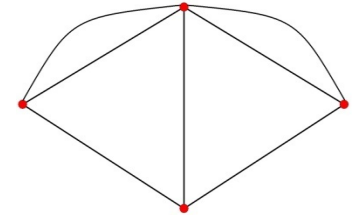


7.1: Games on Networks



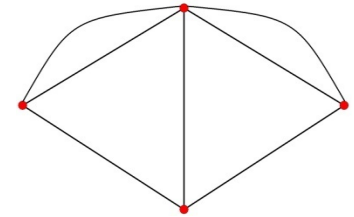
Outline



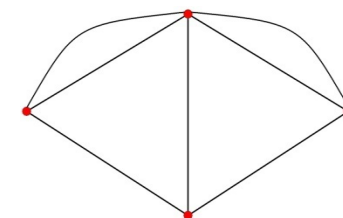
- Part I: Background and Fundamentals
 - Definitions and Characteristics of Networks (1,2)
 - Empirical Background (3)
- Part II: Network Formation
 - Random Network Models (4,5)
 - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
 - Diffusion and Learning (7,8)
 - Games on Networks (9)

Games on Networks

- Decisions to be made
 - not just diffusion
 - not just updating
- Complementarities...
- “Strategic” Interplay
 - Inter-dependencies



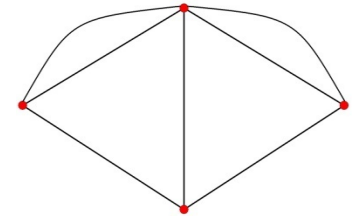
Games on Networks - Outline



- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure
- Equilibrium response to network structure

Games on Networks

- Players on a network - explicitly modeled...
- Care about actions of neighbors
- Early literature: How complex is the computation of equilibrium in worse case games?
- Second branch: what can we say about behavior and how it relates to network structure



Start with a Canonical Special Case:



- Each player chooses action x_i in $\{0,1\}$
- payoff will depend on
 - how many neighbors choose each action
 - how many neighbors a player has

Definitions



- Each player chooses action x_i in $\{0,1\}$
- Consider cases where i 's payoff is

$$u_{d_i}(x_i, m_{N_i})$$

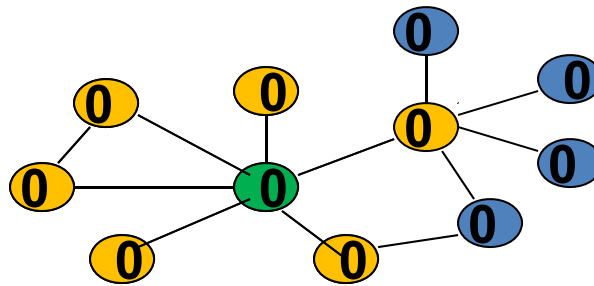
depends only on $d_i(g)$ and $m_{N_i(g)}$ - the number of neighbors of i choosing 1

Example: Simple Complement



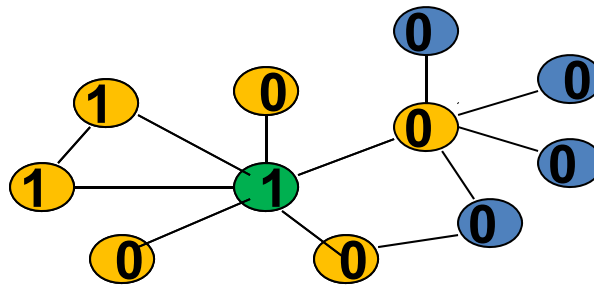
- agent i is willing to choose 1 if and only if at least t neighbors do:
- Payoff action 0: $u_{d_i}(0, m_{N_i}) = 0$
- Payoff action 1: $u_{d_i}(1, m_{N_i}) = -t + m_{N_i}$

Example:



- An agent is willing to take action 1 if and only if at least two neighbors do

Example:



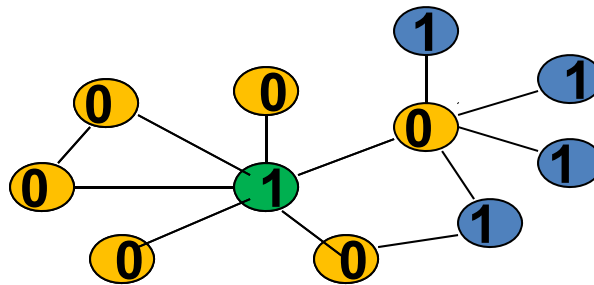
- An agent is willing to take action 1 if and only if at least two neighbors do

Example: Best Shot



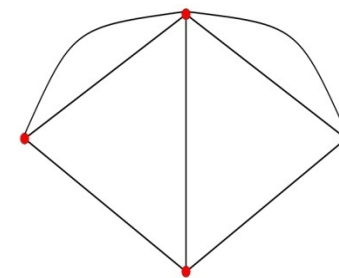
- agent i is willing to choose 1 if and only if no neighbors do:
- Payoff action 0:
$$u_{d_i}(0, m_{N_i}) = \begin{cases} 1 & \text{if } m_{N_i} > 0 \\ 0 & \text{if } m_{N_i} = 0 \end{cases}$$
- Payoff action 1:
$$u_{d_i}(1, m_{N_i}) = 1 - c$$

Another Example: Best Shot Public Goods



- An agent is willing to take action 1 if and only if no neighbors do

Social and Economic Networks: Models and Analysis

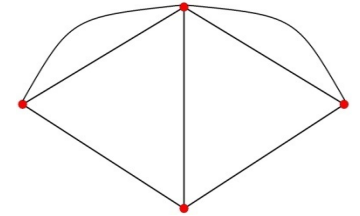


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7.2: Complements and Substitutes



Games on Networks - Outline



- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure
- Equilibrium response to network structure

Complements/Substitutes

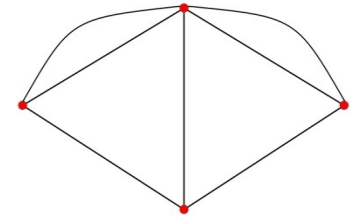
- strategic **complements** -- for all $d, m \geq m'$
 - Increasing differences:
 $u_d(1, m) - u_d(0, m) \geq u_d(1, m') - u_d(0, m')$
- strategic **substitutes** -- for all $d, m \geq m'$
 - Decreasing differences:
 $u_d(1, m) - u_d(0, m) \leq u_d(1, m') - u_d(0, m')$

Externalities:



- Others' behaviors affect my **utility/welfare**
- Others' behaviors affect my ***decisions, actions, consumptions, opinions...***
 - others' actions affect the ***relative*** payoffs to my behaviors

(Strategic) Complements/Substitutes



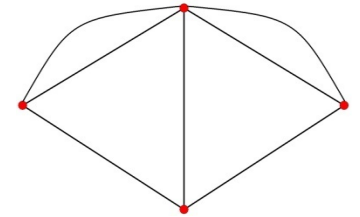
- **Complements:** Choice to take an action by my friends increases my relative payoff to taking that action (e.g., friend learns to play a video game)
- **Substitutes:** Choice to take an action by my friends decreases my relative payoff to taking that action (e.g., roommate buys a stereo/fridge)

Examples

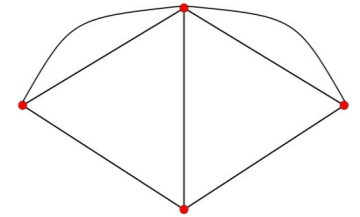
- Complements:
 - education decisions
 - care about number of neighbors, access to jobs, etc. – invest if at least k neighbors do
 - smoking & other behavior among teens, peers, ...
 - technology adoption – how many others are compatible...
 - learn a language, ...
 - cheating, doping
- Substitutes
 - information gathering
 - e.g., payoff of 1 if anyone in neighborhood is informed, cost to being informed ($c < 1$)
 - local public goods (shareable products...)
 - competing firms (oligopoly with local markets)
 - ...

Games on Networks - Outline

- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure
- Equilibrium response to network structure

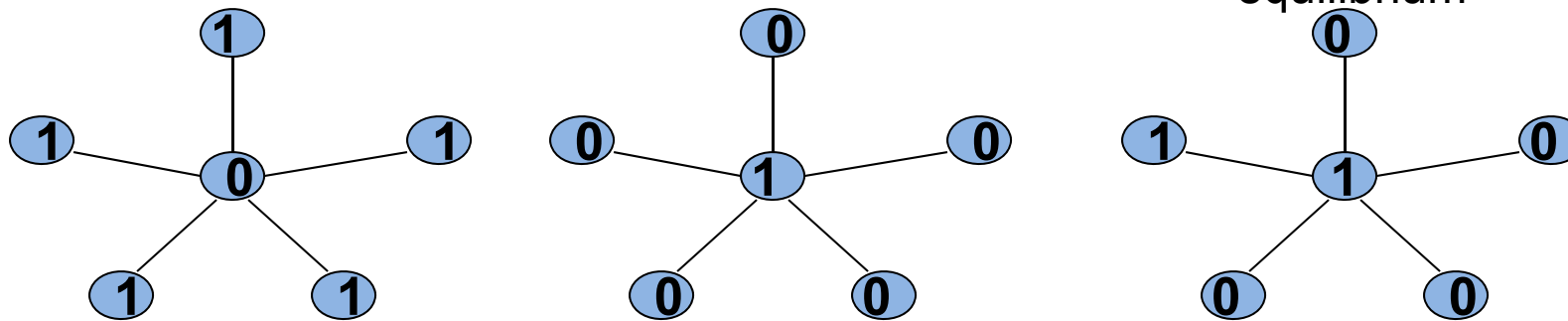


Equilibrium



- Nash equilibrium: Every player's action is optimal for that player given the actions of others
- Often look for pure strategy equilibria
- May require some mixing

Best shot



- Maximal independent set: each 1 has no 1's in its neighborhood, each 0 has at least one 1
- Different distributions of utilities, and different total costs

Maximal Independent Set

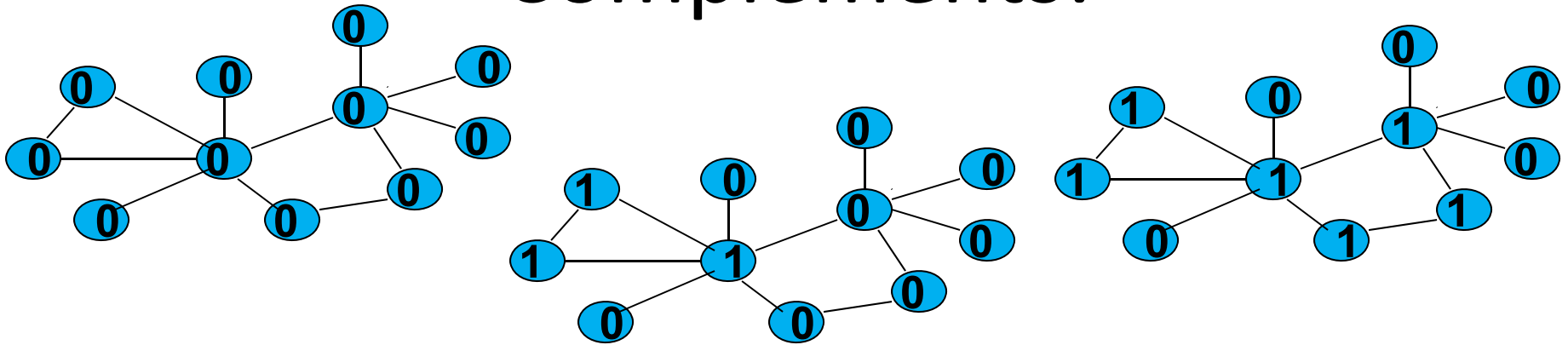
- Independent Set: a set S of nodes such that no two nodes in S are linked,
- Maximal: every node in N is either in S or linked to a node in S

Useful Observation



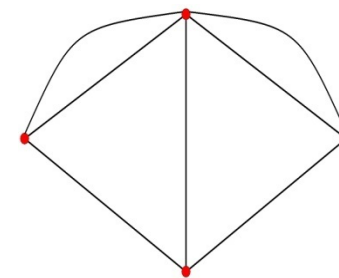
- Complements: there is a threshold $t(d)$, such that i prefers 1 if $m_{N_i} > t(d)$ and 0 if $m_{N_i} < t(d)$
- Substitutes: there is a threshold $t(d)$, such that i prefers 1 if $m_{N_i} < t(d)$ and 0 if $m_{N_i} > t(d)$
- Can be indifferent at the threshold

Complements:



- threshold is two
- multiple equilibria
- lattice structure to set of equilibria

Social and Economic Networks: Models and Analysis

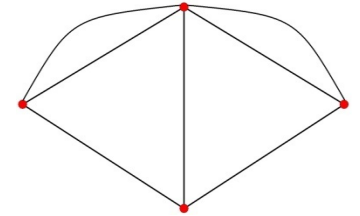


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7.3: Properties of Equilibria

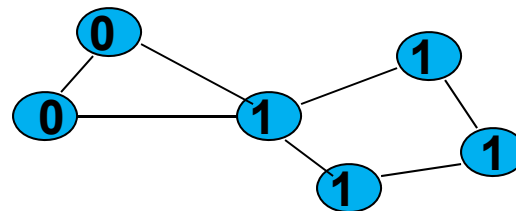
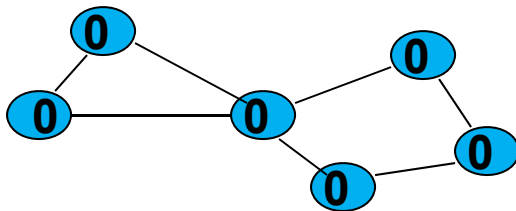
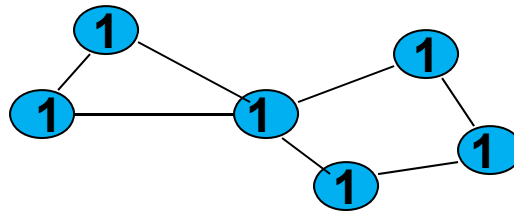
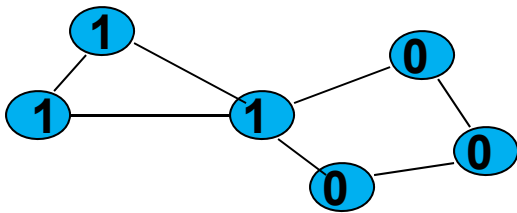


Complete lattice



- Complete Lattice: for every set of equilibria X
 - there exists an equilibrium x' such that $x' \geq x$ for all x in X , and
 - there exists an equilibrium x'' such that $x'' \leq x$ for all x in X .

Lattice:



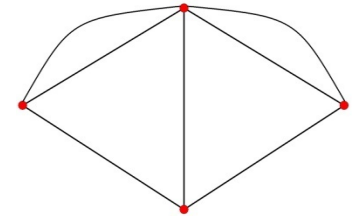
Proposition



In a game of strategic complements where the individual strategy sets are complete lattices:

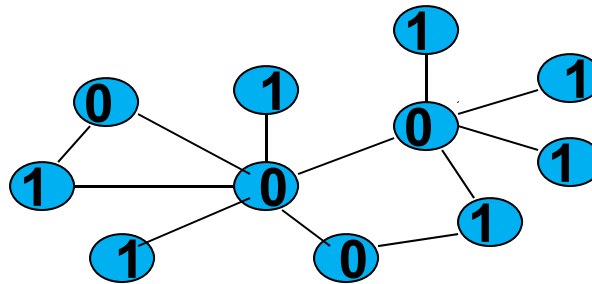
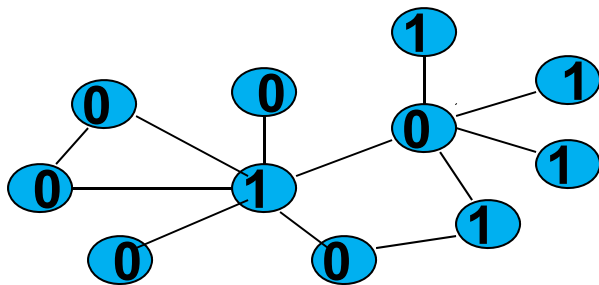
the set of pure strategy equilibria are a (nonempty) complete lattice.

Contrast: Complements and Substitutes



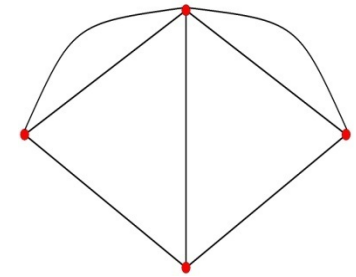
- In a game of complements: pure strategy equilibria are a nonempty complete lattice
- In a game of strategic substitutes:
 - Best shot game: pure strategy equilibria exist and are related to maximal independent sets
 - Others: pure strategy may not exist, but mixed will (with finite action spaces)
 - Equilibria usually do not form a lattice

Best Shot Public Goods



- invest if and only if no neighbors do (threshold is 1)
- again, multiple equilibria
- but, no lattice structure...

Social and Economic Networks: Models and Analysis

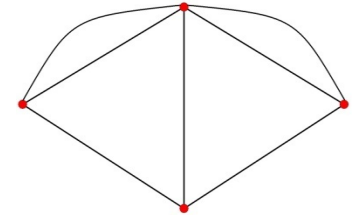


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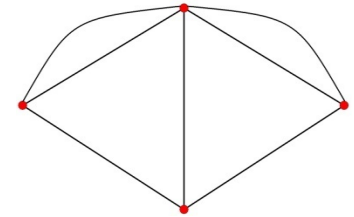
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7.4: Multiple Equilibria

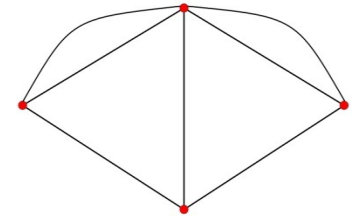


Games on Networks - Outline



- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure
- Equilibrium relation to network structure

When can multiple actions be sustained:

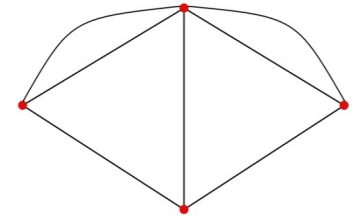


- Morris (2000) Coordination game
- Care only about fraction of neighbors
- prefer to take action 1 if fraction q or more take 1

Equilibrium Structure

Let S be the group that take action 1

- Each i in S must have fraction of at least q neighbors in S
- Each i not in S must have a fraction of at least $1-q$ neighbors outside of S



Cohesion

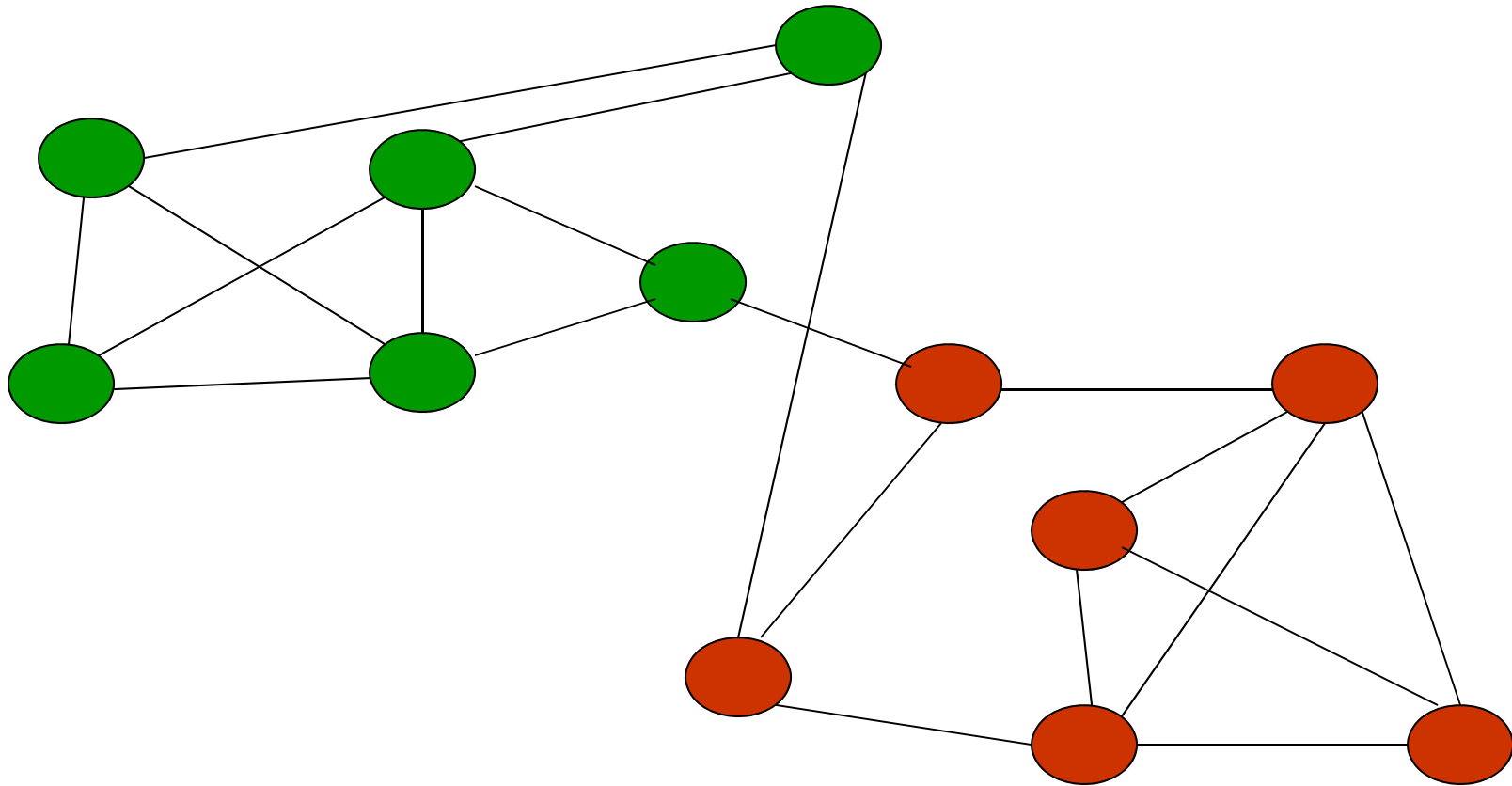


A group S is r -cohesive relative to g if

$$\min_{i \in S} |\{j \in N_i(g) \text{ and } S\}| / d_i(g) \geq r$$

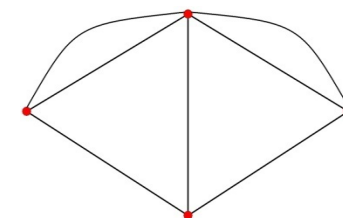
At least a fraction r of each member of S 's neighbors are in S

Cohesiveness of S is $\min_{i \in S} |\{j \in N_i(g) \text{ and } S\}| / d_i(g)$



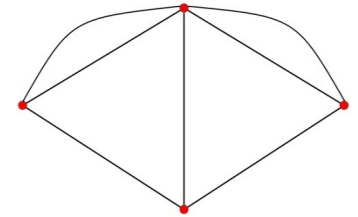
Both groups are 2/3 cohesive

Equilibria where both strategies are played:



Morris (2000): there exists a pure strategy equilibrium where both actions are played if and only if there is a group S that is at least q cohesive and such that its complement is at least $1-q$ cohesive.

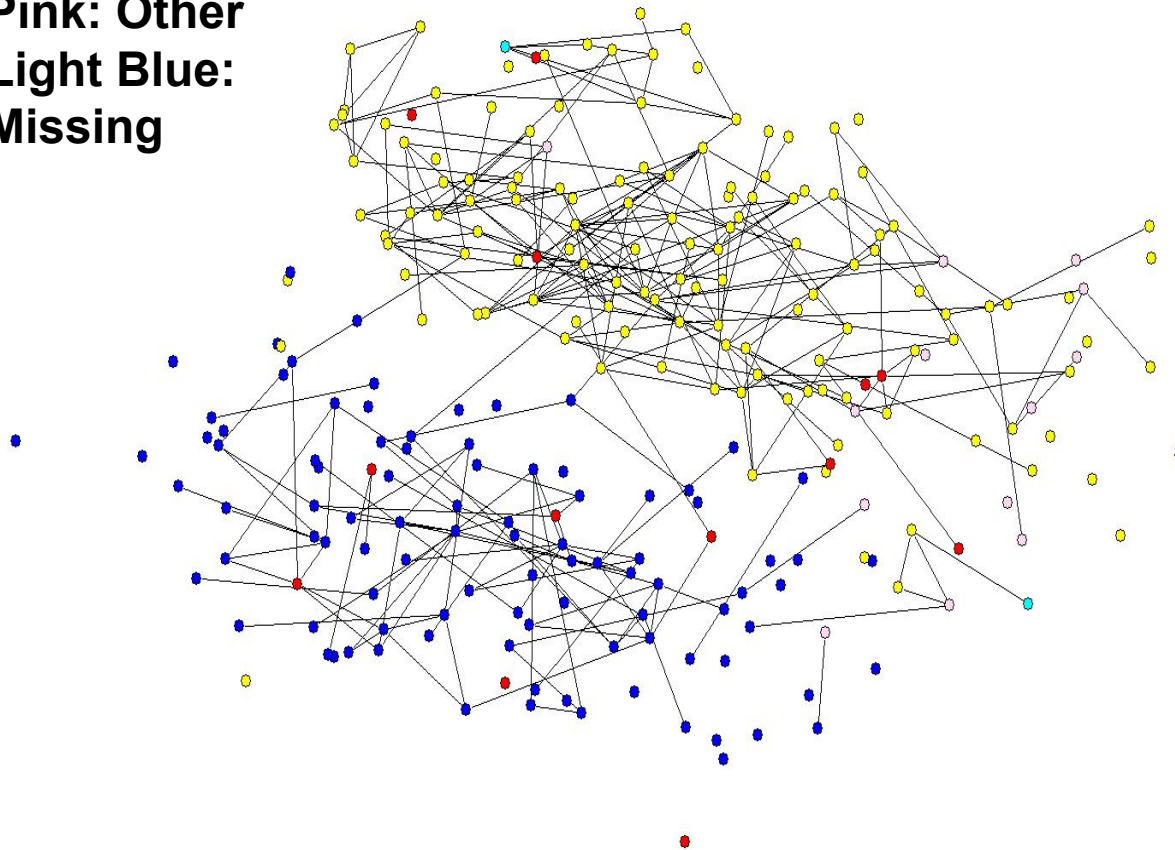
Homophily?



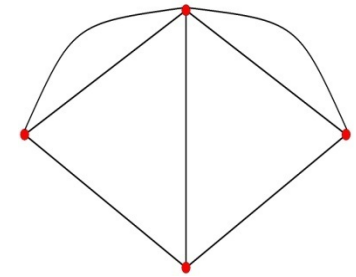
- If $q=1/2$ – players want to match majority
- Then two groups that have more self-ties than cross-ties suffices to sustain both actions
- As q rises (game payoffs become more asymmetric), need more homophilous behavior between the groups to sustain both actions

Blue: Black
Reds: Hispanic
Yellow: White
Pink: Other
Light Blue:
Missing

“strong friendships”
cross group links less than half as frequent
Jackson 07



Social and Economic Networks: Models and Analysis

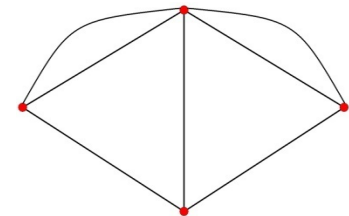


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7.5: An Application



Application:



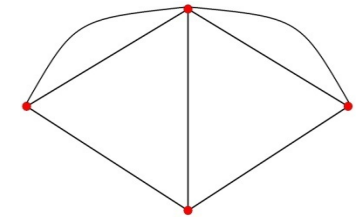
- Drop out decisions
- Strategic complements

Application:



- Drop out decisions
- Strategic complements

Labor Participation Decisions (Calvo-Armengol & Jackson 04,07,09)



- Value to being in the labor market depends on number of friends in labor force, value to non-labor activities depend on number of friends outside of labor market
- Participate if at least some fraction of friends do
- Homophily – and different starting conditions (history) lead to different outcomes for different groups...

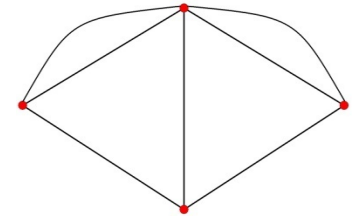
Drop-Out Rates

- Chandra (2000) Census – males 25 to 55

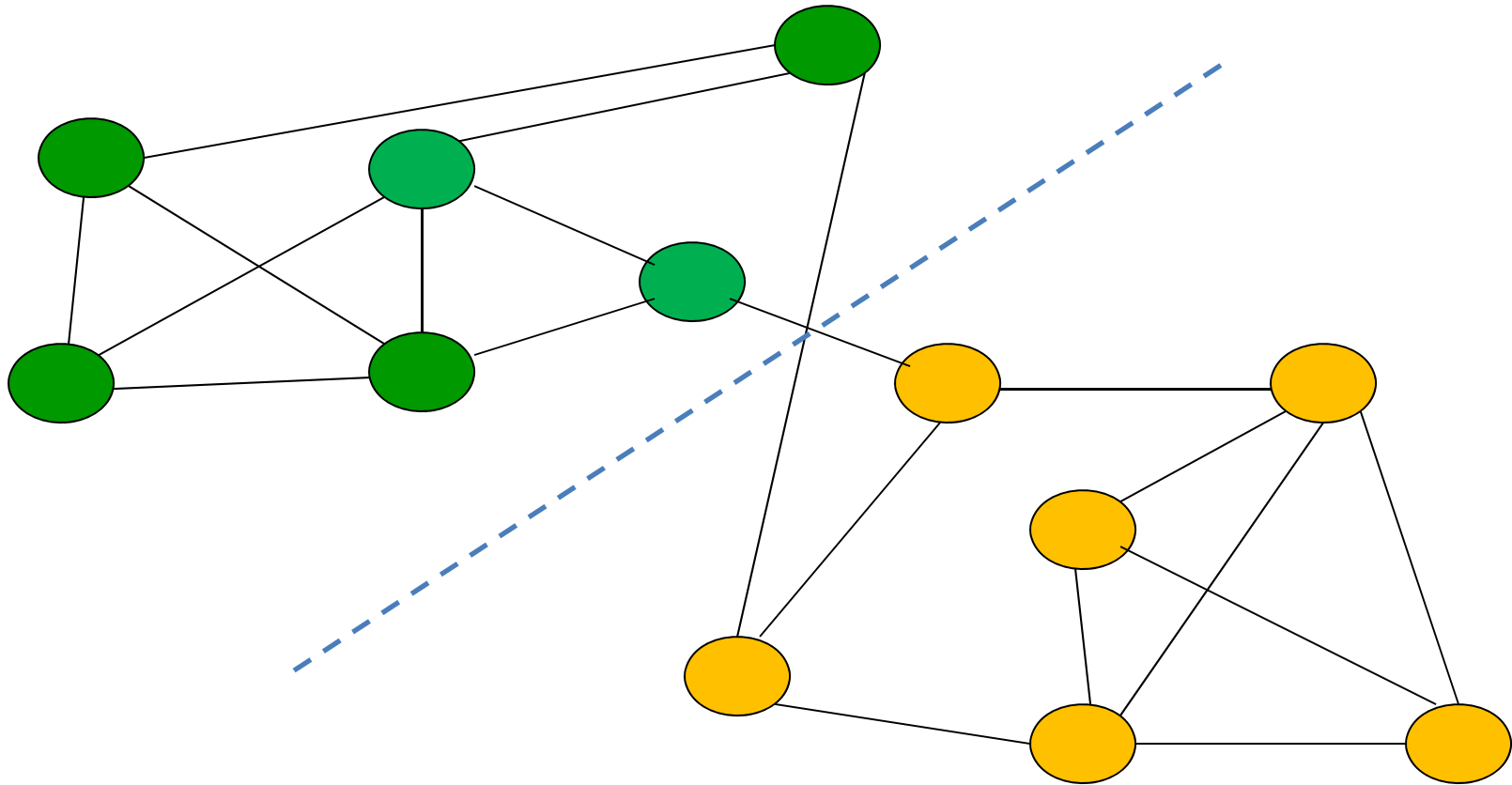
	1940	1950	1960	1970	1980	1990
whites	3.3	4.2	3.0	3.5	4.8	4.9
blacks	4.2	7.5	6.9	8.9	12.7	12.7

See DiCecio et al 2008, data from BLS for more recent, and by gender, including Hispanics

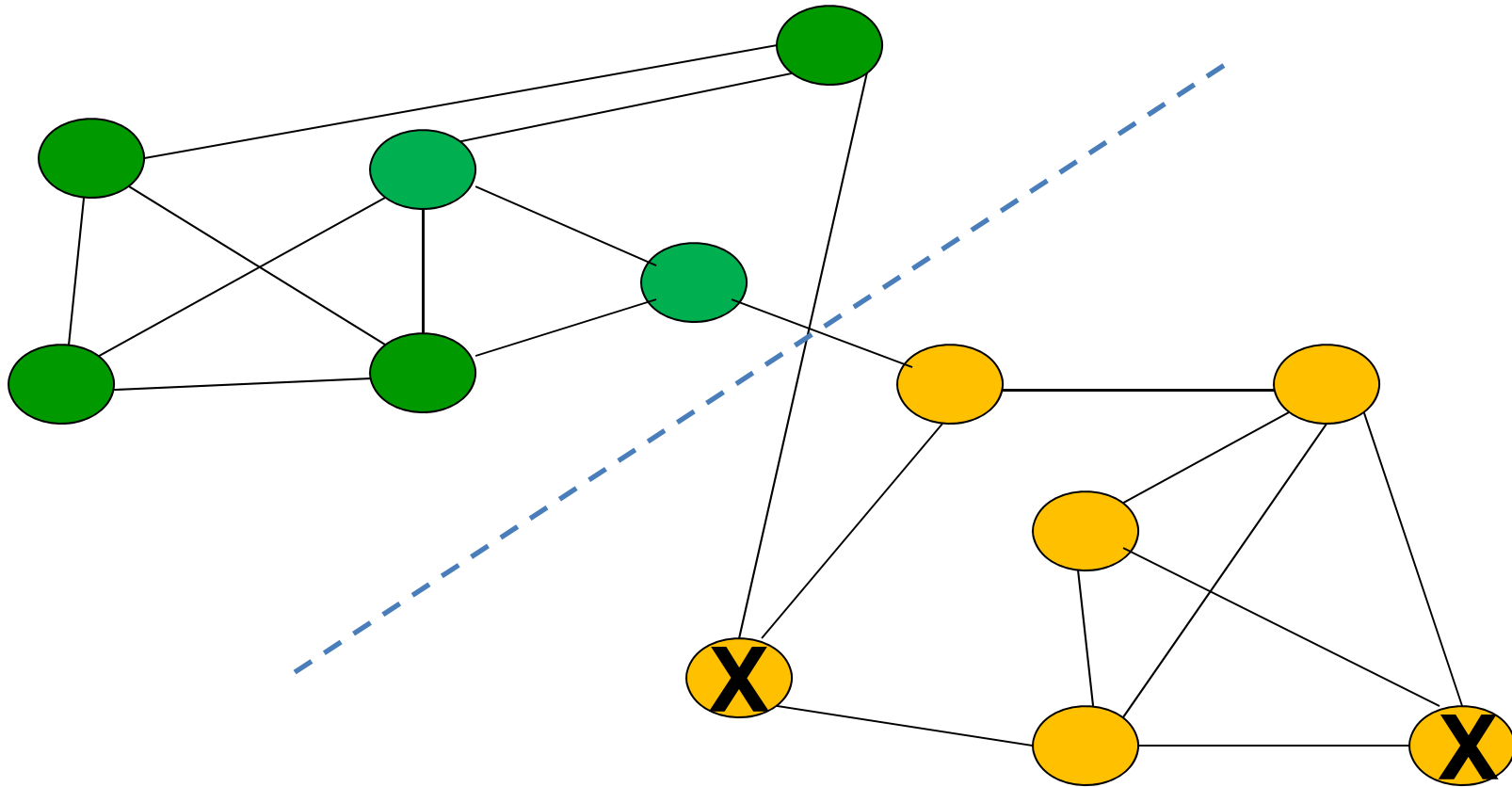
Drop-Out Decisions



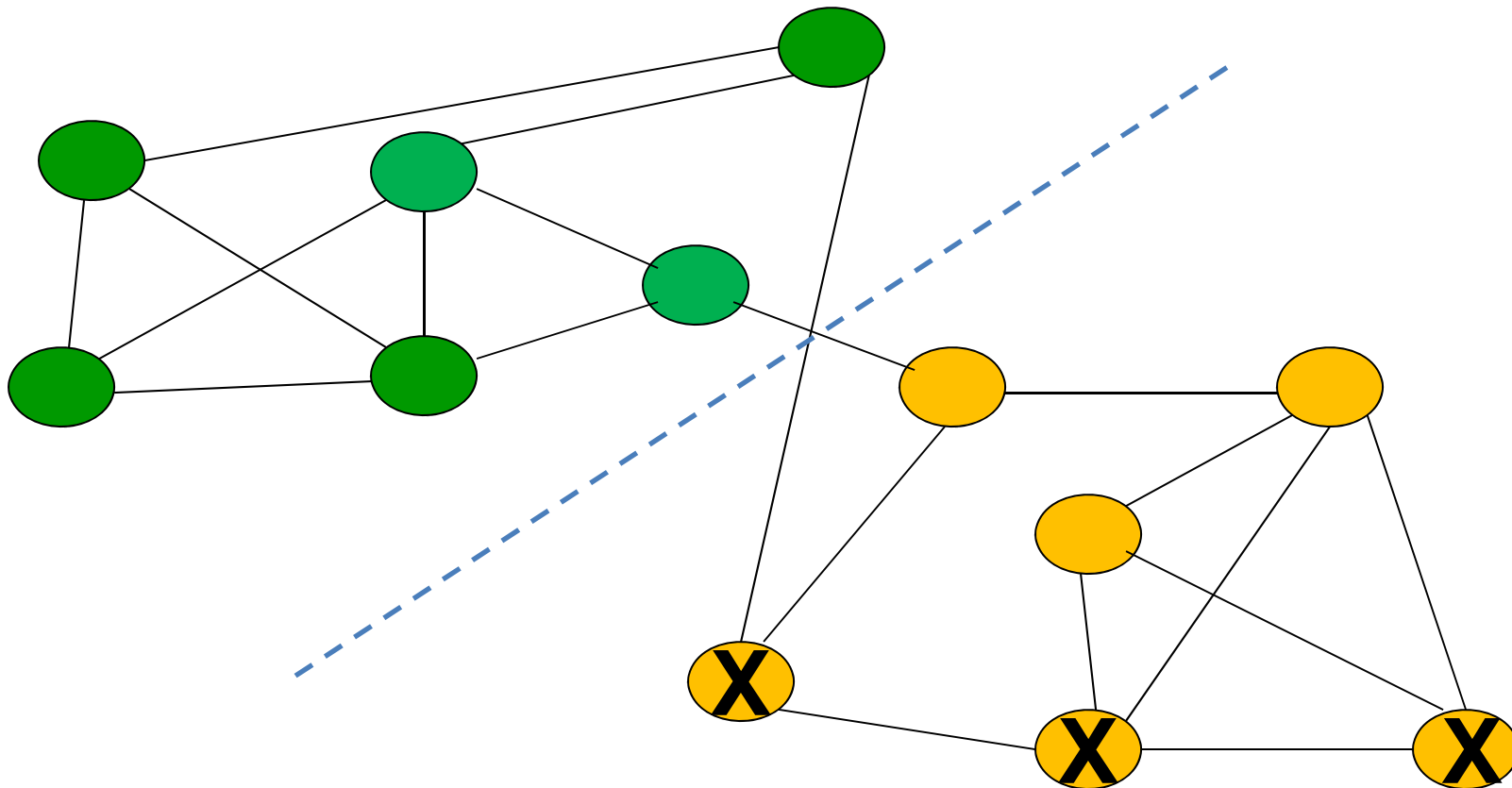
- Value to being in the labor market depends on number of friends in labor force
- Drop out if some number of friends drop out
- Some heterogeneity in threshold (different costs, natural abilities...)
- Homophily – segregation in network
- Different starting conditions: history...



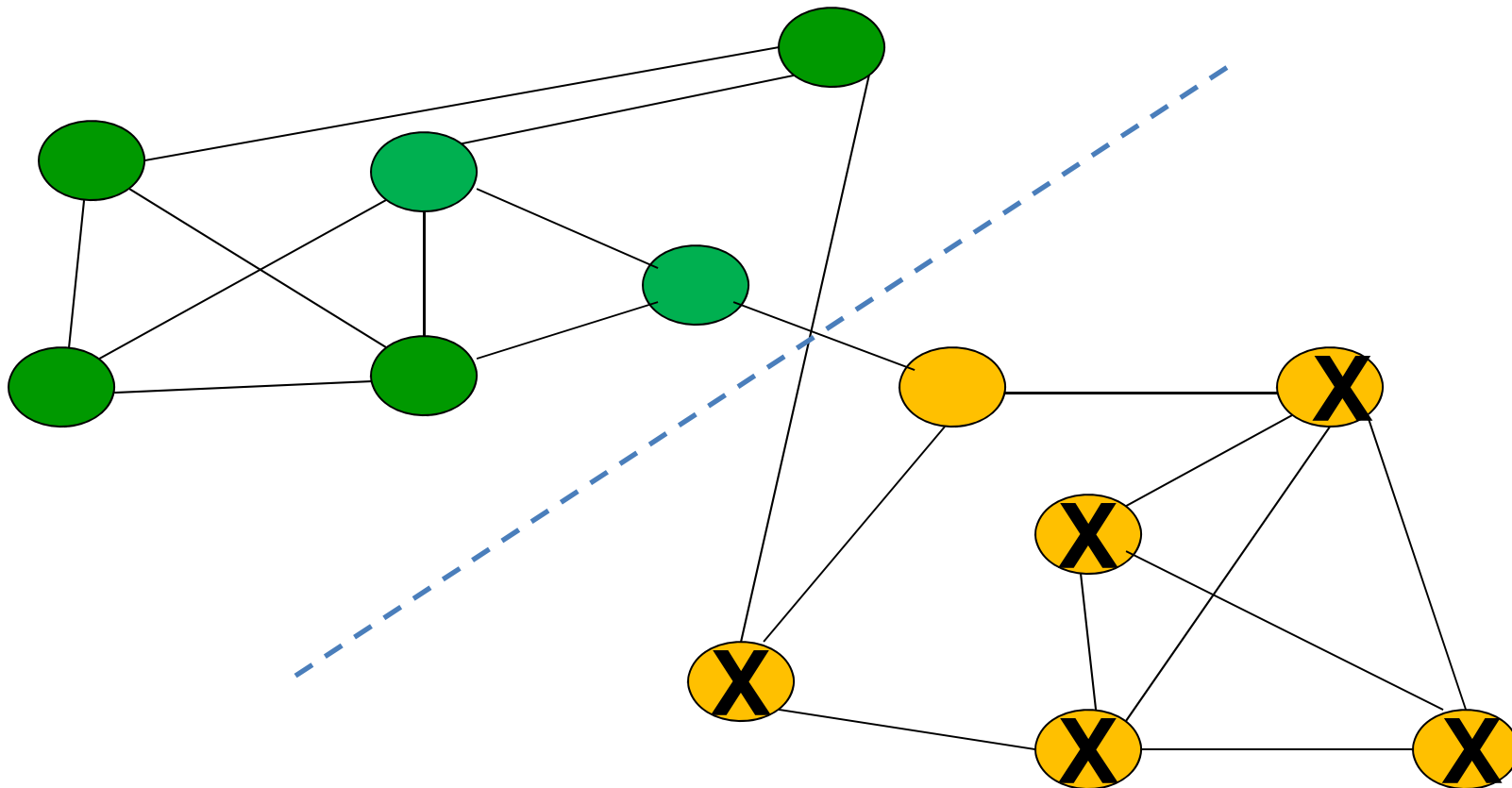
Two groups exhibit homophily



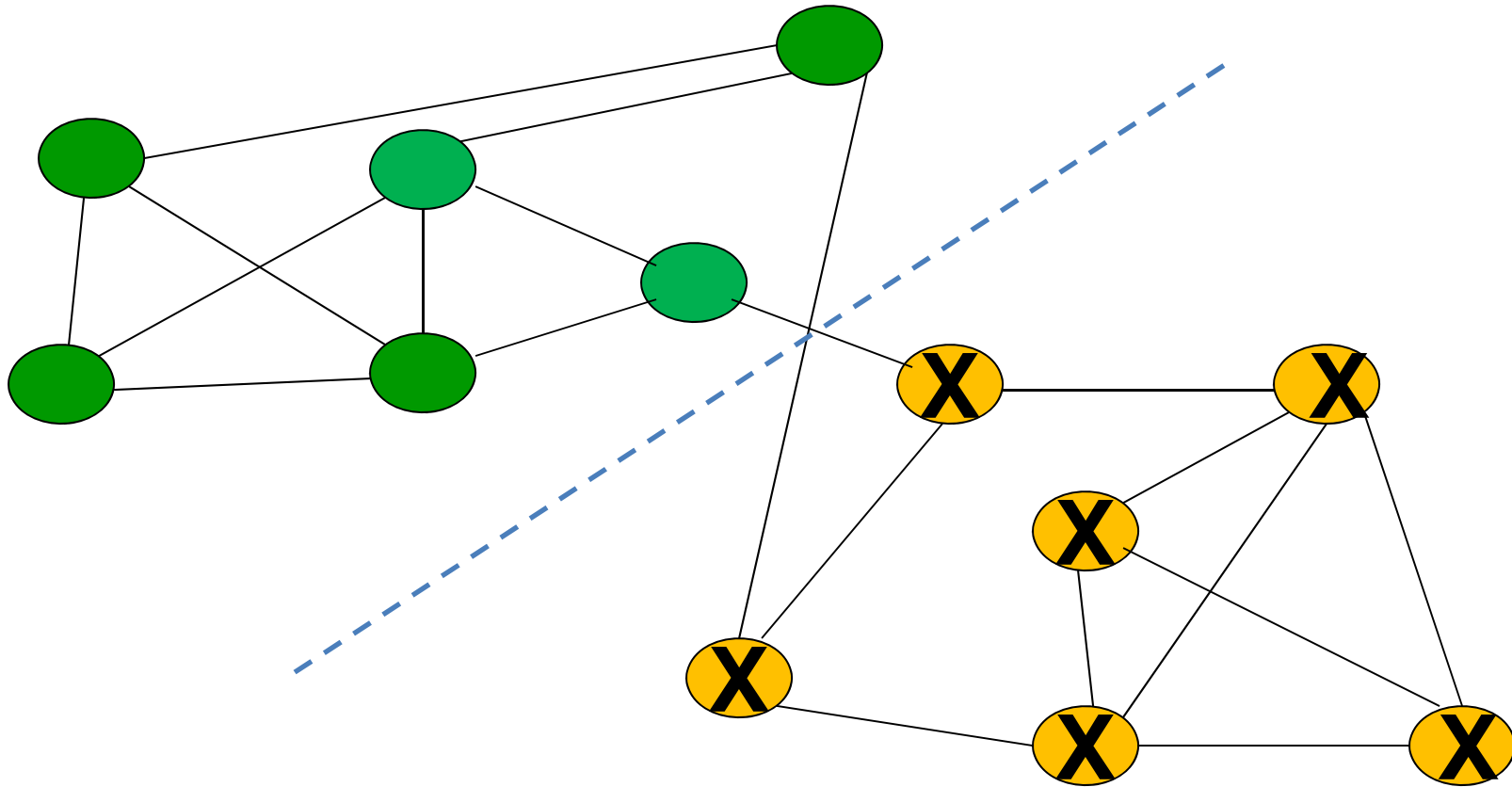
Drop-out if at least half of neighbors
do -- begin with two initial dropouts



Drop-out if at least half of neighbors
do



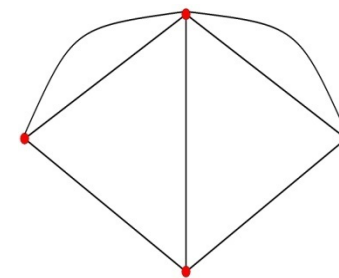
Drop-out if at least half of neighbors
do



End up with persistent differences across groups...
Applications to social mobility, wage inequality, etc.

Social and Economic Networks: Models and Analysis

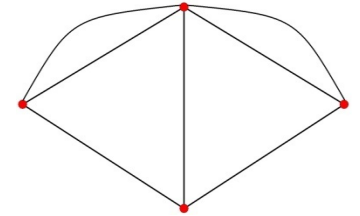
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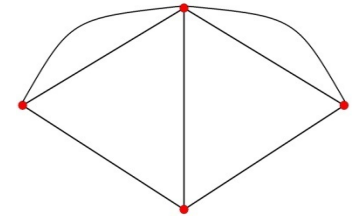
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7.6: Beyond 0-1 Choices



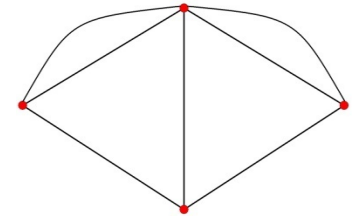
Beyond 0-1 choices



- Graphical game
- x_i in $[0,1]$
- Start with Bramouille and Kranton:
information acquisition

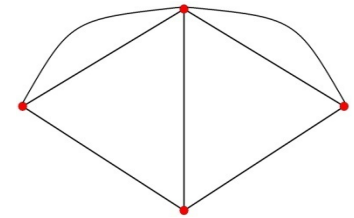
Bramouille-Kranton Setting:

- payoff $f(x_i + \sum_{j \in N_i(g)} x_j) - c x_i$ concave f
- Let $x^* > 0$ solve $f'(x^*) = c$

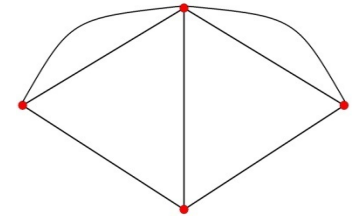


Bramouille-Kranton Setting:

- payoff $f(x_i + \sum_{j \in N_i(g)} x_j) - c x_i$ concave f
- Let $x^* > 0$ solve $f'(x^*) = c$
- In all pure strategy Nash equilibria:
 $x_i + \sum_{j \in N_i(g)} x_j \geq x^*$ for all i , and if $>$, then $x_i = 0$



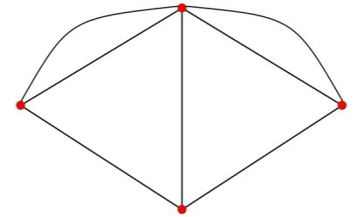
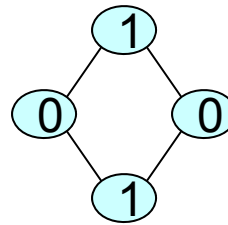
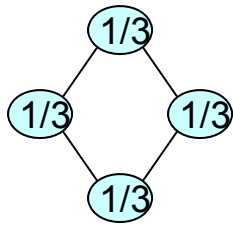
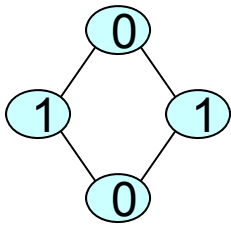
Bramouille-Kranton Setting:

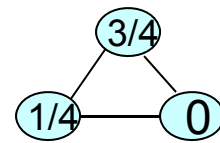
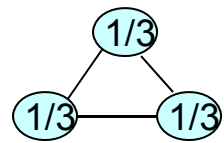
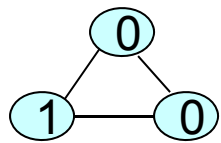
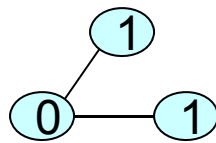
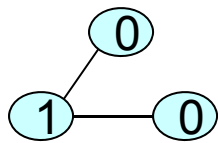
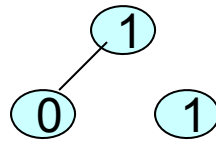
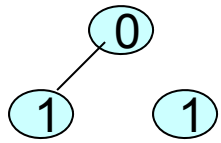
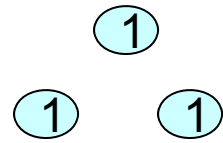


- payoff $f(x_i + \sum_{j \in N_i(g)} x_j) - c x_i$ concave f
- Let $x^* > 0$ solve $f'(x^*) = c$
- In all pure strategy Nash equilibria:
 $x_i + \sum_{j \in N_i(g)} x_j \geq x^*$ for all i , and if $>$, then $x_i = 0$
- Look at two types of pure equilibria
 - **distributed**: $x^* > x_i > 0$ for some i 's
 - **specialized**: for each i either $x_i = 0$ or $x_i = x^*$

Various Equilibria

- Case $x^* = 1$



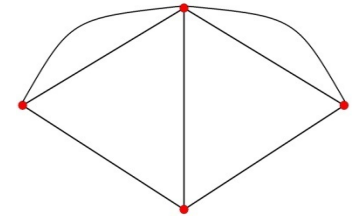


Specialized Equilibria

- Maximal independent set – set S of nodes such that
 - no two nodes in S are linked, and
 - every node in N is either in S or linked to a node in S
- Proposition (B&K): The set of specialized Nash equilibria are profiles such that a maximal independent set = the specialists ($x_i = x^*$)

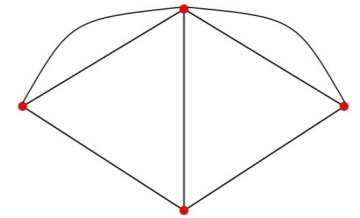
Stability Notion: pure strategy equilibrium

- perturb x to $x^0 = (x_1 + \varepsilon_1, \dots, x_n + \varepsilon_n)$, being sure that all entries are feasible
- Let x^1 be the best response to x^0 , x^t to x^{t-1}
- If for all small enough ε_i 's converge back to x , then “stable”

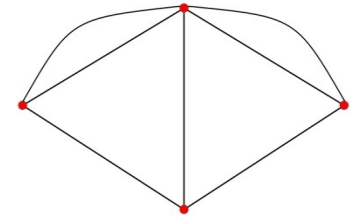


Stable equilibria:

- Dyad: nothing is stable:
- let $x_1 \leq x_2$
- $x_1 + \varepsilon, x_2 - \varepsilon$ stays there

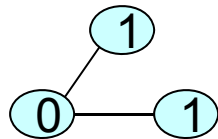


Stable equilibria, BK:

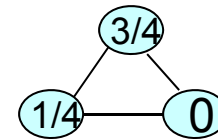
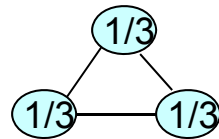
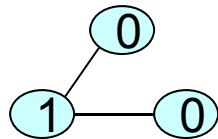


- Only stable equilibria are specialist equilibria such that every non-specialist has two specialists in his or her neighborhood

stable:



unstable:



Sketch of Proof



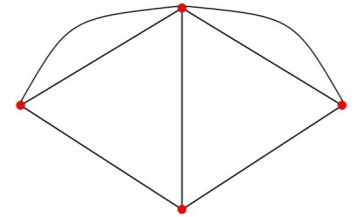
- Stability of such equilibria: for small perturbations, BR of non-specialists is 0, converge right back
- For any other equilibrium, if there is an agent providing is a non-specialists, then perturb the agent up, neighbors go down...
- If all specialists or not – then some non-specialist just has one neighbor as specialist – raise that nonspecialist, lower the specialist...

Stability and Pairwise Stability:



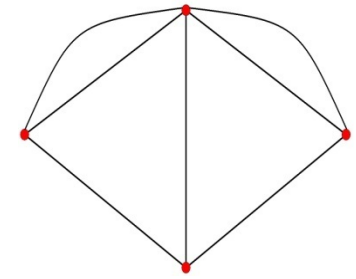
- Suppose links are costly
- specialists drop links to non
- non-specialized equilibria are only ``stable'' ones...

Heterogeneity?



- Introduction of heterogeneous costs and benefits, and some less than perfect spillovers
- Would change the nature of equilibria

Social and Economic Networks: Models and Analysis

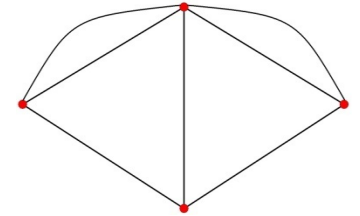


Matthew O. Jackson

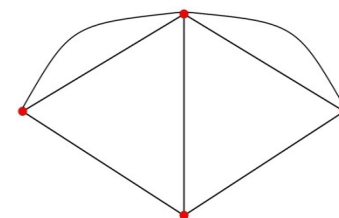
**Stanford University,
Santa Fe Institute, CIFAR,
www.stanford.edu/~jacksonm**

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7.7: A Linear Quadratic Model



A Linear-Quadratic Model



Ballester, Calvo-Armengol and Zenou (2006)

$$u_i(x_i, x_{-i}) = a x_i - b x_i^2 / 2 + \sum_j w_{ij} x_i x_j$$

strategic complements

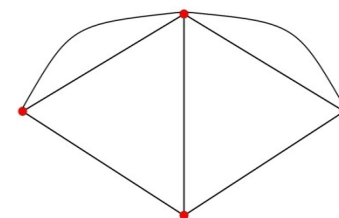
A Linear-Quadratic Model

$$u_i(x_i, x_{-i}) = a x_i - b x_i^2/2 + \sum_j w_{ij} x_i x_j$$

Best response of x_i to x_{-i} :

$$a - b x_i + \sum_j w_{ij} x_j = 0$$

$$(a + \sum_j w_{ij} x_j)/b = x_i$$

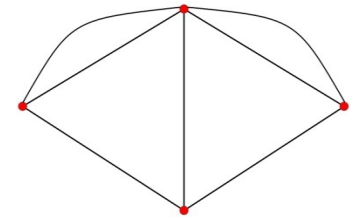


A Linear-Quadratic Model

$$x_i = (a + \sum_j w_{ij} x_j) / b$$

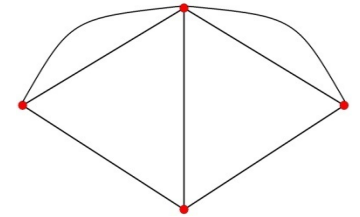
Thus, $\mathbf{x} = \boldsymbol{\alpha} + \mathbf{g} \mathbf{x}$

where $\boldsymbol{\alpha} = (a/b, \dots, a/b)$ and $g_{ij} = w_{ij} / b$



A Linear-Quadratic Model

$$\mathbf{x} = \boldsymbol{\alpha} + \mathbf{g} \mathbf{x}$$

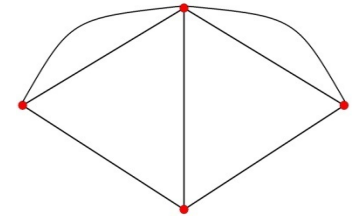


$$\text{or } \mathbf{x} = \boldsymbol{\alpha} + \mathbf{g} (\boldsymbol{\alpha} + \mathbf{g} (\boldsymbol{\alpha} + \mathbf{g} \dots)) = \sum_{k \geq 0} \mathbf{g}^k \boldsymbol{\alpha}$$

$$\text{or } \mathbf{x} = (\mathbf{I} - \mathbf{g})^{-1} \boldsymbol{\alpha} \quad \text{if invertible}$$

(or if $\mathbf{a}=0$, then $\mathbf{x}=\mathbf{g}\mathbf{x}$, so unit eigenvector)

A Linear-Quadratic Model



- Actions are related to network structure:
- higher neighbors' actions, higher own action
- higher own action, higher neighbors actions
- feedback – for solution need b to be large and/or w_{ij} 's to be small

A Linear-Quadratic Model



- Relation to centrality measures:

$$\mathbf{x} = \sum_{k \geq 0} \mathbf{g}^k \boldsymbol{\alpha}$$

$$\text{or } \mathbf{x} = (\mathbf{I} - \mathbf{g})^{-1} \boldsymbol{\alpha}$$

Recall Bonacich centrality:

$$\mathbf{B}(\mathbf{g}) = (\mathbf{I} - \mathbf{g})^{-1} \mathbf{g} \mathbf{1} = \sum_{k \geq 0} \mathbf{g}^{k+1} \mathbf{1}$$

(number of paths from i to j of length $k+1$, summed over all $k+1$, here weighted and directed w_{ij}/b)

A Linear-Quadratic Model



- Relation to centrality measures:

$$\mathbf{x} = \sum_{k \geq 0} \mathbf{g}^k \boldsymbol{\alpha}$$

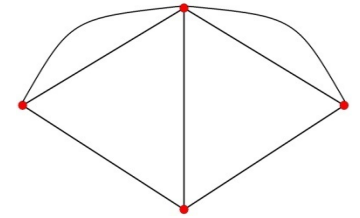
$$\text{or } \mathbf{x} = (\mathbf{I} - \mathbf{g})^{-1} \boldsymbol{\alpha}$$

Bonacich centrality:

$$\mathbf{B}(\mathbf{g}) = (\mathbf{I} - \mathbf{g})^{-1} \mathbf{g} \mathbf{1} = \sum_{k \geq 0} \mathbf{g}^{k+1} \mathbf{1}$$

$$\text{So, } \mathbf{x} = (\mathbf{1} + \mathbf{B}(\mathbf{g}))(\mathbf{a}/b)$$

A Linear-Quadratic Model



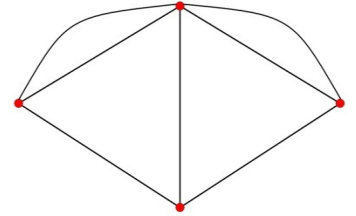
- Natural feedback from complementarities, actions relate to the total feedback from various positions
- Centrality: relative number of weighted influences going from one node to another
- Captures complementarities

Example



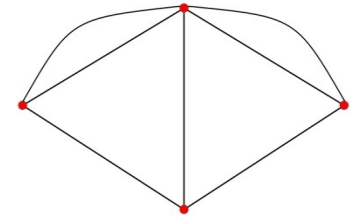
- $\mathbf{x} = (\mathbf{1} + \mathbf{B}(\mathbf{g}))(a/b)$
- Scales with a/b so ignore that
- $g_{ij} = w_{ij} / b$ let us take w_{ij} in $\{0,1\}$ and then only b matters

Example

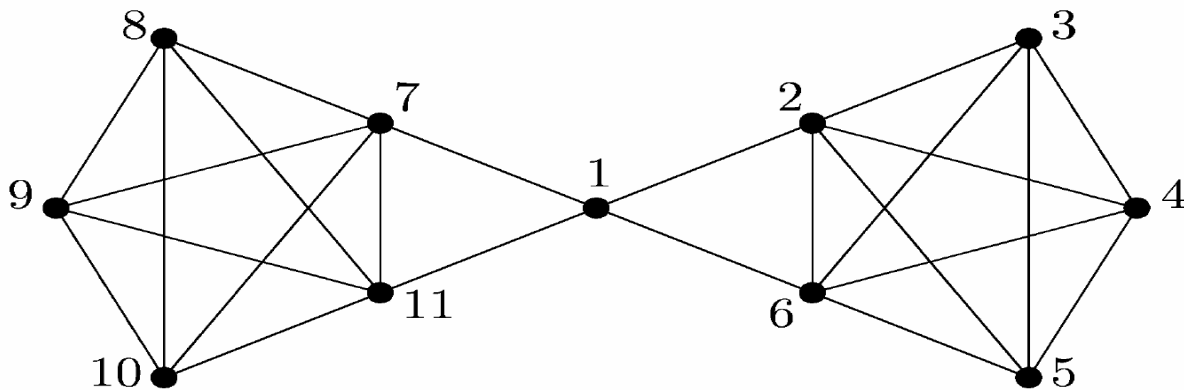


- $\mathbf{x} = (\mathbf{1} + B(\mathbf{g}))(a/b)$
- $g_{ij} = w_{ij} / b$ let us take w_{ij} in $\{0,1\}$ and then only b matters

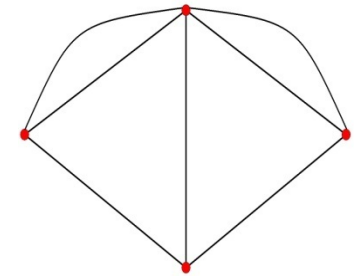
Example



- $x = (1 + B(g))(a/b)$
- $B(g) = 1.75, 1.88, 1.72$ for 1,2,3 if $b=10$
= 8.33, 9.17, 7.88 for 1,2,3 if $b=5$



Social and Economic Networks: Models and Analysis

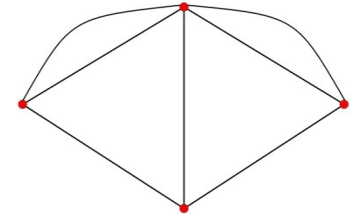


Matthew O. Jackson

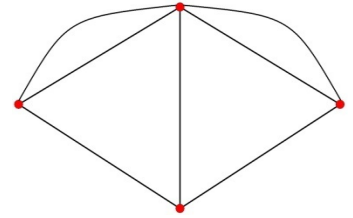
**Stanford University,
Santa Fe Institute, CIFAR,
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7.8: Repeated Games and Networks

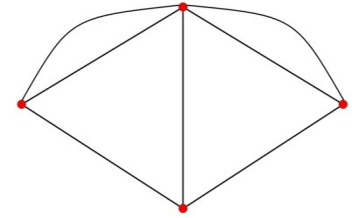


Repeated Games on Networks: Favor Exchange



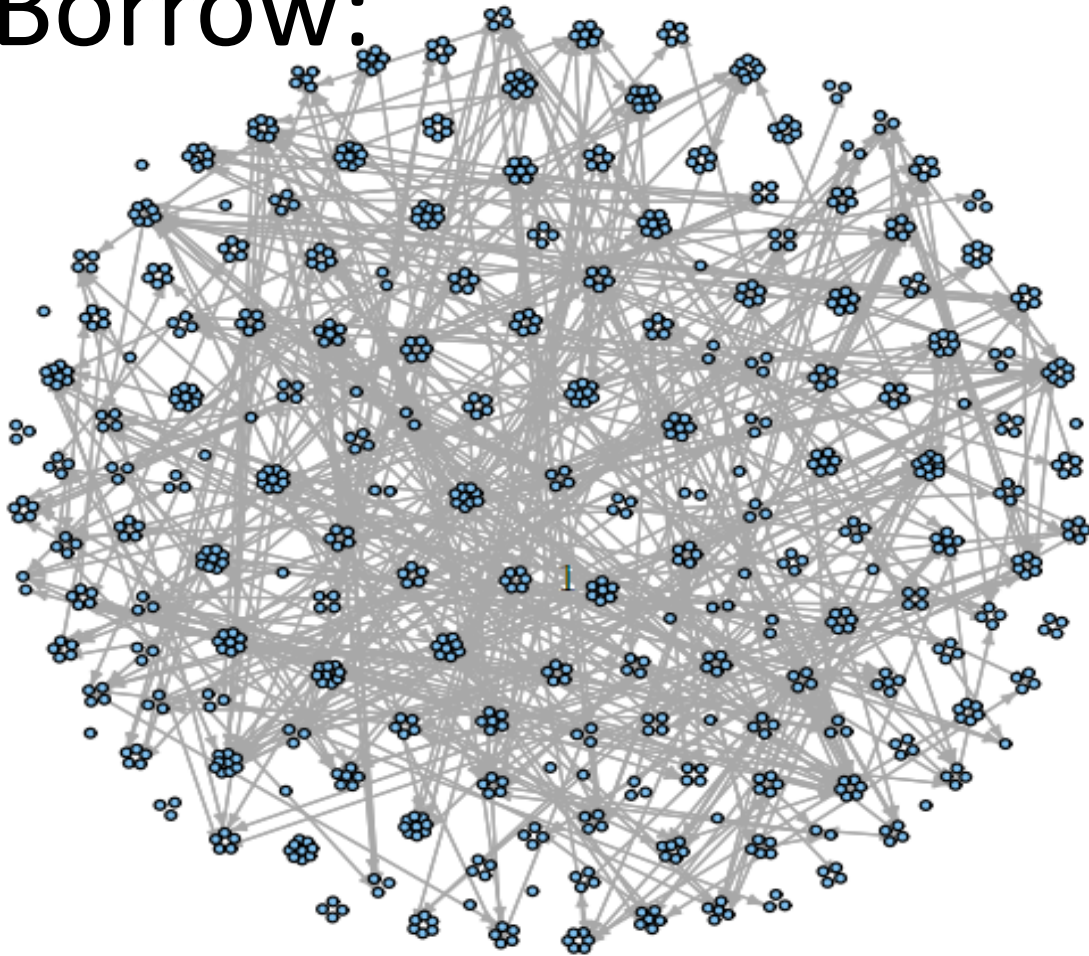
- How does successful favor exchange depend on/influence network structure?
- Co-determination of network and behavior

Repeated Games on Networks: Favor Exchange

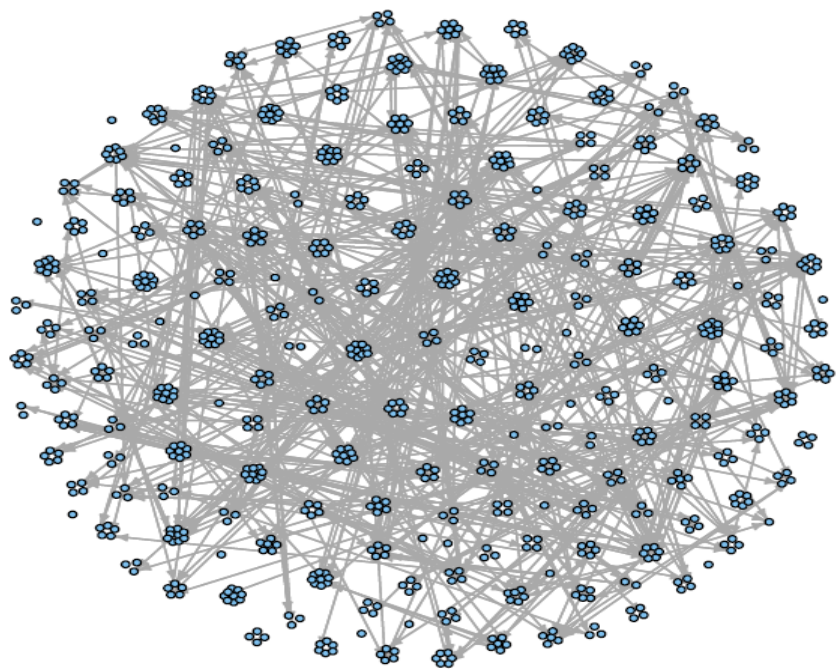


- Jackson, Rodriguez-Barraquer, Tan 12
- Many interactions are not contractible, and need to be self-enforcing
- How does successful favor exchange depend on/influence network structure?

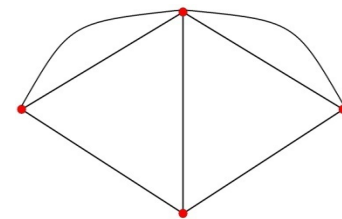
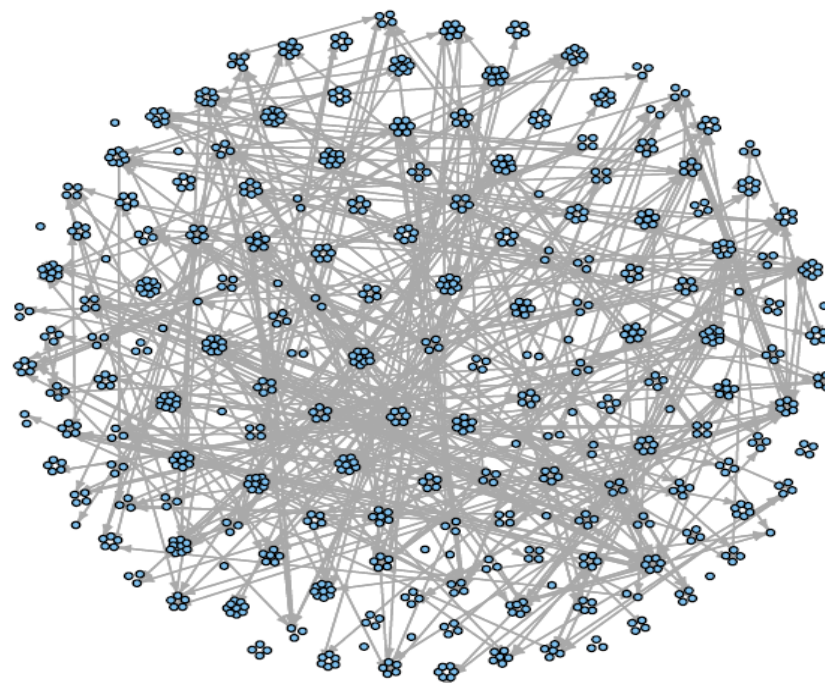
Borrow:



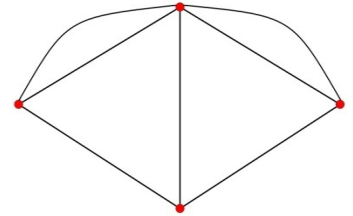
Kero-Come



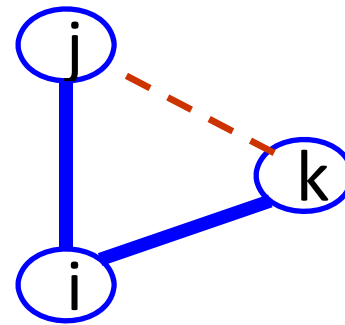
Medic



Social Enforcement



- Social capital literature's (e.g., Coleman, Bourdieu, Putnam...) discussion of enforcement has been interpreted as high clustering/transitivity:
- If we model social pressure and enforcement what comes out?



High? Clustering Coefficients -

- Prison friendships

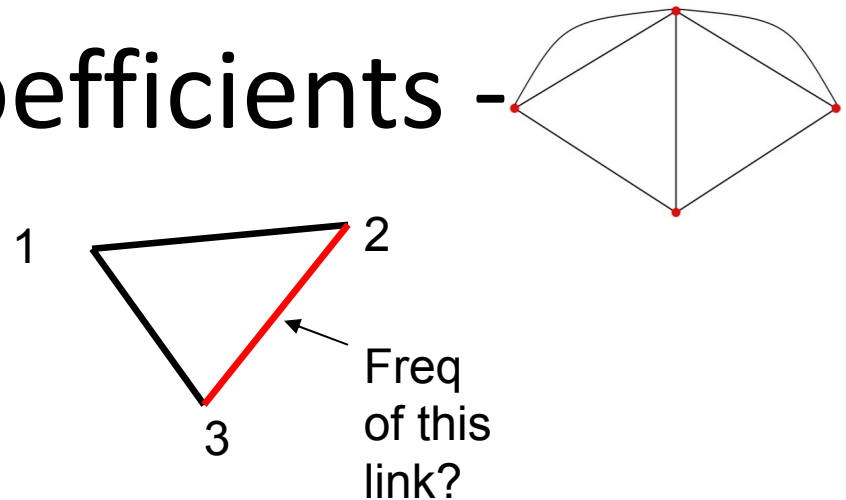
- **.31** (MacRae 60) vs .0134

- co-authorships

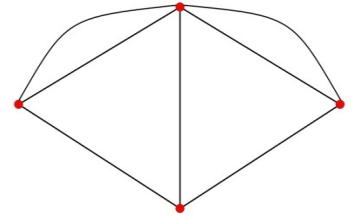
- **.15** math (Grossman 02) vs .00002,
- **.09** biology (Newman 01) vs .00001,
- **.19** econ (Goyal et al 06) vs .00002,

- WWW

- **.11** for web links (Adamic 99) vs .0002



Favors



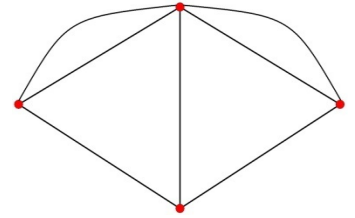
v value of a favor

c cost of a favor, $v > c > 0$

δ discount factor $1 > \delta > 0$

p prob. i needs a favor from j in a period

Repeated Game of Favor Exchange



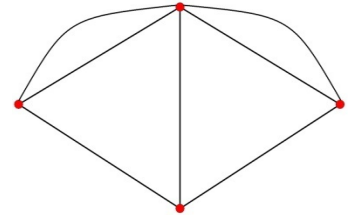
Favor need arises at random to (at most) one of the two agents

Other agent decides whether to provide favor

If provided, value v to receiver, $-c$ to giver

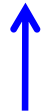
Otherwise, value 0 to both

Favor Exchange



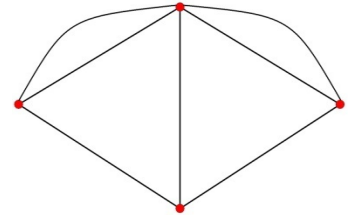
Favor exchange between two agents

$$p_v - p_c$$



expected value of relationship per period

Favor Exchange



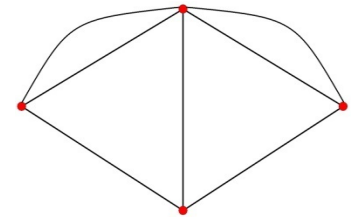
Favor exchange between two agents

$$p(v - c) / (1 - \delta)$$



value of **perpetual** relationship

Favor Exchange



Favor exchange between two agents iff:

$$c < \delta p (v - c) / (1 - \delta)$$

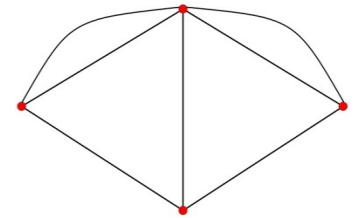


current
cost



value of future relationship

Network: Social Capital - Ostracism

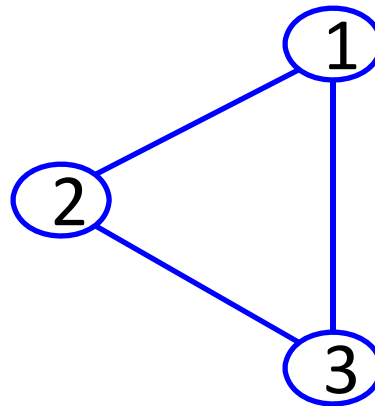


Three agents (a ``triad’’):

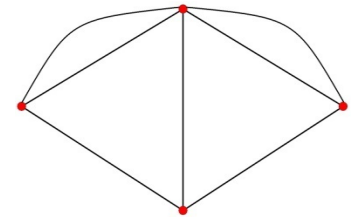
Ostracize agent who does not perform a favor

only need

$$c < 2 \delta p (v - c) / (1 - \delta)$$



Network: Social Capital - Ostracism

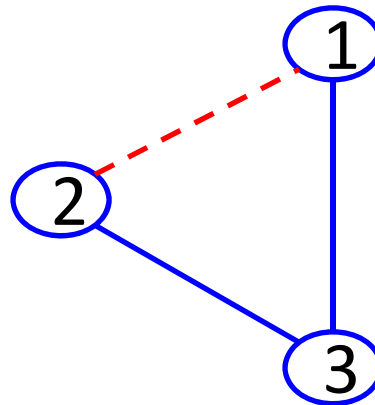


Three agents (a ``triad’’):

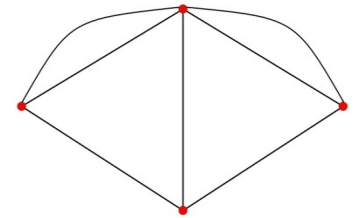
Ostracize agent who does not perform a favor

only need

$$c < 2 \delta p (v - c) / (1 - \delta)$$



Network: Social Capital - Ostracism

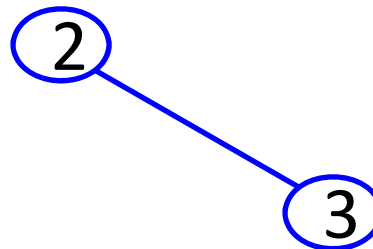


Three agents (a ``triad’’):

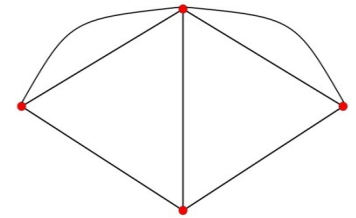
Ostracize agent who does not perform a favor

only need

$$c < \mathbf{2} \delta p (v - c) / (1 - \delta)$$



Network: Social Capital - Ostracism



Three agents (a ``triad’’):

Ostracize agent who does not perform a favor

①

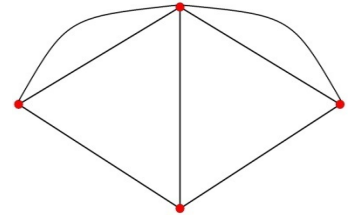
only need

$$c < 2 \delta p (v - c) / (1 - \delta) \quad \text{②}$$

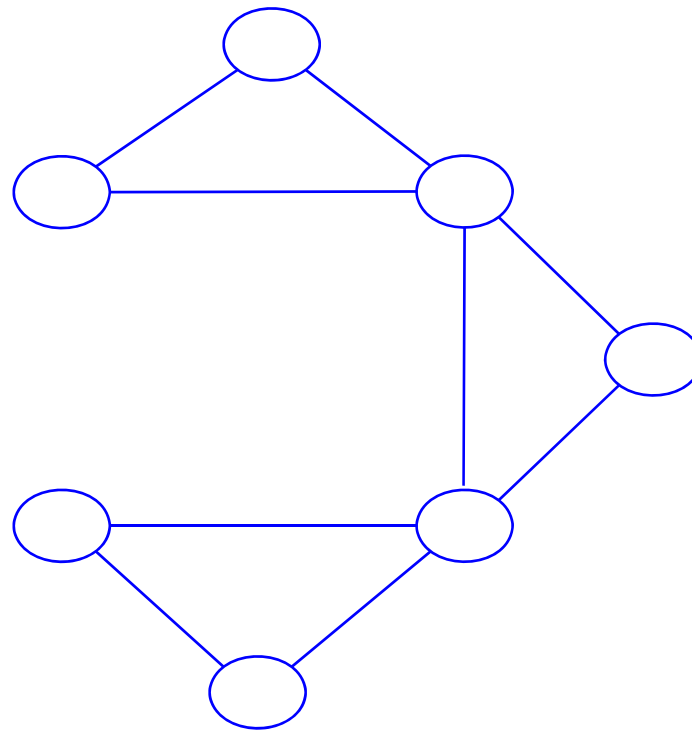
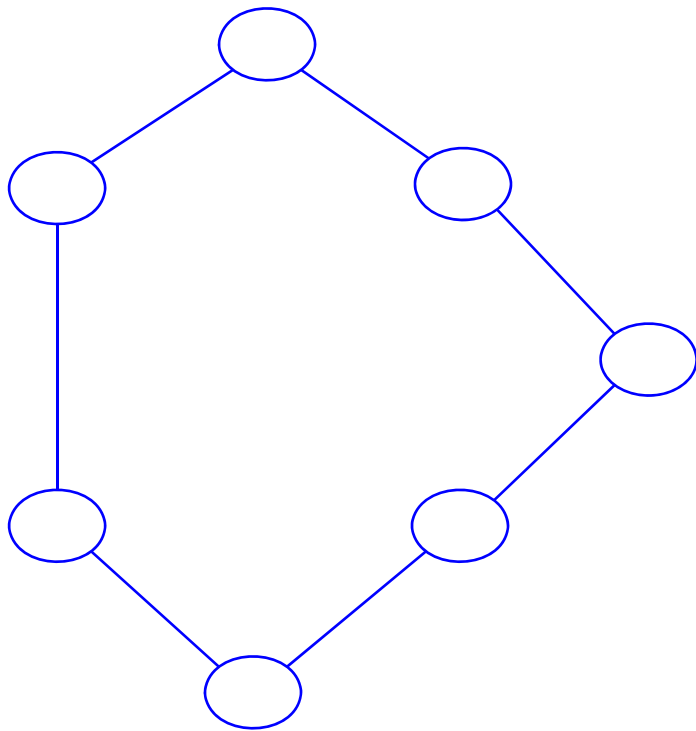
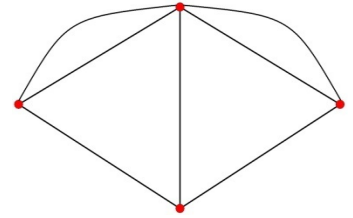
③

Game: Period t

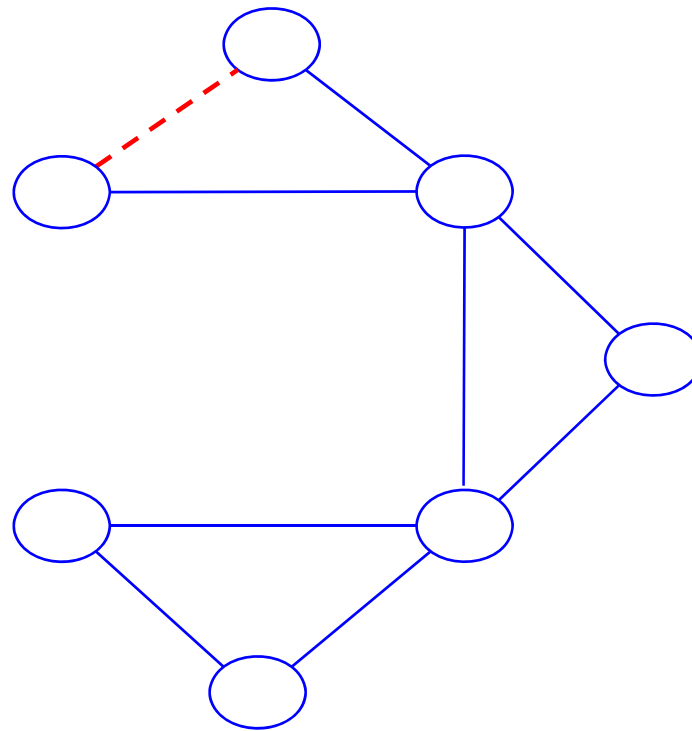
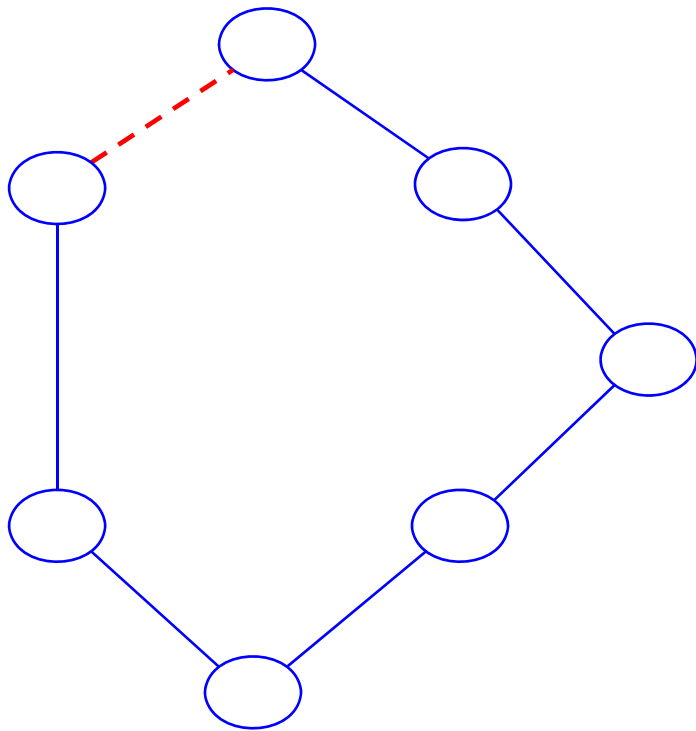
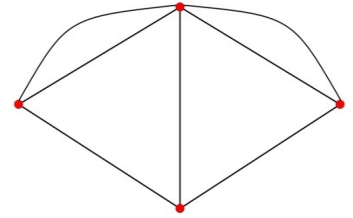
- At most one agent i_t is called upon to perform a favor for $j_t \in N_i(g_t)$ (p small)
- i_t keeps or deletes the link
- Others can respond: announce which (remaining) links they wish to maintain
- Links are retained if mutually agree - resulting network is g_{t+1}



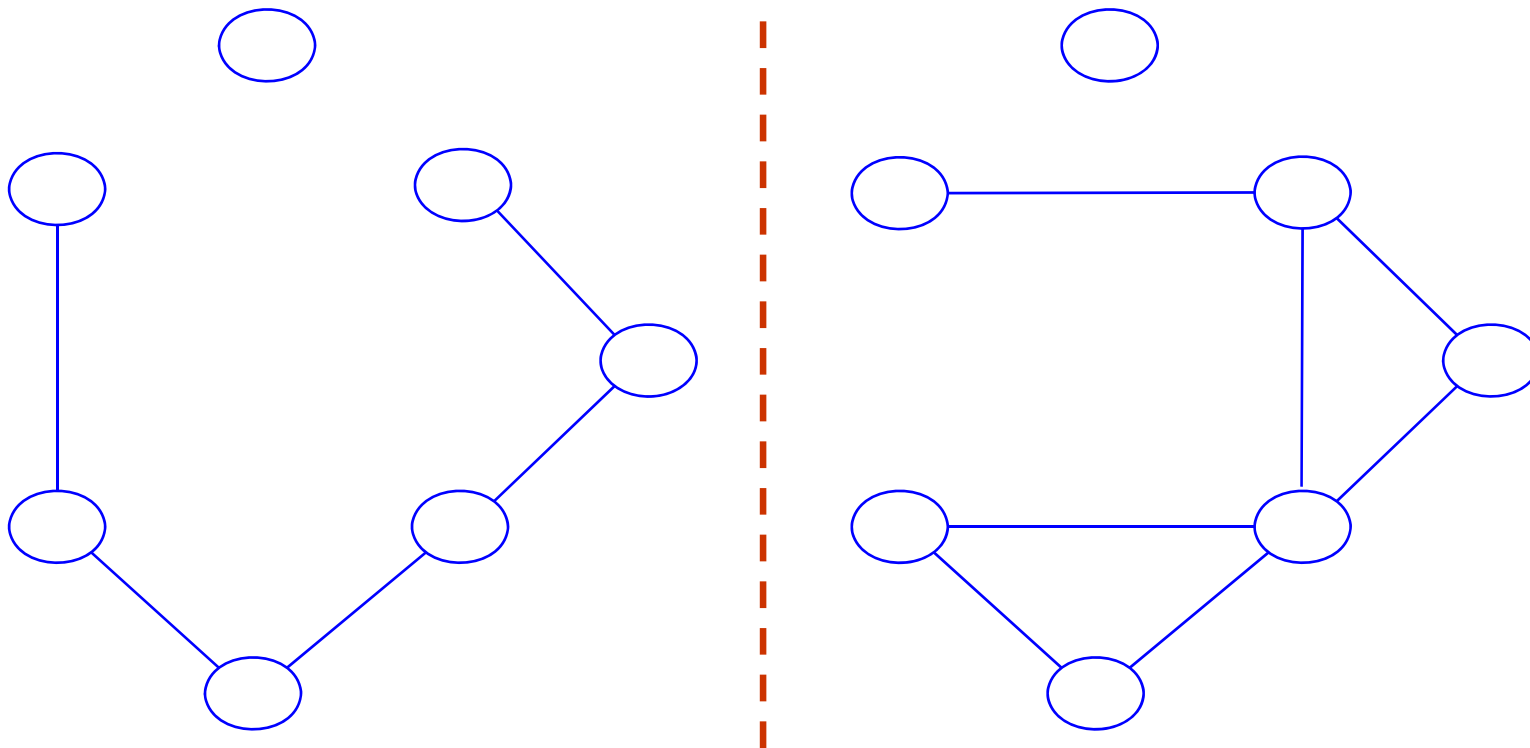
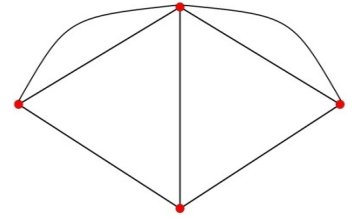
Two ways to support favor exchange:



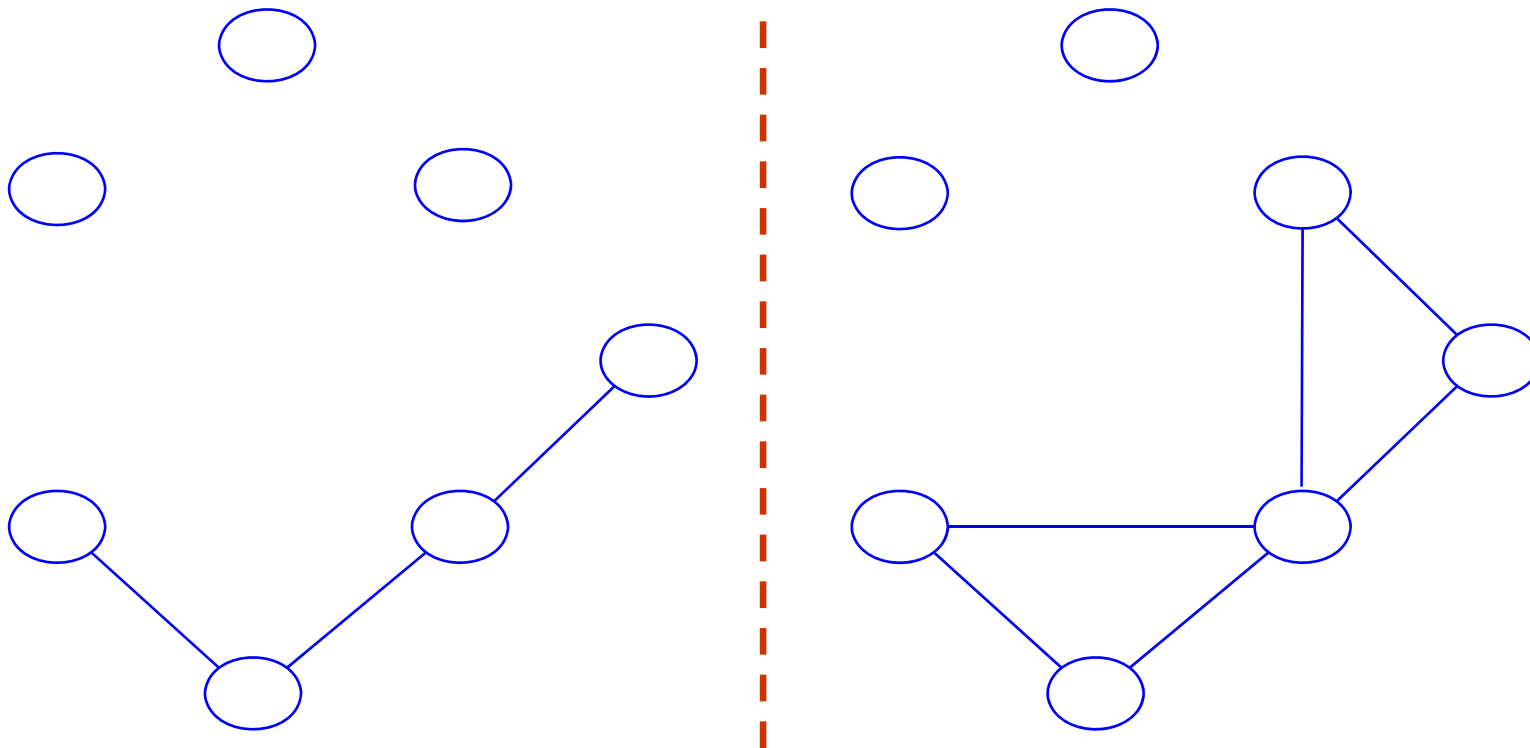
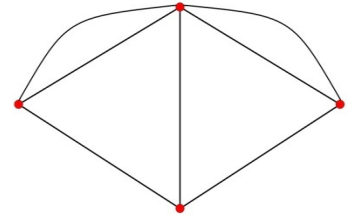
Two ways to support favor exchange:



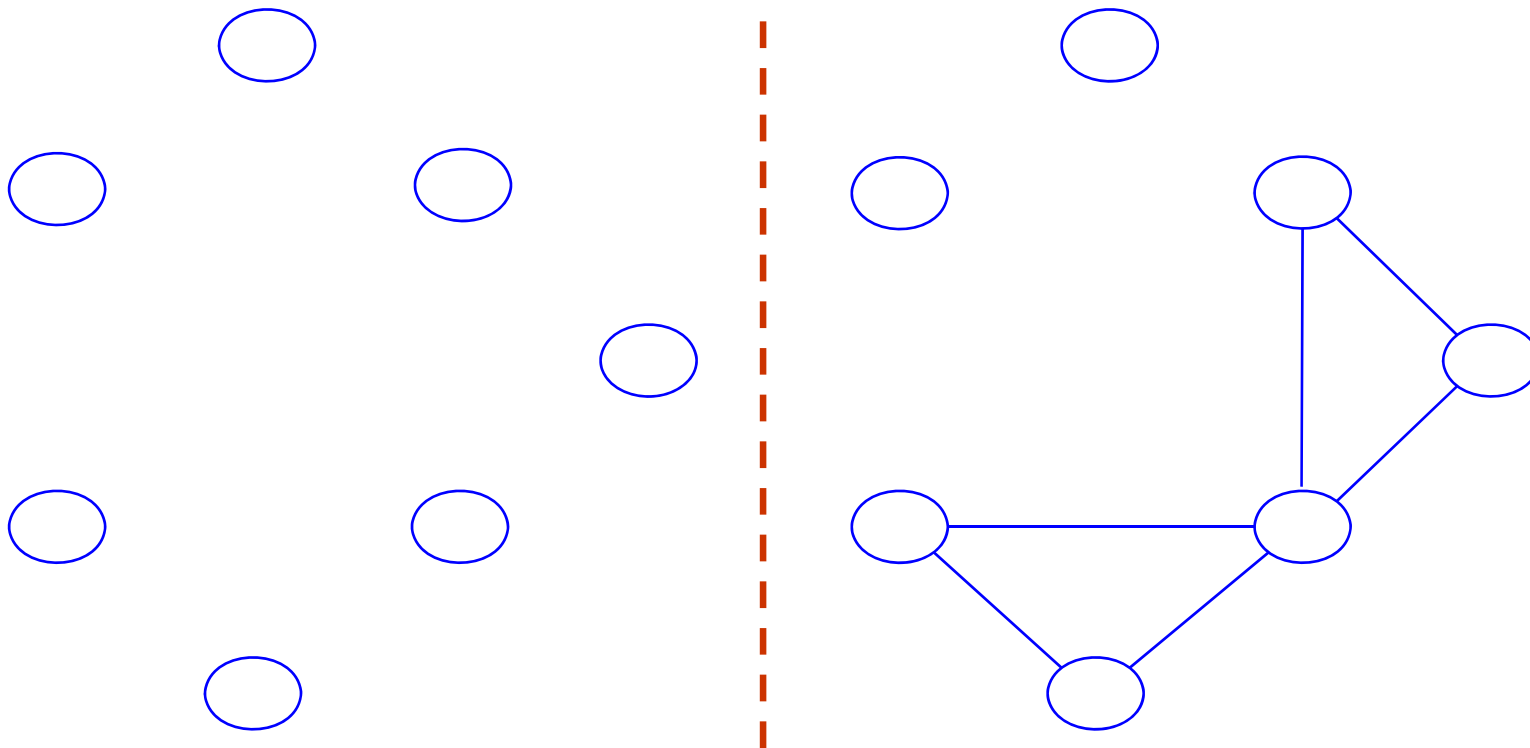
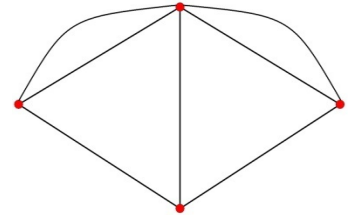
Two ways to support favor exchange:



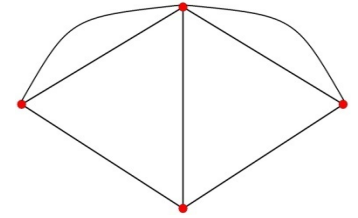
Two ways to support favor exchange:



Two ways to support favor exchange:



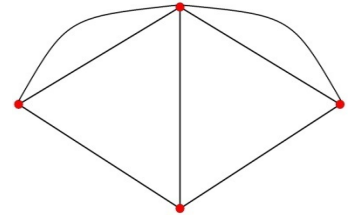
Robustness Against Social Contagion



A network such that the punishment for failing to perform a favor only impacts neighbors of original players lose links

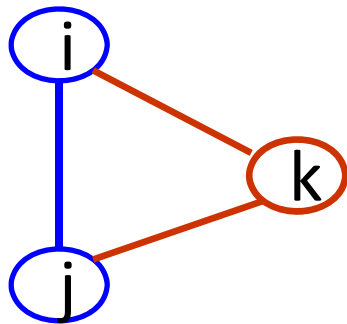
Impact of a deletion/perturbation is local

Supported links:

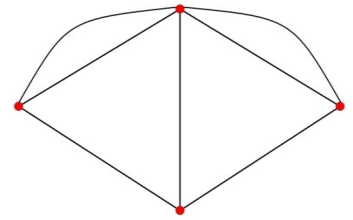


link $ij \in g$ is **supported** if there exists k
such that $ik \in g$ and $jk \in g$

Friend in common:



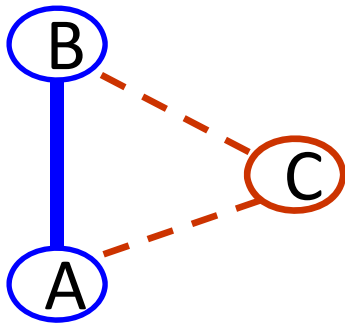
Thm: Implications of the game



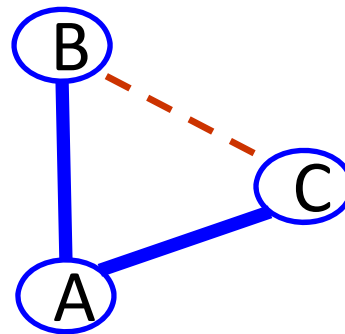
If no pair of players could sustain favor exchange in isolation and a network is robust, then all of its links are supported.

Theory:

Usual Measure:

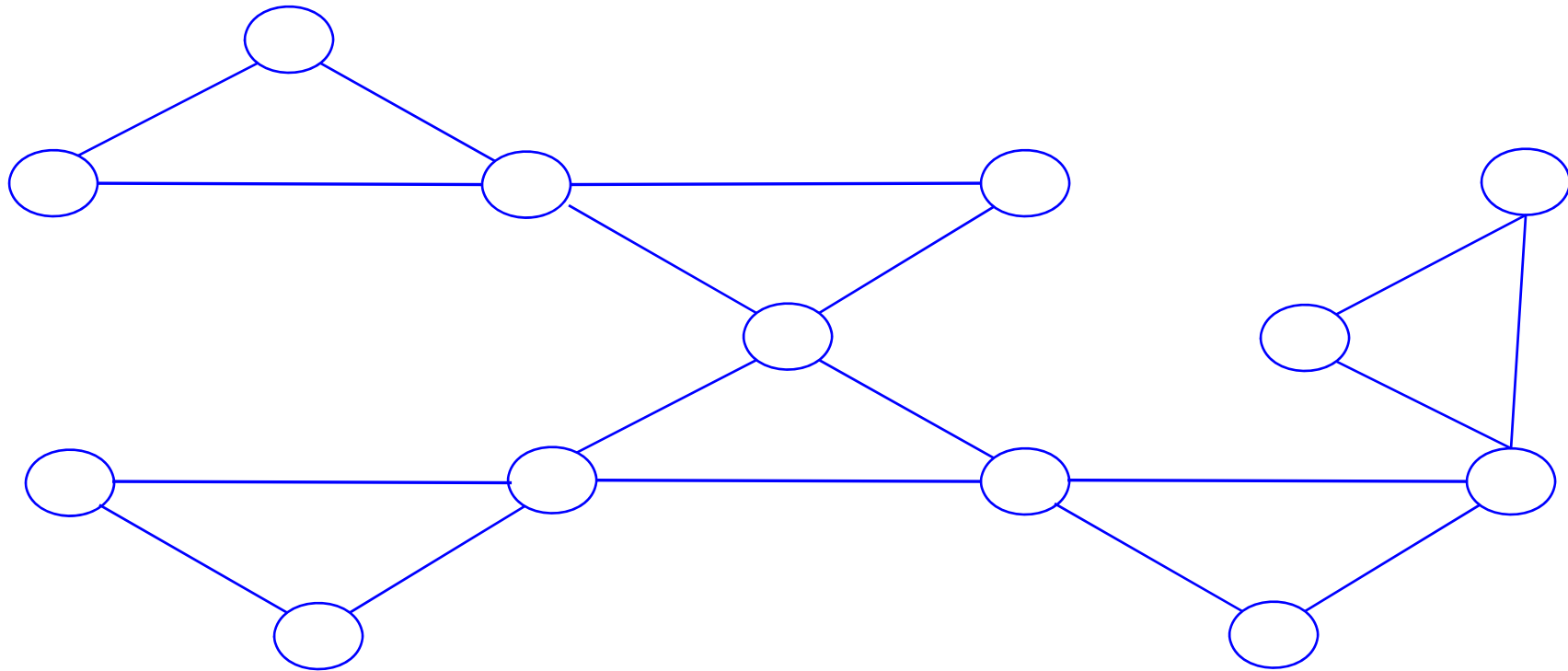
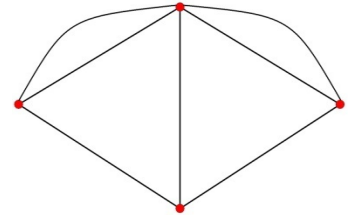


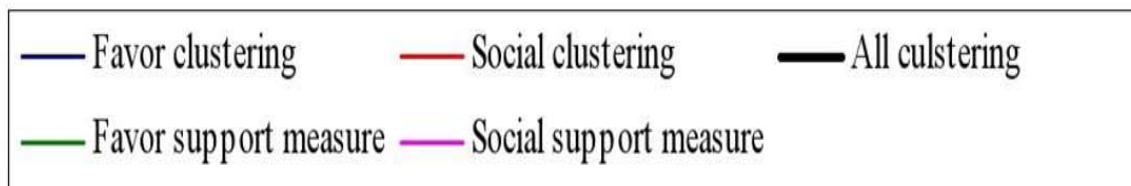
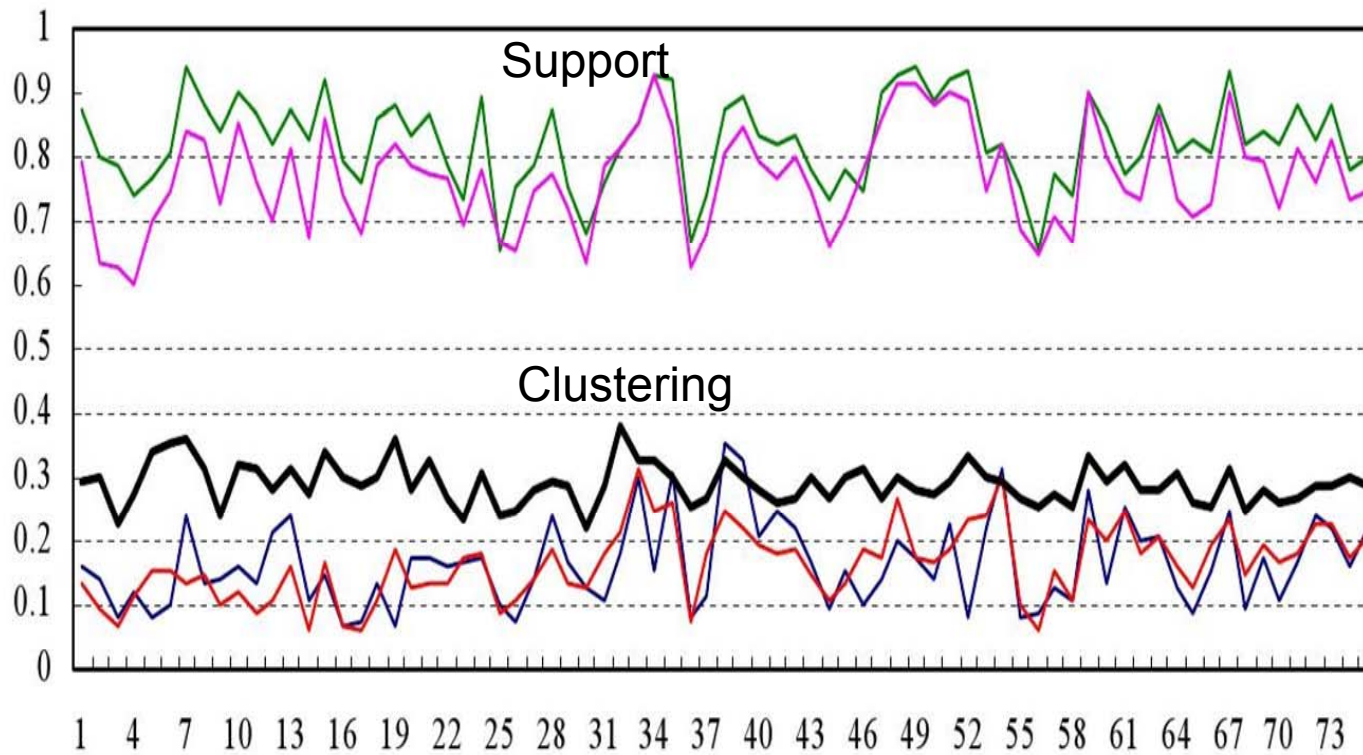
Support: With what frequency do a typical pair of connected nodes, A and B, have a common neighbor?



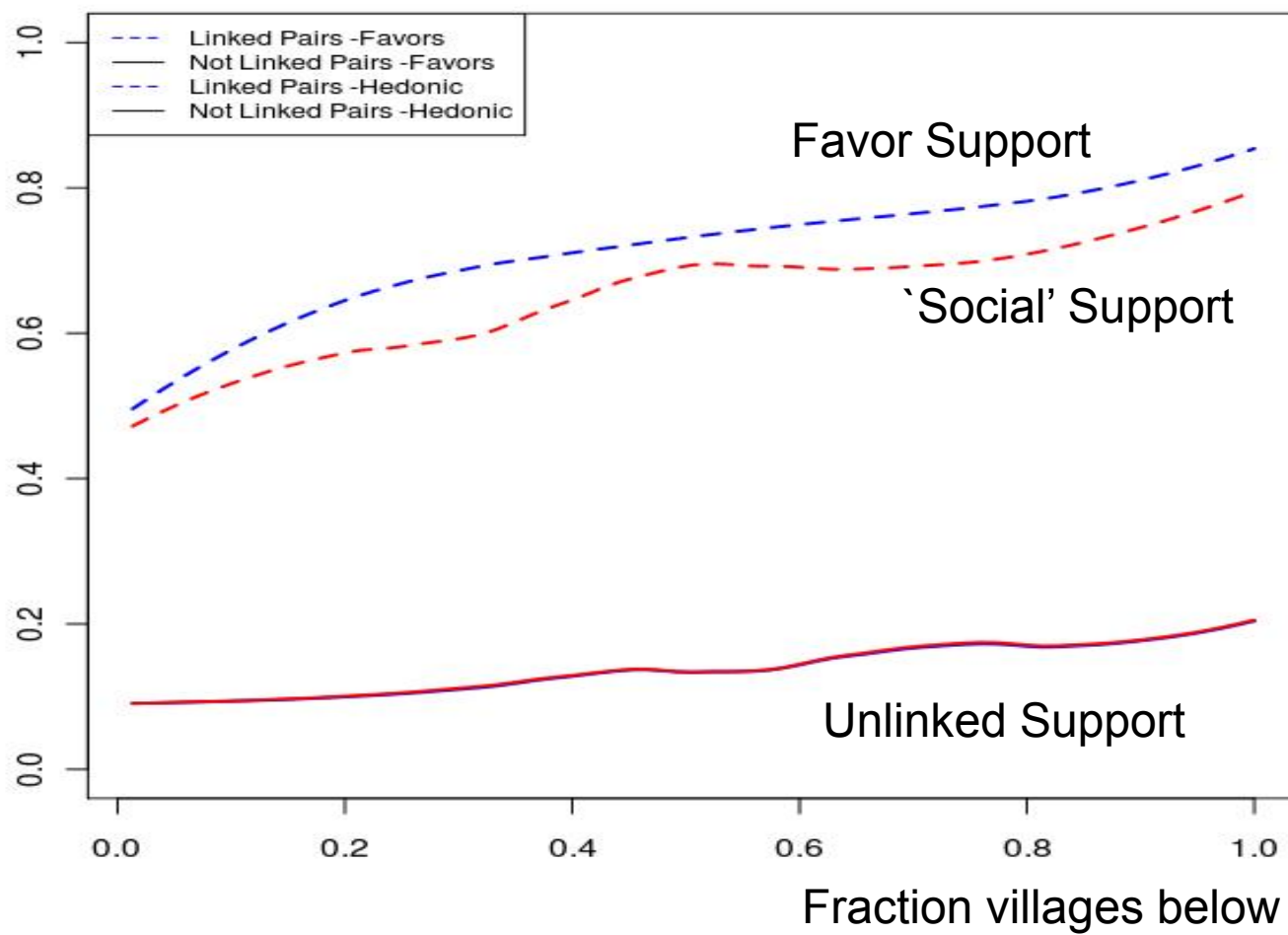
Clustering: With what frequency are a typical node A's neighbors, say B and C neighbors of each other?

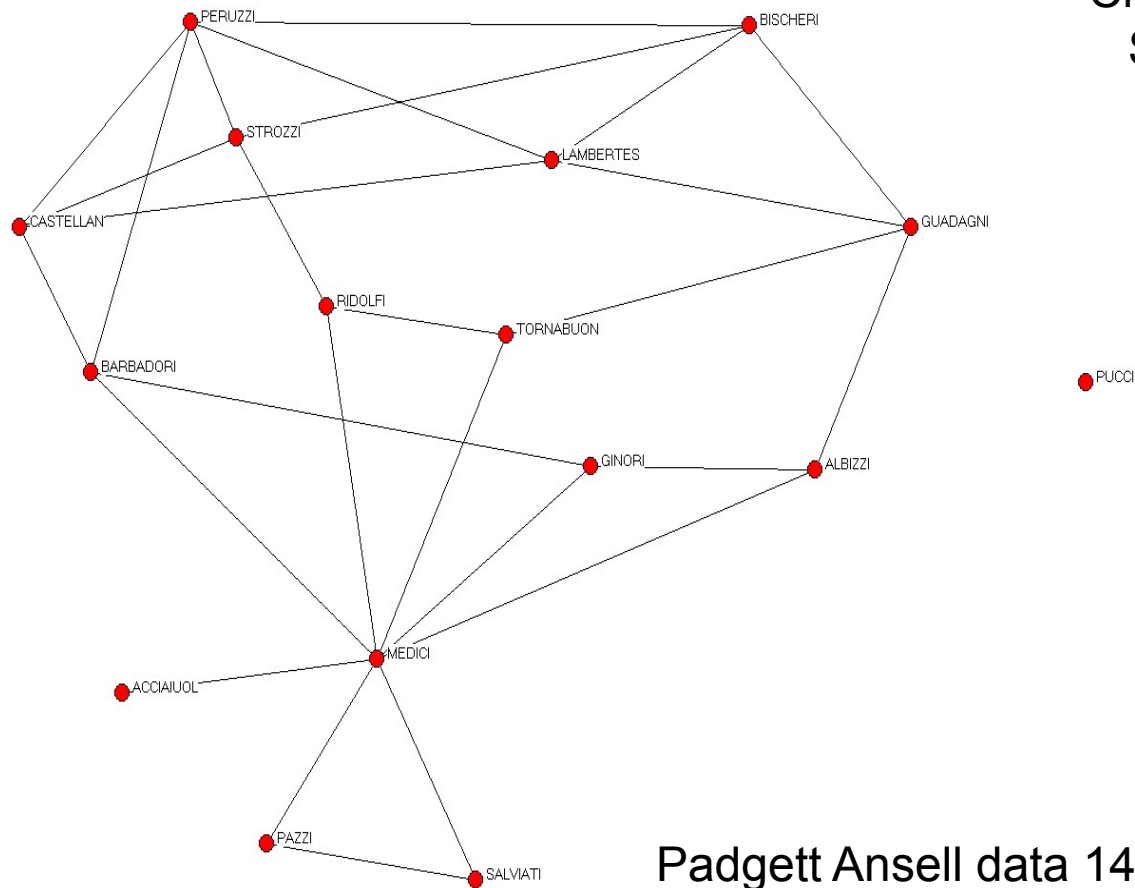
Support=1, Clustering=.47





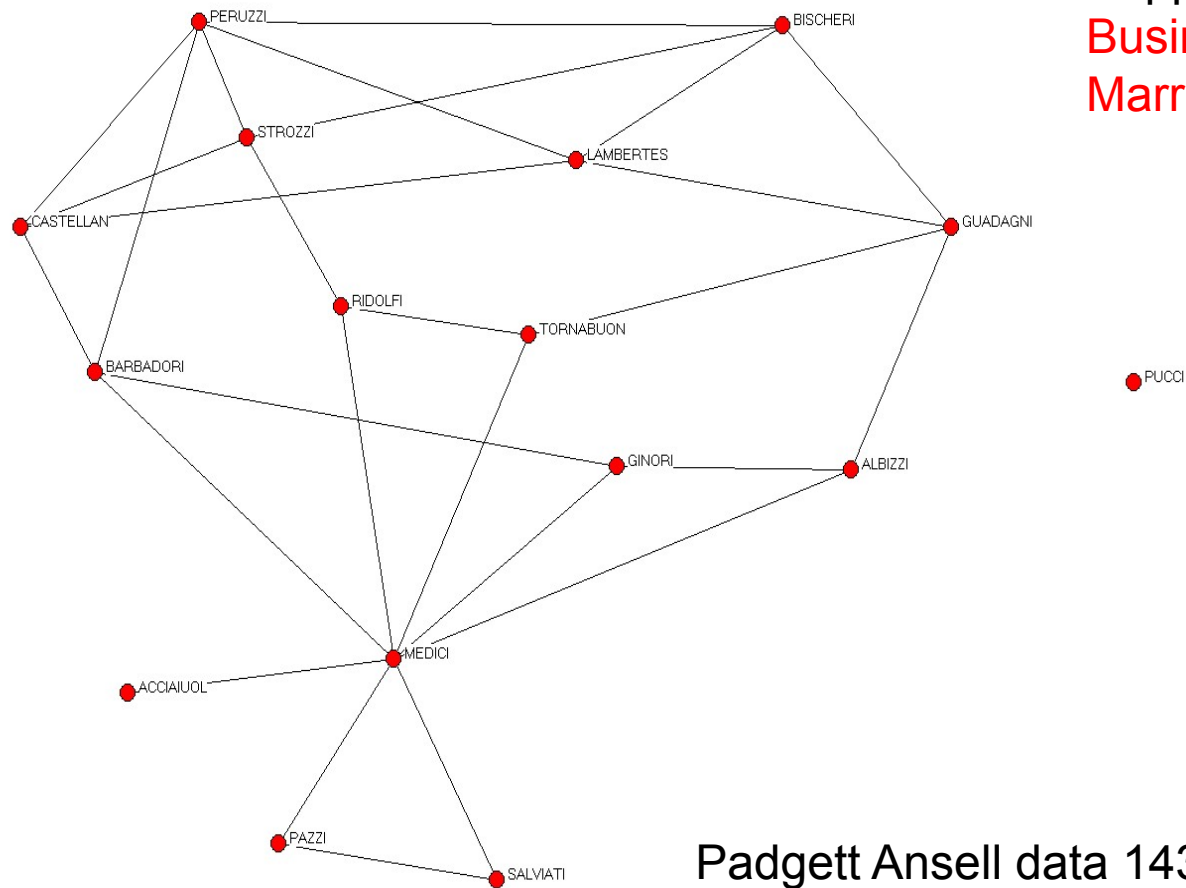
g'=Favors, g=All





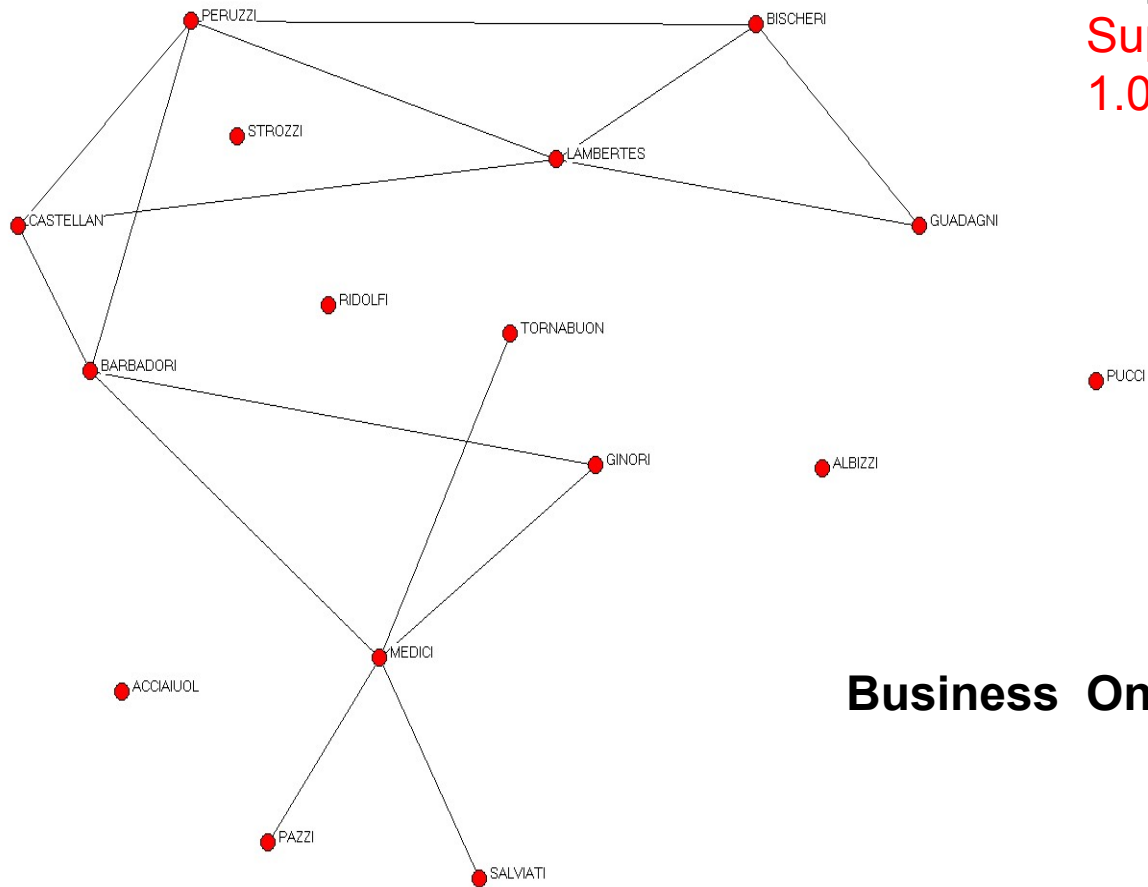
Clustering: .46
Support: .88

Padgett Ansell data 1430's
Florentine marriages
and business dealings



Support: .88
Business 1.0
Marriage .85

Padgett Ansell data 1430's
Florentine marriages
and business dealings

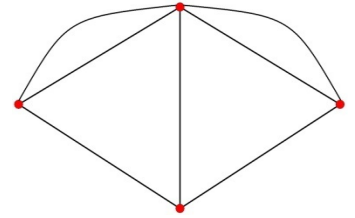


Support: .80

Support with marriage:
1.0

Business Only

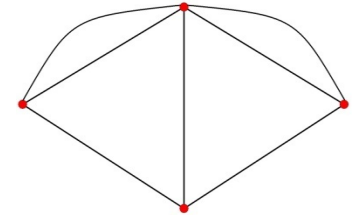
Conclusions



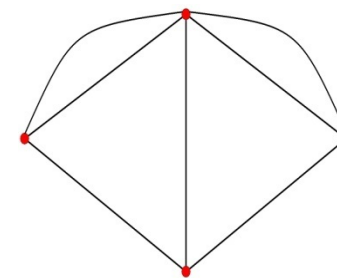
- Robust enforcement gives social quilts;
- Theory for: Support – Friends in Common (which differs significantly from clustering)
- Support is “high” in favor exchange data
 - favor/advice/business networks show significantly more support than purely social

Week 7 Wrap

- Behavior and network structure
 - complements provide nice lattice structure to equilibria
 - substitutes less structured (except best-shot games)
 - comparative statics: higher density – more activity with complements...
 - multiple behaviors related to homophily, cohesion – splits in network allow for different behaviors on different parts of network
 - linear-quadratic games: intensity of behavior depends on position, relates to centrality measures, tractable model



Social and Economic Networks: Models and Analysis

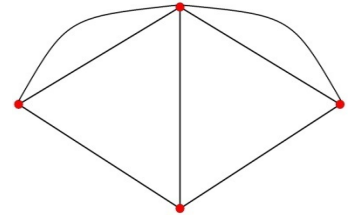


Matthew O. Jackson

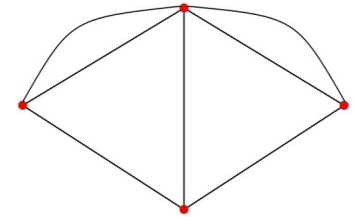
Stanford University,
Santa Fe Institute, CIFAR,
www.stanford.edu/~jacksonm

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7.9: Course Wrap



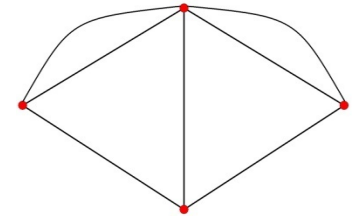
Summary – Games on Networks:



- Strategic Complements and Substitutes exhibit very different patterns
- Position matters:
 - more connected take
 - higher actions in complements (and earlier)
 - lower actions in substitutes
- Structure matters:
 - some networks lead to diffusion of behavior others do not
 - Homophily /cohesion is a critical determinant of diversity of actions

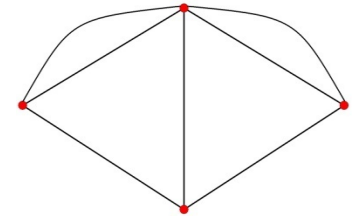
To do list:

- Study impact of homophily, clustering, and other network characteristics on behavior
- More integration behavior with network formation
- Take models of games on networks to data: structural modeling of peer effects

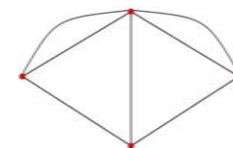


Whither Now?

- Bridging random/economic models of formation
- New statistical models of network formation
- Relate Networks to outcomes –
 - Applications: labor, knowledge, mobility, voting, trade, collaboration, crime, www, risk sharing, ...
 - markets, international trade, growth...
- Co-evolution networks and behavior
- Empirical/Experimental
 - enrich modeling of social interactions from a structural perspective - fit network models to data, test network models
- Foundations and Tools– centrality, power, allocation rules, community structures, ...

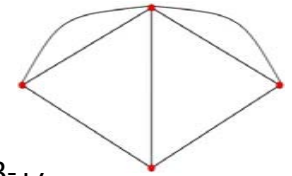


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