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Homework #2, Written Part
Use the spaces provided, scan and turn in as a pdf on Connex. Total of 20 marks.

- [2 marks] What exactly is the least number of comparisons used in the worst case in an unsuccessful search for an element y in an sorted array $x[1..n]$ of distinct elements using binary search? No justification required. Your expression should work for all n; use floor and ceiling as appropriate.

  ANSWER: $\lceil \lg n \rceil$.

- [4 marks] Describe an algorithm that finds the largest and the second largest of a set of n distinct integers using $n + \lceil \lg n \rceil - 1$ comparisons. Explain why it uses that many comparisons. HINT: Think of a tournament; e.g., like in tennis.

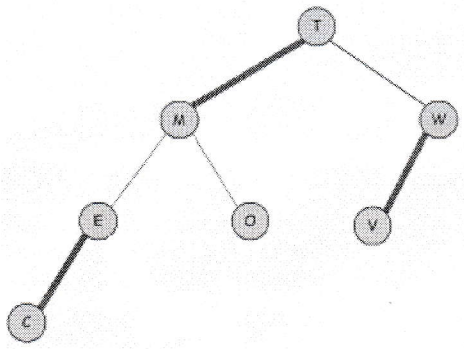
Let there be a set, S, with n distinct integers. Let there also be an empty set S'.

Iteratively compare 2 elements of S, removing the lesser one and placing it into S'. Stop when S has only 1 element.
This takes (n-1) comparisons

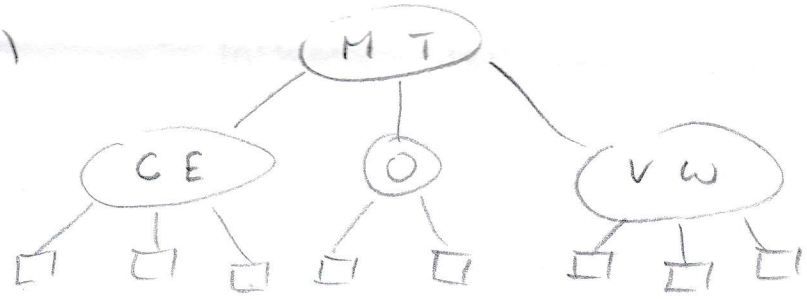
Then iteratively compare 2 elements from S', removing the lesser of the two, until S' has only 1 element.
This takes $\lceil \lg n \rceil$ comparisons.

The remaining elements of S and S' are the largest and second largest of the original set of n elements

[7 marks] A red-black tree $\mathcal{T}$ is shown below. (a) Draw the 2-3 tree corresponding to T. (b) Insert the key Z and then the key A into $\mathcal{T}$. Draw the resulting red-black tree below. Do not show intermediate results.



a)



b)

T'' :