Formation Models:



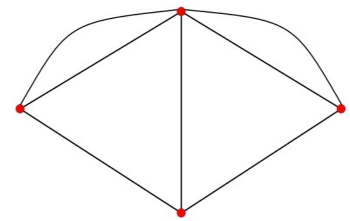
- Saw conflict between stability and efficiency
- Can Transfers help?
- Modeling Stability and Dynamics, etc
  – Refining pairwise stability
  – Dynamic processes
  – Forward looking behavior
- Directed Networks
- Fitting such models
  – Introduce heterogeneity
  – Introduce randomness – meeting processes

Strategic Formation Models:



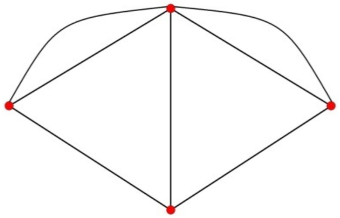
- Saw conflict between stability and efficiency
- Can Transfers help?
- Modeling Stability and Dynamics, etc
  - Refining pairwise stability
  - Dynamic processes
  - Forward looking behavior
- Directed Networks
- Fitting such models
  - Introduce heterogeneity
  - Introduce randomness – meeting processes

What are Transfers ?



- Outside intervention, taxing or subsidizing relationships – e.g., gvt support of R and D relationship
- Bargaining among the individuals involved in the relationships
- Favors exchanged among friends….

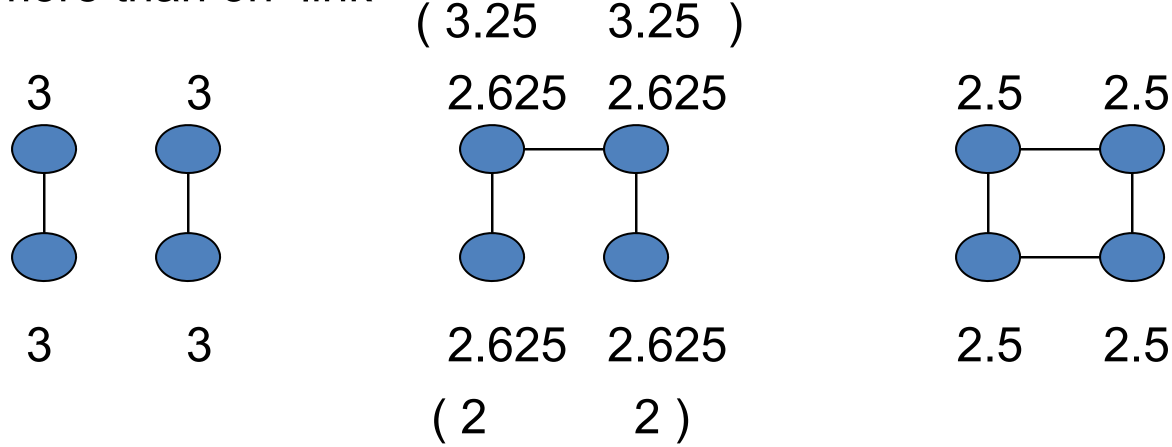
Modeling Transfers



- Change utilities from $u_i(g)$ to $u_i(g)+t_i(g)$
- E.g., peripheral players pay center of star in connections model to maintain connections

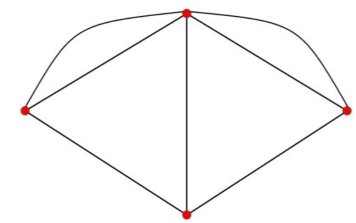
Transfers in Co-author - equalizing works

tax on having more than on link



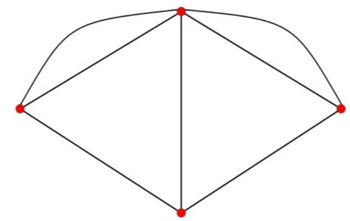
Here: charge players who form links, reallocate it

Egalitarian Transfers



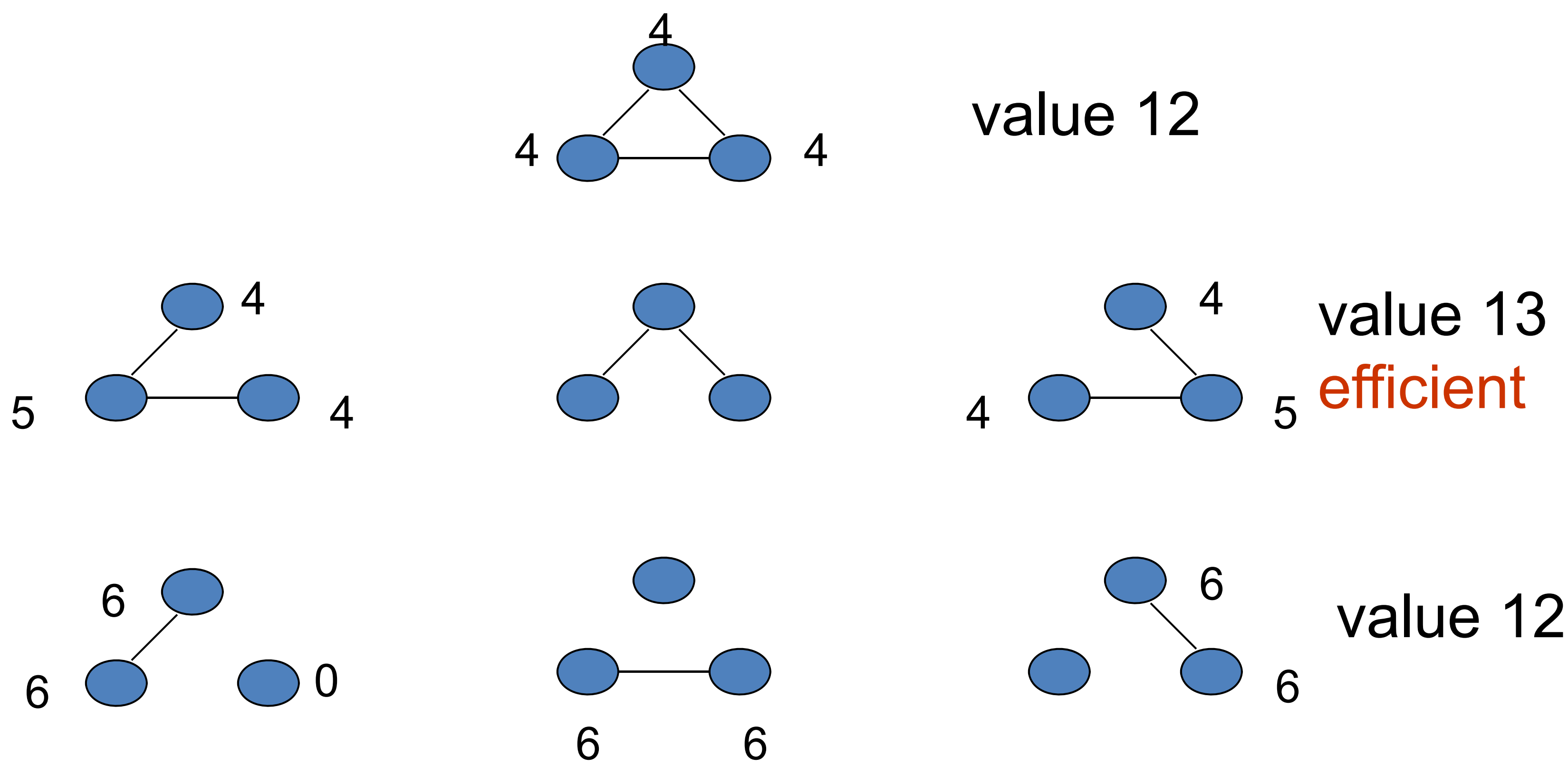
- Set $t_i(g) = \Sigma_j\, u_j(g)/n \;-\; u_i(g)$
- Then $u_i(g) + t_i(g) = \Sigma_j\, u_j(g)/n$
- Now every agent has societal incentives

Transfers can Fail

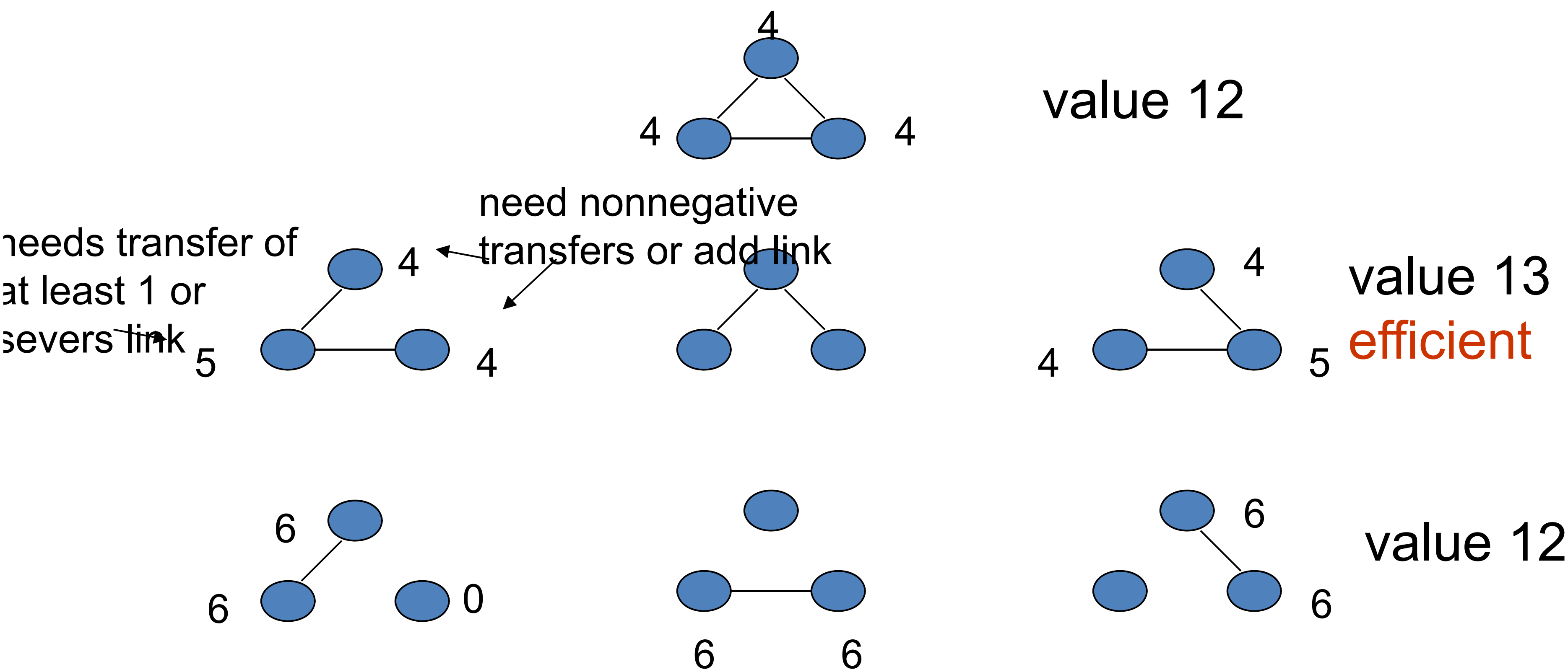


- Put in some basic requirements on transfers:
  - completely isolated nodes that generate no value get 0
  - nodes that are completely interchangeable get same transfers

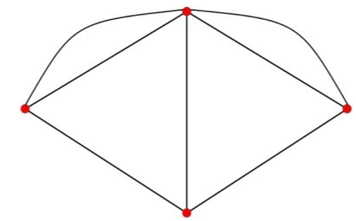
Transfers cannot always help (JW 96)



Transfers cannot always help (JW 1996)



Intuition

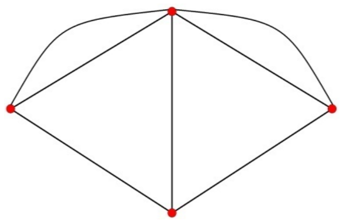


- Coase: without frictions, transfers can solve inefficiencies

- What is special here?

- Combination of multiple externalities that all need to be handled at once.

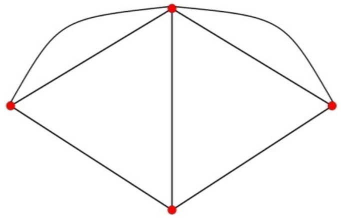
Efficiency and Stability



tension due to externalities, either positive or negative or mixed

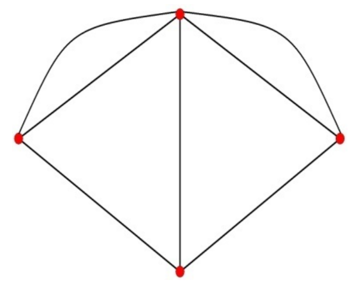
network setting introduces interesting problem: not entirely correctable with bargaining or transfers

Summary So Far



- Efficient networks take some simple forms in a variety of models
- Efficient networks and pairwise stable need not coincide
- Transfers may help, but not always without violating some basic conditions

Social and Economic Networks: Models and Analysis



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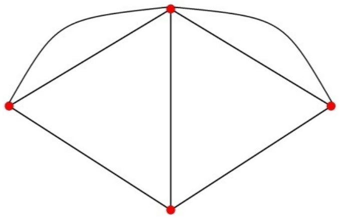
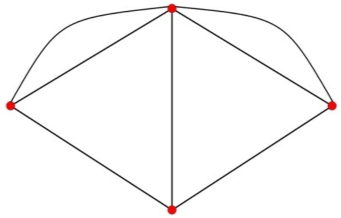
4.8: Heterogeneity in Strategic Models



Outline



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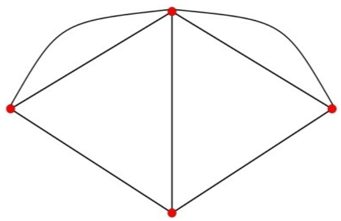
Enriching Such Models



- Costs depend on geography and characteristics of nodes
  - easier to be friends with neighbors
  - easier to relate to people with similar background
- Benefits depend on characteristics of nodes
  - synergies from working together, trading, sharing risk, exchanging favors..
  - complementarities: benefits from diversity...

Can economic models match observables?

- Small worlds derived from costs/benefits
  - low costs to local links – high clustering
  - high value to distant connections – low diameter
  - high cost of distant connections – few distant links

Geographic Connections (Johnson-Gilles (2000), Carayol-Roux (2005), Jackson-Rogers (2005),Galeotti-Goyal- Kamphorst (2006),...)

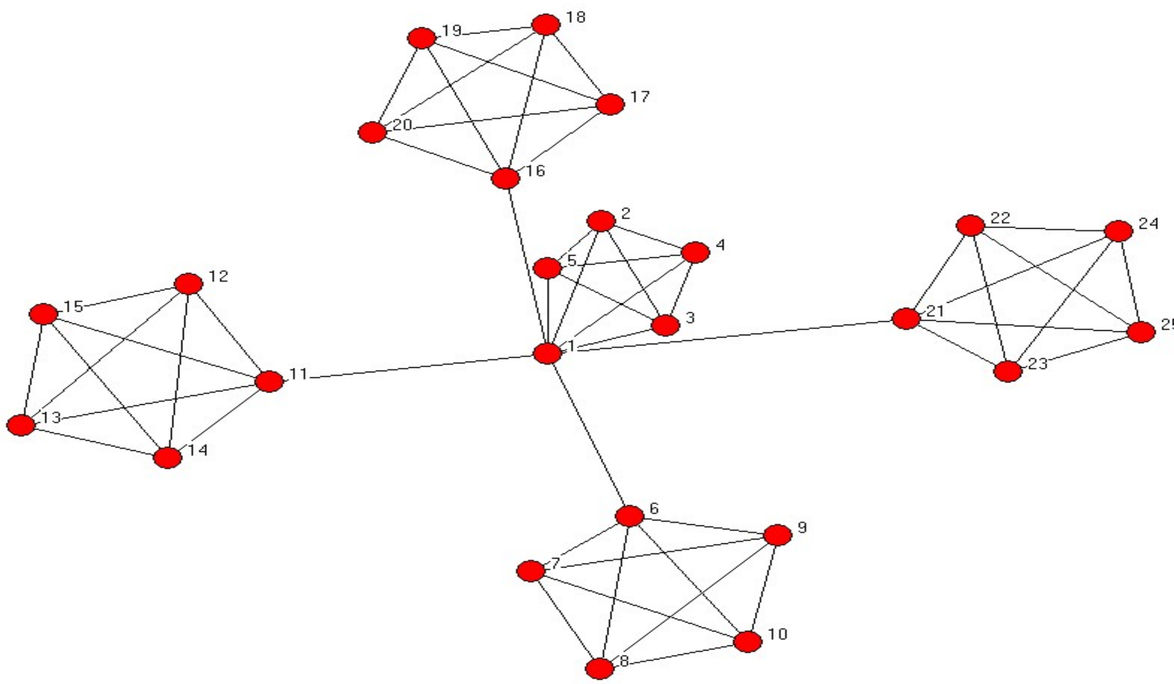
Islands connections model Jackson Rogers-05:

- J players live on an island, K islands
- cost c of link to player on the island
- cost C>c of link to player on another island

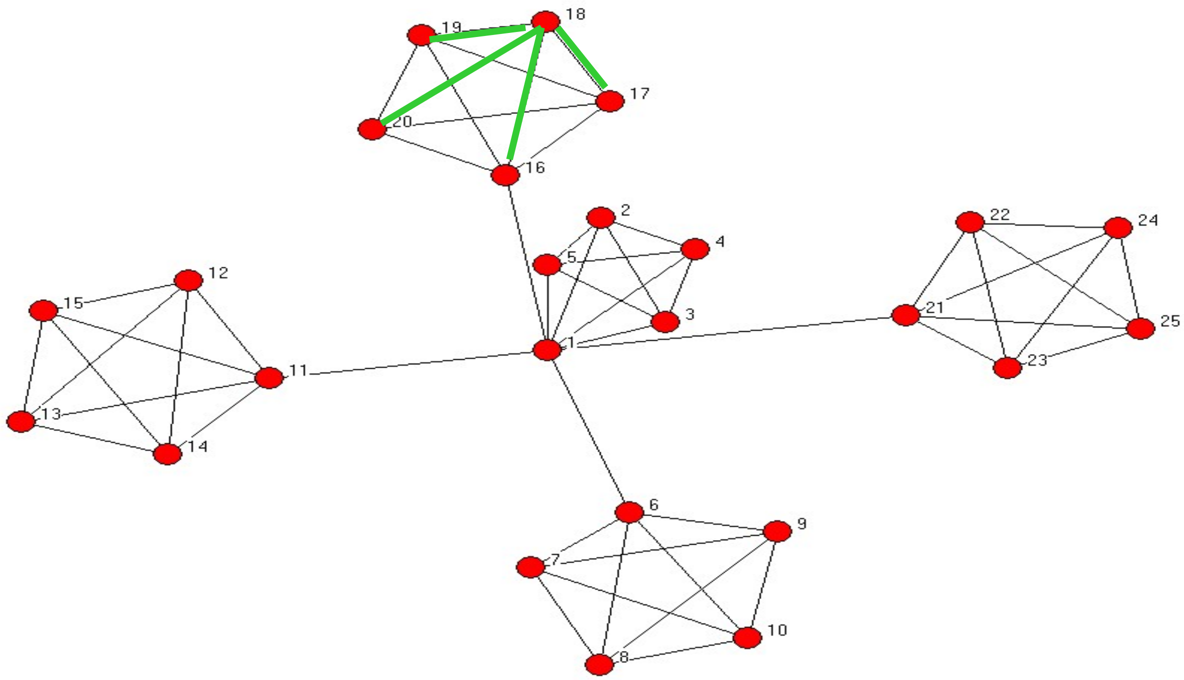
Results:

- High clustering within islands, few links across
- small distances

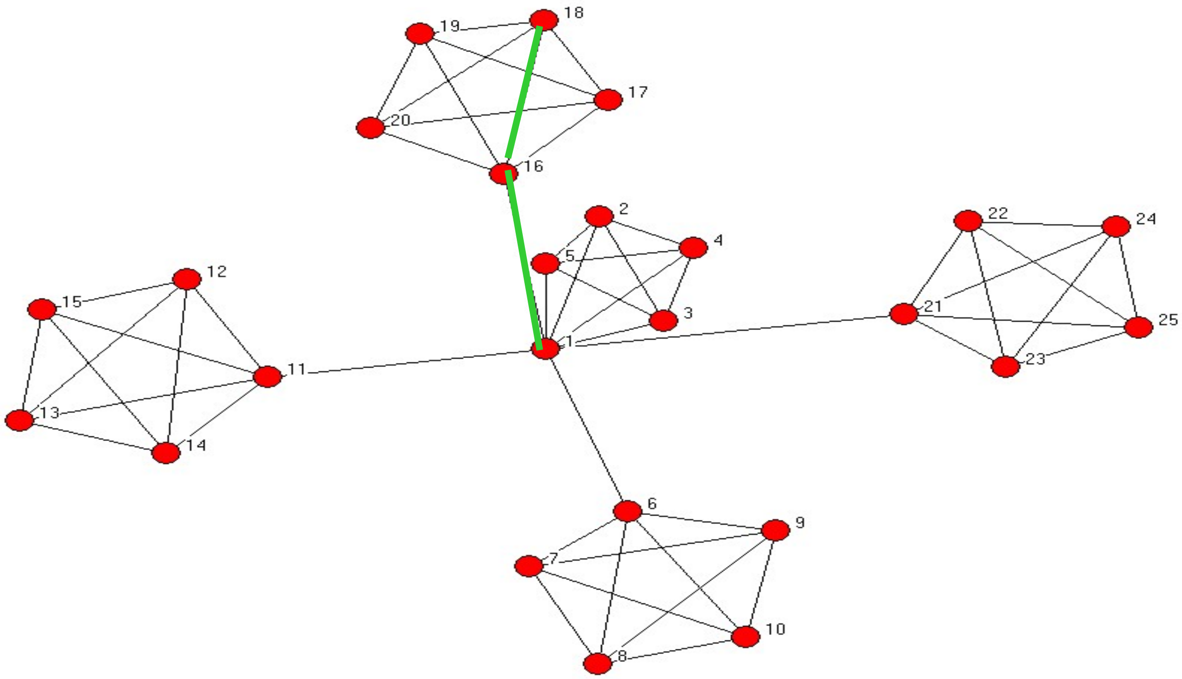
$$4\delta - 4c + \delta^2 + 7\delta^3 + 12\,\delta^4$$



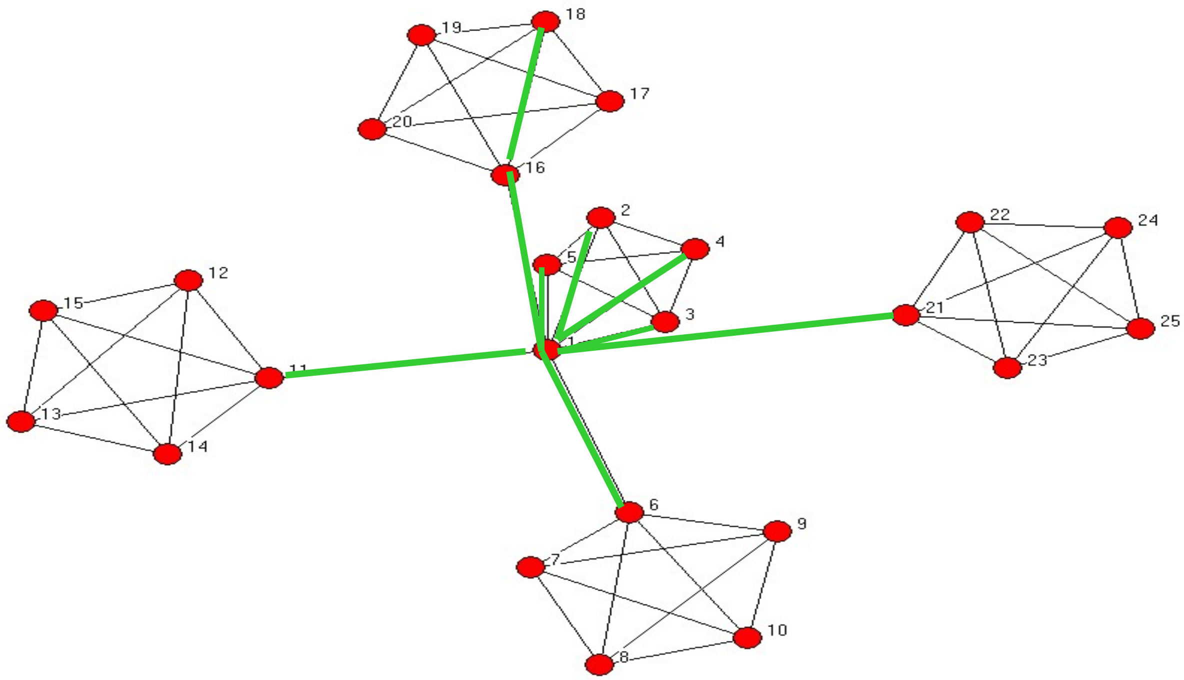
$4\delta - 4c + \delta^2 + 7\delta^3 + 12\,\delta^4$



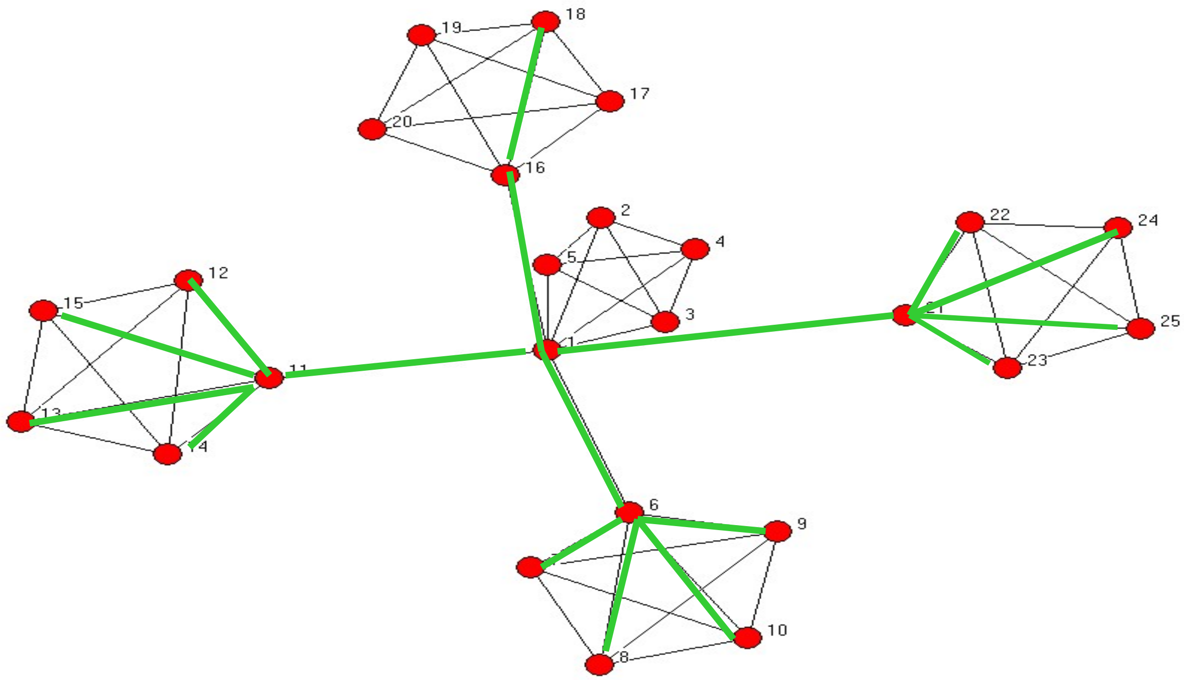
$4\delta - 4c + \delta^2 + 7\delta^3 + 12\,\delta^4$



$$4\delta\ \ - 4c + \delta^2 + 7\delta^3 + 12\ \delta^4$$

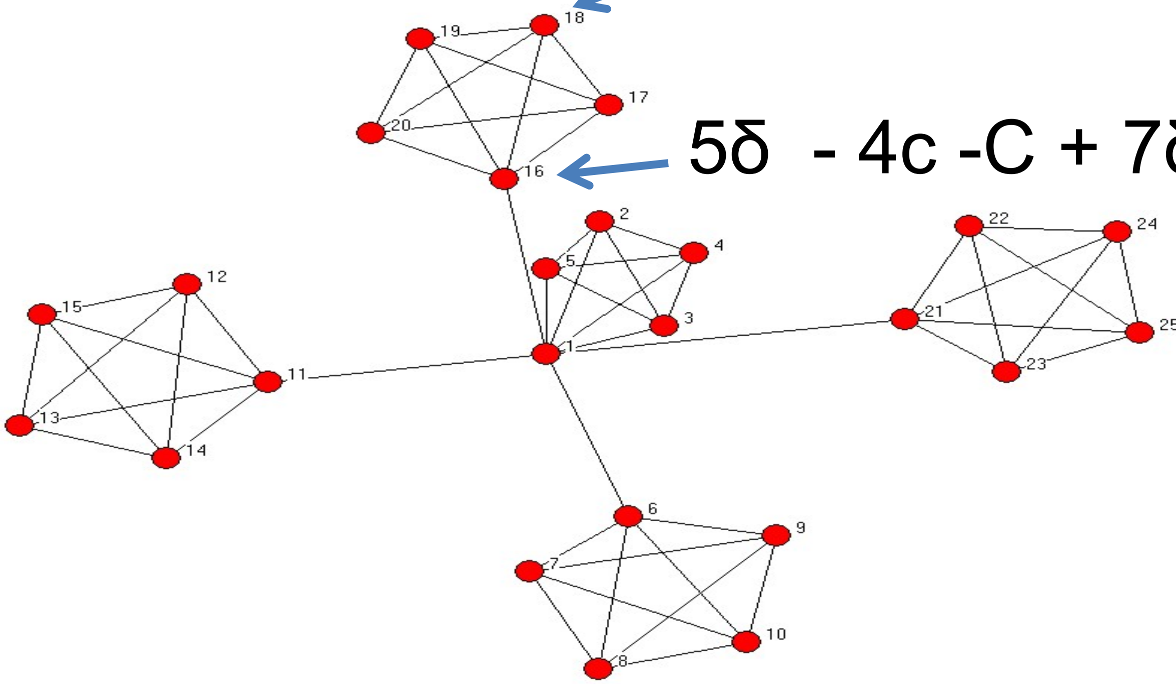


$$4\delta \ - 4c + \delta^2 + 7\delta^3 + 12\ \delta^4$$



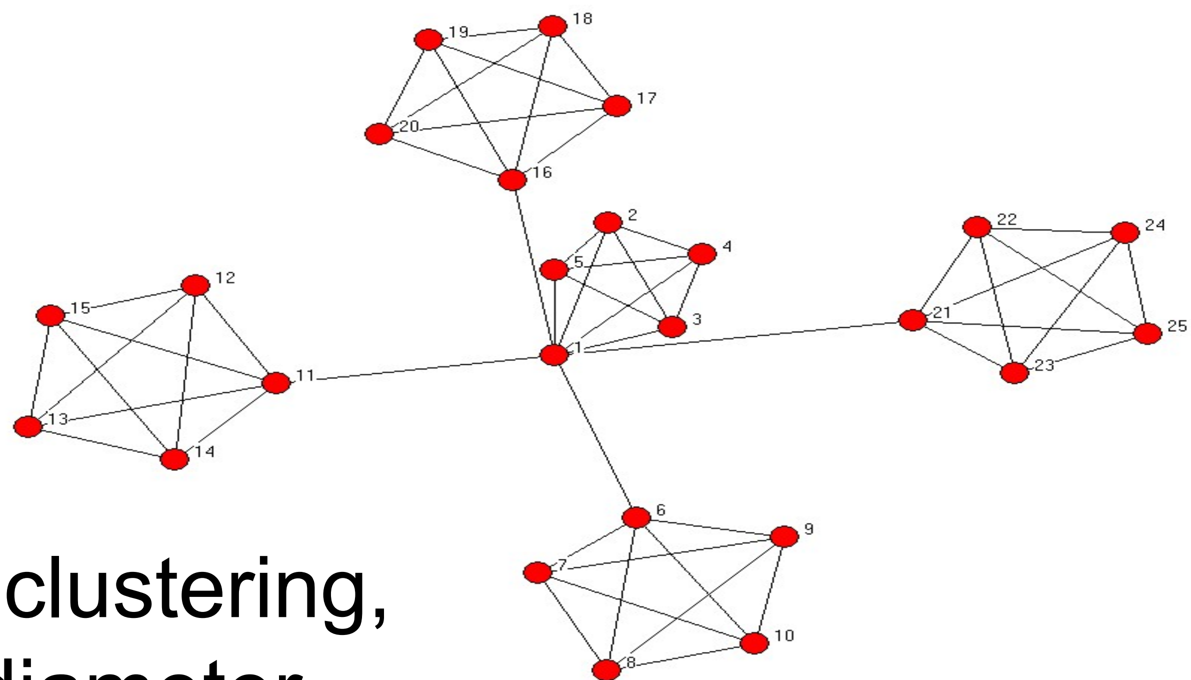
$$4\delta \ - 4c + \delta^2 + 7\delta^3 + 12\,\delta^4$$

$$5\delta \ - 4c - C + 7\delta^2 + 12\delta^3$$



low cost of link to player on own ``island''
– high cost across islands

Pairwise stable: (c<.04, 1<C<4.5, δ=.95)

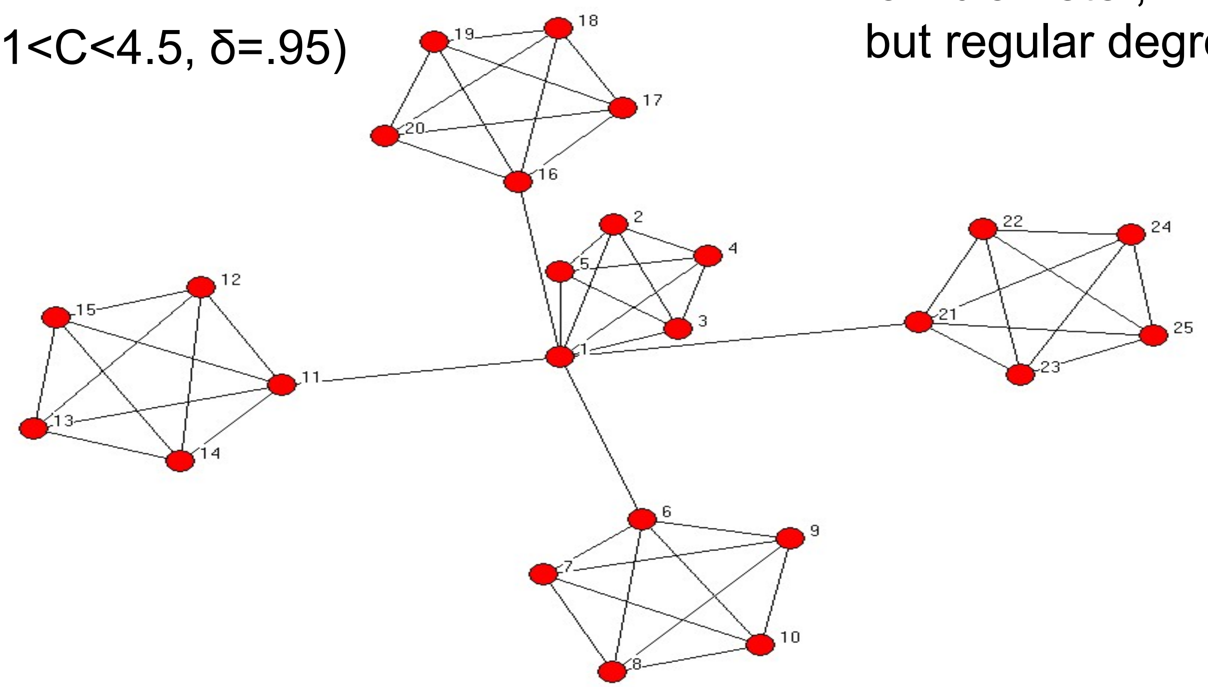


high clustering,
low diameter,

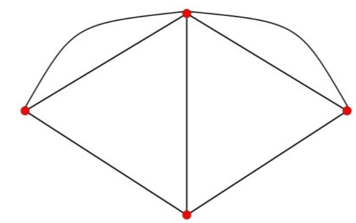
low cost of link to player on own ``island'' – high cost across islands

(c<.04, 1<C<4.5, δ=.95)

high clustering,
low diameter,
but regular degree



Proposition JR05



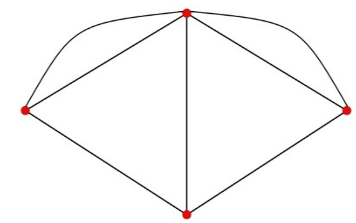
- Truncate connections:

  $u_i(g) = \sum_{j:\, \ell(i,j) \leq D} \delta^{\ell(i,j)} - d_i(g)c$

If $c < \delta - \delta^2$ and $C < \delta + (J-1)\delta^2$ then

- players on each island form a clique
- diameter is bounded by D+1
- $\delta - \delta^3 < C$ implies a lower bound on individual clustering is $(J-1)(J-2)/(J^2K^2)$

Summary Strategic Formation



- Efficient networks and stable Networks need not coincide
- Need not coincide even when transfers are possible, and with complete information
- Depends on
  - setting
  - restrictions on transfers, endogenous transfers...
  - forward looking, errors…
- Can match and explain some observables

Strengths of an economic approach

- Payoffs allow for a welfare analysis
  - Identify tradeoffs – incentives versus efficiency
- Tie the nature of externalities to network formation…
- Put network structures in context
- Account for and *explain* some observables

Challenges to an Economic Approach

- Stark (overly regular) network structures emerge
  - need some heterogeneity
  - simulations help in fitting

- over-emphasize choice versus chance for some (especially large) applications??

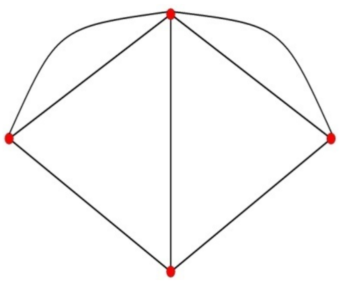
- How to identify payoff structure in applications?
  - relating network structure and outcomes, payoffs

Models that marry strategic with random are needed

- Weaknesses of Random are Strengths of Economic approach, and vice versa.

- Mixed models
  – allow for welfare/efficiency analysis
  – take model to data and fit observed networks
  – do so across applications

Social and Economic Networks: Models and Analysis



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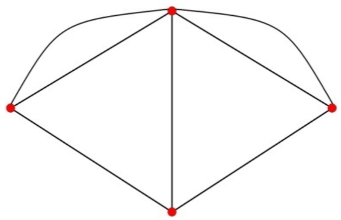
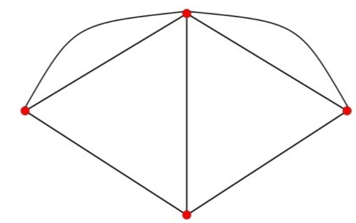
4.9: SUGMs and Strategic Model of Network Formation



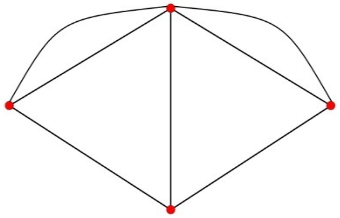
Strategic and Random



Utility from forming subgraphs: links, triangles, etc.

Some randomness in the utility (or decision)

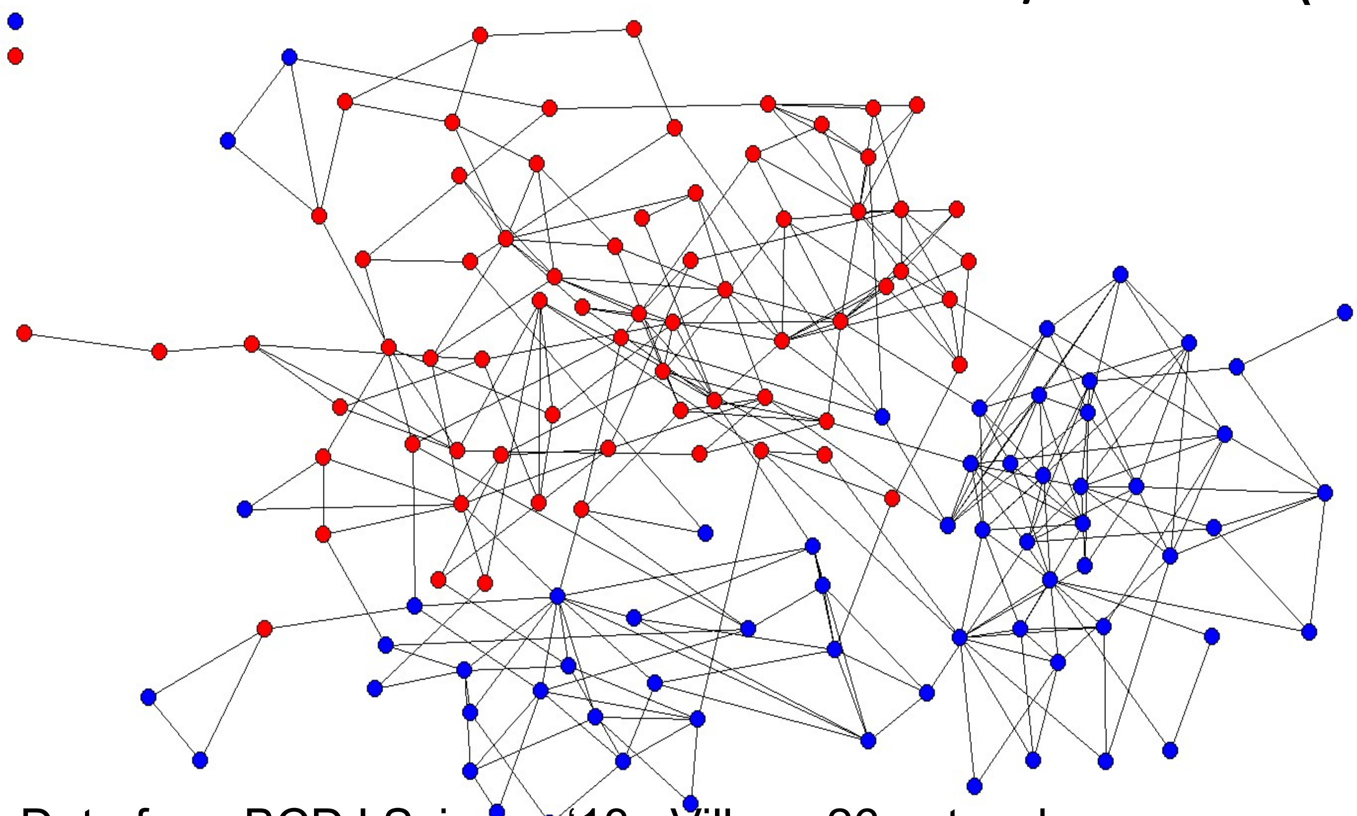
Example: Social Pressure



- Caste relationships
  - Are they more likely to occur ``in private'' with no friends in common
  - Or occur with same frequency in embedded relationships?

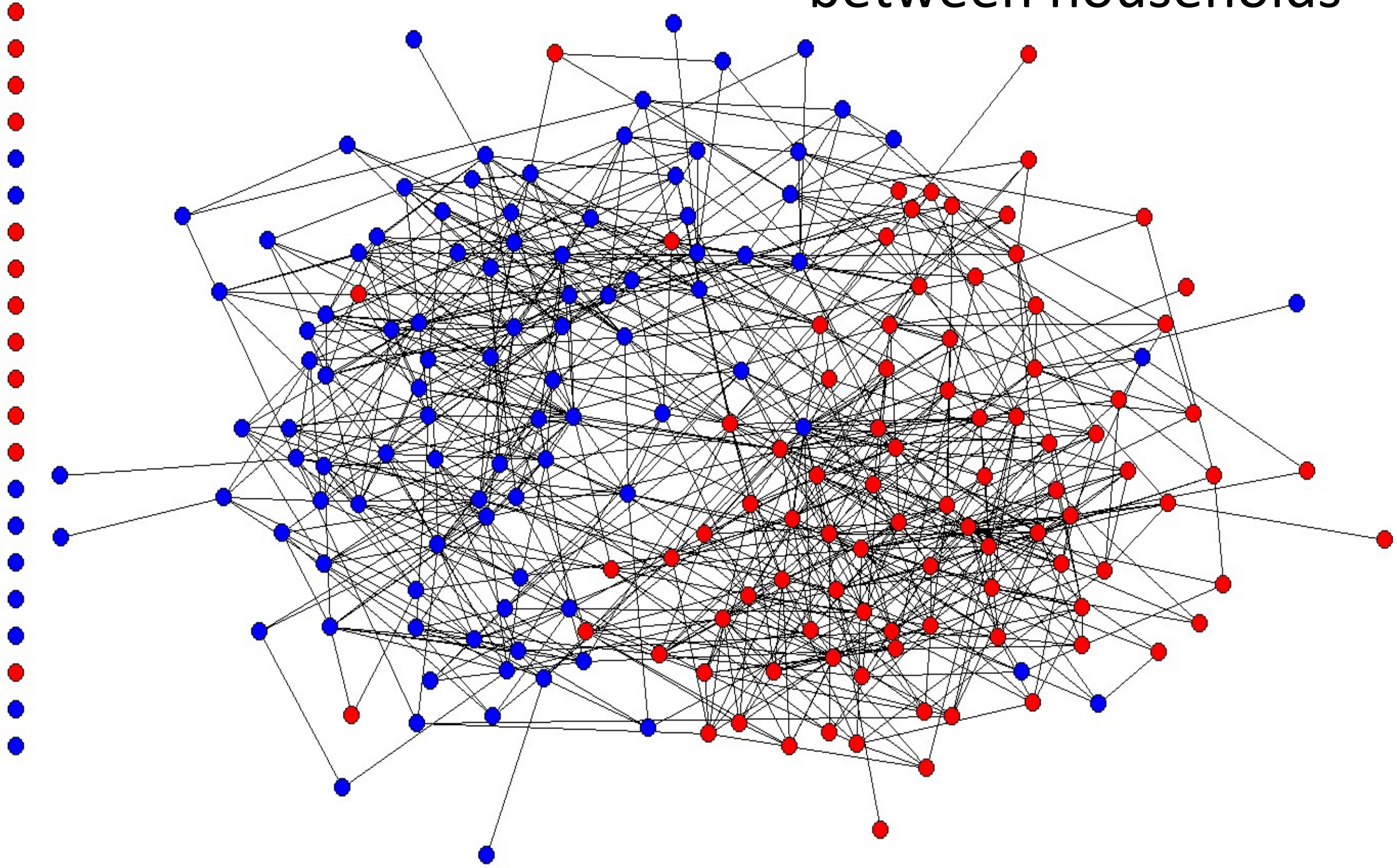
Pcross= .006
Pwithin=.089

Red=General/OBC (adv. castes)
Blue=SC/ST (disadv. castes)

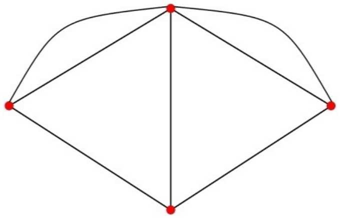


Data from BCDJ Science '13, Village 26 network
of Kerosene-Rice Sharing among households

Red=General/OBC   BCDJ 2013
Blue=SC/ST   Village48 social visits
between households

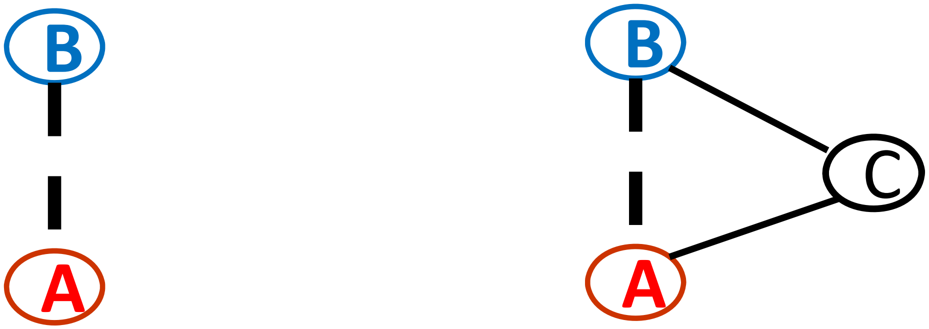


Application : Social Pressure

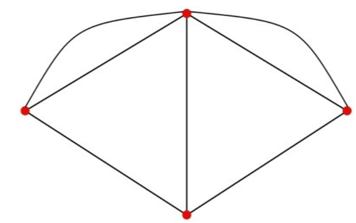


Relatively

Less likely?



Preferences:



Need more consent to form triad than dyad

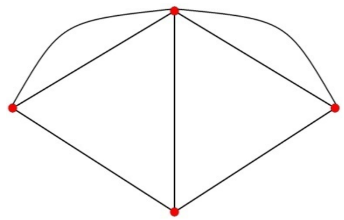
Need to account for preferences, otherwise will naturally find

less desired triads/more desired triads

$<$

less desired dyads/more desired dyads

Preference based SUGM:



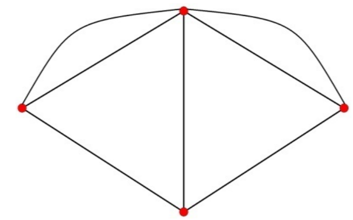
Probability of a link forming depends on likelihood that pair meets, and both wish to form it

X_i i's characteristics

$U_L(X_i, X_j) - \varepsilon_{ij}$ utility of a link between i, j

i benefits from the link iff: $\varepsilon_{ij} < U_L(X_i, X_j)$

Preference based SUGM:



Probability of a link forming depends on likelihood that pair meets, and both wish to form it

X_i i's characteristics

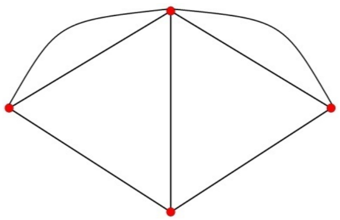
$U_L(X_i, X_j) - \varepsilon_{ij}$ utility of a link between i, j

i benefits from the link iff: $\varepsilon_{ij} < U_L(X_i, X_j)$

pairwise stability: links form if and only if

$\varepsilon_{ij} < U_L(X_i, X_j)$ and $\varepsilon_{ji} < U_L(X_j, X_i)$

Preference based SUGM:



Probability of a link forming depends on likelihood that pair meets, and both wish to form it

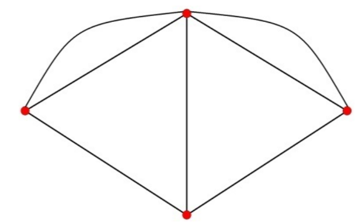
X_i i's characteristics

$U_L(X_{i,}X_j) - \varepsilon_{ij}$ utility of a link between i, j

$F_L(X_{i,}X_j)$ distribution of ε_{ij}

prob link forms prop to $F_L(U_L(X_{i,}X_j))\ F_L(U_L(X_jX_i))$

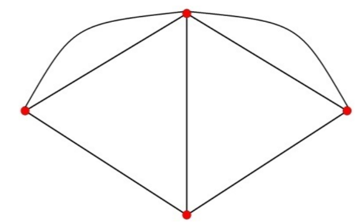
Triangles form:



Probability of a triangle forming proportional to:

$$F_T(U_T(X_{i,}X_j,X_k))\ F_T(U_T(X_{i,}X_j,X_k))\ F_T(U_T(X_{k,}X_j,X_i))$$

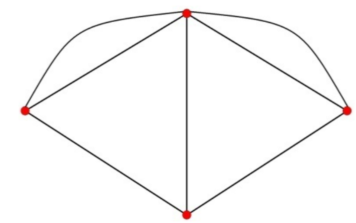
Application: Social Pressure



Null Hypothesis:

$$\frac{\text{Prob prefer across caste triad}}{\text{Prob prefer within caste triad}} = \frac{\text{Prob prefer across caste link}}{\text{Prob prefer within caste link}}$$

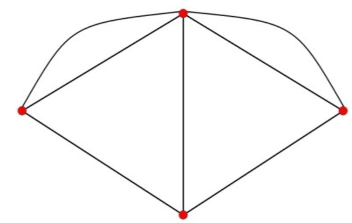
Application: Social Pressure



$$\frac{\text{Freq across caste triad}}{\text{Freq within caste triad}} = \frac{F(U(\text{cross triad}))^3}{F(U(\text{within triad}))^3}$$

$$\frac{\text{Freq across caste link}}{\text{Freq within caste link}} = \frac{F(U(\text{cross link}))^2}{F(U(\text{within link}))^2}$$

Application: Social Pressure

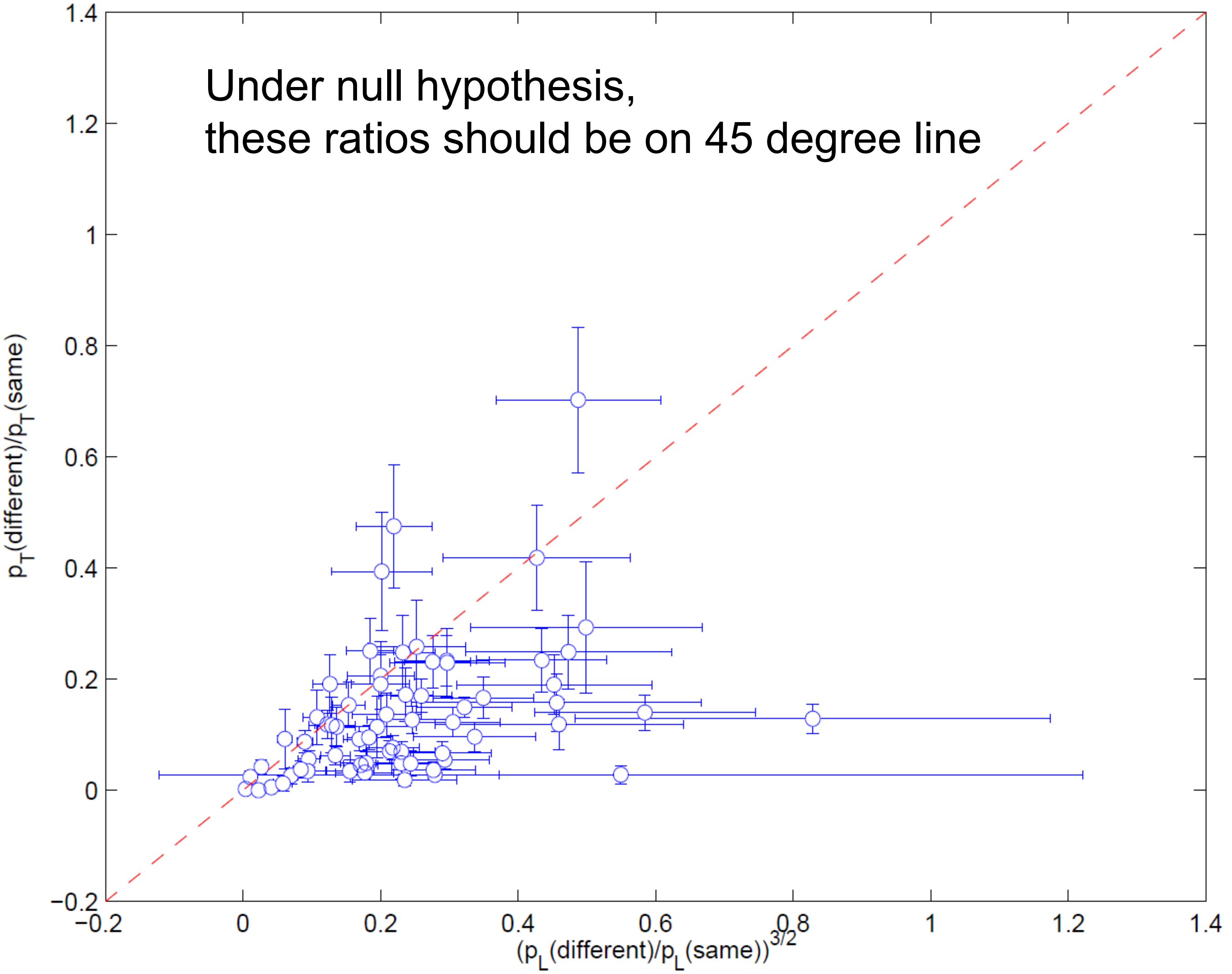


So:

$$\frac{\text{Prob } \textbf{prefer} \text{ form across caste triad}}{\text{Prob } \textbf{prefer} \text{ form within caste triad}} \sim \left[\frac{\text{Freq}_T(\text{cross})}{\text{Freq}_T(\text{within})}\right]^{1/3}$$

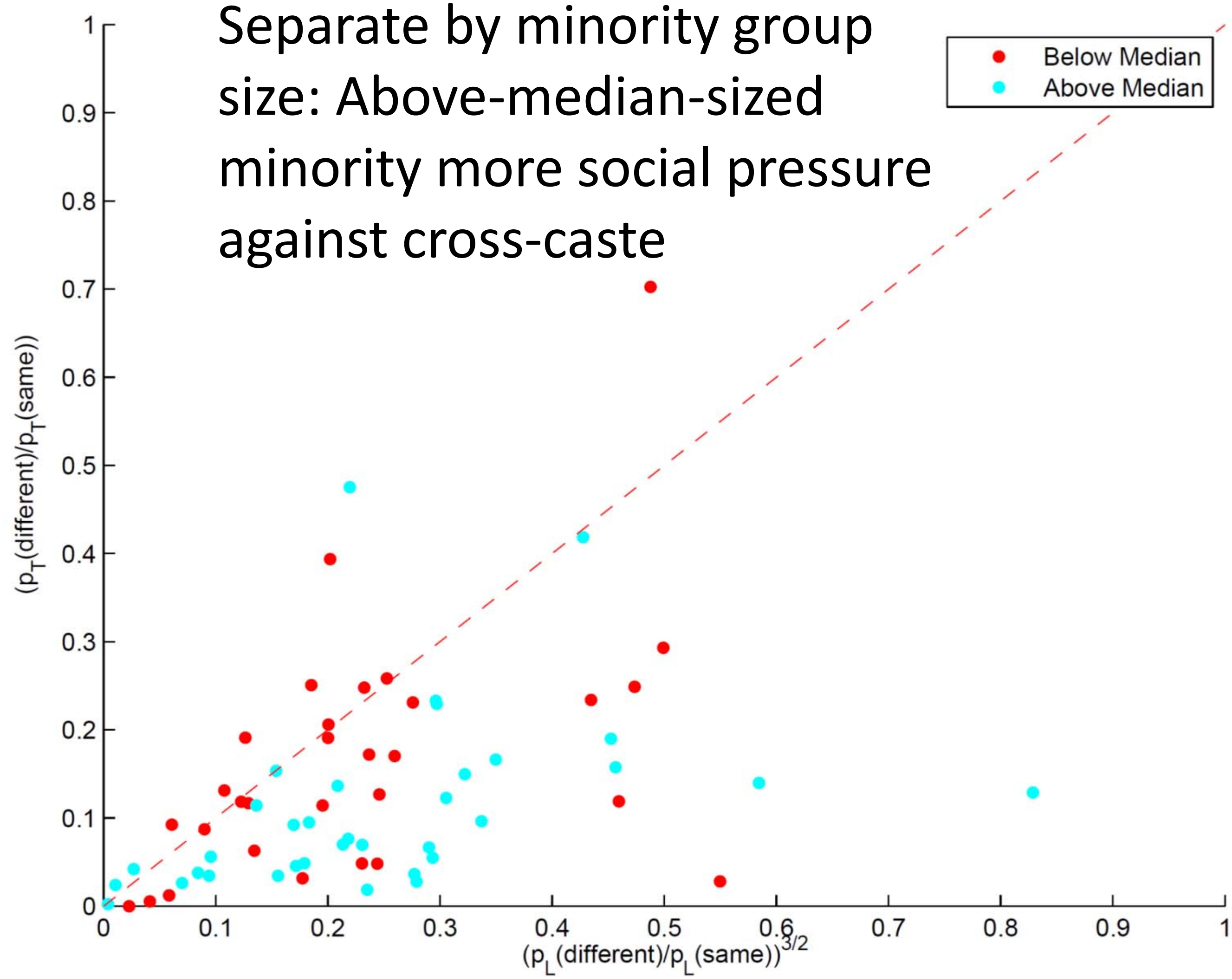
$$=$$

$$\frac{\text{Prob } \textbf{prefer} \text{ form across caste link}}{\text{Prob } \textbf{prefer} \text{ form within caste link}} \sim \left[\frac{\text{Freq}_L(\text{across})}{\text{Freq}_L(\text{within})}\right]^{1/2}$$

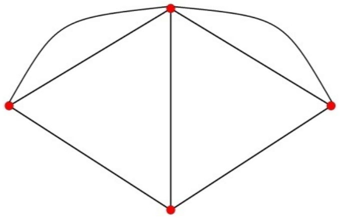


Under null hypothesis,
these ratios should be on 45 degree line
$p_T(\text{different})/p_T(\text{same})$
$(p_L(\text{different})/p_L(\text{same}))^{3/2}$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Separate by minority group size: Above-median-sized minority more social pressure against cross-caste



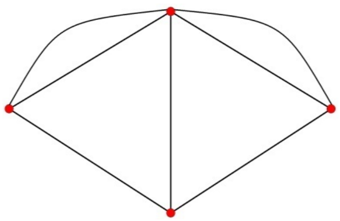
Social Pressure Conclusion:



- Reject the null hypothesis at the 99.9% level

- Based on model, people show a significantly stronger preference for forming cross-caste relationships when the link is in isolation: no friends in common

Modeling Strategic Network Formation in a Statistical Model

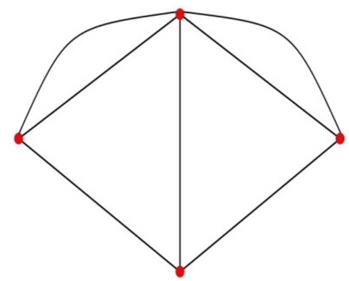


- SUGMs / SERGMs allow for strategic network estimation

- Opens possibilities for systematic estimation

- Subgraphs included, dynamic models...

Social and Economic Networks: Models and Analysis



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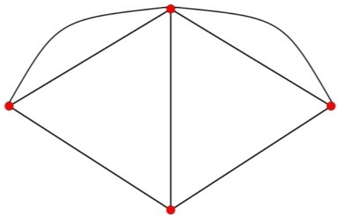
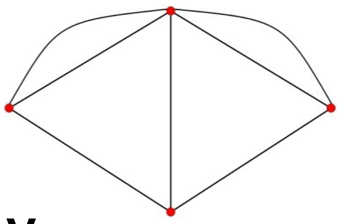
4.10: Pairwise Nash Stability

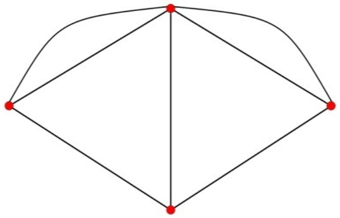


Strategic Formation Models:



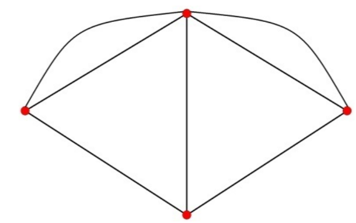
- Saw conflict between stability and efficiency
- Can Transfers help?
- Modeling Stability and Dynamics, etc
  - Refining pairwise stability
  - Dynamic processes
  - Forward looking behavior
- Directed Networks
- Fitting such models

Modeling Stability



- Beyond Pairwise Stability - Allowing other deviations
  - multiple links by individuals
  - coordinated deviations
- Existence questions
- Dynamics
- Stochastic Stability
- Forward looking behavior
- Directed Networks

Nash equilibrium

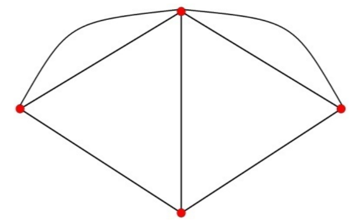


- Myerson's announcement game

- Players simultaneously announce their preferred set of neighbors S_i

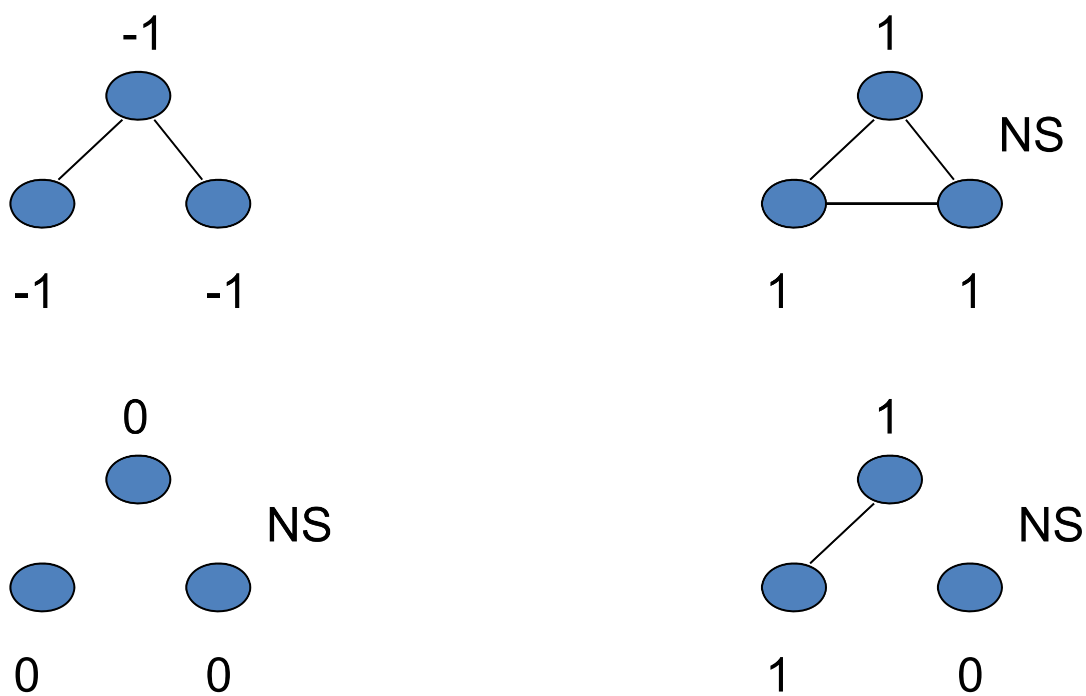
- $g(S)=\{ ij : j \text{ in } S_i \text{ and } i \text{ in } S_j \}$

Nash Stability



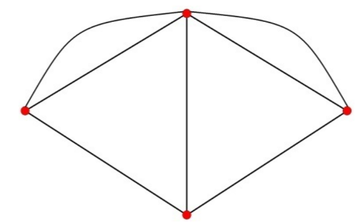
- Nash stable, $u_i(g(S)) \geq u_i(g(S'_i, S_{-i}))$ for all i S'_i

- So, g is Nash stable if and only if no player wants to delete some set of his or her links



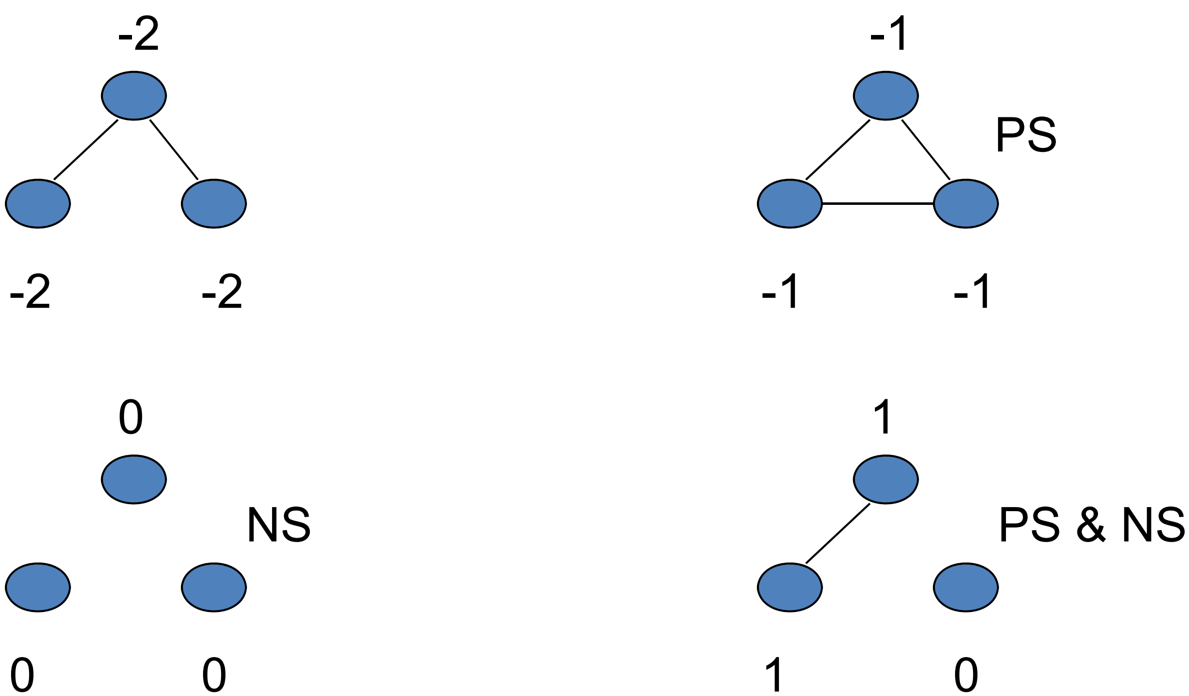
-1
-1
-1
1
NS
1
1
0
NS
0
0
1
NS
1
0

Pairwise Nash Stability



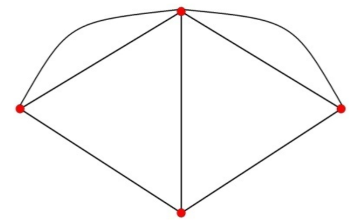
- Both pairwise stable and Nash stable

- Captures multiple link changes



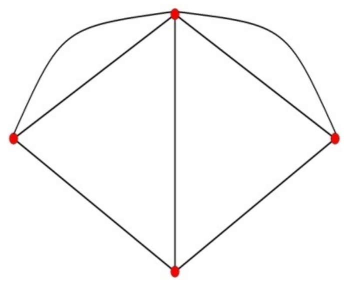
-2
-2
-2
-1
PS
-1
-1
0
NS
0
0
1
PS & NS
1
0

Pairwise Nash Stability



- Both pairwise stable and Nash stable
- Captures multiple link changes
- Other variations: e.g., allow addition of link plus deletion of others at same time, larger coalitions,...)

Social and Economic Networks: Models and Analysis



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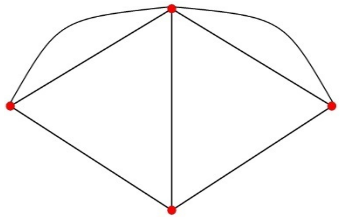
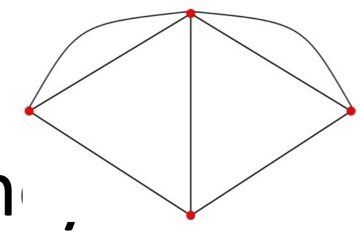
4.11: Dynamic Strategic Network Formation

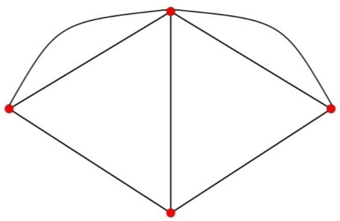


Strategic Formation Models:



- Saw conflict between stability and efficiency
- Can Transfers help?
- Modeling Stability and Dynamics, etc
  - Refining pairwise stability
  - Dynamic processes
  - Forward looking behavior
- Directed Networks
- Fitting such models

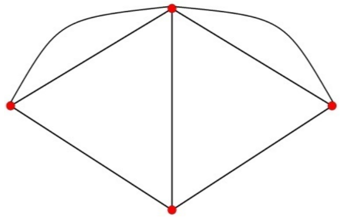
Dynamic Strategic Models



- Explicitly model dynamics and incentives
  - Realism(?)
  - Refine static stable models
  - Incorporate forward looking nature

- Very different approaches:
  - Myopic and error prone
  - Fully forward looking and calculating

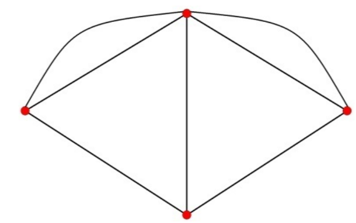
A Dynamic Process



- Even if some pairwise stable networks are efficient, might not reach those:

- A. Watts (01): link is picked uniformly at random
  - added if it benefits both players (at least one strictly)
  - deleted if it benefits either to delete it

Endpoints



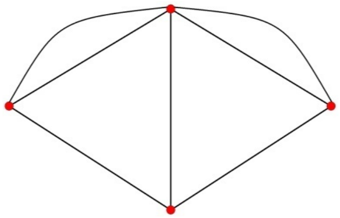
Resting point must be pairwise stable

Proposition (A. Watts (2001)):

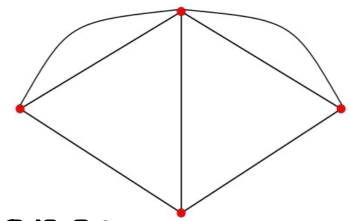
Consider connections model where

$\delta - \delta^2 < c < \delta$, so that a star is efficient and pairwise stable. As n grows, the probability that the above process stops at a star goes to 0.

Ideas:



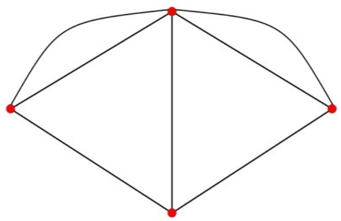
Proof



- In this cost range, once have a link, always have at least one:

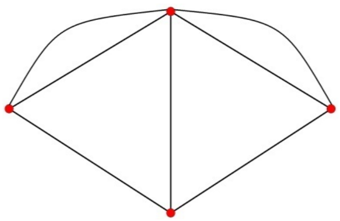
- c< δ so a link is a net benefit, nobody severs a link that would lead a node to be isolated

Proof



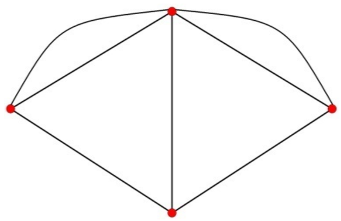
- In this cost range, once have a link, always have one
- If reach a star, relabel 1 as whatever node is the center and 2 through n labeled by last date attached to 1
- n could not have been attached to another node when 1 attached to it to form the star, or n and 1 would have been already been at a distance of 2, and would not have formed the link. So n was never attached to before attaching to 1

Proof



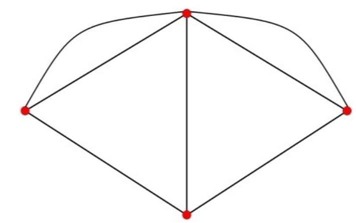
- In this cost range, once have a link, always have one
- If reach a star, relabel 1 as whatever node is the center and 2 through n be ordered by last date attached to 1
- n could not have been attached to another when 1 attached to it last, or would have been already been at a distance of 2. So n was never attached to before attaching to 1
- induct – same for n-1, etc.
- So must form star directly

Proof



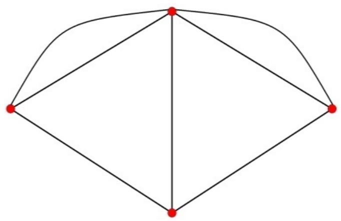
- So must form star directly
- If link ij is first one identified, then next one must involve i or j to get a star.
  - there are 2(n-2) such links
  - there are n(n-1)/2 – 2(n-2)-1 other links

Proof



- So must form star directly
- If link ij is first one identified, then next one must involve i or j to get a star.
  - there are 2(n-2) such links
  - there are n(n-1)/2 – 2(n-2)-1 other links
- So, the chance that even take the first step to forming the star is no more than (2n-4)/ [n(n-1)/2 - 2n+3] which goes to 0 at rate 1/n
- Chance that actually form a star is much lower than this: on the order of $1/n^n$ since the same is true on each step...

A Dynamic Process

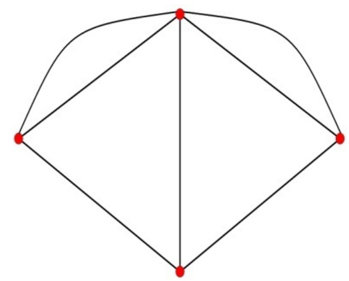


- Natural dynamics: link is picked at random
  - added if it benefits both players (at least one strictly)
  - deleted if it benefits either to delete it

- Will find pairwise stable networks (if they exist)

- Even if efficient networks are pairwise stable, may have low chance of reaching them....

Social and Economic Networks: Models and Analysis



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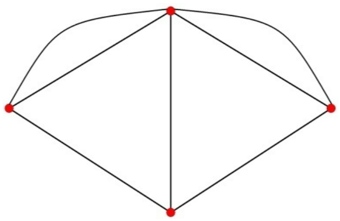
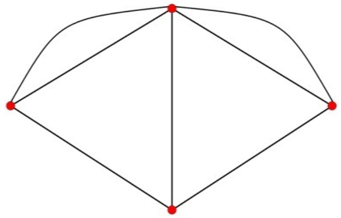
4.12: Evolution and Stochastics

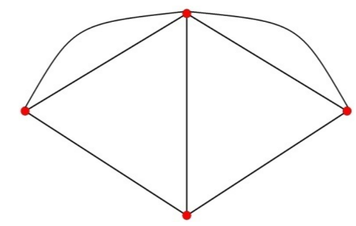


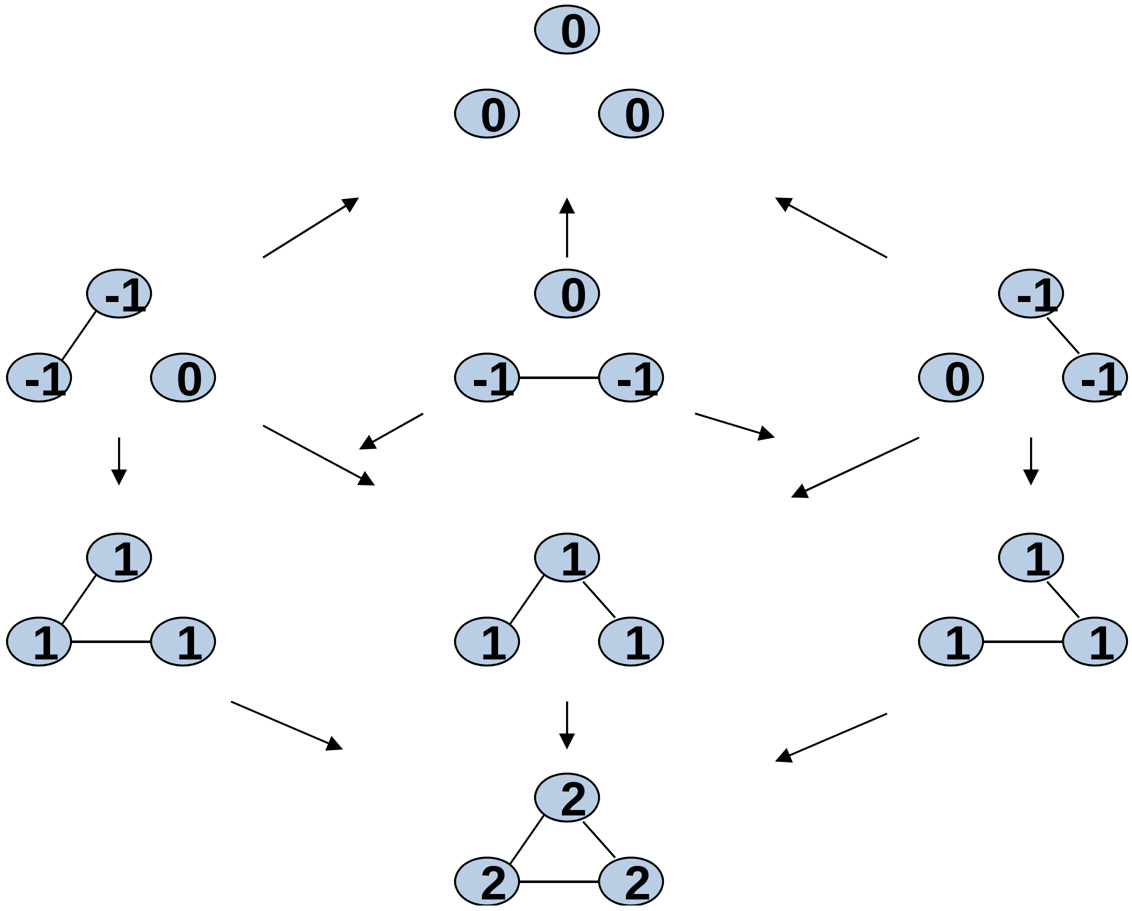
Improving path:



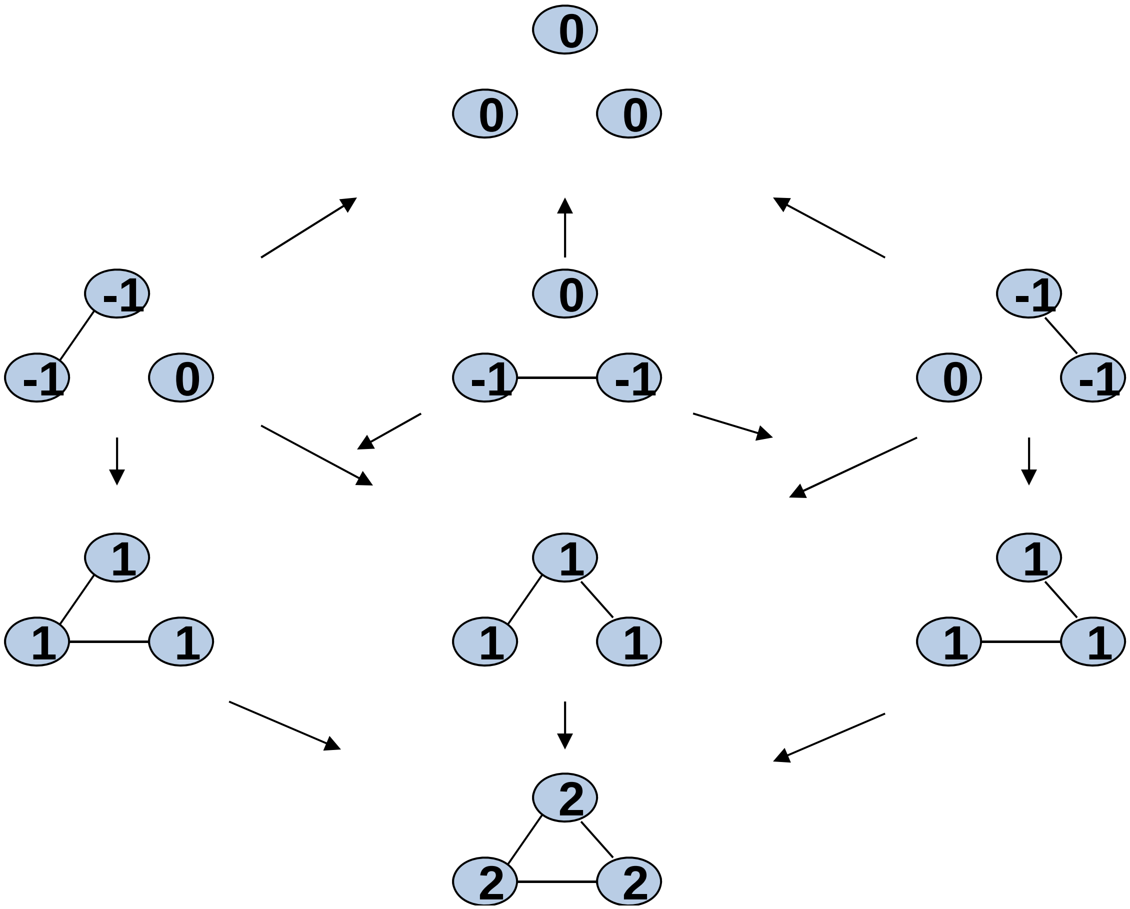
- Sequence of adjacent networks:
  - Link is added if it benefits both agents, at least one strictly
  - Link is deleted if either agent benefits from its deletion

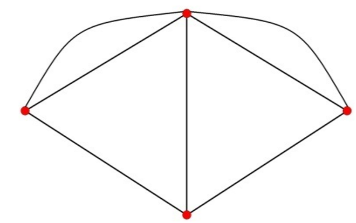
Improving paths:



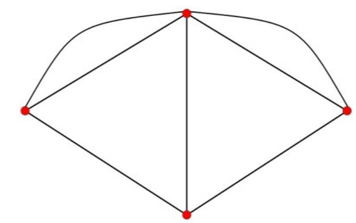


Two PNS networks:



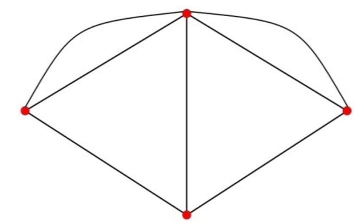


Stochastic Stability



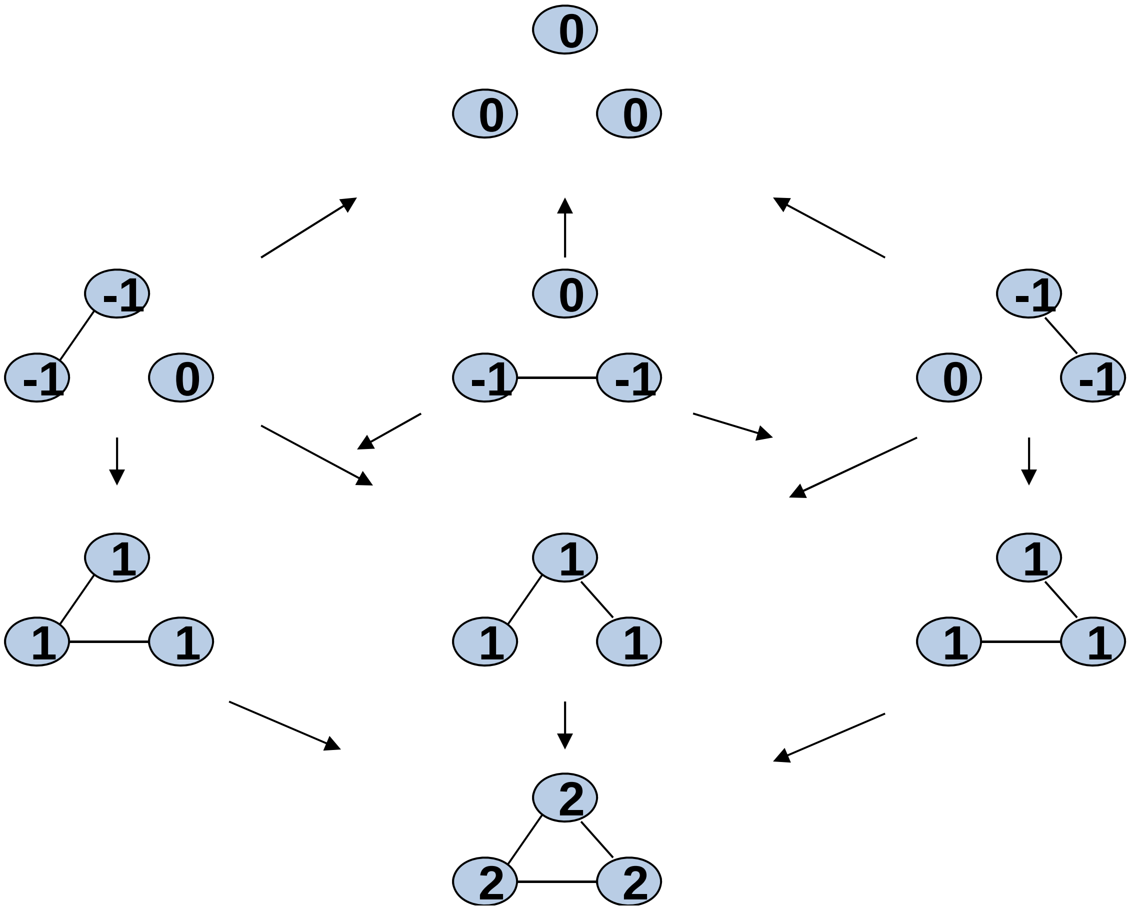
- Add trembles/errors to improving paths:
- Start at some network and with equal probability on all links choose a link:
  - Add that link if it is not present and both agents prefer to add it (at least one strictly)
  - delete that link if it is present and one of the two agents prefers to delete it.
  - *Reverse the above decision with probability $\varepsilon>0$*

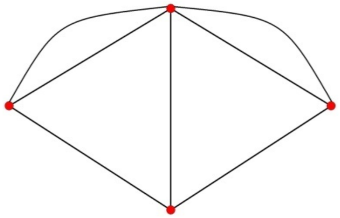
Stochastic Stability



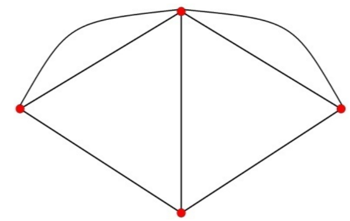
- Add trembles/errors to improving paths
- Trembles/errors: with probability $\varepsilon>0$
- Finite state, irreducible, aperiodic Markov chain

Improving Paths:





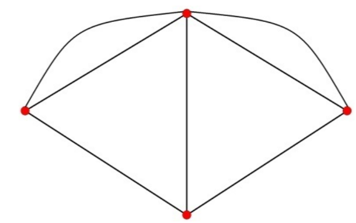
Associated Markov Chain



- State be the number of links (more generally states are the networks)

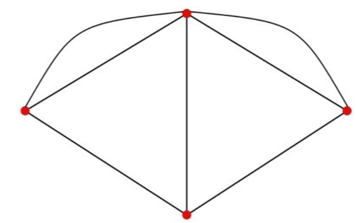
$$\Pi(\varepsilon) = \begin{pmatrix} 1-\varepsilon & \varepsilon & 0 & 0 \\ \frac{1-\varepsilon}{3} & \varepsilon & \frac{2(1-\varepsilon)}{3} & 0 \\ 0 & \frac{2\varepsilon}{3} & \frac{2-\varepsilon}{3} & \frac{1-\varepsilon}{3} \\ 0 & 0 & \varepsilon & 1-\varepsilon \end{pmatrix}.$$

Unique steady state distribution



$$\mu(\varepsilon) = \left(\frac{\varepsilon(1-\varepsilon)}{1+2\varepsilon}, \frac{3\varepsilon^2}{1+2\varepsilon}, \frac{3\varepsilon(1-\varepsilon)}{1+2\varepsilon}, \frac{(1-\varepsilon)^2}{1+2\varepsilon} \right)$$

Unique steady state distribution

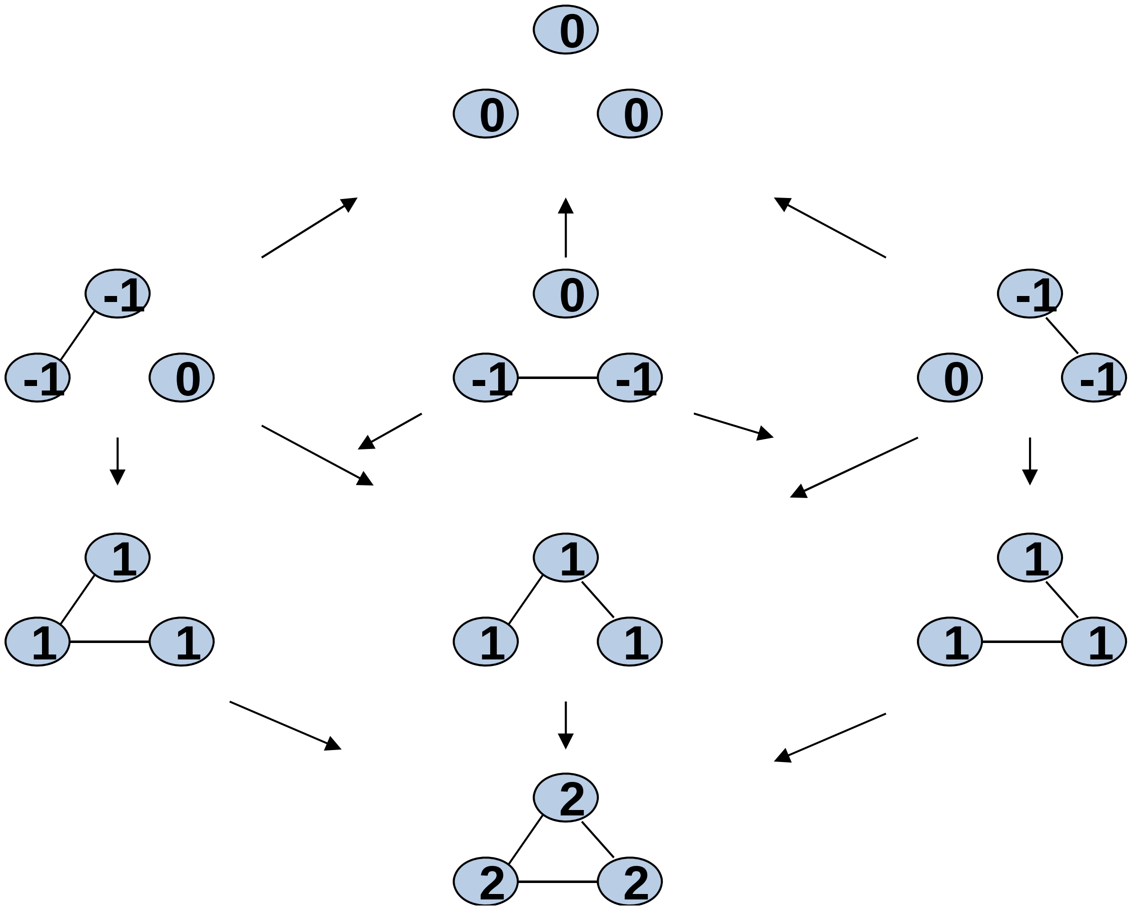


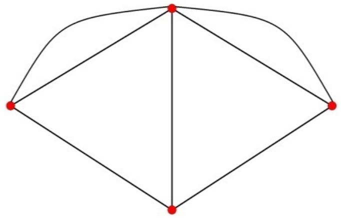
$$\mu(\varepsilon) = \left(\frac{\varepsilon(1-\varepsilon)}{1+2\varepsilon}, \frac{3\varepsilon^2}{1+2\varepsilon}, \frac{3\varepsilon(1-\varepsilon)}{1+2\varepsilon}, \frac{(1-\varepsilon)^2}{1+2\varepsilon} \right)$$

Limit place probability 1 on the complete network

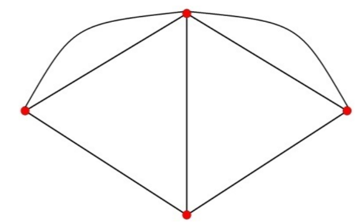
Not the same as the steady states of limit process – which are any with probability on both the complete and empty

Two PNS networks:





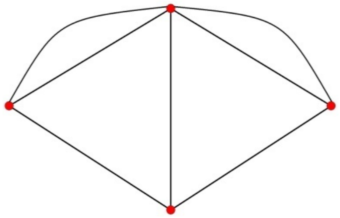
Ideas:



- More errors to leave basin of attraction of the complete network than to leave the basin of attraction of empty network

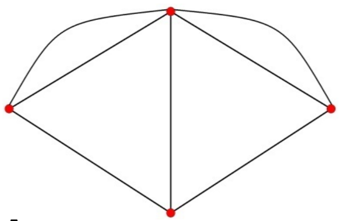
- More generally need to keep track of basins of attraction of many states (via a theorem of Friedlin and Wentzel (1984))

Turns Game into Markov Chain



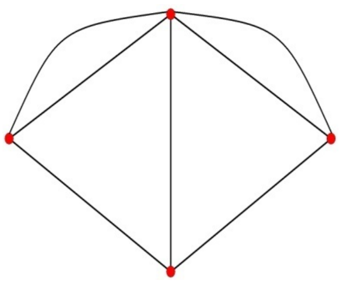
- Much is known about Markov chains
- Can leverage that to prove results
- Can be difficult to get analytic solutions in some contexts with large societies, but can simulate

Modeling Stability



- Beyond Pairwise Stability - Allowing other deviations
  - multiple links by individuals
  - coordinated deviations
- Existence questions
- Dynamics
- Stochastic Stability
- Forward looking behavior
- Build transfers into the formation process

Social and Economic Networks: Models and Analysis



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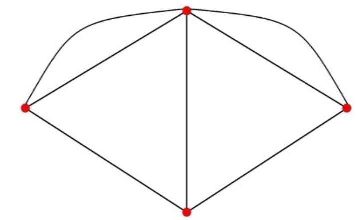
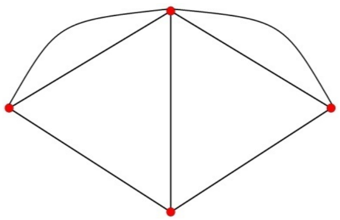
4.13: Directed Network Formation

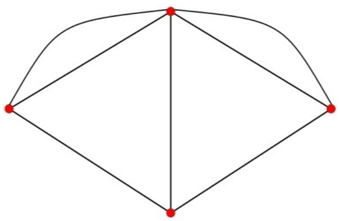


Strategic Formation Models:



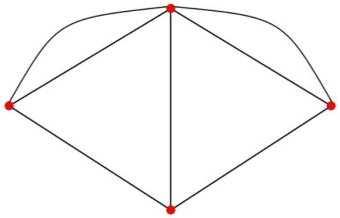
- Saw conflict between stability and efficiency
- Can Transfers help?
- Modeling Stability and Dynamics, etc
  - Refining pairwise stability
  - Dynamic processes
  - Forward looking behavior
- Directed Networks
- Fitting such models

Directed Networks



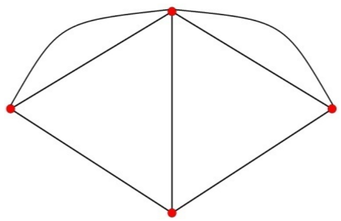
- Formation game easy:
- Players simultaneously announce their preferred set of neighbors S_i
- $g(S) = \{ ij : j \text{ in } S_i \}$ keeping track of *ordered* pairs
- Nash equilibrium

Flow of Payoffs?



- One way flow – get information but not vice versa
- Two way flow – one player bears the cost, but both benefit from the connection (link on internet, phone call??)

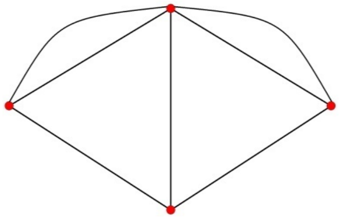
Two Way Flow



- Directed Connections: Bala and Goyal (00)

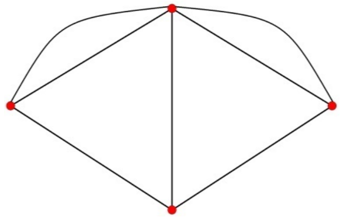
- Same as JW96, but only need link present in either direction

Two Way Flow



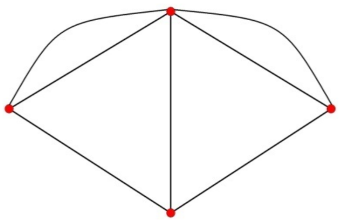
- Efficiency as in the undirected connections model, except c/2 and link in either direction (but not both)
  - low cost: $c/2 < \delta - \delta^2$
    - ``complete'' networks
  - medium cost: $\delta - \delta^2 < c/2 < \delta + (n-2)\delta^2/2$
    - ``star'' networks
  - high cost: $\delta + (n-2)\delta^2/2 < c/2$
    - empty network

Two Way Flow



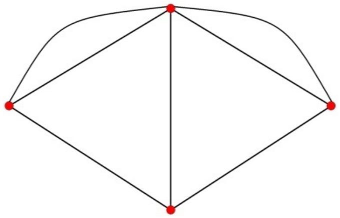
- Nash Stable:
  - low cost: $c < \delta-\delta^2$
    - two-way ``complete'' networks are Nash stable
  - medium/low cost: $\delta-\delta^2 < c < \delta$
    - all star networks are Nash stable, plus others
  - medium/high cost: $\delta < c < \delta+(n-2)\delta^2/2$
    - peripherally sponsored star networks are Nash stable (no other stars, but sometimes other networks)
  - efficient and stable can be empty:
    - $\delta-\delta^2 < c < 2(\delta- \delta^2)$ complete is efficient, not equilibrium

One Way Flow



- Keep track of directed flows, and in links are not (always) useful

An Example

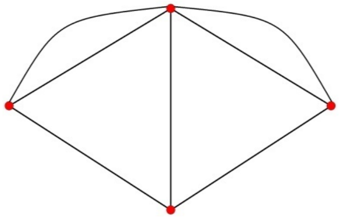


- Bala and Goyal (00) - Directed connections model with no decay:

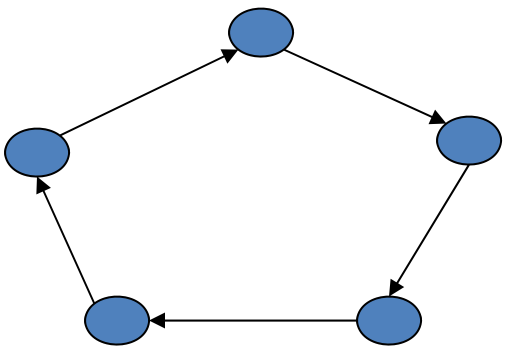
- $u_i(g) = R_i(g) – d_i^{out}(g)c$

where $R_i(g)$ is the number of players reached by directed paths from i

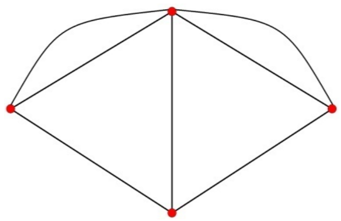
Efficient Networks



- n-player ``wheels'' if c< n-1, empty otherwise:



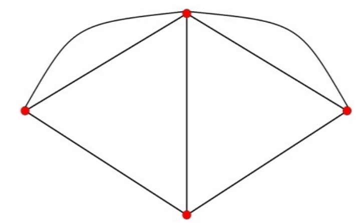
Stable Networks



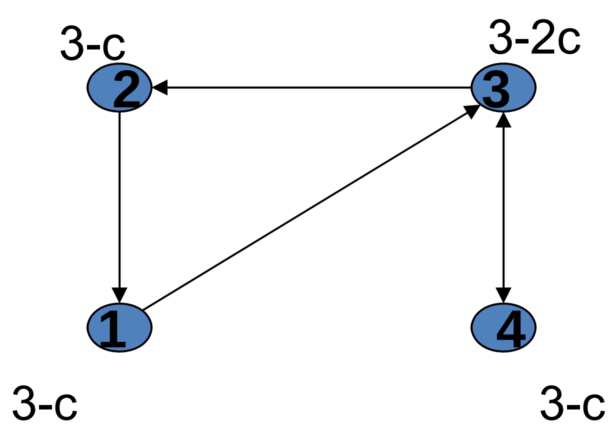
- If c<1 then n-player wheels are the only *strictly* Nash stable network

- If 1<c<n-1 n-player wheels and empty networks are the only *strictly* Nash stable networks

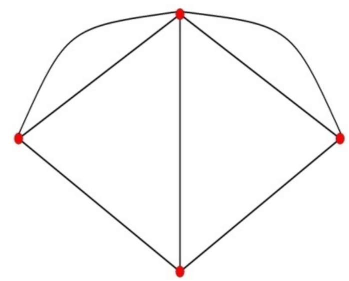
Strictness:



- Nash Stable, but not strictly so: 1 is indifferent between switching link from 3 to 4



Social and Economic Networks: Models and Analysis



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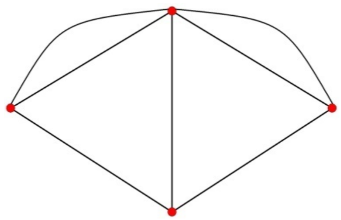
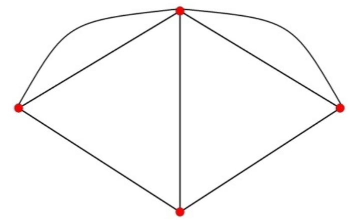
4.14: Structural Estimation of Strategic + Random Formation

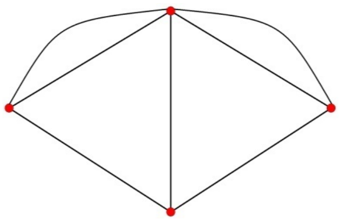


Hybrid Network Models



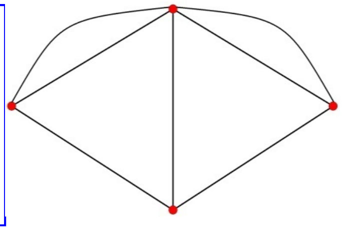
- Most networks involve both choice and chance in formation
- What are the relative roles?
- Random/Strategic models can be too extreme
- Can we see relative roles in homophily?

Application - Homophily:



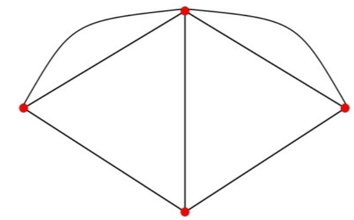
- Group A and Group B form fewer cross race friendships than would be expected given population mix
  - Is it due to structure: few meetings?
  - Is it due to preferences of group A?
  - Is it due to preferences of group B?

Currarini, Jackson, Pin (09,10)



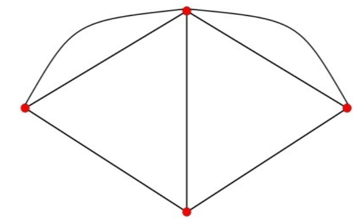
- Utilities specified as a function of friendships
- Meeting process that incorporates randomness
- Allow both utilities and meeting process to depend on types

Revealed Preference Theory



- Common to Consumer Theory
- Use it in mapping social/friendship choices too!
- Different information than surveys on racial attitudes

Model



Types: $i \in \{1,\ldots.,K\}$

s_i = # same-type friends

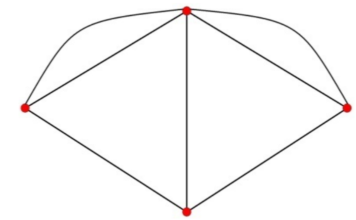
d_i = # different-type friends

$U_i = (s_i + \gamma_i\, d_i)^{\alpha}$ utility to type i

γ_i is the preference bias

$\alpha<1$ captures diminishing returns

Individual Choice

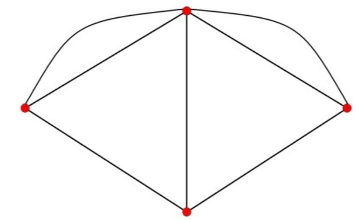


- t_i number of friends -- proportional to time spent socializing -- i is ``type''

- q_i fraction of friends that will be of own type

- t_i maximizes $(q_i t_i + \gamma_i (1-q_i)t_i)^{\alpha} - ct_i$

Individual Choice



- t_i maximizes $(q_i t_i + \gamma_i (1\text{-}q_i)t_i)^{\alpha} - ct_i$

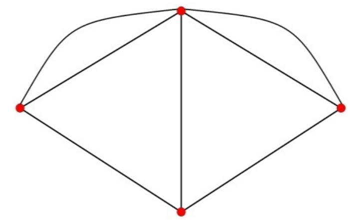
- Solution:

$$t_i = (\alpha/c)^{1/(1-\alpha)} (q_i + \gamma_i (1\text{-}q_i))^{\alpha/(1-\alpha)}$$

- Add noise for particular agent a of type i:

$$t_{ai} = (\alpha/c)^{1/(1-\alpha)} (q_i + \gamma_i (1\text{-}q_i))^{\alpha/(1-\alpha)} + \varepsilon_a$$

How to identify preference parameters from data?

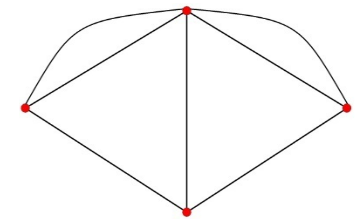


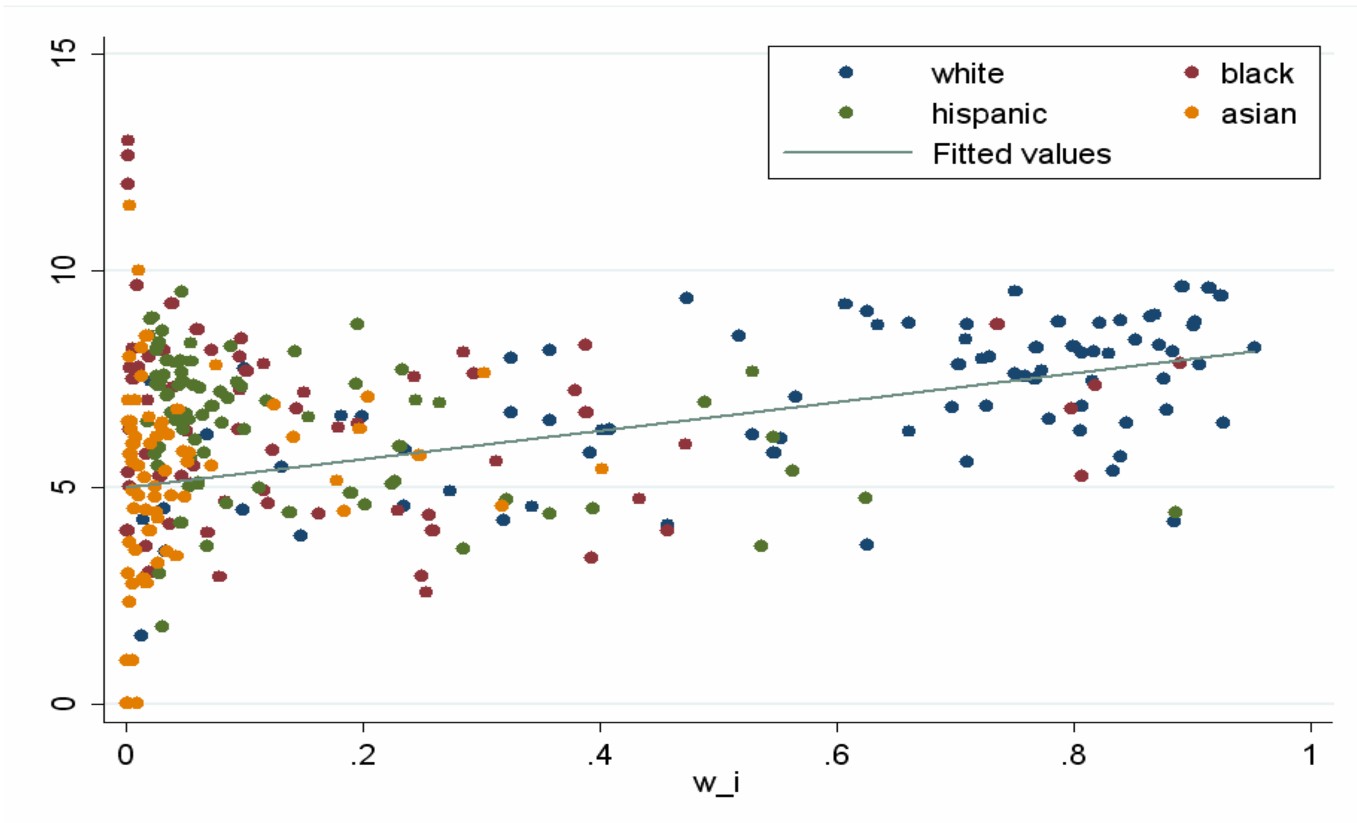
- $t_{ai} = (\alpha/c)^{1/(1-\alpha)} (q_i + \gamma_i (1-q_i))^{\alpha/(1-\alpha)} + \varepsilon_a$

- This is observed directly in the data and will vary with q_i

- If $\gamma_i < 1$ then this is increasing in q_i

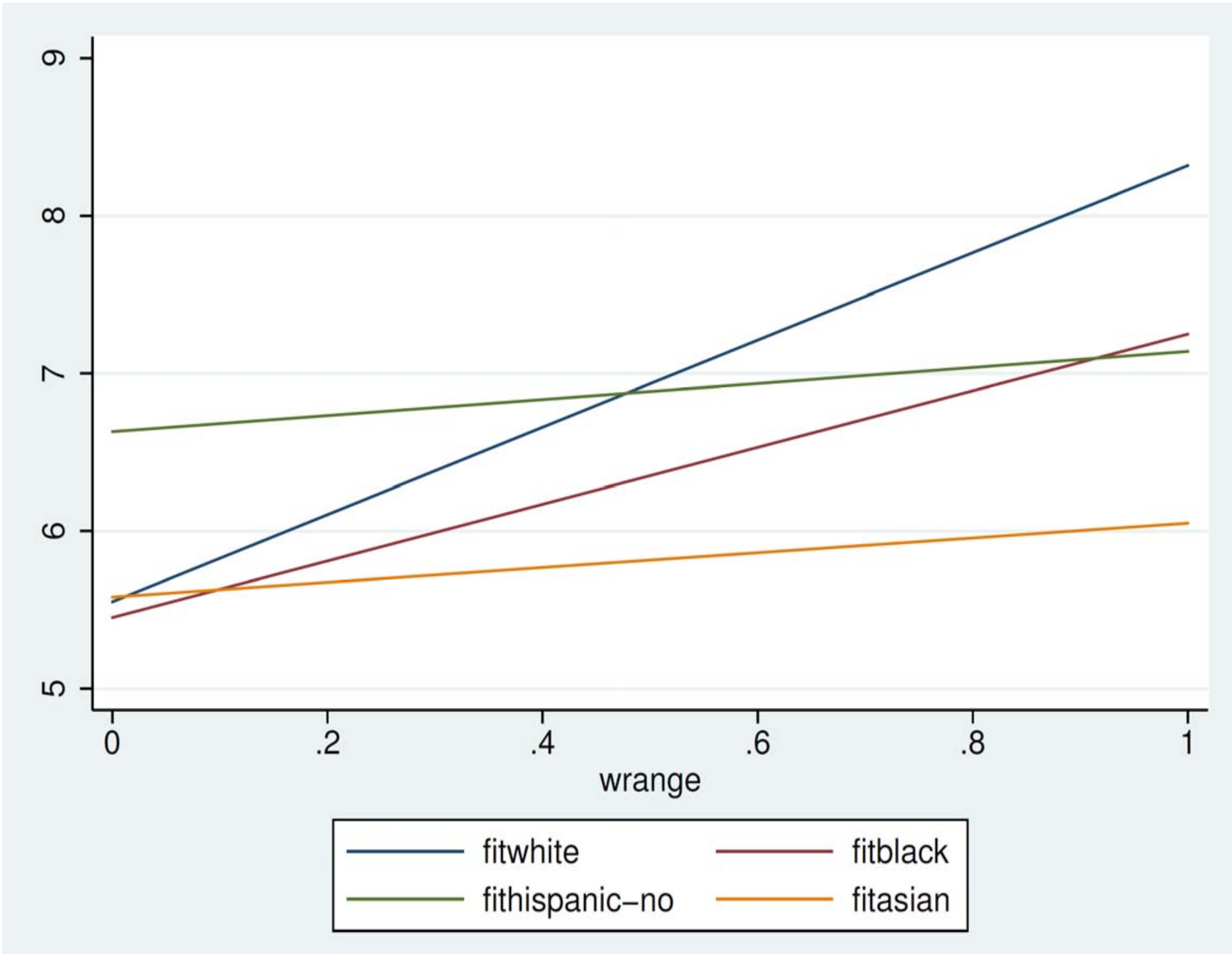
Larger Group=More Friends





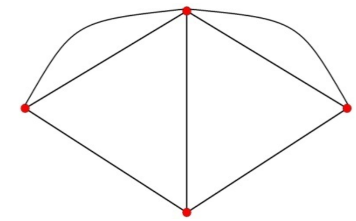
Group size (fraction)

slope 2.3
t=7.3
int= 5.5
t=28



5
6
7
8
9
0
.2
.4
.6
.8
1
wrange
fitwhite
fitblack
fithispanic–no
fitasian

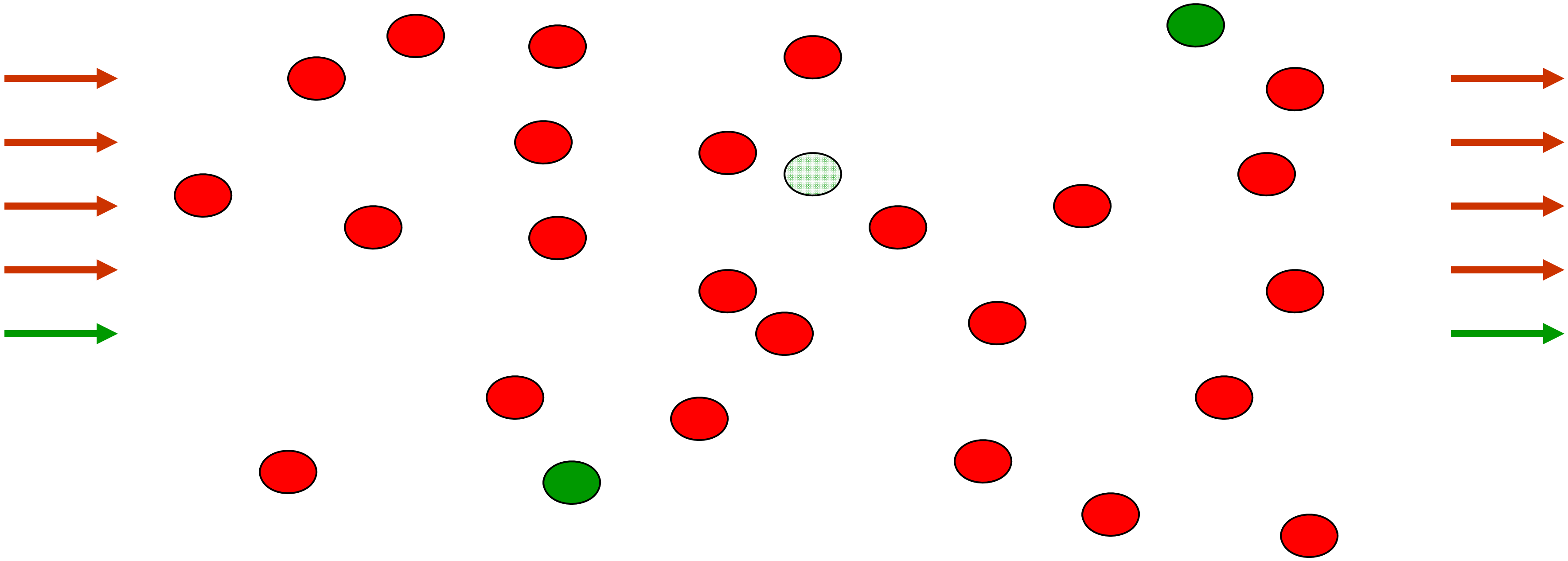
Meeting Process

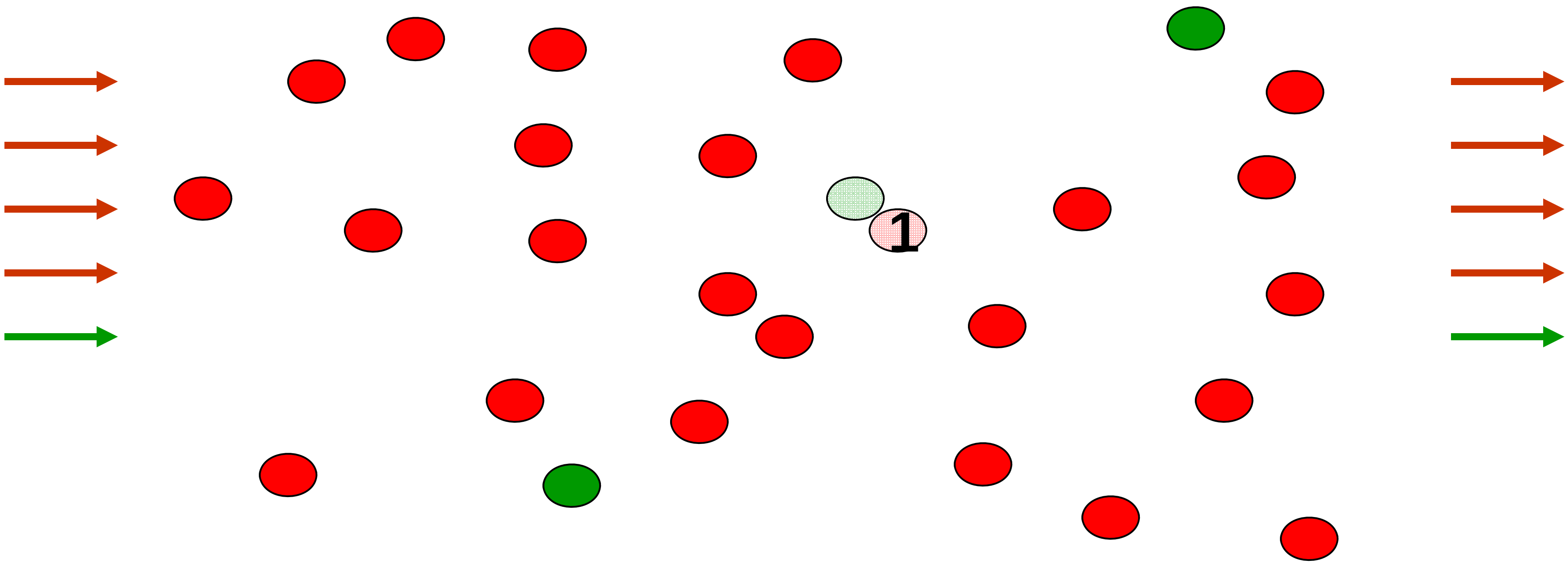


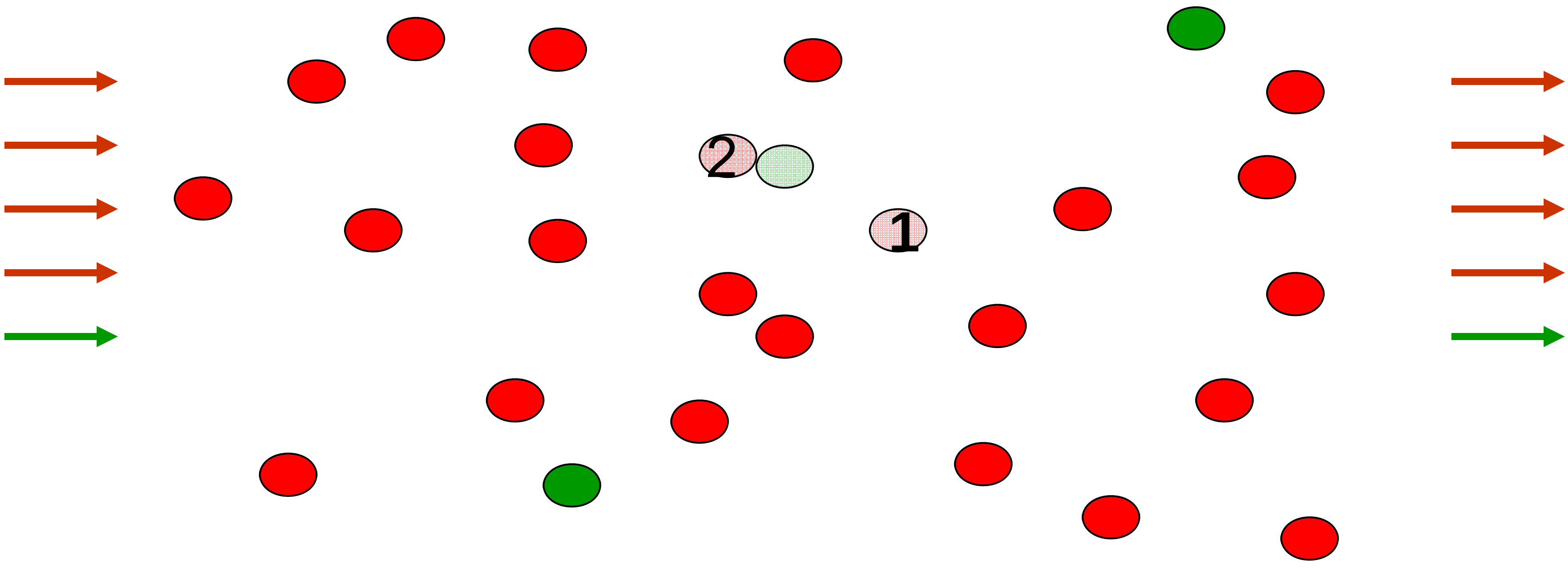
Where do q_i s come from?

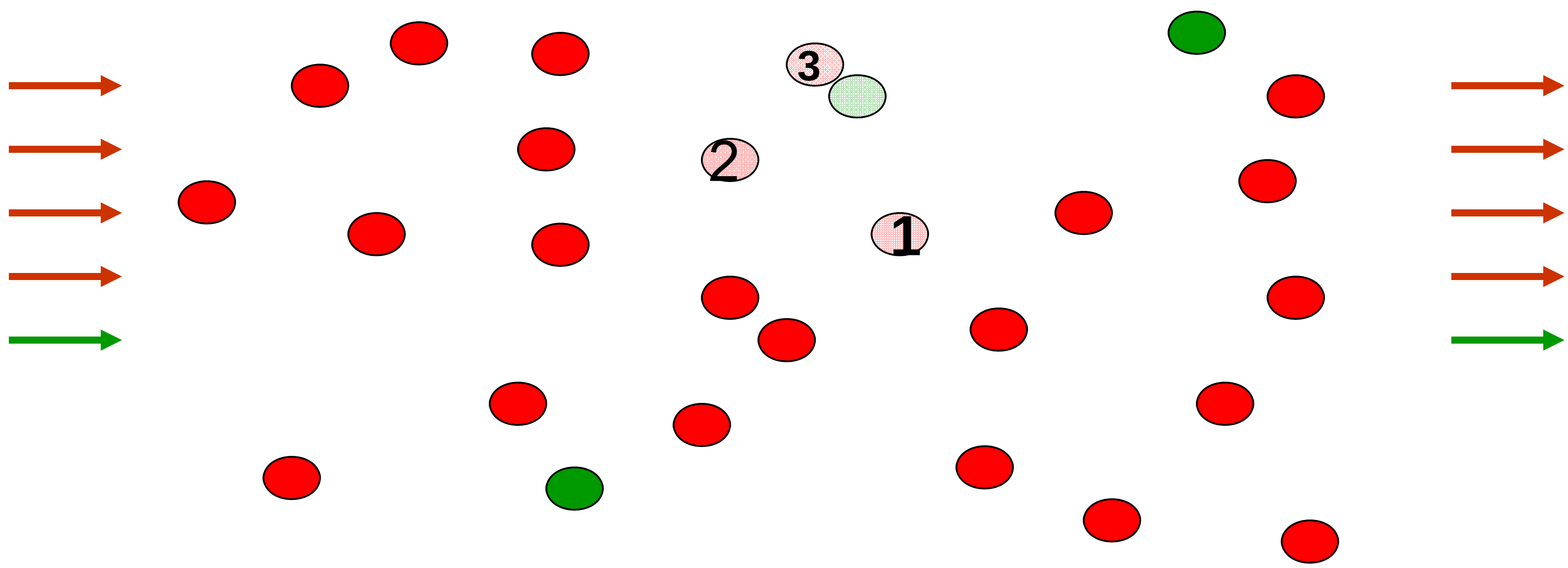
Randomness in meetings, but also have q_i s determined by the decisions of the agents

Meeting Process: ``Party''

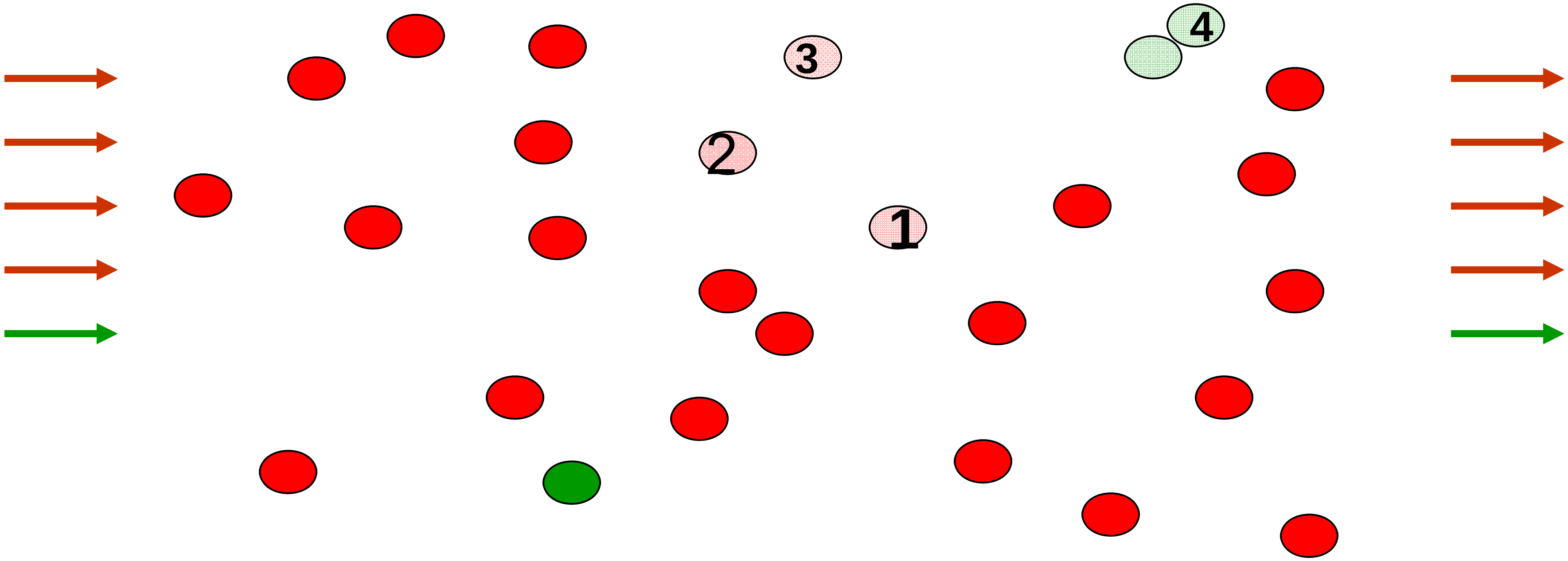


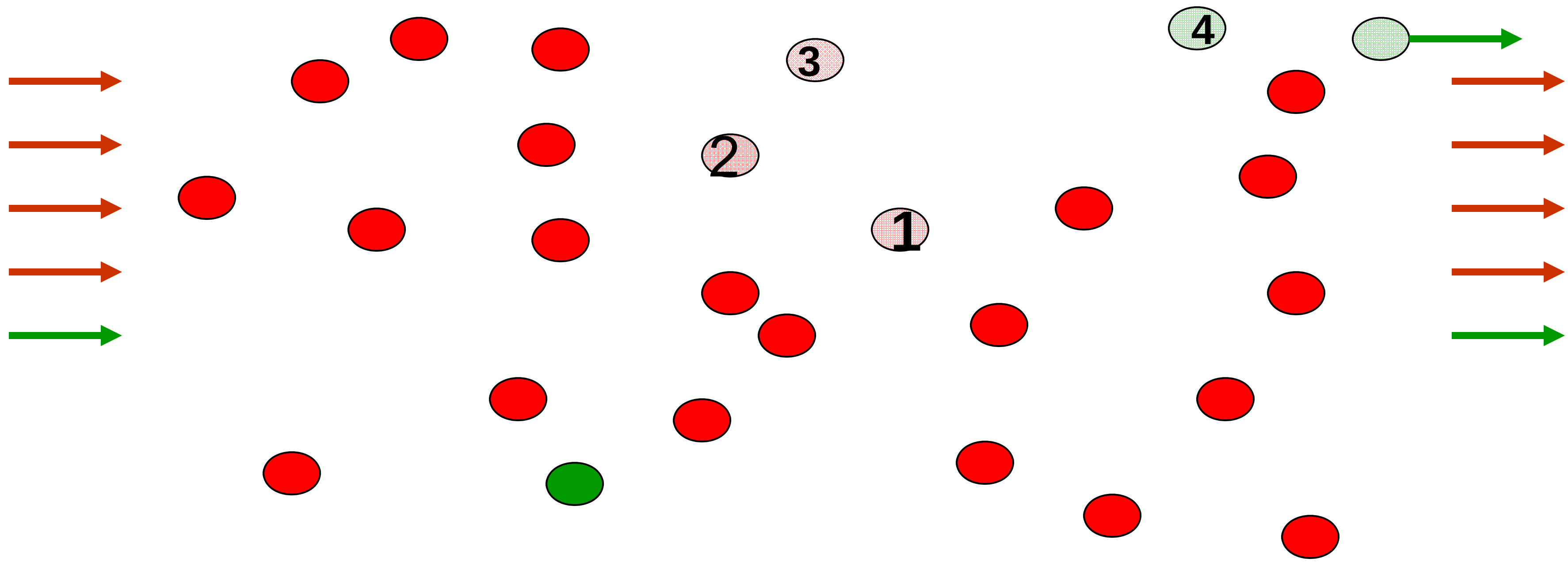

1


2
1

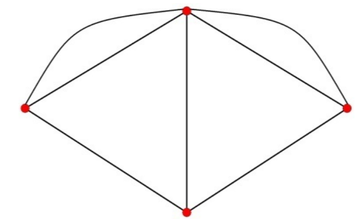


3
2
1


3
4
2
1


3
4
2
1

Bias in Meeting Process



q_i rate at which type i meets type i,

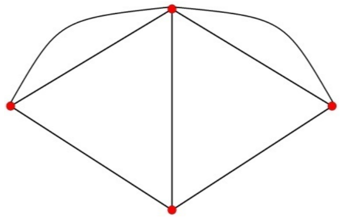
$1-q_i$ rate at which type i meets other types

$$q_i = (\text{stock}_i)^{1/\beta_i}$$

$\beta_i = 1$ ``unbiased'': $q_i = \text{stock}_i$

$\beta_i > 1$ meet own types faster than stocks

Meeting Process



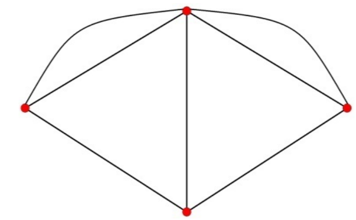
$q_i = (stock_i)^{1/\beta i}$

$\beta_i = 1$ if $stock_i = 1/2$ then $q_i = (1/2)^{1/1} = 1/2$

$\beta_i = 2$ if $stock_i = 1/2$ then $q_i = (1/2)^{1/2} = .707$

$\beta_i = 7$ if $stock_i = 1/2$ then $q_i = (1/2)^{1/7} = .906$

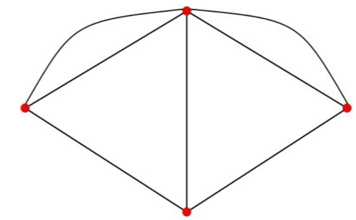
Equilibrium Conditions:



- t_i maximizes $(q_i t_i + \gamma_i (1-q_i)t_i)^{\alpha} - ct_i$
- $stock_i = w_i t_i / \Sigma w_j t_j$ fraction type i in the meeting
- $q_i=(stock_i)^{1/\beta i}$ meetings determined by stocks
- $q_i^{\beta i} = stock_i$ and $\Sigma\, stock_i = 1$ imply that

  $\Sigma q_i^{\beta i} = 1$(balanced meetings)
- atomless population (ignore individual errors)

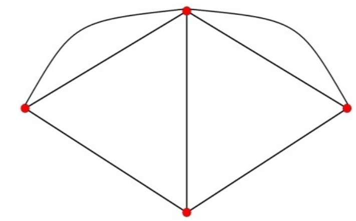
Fitting: Equilibrium Conditions



$$\max_{t_i} (q_i t_i + \gamma_i (1 - q_i) t_i)^\alpha - c t_i$$

$$\sum_i q_i^{\beta_i} = 1$$

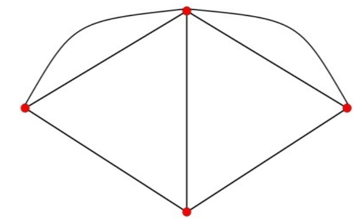
Fitting the model



- $t_i - \varepsilon_i = (\alpha/c)^{1/(1-\alpha)} (q_i + \gamma_i (1-q_i))^{\alpha/(1-\alpha)}$

- $\Sigma\, q_i^{\beta_i} - \varepsilon = 1$

Eliminate (unobserved) costs:

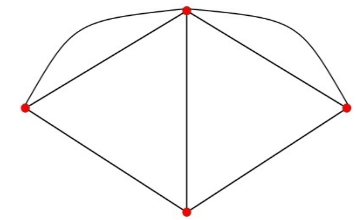


- $t_i - \varepsilon_i = (\alpha/c)^{1/(1-\alpha)} (q_i + \gamma_i (1-q_i))^{\alpha/(1-\alpha)}$
- $(t_i - \varepsilon_i) / (t_j - \varepsilon_j)$

  $= (q_i + \gamma_i (1-q_i))^{\alpha/(1-\alpha)} / (q_j + \gamma_j (1-q_j))^{\alpha/(1-\alpha)}$

$t_i (q_j + \gamma_j (1-q_j))^{\alpha/(1-\alpha)} - t_j (q_i + \gamma_i (1-q_i))^{\alpha/(1-\alpha)} = \text{error}$

Fitting Technique:



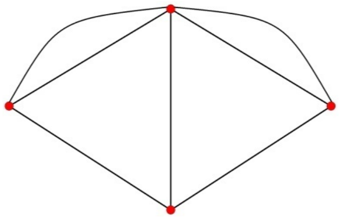
Search on grid of biases in preferences and meetings:

- For each network (school) and specification of biases, calculate an error in terms of total deviation from fitting equations

- Sum squared errors across networks (schools)

- Choose biases to minimize (weighted) sum of squared errors

Fitted Values



ALPHA = .55

	A	B	H	W	O
GAMMA =	0.9	0.55	0.65	0.75	0.9
BETA =	7	7.5	2.5	1	1

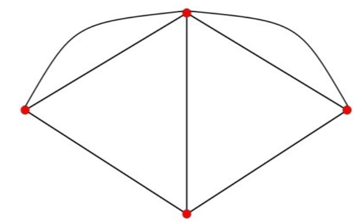
Preference Biases

ALL SCH	alph	A	B	H	W	O	RSS free	RSS cnst	F	thres %95	thres %99
	0.55	**0.9**	**0.55**	**0.65**	**0.75**	**0.90**	4704			3.963	6.971
a=b	0.70	0.8	0.8	0.80	0.85	0.95		5303	**9.93**		
a=h	0.65	0.75	0.7	0.75	0.80	0.95		5197	**8.17**		
a=w	0.65	0.85	0.70	0.75	0.85	0.95		4864	**2.65**		
b=h	0.65	0.90	0.70	0.70	0.80	0.95		4798	**1.56**		
b=w	0.55	0.80	0.65	0.60	0.65	0.90		5333	**10.43**		
h=w	0.60	0.90	0.55	0.70	0.70	0.90		4911	**3.43**		
all =1	0.20	1.00	1.00	1.00	1.00	1.00		17554	**42.61**	2.3	3.3
all=	0.55	0.80	0.80	0.80	0.80	0.80		6175	**6.10**	2.5	3.6

Meeting Bias

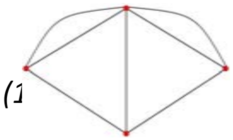
All Sch	A	B	H	W	O	RSS free	RSS cnst	F	thrsh 95	thrsh 99
	7.0	**7.5**	**2.5**	**1.0**	**1.0**	1.726			3.962	6.967
a=b	7.5	7.5	2.5	1.0	1.0		1.727	**0.04**		
a=h	3.5	7.5	3.5	1.0	1.0		1.834	**4.95**		
a=w	1.5	6.5	3.5	1.5	1.0		2.774	**47.97**		
b=h	3.5	5.5	5.5	1.0	1.0		2.148	**19.31**		
b=w	9	3	1	3.0	1.0		4.448	**124.5**		
h=w	8.5	7	1.5	1.5	1.0		2.236	**23.34**		
all =1	1	1	1	1.0	1.0		25.84	**220.6**	2.3	3.3
all =	2.0	2.0	2.0	2.0	2.0		6.207	**51.25**	2.5	3.6

Week 4 Wrap



- Strategic models : choice based formation, welfare analysis
- Formation:
  - pairwise stability...
- Welfare
  - Pareto efficiency, utilitarian measure/efficiency
- Tension: stable and efficient networks need not coincide
  - Positive externalities – under-connected
  - Negative externalities – over-connected
  - transfers cannot always help
- Small worlds: low costs of local links gives clustering
  - high benefits from distant links give short paths
- Adding heterogeneity can lead to estimable models

Week 4: In order mentioned



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