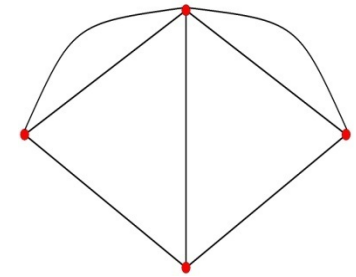


Social and Economic Networks: Models and Analysis

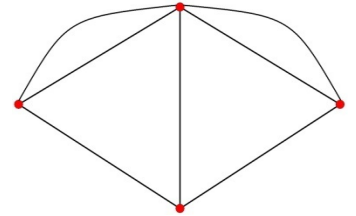


Matthew O. Jackson

**Stanford University,
Santa Fe Institute, CIFAR,
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5.1: Diffusion

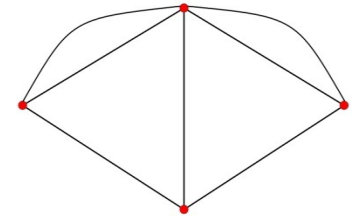


Outline



- Part I: Background and Fundamentals
 - Definitions and Characteristics of Networks (1,2)
 - Empirical Background (3)
- Part II: Network Formation
 - Random Network Models (4,5)
 - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
 - Diffusion and Learning (7,8)
 - Games on Networks (9)

Networks and Behavior



- How does network structure impact behavior?
- Simple infections, contagion – diffusion
- Opinions, information – learning
- Choices, decisions – games on networks

Diffusion



- disease
- Ideas basic information (know or not know)
- Buy a product or not (come back to complementarities later...)

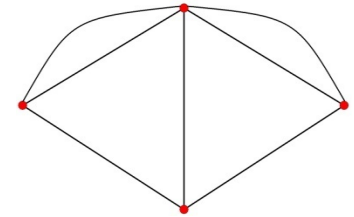
Diffusion



- Questions and Background
- Bass Model – no networks
- Bring in interaction structure

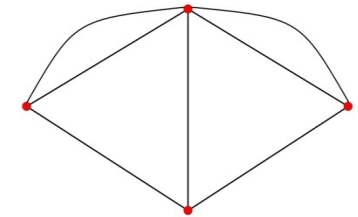
S-Shape Adoption

- Diffusion over time and space
 - Griliches economic story: variation in cost effectiveness by geography
- Initial adopters
 - Who are they? High degree? Innovators?
- Increase in speed
 - Word of mouth, observations of neighbors
- Eventual slowdown
 - Saturation



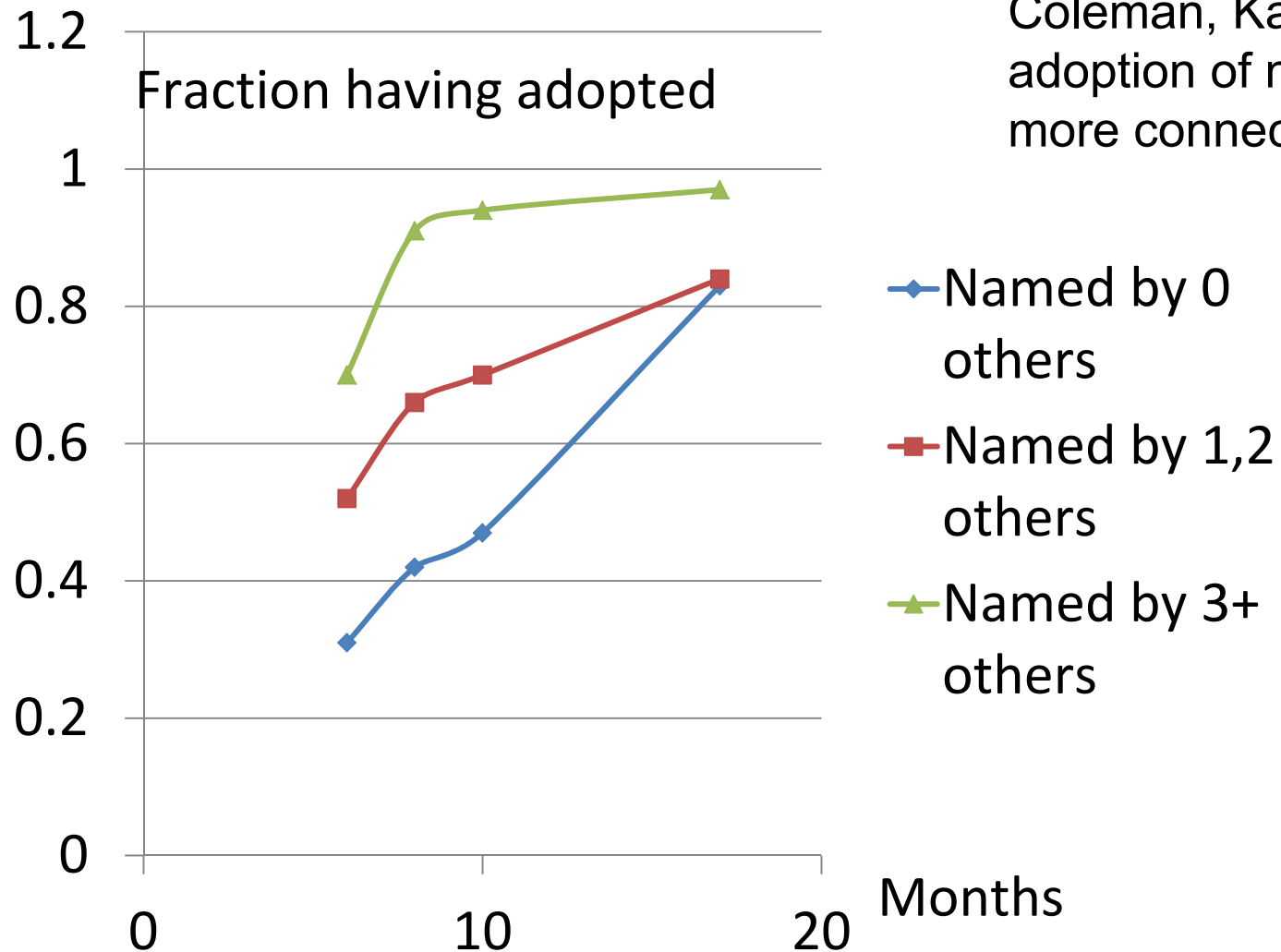
Diffusion: Coleman, Katz, Menzel (1966)

adoption (prescribing) of new drug by doctors:
more connected are earlier adopters

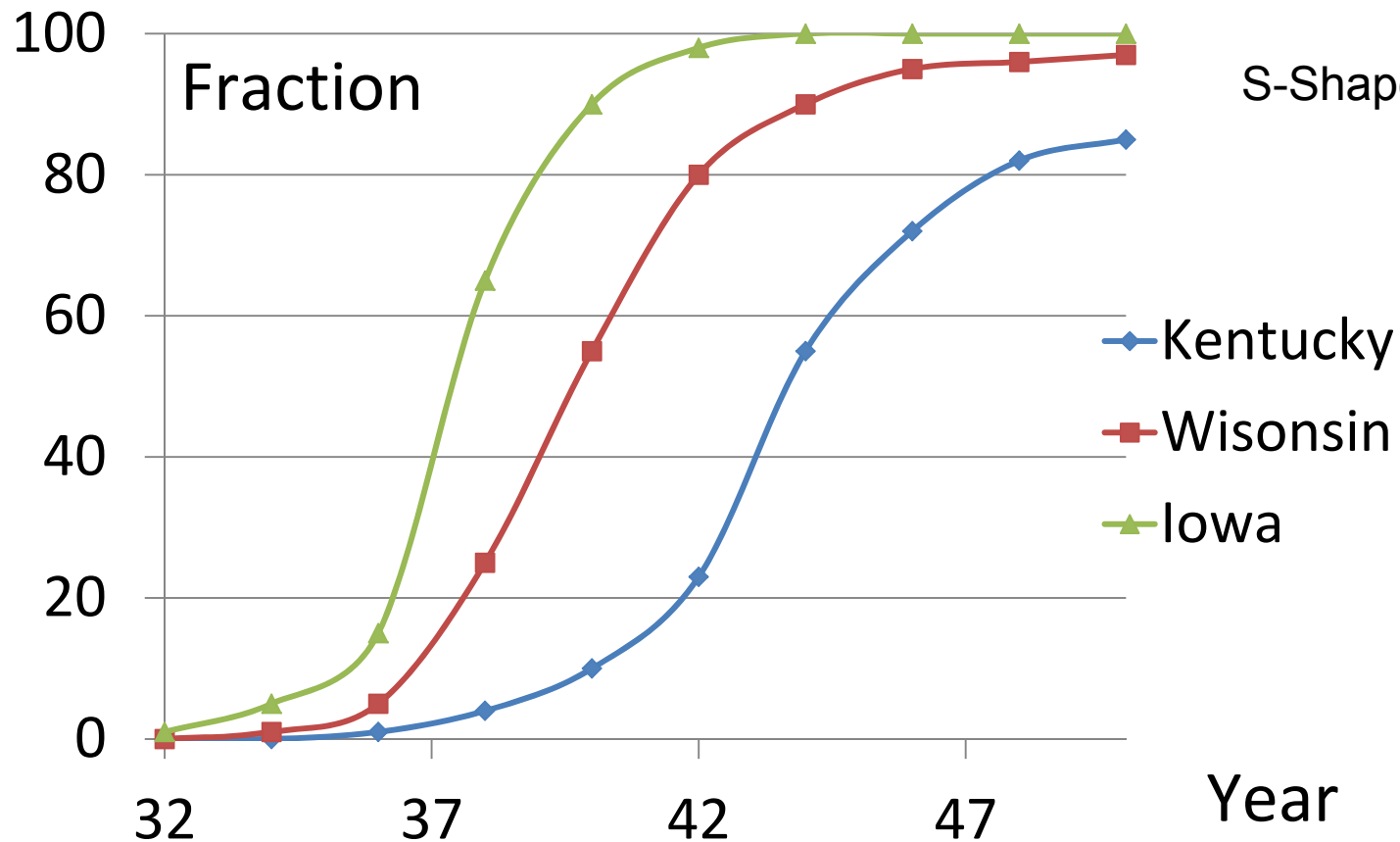
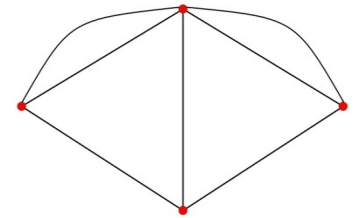


Fraction Adopting by:	names by 0 others (36)	named by 1 or 2 others (56)	named by 3+ others (33)
6 months	.31	.52	.70
8 months	.42	.66	.91
10 months	.47	.70	.94
17 months	.83	.84	.97

Coleman, Katz, Menzel (1966)
adoption of new drug by doctors
more connected earlier adopters

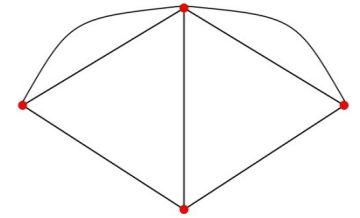


Griliches (1957): Hybrid Corn Diffusion

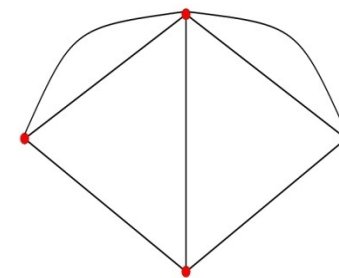


Questions:

- Extent of diffusion?
- How does it depend on the particulars of the process as well as the network?
- Time patterns? S-shape?
- Welfare analysis?



Social and Economic Networks: Models and Analysis

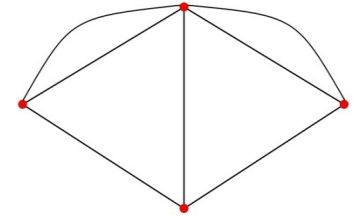


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5.2: Bass Model



Diffusion



- Questions and Background
- Bass Model – no networks
- Bring in interaction structure

Bass Model



- A benchmark model with no explicit social structure
- Two actions/states/behaviors 0 and 1
- $F(t)$ fraction of the population who have adopted action 1 at time t

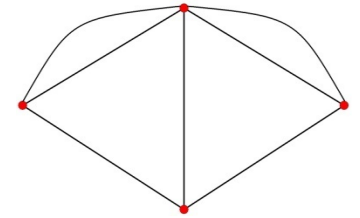
Bass Model



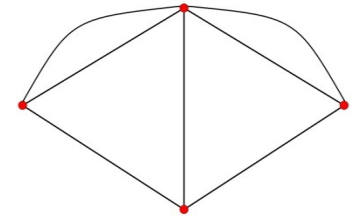
- p rate of spontaneous innovation/adoption
- q rate of imitation of adoption
- $\frac{dF(t)}{dt} = (p + q F(t))(1-F(t))$

Solution:

- p rate of spontaneous innovation/adoption
- q rate of imitation of adoption
- $dF(t)/dt = (p + q F(t))(1-F(t))$
- $F(t) = (1-e^{-(p+q)t}) / (1+qe^{-(p+q)t}/p)$



Getting the S-shape

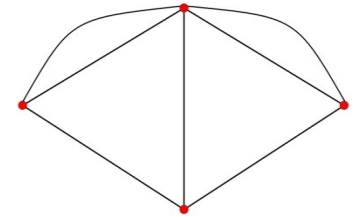


- Gives S-shape (if $q > p$) and tends to 1 in the limit
- Initially only p matters, then q takes over
- Eventually change slows as $F(t)$ approaches 1

Getting the S-shape

$$dF(t)/dt = (p + q F(t))(1-F(t))$$

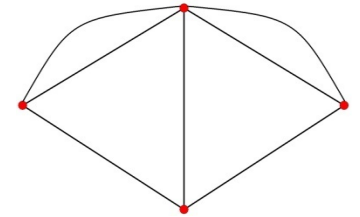
- when $F(t)$ nears 1 , $dF(t)/dt = 0$
- when $F(t)=0$, $dF(t)/dt = p$

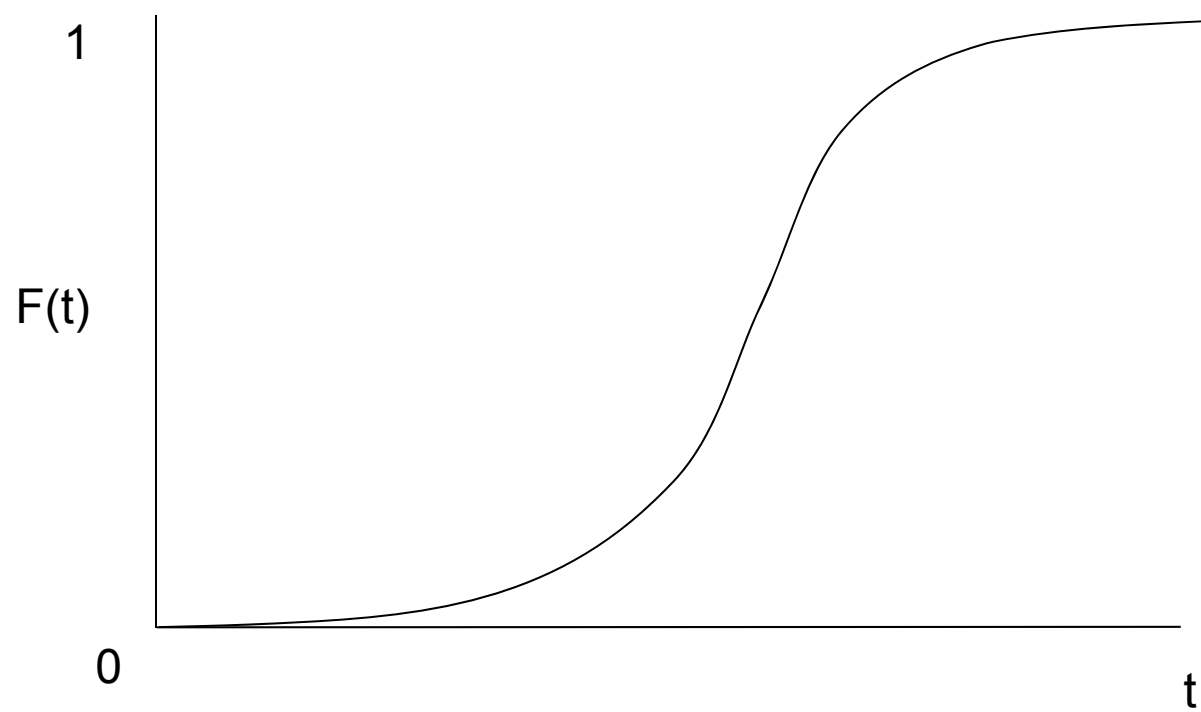


Getting the S-shape

$$dF(t)/dt = (p + q F(t))(1-F(t))$$

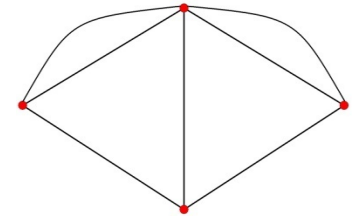
- when $F(t)$ nears 1 , $dF(t)/dt = 0$
- when $F(t)=0$, $dF(t)/dt = p$
- when $F(t)=\varepsilon$, $dF(t)/dt = (p + q \varepsilon)(1 - \varepsilon)$
- to get initial convexity: need $(p + q \varepsilon)(1 - \varepsilon) > p$
- $q(1 - \varepsilon) > p$ so initially need $q > p$



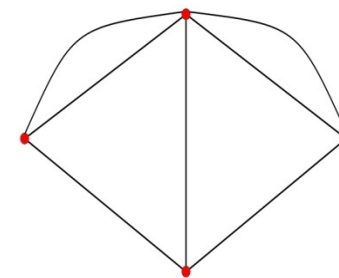


Next: Beyond Bass - Component Structure

- Reach of diffusion is bounded by the component structure
- Some players or nodes are immune
- Some links fail to transmit...
- Answers questions of when get diffusion, and its extent (neither answered by simple Bass)



Social and Economic Networks: Models and Analysis

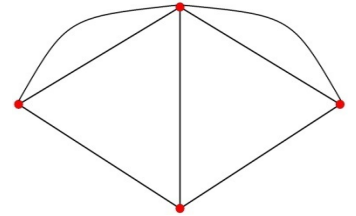


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5.3: Diffusion on Random Networks



Diffusion



- Questions and Background
- Bass Model – no networks
- Bring in interaction structure

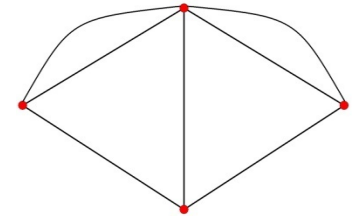
Random Networks and Diffusion



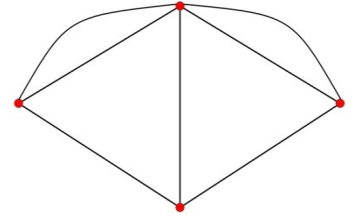
- Idea, disease, computer virus spreads via connections in the network
- Nodes are linked if one would ``infect'' the other
- Will an infection take hold?
- How many nodes/people will it reach?

Questions:

- When do we get diffusion?
- What is the extent of diffusion?
- How does it depend on the particulars of the process as well as the network?
- Who is likely to be infected earliest?

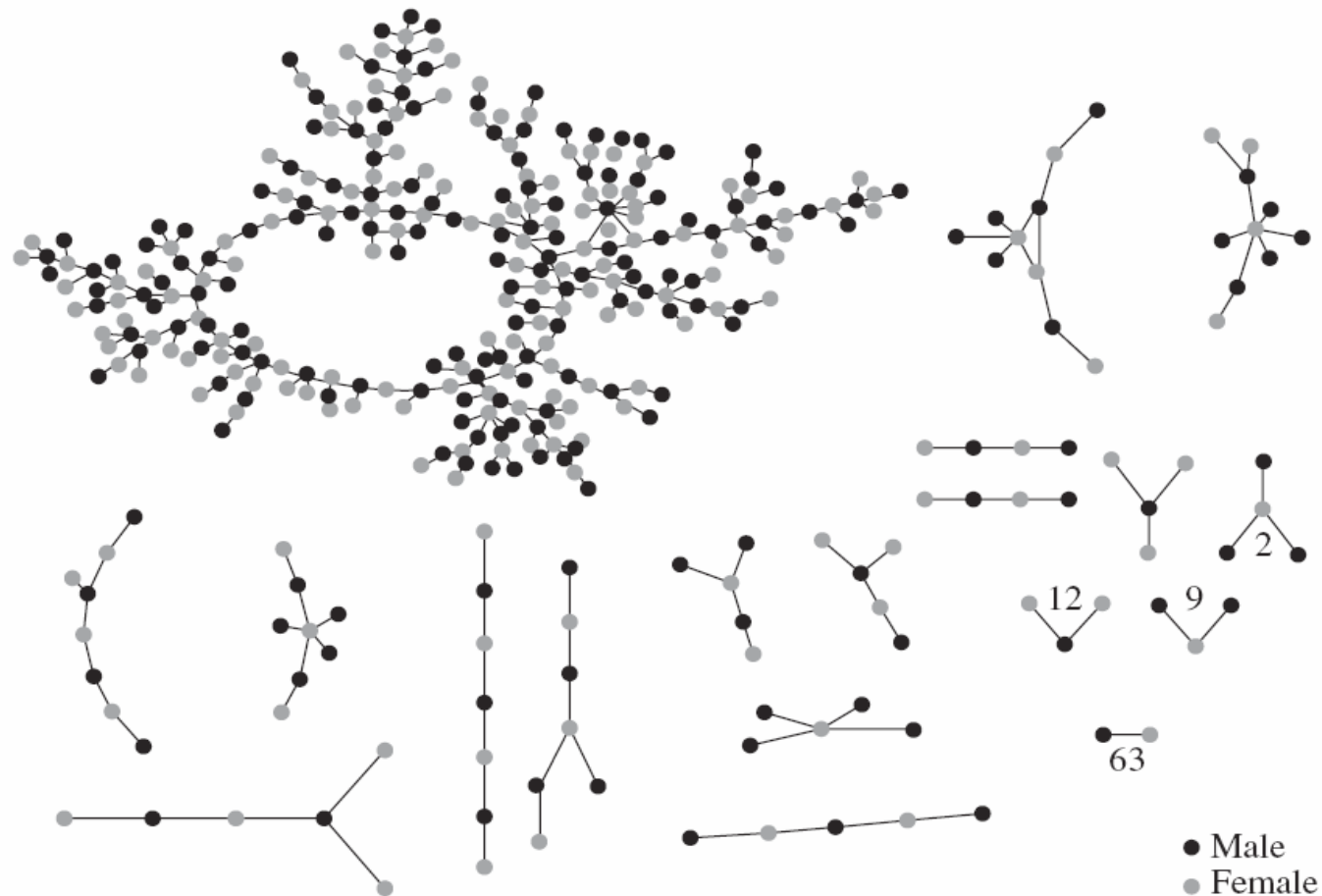


Component Structure



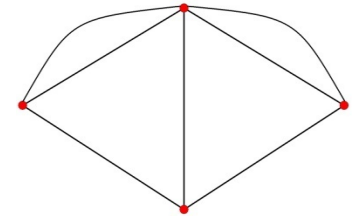
- Reach of contagion is determined by the component structure
- Some players or nodes are immune, Some links fail to transmit...
- What do components look like of those who are susceptible and given links that work

Bearman, Moody, and Stovel's
04 High School Romance

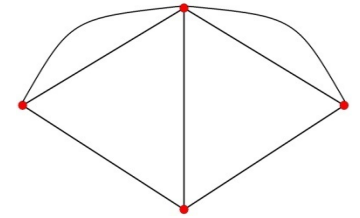


Extent of Diffusion

- Get nontrivial diffusion if someone in the giant component is infected/adopts
- Size of the giant component determines likelihood of diffusion and its extent
- Random network models allow for giant component calculations



Extent of Diffusion



- Simple example of such a calculation
- Work with Erdos-Renyi random network
- How big is the giant component??

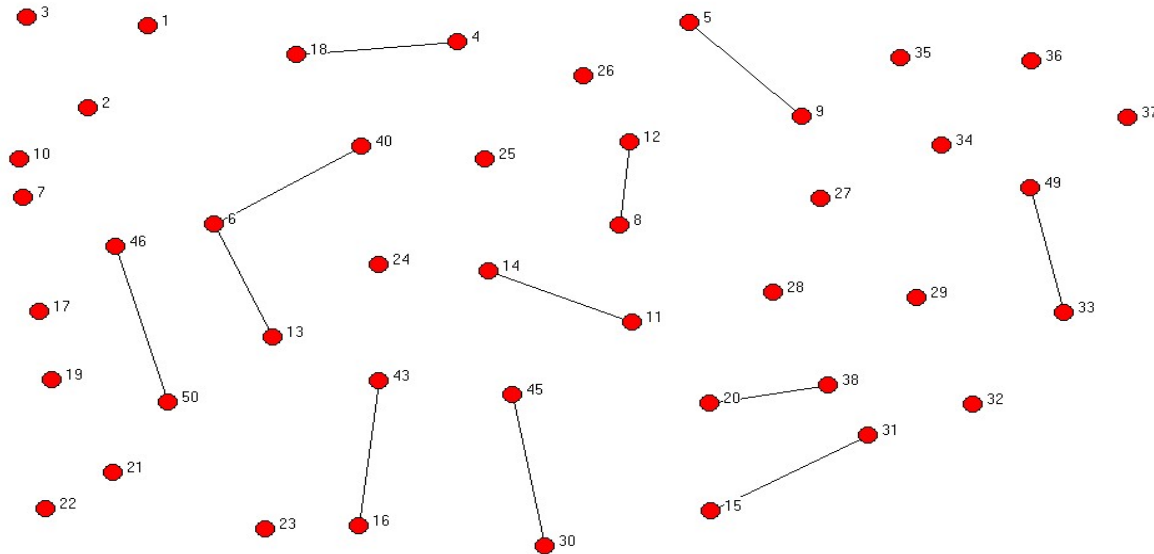
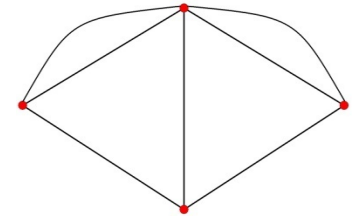
Size of the Giant Component:



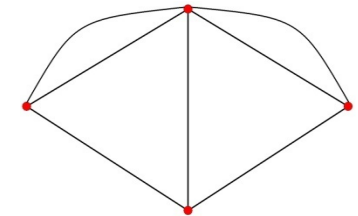
- How big is the giant component when there is one?
- Size of the giant component when
$$1/n < p < \log(n)/n$$

[know that if $p \ll 1/n$ all isolated, and if $\log(n)/n \ll p$ then all path connected]

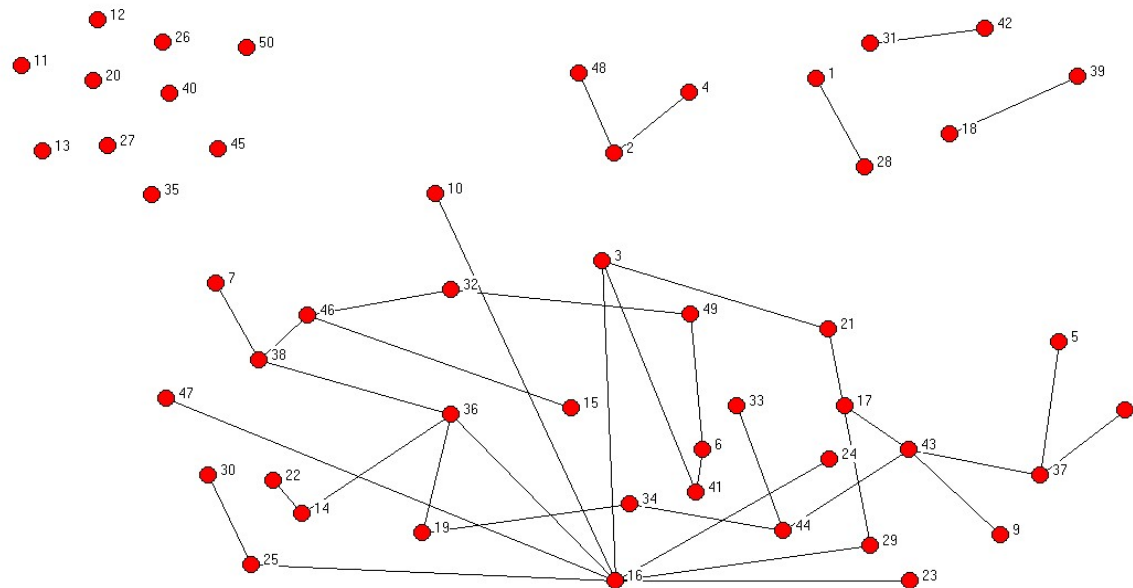
Poisson $p=.01$, 50 nodes



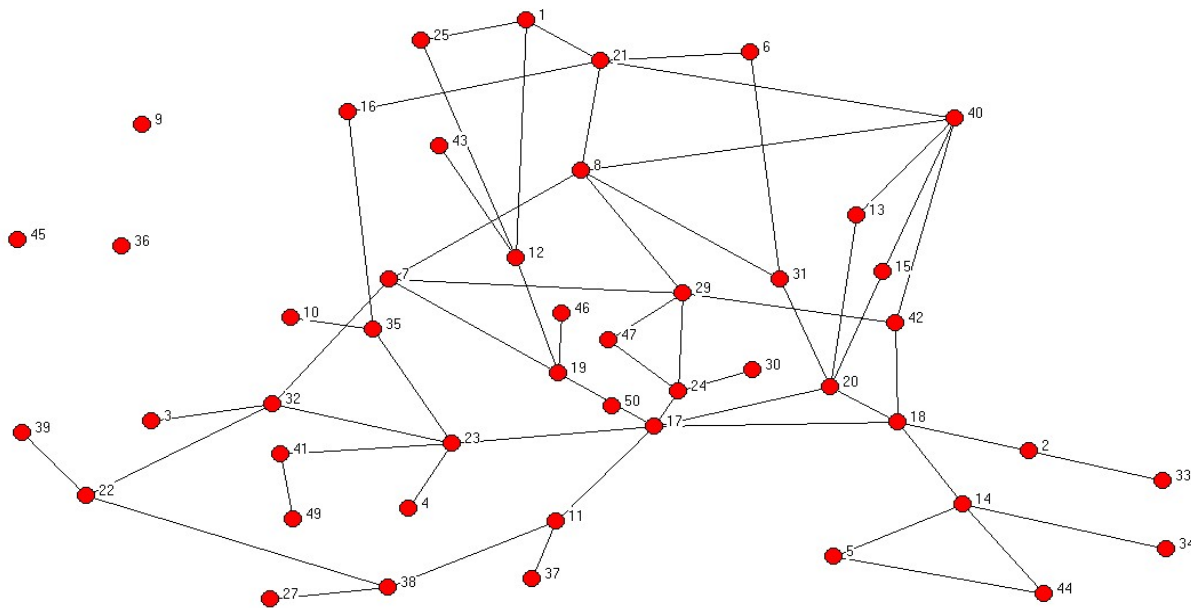
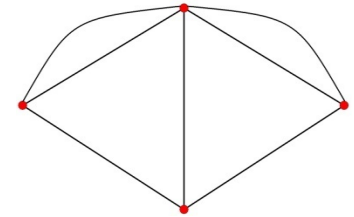
Poisson $p=.03$, 50 nodes



.02 is the threshold for emergence of cycles and a giant component

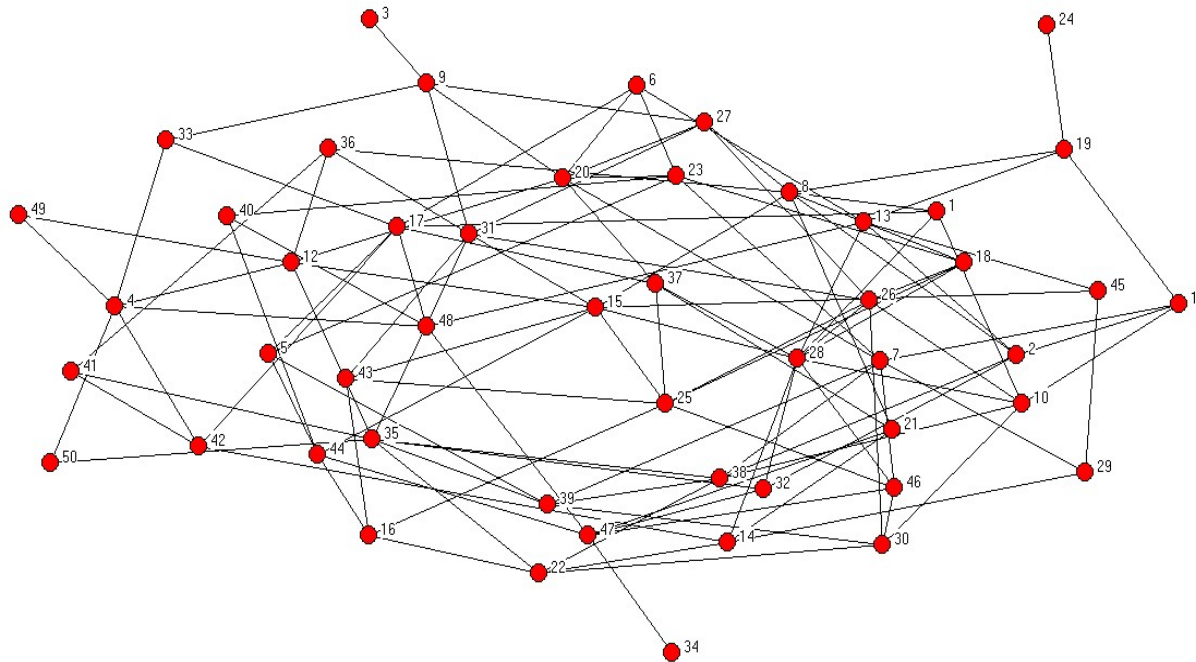
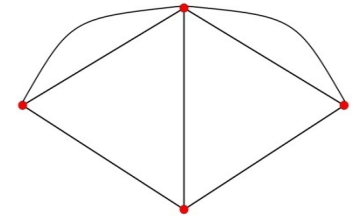


Poisson $p=.05$, 50 nodes

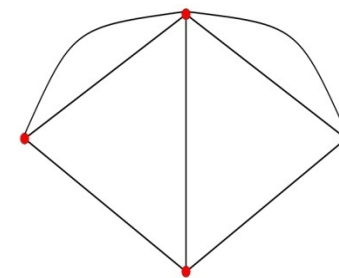


Poisson $p=.10$, 50 nodes

.08 is the threshold for connection



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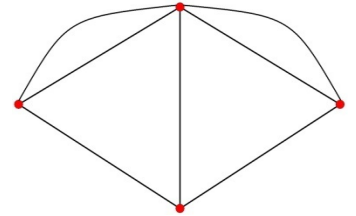


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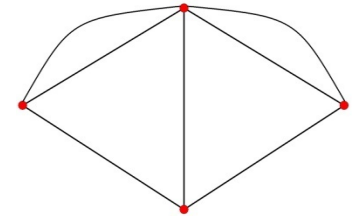
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5.4: Giant Component Poisson Case

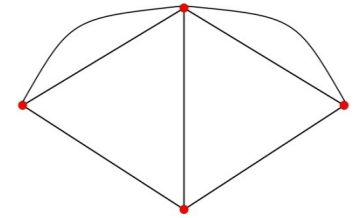


Calculating the Size of the Giant Component



- q is fraction of nodes in largest component
- look at any node: chance it is in the giant component is q
- chance that this node is outside of the giant component is *the chance that all of its neighbors are outside of the giant component*

Calculating the Size of the Giant Component



- Probability that a node is outside of the giant component = $1-q$

= probability that all of its neighbors are outside

= $(1-q)^d$ where d is the node's degree

Giant Component Size

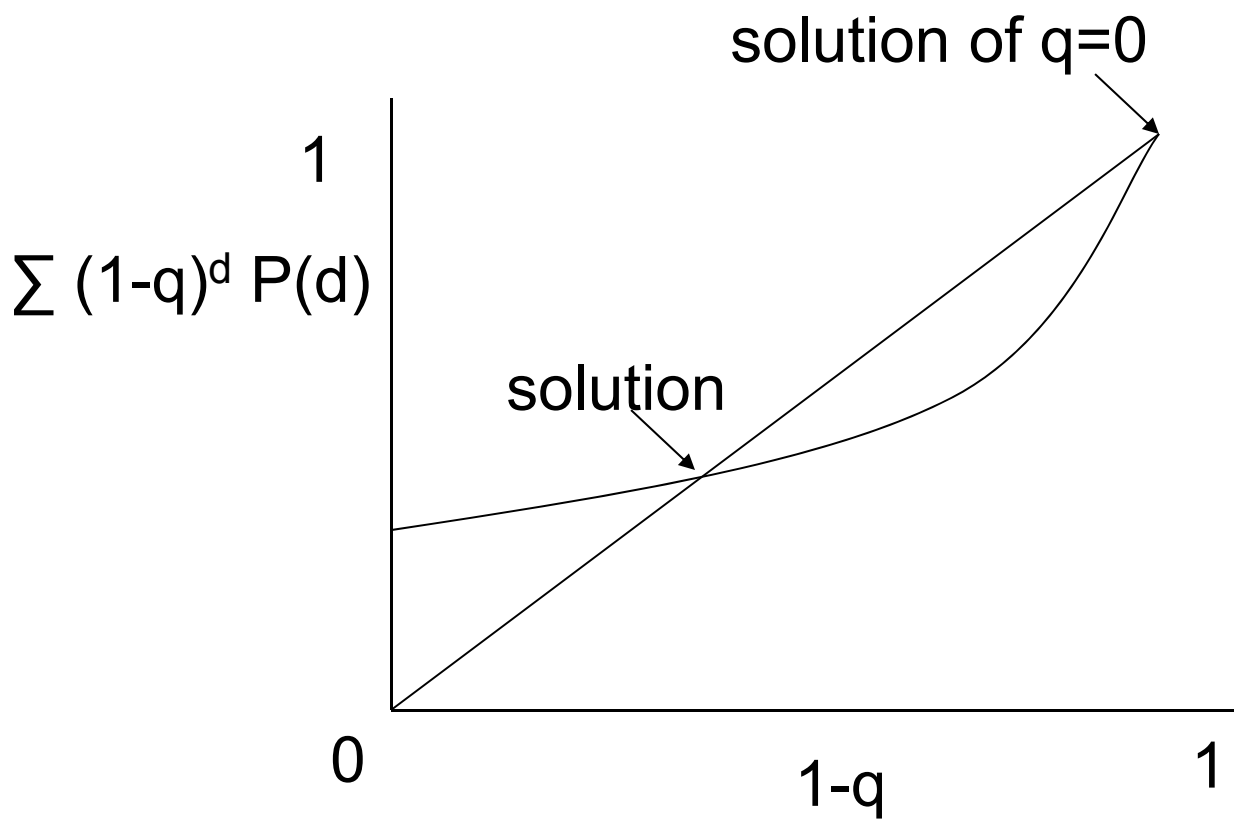


- So, probability $1-q$ that a node is outside of the giant component is

$$1-q = \sum (1-q)^d P(d)$$

Where $P(d)$ is the chance that the node has d neighbors

- Solve for q ...



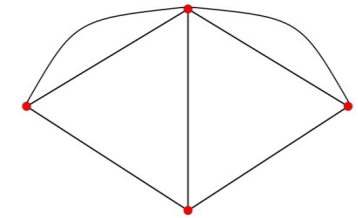
Giant Component Size: Poisson Case



Solve $1-q = \sum (1-q)^d P(d)$

when $P(d) = [(n-1)^d / d!] p^d e^{-(n-1)p}$

Giant Component Size: Poisson Case

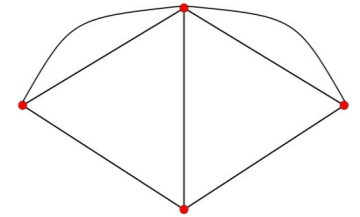


Solve $1-q = \sum (1-q)^d P(d)$

when $P(d) = [(n-1)^d / d!] p^d e^{-(n-1)p}$

so $1-q = e^{-(n-1)p} \sum [(1-q) (n-1)p]^d / d!$

Useful Approximations

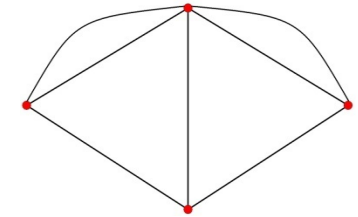


Taylor series approximation:

$$\begin{aligned} e^x &= 1 + x + x^2/2! + x^3/3! \dots \\ &= \sum x^d / d! \end{aligned}$$

$$[f(x) = f(a) + f'(a)(x-a)/1! + f''(a)(x-a)^2/2! \dots]$$

Giant Component Size: Poisson Case



Solve $1-q = \sum (1-q)^d P(d)$

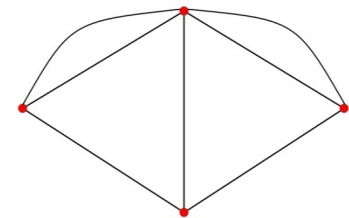
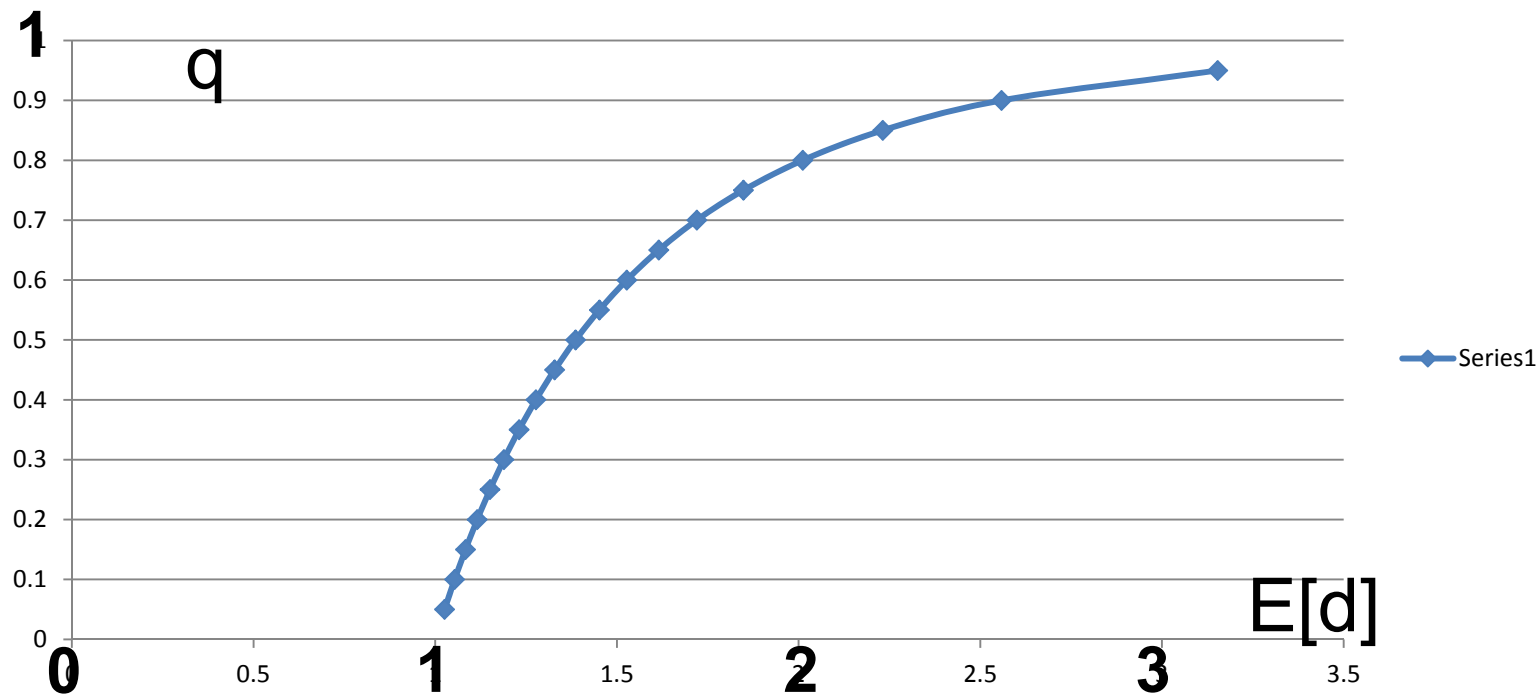
when $P(d) = [(n-1)^d / d!] p^d e^{-(n-1)p}$

so
$$\begin{aligned} 1-q &= e^{-(n-1)p} \sum [(1-q) (n-1)p]^d / d! \\ &= e^{-(n-1)p} e^{(n-1)p(1-q)} \\ &= e^{-q(n-1)p} \end{aligned}$$

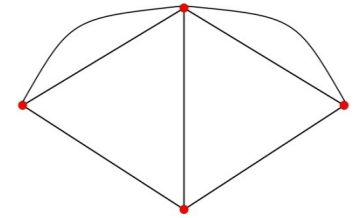
or $-\log(1-q) / q = (n-1) p = E[d]$

Giant Component Size:

$$-\log(1-q) / q = E[d]$$



Who is infected?



- Probability of being in the giant component:
- $1-(1-q)^d$ increasing in d
- More connected, more likely to be infected

(more likely to be infected at any point in time...)

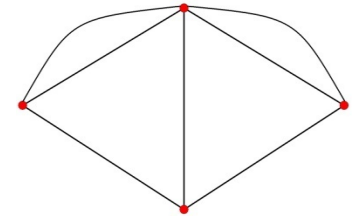
Extensions:



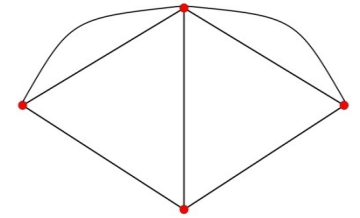
- **Immunity:** delete a fraction of nodes and study the giant component on remaining nodes
- **Probabilistic infection**
 - Random infection: have some links fail, just lower p

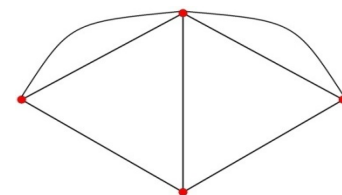
Contagion with Immunity and Link Failure

- Some node is initially exposed to infection
- π of the nodes are immune naturally
- only some links result in contagion – fraction f
- What is the extent of the infection?



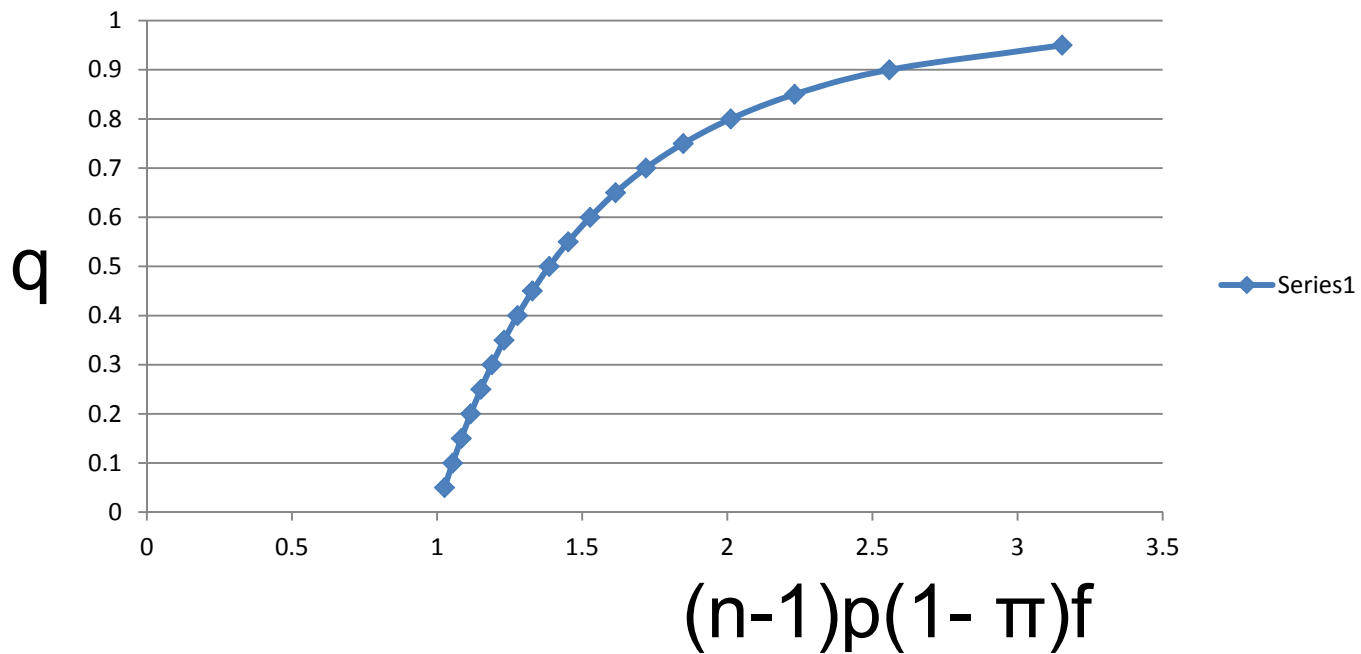
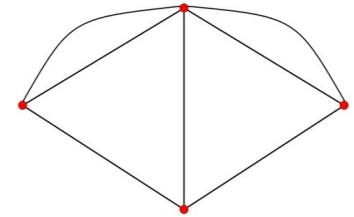
- Consider a random network on n nodes
- Delete fraction π of the nodes
- Delete fraction $1-f$ of the links
- If starts at a node in giant component of the remaining network, then the giant component of that network is the extent of the infection; otherwise negligible





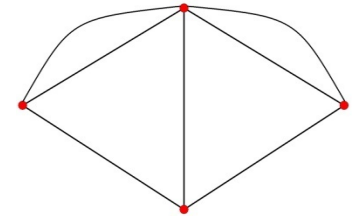
- Let q be the fraction of nodes of the remaining network in its giant component
- $q(1-\pi)$ is the probability of a nontrivial contagion
- Conditional on a contagion it infects $q(1-\pi)$ of the original nodes
- q solves $-\log(1-q)/q = (n-1)p(1-\pi)f$

Infected Fraction of Nodes:



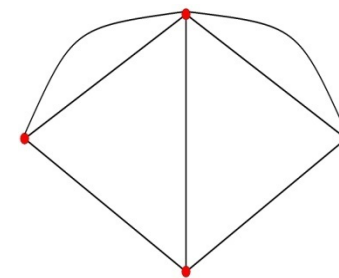
Implications:

- Infection can fail if π is high enough or f or p are low enough
- High π - immunization, low virulence
- Low f - low contagiousness
- Low p - low contact among population



Social and Economic Networks: Models and Analysis

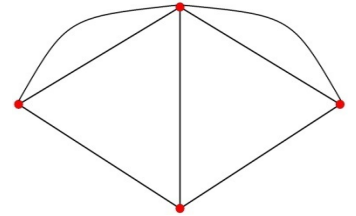
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5.5: SIS Model

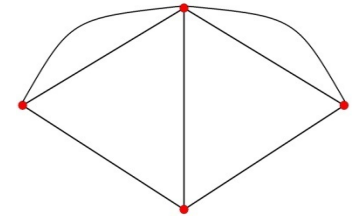


SIS Model



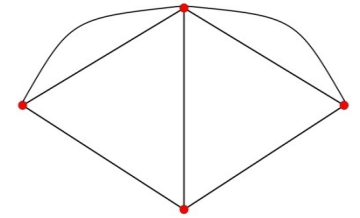
- An extensively studied model in epidemiology
- Allows nodes to change behaviors back and forth over time
- Model of catching some recurring diseases, who to vote for, etc.

SIS Model



- Nodes are infected or susceptible
- Probability that get infected is proportional to number of infected neighbors with rate $\nu > 0$, plus spontaneous ϵ
- get well randomly in any period at rate $\delta > 0$
- Let p be the percent infected

SIS Model

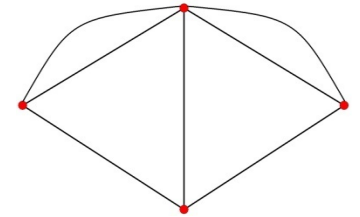


- Start with benchmark where all players mix with even probabilities
- Randomly meet an individual each period
- Large Markov chain
- Steady state mean-field: $dp/dt = 0$

Mean-Field

$$d\rho/dt = (1-\rho)(v\rho+\varepsilon) - \rho\delta = 0$$

$$\rho = [(v-\delta-\varepsilon) + ((v-\delta-\varepsilon)^2 + 4\varepsilon v)^{1/2}] / 2v$$



``Mean-Field'' drop ε

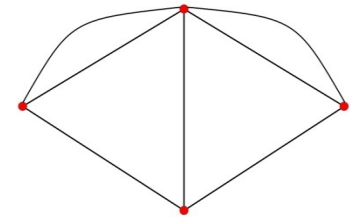
$$d\rho/dt = (1-\rho)v\rho - \rho\delta$$

$$(1-\rho)v\rho - \rho\delta = 0$$

Two solutions:

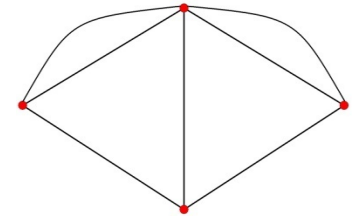
$$\rho = 1 - \delta/v \quad (\text{if } >0)$$

$$\rho = 0$$



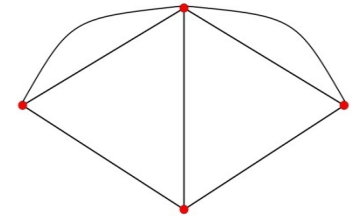
Implications:

- $\rho = 1 - \delta/v$
- If $\delta > v$ then recover faster than get sick, no infection stays
- Otherwise, infection stays at some level, for low recovery rates can lead to large infections

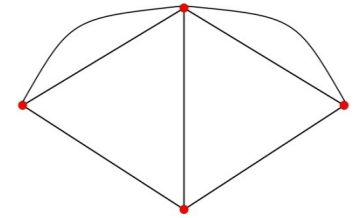


Where's the network?

- so far uniformly random interaction
- missing heterogeneity in degree
- missing local patterns
- we can at least address the first concern...



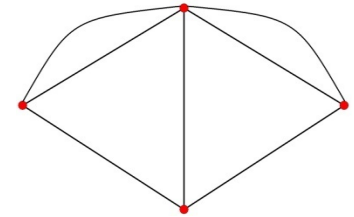
Explore Degree Distribution Influence



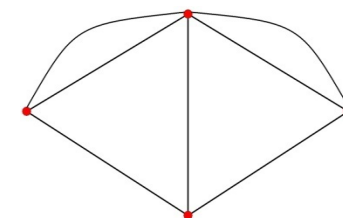
- random matching with d_i matches for node i
- $\rho(d)$ fraction of nodes of degree d infected
- θ fraction of randomly chosen neighbors infected

Chance that meet an infected node

- $P(d)$ fraction of nodes that have d meetings
- More likely to meet someone who has high d
- likelihood of meeting node of degree d is
$$P(d) d / E[d]$$
- So likelihood of meeting infected node is:
$$\theta = \sum \rho(d) P(d) d / E[d]$$



Mean Field: Pastor-Satorras and Vespignagi 2000



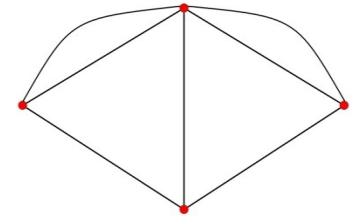
- $\theta = \sum \rho(d) P(d) d / E[d]$ fraction of infected neighbors/random partners
- Steady state: for each d
$$0 = d\rho(d) / dt = (1 - \rho(d)) \nu \theta d - \rho(d) \delta$$

Solving

- Steady state: for each d

$$0 = d\rho(d) / dt = (1 - \rho(d))v\theta_d - \rho(d)\delta$$

$$\rho(d) = \lambda\theta_d / (\lambda\theta_d + 1) \text{ where } \lambda = v/\delta$$



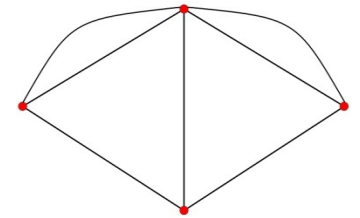
Solving

$$\rho(d) = \lambda \theta d / (\lambda \theta d + 1) \text{ where } \lambda = v/\delta$$

$$\begin{aligned} \theta &= \sum \rho(d) P(d) d / E[d] \\ &= \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]] \end{aligned}$$

- Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$



Solving



Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

What can we say about how this depends on the ``network structure''?

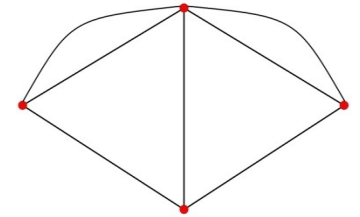
How does infection rate of neighbors θ depend on $P(d)$, $E(d)$?

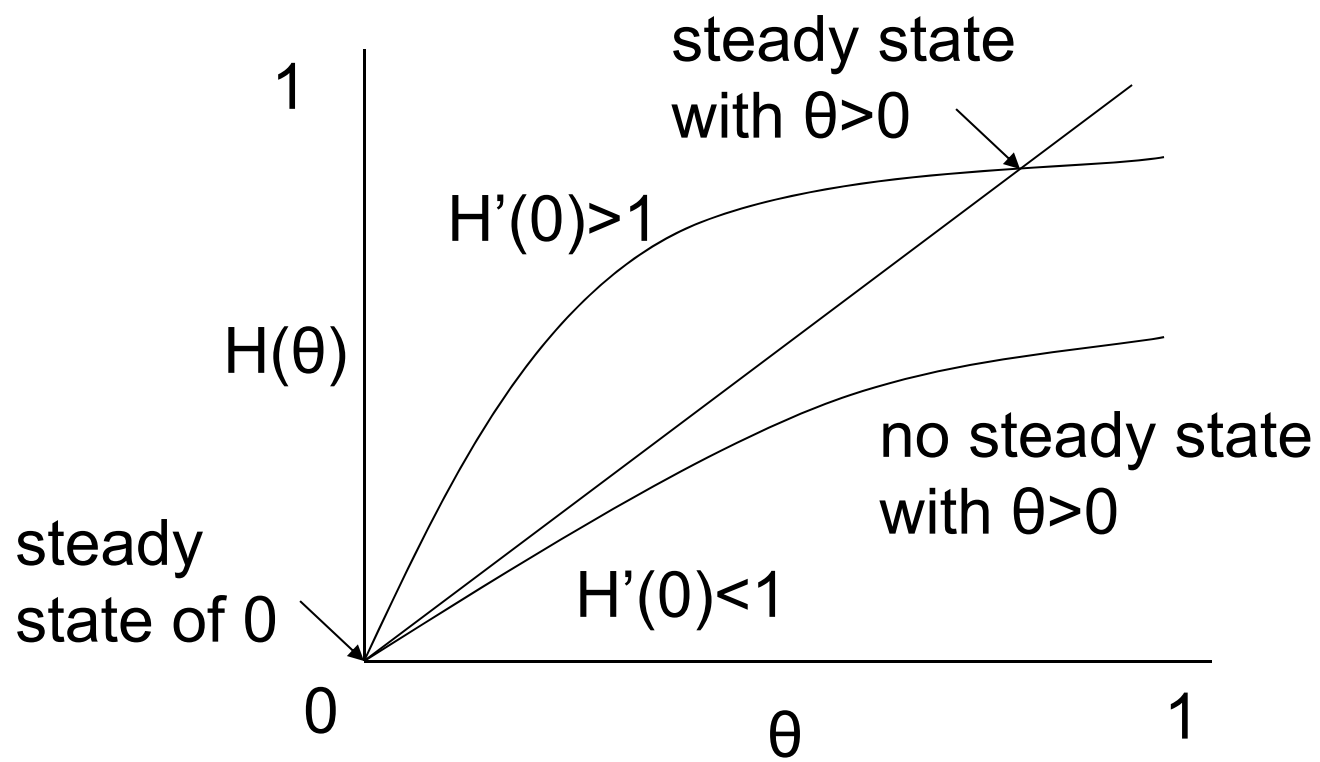
Solving

Steady state infection rate of people you meet is the solution to

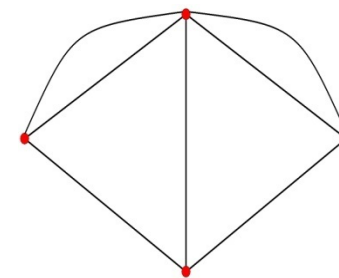
$$\theta = H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

See what $H(\theta)$ looks like and how it depends on $P(d)$, $E[d]$ etc.





Social and Economic Networks: Models and Analysis

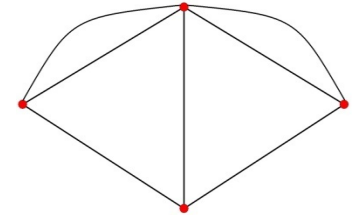


Matthew O. Jackson

**Stanford University,
Santa Fe Institute, CIFAR,
www.stanford.edu/~jacksonm**

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5.6: Solving the SIS Model

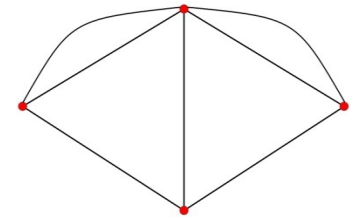


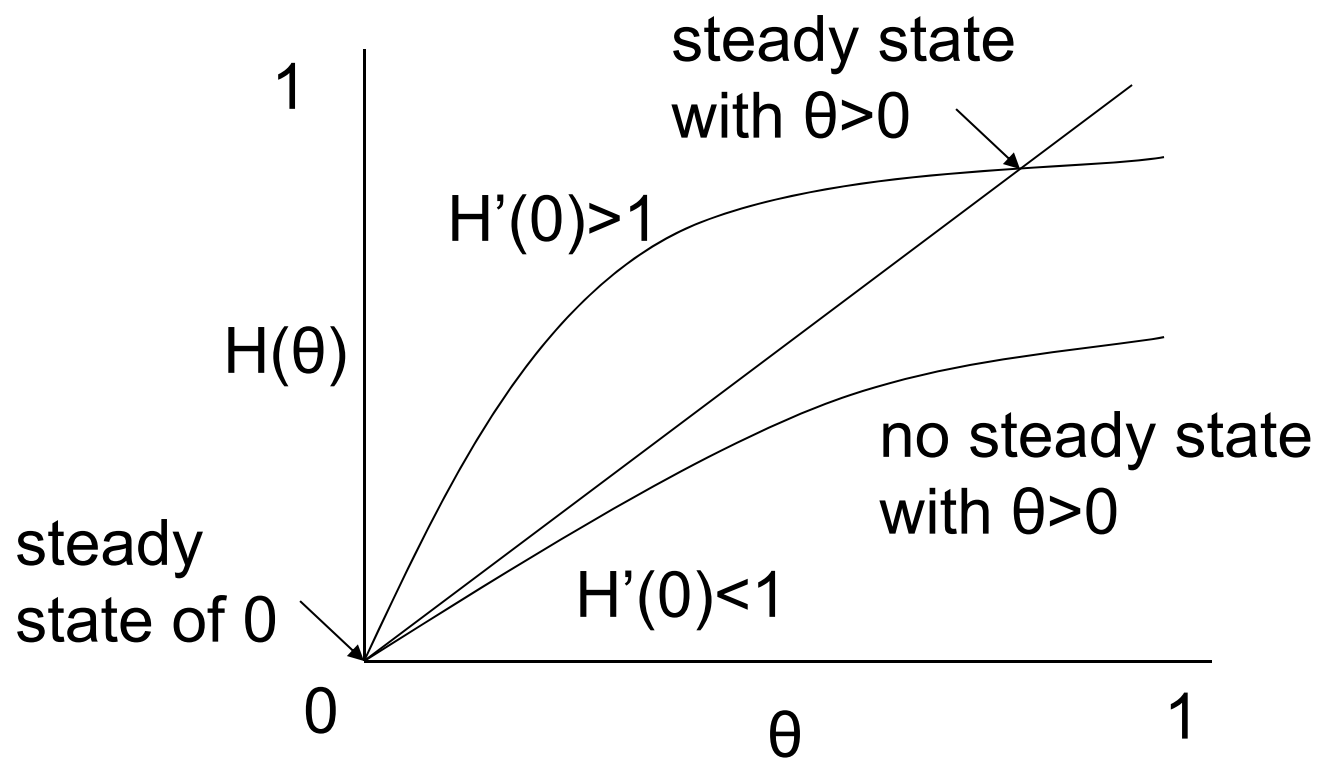
Solving

Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

See what $H(\theta)$ looks like and how it depends on $P(d)$, $E[d]$ etc.



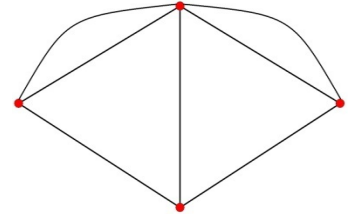


Properties of H



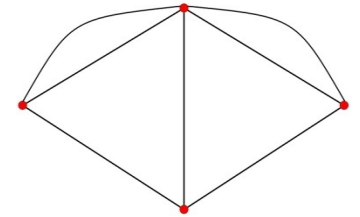
- $H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$

Properties of H

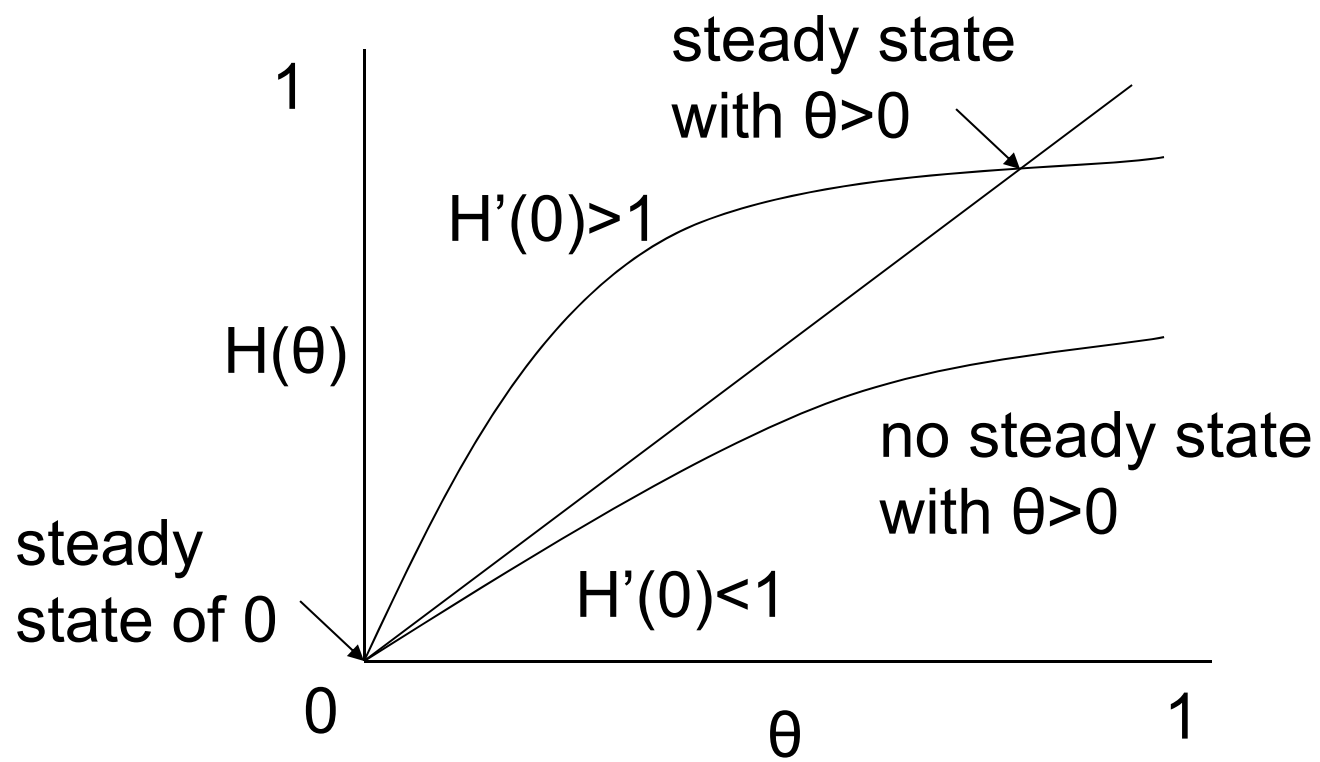


- $H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$
- $H'(\theta) = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1)^2 E[d]] > 0$
so H is increasing

Properties of H



- $H(\theta) = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$
- $H'(\theta) = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1)^2 E[d]] > 0$
so H is increasing
- $H''(\theta) = - 2 \sum P(d) \lambda^2 d^3 / [(\lambda \theta d + 1)^3 E[d]] < 0$
so H is strictly Concave



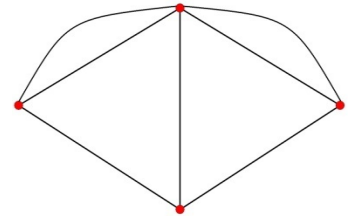
Nonzero Steady State: Lopez-Pintado (08)

- Look at $H'(0)$:

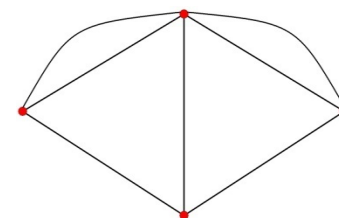
$$H'(\theta) = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1)^2 E[d]]$$

$$\begin{aligned} H'(0) &= \sum P(d) \lambda d^2 / E[d] \\ &= \lambda E[d^2] / E[d] \end{aligned}$$

(recall $\lambda = v/\delta$)



Theorem: Conditions for Steady State of Mean-Field SIS Process



There exists a nonzero steady-state if and only if $\lambda > E[d]/E[d^2]$

So need infection/recovery rate to be high enough relative to average degree divided by second moment (roughly variance)

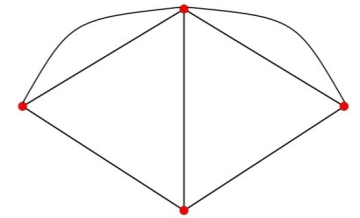
Conditions for Steady State

Iff $\lambda > E[d]/E[d^2]$ have a nonzero steady state

In a **regular network**, need $\lambda > 1/E[d]$

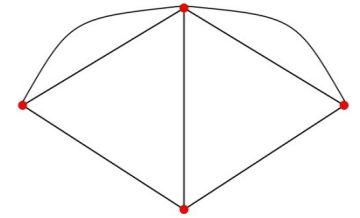
In a **E-R network**, need $\lambda > 1/(1+E[d])$

In a **power-law network**, $E[d^2]$ diverges –
always have a nonzero steady state

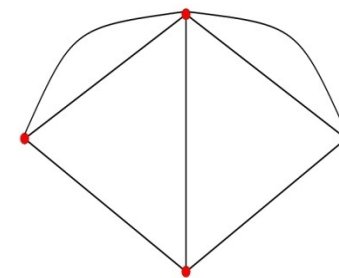


Ideas:

- High degree nodes are more prone to infection
- Serve as conduits
- Higher variance, more such nodes to enable infection



Social and Economic Networks: Models and Analysis

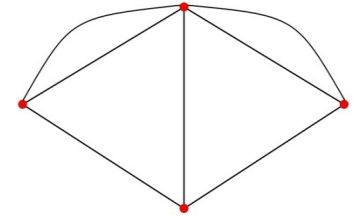


Matthew O. Jackson

**Stanford University,
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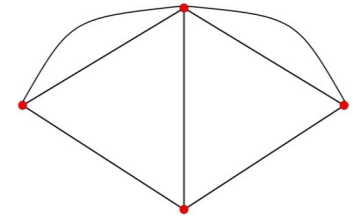
5.7: Solving the SIS Model



Solving for the Positive Steady State when it Exists

$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

$$1 = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1) E[d]]$$

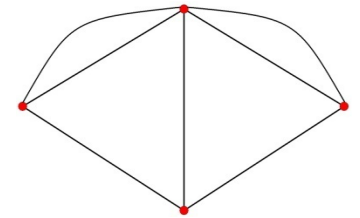


Solving for the Positive Steady State when it Exists

$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

$$1 = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1) E[d]]$$

Regular: $1 = \lambda E[d] / (\lambda \theta E[d] + 1)$; $\theta = 1 - 1/(\lambda E[d])$;



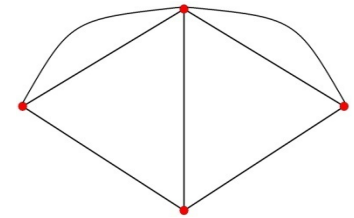
Solving for the Positive Steady State when it Exists

$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

$$1 = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1) E[d]]$$

Regular: $1 = \lambda E[d] / (\lambda \theta E[d] + 1)$; $\theta = 1 - 1/(\lambda E[d])$;

θ is increasing in $\lambda E[d]$ Need $\lambda E[d] > 1$

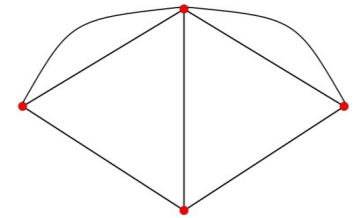


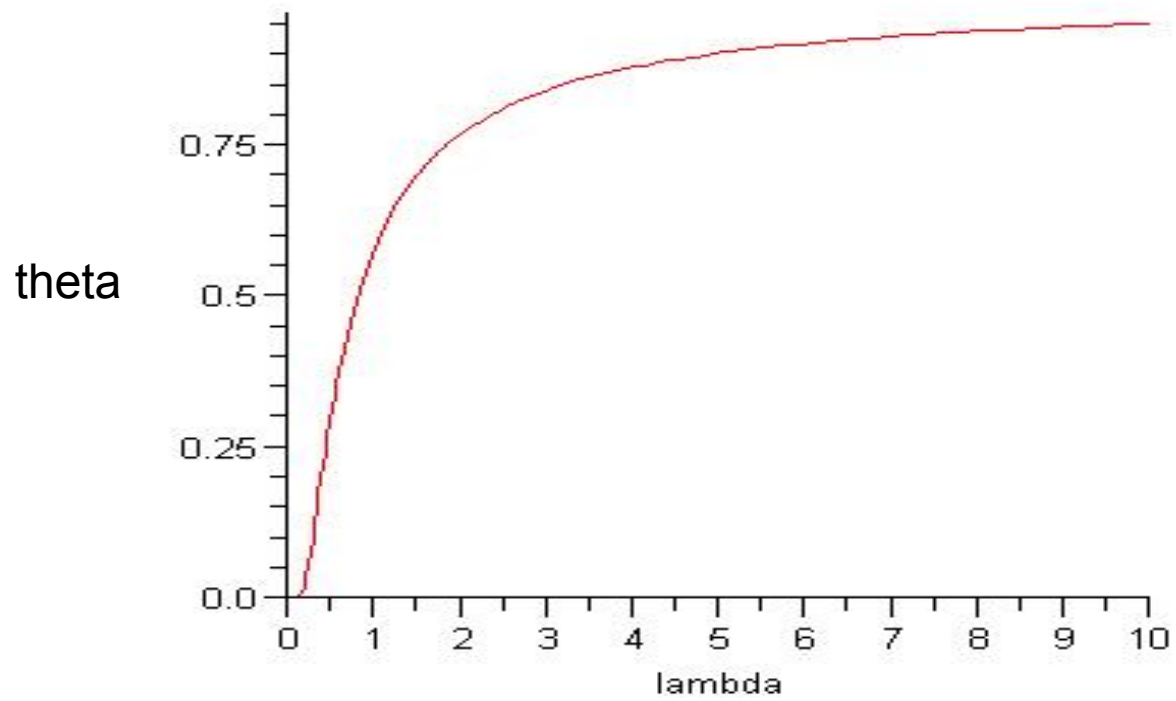
Solving for the Positive Steady State when it Exists

$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

$$1 = \sum P(d) \lambda d^2 / [(\lambda \theta d + 1) E[d]]$$

Power: $P(d) = 2d^{-3} \dots \theta = 1/(\lambda(e^{1/\lambda} - 1))$;





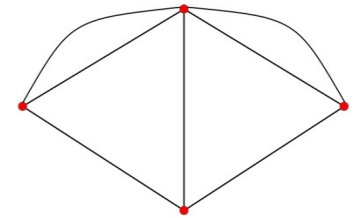
neighbor infection rate for power distribution

Ordering Networks

Jackson Rogers (07b)

$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

How does the right side shift with $P(d)$?



Ordering Networks

Jackson Rogers (07b)



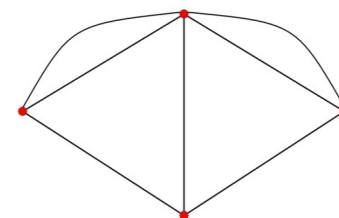
$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

How does the right side shift with $P(d)$?

$\lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$ is increasing in d

Ordering Networks

Jackson Rogers (07b)

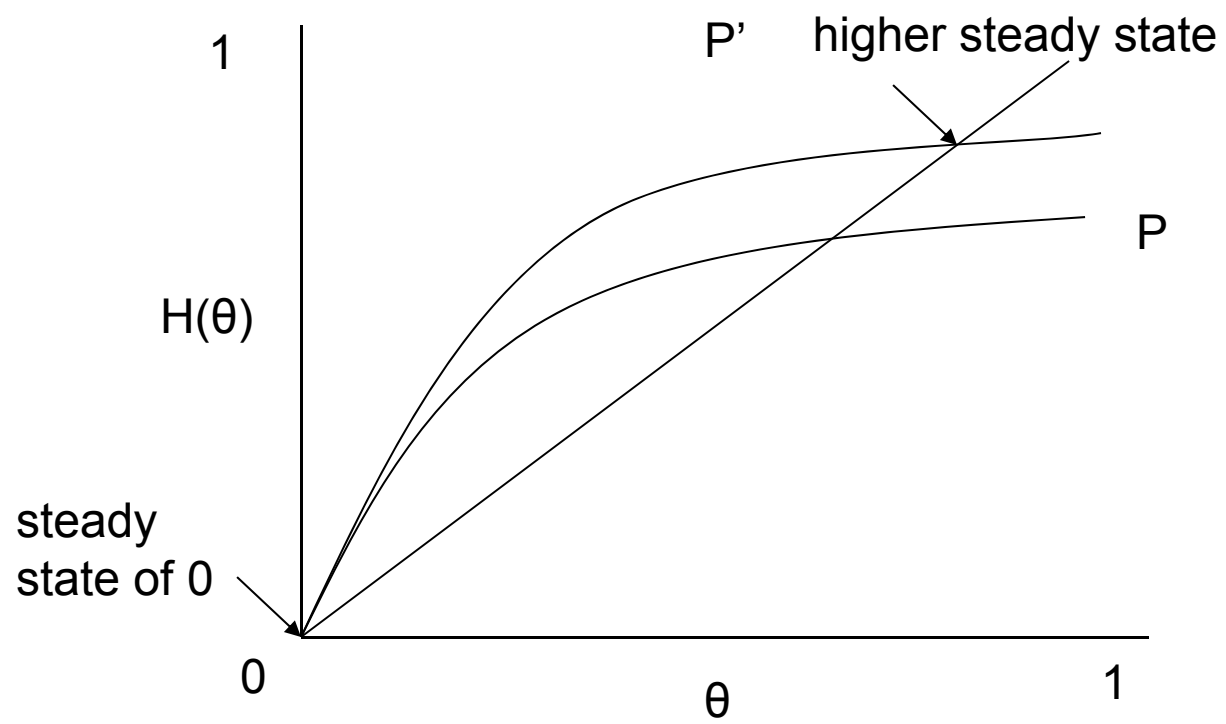


$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

How does the right side shift with $P(d)$?

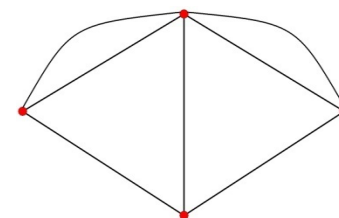
$\lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$ is increasing in d

If P' first order stochastic dominates P , then
rhs increases at every θ



Ordering Networks

Jackson Rogers (07b)



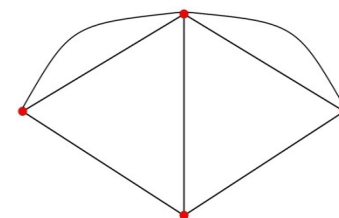
$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

How does the right side shift with $P(d)$?

$\lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$ is convex in d

Ordering Networks

Jackson Rogers (07b)

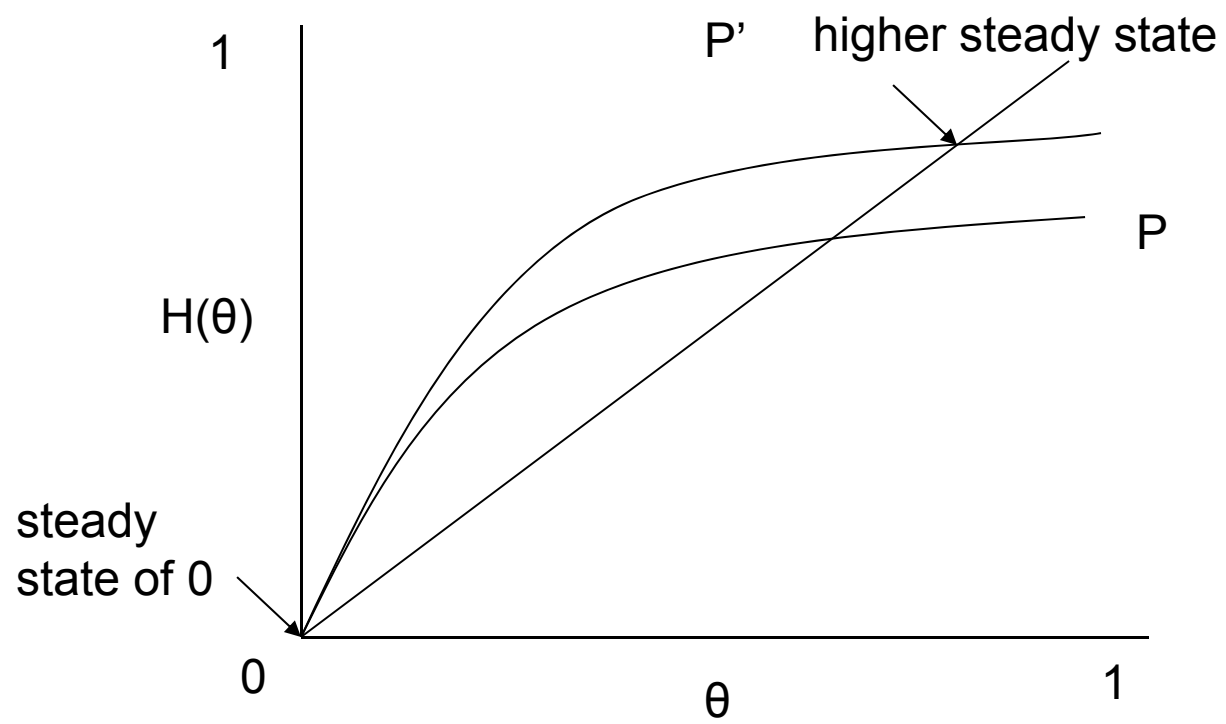


$$\theta = \sum P(d) \lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$$

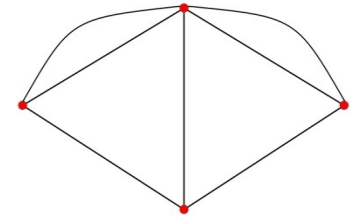
How does the right side shift with $P(d)$?

$\lambda \theta d^2 / [(\lambda \theta d + 1) E[d]]$ is convex in d

If P' is a mean-preserving spread of P , then
rhs increases at every θ



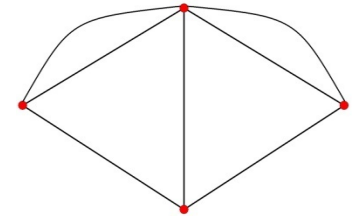
Ideas:



- Mean preserving spread – more high degree nodes and low degree nodes
- Higher degree nodes are more prone to infection
- Neighbors are more likely to be high degree
- So, either first order stochastic dominance, or mean-preserving spreads in P increase θ

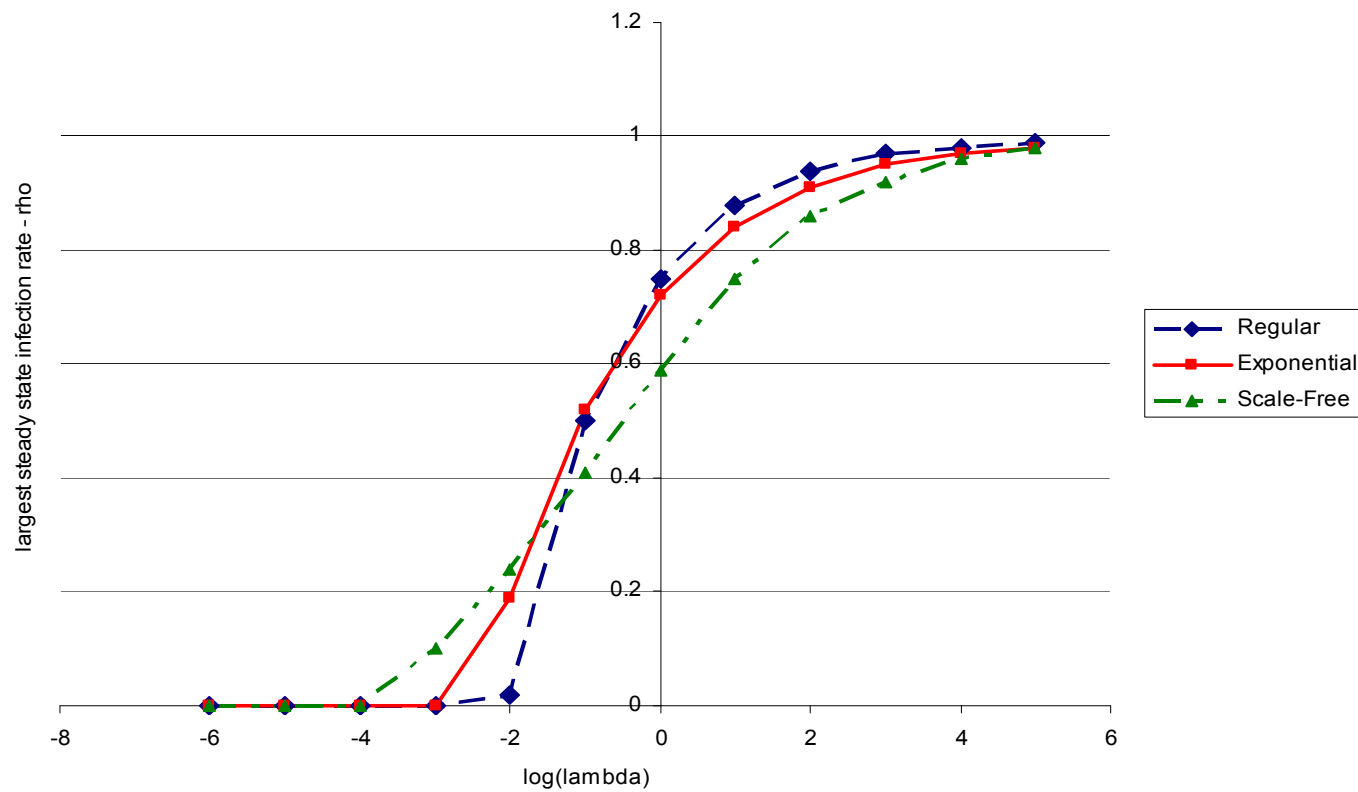
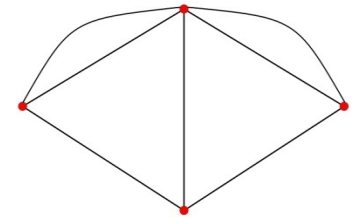
What about Average?

- infection rate of neighbors is not the same as infection rate of the population



Theorem JR (2007): If P' is a mean preserving spread of P , then the highest steady state $\theta' > \theta$, but the corresponding $\rho' > \rho$ if λ is low, while $\rho' < \rho$ if λ is high

Steady States



Proof



$$0 = d\rho(d) / dt = (1 - \rho(d)) v \theta d - \rho(d) \delta$$

Expecting over d :

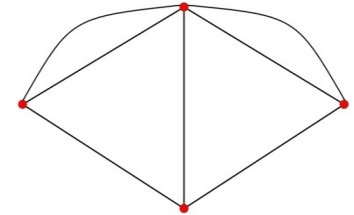
$$\begin{aligned} 0 &= v \theta E[d] - \sum P(d) \rho(d) v \theta d - \rho \delta \\ &= v \theta E[d] - v \theta^2 E[d] - \rho \delta \end{aligned}$$

$$\rho = \lambda \theta E[d] (1 - \theta)$$

rhs is increasing in θ iff $\theta < 1/2$

θ is increasing in λ

SIS Diffusion Model

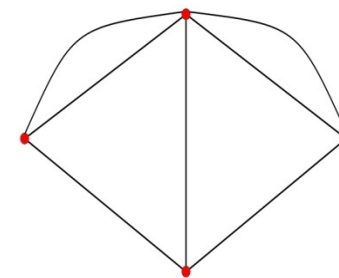


▲
Simple and tractable model

Bring in relative meeting rates

Can order infections by properties of ``network''

Social and Economic Networks: Models and Analysis

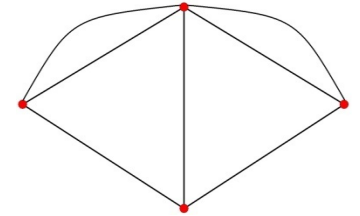


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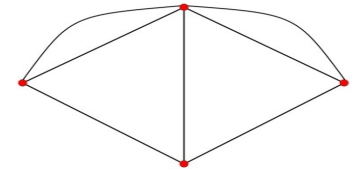
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5.8:Fitting a Diffusion Model to Data

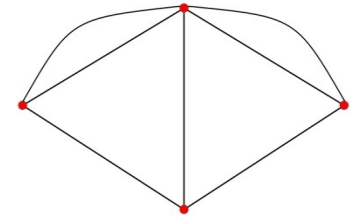


Estimating Models



- **Banerjee, Chandrasekhar, Duflo, Jackson (2013)** Study of Diffusion:
- Map network structure via surveys, observe behavior
- Model diffusion and fit the model from observed networks and behaviors

Questions

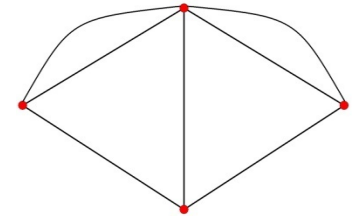


- What determines behavior:
 - Pure access to information (no strategic effects)?
 - Complementarities (strategic affects)?
- Are non-participants important in diffusion?
 - Model information passing by participants (usual contagion)
 - Information passing by non-participants too

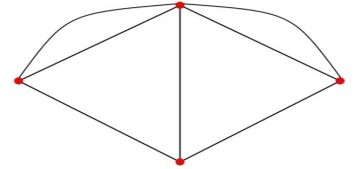
Estimate structural models of diffusion and behavior

Modeling diffusion:

- Know the set of initially informed nodes
- Informed nodes (repeatedly) pass information randomly to their neighbors over discrete times
- Once informed (just once), nodes choose to participate depending on their characteristics and their neighbors' choices



Background

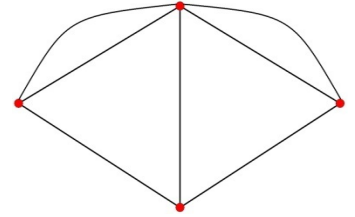


75 rural villages in Karnataka, relatively isolated from microfinance initially

- BSS entered 43 of them and offered microfinance
- We surveyed villages before entry, observed network structure and various demographics
- Tracked microfinance participation over time

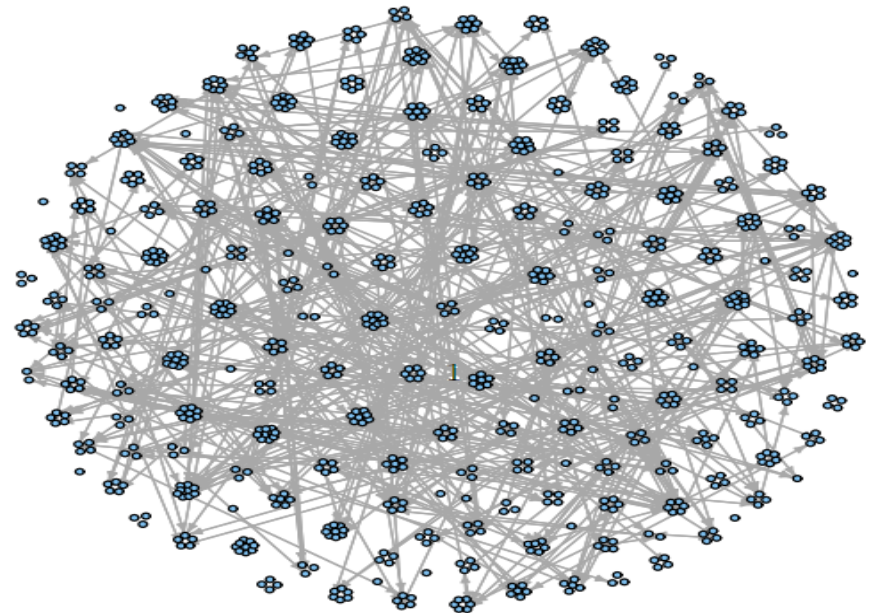


Background: 75 Indian Villages – Networks

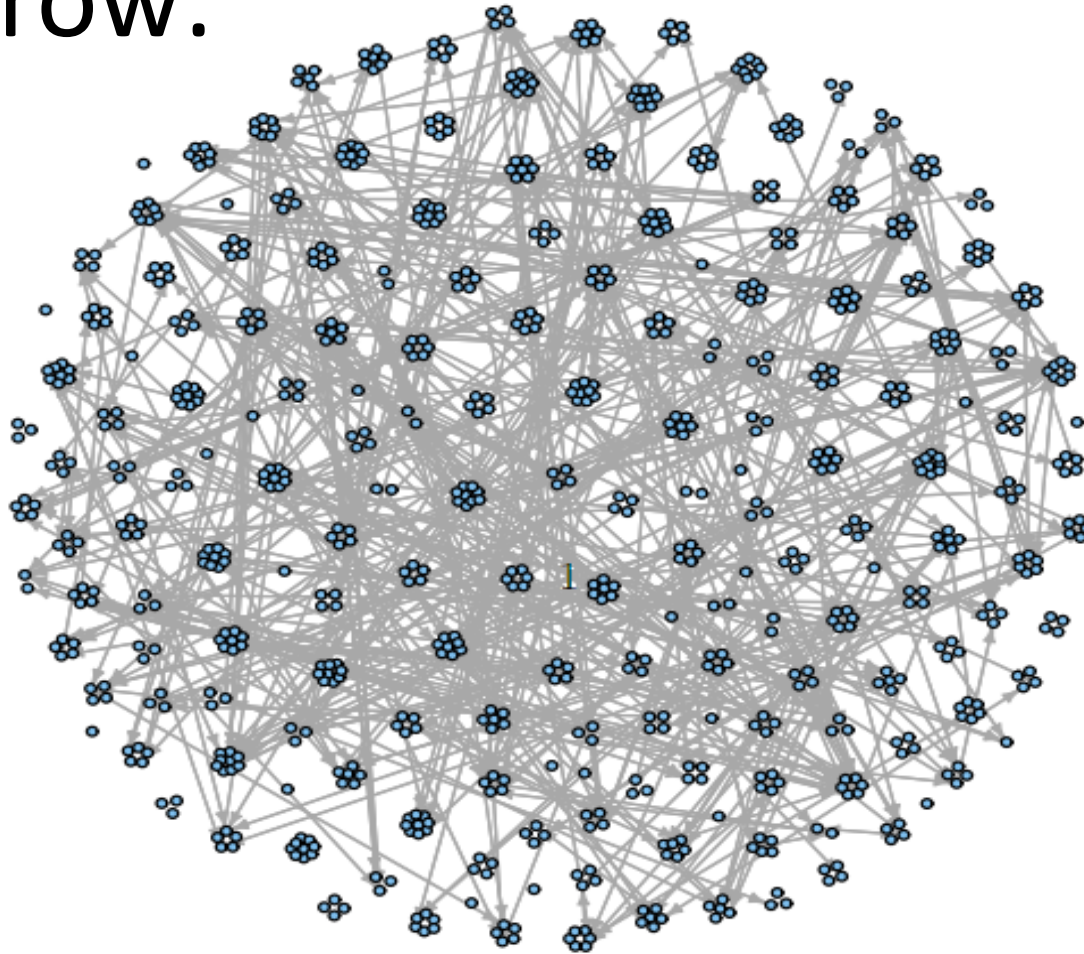


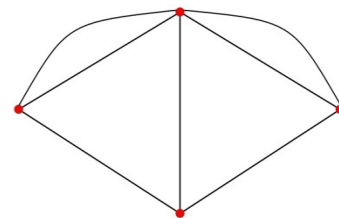
Borrow:

- ``Favor'' Networks:
 - both borrow and lend money
 - both borrow and lend kero-rice
- ``Social'' Networks:
 - both visit come and go
 - friends (talk together most)
- Others (temple, medical help...)

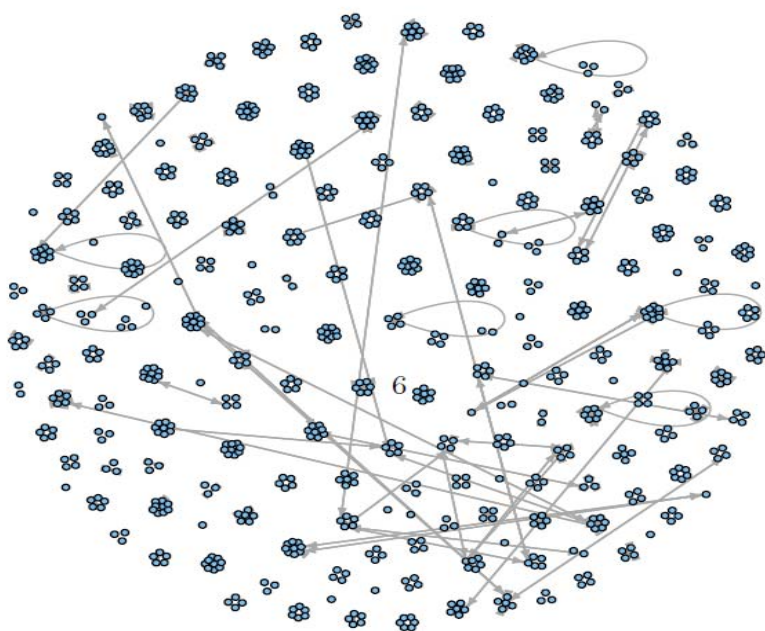


Borrow:

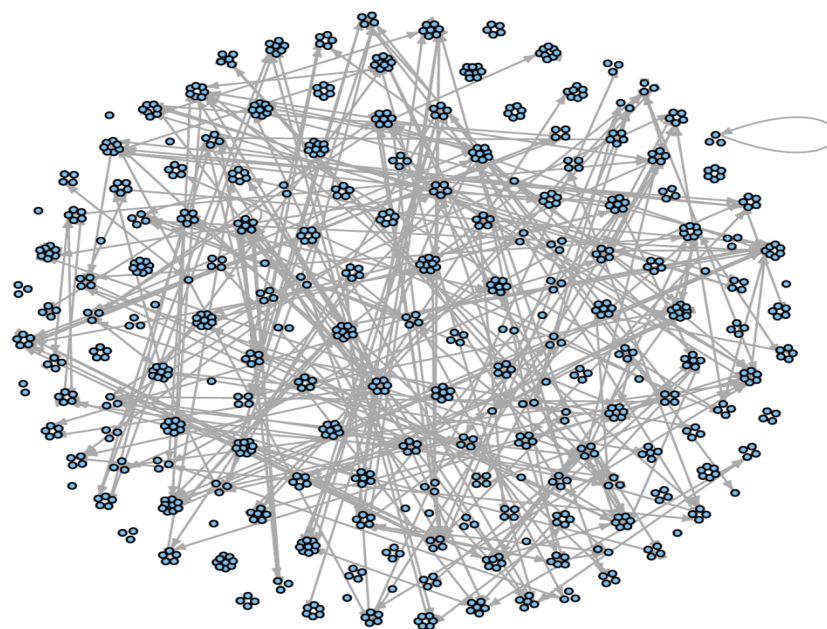




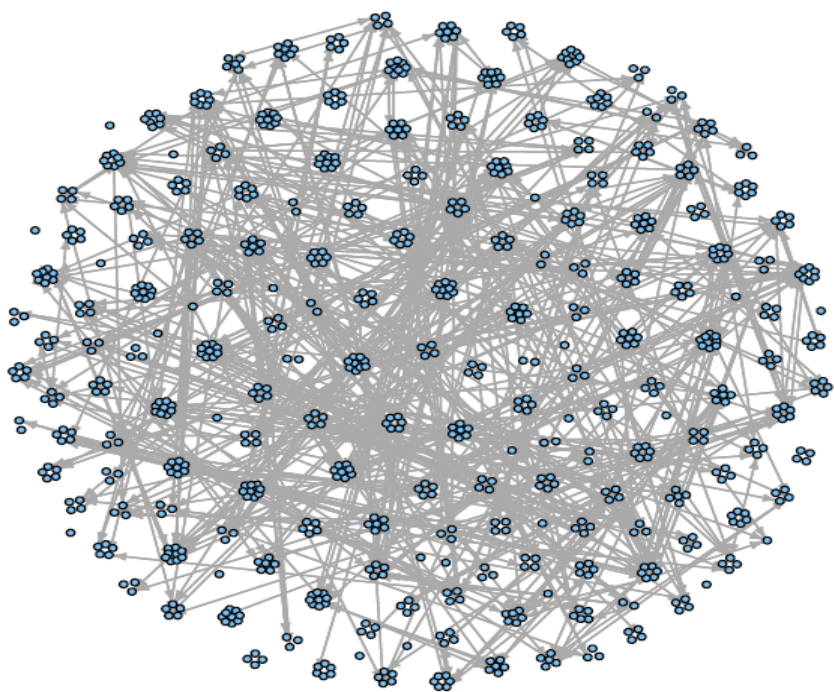
Temple



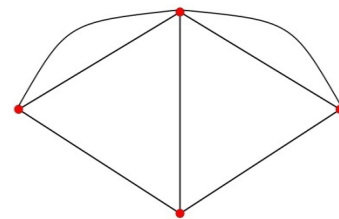
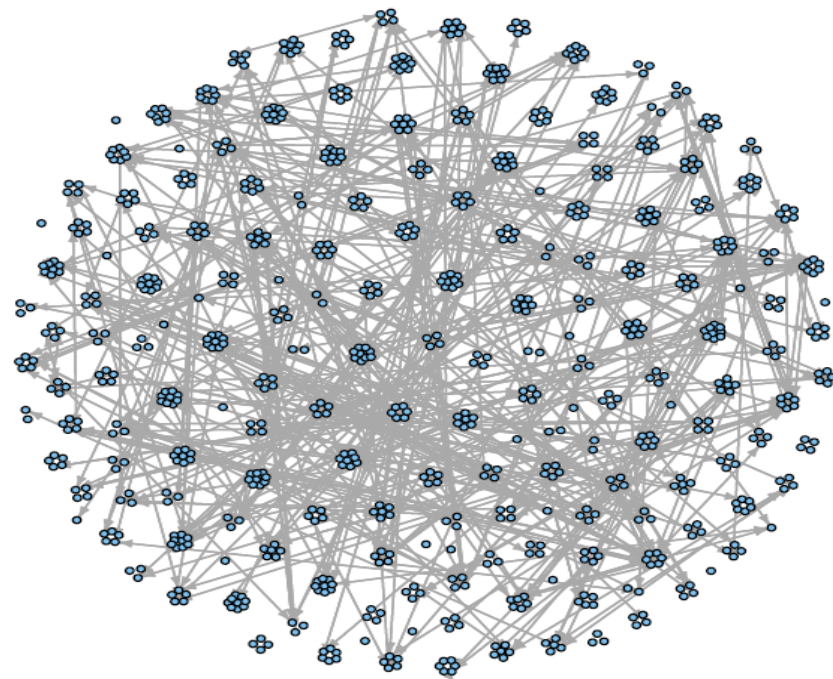
Advice



Kero-Come

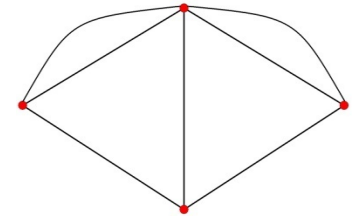


Medic

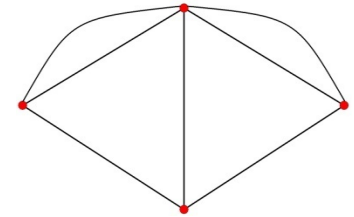


Data also include

- Microfinance participation by individual, time
- Number of households and their composition
- Demographics: age, gender, subcaste, religion, profession, education level, family...
- Wealth variables: latrine, number rooms, roof,
- Self Help Group participation rate, ration card, voting
- Caste: village fraction of ``higher castes'' (GM/FC and OBC, remainder are SC/ST)



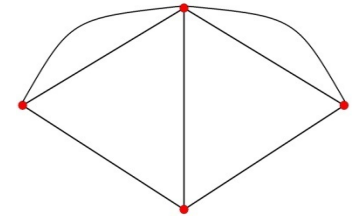
Standard Peer-effects analysis:



Let p_i be prob i participates

- $\text{Log}(p_i/(1-p_i))$
= b_0
+ b_{char} characteristics _{i}
+ b_{Peer} frac _{i} friends participate

Standard Peer-effects analysis:



Let p_i be prob i participates

- $\text{Log}(p_i/(1-p_i))$
= b_0
+ b_{char} characteristics _{i}
+ **2.5***** frac _{i} friends participate

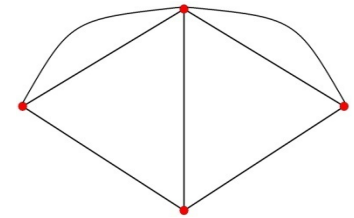
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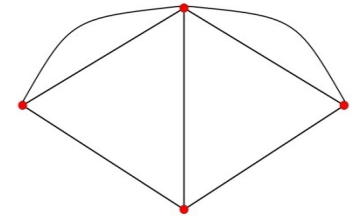
- $\text{Log}(p_i/(1-p_i))$
= b_0
+ b_{char} characteristics _{i}
+ **2.5***** frac _{i} friends participate

frac 0 to 1 increases $p_i/(1-p_i)$ by factor 12.2,

frac .1 to .3 increases $p_i/(1-p_i)$ by factor 1.65,



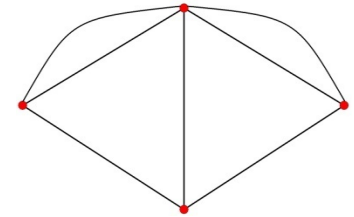
Modeling behavior/information diffusion:



- Use network information for diffusion, not just who friends are:
 - People who hear about microfinance randomly pass to friends – diffusion in network
 - Once hear, decide whether to participate – friends might matter

Participation Decision

- Once informed, make choice of whether to participate
- Choice allowed to depend on personal characteristics and fraction of informed neighbors who participate
 - Complementarity?
 - Substitution?



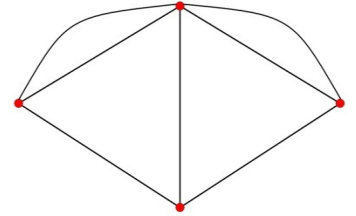
Choice Decision



Let p_i be i 's choice of whether to participate

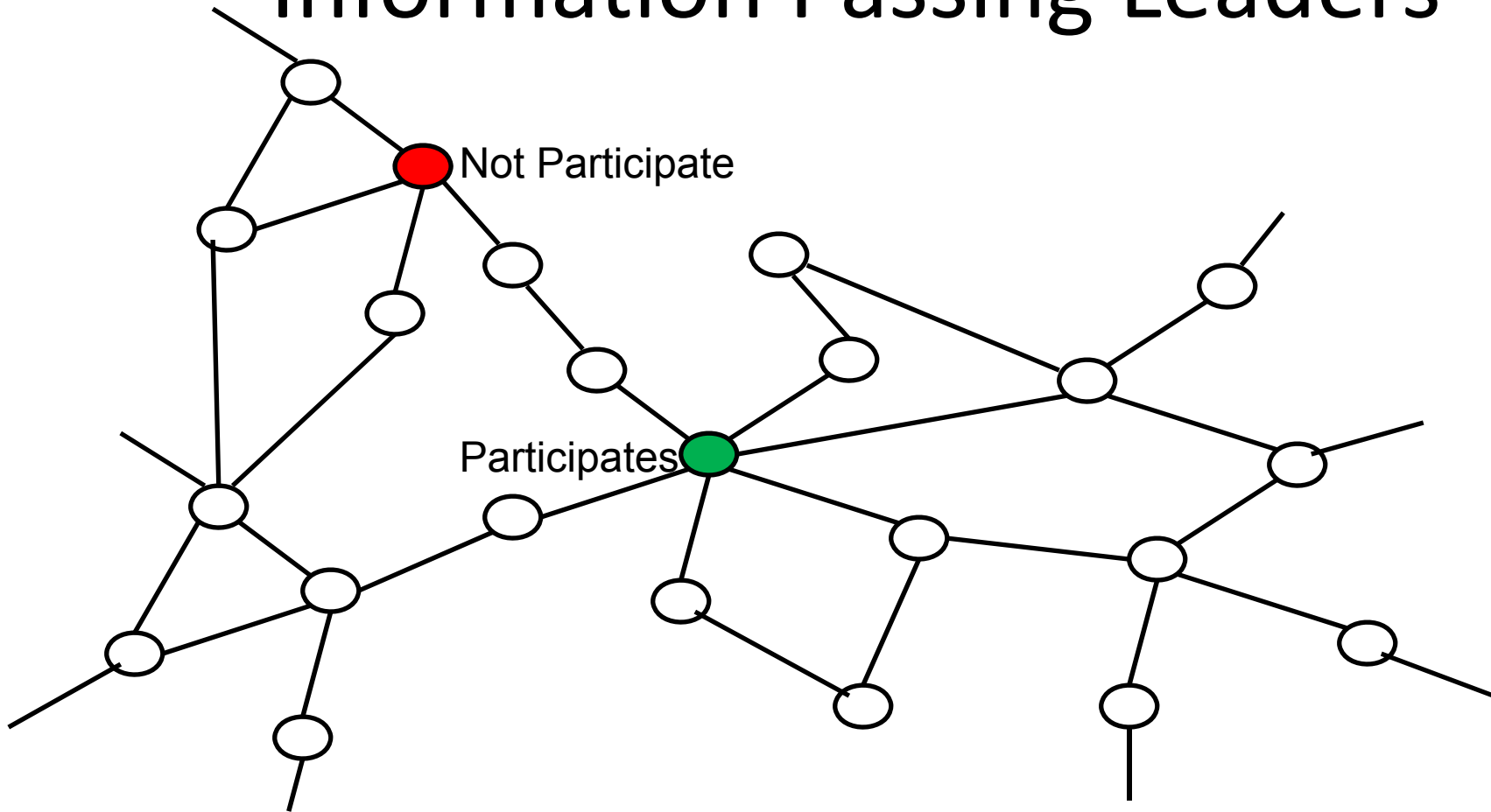
- $\text{Log}(p_i/(1-p_i))$
= b_0
+ b_{char} characteristics _{i}
+ b_{peer} frac _{i} informing friends participating

Modeling behavior/information diffusion:

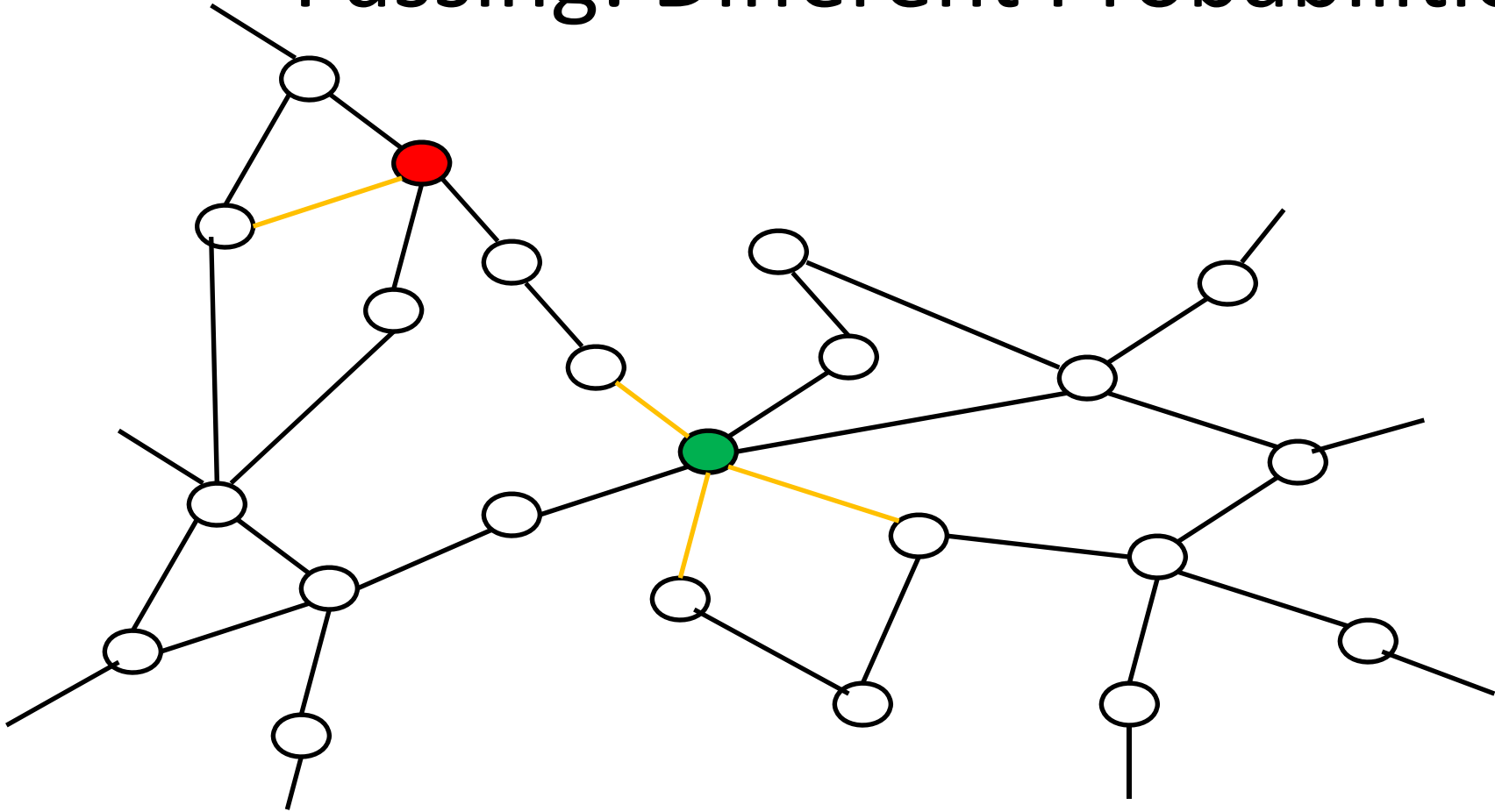


- Probability of passing to a given individual:
 - q^N if did Not participate
 - q^P if did Participate

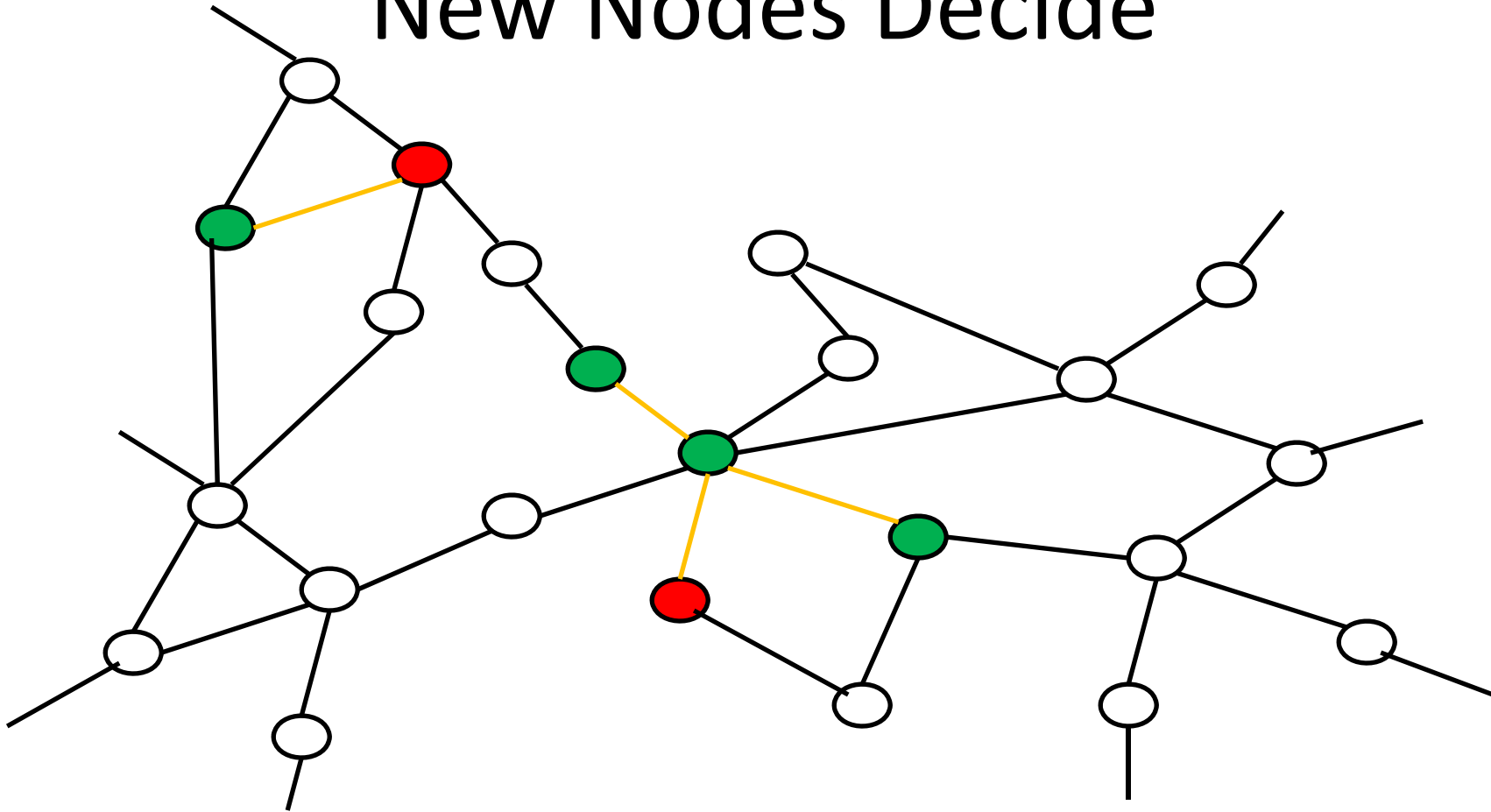
Information Passing Leaders



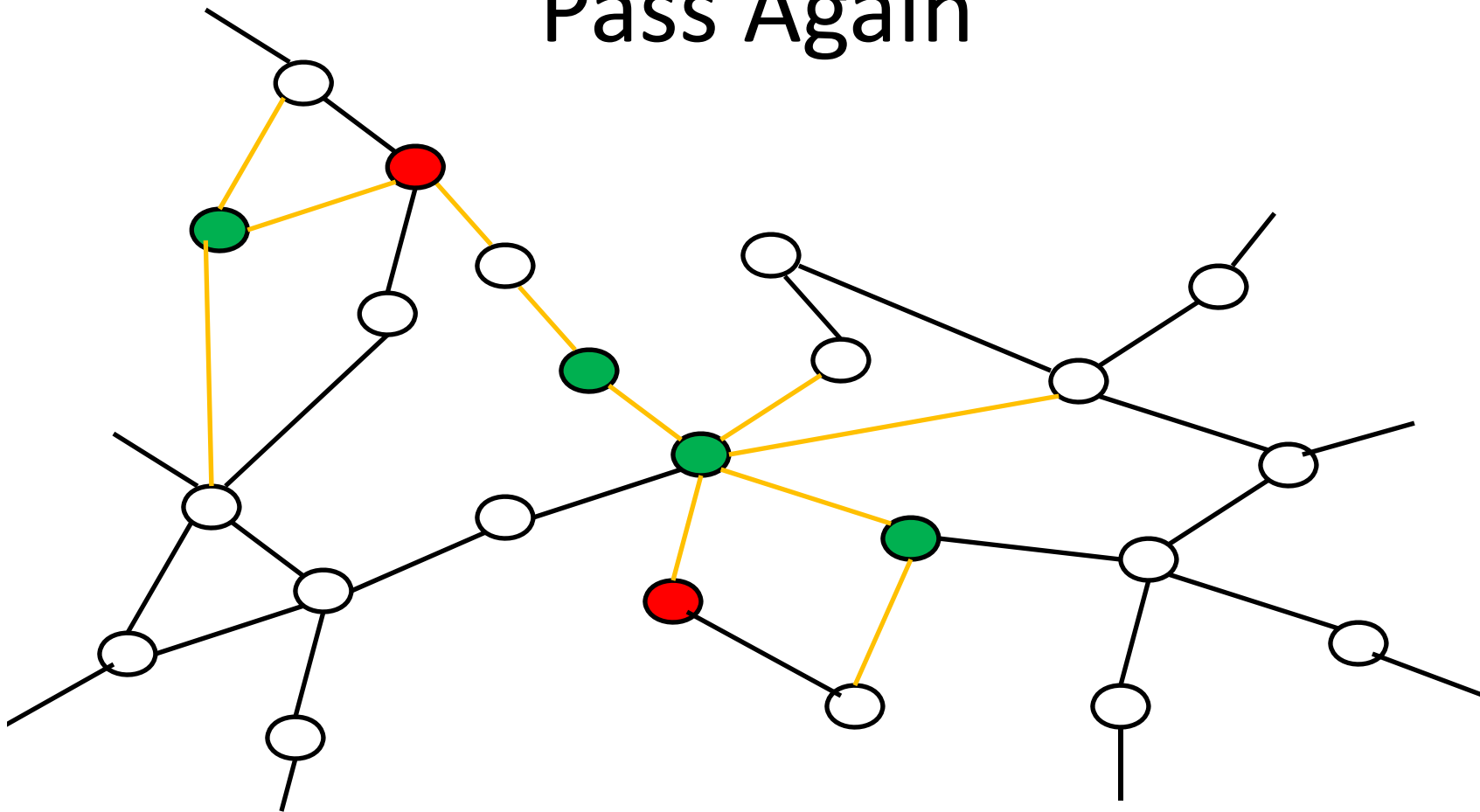
Passing: Different Probabilities



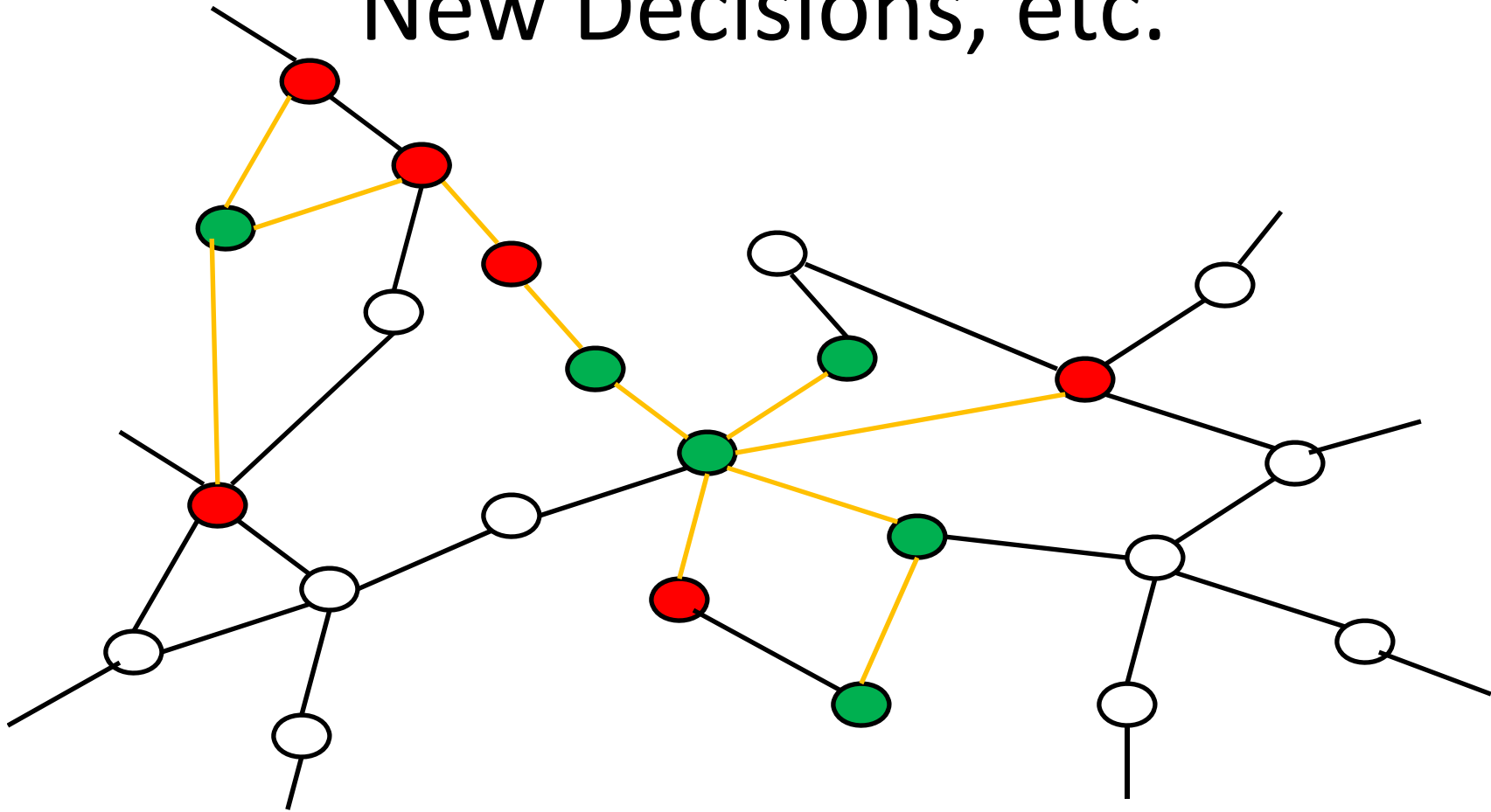
New Nodes Decide



Pass Again

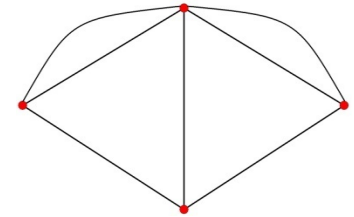


New Decisions, etc.



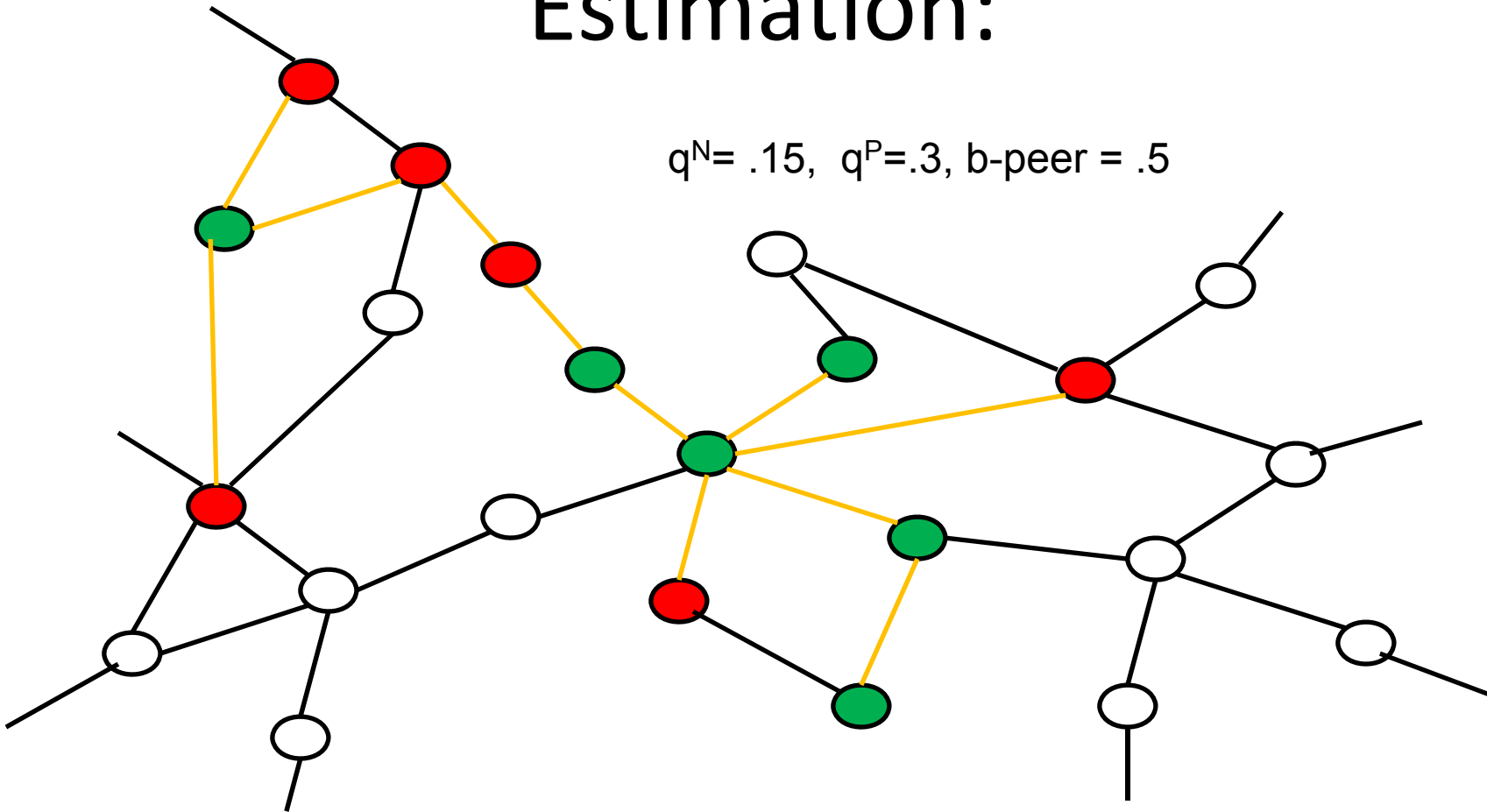
Estimation technique:

- Estimate b_0 , b_{char} from initially informed (saves on computation size of grid)
- q^N , q^P , b_{peer} - For each choice of parameters, simulate on the actual networks of the villages for time period proportional to number of trimesters in data for village (3 to 8 times)
- Choose parameters to best match simulated participation rates and various moments to observed moments (GMM)



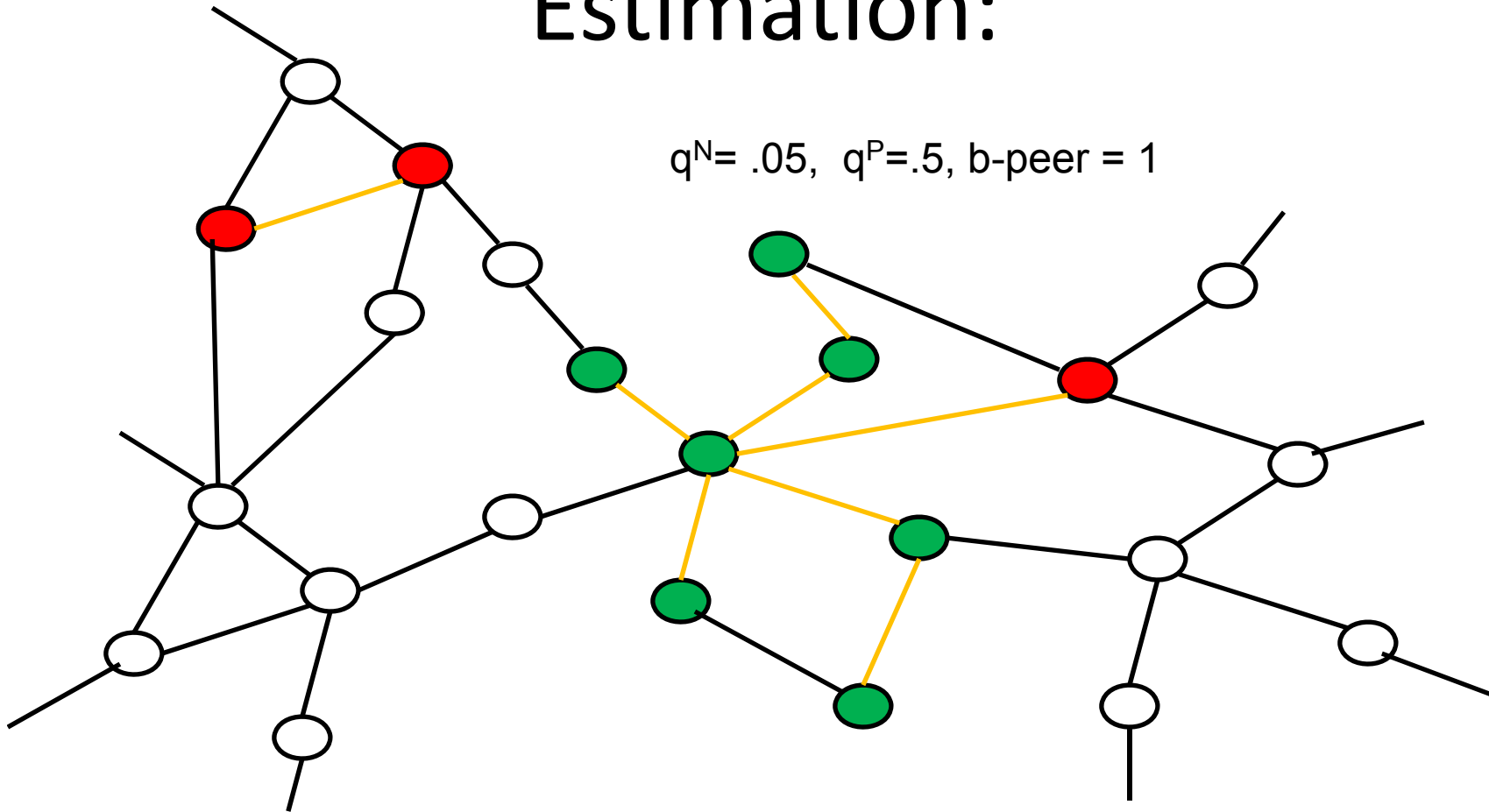
Estimation:

$q^N = .15$, $q^P = .3$, $b\text{-peer} = .5$



Estimation:

$q^N = .05$, $q^P = .5$, $b\text{-peer} = 1$



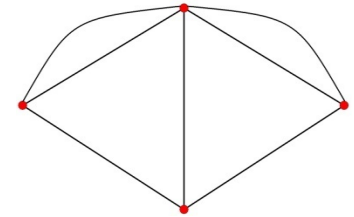
Estimated parameters:



- Information significant, peer/endorse effect not

	qN	qP	b-peer	qN – qP
Diffusion and peer	0.05*** [0.01]	0.55*** [0.13]	-0.20 [0.16]	-0.50*** [0.13]

Estimated parameters:



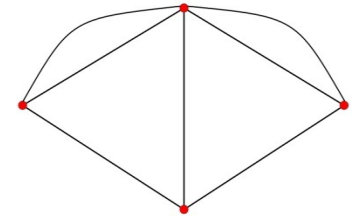
- Information significant, peer/endorse effect not

	qN	qP	b-peer	qN – qP
Diffusion and peer	0.05*** [0.01]	0.55*** [0.13]	-0.20 [0.16]	-0.50*** [0.13]

just peer!:

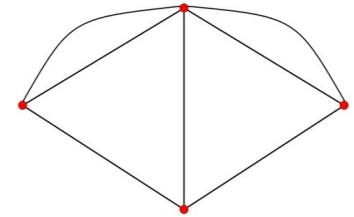
2.5***

Network Effects:



- Significant information passing parameters
- Information passing depends on whether participate: more likely if participate
- Slight complementarities, but insignificant

Information Passing



- What fraction of eventual informed agents are accounted for by information passing of non-participants?
- Hold all else constant, but rerun the model with $q^N=0$
- See what happens to information and participation rates

Information Passing



Median for model as fit:

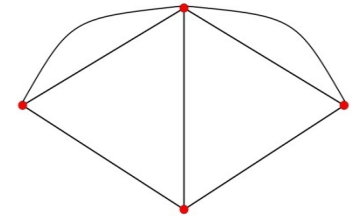
Informed 85.8% Participation 20.7%

Median for model re-setting $q^N = 0$

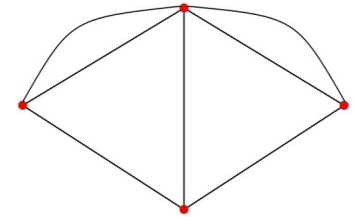
Informed 58.9% Participation 13.8%

Results from Fitting Models of Diffusion:

- Significant information passing parameters
- Insignificant, limited Peer Effects
- Information passing depends on whether participate: more likely if participate
- Nonparticipants play a substantial role (1/3 of total)

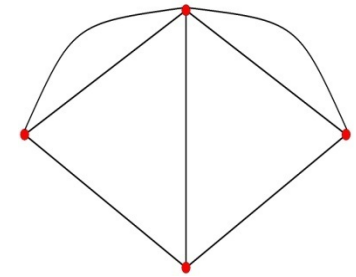


Conclusions



- **Models of diffusion can help us disentangle effects**
- **Important for policy**
 - Enhance information spreading?
 - Help overcome/enhance peer influences
- **Relate back to network structure**
 - homophily
 - degree distribution, clustering ...

Social and Economic Networks: Models and Analysis

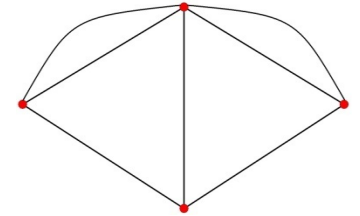


Matthew O. Jackson

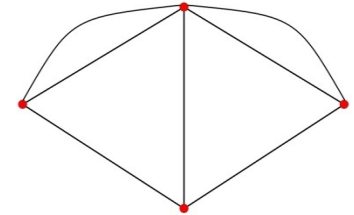
**Stanford University,
Santa Fe Institute, CIFAR,
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5.8b Application: Financial Contagions



Explore Contagions



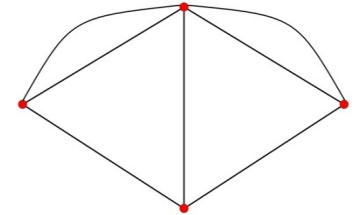
Simple model of Elliott, Golub Jackson 13:

Companies are linked to each other via various contracts: debts, promised deliveries, equity,

That exposes each company to others' investments and values

First, let us see how to use networks to model exposures

Explore Contagions



An organization has direct investments:

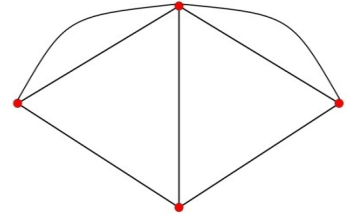
Fraction c_i of value accrues directly to them

Fraction $1-c_i$ is owed to others

Also hold obligations of d_i other organizations:

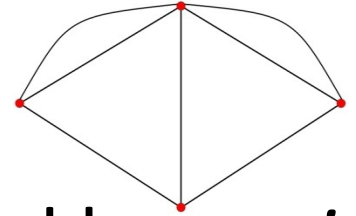
Have claims to those other organizations' investments

Model



- $\{1, \dots, n\}$: Organizations (countries, firms, banks...)
- p_i : price of investments of organization i

Cross Holdings:



- C_{ij} : cross holdings: fraction of org j owned by org i
- $C_{ii} = 0$: (don't own yourself)
- $\hat{C}_{ii} = 1 - \sum_j C_{ji}$:
fraction of org i privately held

Value of an Organization

book value:

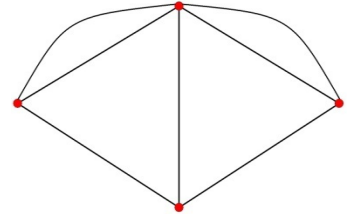
$$V_i = p_i + \sum_j c_{ij} V_j$$



direct asset
holdings



cross-
holdings

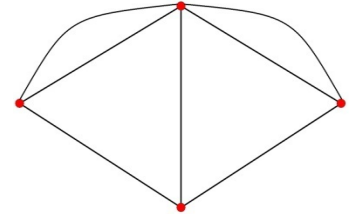


Value of an Organization

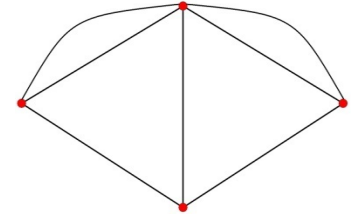
$$V_i = p_i + \sum_j c_{ij} V_j$$

$$V = p + CV$$

Leontief
calculation of
book value



Value of an Organization



Book value:

$$V = (I - C)^{-1} p$$

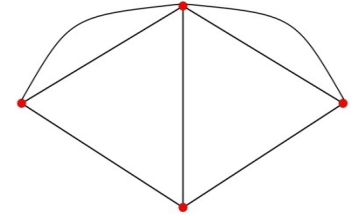
Market value – value to final (private) investors.

$$v_i = \hat{C}_{ii} V_i$$

$$v = \underbrace{\hat{C} (I - C)^{-1}}_{\downarrow} p$$

$$v = \mathbf{A} p$$

Value of an Organization



Book value:

$$V = (I - C)^{-1} p$$

Market value – value to final (private) investors.

$$v_i = \hat{C}_{ii} V_i$$

$$v = \underbrace{\hat{C} (I - C)^{-1}}_{\downarrow} p$$

$$v = A p$$

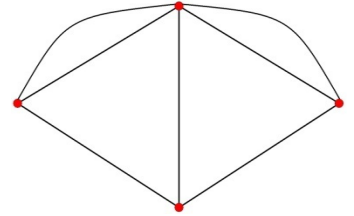
A_{ij} :

fraction of the investments owned by org j that ultimately accrue to private shareholders of i

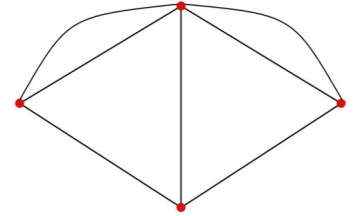
Example

- Two organizations: $n = 2$
- Each owns half of the other: $\mathbf{C} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$
- Implied holdings by private investors:

$$\hat{\mathbf{C}} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$



Example

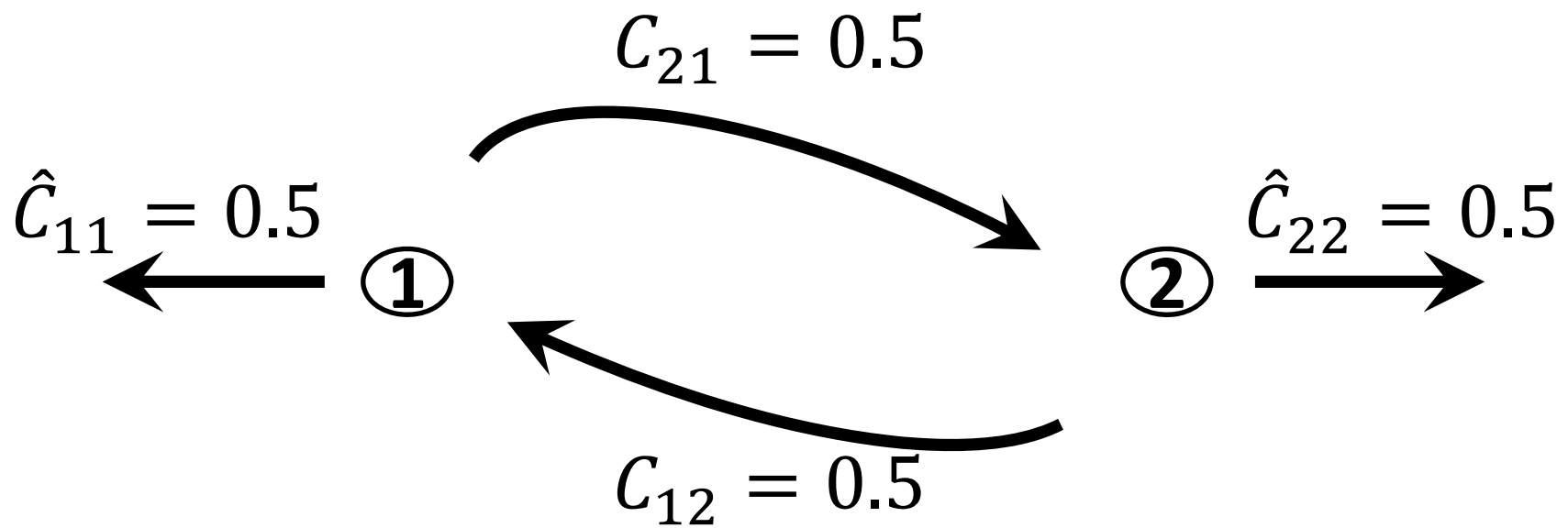
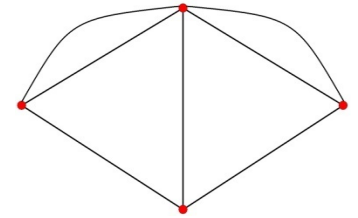


- Two organizations: $n = 2$
- Each owns half of the other: $\mathbf{C} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$
- Final investors' claims on assets:

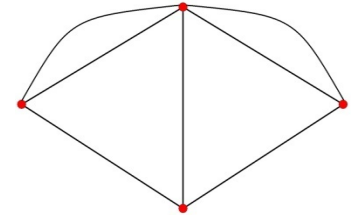
$$\hat{\mathbf{C}} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$\mathbf{A} = \hat{\mathbf{C}} (\mathbf{I} - \mathbf{C})^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

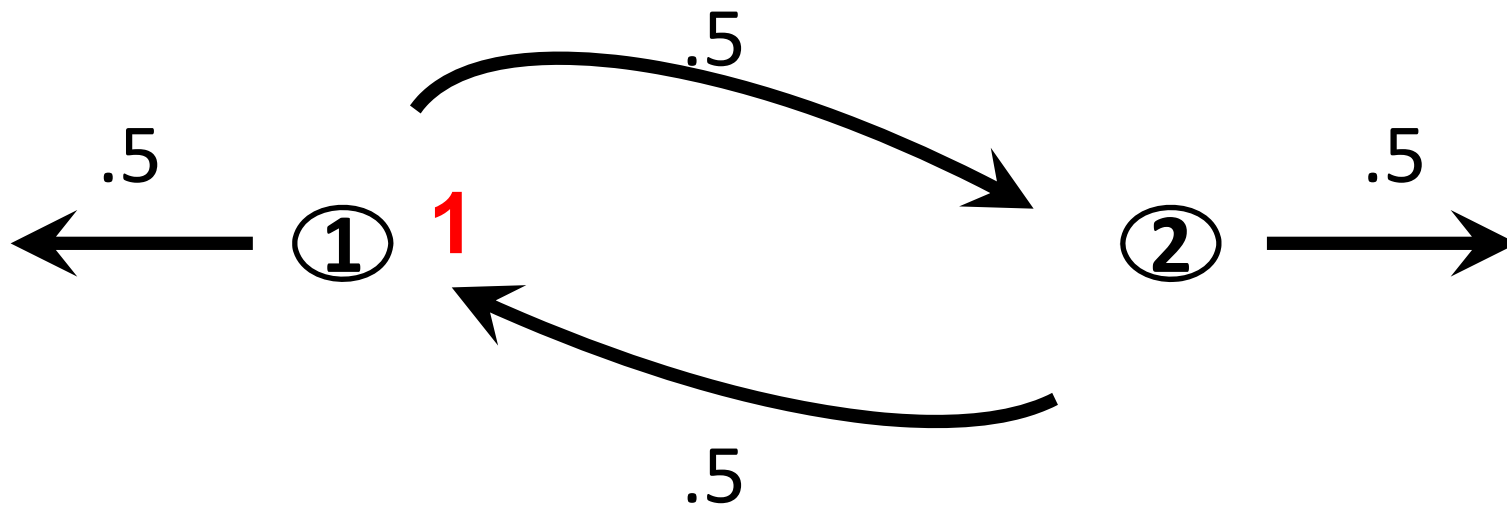
Example



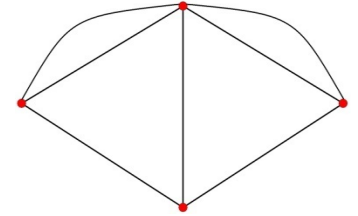
Example



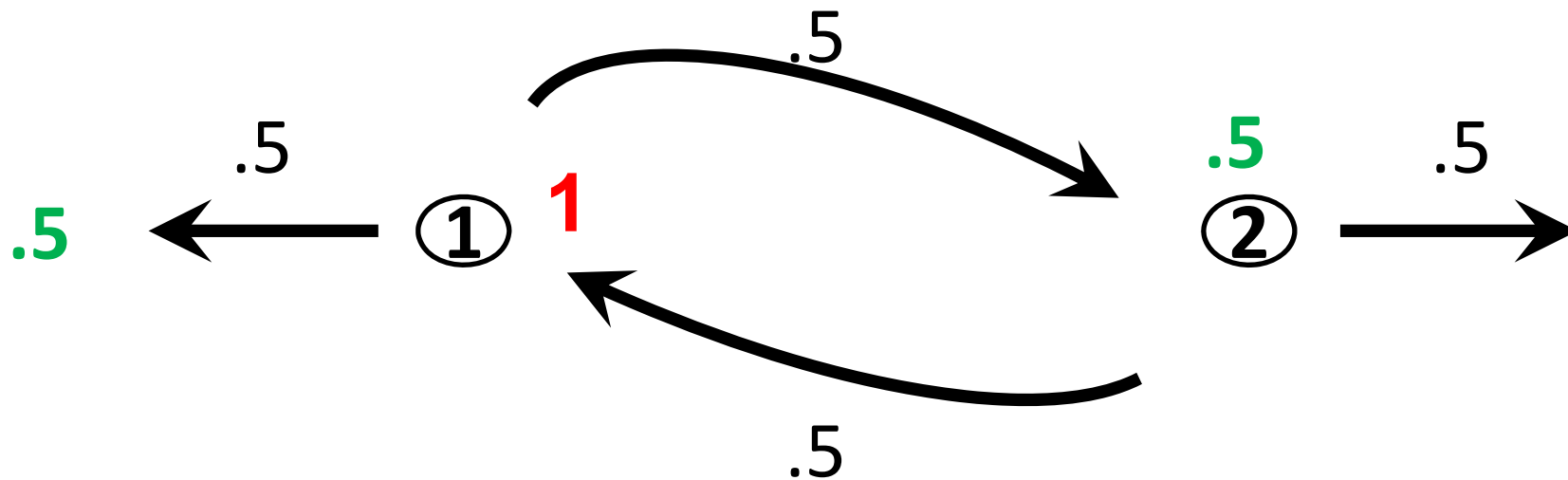
What happens to \$1 of investment income to 1?



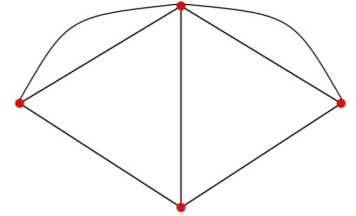
Example



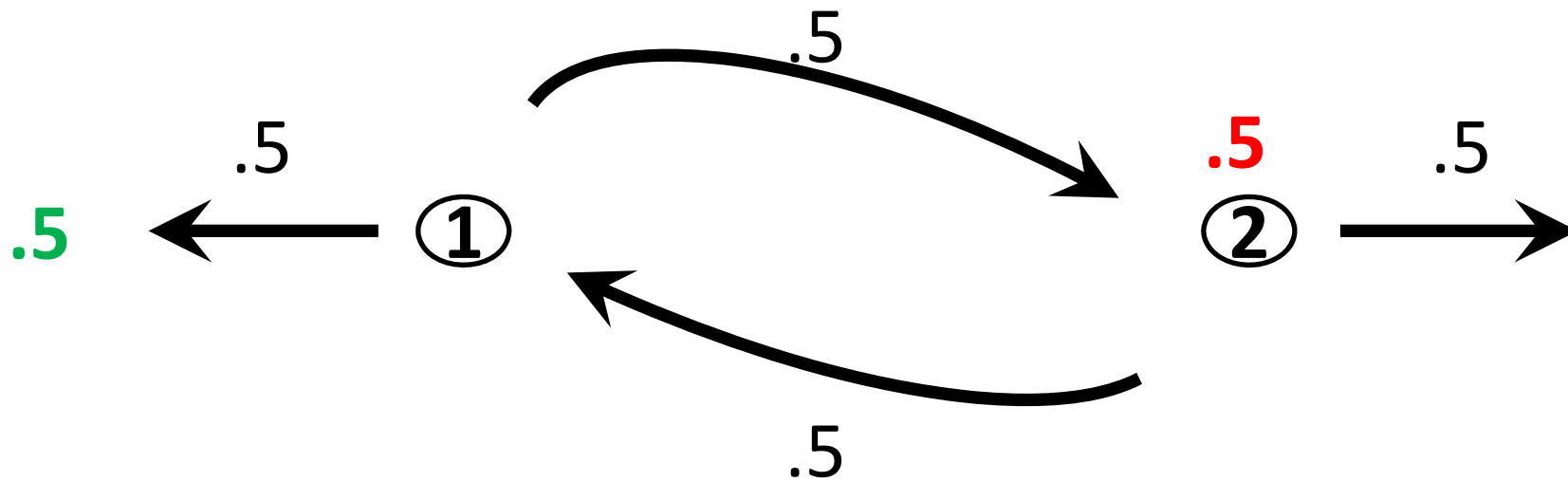
What happens to \$1 of investment income to 1?



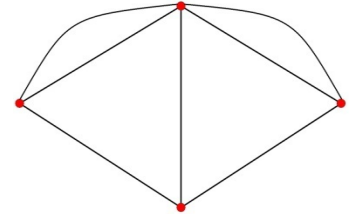
Example



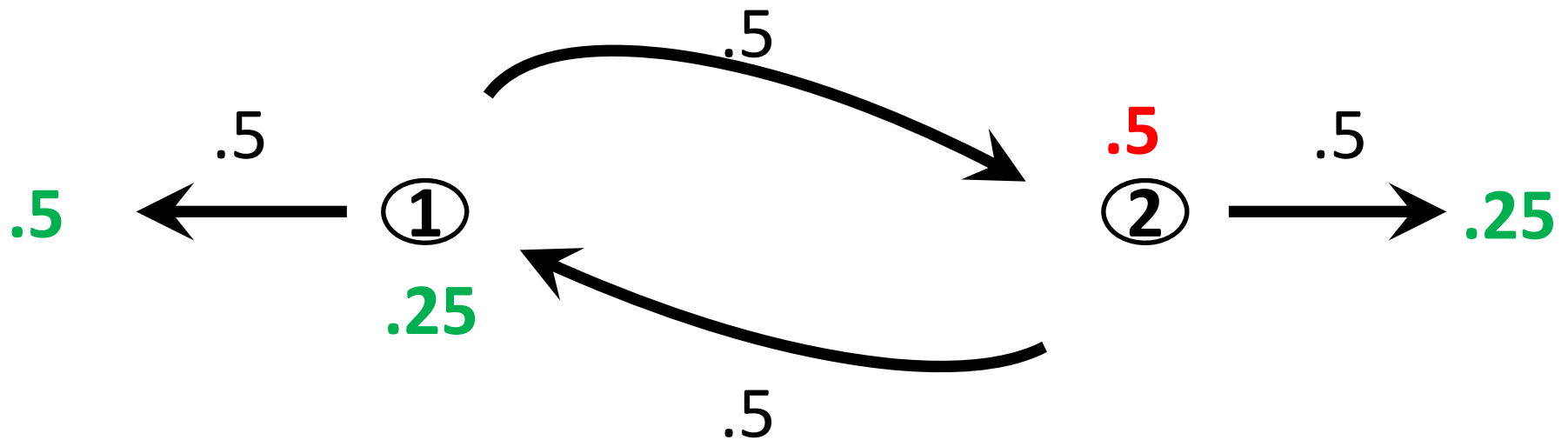
What happens to \$1 of investment income to 1?



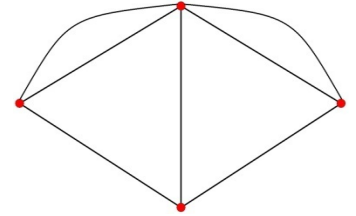
Example



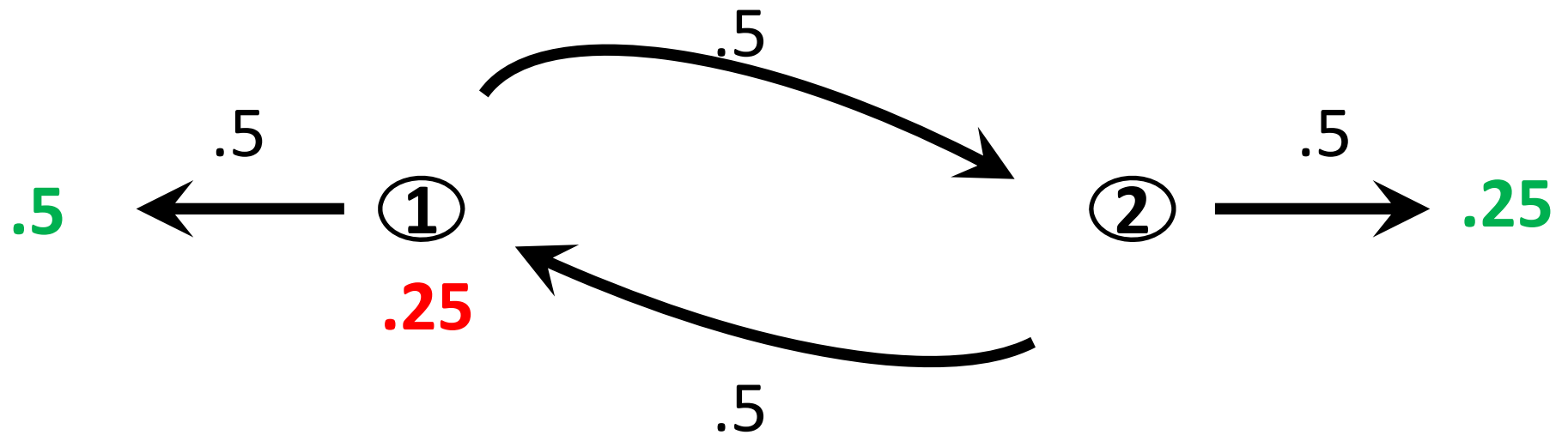
What happens to \$1 of investment income to 1?



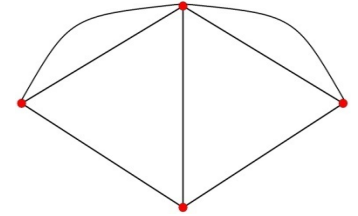
Example



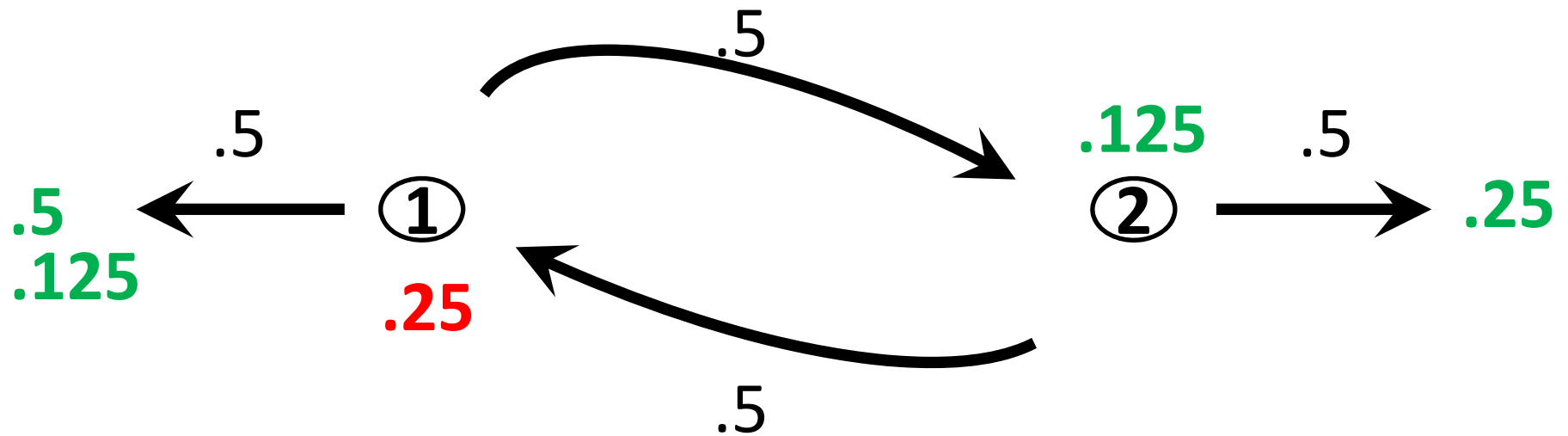
What happens to \$1 of investment income to 1?



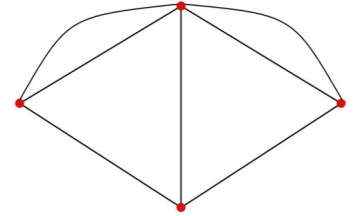
Example



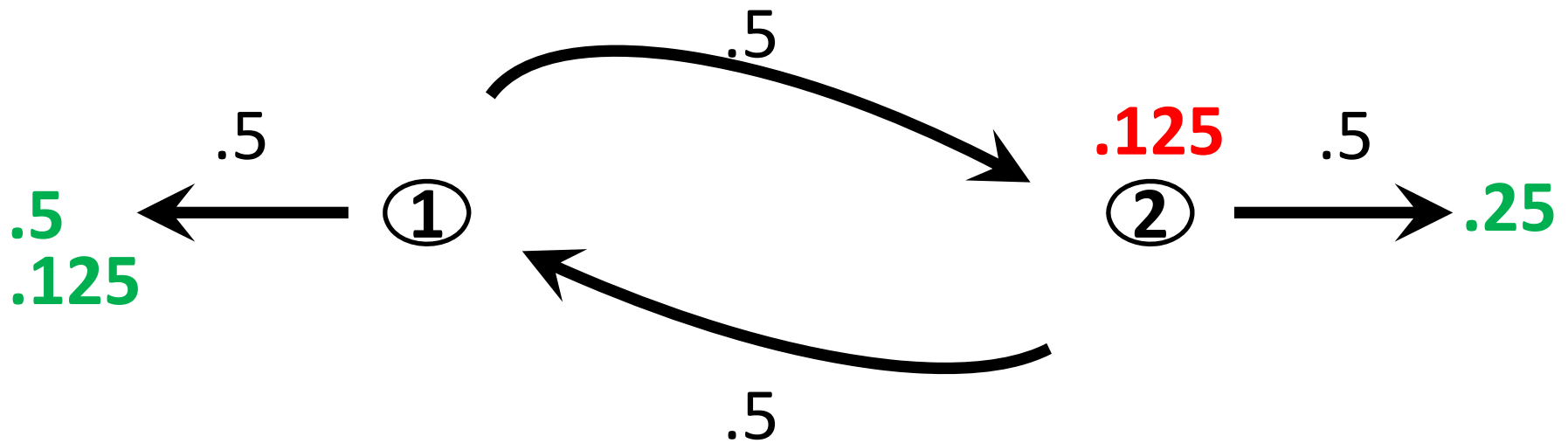
What happens to \$1 of investment income to 1?



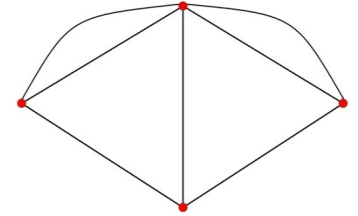
Example



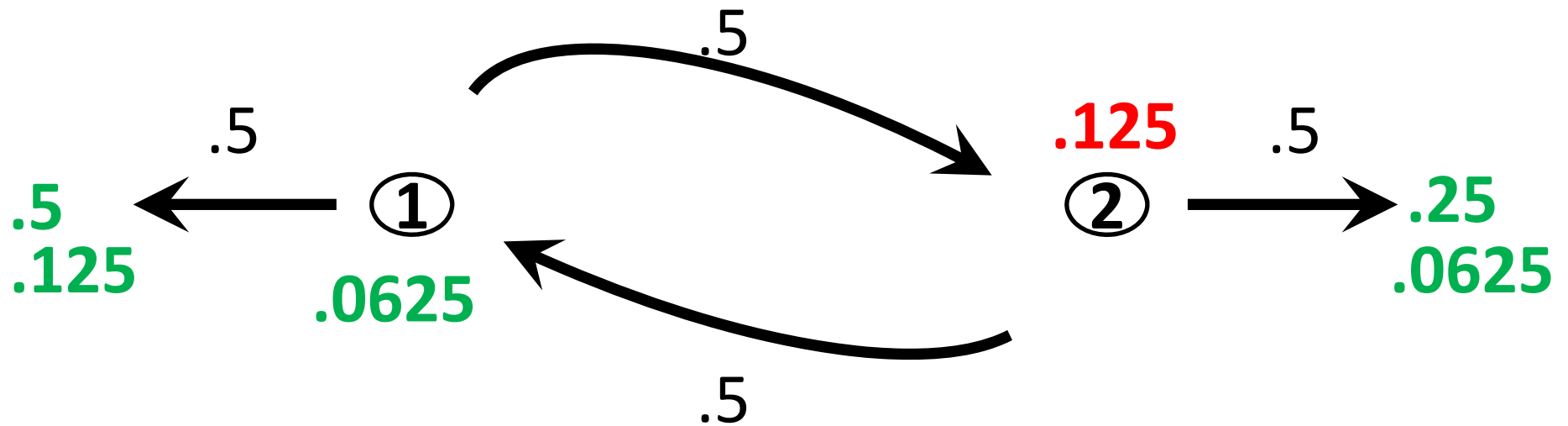
What happens to \$1 of investment income to 1?



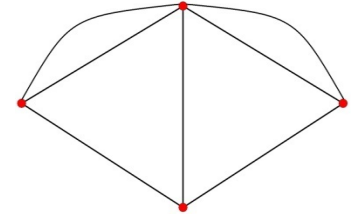
Example



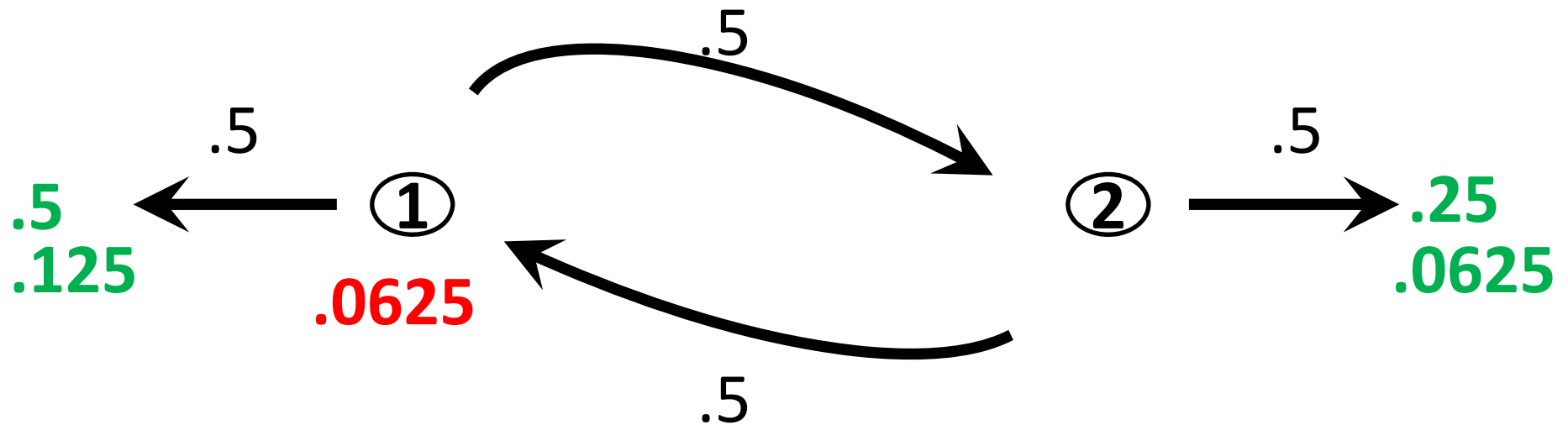
What happens to \$1 of investment income to 1?



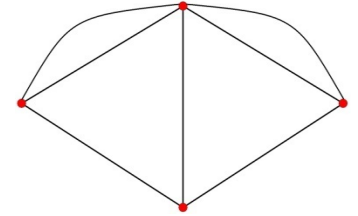
Example



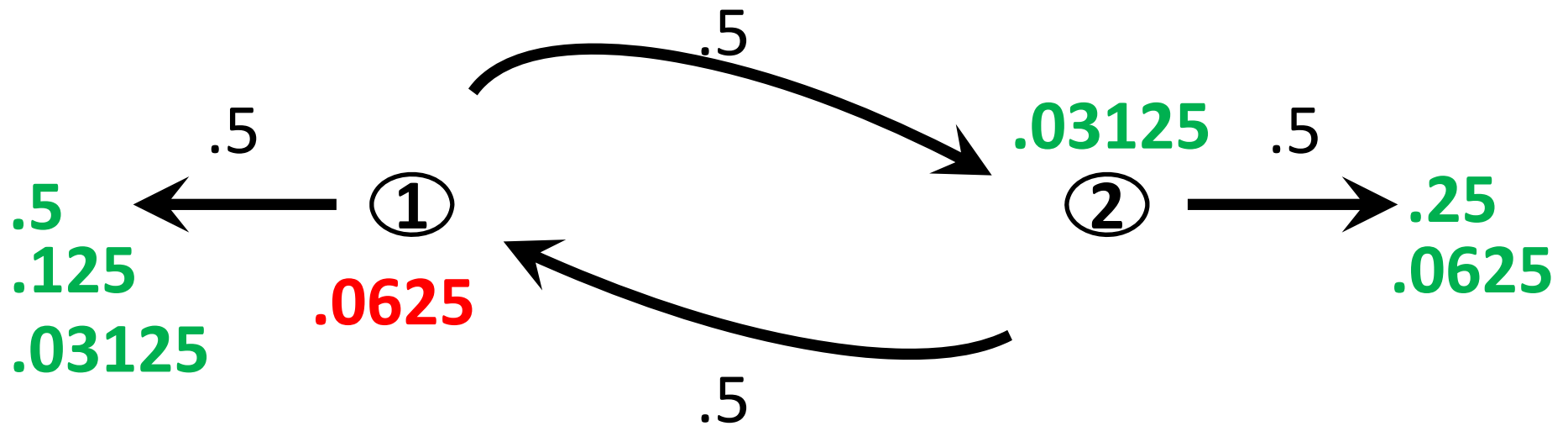
What happens to \$1 of investment income to 1?



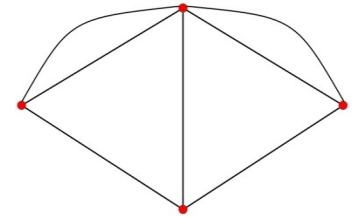
Example



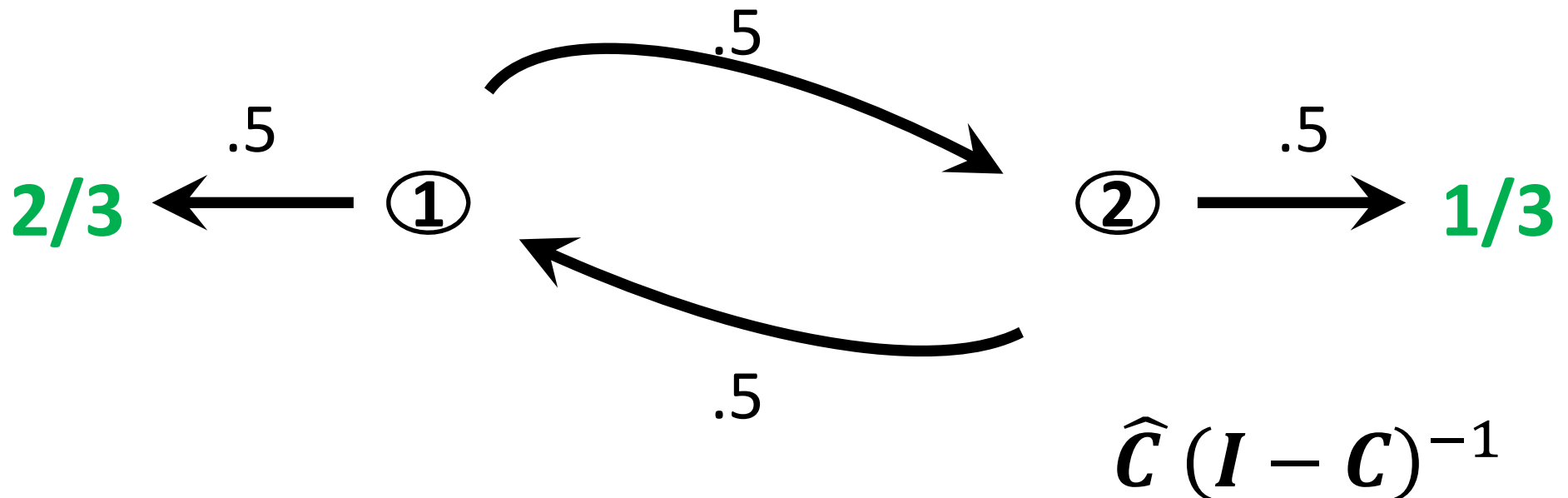
What happens to \$1 of investment income to 1?



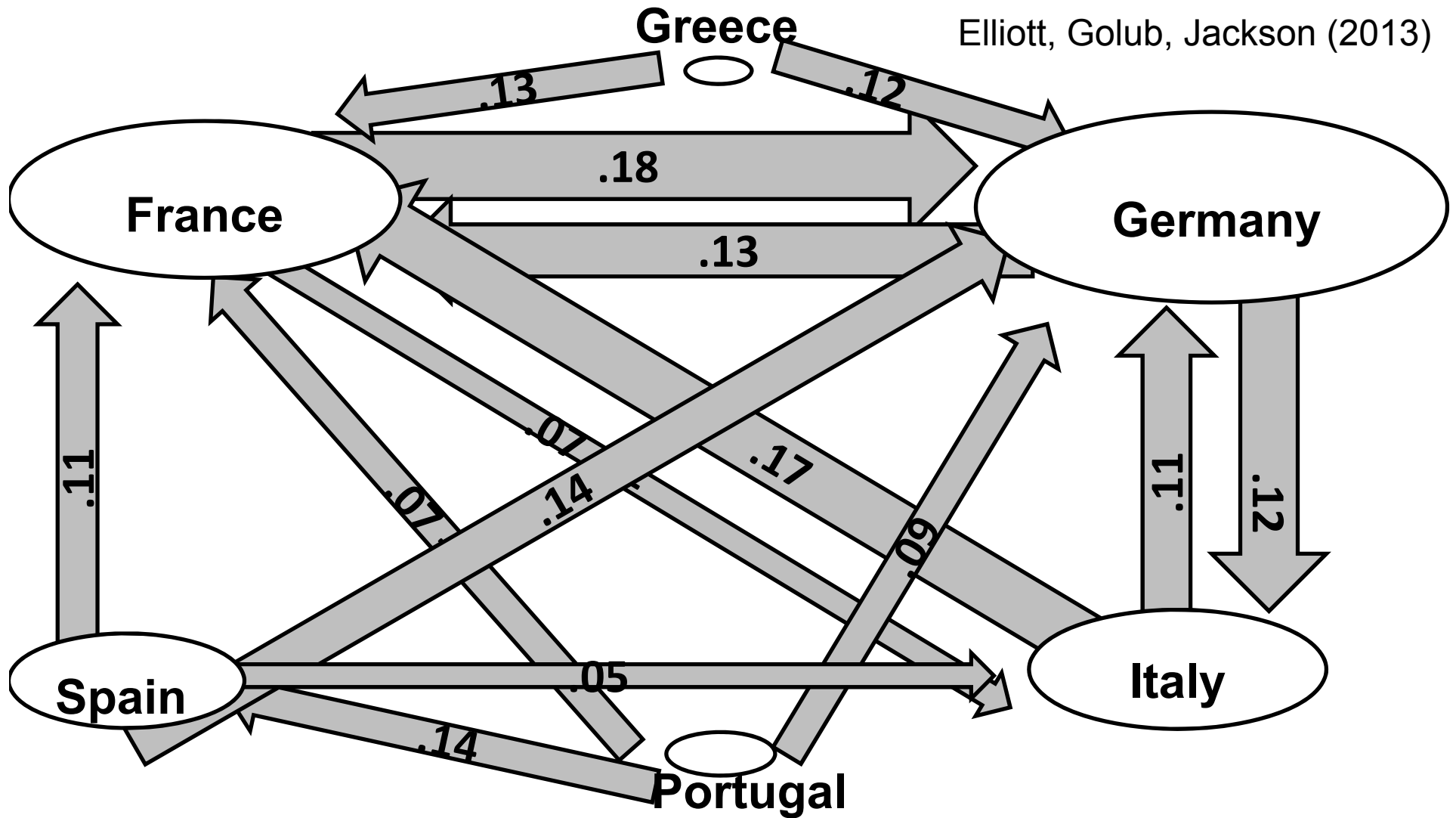
Example



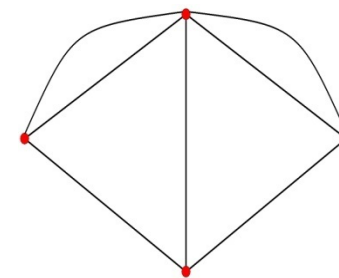
What happens to \$1 of investment income to 1?



Elliott, Golub, Jackson (2013)



Social and Economic Networks: Models and Analysis

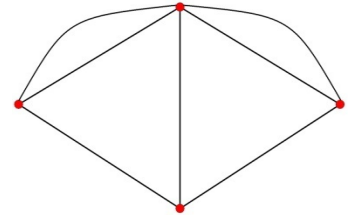


Matthew O. Jackson

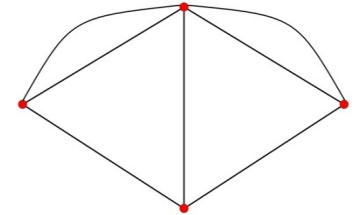
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5.8c Application: Financial Contagions



Explore Contagions



Simple model of Elliott, Golub Jackson 13:



An organization has direct investments:

Fraction c_i of value accrues directly to them

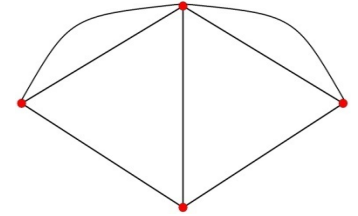
Fraction $1-c_i$ is owed to others

Also hold obligations of d_i other organizations:

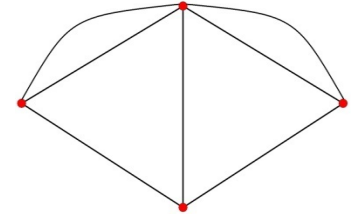
Have claims to those other organizations'
investments

Simulation Setup

- $n=100$ organizations
- Random network g with $\Pr(g_{ij} = 1) = d/(n-1)$
- d = expected # other organizations that an organization cross holds (**$d = \text{level of diversification}$**)
- Fraction c of org cross-held (evenly split among those holding it), $1-c$ held privately (**$c = \text{level of integration}$**)
- So, claim i has on j : $C_{ij} = cg_{ij}/d_j$

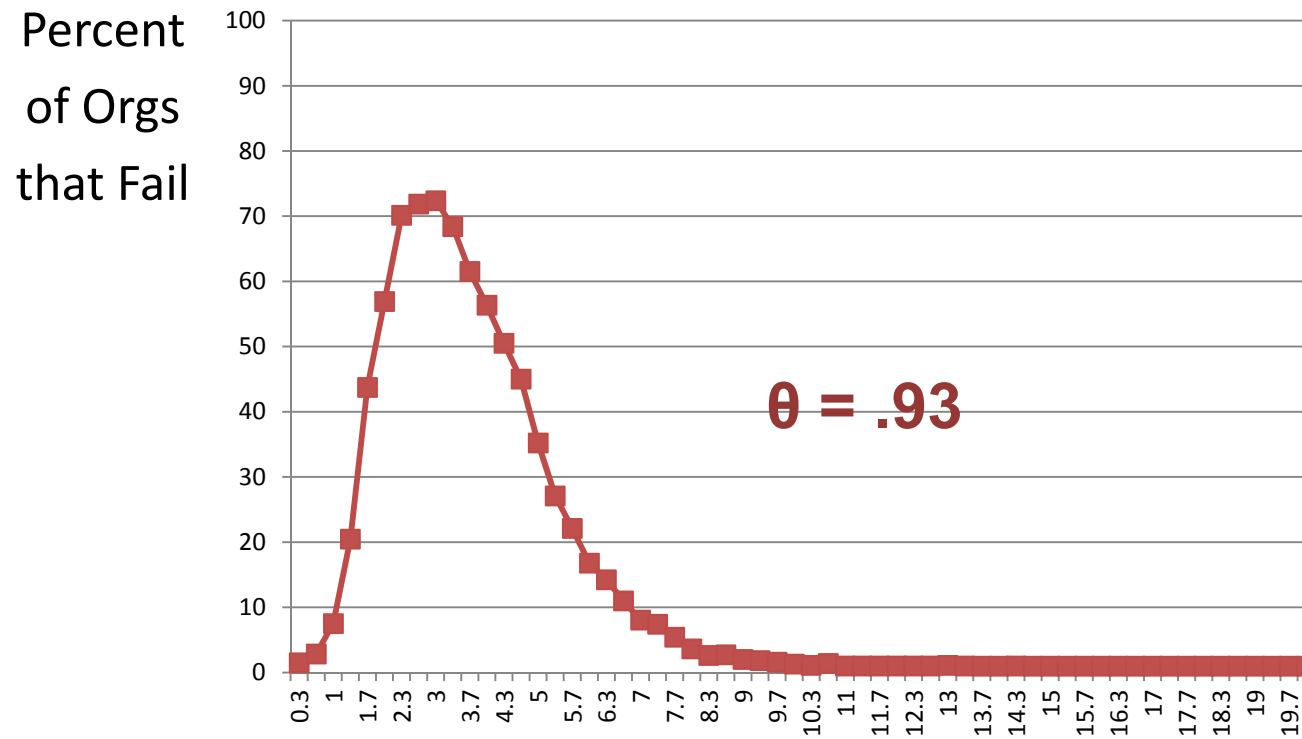
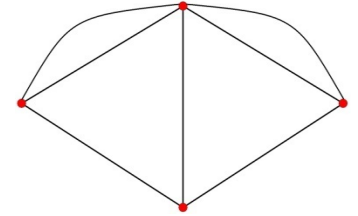


The Exercise



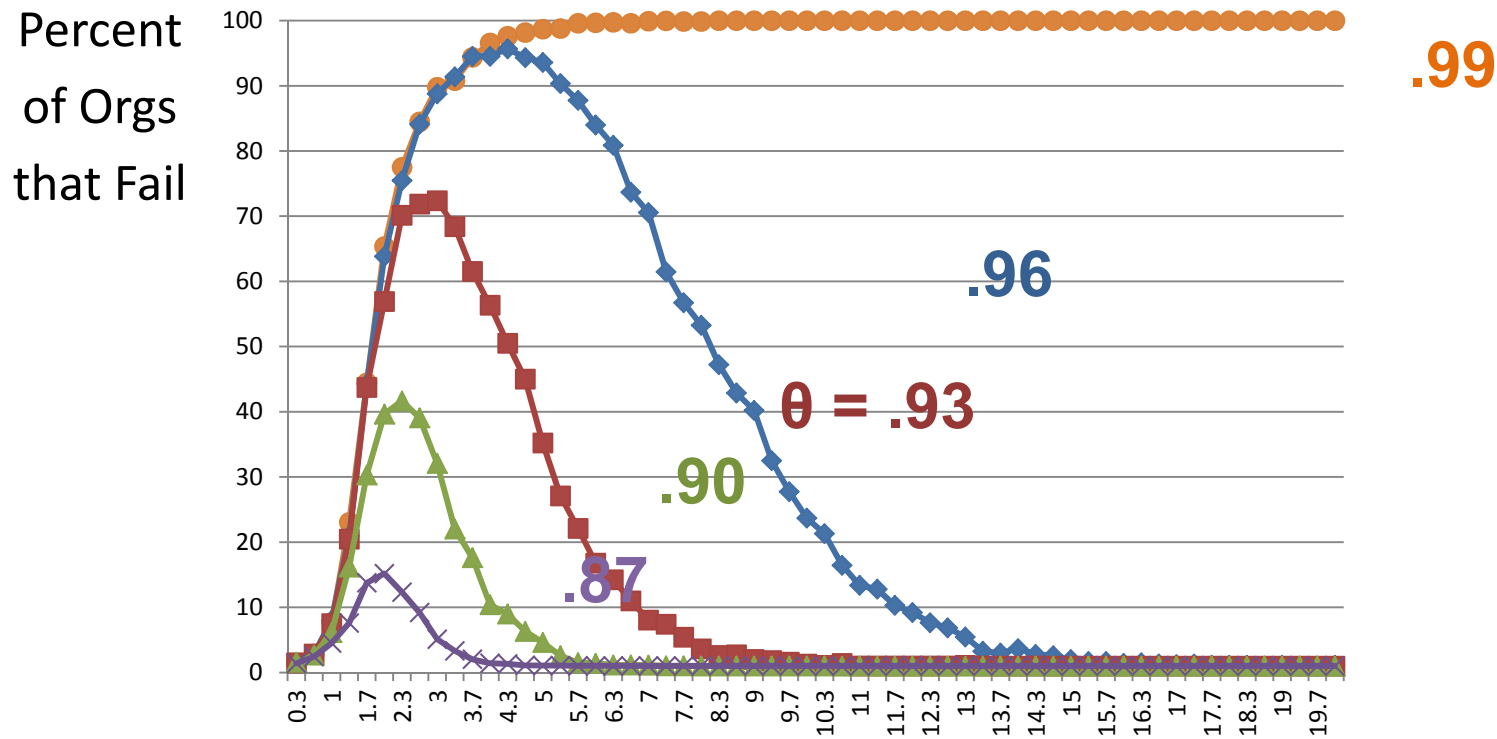
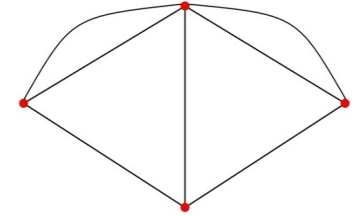
- One asset per organization (their investments), starts at value 1
- Pick one organization's investment to devalue to 0
- If an organization's value drops below θ of its starting value, it fails.
- Look at resulting cascade

Diversification and Contagion: 93% threshold, $c=.5$



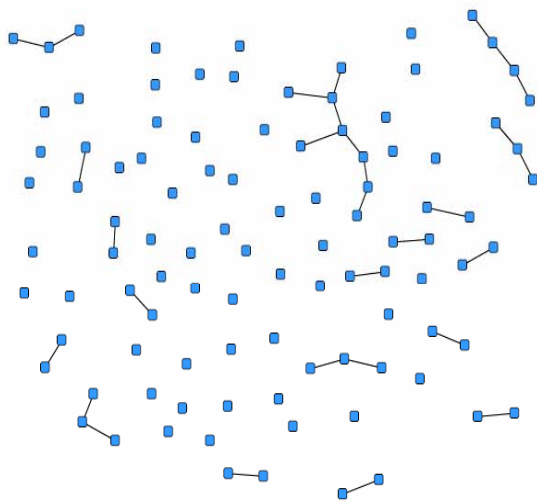
Degree: Expected # of cross-holdings

Diversification and Contagion: Various Thresholds

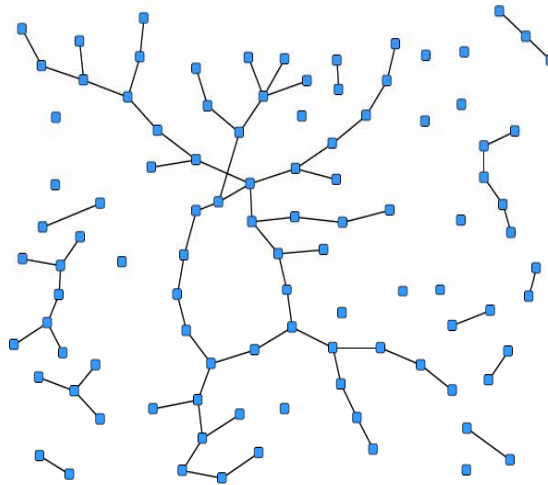


Degree: Expected # of cross-holdings

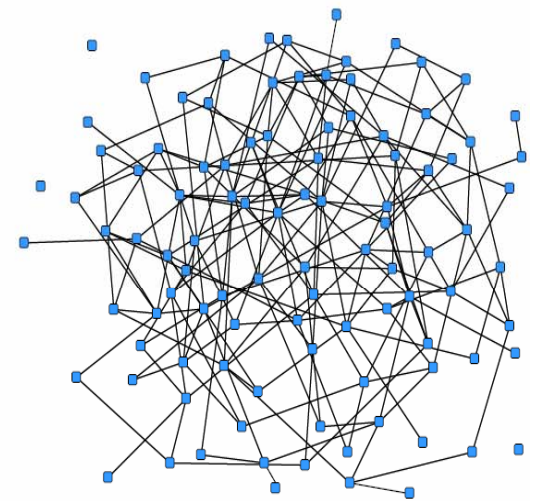
Intuition



(a) Low diversification

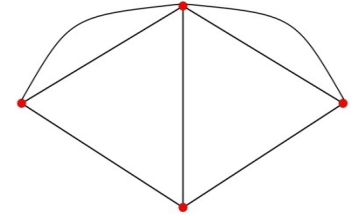


(b) Medium diversification



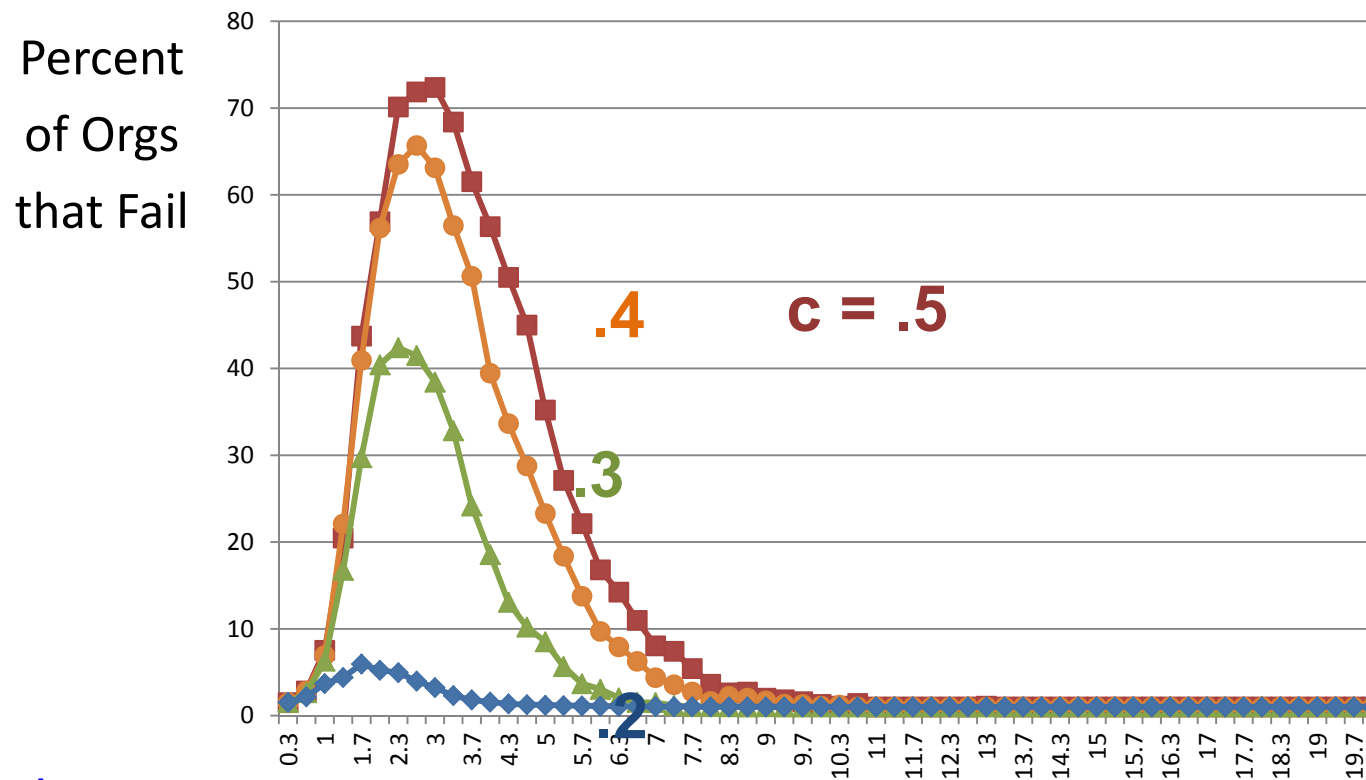
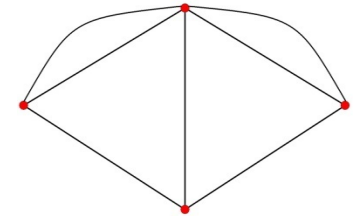
(c) High diversification

Diversification : Dangerous Middle Levels



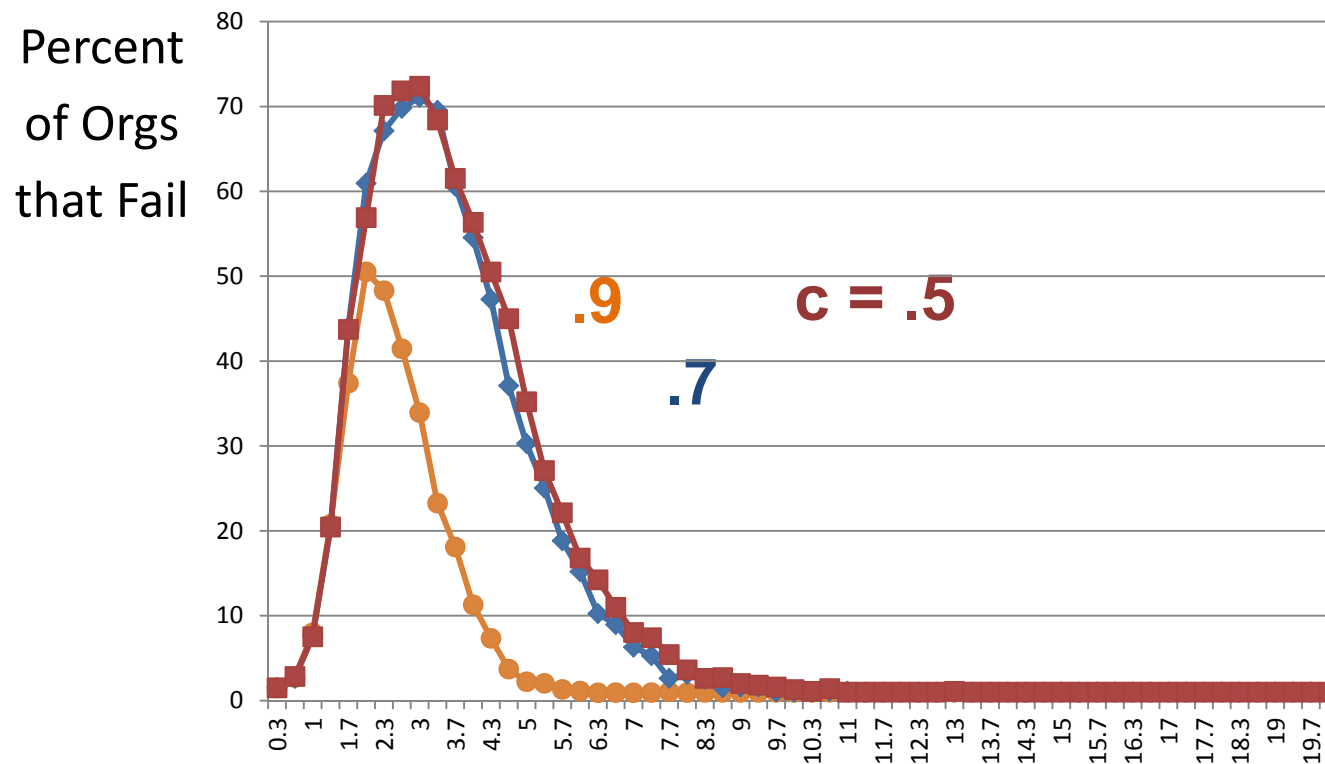
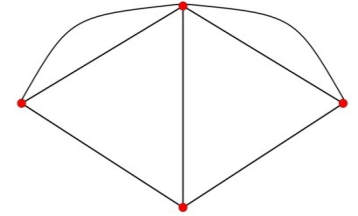
- Low diversification:
 - fragmented network, no widespread contagion
- Medium diversification
 - Connected network, contagion is possible
 - Exposure to only a few others makes it easy to spread
- High diversification
 - Little exposure to any single other organization
 - Failures do not spread

Integration: .93 Threshold



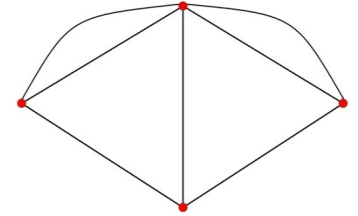
Degree: Expected # of cross-holdings

Integration: .93 Threshold



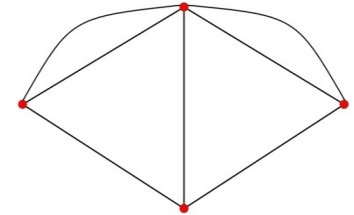
Degree: Expected # of cross-holdings

Integration



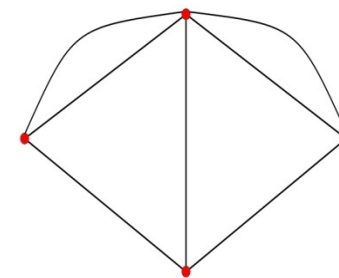
- Low integration: little exposure to others, failures don't trigger others
- Middle integration: exposure to others substantial enough to trigger contagion
- High integration: difficult to get a *first* failure – failure of own assets does not trigger failure

Analysis



- Analyze richer networks
 -
- Understand indirect holdings and how valuations/devaluations spread
- Understand effects of diversification, integration, size of shocks, correlation of shocks, heterogeneity in networks!...

Social and Economic Networks: Models and Analysis

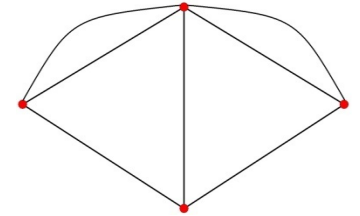


Matthew O. Jackson

**Stanford University,
Santa Fe Institute, CIFAR,
www.stanford.edu/~jacksonm**

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5.9: Diffusion Summary



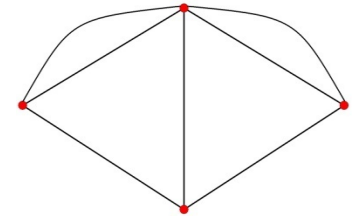
Lessons:



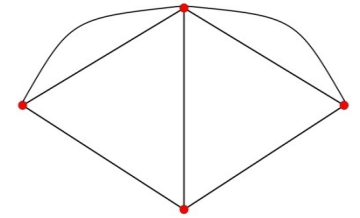
- Thresholds/“Phase Transitions”:
 - low density no contagion
 - middle density some probability of infection, part of population infected – reach most of population even with average degree around 3...
 - high density sure infection and all infected
- Degree affects who is infected and when

General Points

- Diffusion modeling
 - Important to model both information and peer effects:
 - Not simply an infection model: nonparticipants communicate - Distinguishes such models from epidemiology
- Need more studies that identify the details of what matters in interactions: information, learning, complementarities/substitution, peer pressure, ...

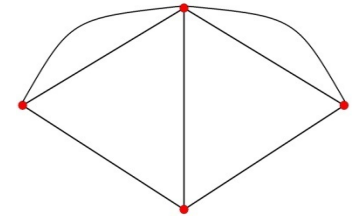


Diffusion



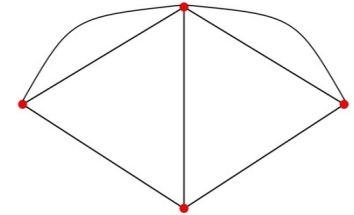
- Network structure matters
- Tractable, and simulations can go a long way to offering predictions
- experiment with changes in network structure...

Outline



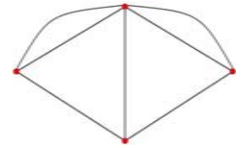
- Part I: Background and Fundamentals
 - Definitions and Characteristics of Networks (1,2)
 - Empirical Background (3)
- Part II: Network Formation
 - Random Network Models (4,5)
 - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
 - Diffusion and **Learning** (7,8)
 - Games on Networks (9)

Week 5 Wrap



- Adoption curves: s-shapes of diffusion
- S-shape: combination of imitation/complementarities and eventual saturation
- Initial contagion:
 - Depends on density and variance: high degree nodes serve as hubs and enable diffusion
- Extent of diffusion
 - relates to component structure, density – beyond one friend, (homophily...)
- Diffusion modeling
 - Can help dissect peer effects
 - underlies many relations: sheds light on financial contagions...

Week 5: References in order mentioned



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