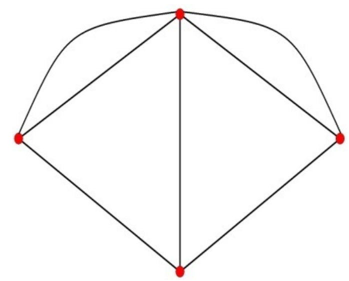
Social and Economic Networks: Models and Analysis



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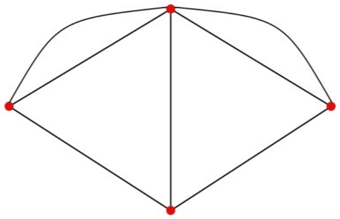
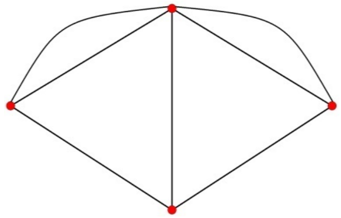
6.1: Learning

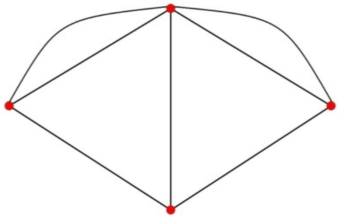


Outline



- Part I: Background and Fundamentals
  - Definitions and Characteristics of Networks (1,2)
  - Empirical Background (3)
- Part II: Network Formation
  - Random Network Models (4,5)
  - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
  - Diffusion and Learning (7,8)
  - Games on Networks (9)

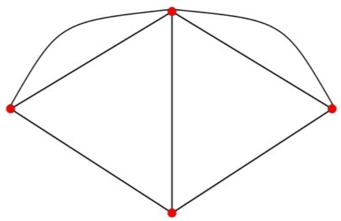
Outline



- Bayesian learning
  - repeated actions, observe each other

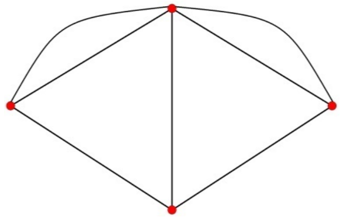
- DeGroot model
  - repeated communication, ``naïve'' updating

Bayesian Learning



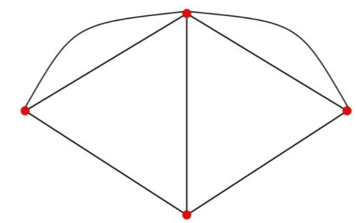
- Will society converge
- Will they aggregate information properly? …

Bala Goyal 98



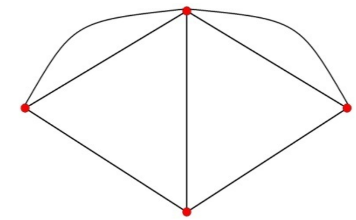
- n players in an undirected component g
- Choose action A or B each period
- A pays 1 for sure, B pays 2 with probability p and 0 with probability 1-p

Learning

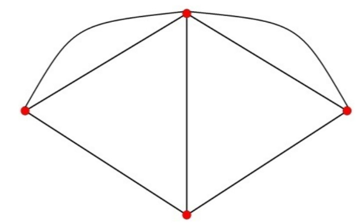


- Each period get a payoff based on choice
- Also observe neighbors' choices
- Maximize discounted stream of payoffs
  $E [\Sigma_t \delta^t \pi_{it}]$
- p is unknown takes on finite set of values

Challenges Bayesian Learning:

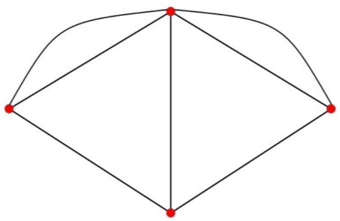


Proposition



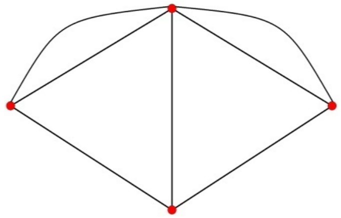
If p is not exactly 1/2, then with probability 1 there is a time such that all agents in a given component play just one action (and all play the same action) from that time onward

Sketch of Proof



- Suppose contrary
- Some agent in some component plays B infinitely often
- That agent will converge to true belief by the law of large numbers
- Must be that belief converges to $p > 1/2$, or that agent would stop playing B

Proof continued

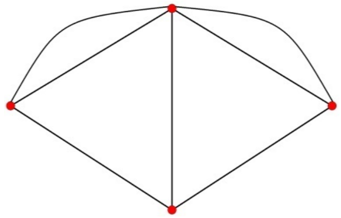


- With probability 1, all agents who see B played infinitely often converge to a belief that B pays 2 with prob p>1/2

- Neighbors of agent must play B, after some time, and so forth

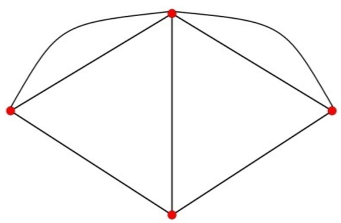
- All agents must play B from some time on

Play the right action?



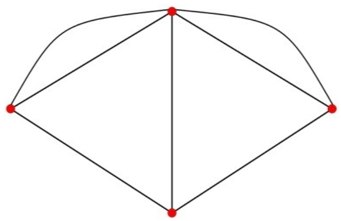
- If B is the right action then play the right action if converge to it, but might not
- If A is the right action, then must converge to right action

Probability of Converging to ``correct'' action



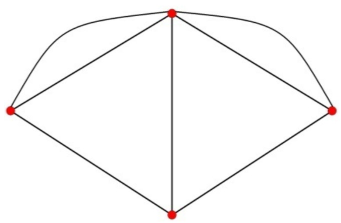
- Arbitrarily high if each action has some agent who initially has arbitrarily high prior that the action is the best one

Conclusions



- Consensus action chosen
- Not necessarily consensus belief
- Speed of convergence?

Limitations



- Homogeneity of actions and payoffs across players
- What if heterogeneity?
- Repeated actions over time
- Stationarity
- Networks are not playing role here!

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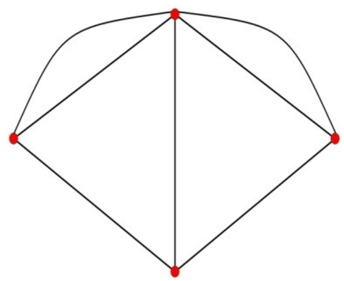

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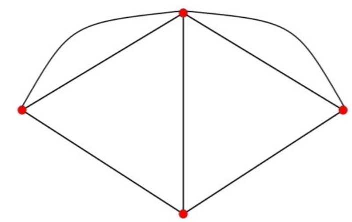
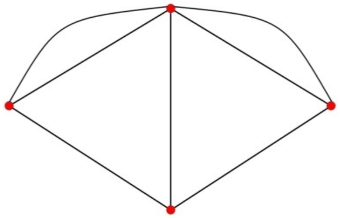
6.2:DeGroot Model



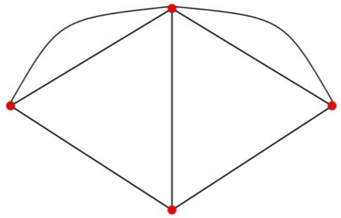
Outline



- Bayesian learning
  - repeated actions, observe each other

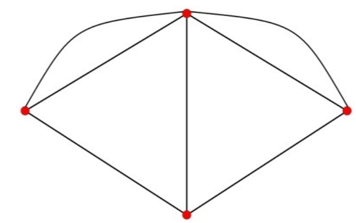
- DeGroot model
  - repeated communication, ``naïve'' updating

Outline: DeGroot Model



- Basic Definitions
- When is there convergence?
- When is there a consensus?
- Who has influence?
- When is the consensus accurate?

Network Structure and Learning



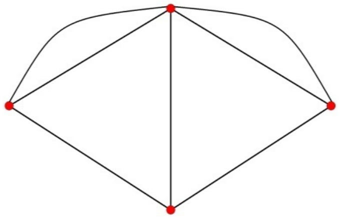
- Repeated communication

- Information comes only once

- See how information disseminates

- Who has influence, convergence speed, network structure impact…

Bounded Rationality Model

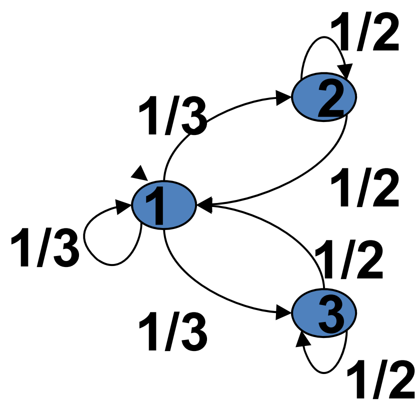


- Repeatedly average beliefs of self with neighbors
- Non-Bayesian if weights do not adjust over time
- Can under-weight neighbors (just as in experiments)

DeGroot (1974) Social Interaction Model

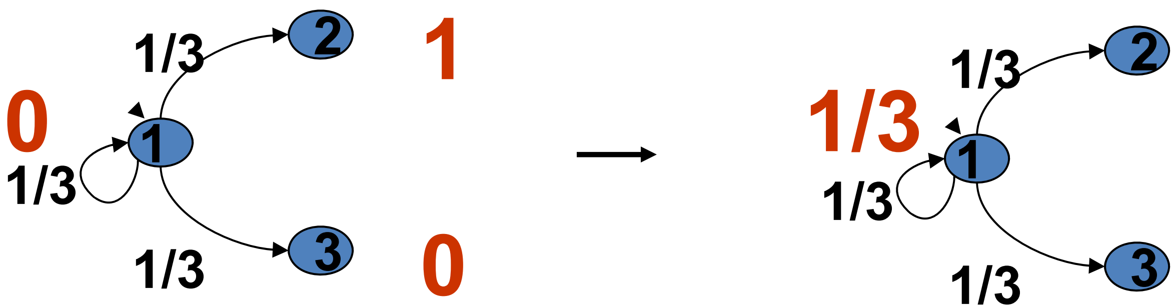
- Individuals {1, ... n}
- **T** weighted directed network, stochastic matrix
- Start with beliefs, attitude, etc. $b_i(0)$ in [0,1]
  - can also have these be vectors…
- Updating: $b_i(t) = \sum_j T_{ij}\, b_j(t-1)$

Example



$$T = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Updating



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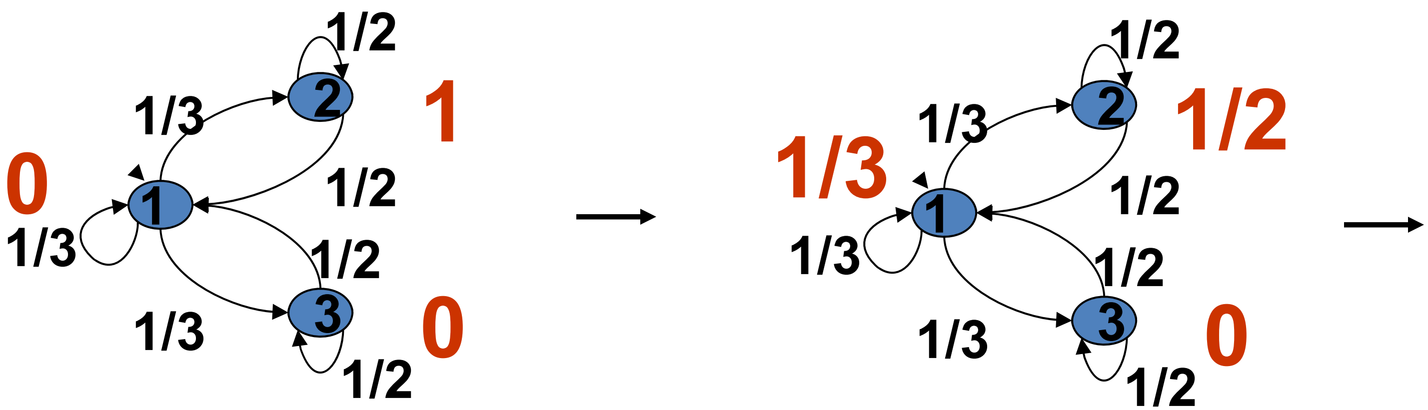

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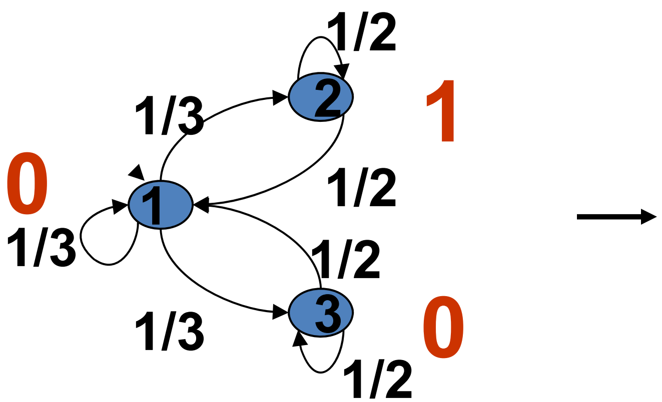

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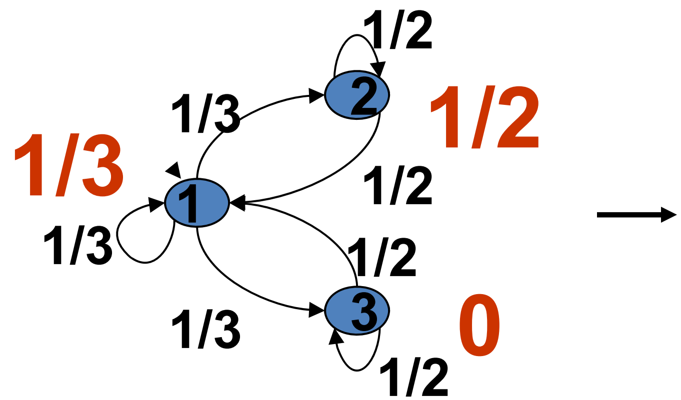

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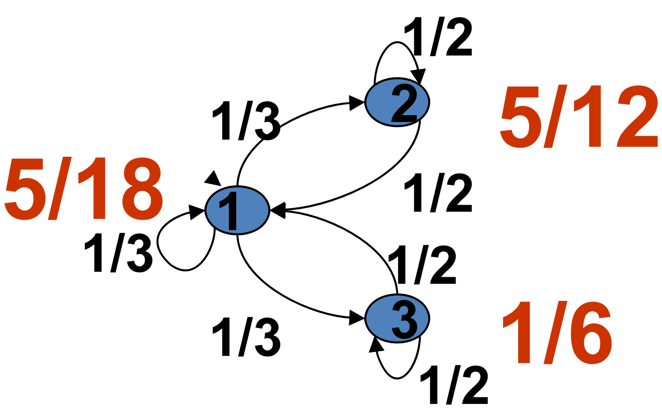

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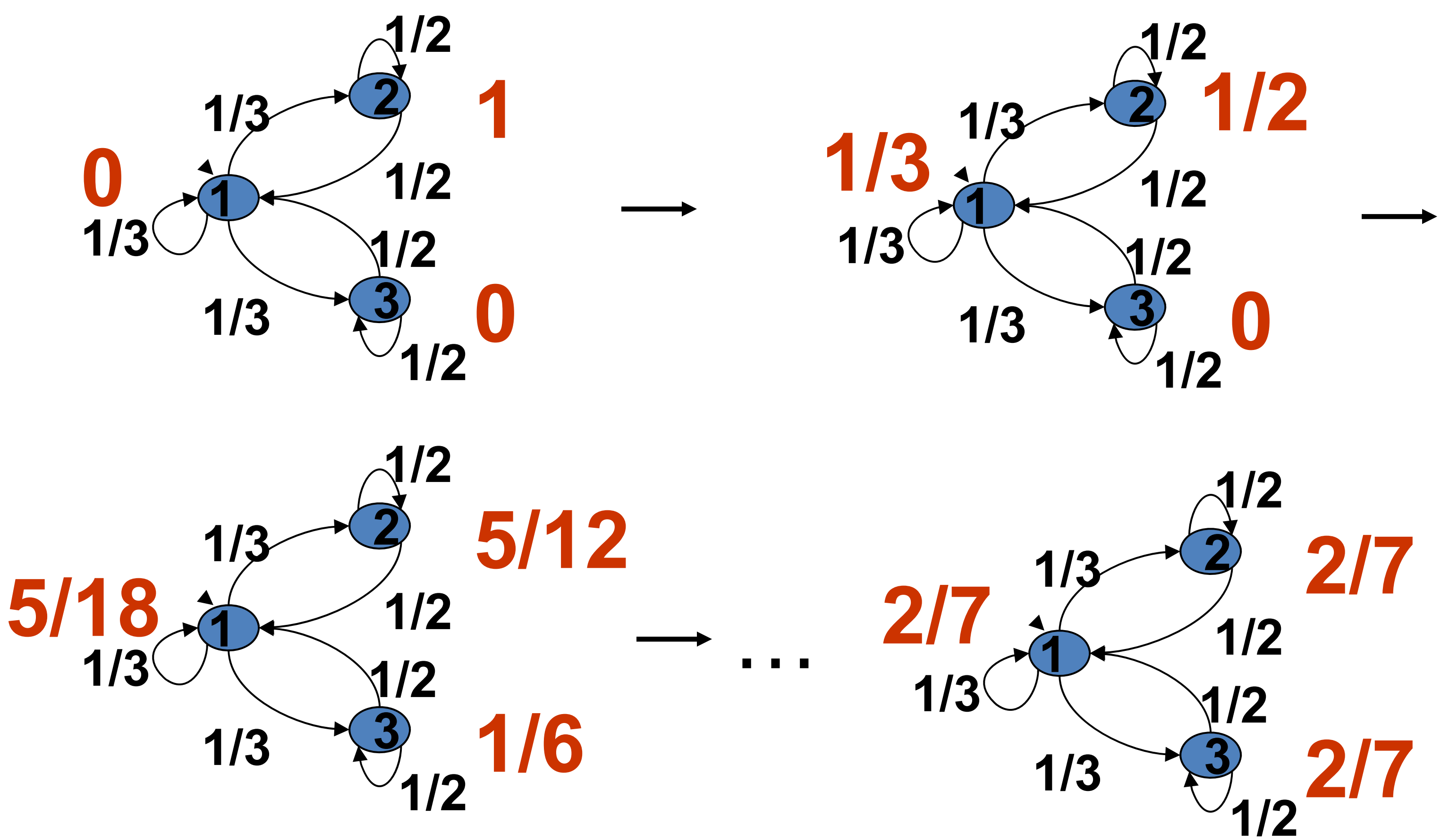
Updating



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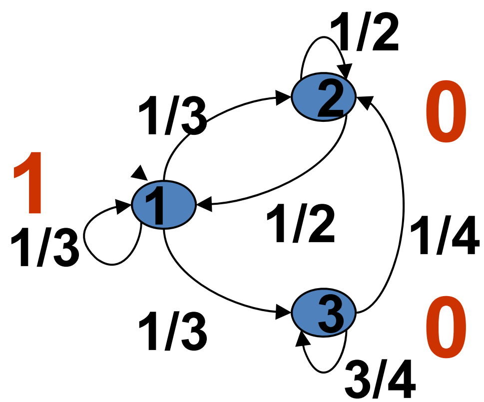
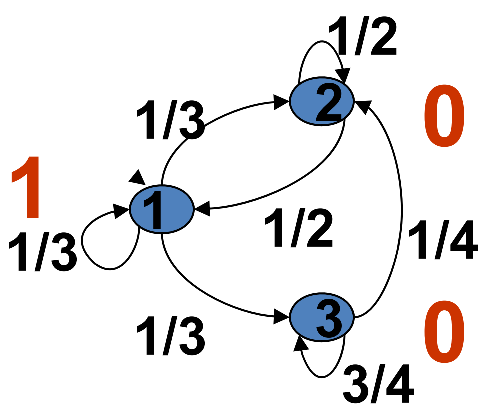
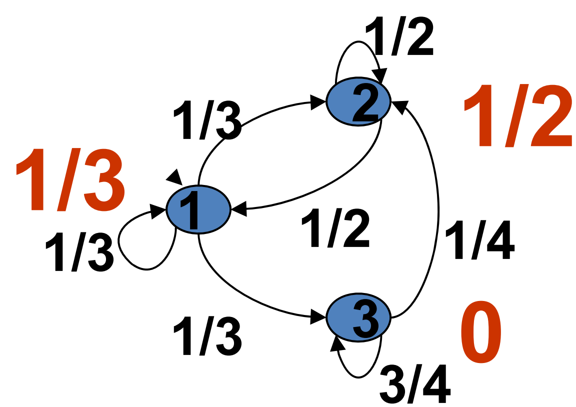
Example



$$T = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}$$

Updating





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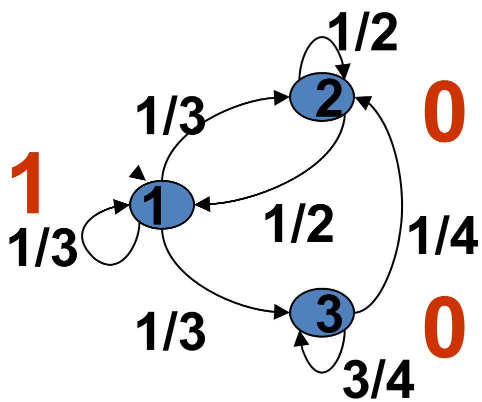
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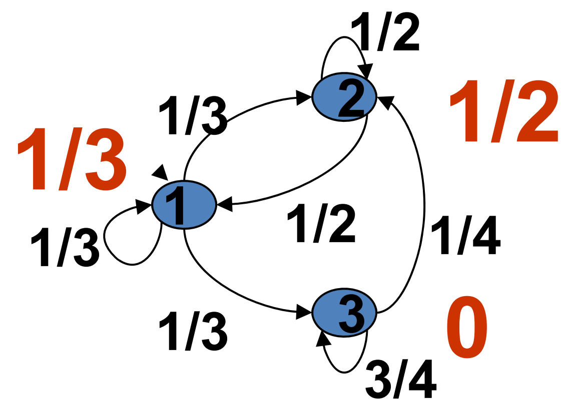
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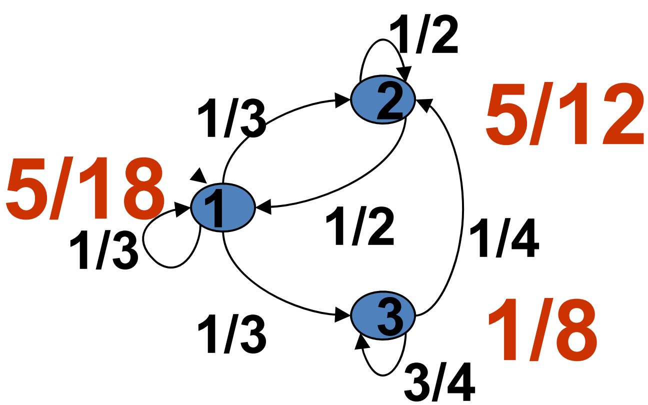
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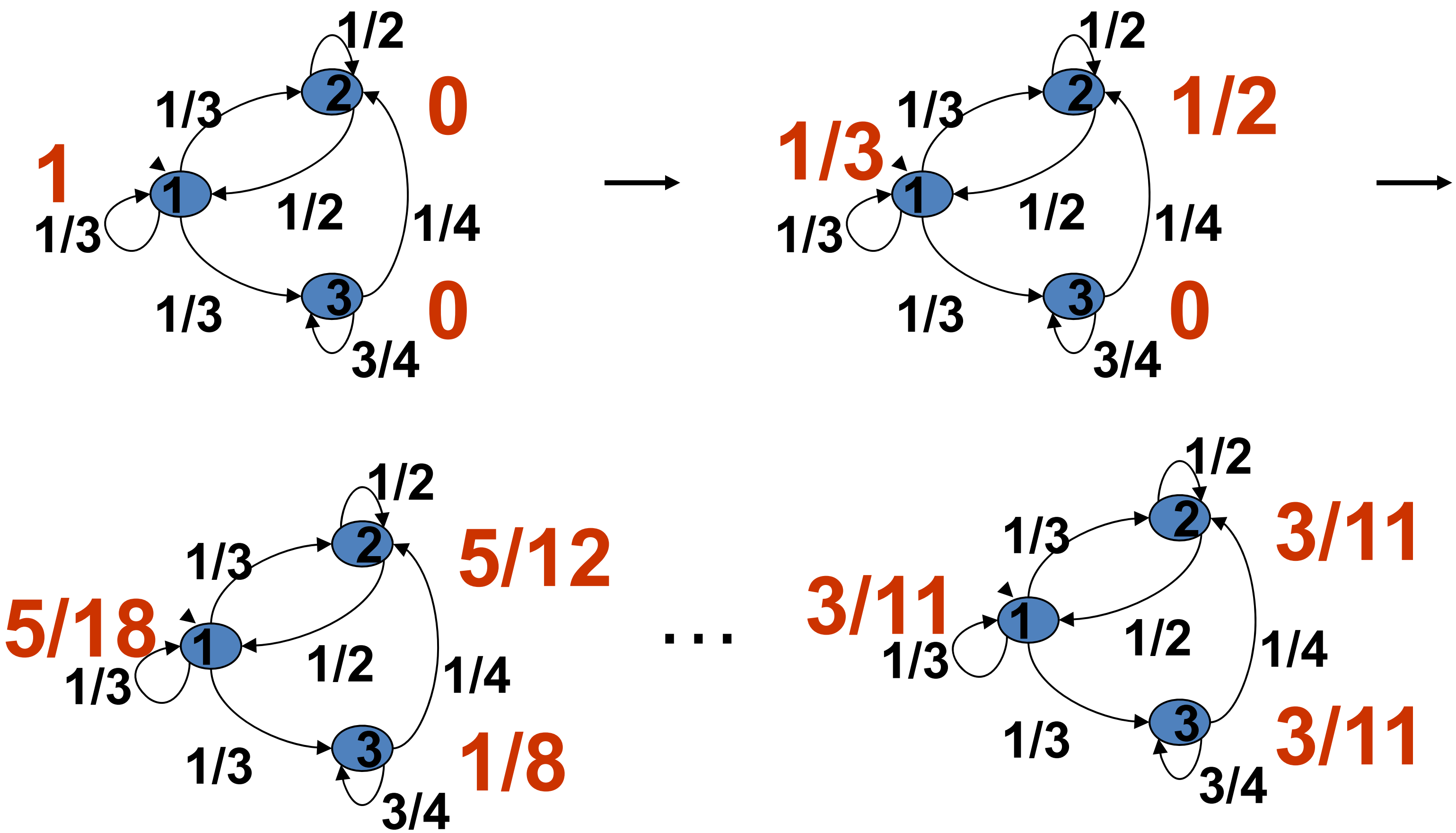
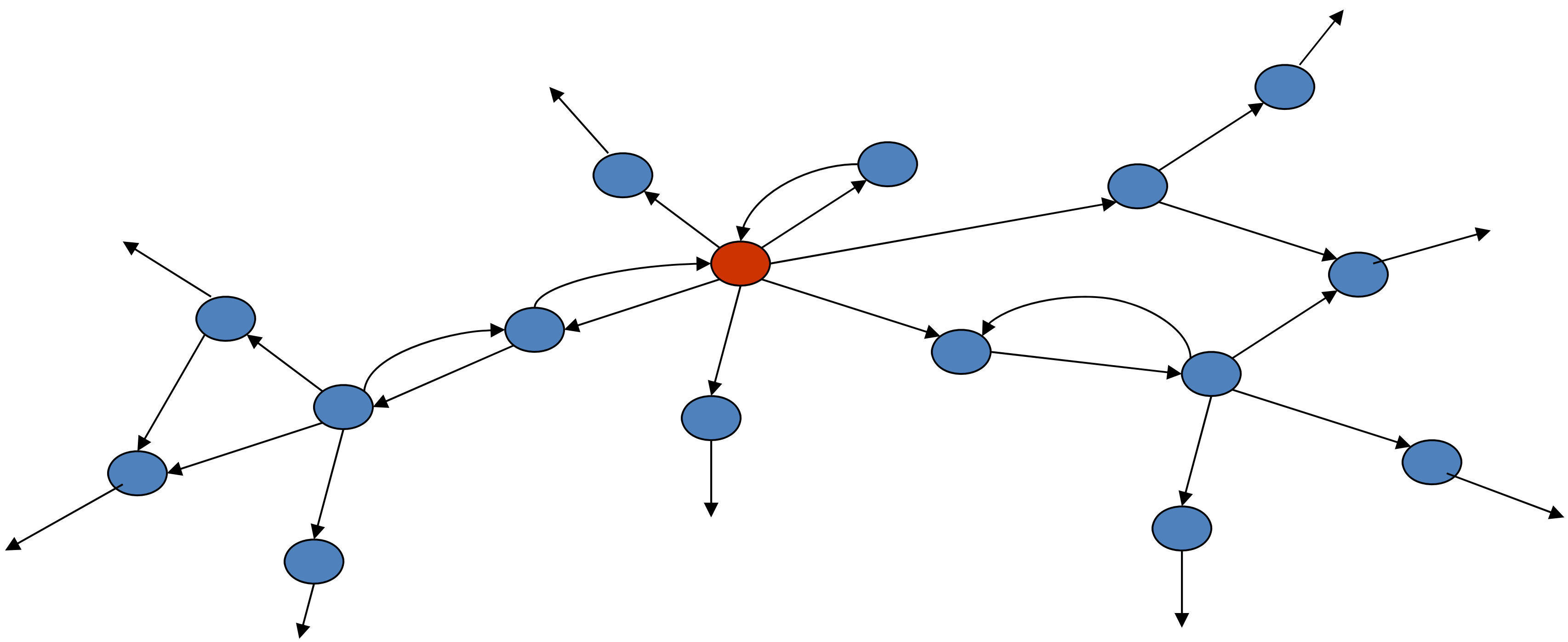
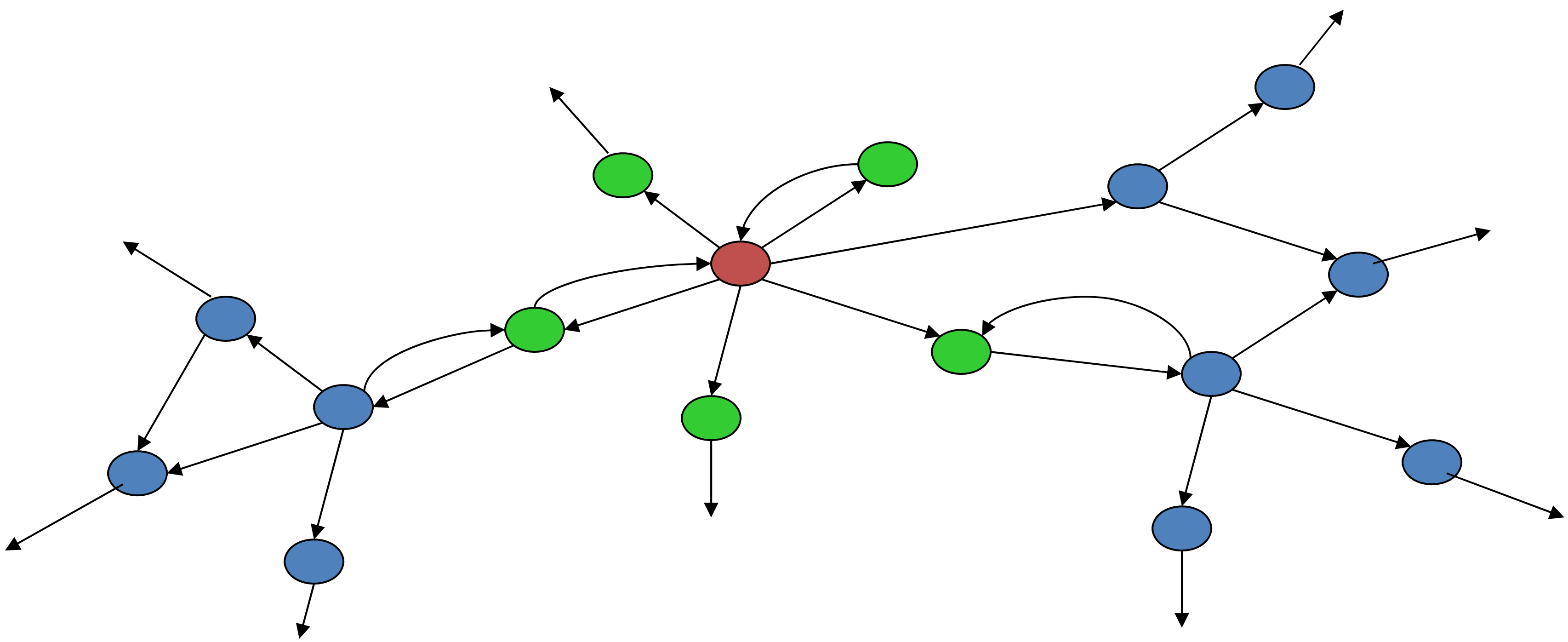
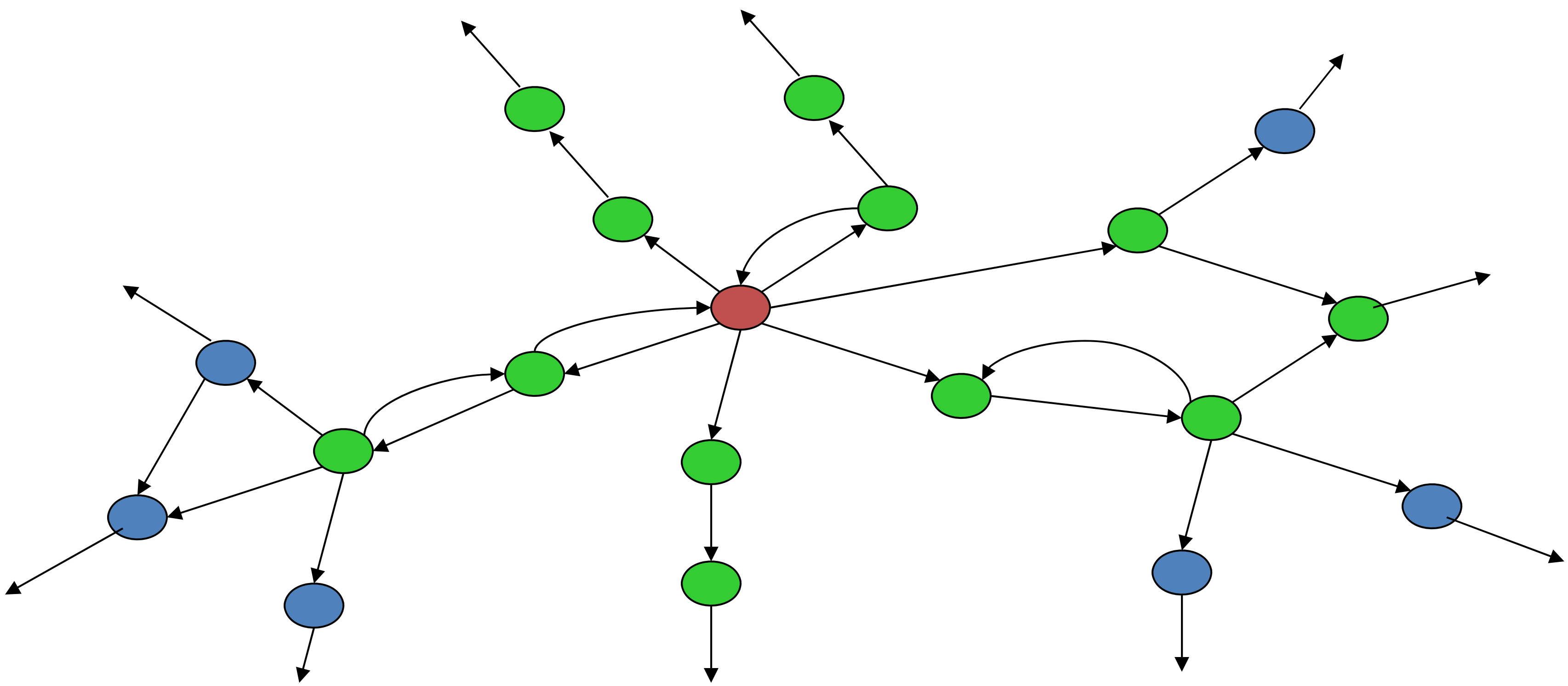
Updating

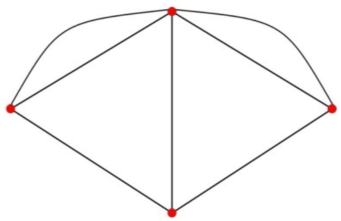








Other interpretations



- Social influence on actions
- Random actions (Markov process…)
- Page ranks…

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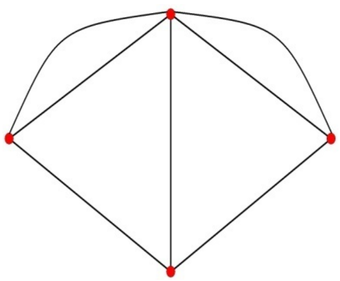

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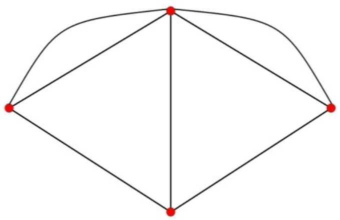
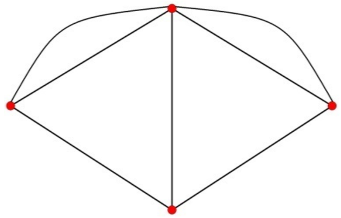
6.3: Convergence in DeGroot Model

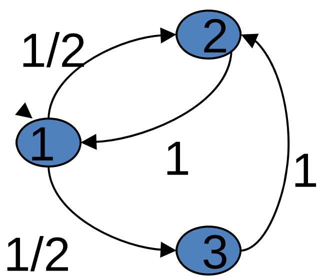


Outline: DeGroot Model



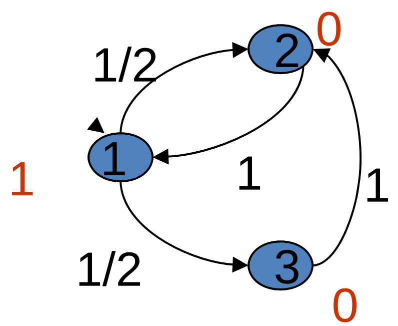
- Basic Definitions
- When is there convergence?
- When is there a consensus?
- Who has influence?
- When is the consensus accurate?

Example - Convergence:



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

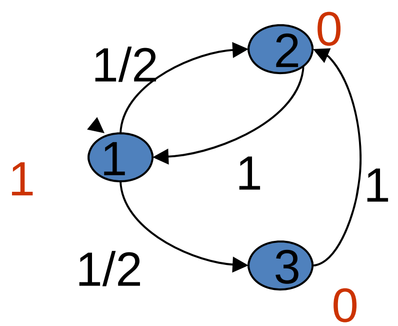
Example - Convergence:



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

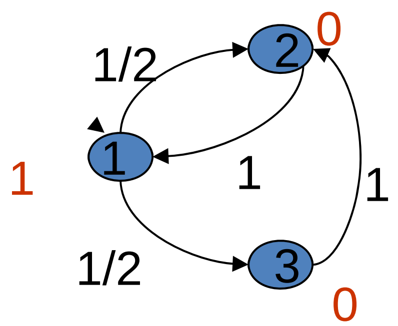
Example - Convergence:



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

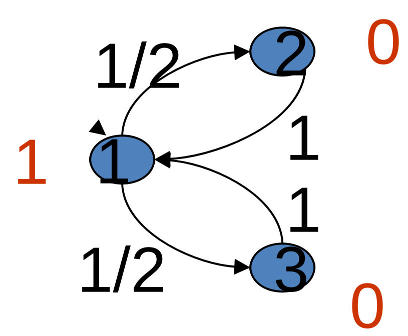
Example - Convergence:



$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad b(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3/4 \\ 1/2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/4 \\ 3/4 \\ 1/2 \end{pmatrix} \cdots \rightarrow \begin{pmatrix} 2/5 \\ 2/5 \\ 2/5 \end{pmatrix}$$

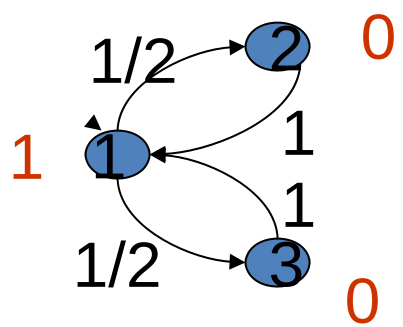
Example – No Convergence:



$$T=\begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

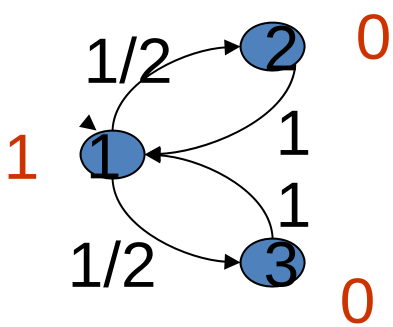
Example – No Convergence:



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

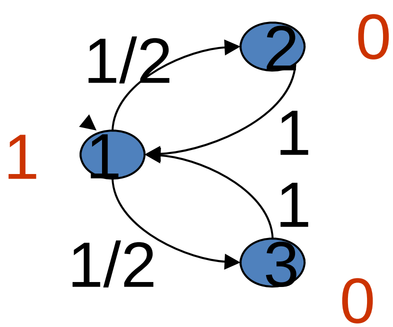
Example – No Convergence:



$$T=\begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Example – No Convergence:



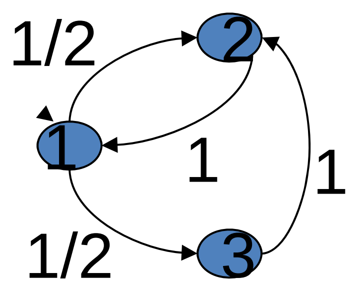
$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ldots \rightarrow$$

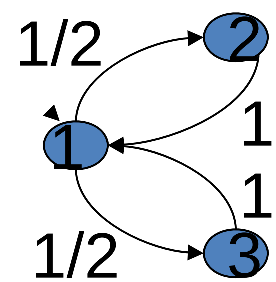
Convergence

- T *converges* if $\lim T^t b$ exists for all b

- T is *aperiodic* if the greatest common divisor of its cycle lengths is one



aperiodic



periodic

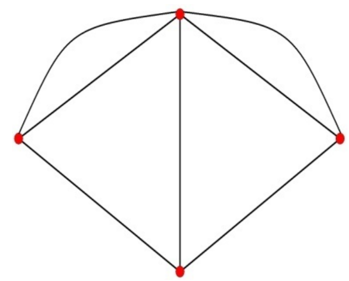
Theorem

Suppose T is strongly connected.

T is convergent if and only if it is aperiodic.

T is convergent if and only if: lim $T^t = (1,1,\ldots.,1)^T s$ where s is the unique lhs eigenvector with eigenvalue 1

Social and Economic Networks: Models and Analysis



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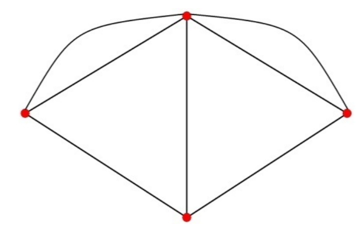
6.4: Proof of Convergence Theorem



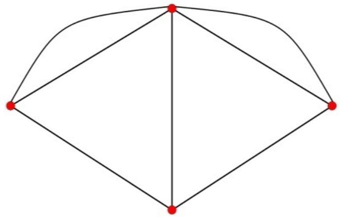
Theorem

Suppose T is strongly connected.

T is convergent if and only if it is aperiodic.

T is convergent if and only if: lim $T^t = (1,1,\ldots.,1)^T s$ where s is the unique lhs eigenvector with eigenvalue 1

Proof :



Defn: T *is primitive* if $T^t_{ij}>0$ for all ij after some t

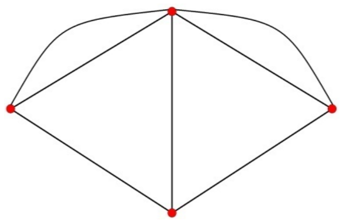
- If T is strongly connected and stochastic then it is aperiodic if and only if it is primitive. (Perkins (1961))

- If T is strongly connected and primitive then

$$\lim T^t = (1,1,\ldots,1)^T s$$

where s is the unique lhs eigenvector with eigenvalue 1 (e.g., see Meyer (2000))

Proof Cont'd

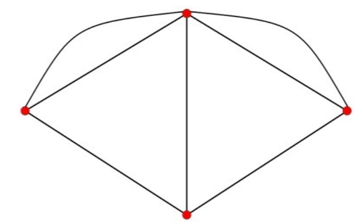


- So, strongly connected and aperiodic implies convergence.

- Converse comes from showing:

If T is strongly connected, stochastic and convergent, then it is primitive.

Proof Cont'd



- Show:

If T is strongly connected, stochastic and convergent, then it is primitive.

Let $S = \lim T^t$ by convergence

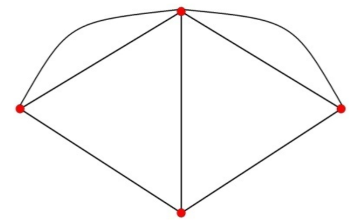
Then $ST = \lim T^t T = S$

So each row is a lhs eigenvector with eigenvalue 1: it is a positive vector by Perron-Frobenius theorem (An eigenvector of an irreducible non-negative matrix is strictly positive *if* (and only if) it is associated with its largest eigenvalue. This vector is unique if the matrix is primitive)

So since S is all positive, T is primitive.

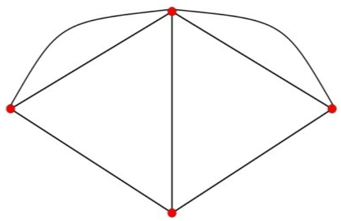
Since, T is primitive then Perron-Frobenius implies the eigenvector is unique, and all rows of S are the same s

Aperiodicity Easy



- Aperiodicity is easy to satisfy:
  - Have some agent weight him or herself
  - Or have at least one communicating dyad and a transitive triple…

Outline: DeGroot Model



- Basic Definitions
- When is there convergence?
- When is there a consensus? - Just answered: convergence is sufficient, aperiodicity (see G&J11 for details more generally)
- Who has influence?
- When is the consensus accurate?

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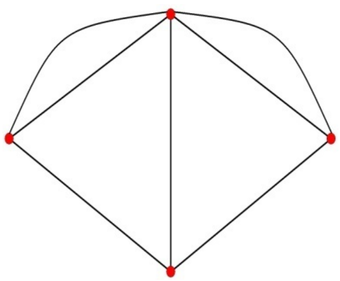

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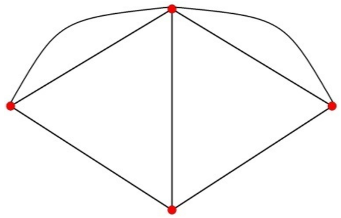
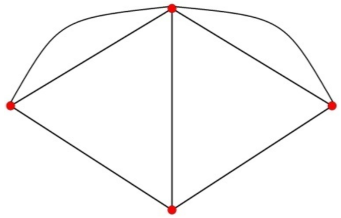
6.5: Influence

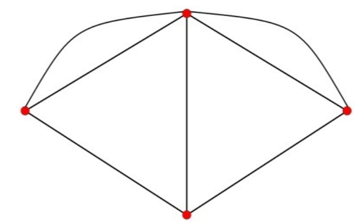


Outline: DeGroot Model



- Basic Definitions
- When is there convergence?
- When is there a consensus?
- Who has influence?
- When is the consensus accurate?

Consensus



- Converge to (normalized) eigenvector weighted sum of original beliefs.

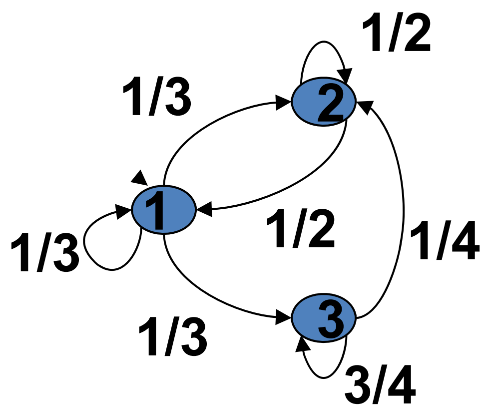
Consensus

$$T = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad T^2 = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T^3 = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix} \qquad T^4 = \begin{pmatrix} 1/4 & 1/2 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

$$T^5 = \begin{pmatrix} 1/2 & 3/8 & 1/8 \\ 1/4 & 1/2 & 1/4 \\ 1/2 & 1/4 & 1/4 \end{pmatrix} \qquad T^\infty = \begin{pmatrix} 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 \end{pmatrix}$$

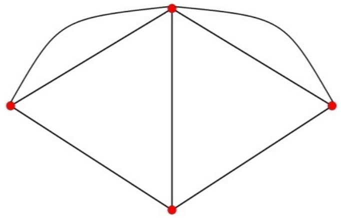
Consensus



$$\lim_t \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}^{t} \mathbf{b(0)}$$

$$\lim_t \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}^{t} = \begin{pmatrix} 3/11 & 4/11 & 4/11 \\ 3/11 & 4/11 & 4/11 \\ 3/11 & 4/11 & 4/11 \end{pmatrix}$$

What are Limiting beliefs?



- When group reaches a consensus, what is it?

- Who are the influential agents in terms of steering the limiting belief?

- Must be that the rows of $\mathbf{T}^t$ converge to same thing since beliefs converge to same thing for all initial vectors

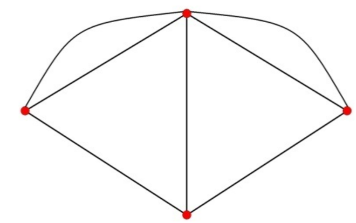
Influence:

$$\text{Limit} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}^{t} \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{3/11} \\ \mathbf{3/11} \\ \mathbf{3/11} \end{pmatrix}$$

$$\text{Limit} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}^{t} \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{4/11} \\ \mathbf{4/11} \\ \mathbf{4/11} \end{pmatrix}$$

$$\text{Limit} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}^{t} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{4/11} \\ \mathbf{4/11} \\ \mathbf{4/11} \end{pmatrix}$$

Influence Measure



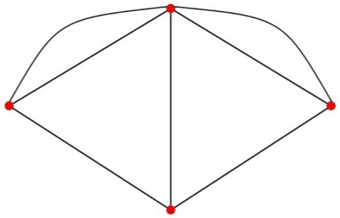
- What do rows of $\mathbf{T}^t$ converge to?

- Look for a row vector **s** indicating the *relative influence* each agent has – *limit belief* is **s b**

- Note that **s b** = **s T b**

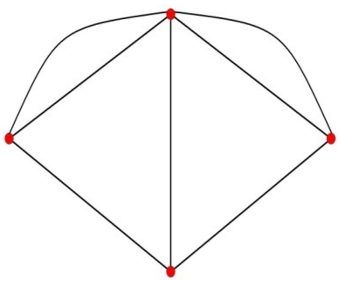
- So, **s** = **s T** : **s** is the **left unit eigenvector**

Who has Influence



- $s_i = \sum_j T_{ji}\, s_j$
- High influence from being paid attention to by people with high influence...
- Power measures, Google Page Ranks
- Related to eigenvector centrality…

Social and Economic Networks: Models and Analysis



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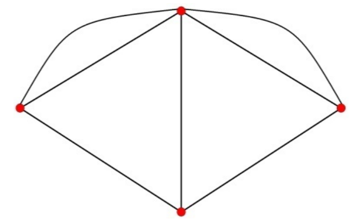
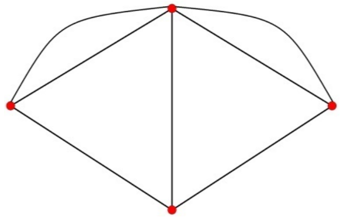
6.6: Examples of Influence

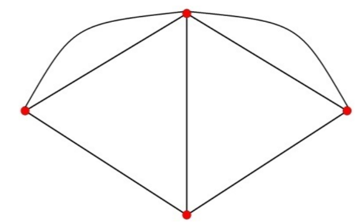


Outline: DeGroot Model



- Basic Definitions
- When is there convergence?
- When is there a consensus?
- Who has influence?
- When is the consensus accurate?

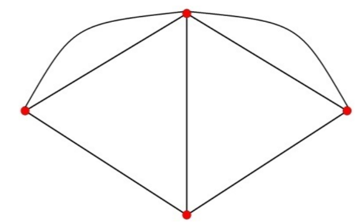
Stubborn Agents



- An agent who places high weight on self will maintain belief while others converge to that agent's belief

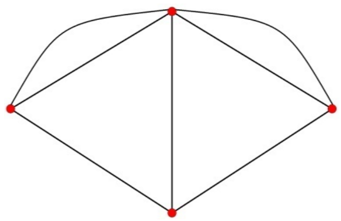
- Groups that are highly introspective will have substantial influence.

Another Example: Influence



- Suppose equally weight connections
- Suppose also that $T_{ij}>0$ if and only if $T_{ji}>0$
- d_i is i's out degree
- So, $T_{ij}=1/d_i$ for each i and j that i has a (directed) link to: so weight friends and weight them all equally

Example: Influence



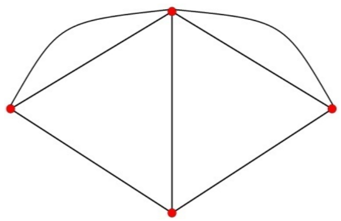
Let $D = \sum_k d_k$

Claim: $s_i = d_i / D$ for each i

- Recall s is unit eigenvector: $s_i = \sum_j T_{ji} s_j$

- Verify that $s_i = \sum_j T_{ji} s_j$

Example: Influence



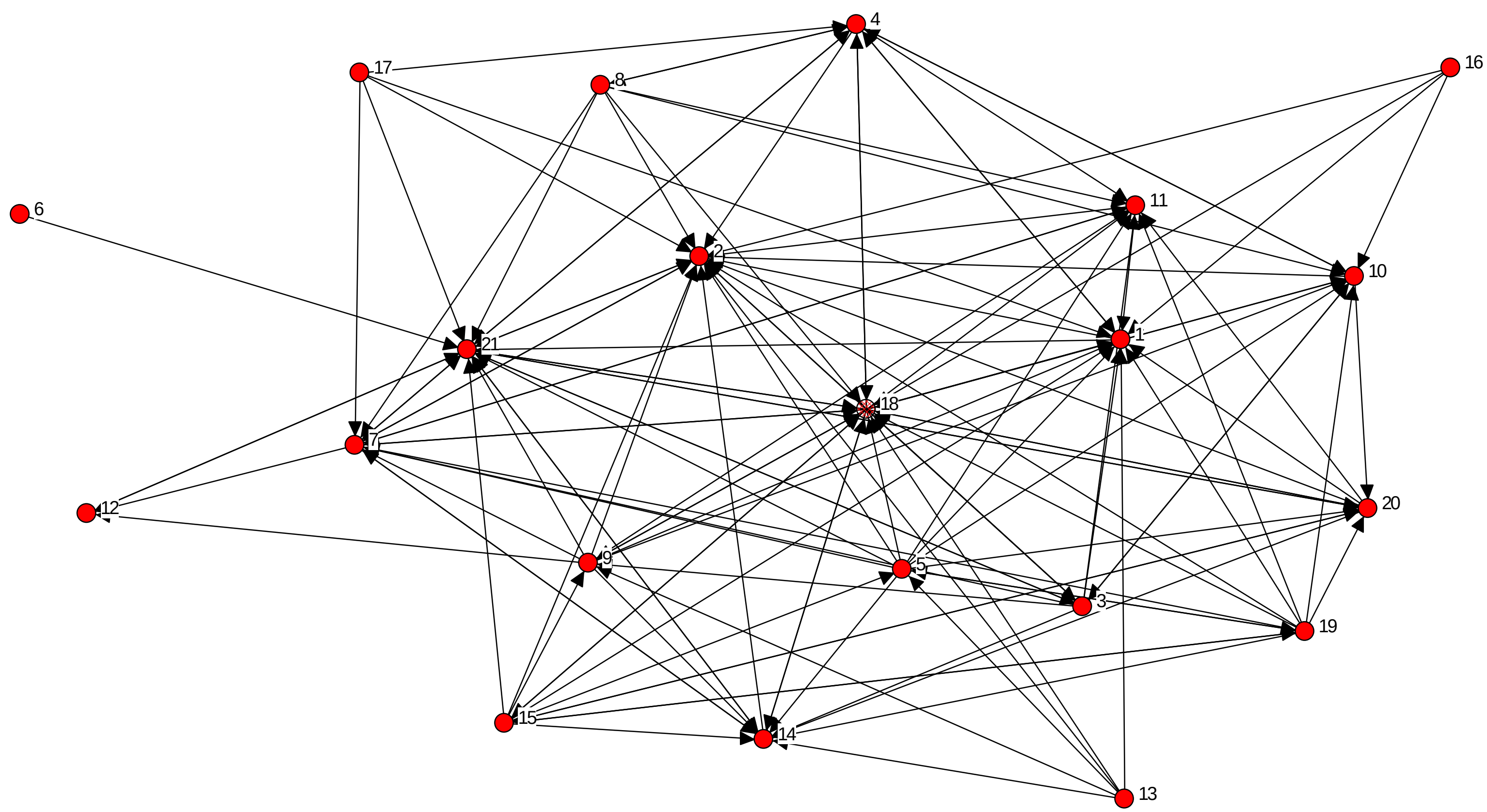
Claim: $s_i = d_i / D$ for each i

- Recall s is unit eigenvector: $s_i = \sum_j T_{ji} s_j$

- Verify that $s_i = \sum_j T_{ji} s_j$

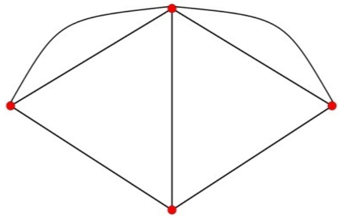
- $s_i = \sum_j T_{ji} s_j = \sum_{j:\, Tij>0} (1/d_j)\, d_j / D = d_i/D$

Krackardt’s (1987) advice network:



label	s	level	dept.	age	tenure
1	0.048	3	4	33	9.3
2	0.132	2	4	42	19.6
3	0.039	3	2	40	12.8
4	0.052	3	4	33	7.5
5	0.002	3	2	32	3.3
6	0.000	3	1	59	28
7	0.143	1	0	55	30
8	0.007	3	1	34	11.3
9	0.015	3	2	62	5.4
10	0.024	3	3	37	9.3
11	0.053	3	3	46	27
12	0.051	3	1	34	8.9
13	0.000	3	2	48	0.3
14	0.071	2	2	43	10.4
15	0.015	3	2	40	8.4
16	0.000	3	4	27	4.7
17	0.000	3	1	30	12.4
18	0.106	2	3	33	9.1
19	0.002	3	2	32	4.8
20	0.041	3	2	38	11.7
21	0.201	2	1	36	12.5

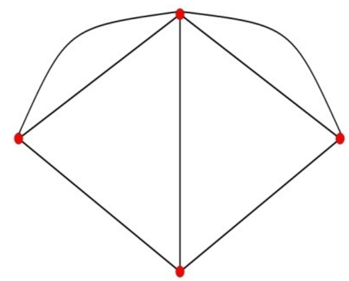
Influence



- Provides foundation for eigenvector-based centrality or power measures

- Saw relation of eigenvector to walks - g^t gives measure of all walks of length t...

Social and Economic Networks: Models and Analysis



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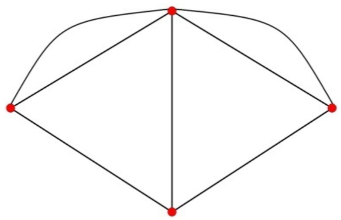
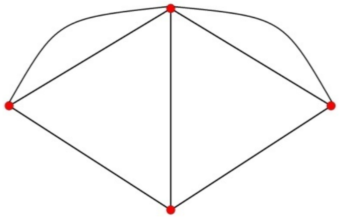
6.7: Information Aggregation

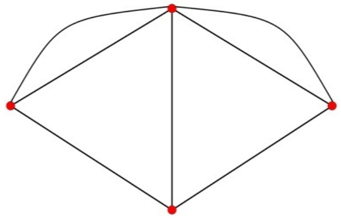


Outline: DeGroot Model



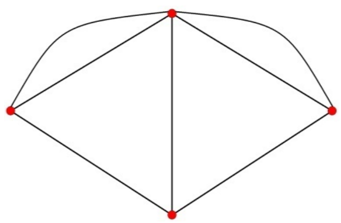
- Basic Definitions
- When is there convergence?
- When is there a consensus?
- Who has influence?
- When is the consensus accurate?

When is Information Aggregation Accurate:



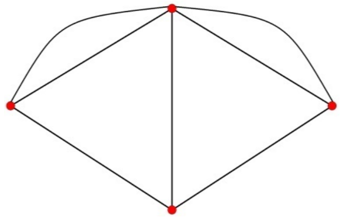
- How does this depend on network structure?
- How does it depend on influence?
- How does it relate to speed of convergence

Uncertainty Structure



- Suppose true state is μ
- Agent i sees $b_i(0) = \mu + \varepsilon_i$
- ε_i has 0 mean and finite variance, bounded below and above,
- signal distributions may differ across agents, but are independent conditional on μ

Wise Crowds



- Consider large societies

- If they pooled their information, they would have an accurate estimate of μ

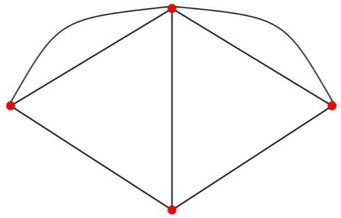
- For what sequences of societies indexed by n does

  Prob $\lim_t [\ |\ b_j^n(t) - \mu | > \delta\ ] \rightarrow_n 0$ for all δ, j?

A Weak Law of Large Numbers:

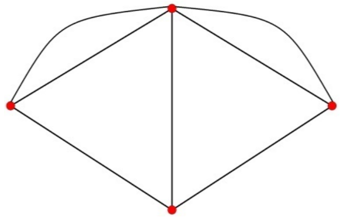
Let ε_i's be independent, zero mean, and each have finite variance (bounded below). Then:

$$\text{plim} \sum s_i^n \varepsilon_i = 0 \qquad \textit{iff} \qquad \max_i s_i^n \to 0$$



Wise crowds iff max influence vanishes

A Weak Law of Large Numbers:



Let ε_i's be independent, zero mean, and each have finite variance (bounded below). Then:

$$\text{plim} \sum s_i^n \varepsilon_i = 0 \qquad \textit{iff} \qquad \max_i s_i^n \to 0$$

Wise crowds iff max influence vanishes: recall that

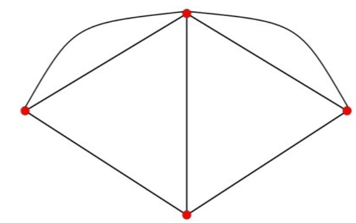
$$\lim_t b_j^n(t) = \sum s_i^n b_i^n(0)$$

$$= \sum s_i^n (\mu + \varepsilon_i)$$

$$= \mu + \sum s_j^n \varepsilon_j$$

So: $\text{plim}(\lim_t b_j^n(t)) = \mu$ iff $\text{plim}(\sum s_j^n \varepsilon_j) = 0$, iff $\max_i s_i^n \to 0$

Reciprocal Attention:

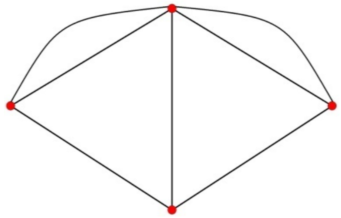


Suppose that T is column stochastic (so each agent receives weight one). Then s=(1/n,...1/n) is a unit lhs eigenvector, and so T is wise.

So, **reciprocal trust** implies wisdom.

But that is a very strong condition…

No Opinion Leaders

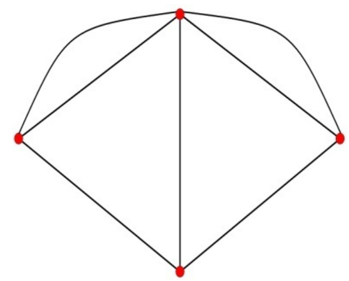


- $s_i = \sum_j T_{ji} s_j$

- If there is some i with $T_{ji} > a > 0$ for all j, then $s_i > a$

- So clearly cannot have too strong an ``opinion leader''

Social and Economic Networks: Models and Analysis



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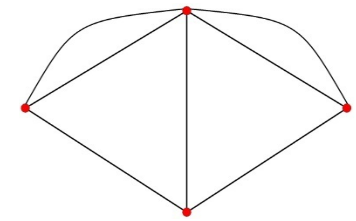
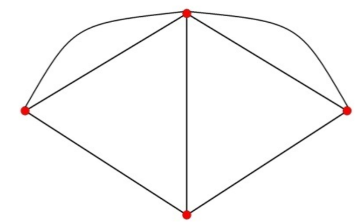
6.8: Learning Summary



Summary

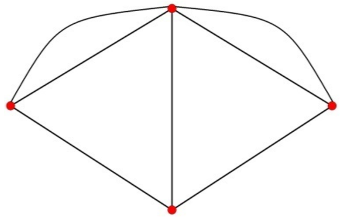


- **Convergence/Consensus** if and only if **aperiodicity**

- **Limiting influence** related to eigenvectors and weights from **influential neighbors**

- **Wise crowds**: nobody retains too much influence

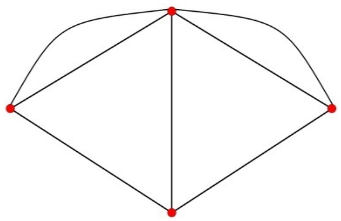
Learning Models



- Bayesian is computationally demanding in network settings
- Restricted Bayesian gives consensus

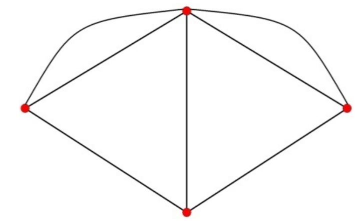
  network not much of a role
- DeGroot and other myopic models bring network into play
- Can reach consensus, can be wise
- Influence and speed are tractable…

To do list:



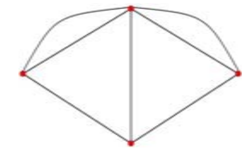
- Between myopic and rational?
- Richer settings with strategic considerations (political…)
- Translate social structure to learning conclusions: homophily, etc.

Week 6 Wrap



- Rational/Bayesian learning: complex but leads to consensus actions if: homogeneous, repeated observations, stationary
- Network Structure: DeGroot model
  - tractable repeated discussions
  - eventual consensus for many structures (speed depends on homophily)
  - influence depends on how much listened to – rationalizes eigenvector-style centrality measures
  - accurate beliefs depend on balance

Week 6: References in order mentioned



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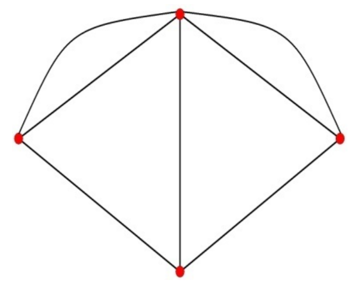

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