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Performance Analysis of Unslotted CSMA/CA in Wireless Networks

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Abstract In this paper a novel analytical model for the saturation throughput of unslotted Carrier Sensing Multiple Access with Collision Avoidance (CSMA/CA) in wireless networks is proposed. A fixed point procedure is developed based on the interaction of the Physical layer (PHY) and the Medium Access Control sublayer (MAC). The output of the Clear Channel Assessment (CCA), i.e. idle or busy medium in the neighborhood of a node, serves as a feedback mechanism for the dynamical scheduling rate controlled by the back-off procedure. The PHY is described by a renewal process between successful transmissions with failed attempts and collided packets in between. A semi-Markov process of the internal states of a node is used as a model for the MAC. An event-driven simulator for the non-beacon enabled IEEE Std 802.15.4$^{\text{TM}}$MAC is developed to verify the numerical results of the analytical method. A detailed analysis of the idle period after a transmission is carried out based on the proposed analytical approach. The probability that the CCA senses the channel idle depends clearly on the actual back-off stage and the first back-off expiration after a transmission cannot be modeled by a exponential distribution when a finite number of nodes are in contention. The output of the event-driven simulations confirms both statements in great detail and the saturated throughput evaluated with the analytical procedure is verified by event-driven simulations.

Keywords saturation throughput · wireless · unslotted CSMA/CA · IEEE Std 802.15.4$^{\text{TM}}$ · analytical modeling · semi-Markov process · event-driven simulation

1 Introduction

The use of wireless transmissions has known an enormous growth since the start of mobile telephony. Large scale wireless networks transporting data and voice are nowadays available. One of the disadvantages of the existing single-hop wireless networks is the overhead in infrastructure. Only transmissions between a terminal and a kind of base station or coordinator are allowed. In a multi-hop network a terminal, called a node, can also play the role of relay. Less infrastructure is needed but the scheduling of transmissions can no longer be coordinated in a centralized way. A main characteristic of the different types of wireless networks is the way how they control the access to the medium. Several access mechanisms are known, e.g. Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA) and Code Division Multiple Access (CDMA). In case of a shared medium for which each node can transmit to every node in its transmission range, FDMA is difficult to implement and CDMA needs a scheme to distributed the codes. TDMA is not possible without external synchronization, e.g. GPS signals or beacons. A new kind of medium access was born in 1970 with the ALOHA protocol. The novel idea is twofold. The data is split into packets and the nodes self make their own decision whether to transmit or not. However if

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two or more nodes transmit at the same time, a collision can occur and the payload of the packet gets lost. These transmission failures can be remedied by a positive acknowledgment scheme (ACK), i.e. retransmitting the packet if no acknowledgment from the receiver is heard by the transmitter. The effective data rate decreases due to the retransmission of the data packets and the transmission of the ACK packets. The node can make a more intelligent decision by first sensing the shared medium before starting a transmission. With Carrier Sensing Multiple Access (CSMA) collisions are avoided by not transmitting if the medium is sensed busy; collisions due to hidden nodes are however still possible. The best known sibling of CSMA is Carrier Sensing Multiple Access with Collision Detection (CSMA/CD) used by the IEEE Std 802.3$^{\mathrm{TM}}$-2005 [1]. In a wireless medium this collision detection is not feasible. In order to further reduce the number of collisions in a wireless channel a random back-off scheme can be implemented to limit the probability that two or more nodes start transmitting at almost the same time. This is called Carrier Sensing Multiple Access with Collision Avoidance (CSMA/CA). The CSMA family of protocols, including ALOHA, comes in two variants: slotted and unslotted. In slotted CSMA the time is divided into time intervals, called slots, and a transmission can only start at a slot boundary. Timing information is included in the transmissions or beacons can be used to synchronize the different nodes in the network. It can be shown that the performance is better with slotted CSMA because the vulnerable period, i.e. the time interval in which two or more nodes can start to transmit simultaneously, is reduced due to the alignment of the time slots. In this paper however the throughput performance of unslotted CSMA/CA is discussed. The disadvantage of the unslotted version is also its forte: the lack of synchronization. The overhead is minimized and the medium access control is completely decentralized. A second reason to model the unslotted CSMA/CA is the impact of collisions due to hidden nodes in a multi-hop wireless network. Two transmitters that are hidden for each other but having a mutual receiver will no longer transmit in a synchronized way and the vulnerable period will be the same for the slotted and the unslotted version. The unslotted CSMA/CA is also easier to implement and the throughput can be higher due to less synchronization overhead.

The throughput of a node in a wireless network depends on the interaction between the Physical layer (PHY) and the Medium Access Control sub-layer (MAC) [3]. On the one hand, the Carrier Sensing Multiple Access / Collision Avoidance (CSMA/CA) functionality of the MAC controls the attempt scheduling discipline of the nodes to access the shared channel through a back-off procedure. On the other hand, the PHY senses the state of the channel and encodes the data to ensure the correct reception of frames. The likelihood of a successful reception depends on how well the channel coding and the modulation defends against noise and interference. The Clear Channel Assessment (CCA) of the PHY can be regarded as a feedback for the dynamic scheduling rate governed by the MAC. The connectivity of the network is determined by the topology of the network, i.e. the relative position of the nodes, and the sensitivity of the receiver. The idle probability of the medium as seen by the sensing mechanism of a node depends on this connectivity and the scheduling rates of the adjoining nodes. With a contention based MAC the relationship between PHY and MAC is not trivial. Besides the collisions due to hidden nodes, they can be caused by the finite propagation delay or by the timing constraints to switch from receive mode to transmit mode and vice-versa. In a real-world implementation the CCA is not a point process and takes a finite amount of time. The simulations and calculations hereafter are modeled after the non-beacon enabled IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] MAC, which uses unslotted non-persistent CSMA with a truncated exponential back-off procedure. The different algorithms and timings are respected as specified in the standard. The used techniques however are fairly general and can easily be adapted to include other unslotted CSMA implementations.

In this paper two relations are proposed to model the basic interaction between the PHY and the MAC. Firstly, the channel occupation as seen by a node can be regarded as a renewal process between successful transmissions with failed attempts in between. The success rate of an individual node is given as a function of the mean back-off delay after a transmission and the probability that this node breaks the idle period and initiates the transmission. Secondly, a semi-Markov chain analysis of the back-off algorithm gives the steady-state back-off probabilities as a function of the probability that the medium is sensed idle by the CCA. This idle probability depends however on the back-off stage. A fixed point procedure can be applied to the complete set of non-linear equations, including the mean back-off delay and the idle probabilities. After convergence, the saturation throughput, i.e. each node has always a packet ready to transmit, can be calculated.

The following hypotheses are made in this paper to make the analytical calculation tractable:

- the physical sensing range of each node equals the transmission range;
- there are no power capture effects;

- the communication channel is error-free;
- the propagation delay is zero;
- packets have a fixed length;
- acknowledgments always arrive reliable and at not cost;
- the internal states of a node are ergodic and
- the system is decoupled, i.e. the carrier sensing probabilities of the nodes are mutually independent.

The first three restrictions make the resulting equations tractable. A method similar to [3] can be used to model a more realistic PHY with noise and fading but the collision probabilities are linearized with a loss in accuracy as a consequence. For moderate values of the fading parameters and a relatively low noise floor the results of the event-simulations are not significantly different compared to the simplified model. In this paper the analysis of the MAC is more important than possible PHY effects. The vulnerable period due to a non-zero propagation delay is much smaller than the vulnerable period due to the time interval needed to switch between receive and transmit mode. The number of collisions in the former case is negligible compared with the latter case and event-driven simulations confirm this hypothesis. The next assumption simplifies the MAC protocol and eliminates collisions with ACKs. The state space of both the renewal process and the semi-Markov chain increases enormously. The ergodic hypothesis is needed to use the steady-state probabilities of the internal states of a node as time average for the N nodes in contention. The decoupling feature is needed to model the internal state of a node independently of the internal states of the others nodes. In [4] a mean field analysis provides a theoretical justification in the case of a slotted CSMA with an explicit exponential back-off algorithm.

The main contributions in this papers are twofold. Firstly, a novel method is proposed to study the interaction between the PHY and the MAC for unslotted CSMA/CA. The PHY is modeled as a renewal process between successful transmission and the MAC is described by a semi-Markov chain. The saturation throughput of the nodes in the network can be calculated. The model is also valid in a multi-hop scenario where the saturation throughput of a node is highly dependent on its connectivity. Secondly, a detailed analysis of the back-off delay for the IEEE Std 802.15.4$^{\text{TM}}$-2006 [2] unslotted MAC is provided, based on both simulations and analytical expressions. The main results are that the output of the CCA depends on the back-off stage and that the use of an exponential distribution to model the idle period after a transmission is not appropriate.

The main differences with existing models are due to the continuous time analysis needed to model the non-persistent unslotted CSMA/CA used in the non-beacon enabled IEEE Std 802.15.4$^{\text{TM}}$-2006 [2] whereas the IEEE Std 802.11$^{\text{TM}}$-2007 [15] uses a persistent slotted CSMA/CA:

- the MAC is described by a semi-Markov chain having states with variable duration instead of an ordinary Markov chain for which the states have all the same duration;
- the conditional collision probability is replaced by the conditional idle probability of a CCA to model the transitions between the states of the semi-Markov process;
- the state transition probability depends on the back-off stage and can be non-homogeneous between different nodes whereas the existing models assume that the state transition probability is identical for all back-off stages and for all nodes and
- the use of an exponential distribution to model the idle period after a transmission is not appropriate and the calculcation scheme uses a realistic description of the MAC to evaluate the idle period after a transmission.

The non-homogeneousness of the state transition probability allows a straightforward generalization to both nodes with different back-off settings and to multi-hop networks for which the topology introduces a heterogeneity between nodes. This is the first paper presenting a realistic saturation throughput model for the non-beacon enabled IEEE Std 802.15.4$^{\text{TM}}$-2006 [2] so that the results can only be verified by simulations.

This paper is organized as follows. In section 2 the related work is discussed and the differences with the approach outlined in this paper are commented. The non-beacon enabled version of the IEEE Std 802.15.4$^{\text{TM}}$-2006 [2] is detailed by calculating the maximum daterate in the case of one transmitter-receiver pair in section 3. To validate the novel analytical method an event-driven simulator is developed. Section 4 describes briefly the implementation of the events and some graphs are presented which give the maximum data-rate as a function of the payload length in the case of one transmitter-receiver pair. Section 5 starts with a description of the channel state as seen by a node. A renewal process between successful transmission attempts is proposed. Then the back-off algorithm based on a semi-Markov model is analyzed and the density function of the minimum back-off delay is established. The probability that the CCA sense the medium idle is evaluated for each back-off stage. The section ends with a description of the fixed point procedure to calculate the saturation throughput. Some graphs of the saturation throughput as a function of the number of competing nodes are shown for different settings of the back-off parameters. The analytical results are verified with the event-simulator. In section 6 a conclusion is formulated and future work is discussed.

2 Related work

The seminal work of Kleinrock and Tobagi, published in several parts [5,6,7,8], gives the basis for the performance analysis of contention based MAC protocols. Both the slotted and the unslotted version of ALOHA and the CSMA variants[1] are detailed and analyzed. They assume that the traffic is generated by an infinite number of nodes who collectively form an independent Poisson source. All packets are of constant length and are transmitted over a noiseless channel. Another assumption states that the interarrival times of the point process defined by the start time of all the packets plus retransmissions are independent and exponentially distributed. Collisions are caused by a small, but non-zero propagation delay which is identical for all transmitter-receiver pairs. Their analysis is based on renewal theory and probabilistic arguments requiring the independence of random variables. The performance is measured as the maximum achievable throughput, called the capacity. The effect of acknowledgment traffic is analyzed in [9] by the same authors. The delay due to the acknowledgment scheme is studied but the turnaround time to switch from a transmission mode to a reception mode and vice versa is considered to be negligible. Collisions with an ACK packet are not modeled. In [10] Takagi and Kleinrock extend the previous works to include a finite number of nodes in the case of non-acknowledged traffic. Not only the mean transmission rate is calculated but also the variance. This allows to determine the coefficients in the diffusion process approximation of a node's queue length distribution. The renewal process between successful transmissions in our paper is based on the previous work. The vulnerable period is no longer the propagation delay but the much larger[2] turnaround time needed to switch from a transmission mode to a reception mode. The duration of the delay between two successive transmission is not modeled as an exponential distribution but evaluated by a semi-Markov process of the internal states of the nodes.

CSMA techniques applied to ground-based packet radio systems showed that there are many situations in which some users cannot hear transmissions from certain other users. Such a situation is called a hidden-terminal problem. Besides the basic introduction of this phenomenon in [6], Takagi and Kleinrock give an approximate output process in the case of hidden nodes in [11]. Not only collisions due to a non-zero propagation delay are taken into account but also the probability that hidden nodes cause collisions is approximately evaluated. The main problem is the dependent behavior of nodes once any transmission in their neighborhood has started, i.e. a node stops initiations of transmissions when a transmission is heard. A solution is proposed by Boorstyn, et al, in [12]. They consider a CSMA with perfect capture and use a Markov model to develop a product form solution to efficiently analyze the throughput of arbitrary topology multi-hop packet radio networks. In [13] the validity of the method is analyzed. A product form solution is only accurate if the corresponding Markov chain is reversible and the CSMA protocol may not introduce a bias in the length statistics of packets rescheduled because of collisions. The first assumption excludes an asymmetric channel and the second states that only fixed length packets can be modeled. In the case of the IEEE Std 802.11™-2003 [15] the latter condition is always met using Request To Send (RTS) and Clear To Send (CTS) messages with the Distributed Coordination Function (DCF). The effect of acknowledgment traffic is analyzed by Boorstyn, et al, in [14] and includes the possibility of collisions with ACK messages. Our analysis of the scheduling rate is based on the view of a specific node, i.e. the method can easily be combined with the product form solution of the Markov model for a multi-hop network with an arbitrary topology. Due to paper size limits this is not discussed in detail.

The analysis of real-world wireless systems has known a breakthrough with the work of Bianchi [16]. He calculated the saturation throughput of a single-hop IEEE Std 802.11™[15] network when N nodes compete. A decoupling strategy is applied and a single tagged node is analyzed. The state of a node is modeled as a Markov process based on the assumption that the conditional collision probability is the same regardless the number of previous attempts. In [17] the saturation throughput is evaluated in non-saturated conditions for which the number of idle states is described by a geometrical distribution. A linearization procedure is introduced in [3] to model a non-error free channel with fading effects. An overview of the fixed point analysis is presented in [18] which uses the decoupling approximation to evaluate the attempt rate per node as the long run average back-off rate per slot. A complete analysis of the impact of hidden nodes on the DCF is given in [19] with an emphasis on the starvation effects. In [20] the concept of a

[1] The different CSMA variants are known as 1-persistent, non-persistent and p-persistent. The protocols differ in their way of handling a busy channel.

[2] In this paper the vulnerable period of 64 μs is large compared to a typical propagation delay of 1 μs.

transmission link, a specific transmitter-receiver pair, is introduced to evaluate the saturation throughput which is highly directional in the case of a network with an arbitrary topology.

Several papers, e.g. [21,22,23], analyze the performance of the IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] with the beacon enabled access mode, i.e. the slotted CSMA/CA. In most cases a similar Markov model as introduced by Bianchi is proposed. However the conditional collision probability is replaced by the conditional idle probability of a CCA and this probability is also the same regardless the back-off stage. An extension to the multi-hop case has not yet been found. The excellent results of the previous papers, inspired several authors to apply the same methodology to the non-beacon enabled access mode. In [24] an overview of the standard is presented and the data rate of a single transmitter-receiver pair is calculated. In section 3 of our paper this data rate is reevaluated to include a real-world CCA which is not a point process and similar results are obtained. The authors of [25] present a model for the non-beacon enabled mode in a star topology. Two successive CCAs, as for the beacon-enabled mode, are considered which is not compliant to the IEEE Std 802.15.4$^{\mathrm{TM}}$-2006. The back-off expiration after a transmission is given by an exponential distribution which is shown to be faulty in our paper. The same principal author proposes a mathematical model based on the busy cycle of a $M/G/1$ queueing system to analyze the throughput and the energy consumption. The turnaround time is considered to be smaller than the CCA so that collisions with ACK messages are avoided which in reality do happen. A non-stationary Markov model is used for which the attempt probabilities are time-dependent in [27]. The idle probability of the CCA is considered to be independent of the back-off stage and the CCA is modeled as a point process. In this paper a stationary semi-Markov process is proposed for which the idle probabilities of the CCA depends on the back-off stage. The non-zero duration of the CCA is analyzed by a renewal process with a non-exponential distribution for the first back-off expiration after a transmission. Each node can be modeled independently so that the analysis can be extended to a product form solution in the case of a multi-hop wireless network.

3 Non-beacon enabled IEEE802.15.4

The IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] specifies the MAC sub-layer and the PHY layer for Low-Rate Wireless Personal Area Networks (LR-WPANs).

The PHY layer is responsible for the data transmission and the reception using a certain radio channel and according to a specific modulation and spreading technique. The IEEE802.15.4 offers three operational frequency bands and corresponding data rates (see Table 1). A Direct Sequence Spread Spectrum (DSSS) technique is used for all frequency bands. The PHY layer is also in charge of the activation and deactivation of the radio transceiver. The turnaround time from transmitting to receiving and vice versa should be no more than 12 symbol periods according to the standard. The PHY layer also performs the Clear Channel Assessment (CCA). The CCA reports the medium activity state: busy or idle. In this paper it is assumed without loss of generality that the medium is busy when the mean power received during the CCA is above the Energy Detection threshold (ED). This is one of the three methods allowed by the IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] to perform the CCA.

Table 1 Frequency bands, data rates and modulation types of IEEE802.15.4$^{\mathrm{TM}}$-2006 [2]

Frequency	Spreading Parameters		Data Parameters	
	Chip Rate	Modulation	Bit Rate	Symbols
868 MHz	300 kchip/s	BPSK	20 kb/s	Binary
915 MHz	600 kchip/s	BPSK	40 kb/s	Binary
2400 MHz	2000 kchip/s	O-QPSK	250 kb/s	16-ary

The MAC sub-layer provides an interface between the PHY layer and higher layer protocols. The MAC sub-layer of IEEE802.15.4 uses Carrier Sense Multiple Access / Contention Avoidance (CSMA/CA) as channel access protocol. The MAC protocol supports two operational modes: beacon-enabled and non beacon-enabled mode. The latter is modeled in this paper. In the non-beacon enabled mode, the nodes can simply send their data by using unslotted CSMA/CA. All messages which have to be transmitted, with the exception of acknowledgment (ACK) frames, must be dispatched according to this mechanism. The use of ACK frames is optional.

The IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] supports a maximum over-the-air data rate of 250 kbps in the 2.4 GHz band, the reference implementation used in this paper. In practice, the effective data rate is lower due to the frame structure and the timings given by the MAC and the PHY.

3.1 Channel Access Timing

The unslotted CSMA/CA mechanism of the IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 [2] uses two variables to schedule the access to the medium:

NB is the number of times the CSMA/CA algorithm was required to back-off while attempting to access

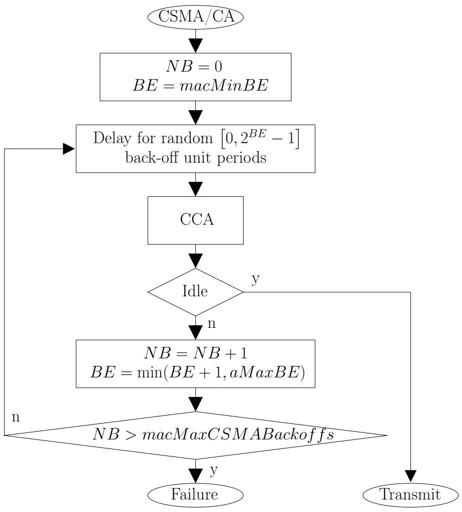

Fig. 1 The unslotted CSMA/CA channel access mechanism of the IEEE Std 802.15.4™-2006 [2]

the current channel. This value is initialized to zero before each new transmission attempt.

BE is the back-off exponent, which is related to how many back-off unit periods a node must wait before attempting to perform a CCA.

Figure 1 depicts a flowchart describing the unslotted version of the CSMA/CA access protocol. It can be summarized in five steps:

1. Initialization of NB and BE: NB is initialized to 0 and BE is set to $macMinBE$, which is equal to 3 by default.
2. Random delay for collision avoidance: the algorithm attempts to avoid collisions by waiting during a given delay randomly generated in the range $\left[0, 2^{BE}-1\right]$ back-off unit periods. One back-off unit period equals $20\,T_s$ with $T_s = 16\,\mu$s.
3. Clear channel assessment: the CCA starts immediately after the expiration of the random delay. If the channel is detected to be in a busy state, the algorithm goes to step 4, otherwise, i.e. the channel is idle, the algorithm goes to step 5.
4. Busy channel: the values of NB and BE are increased by one. However, BE cannot exceed *macMaxBE*, which has a default value of 5. If the number of retries is less than $macMaxCSMABackoffs$, with a default value of 4, the algorithm returns to step 2, otherwise the algorithm terminates with a channel access failure status.
5. Idle channel: the MAC sub-layer starts immediately transmitting its current frame just after the channel is assessed to be idle by the CCA procedure.

Initially the back-off exponent BE is set to $macMinBE$. Using the default value of 3 for $macMinBE$ and assuming the channel is found to be free, the mean access time of the first back-off stage can be calculated as:

$$\overline{T}_{ACCESS} = \overline{T}_{B_1} + T_{CCA} + T_{Rx\to Tx} = 1312\,\mu\text{s} \quad (1)$$

with $\overline{T}_{B_1} = \frac{0+\left(2^3-1\right)}{2}\,20\,T_s$ and $T_{CCA} = 8\,T_s$. $T_{Rx\to Tx}$ is the turnaround time needed to switch from receive mode to transmit mode. The clear channel assessment is performed during the turnaround time in the CC2420 and CC2520 chipsets of Texas Instruments[3]. The use of the STXONCCA command strobe is advised in the non-beacon enabled mode with the unslotted CSMA/CA channel access mechanism and takes $aTurnaroundTime$ ($= 12\,T_s$) as specified by the IEEE Std 802.15.4™-2006 [2]. This behavior is simulated and modeled in this paper.

3.2 Frame Structure

The IEEE Std 802.15.4™-2006 [2] specifies the maximum number of bytes that can be transmitted in the MAC data payload (MSDU) as 114 byte using short addressing (16-bit instead of 64-bit addresses). Fig. 2 gives a schematic view of a data frame. The PPDU is composed of the MPDU (127 byte), the PHR (1 byte) and the SHR (5 byte).

The maximal frame transfer time can be calculated as:

$$T_{DATA} = \frac{(L_{MPDU} + L_{PHR} + L_{SHR})\,8}{R_{PHY}} = 4256\,\mu\text{s} \quad (2)$$

with $R_{PHY} = 250$ kbps.

3.3 Acknowledged Transmission Timing

The use of acknowledgment frames is optional in the IEEE Std 802.15.4™-2006 [2]. The ACK transmission time can be evaluated as:

$$T_{ACK} = \frac{L_{ACK} 8}{R_{PHY}} = 352\,\mu\text{s} \quad (3)$$

where the ACK frame size equals 11 byte. The transmission of an ACK does not use CSMA/CA and starts $aTurnaroundTime$ after the reception of the last byte of the data frame.

To allow the received data to be processed, an inter frame space (IFS) must follow the reception of the ACK frame by the initial transmitter respectively the transmission of the DATA frame in case of an unacknowledged transmission. For MPDUs with a length of up to 18 byte, a short IFS (SIFS) of at least $12\,T_s$ is waited, for larger MPDUs a long IFS (LIFS) of at least $40\,T_s$ is introduced before starting a new transmission. When the back-off delay is in the range $[0, 1]$ back-off

[3] Former Chipcon.

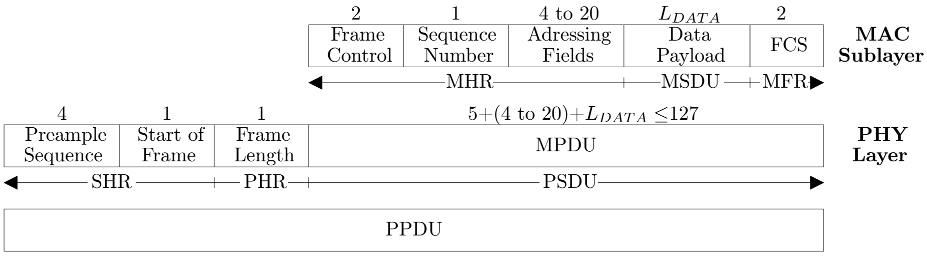

Fig. 2 Schematic view of a data frame

unit periods a delay of 448 μs respectively 128 μs is introduced before the back-off interval. This increases the mean access time after a successful transmission with 72 μs in the case of a maximal data payload:

$$\overline{T}_{ACCESS} = 1384\,\mu\text{s} \quad . \tag{4}$$

It is unlikely that a 0 % packet error rate (PER) will be achieved. The transmitting node will wait a *macAckWaitDuration* (=54 T_s) for an acknowledgment before it attempts a retry. The number of retries, *macMaxFrameRetries* (default value of 3), can be specified.

3.4 Data Rate of a Single Transmitter-Receiver Pair

The maximum effective data rate between a single transmitter receiver pair without acknowledgment can be calculated based on the following assumptions:

- CSMA/CA algorithm never finds that the channel is busy and
- there are no collision between packets,

i.e. the transmitter senses the channel always idle during the first back-off stage:

$$R_{DATA} = \frac{L_{MSDU} 8}{\overline{T}_{ACCESS} + T_{DATA}} = 161.7\,\text{kbps} \tag{5}$$

where $L_{MSDU} = 114$ byte.

With acknowledgment the previous assumptions are extended:

- No retries are required.

$$R_{DATA,ACK} = \frac{L_{MSDU} 8}{\overline{T}_{ACCESS} + T_{DATA} + T_{Rx\to Tx} + T_{ACK}} = 147.5\,\text{kbps} \tag{6}$$

where $L_{MSDU} = 114$ byte.

4 Simulator

4.1 Main Simulation Loop

A MAC event-driven simulator has been developed to validate the analytic results. It mimics with a high degree of precision the behavior of the IEEE Std 802.15.4$^{\text{TM}}$-2006 [2] MAC in non-beacon enabled mode with the unslotted CSMA/CA procedure.

The different steps during the transmission and reception of a frame are implemented as events containing a timestamp. An efficient binary heap is used as a queue for future events. In the main simulation loop, the following sequence is repeated until the end of the simulation: the event with the smallest timestamp is popped out of the heap, the current simulation time, $t_{current}$, is updated and the event is executed. During the execution of the event, new events can be pushed into the heap.

The simulator is written in fortran 2003 [30] and the object oriented features of this recent ISO standard are used to ease the implementation, e.g. events are implemented as fortran types. Both data and procedures can be linked to a type and inheritance can be used to eliminate redundancy in the code.

4.2 Channel matrix

The channel is modeled by a matrix where the rows correspond to transmitting nodes and the columns to receiving nodes. In each cell of this matrix the state of the transmitting node, the last modification time and the received power are stored.

The propagation model used in section 5 is based on a fixed transmission range, which also equals the sensing range. The code is however pluggable and a module with a Rayleigh fading model with additive Gaussian noise is also implemented. Distances are calculated from 2D coordinates and the sensing range can be different from the transmission range. For moderate values of the fading parameters and a relative low noise floor the results are not significantly different than with the basic propagation model.

4.3 Events

The unslotted version of the CSMA/CA MAC with ACK can be implemented in only 5 events, which are always linked to the node transmitting the DATA frame. Fig. 3 represents the event generation flow.

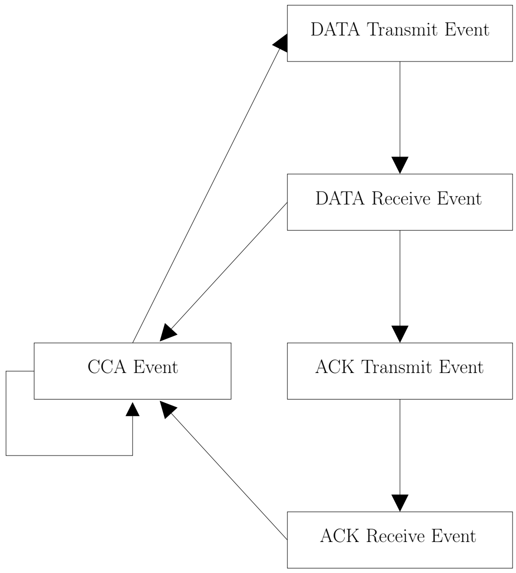

Fig. 3 Schema depicting the event generation flow in the simulator.

4.3.1 CCA Event

The channel matrix is checked for an ongoing transmission between $[t_{current} - T_{CCA},\ t_{current}]$ by comparing the mean received power to the ED. If the channel is idle a DATA Transmit Event is pushed into the heap to be scheduled at $t = t_{current} + T_{Rx\rightarrow Tx}$, else BE and NB are updated, a new random back-off period is chosen and a CCA event is pushed into the heap with the timestamp set to $t = t_{current} + T_B + T_{CCA}$.

4.3.2 DATA Transmit Event

The mean received power is calculated by the pluggable propagation model. The state of the cells in the channel matrix with the rows corresponding to the transmitting node is updated and the modification time and received power are changed. A DATA Receive Event is scheduled at time $t = t_{current} + T_{DATA}$.

4.3.3 DATA Receive Event

The channel matrix is checked for simultaneous transmissions heard by the receiver between $[t_{current} - T_{CCA},\ t_{current}]$. The received power indicated in the cells of the channel matrix corresponding to the transmitting node, is set to the noise level and the state is toggled. BE and NB are reset and a new random back-off period is chosen. Without the use of acknowledgment frames, a CCA Event with a new packet is pushed into the heap with the timestamp set to $t = t_{current} + \max{(T_B + T_{CCA}, t_{IFS})}$. The packet is lost if a collision happens. When the ACK procedure is used, an ACK Transmit Event is scheduled at time $t = t_{current} + aTurnaroundTime$ if no other transmission collides at the receiver, else a CCA Event is pushed into the heap to be scheduled at $t = t_{current} + macAckWaitDuration + T_B + T_{CCA}$ and the retry count is incremented. If the retry limit is exceeded, the packet is lost and a new packet is created.

4.3.4 ACK Transmit Event

The mean received power is calculated by the pluggable propagation model. The state of the cells in the channel matrix with the rows corresponding to the node, which is sending the ACK, is updated and the modification time and received power are changed. An ACK Receive Event is scheduled at time $t = t_{current} + T_{ACK}$.

4.3.5 ACK Receive Event

The channel matrix is checked for simultaneous transmissions heard by the receiver between $[t_{current} - T_{CCA},\ t_{current}]$. The received power indicated in the cells of the channel matrix corresponding to the transmitting node, is set to the noise level and the state is toggled. BE and NB are reset and a new random back-off period is chosen. If the ACK is successful received, a CCA Event with a new packet is pushed into the heap with the timestamp set to $t = t_{current} + \max{(T_B + T_{CCA}, t_{IFS})}$, else a CCA Event is scheduled at time $t = t_{current} - aTurnaroundTime - T_{ACK} + macAckWaitDuration + T_B + T_{CCA}$ and the retry count is incremented. In the latter case if the retry limit is exceeded, the packet is lost and a new packet is created.

4.4 Data Rate of a Single Transmitter-Receiver Pair

To validate the simulator, the saturation data-rate as a function of the payload length is simulated and the results are shown in figure 4. Each simulation run simulates the behavior of the saturated node during 1000 seconds. Both the scheme with acknowledgment and without acknowledgment are evaluated by equations 5 and 6 for different settings of the back-off parameters and the results are compared with the simulations. No anomalies can be observed. The bump at $L_{DATA} = 18\,\text{byte}$ is due to the IFS which changes from SIFS to LIFS.

One run of the simulator takes less than 0.3 s CPU time on a recent 3 GHz Intel processor and during that time almost one million events are executed.

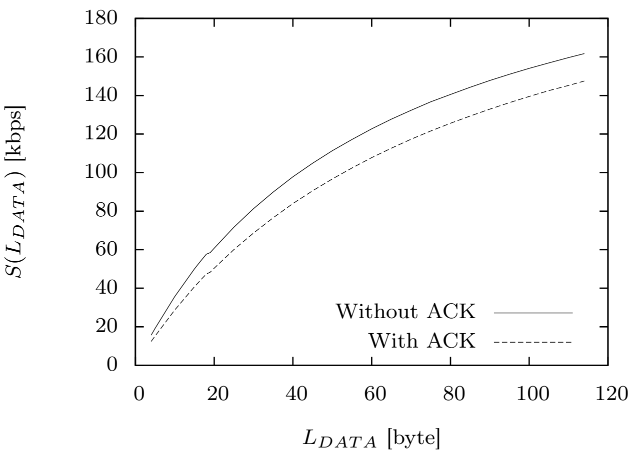

Fig. 4 The saturation throughput of a single transmitter-receiver pair as a function of the length of the payload L_{DATA}

5 Analytical Model of the Interaction between PHY and MAC

5.1 Non-persistent CSMA Renewal Process

Let $\left\{X^{(k)}; k \in \mathbb{N}_0\right\}$ be a sequence of packet interdeparture times (from the entire system) beginning at the end of an arbitrarily chosen successful transmission ($t = 0$). These interdeparture times are independent and identically distributed. If the CSMA protocol is memoryless then

$$Z^{(n)} = \sum_{k=1}^{n} X^{(k)} \quad n \in \mathbb{N}_0 \tag{7}$$

defines the time at which the nth successful transmission completes. By definition $\left\{Z^{(n)}; n \in \mathbb{N}_0\right\}$ is a renewal process; see for example [31]. For time $t > 0$, let $D(t)$ be the number of successful transmissions completed during an interval $[0, t]$:

$$D(t) = \max_{Z^{(n)} \leq t} (n) \quad . \tag{8}$$

Renewal theory states that

$$\lim_{t \to \infty} \frac{D(t)}{t} = \frac{1}{\overline{X}} = S \tag{9}$$

where S is the channel throughput expressed in packets per second.

Let $\forall i \in \{1, \ldots N\} : D_i(t)$ be the number of successful transmissions completed by node i and let d_i be the probability that a successful transmission is achieved by node i; $\sum_{i=1}^{N} d_i = 1$. In the case where all the nodes have the same back-off settings, d_i equals $\frac{1}{N}$. In the general case the interdeparture times between successful transmission of the same node are not identically distributed although they are independent. In [10] it is shown that for

$$D_i(t) = \max_{Z_i^{(n)} \leq t} (n) \tag{10}$$

the asymptotes still exist

$$\lim_{t \to \infty} \frac{D_i(t)}{t} = \frac{1}{\overline{X_i}} \tag{11}$$

with

$$D_i(t) = d_i D(t) \tag{12}$$

and the throughput of an individual node is then

$$S_i = d_i S \tag{13}$$

5.2 Modeling the IEEE Std 802.15.4$^{\text{TM}}$-2006 CSMA/CA

Figure 5 illustrates how X is composed of $j-1$ unsuccessful transmissions and one successful transmission $Y^{(j)}$. The channel throughput can then be evaluated by

$$S = \frac{\gamma}{\overline{T}_{ACCESS} + T_{DATA} + (1-\gamma)\overline{T}_{COL}} \tag{14}$$

where γ is the success probability of a transmission and $\overline{T}_{COL}$ is the mean transmission start delay of the last colliding packet in reference to the start of the transmission breaking the channel idle period. The mean access period $\overline{T}_{ACCESS}$ equals the sum of the delay between the previous transmission and the first back-off timer expiration, the time to evaluate a CCA and the time to switch from receive mode to transmit mode:

$$\overline{T}_{ACCESS} = \overline{\min_i (t_i)} + T_{CCA} + T_{Rx \to Tx} \quad . \tag{15}$$

To calculate the first term, the classic CSMA literature [5] assumes that the interarrival times of the point process defined by the CCA attempts are independent among the different nodes and that they are all exponentially distributed. Simulations have shown however that the back-off expiration times after a transmission are mutually independent but not exponentially distributed. In subsection 5.4 the exact density function $f_i(t)$ of the point process is analytically established and compared with the simulation results.

After a transmission, successful or failure, all nodes have a packet ready to transmit due to the saturation scenario and are in a back-off state. Since the first back-off expiration after a transmission is the minimum of all nodes back-off delays, its distribution is given by

$$\begin{aligned} F(t) = &= \Pr\left(\min_i (t_i) \leq t\right) = 1 - \prod_{i=1}^{N} \Pr(t_i > t) \\ &= 1 - \prod_{i=1}^{N} \left(1 - \int_0^t f_i(t_i)\,\mathrm{d}t_i\right) \quad . \end{aligned} \tag{16}$$

The mean value of the minimum access delay $\overline{T}_{ACCESS}$ becomes

$$\overline{T}_{ACCESS} = \int_0^{+\infty} t\mathrm{d}F(t) + T_{CCA} + T_{Rx \to Tx} \quad . \tag{17}$$

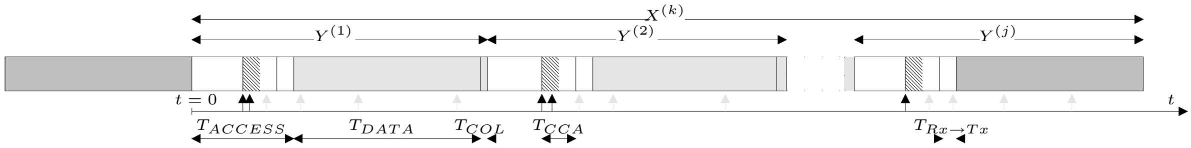

Fig. 5 Packet interarrival times X for the IEEE Std 802.15.4$^{\mathrm{TM}}$-2006 CSMA; CCA attempts are indicated by arrows and the vulnerable period is shaded, i.e. more than one CCA attempt during this interval causes a collision

The probability that node i among others breaks the idle period can be calculated

$$\nu_i = \Pr(t_i \leq \min_{j \neq i} t_j) = \int_0^{+\infty} f_i(t_i) \int_{t_i}^{+\infty} \mathrm{d}F^*(t)\mathrm{d}t_i \quad (18)$$

where $F^*(t)$ is the minimum back-off delay after a transmission of all nodes except the node i

$$F^*(t) = 1 - \prod_{j=1; j \neq i}^{N} \left(1 - \int_0^t f_j(t_j)\,\mathrm{d}t_j\right) \quad . \quad (19)$$

Collisions can occur if one or more nodes start a CCA after the CCA attempt of a first node and if their CCAs ends before the start of the transmission of that first node. The vulnerable period in which simultaneous CCAs can happen has a duration $T_{Rx \to Tx}$ after the start of the first CCA. The probability of success in a cycle where node i initiates the transmission period is given by

$$\gamma_i = \int_0^{+\infty} f_i(t_i) \int_{t_i + T_{RX \to Tx}}^{+\infty} \mathrm{d}F^*(t)\mathrm{d}t_i \quad (20)$$

and the probability that a transmission is successful, γ, can be evaluated

$$\gamma = \sum_{i=1}^{N} \nu_i \gamma_i \quad . \quad (21)$$

The time between the start of the first and the last colliding transmission, $\overline{T}_{COL}$, has a value in the range $[0; T_{Rx \to Tx}]$ which is small compared to the cycle length. An upper bound and a lower bound of the effective data rates are obtained by setting respectively $\overline{T}_{COL} = 0$ and $\overline{T}_{COL} = T_{Rx \to Tx}$.

5.3 Internal Semi-Markov Model

The internal states of a node can be described by a semi-Markov model. In [31] a semi-Markov process is defined as a process with a finite number of states. Each time the process enters a state i it remains there for a random amount of time with mean μ_i and then makes a transition into state j with probability P_{ij}. Fig. 6 depicts the semi-Markov chain for a node which does not use the acknowledgment scheme. Two logical states can be distinguished:

- Back-off and Clear Channel Assessment state, noted by CB_i where i indicates the back-off stage, and
- Transmitting and Inter Frame Space state, noted by T.

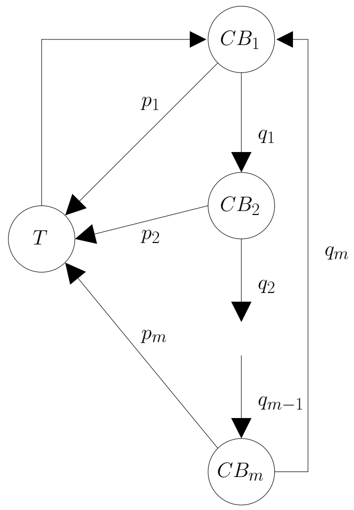

Fig. 6 Internal Semi-Markov Chain of a node without ACK

Table 2 The idle probabilities of the different back-off stages in the internal semi-Markov model without the use of acknowledgments

$macMinBE$	3	3	3	4	3	4
$macMaxBe$	5	5	5	5	3	4
m	5	5	5	5	5	5
N	3	5	10	5	5	5
L_{MSDU}	114	114	114	114	114	114
p_1	0.47	0.27	0.12	0.23	0.09	0.15
p_2	0.19	0.14	0.10	0.20	0.06	0.14
p_3	0.20	0.15	0.10	0.20	0.09	0.14
p_4	0.20	0.15	0.10	0.20	0.10	0.14
p_5	0.20	0.15	0.10	0.20	0.09	0.14

The transition probabilities are expressed as a function of p_i and $q_i = 1 - p_i$, where i is the index of the previous back-off stage. The probability p_i is called the idle probability of the ith back-off stage. Table 2 shows the corresponding idle probabilities from the event simulation for different values of $macMinBE$, $macMaxBE$, $macMaxCSMABackoffs$ $(= m-1)$ and N, the number of transmitting nodes. One of the main assumptions in the literature of CSMA modeling states that $p_1 = p_2 = \ldots = p_m$ and that this value equals the steady-state idle probability of the medium, P_I. In this paper the idle probabilities p_i can be different for each back-off stage. Figure 7 illustrates this behavior. For a large number of nodes in contention the idle probability no longer depends on the back-off stage. The idle prob-

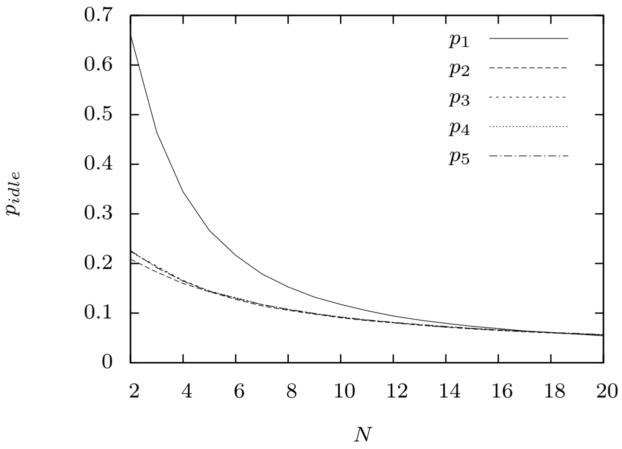

Fig. 7 Event-driven simulation of the idle probabilities for each back-off stage with $macMinBE = 3$, $macMaxBE = 5$ and $macMaxCSMABackoffs = 4$ as a function of the number of nodes N

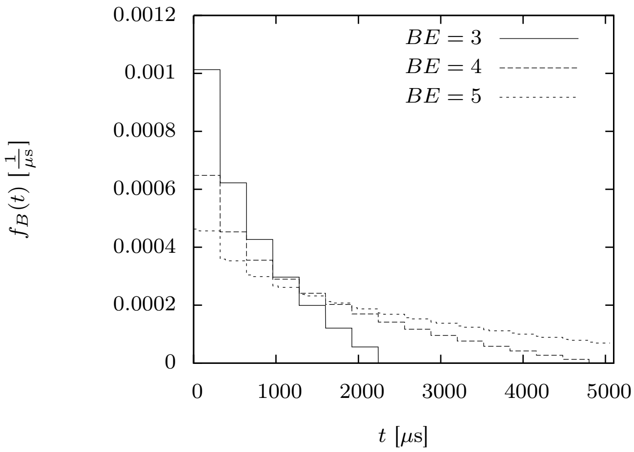

Fig. 8 The density function $f_B(t)$ for $BE = 3$, $BE = 4$ and $BE = 5$ with $\overline{T}_{ACCESS} + T_{DATA} = 5055$

abilities for each back-off stage are analytically modeled in subsection 5.5. The figure shows clearly that the idle probability of the first back-off stage differs significantly from the other idle probabilities for a small number of nodes N.

The transmission scheduling rate of a node is calculated by first expressing the steady-state probabilities of the embedded Markov-Chain:

$$\begin{aligned}
\pi_{CB_{i+1}} &= q_i \pi_{CB_i} \\
\pi_T &= \sum_{i=1}^{m} p_i \pi_{CB_i} \\
1 &= \sum_{i=1}^{m} \pi_{CB_i} + \pi_T
\end{aligned} \tag{22}$$

with $m = macMaxCSMABackoffs + 1$. The steady-state probabilities of the different states are then given by

$$\begin{aligned}
P_{CB_i} &= \frac{\pi_{CB_i} T_{CB_i}}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T} \\
P_T &= \frac{\pi_T T_T}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T}
\end{aligned} \tag{23}$$

where $T_{CB_i} = T_{B_i} + T_{CCA}$ and $T_T = T_{Rx \to Tx} + T_{DATA} + T_{IFS}$. The scheduling rate can be evaluated considering the steady-state probability that a node enters a state

$$g = \frac{\pi_T}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T} \quad . \tag{24}$$

5.4 Distribution of the Back-off Delays

After a successful transmission or a failure, all nodes have a packet ready to transmit due to the saturation assumptions. The nodes involved in the previous transmission are in the inter frame space (IFS) state while the others are in one of the back-off states. The steady-state probabilities of the internal node state can be expressed as a function of the probabilities of the embedded Markov-Chain:

$$\begin{aligned}
P_T &= \frac{\pi_T T_T}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T} \\
P_{B_1} &= \frac{(\pi_{CB_1} - \pi_T)\, T_{B_1}}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T} \\
P_{B_i} &= \frac{\pi_{CB_i} T_{B_i}}{\sum_{i=1}^{m} \pi_{CB_i} T_{CB_i} + \pi_T T_T}; \; i \neq 1 \quad .
\end{aligned} \tag{25}$$

In case of back-off stage i, the back-off period has a discrete uniform distribution with values in the range of $\left[0, 2^{BE_i} - 1\right]$ back-off unit periods. Considering the fact that the node is in the specific back-off state, the density function of the back-off delay after a transmission and before performing a CCA is then given by

$$f_{B_i}(t) = \frac{1}{2^{BE_i}} \sum_{j=0}^{2^{BE_i}-1} \chi\left(0 \le t \le j\, 20\, T_S\right) \left(\frac{1}{j\, 20\, T_S}\right) \tag{26}$$

where χ is the indicator function. If $\overline{T}_{ACCESS} + T_{DATA} > j\, 20\, T_S$ the density function is truncated at $\overline{T}_{ACCESS} + T_{DATA}$ and the remainder is added at $t = 0$. The part of the density function after the truncation attributes to the delay of the next CCA. Figure 8 shows the density function of the back-off delay after a transmission for different values of BE when the node is in a back-off state.

In case the node has just finished a transmission the density function is discrete

$$f_T(t) = \frac{n}{2^{macMinBE} - 1} \tag{27}$$

where $n = 1$ for values of $t = 2^i\, 20\, T_S > T_{IFS} - T_{CCA} - T_{Rx \to Tx}$ and n equals the number of times $2^i\, 20\, T_S \le T_{IFS} - T_{CCA} - T_{Rx \to Tx}$ for $t = T_{IFS} - T_{CCA} - T_{Rx \to Tx}$. Figure 9 shows the density function of the back-off delay

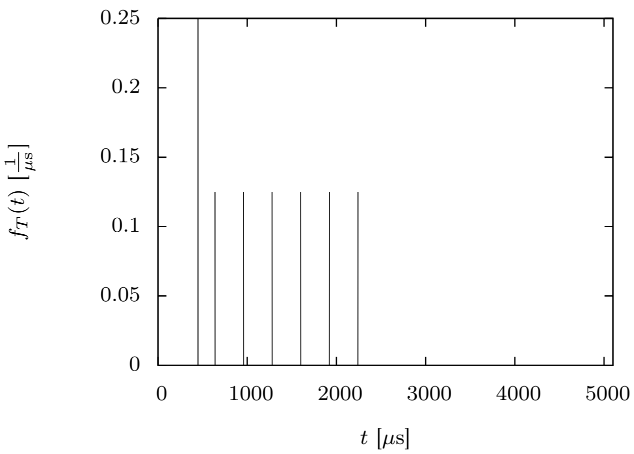

Fig. 9 The density function $f_T(t)$ for $macMinBE = 3$ with $\overline{T}_{ACCESS} + T_{DATA} = 5055$

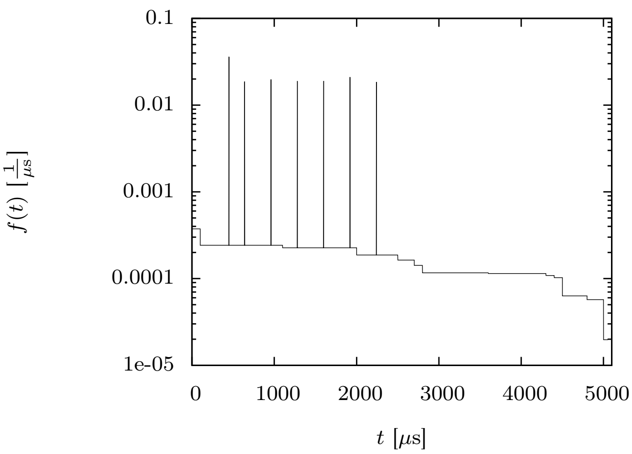

Fig. 11 The simulated weighted density function $f(t)$ with $macMinBE = 3$, $macMaxBE = 5$, $macMaxCSMABackoffs = 4$

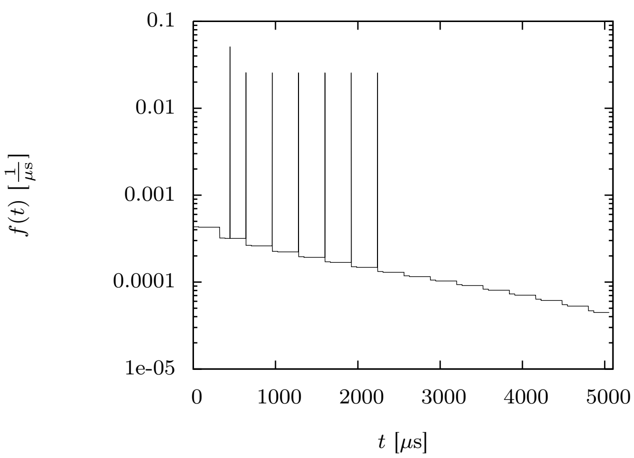

Fig. 10 The weighted density function $f(t)$ with $macMinBE = 3$, $macMaxBE = 5$, $macMaxCSMABackoffs = 4$

after a transmission for $macMinBE$ when the node is in an IFS state after the previous transmission.

By taking the weighted sum of all these functions the density function of the back-off delay of one node after a transmission can be expressed as

$$f(t) = P_T^* f_T(t) + \sum_{i=1}^{m} P_{B_i}^* f_{B_i}(t) \tag{28}$$

with

$$P_T^* = \frac{P_T}{P_T + \sum_{i=1}^{m} P_{B_i}}$$

$$P_{B_I}^* = \frac{P_{B_i}}{P_T + \sum_{i=1}^{m} P_{B_i}} \quad .$$

Figure 10 shows the weighted density function which is a superposition of an almost exponential function with a discrete spectrum. Figure 11 gives the same graph obtained by simulation. The discrete part of the density function is due to the first back-off stage after a transmission and is the main reason for the high value of p_1.

5.5 Calculation of the Idle Probabilities

Figure 7 illustrates the variance of the idle probability of each back-off stage p_i as a function of the number of nodes N. For small values of N the idle probability of the first back-off stage is significantly higher than the others. In the previous subsection it is shown by simulation and analytical modeling that this behavior is caused by the discrete part of the weighted density function.

The probability that a CCA attempt of a node which has not participated in the previous transmission, is idle, is found to be independent of the back-off stage. This idle probability can be approximated by the steady state channel idle probability P_I

$$P_I = \frac{\overline{T}_{ACCESS} - T_{CCA}}{\overline{T}_{ACCESS} + T_{DATA} + (1-\gamma)\overline{T}_{COL}} \tag{29}$$

where $\overline{T}_{COL}$ can be approximated by $\frac{T_{Rx \to Tx}}{2}$.

The probability that a CCA attempt of a node which has just finished a transmission, is idle, needs a more involved calculation. Let ν_i^* be the probability that this node, indicated by i, among all others breaks the idle period

$$\nu_i^* = \Pr(t_i \leq \min_{j \neq i} t_j) = \int_0^{+\infty} f_T(t_i) \int_{t_i}^{+\infty} \mathrm{d}F^*(t)\mathrm{d}t_i \tag{30}$$

where $F^*(t)$ is the minimum back-off delay after a transmission of all nodes except the node i

$$F^*(t) = 1 - \prod_{j=1; j \neq i}^{N} \left(1 - \int_0^t f_j(t_j)\,\mathrm{d}t_j\right) \quad . \tag{31}$$

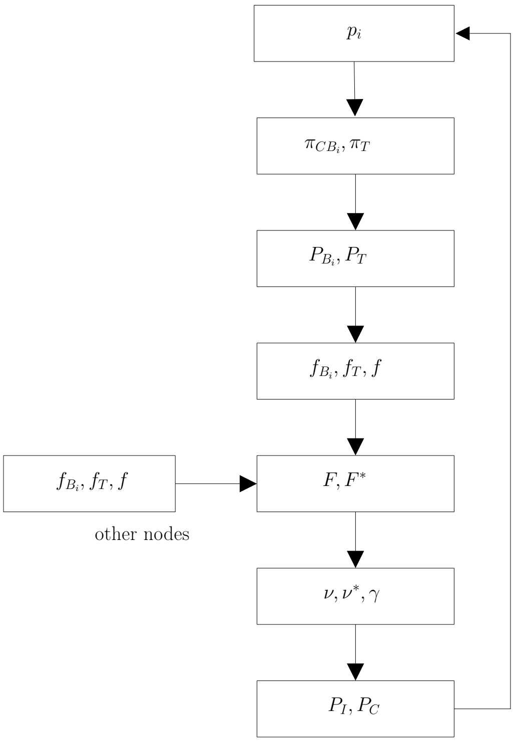

Fig. 12 Illustration of the iterative method for one node

and $f_T(t_i)$ is the discrete density function calculated in the previous section.

If this nodes does not break the idle period, the node can sense the medium as idle when the CCA attempt happens in the vulnerable period of another transmission attempt. This probability P_C can be evaluated by

$$P_C = \int_{\overline{T}_I}^{\overline{T}_I + T_{Rx\to Tx}} f_T(t)\mathrm{d}t \tag{32}$$

with

$$\overline{T}_I = \int_0^{+\infty} t\mathrm{d}F^*(t) \quad . \tag{33}$$

The idle probability of the first back-off stage can then be calculated by

$$p_1 = P_T^* \left(\nu_i^* + (1-\nu_i^*)P_C\right) + P_{B_1}^* P_I \tag{34}$$

with

$$P_T^* = \frac{P_T}{P_T + P_{B_1}}$$

$$P_{B_I}^* = \frac{P_{B_1}}{P_T + P_{B_1}} \quad .$$

5.6 Fixed Point Procedure

The calculation detailed in this section cannot be done in one step. The resulting equations are non-linear and the variables are not separable. A fixed point procedure is proposed to calculate the saturation throughput. Figure 12 illustrates the iterative method for one node. The calculation loops for all nodes are done in parallel. If all nodes are coordinated, i.e. they have the same view of the channel, the method can be simplified because the function evaluations for all nodes are identical. In the case of a multi-hop network however, this is no longer true and all the equations have to be evaluated for each node separately.

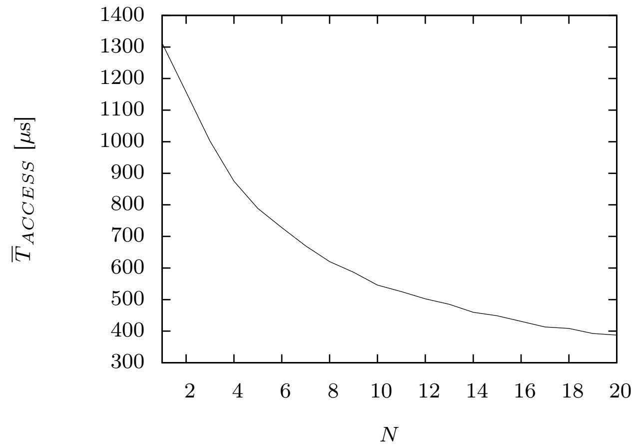

Fig. 13 The mean time between two successive transmission, $\overline{T}_{ACCESS}$, as a function of the number of nodes N with $macMinBE = 3$, $macMaxBE = 5$, $macMaxCSMABackoffs = 4$

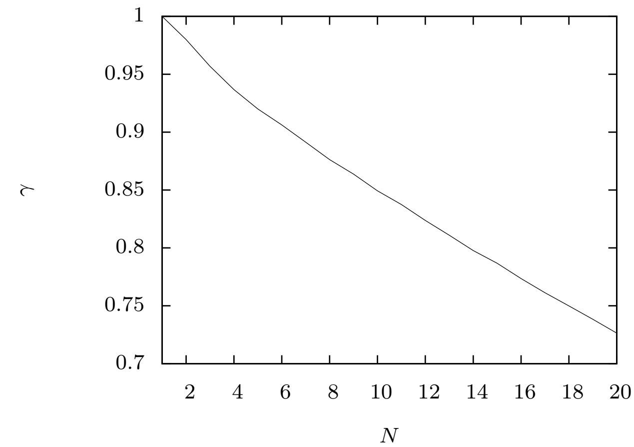

Fig. 14 The probability that a transmission is successful,γ, as a function of the number of nodes N with $macMinBE = 3$, $macMaxBE = 5$, $macMaxCSMABackoffs = 4$

Once the fixed point procedure has converged, $\overline{T}_{ACCESS}$ and γ are used to calculate the saturation throughput. Figure 13 shows the analytical evaluation of $\overline{T}_{ACCESS}$ as a function of the number of contending nodes N for the default values specified in the IEEE Std 802.15.4$^{\mathrm{TM}}$ and figure 14 the probability that a transmission is successful, γ. As a final result the saturation throughput as a function of the number of nodes is plotted in figure

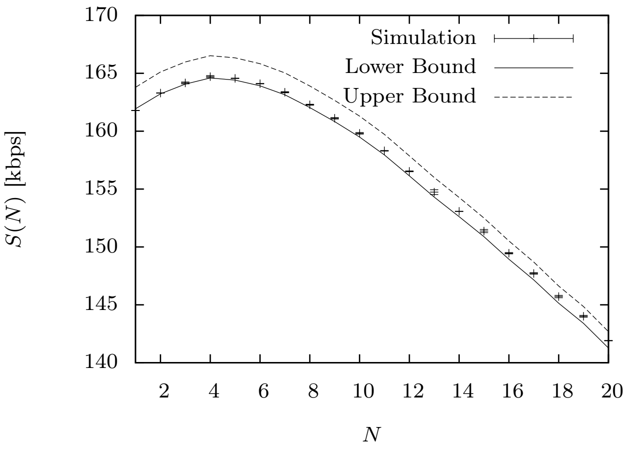

Fig. 15 Saturation throughput $S(t)$ with $macMinBE = 3$, $macMaxBE = 5$, $macMaxCSMABackoffs = 4$ as a function of the number of nodes N

15. This graph shows also the 95% confidence interval of 25 simulation runs. The upper and lower bound fit closely the simulated results. For a large value of N both bounds coincide.

6 Conclusion

In this paper a analytical model for the saturation throughput of N contending nodes in a wireless network is proposed. The unslotted CSMA/CA scheme is analyzed with a non-zero duration of the CCA and the parameters of the non-beacon enabled access mode of the IEEE Std 802.15.4$^{\mathrm{TM}}$MAC are used. To evaluate the impact of the contention based MAC a renewal process between successful transmission is defined. The internal states of the nodes are assumed to be independent. A node state is modeled as a semi-Markov chain. An event-simulator is also developed to analyze the unslotted CSMA/CA and to verify the results of the analytical calculations. The simulations show that the idle probability of a CCA is clearly dependent on the back-off stage and that the first back-off expiration after a transmission cannot be modeled as an exponential distribution. These results are used in the analytical model. The saturation throughput of each node can be independently calculated so that an extension to a multi-hop wireless network is straightforward. A good fit between the analytical model and the simulations is obtained.

This methodology will be used to analyze the queueing behavior in a multi-hop wireless network. The analysis in [32] can be adapted to the analytical procedure in this paper so that the ideal CSMA protocol can be replaced by a realistic unslotted CSMA/CA scheme with truncated exponential back-offs. Topics for further research also include:

Implementation of the ACK scheme. The semi-Markov model has to be adapted to include two states: a successful DATA and ACK message reception state and a waiting state in which the DATA message or the ACK message has collided and is lost. This will have an impact on the density function of the first expiration of a back-off after a transmission and will also alter the idle probability analysis. The collision probability will be extended to include the case of a collided ACK message. Simulations have already shown that this collision probability is mildly dependent on the specific back-off stage.

Extension to the multi-hop scenario. A Markov chain of simultaneous transmissions can be formed to analyze the impact of hidden nodes [12, 14]. The output of the product form analysis serves as a feed-back for the calculation of the back-off delay before a transmission. The saturation throughput of the nodes in a network with an arbitrary topology is highly depend on the connectivity of each node.

More realistic physical layer. The effect of an error-prone PHY can be evaluated by linking the Signal / Noise ratio in a Rayleigh of Ricean fading channel with additive Gaussian noise to the Bit Error Rate (BER) [3]. The analysis of the probability that a transmission is successful will be extended to include the supplementary packet loss which depends on the specific modulation type and the receiver sensitivity.

The developed event-driven simulator is ready to tackle these problems: the propagation model is pluggable, the channel is modeled as a matrix which contains specific information for each transmitter-receiver pair and the ACK scheme is completely implemented. Without the development of this event-driven simulator the analytical results of this paper would be very difficult to validate and the simulator provides the information to abstract the complex interaction of the contending nodes based on an unslotted CSMA/CA scheme in order to obtain a tractable but still valid model.

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