A Theory of Small Campaign Contributions*

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Abstract

Popular and academic discussions have mostly concentrated on large donors, even though small donors are a major source of financing for political campaigns. We propose a theory of small donors with a key novelty: it centers on the interactions between small donors and the parties' fund-raising strategy. In equilibrium, parties microtarget donors with a higher contribution potential (i.e., richer and with more intense preferences) and increase their total fundraising effort in close races. The parties' strategic fundraising amplifies the effect of income on contributions, and leads to closeness, underdog and bandwagon effects. We then study the welfare effects of a number of common campaign finance laws. We find that, due to equilibrium effects, those tools may produce outcomes opposite to intended objectives. Finally, we identify a tax-and-subsidy scheme that mutes the effect of income while still allowing donors to voice the intensity of their support.

Keywords: Campaign contributions, Small donors, Fundraising, Campaign finance laws, Elections

JEL codes: D71, D72, H31

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An informed public of small contributors "would make the millions feel that it was their government, as it is; and that you and your administration were beholden to the many, not to the few." — Lincoln Steffens to Theodore Roosevelt, September 21, 1905 (Doris Kearns Goodwin, *The Bully Pulpit*, p. 417)

1 Introduction

Popular and academic discussions have mostly concentrated on large donors, who may buy undue influence on policy. This focus shapes the thinking and design of campaign finance regulation. Yet, in reality, contributions are much smaller than commonly believed: in the U.S., for instance, Ansolabehere et al. (2015) show that both newspapers and survey respondents believe that the average level of a contribution is at least eight times as large as actual Federal Election Commission figures. The FEC reported that in the 2016 presidential races, out of a total of $1.4 billion for the main candidates, $815 million were in small contributions (i.e., below $200).[1] Bernie Sanders raised almost 90% of his total campaign budget of 228 million in small contributions. The 2020 presidential campaign broke new records: small contributions totaled more than $1.6 billion.[2]

There is a clear conceptual distinction between large and small donors. Large donors are seen as making an investment: they either have an *electoral motive* (their contributions are meant to affect election outcomes, as in, for example, Poole and Romer 1985, Wand 2007) or an *influence motive* (their contributions are made in exchange for shifts in policy, as in, for example, Baron 1989, Snyder 1990, Grossman and Helpman 1994). However, as aptly put by Bonica (2014, p. 370), "the vast majority of donors give amounts so diminutive that it is difficult to conceive of the contribution as an investment". Small donors are thus viewed as only having a *consumption motive*, that is, their contributions are a consumption good to the donor (e.g., Ansolabehere, de Figueiredo, and Snyder 2003, and Gimpel, Lee, and Pearson-Merkowitz 2008).[3]

Embracing this view, we propose a model of small donor contributions in which donors

[1] http://www.fec.gov/disclosurep/pnational.do.

[2] For a more systematic analysis of the importance of small donors, see Bouton et al. (2022) for the U.S., and Cagé (2020) for other countries.

[3] Ansolabehere et al. (2003) argue that the "tiny size of the average contribution made by private citizens suggests that little private benefit could be bought with such donations" (p117). They support their claim with the finding that "income is by far the strongest predictor of giving to political campaigns and organizations, and it is also the main predictor of contributing to nonreligious charities" like other normal consumption goods.

have a consumption motive which responds positively to fundraising effort by the parties.[4] Our distinction between large and small donors is thus in terms of the drivers of their contributions rather than the size of those contributions. The crucial role of solicitation means that the decisions of fundraisers –candidates, parties, PACs, etc.– are key to understanding contributions made by small donors. Since their resources are not unlimited,[5] fundraisers must decide how to target their costly efforts to different donors. That is, though donors may not make strategic decisions, those who solicit their contributions do.

Central to the interaction between parties and donors in our model is the idea that parties will strategically allocate their solicitation efforts depending both on the anticipated response of small donors and on the overall characteristics of the electoral race. Several predictions emerge from this approach. First, in line with conventional wisdom, solicitation resources are targeted toward individuals with high "contribution potential". In the model, consistent with empirical regularities, these are individuals with high income or high preference intensity for one candidate over the other. This leads to an *income effect*: richer donors are willing to contribute more, and are therefore solicited more. This inflates the observed elasticity of contributions with respect to income, with the trailing party displaying higher elasticities than the leading party. We relate such effects to a number of empirical challenges and results.

Second, races involving parties with more balanced support (in terms of donors' contribution potential) have two characteristics in our model: (i) they end up being electorally closer, and (ii) they involve more solicitation effort because the return of a marginal dollar of campaign spending is higher. Hence, in line with empirical regularities, our model predicts a comovement between small campaign contributions and electoral closeness.

Second, parties in close races increase their solicitation effort because the return of a marginal dollar of campaign spending is higher in those races. Hence, in line with empirical regularities, total contributions are higher in closer elections, a closeness effect.

[4] Though some donors give "without being asked" (Jones and Hopkins, 1985), Gimpel et al. (2008, pp. 375-376) argue that "[t]he literature on campaign finance has established that individuals contribute to campaigns in great part because they are asked to do so (Brown, Powell, and Wilcox 1995; Francia *et al.* 2003; Grant and Rudolph 2002)."

[5] A key limitation is the time available. As former Senator, Vice President, and presidential candidate Walter Mondale put it, "Candidates and officeholders spend far too much time raising money [in order] to run for office," a complaint echoed not only by many politicians, but also by observers of the political process. In his dissent to Randall v. Sorell, 528 US 230 (2006), Supreme Court Justice John Paul Stevens wrote, "Fundraising devours the time and attention of political leaders, leaving them too busy to handle their public responsibilities effectively." See also Daley and Snowberg (2011).

Third, the probability of winning responds asymmetrically to a party's *popularity* (the number of its donors) as opposed to the donors' *preference intensity*. Depending on which is the source of its advantage, a party's probability of winning may end up displaying either a *bandwagon effect* (the party's relative contributions increase more than its standing among donors), an *underdog effect* (it improves less than its standing among donors), or even an "overpowered underdog effect" (the party with the higher standing among donors may actually end up trailing in contributions and hence in the polls).

We then study the potential effects of a number of common campaign finance laws. To perform this analysis, we use a Benthamite social welfare benchmark that depends on the number of donors for each party and their preference intensity. Since small donors do not internalize their influence on the election outcome, they produce both a positive externality on donors who support their own party, and a negative externality on those supporting the opposing party. Regulation should aim to correct these externalities. We derive implications for a regulator who operates under a veil of ignorance: in the presence of income and preference shocks, she does not know *ex ante* which party will dominate the election *ex post*, nor which will be socially preferred.

We study the effects of capping the amount each donor may contribute. When the main source of the regulator's uncertainty is donor income, the best cap is intermediate `(neither too low nor too high)`. However, when the main source of uncertainty is popularity, the best cap is very low. And, when the main source is preference intensity, a policy of no restrictions ("laissez-faire") dominates. That is, no cap on individual contributions that may be appropriate when there is uncertainty along all these dimensions at once. We further show that other oft-discussed campaign regulations, such as caps on total campaign spending, feature similar weaknesses.

The model points to a novel type of regulation that corrects the distortions caused by income differences. By subsidizing low-income donors and taxing high-income ones, a regulator can fully neutralize the effect of income on individual contributions (i.e., donors behave as if they had the same income). We show that such a tax-and-subsidy scheme is welfare-improving when income shocks are large.

In the next section, we review the literature. We then present in Section 3 a model of strategic fundraising to influence election outcomes from behaviorally motivated small donors. In Section 4 we characterize the equilibrium, and derive the results discussed above. We analyze the effects of various campaign finance laws on small donors in section

5. Section 6 presents conclusions. All proofs are in the Appendix.

2 Related Literature

To the best of our knowledge, there are only few formal models of political fundraising. McCarty and Rothenberg (2000) and Ashworth (2006) focus on the influence motive and Grossman and Helpman (1996) focus on the electoral motive, so that those models are more relevant for large donors. The only model of fundraising from small contributors of which we are aware is Walker and Nowlin (2018). They argue that big data analytics allows the microtargeting of fundraising efforts, but collecting and analyzing large data sets is costly. Hence, data-driven political fundraising should be selective and precise in choosing whom to solicit. In this respect, they focus on fundraisers discovering the ideology of potential donors in order to solicit those whose ideology is most aligned with that of the candidate. Our model includes their insights by allowing "contribution potential" to vary across donor groups, but we assume that donor characteristics are observable. Perhaps more importantly, they do not consider the implications of strategic political fundraising for campaign finance laws, an important part of our paper.

Other papers stress the importance of the resource cost of solicitation, or the "organizational effort" required for successful fundraising (Hinckley and Green 1996, Gimpel *et al.* 2008, Magleby, Goodliffe, and Olsen 2018). Like Walker and Nowlin (2018), Nickerson and Rogers (2014) also consider how the cost of fundraising shapes microtargeting in solicitation effort. In short, political fundraisers are characterized as "rational prospectors" (Brady *et al.* 1999), targeting their limited solicitation resources toward individuals with high "participation potential." Our model adopts this approach: we allow candidates to condition their fundraising effort on donor characteristics.

Not only ideology, but also income or wealth, is a key determinant of contribution potential. For instance, using survey data, Grant and Rudolph (2002) find that wealthier individuals are much more often asked to donate than poorer individuals. They then estimate a logit model and find that "[c]onsistent with expectations, solicitors selectively target those with greater financial and informational resources" (p.42). Similarly, Brady *et al.* (1999), Magleby *et al.* (2010), and Hassell and Monson (2014) find that wealth or income is a strong predictor of whether an individual is asked to donate.

Aspects of the political race itself also influence contributions. Erikson and Palfrey

(2000) consider parties which choose how much to spend, in a model where campaign spending enhances the party's probability of winning. Like them, we use a contest success function setup and find that close elections result in bigger campaigns. However, our model focuses on contributions and produces different predictions for bandwagon and underdog effects that stem from the parties' ability to microtarget their donors. We return to these issues and associated empirical results in Section 4.

Baron (1994), Erikson and Palfrey (2000), and others, including us, assume a pre-defined relationship between campaign advertising and electoral outcomes, often under the shape of a contest success function. This begs the question of microfounding that relationship. In Baron (1994), the assumption is that, like with persuasive advertising, campaign money favorably modifies the behavior of uninformed voters. Like with informative advertising, Prat (2002), Coate (2004a,b), and Morton and Myerson, (2012) instead propose models with rational voters. Parties trade off the demands of well-funded special interests with the implicit signals conveyed to the electorate. Coate (2004a) shows that such funding "redistribute[s] welfare from ordinary citizens to members of special interest groups" (p772).

The empirical literature about the effect of money on electoral outcomes can be divided into two sets of studies. The first focuses on the effect of specific campaign spending (e.g., TV ads): recent studies, with well-defined identification strategies, find positive and significant effects (see, e.g., Da Silveira and De Mello 2011; Kendall *et al.* 2015; Larreguy *et al.* 2018; Spenkuch and Toniatti 2018; and Bekkouche *et al.* 2020). The second analyzes the effects of *total* spending. Here, the evidence is mixed: spending by challengers appears more effective than spending by incumbents and, for the latter, no consensus has been reached as to whether the effect of money is economically significant (see, e.g., Levitt 1994; Erikson and Palfrey 2000; Gerber 2004; Benoit and Marsh 2008; Stratmann 2009; Bombardini and Trebbi 2011 and Kawai and Sunada 2015). A way to reconcile the apparent contradiction between these two sets of studies is provided by Sprick Schuster (2020, 2021): Using detailed transaction-level data on candidate disbursements, he finds systematic differences in the way incumbents and challengers allocate their campaign resources. In particular, he finds that incumbents spend a smaller share of their total spending than challengers on "messages to voters" (i.e., advertising and events), and a larger share on other types of spending, for example transferring contributions to other campaigns. Such actions arguably have no effect on their chances of winning their own race.

Finally, as we discuss in Section 4, our model and some of our findings are related to the literature on voter turnout. The so-called *group voting* models focus on costly voting and include mobilization models (Cox and Munger 1989, Morton 1991, and Shachar and Nalebuff 1999) and ethical voting models (Coate and Conlin 2004 and Feddersen and Sandroni 2006). As in those models, we allow the total "effort" (in our case, contributions) of a group to be controlled by a leader/planner (in our case, the party). A key difference with those models is that we allow for (i) different sub-groups of donors, each with its own contribution potential and number of donors, (ii) the leaders/planners can tailor the effort devoted to each of these subgroups, rather than assuming a single level of effort for all supporters of a given party, and (iii) by focusing on contributions, we introduce an intensive margin (how much to contribute?) beyond the extensive margin present in models of costly voting (vote v. abstain). As we discuss in the next sections, some of our predictions are similar to voting models (e.g., the closeness effect) and other are fairly different (e.g., the bandwagon and underdog effects). The focus on campaign contributions, for which heterogeneity in income is crucial, also opens the door to an analysis of the effect of income and income inequality, and of campaign regulations different than those considered for voting.

3 The Model

There are two parties (or candidates), $P = A, B$, each supported by n_P small donors. Write P for one party and $-P$ for the other party. Each donor for party P values electing party P over $-P$ as $v_P > 0$. Each donor has a type $i \in \{0, ..., I\}$, to be thought of as an income class. The income of each donor in that income class is y_i, and we let $n_{P,i}$ denote the number of P-donors with type i (with $\sum_i n_{P,i} = n_P$). Write $\mathbf{y} = (y_0, ..., y_I)$ for the vector of income classes and $\mathbf{n}_P = (n_{P,0}, ..., n_{P,I})$ for the donor vector of party P.

Parties simultaneously choose the amount of effort they will exert to obtain donations from each income class. In particular each party P chooses a vector $e_P = (e_{P,0}, ..., e_{P,I}) \in \mathbb{R}^I_+$, where $e_{P,i}$ represents the effort of P into obtaining donations from a member of income class i. We denote *total effort* by $E(\mathbf{e}_P|\mathbf{n}_P) = \sum_i n_{P,i}e_{P,i}$, which we denote by $E_P(\mathbf{e}_P)$ when $\mathbf{n}_P$ is fixed.

Effort level $e_{P,i}$ triggers contributions among donors of group i. In particular, for each

$i \in \{0, ..., I\}$, let $\sigma : \mathbb{R}^3_+ \to \mathbb{R}_+$ define a contribution schedule. That is, $\sigma(e_{P,i}|v_P, y_i)$ specifies how much a donor of income class i (of party P) contributes as a function of effort $e_{P,i}$, given that their income is y_i and their value of electing party P is v_P. This function is treated as a black box and assumed to be defined by the following functional form:

$$\sigma(e_{P,i}|v_P, y_i) = \lambda(v_P, y_i) \frac{e^{\rho}_{P,i}}{\rho}, \text{ with } 0 < \rho < 1, \tag{1}$$

where ρ captures the elasticity of contributions with respect to fundraising effort and, borrowing from Brady *et al.* 's (1999) terminology, $\lambda : \mathbb{R}^2_+ \to \mathbb{R}_+$ reflects the donor's *contribution potential*. It is assumed that λ is twice continuously differentiable and a strictly increasing function of income and of the value of the party. When the realization of (v_P, y_i) is fixed, we can suppress reference to the value of the party (v_P) and the level of income (y_i) and simply write $\lambda_{P,i}$ for $\lambda(v_P, y_i)$. In that case, write $\lambda_P = (\lambda_{P,0}, ..., \lambda_{P,I})$.

This specification makes three implicit assumptions. First, small donors increase their contributions when their party solicits them. This is in line the empirical finding discussed in the previous section. Second, to contrast with large donors,[6] we assume that small donors *only* contribute in response to their party soliciting their contributions. Third, party solicitation efforts and the donor's contribution potential are complementary: $\partial^2\sigma/\partial\lambda_{P,i}\partial e_{P,i} > 0$. As we detail in Appendix 4, this is similar to the complementarity assumptions made in the advertising literature (e.g., Becker and Murphy 1993). In that literature, consumers (here: small donors) derive utility from two types of goods. For one of them (here: campaign contribution), the marginal utility of consumption increases both in the consumer's intrinsic valuation of the good *and* in advertising (here: solicitation effort). The complementarity in (1) then arises as a result of the consumer's optimization problem. Also in that Appendix, we provide a specific formulation of that utility function that produces exactly (1).

We refer to $\sum_j n_{P,j}\lambda_{P,j}$ as the *intrinsic support* for party P. The total level of small contributions raised by party P is:

$$S(\mathbf{e}_P|\mathbf{n}_P, \mathbf{y}, v_P) = \sum_i n_{P,i}\lambda_{P,i}\frac{e^{\rho}_{P,i}}{\rho}.$$

When the parameters $\mathbf{n}_P, \mathbf{y}$, and v_P are clear, write $S_P(\mathbf{e}_P)$ for $S(\mathbf{e}_P|\mathbf{n}_P, \mathbf{y}, v_P)$. The

[6] Large donors defined as either contributing to influence the outcome of the election (*electoral motive*) or to influence policy (*influence motive*).

likelihood that party P wins the election is:

$$\pi_P(\mathbf{e}_P, \mathbf{e}_{-P}) = \frac{S_P(\mathbf{e}_P)^\gamma}{S_P(\mathbf{e}_P)^\gamma + S_{-P}(\mathbf{e}_{-P})^\gamma} = \frac{1}{1 + \left(\frac{S_P(\mathbf{e}_P)}{S_{-P}(\mathbf{e}_{-P})}\right)^\gamma}, \text{ where } \gamma > 0. \tag{2}$$

Party P's cost of fundraising is strictly increasing and weakly convex in total effort $E(\mathbf{e}_P|\mathbf{n}_P)$:

$$c(\mathbf{e}_P|\mathbf{n}_P) = \frac{E(\mathbf{e}_P|\mathbf{n}_P)^\tau}{\tau}, \text{ where } \tau \geq 1.$$

When $\mathbf{n}_P$, the donor vector of party P, is fixed, simply write $c_P(\mathbf{e}_P)$ to indicate party P's cost as a function of the effort vector. Thus, P's payoffs from $(\mathbf{e}_P, \mathbf{e}_{-P})$ are:

$$\pi_P(\mathbf{e}_P, \mathbf{e}_{-P}) - c_P(\mathbf{e}_P). \tag{3}$$

Note that the functions σ, λ, and c are identical for the two parties. Hence, the two parties are, ex ante, on equal footing conditional on having the same donor vectors $(\mathbf{n}_P, \mathbf{y}, v_P)$.

4 Equilibrium

In this section, we solve for the pure strategy Nash equilibrium of the fundraising game between the two parties and analyze its properties. As discussed in Section 2, the parties' ability to microtarget their potential donors is an important determinant of their fundraising strategy. We formalized microtargeting as the parties' ability to select a different level of effort $e_{P,i}$ for each donor group i (parties have no incentive to exert a different level of effort on two donors with identical characteristics). However, as we are about to show, this large-dimensional optimization problem can be reduced to a much simpler game in which each party selects its total effort level E_P and then allocates this "total budget" across its donors.

We proceed in two steps: first, we consider a party with an exogenously given total effort $\bar{E}_P > 0$ that must be allocated across all its donors, so that $\sum_i n_{P,i} e_{P,i} = \bar{E}_P$. In this constrained problem, $c_P(\mathbf{e}_P)$ becomes a constant and the best response is then a dominant strategy: for any $\bar{E}_{-P} > 0$, party P maximizes its payoff (3) by maximizing $S_P(\mathbf{e}_P)$. We will find that the expansion paths of each $e_{P,i}$ is linear in $\bar{E}_P$, which means that the game can be reduced to the parties optimizing over their total effort levels, E_A

and E_B. Our second step will then be to solve for the equilibrium levels of total efforts, E_A^* and E_B^*. We identify a sufficient condition for the existence of a pure strategy equilibrium and characterize equilibrium effort levels.

Finally, we explore three concrete implications of the equilibrium: (1) we show that the equilibrium is coherent with stylized facts on the link between election closeness and budget size; (2) we uncover novel underdog and bandwagon effects for small donors' contributions; and (3) we detail the effects of income and income inequality on the parties' electoral performance. As we will indicate, some of these effects are known to the literature, while we believe that others are novel. Appendices 1 and 2 provide more detail as well as the proofs of the lemmas and propositions.

4.1 Step 1: fixed total efforts

As a first step, consider a party who must allocate an exogenously given level of total fundraising effort $\bar{E}_P$:

Lemma 1 *For any given total efforts $\bar{E}_P$ and $\bar{E}_{-P}$, party P maximizes its payoff by setting $e_{P,i} = \frac{(\lambda_{P,i})^{\frac{1}{1-\rho}}}{W_P}\bar{E}_P$, with $W_P = \sum_j n_{P,j} \ (\lambda_{P,j})^{\frac{1}{1-\rho}}$.*

Hence, microtargeting induces parties to allocate a higher share of their total effort to donors with a higher contribution potential $\lambda_{P,i}$. With contribution potential modeled to be increasing in donor income, this result aligns well with the empirical regularities identified by Grant and Rudolph (2002), Brady *et al.* (1999), Magleby *et al.* (2010) among others (see Section 2).

The rationale for this result is that, while the costs of reaching out to each donor are equal, their marginal contributions are not. For a given level of effort $e_{P,i}$, donors with a higher $\lambda_{P,i}$ contribute more, meaning that the implicit cost-per-dollar raised is lower. Conversely, the more a donor is solicited, the lower her marginal contribution. The optimal allocation of efforts across donors balances these two forces up until the point where their marginal cost-per-dollar raised become equal.

As a result of this equalization, Lemma 1 uncovers three novel relationships that hold in equilibrium. First, remember that total contributions are given by $S_P = \sum_i n_{P,i}\lambda_{P,i}e_{P,i}^{\rho}/\rho$, which effectively is a CES function (Dixit and Stiglitz, 1977) of individual effort levels $e_{P,i}$. This results in an elasticity of substitution between donors that is constant and equal to $1/(1-\rho)$: a 1% increase in a donor's relative contribution potential will result in a $1/(1-\rho)\,\%$ shift of solicitation effort towards that donor. For $\rho \to 1$, solicitation effort becomes infinitely elastic: the party concentrates all its fundraising effort on donors with the highest contribution potential (i.e., the richest donors). A party's resulting campaign budget then mainly depends on how rich are its richest donors. At the other extreme, $\rho \to 0$ brings us to a "pure consumption motive" model, with donors becoming unresponsive to solicitation effort. In that case, a party's resulting campaign budget mainly depends on the number of it donors.

Second, we note that $e_{P,i}$ is linear in $\bar{E}_P$: the Engel curves for individual solicitation efforts are linear independently of the parameters of the model. This means that total efforts do not affect relative microtargeting.

Finally, Lemma 1 identifies a new parameter, W_P, which compounds the number and contribution potential of the different donors. The weight on each donor type is $(\lambda_{P,j})^{\frac{1}{1-\rho}}$, which depends on the elasticity of substitution. For a low elasticity of substitution ($\rho \to 0$), W_P weighs all donors' contribution potential equally. For larger elasticities, W_P puts a larger weight on the richest donors, because parties target a larger share of their total effort on them.

4.2 Step 2: equilibrium total efforts

Lemma 1 reduces the dimensionality of the optimization problem from $2 \times (I+1)$ to just 2: we can now focus on how much total fundraising effort each party wants to exert, knowing that total contributions follow directly (see Appendix 1 for additional detail). This allows us to prove that:

Proposition 1 *When $\gamma\rho < \tau$, a unique equilibrium exists. This equilibrium is in pure strategies.*

This provides a relatively general sufficient condition for the existence of a unique pure strategy equilibrium (PSE). A standard assumption in the literature (see e.g. Baron 1994, Erikson and Palfrey 2000, and Esteban and Ray 2001) is to impose that each party's payoff function be globally concave for any effort level of the opponent. This is typically done by setting $\gamma = 1$, which implies that winning probabilities are everywhere concave. We instead allow for values of $\gamma > 1$, which capture the presence of setup costs in the electoral campaign.[7] While setup costs are realistic for the application we consider, they may prevent the existence of a PSE.[8] The condition identified in Proposition 1 excludes this possibility, ensuring that the marginal benefit of effort is larger than its marginal cost for very small campaign sizes, and hence that both parties' best responses are to exert strictly positive effort. Roughly speaking, we are focusing on situations where the two parties indeed want to compete in the election.

We can characterize that unique equilibrium. As a convention, henceforth, we add the superscript '*' to denote the equilibrium value of an object. In particular, $\mathbf{e}_P^*$ is the vector of effort levels by party P in equilibrium, and we write π_P^* for $\pi_P\left(\mathbf{e}_P^*, \mathbf{e}_{-P}^*\right)$ and S_P^* for $S\left(\mathbf{e}_P^* | \mathbf{n}_P, \mathbf{y}, v_P\right)$.

Proposition 2 *When $\gamma\rho < \tau$, in equilibrium, winning probabilities are:*

$$\pi_P^* = \left(1 + \left(W_{-P}/W_P\right)^{(1-\rho)\gamma}\right)^{-1}. \tag{4}$$

This implies that the probability that party P wins is increasing in W_P and decreasing in W_{-P}. This stems from the fact that, in equilibrium, total contributions for party P are increasing in W_P (see Lemma A.2 in Appendix 2):

$$S_P^* = (\gamma\rho\omega)^{\frac{\rho}{\tau}} \frac{W_P^{1-\rho}}{\rho}, \tag{5}$$

$$\text{with } \omega = \left(\frac{W_A}{W_B}\right)^{\gamma(\rho-1)} / \left(\left(\frac{W_A}{W_B}\right)^{\gamma(\rho-1)} + 1\right)^2 \in \left[0, \frac{1}{4}\right].$$

As a consequence, $S_A^*/S_B^* = (W_A/W_B)^{1-\rho}$. That is, the party with the highest W_P necessarily has the highest total campaign budget in equilibrium, and hence the highest probability of winning. In what follows, we systematically label by A the alternative with the largest W_P, i.e., $W_A \geq W_B$. We call A the *frontrunner*, and B the *runner-up*.

As we will see in Sections 4.4 and 5, A's advantage may, or may not, align with the parties' intrinsic support $\sum n_{P,i}\lambda_{P,i}$, a fact that will have important implications for welfare

[7] For values of $\gamma > 1$, the probability that party P wins, π_P, is convex for $S_P(\cdot) < \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma}} S_{-P}(\cdot)$: P's campaign must reach that level for additional contributions to have maximal effect.

[8] If the marginal cost of starting a campaign is too high and its marginal benefit too low, one of the two parties' best response may be to exert zero effort, effectively withdrawing from the race. This is typically what happens if $\gamma\rho$ is large and τ is low. In that case, the equilibrium would be in mixed strategies: a high level of $S_B(\cdot)$ would deter party A from exerting any effort. But, in the absence of competition, party B would drastically reduce its effort and hence $S_B(\cdot)$, which induces A to increase its effort, etc.

and campaign finance regulation. Before that, we extensively discuss the determinants of total contributions in (5). These include: election closeness; the effect of changes in the number of supporters and their preference intensity (leading to novel underdog and bandwagon effects); and the effects of changes in income and income inequality.

4.3 Election Closeness

A classical result is that political activities tend to intensify in closer elections. For instance, Erikson and Palfrey (2000) predict that candidates should allocate more of their campaign budget to close races, a prediction that they validate empirically. In our setup, election closeness is endogenous to the contributions flowing to the two parties. However, Proposition 2 and condition (5) identified a new parameter ω, which captures the closeness between the two parties' donors financial willingness to contribute. Indeed, ω is maximized in $W_A/W_B = 1$ and it decreases towards zero as W_A/W_B moves away from 1 (in either direction). The following corollary identifies a co-movement between election closeness and the size of the campaign when ω changes:

Implication 1 *For any W_P, increasing ω makes the election closer,* i.e. *π_A^* gets closer to $1/2$, and increases equilibrium contributions S_P^*.*

Empirically, the comovement between election closeness and campaign contributions appears quantitatively important: combining survey data on US donors with FEC data, Barber *et al.* (2017, p.17) show that "a standard deviation increase [in competitiveness] raises the likelihood a donor gives to that campaign by 43%." Similarly, Gimpel *et al.* (2008, p. 376) argues that "[t]hroughout an election cycle, party leaders monitor the closeness of elections and assess the need for cash infusions in particular races." Jacobson (1985) studies how the expected closeness of US congressional elections affected campaign contributions between 1972 and 1982. He finds that the closer the race, the larger are contributions to both the challenger and the incumbent. Culberson *et al.* (2019) focuses on small contributions and finds similar results for US House elections between 2006 and 2010. To control for hidden heterogeneity, Mutz (1995) and Fuchs *et al.* (2000) study the dynamics of a given campaign to see how shocks to perceived closeness and other events influencing the marginal effect of contributions affect contributions. These papers consistently find that, when the race between the front-runner and the runner-up narrows, contributions to both candidates increase. Relying on administrative individual-level data and regression analysis, Bouton *et al.* (2022) find a similar positive correlation between election closeness and campaign contributions, even if lower for small than for large donors.

The literature identified a similar link between closeness and voter turnout, which holds both in the pivotal voter model (see e.g. Palfrey and Rosenthal 1985, and Herrera *et al.* 2014), and the group-based voter model (see e.g. Morton 1987, 1991, Shachar and Nalebuff 1999, Feddersen and Sandroni 2006, and Ali and Lin 2013). For empirical evidence, see Bursztyn *et al.* (2021) and reference therein. Our result generalizes the voting result by showing that the closeness effect also affects the intensive margin of political participation (i.e., how much to contribute). Voting models instead have to focus on the extensive margin (i.e., whether to vote or not).

4.4 Bandwagon and Underdog Effects

The second comparative static result is:

Implication 2 *Multiplying the number of donors of one candidate, say A, in each income group by a factor n multiplies the equilibrium contribution ratio* $\frac{S_A^*}{S_B^*}$ *by a factor* $n^{1-\rho} < n$. *By contrast, multiplying each A-donor's contribution potential by a factor* λ *multiplies the equilibrium contribution ratio* $\frac{S_A^*}{S_B^*}$ *by the same factor* λ.

This identifies a novel asymmetry between the two main components of a party's intrinsic support ($\sum_i n_{P,i}\lambda_{P,i}$): *popularity* (the number of donors, n_P) and the donors' *contribution potential*, $\lambda_{P,i}$. This asymmetry will prove important to the welfare effects of campaign finance laws in the next section.

When either all $n_{P,i}$ or all $\lambda_{P,i}$ are multiplied by a given number, intrinsic support increases by the same factor. Hence, the source of the asymmetry between these two must be in how the parties adapt their strategy. When a party becomes more popular, it can reach out to more supporters. Even for a fixed level of total effort, having more donors increases total contributions: since marginal returns to effort are decreasing at the individual level, the party collects more contributions by calling two identical donors only once than by calling either donor twice. Hence, the party that becomes more popular collects more funds in equilibrium. But, intuitively, reaching out to more donors to earn less from each individual puts a lid on this increase.

Contrast this with an increase in each donor's contribution potential. Now, each donor contributes more: the marginal cost of fundraising a dollar decreases one-for-one. Not only does the marginal return to effort increase, the party saves itself from having to spread its total effort over more donors.

This result shows that the classic underdog effect for turnout need not hold for small donor contributions.[9] To understand why, consider a simplified case with only one income group in each party. There are n_A (respectively n_B) donors with contribution potential λ_A (λ_B) to A's (B's) campaign, and let $n_A/n_B = n$ and $\lambda_A/\lambda_B = \lambda$. Also, let $n\lambda = k > 1$: the intrinsic support for A is k times higher than for B. The classic underdog effect would imply $k > S_A^*/S_B^* \geq 1$: equilibrium support should move in the direction of the population's intrinsic support, although in a muted way.

For small donor contributions, however, Proposition 2 implies that the contribution ratio is $S_A^*/S_B^* = n^{1-\rho} \times \lambda = k/n^\rho$. There are two possible departures from the classic underdog effect. First, for $n < 1$, that is, whenever party A's donor base is narrower than party B's, but A's donors compensate for that narrowness with their higher contribution potential, we have a *bandwagon effect*:

$$\frac{S_A^*}{S_B^*} > k > 1.$$

That is, the advantage of A in terms of contributions becomes even stronger than its advantage in terms of intrinsic support.[10]

Still focusing on the case $n\lambda = k > 1$, the second possible departure is when $n > k^{1/\rho}$, that is when party A has a much broader donor base than party B, but B's supporters have a higher individual contribution potential. In that case, we have an *overpowered underdog effect*:

$$k > 1 > \frac{S_A^*}{S_B^*}.$$

The underdog effect on A's popularity becomes so strong that the party with the highest intrinsic support, A, ends up trailing behind in terms of total contributions.[11]

[9] In that literature, the underdog effect is the phenomenon that the frontrunner's relative support in the voting booth is lower than its intrinsic support in the population. See e.g. Palfrey and Rosenthal 1985, Castanheira 2003, Feddersen and Sandroni 2006, Agranov *et al.* 2018, and, for models that use the contest success function, Herrera *et al.* 2014, and Kartal 2015.

[10] A bandwagon effect has been identified in models of voting when either there are multiple candidates (see, e.g., Myerson and Weber 1993, Cox 1997, Fey 1997, Myerson 2002, and Bouton and Ogden 2021) or in two-candidate elections when voters have an intrinsic preference for voting for the winning candidate (see, e.g., Callander 2008, and Agranov et al. 2018). The mechanism underlying the bandwagon effect in campaign contributions that emerges from our model is thus fundamentally different than in those cases.

[11] As discussed in Herrera *et al.* (2014), and Kartal (2015), such an overpowered underdog effect cannot happen in models of turnout. At most, the underdog effect in voting leads to an equal expected number of votes for the two parties.

The empirical evidence on campaign contributions supports either underdog or bandwagon effects depending on the type of election or the point in the election cycle: In multicandidate races where some candidates' viability may be in doubt, one may expect a bandwagon effect happening at the expense of the weakest candidates. In primaries for instance, most donors want to focus on the top two or three candidates (Hall and Snyder 2014). This temporarily creates significant bandwagon effects when the names mentioned for the top two or three change over the course of the campaign, while the underdog effect remains dominant among the frontrunners (Mutz 1995, Fuchs *et al.* 2000, Feigenbaum and Shelton 2013). Yet, as explained by Mutz (1995, p. 1019), "[i]n fact, many studies of bandwagon phenomena have ended up demonstrating strong underdog patterns rather than movement in the direction of majority opinion."

In two-candidate races, Bonica (2016, Figure 2) compares the behavior of small individual donors to other donor types, in particular from Corporate PACs. Small individual contributions disproportionately flow to underdogs: depending on the election cycle, only 48 to 55% of their funds go to the winner, instead of 80-90% for Corporate PACs, to be compared to an average vote share of 60 to 65% — a rough proxy for $n_A/(n_A+n_B)$.[12]

4.5 Income Effects

Key to understanding the effects of income and income inequality on electoral outcomes is unpacking how they affect campaign contributions. Precisely with that idea in mind, an empirical literature aims at estimating the elasticity of a donor's contributions with respect to her income or wealth (Ansolabehere *et al.* 2003, Gordon *et al.* 2007, and Bonica and Rosenthal 2018). As we will see in the next section, having precise estimates of this intrinsic elasticity may prove crucial to design campaign finance regulations aimed at mitigating the effect of income and income inequality on campaign finance. The goal is then to estimate a donor's intrinsic income elasticity, defined as:

$$\theta_{P,i} = \frac{\partial \lambda\left(v_P, y_i\right)}{\partial y_i} \frac{y_i}{\lambda\left(v_P, y_i\right)}.$$

However, as the above equilibrium results made clear, the parties' fundraising efforts themselves depend on donor income. Hence, this income elasticity of campaign contri-

[12] Authors' computation based on Bonica's dataset (Bonica, Adam. 2016. Database on Ideology, Money in Politics, and Elections: Public version 2.0. Stanford, CA: Stanford University Libraries. <https://data.stanford.edu/dime>). We thank Moritz Hennicke for his thorough work on these data.

butions may get entangled with the parties' fundraising strategy. When that happens, the econometrician would observe the *equilibrium* effect on an individual's contributions from a shock to her income, which conflates the two effects, leading to a potential omitted variable bias in that the fundraising effort targeted at i is not observed. We indeed find that:

Proposition 3 *For an arbitrarily large number of donors, such that a single donor's income has a vanishingly small impact on W_P, the equilibrium elasticity of her equilibrium contribution with respect to her income is $\frac{\theta_{P,i}}{1-\rho}\,(>\theta_{P,i})$.*

An increase in the income of one individual donor has a direct, positive, effect on the contribution of that donor. The magnitude of that effect is captured by the parameter θ. This direct effect is amplified by the reaction of the party, whose solicitation effort will shift towards this donor as her income increases. This creates a multiplier effect of θ, commensurate to the elasticity of contributions to solicitation, ρ. Taking account of these two effects results in an overall elasticity of $\frac{\theta}{1-\rho}$.

This result is for the case in which the econometrician observes uncorrelated individual-specific income shocks. Suppose however that income shocks *are correlated* across individuals, for instance when income shocks affect the incomes of everyone in an income class. This would also happen when the workers in some industry a support party A whereas those in industry b support party B. Sectorial demand shocks may then increase or decrease the incomes of all workers in one or the other industry. Our next proposition derives the equilibrium response of their contributions following such aggregate shocks. We let η_X^Z denote the elasticity of some variable Z with respect to another variable X, and $s^*_{P,i}$ denote the equilibrium level of contributions when both parties exert equilibrium effort levels $e^*_{P,i}$ (i.e., $s^*_{P,i} = \sigma\left(e^*_{P,i}|v_P, y_i\right)$):

Proposition 4 *Let the income y_i of all $n_{P,i}$ donors of income class i increase. Given $\theta_{P,i}$, the income elasticity of their equilibrium contributions $s^*_{P,i}$ with respect to their income is then:*

$$\eta_{y_i}^{s^*_{P,i}} = \frac{\theta_{P,i}}{1-\rho} + \left(\eta_{W_P}^{S^*_P} - 1\right)\eta_{y_i}^{W_P} > 0,$$

Moreover, $\eta_{W_A}^{S^*_A} < \eta_{W_B}^{S^*_B}$, implying that, ceteris paribus, the income elasticity of an income class $\eta_{y_i}^{s^*_{P,i}}$ is higher for the challenger, B, than for the frontrunner, A.

When we consider a change in the income of an entire income class (in which case

$\eta_{y_i}^{W_P} > 0$),[13] we need to incorporate their impact on total campaign size. When the donors who experience a positive income shock support A, the race becomes less close (i.e., W_A/W_B increases since W_A increases but W_B remains constant). This decreases both candidates' incentives to solicit contributions. The income elasticity then drops below $\frac{\theta}{1-\rho}$, since $\eta_{W_P}^{S_P^*} < 1$. If instead the income increase is for B supporters, then the race becomes closer and both candidates increase their effort. The income elasticity then increases above $\frac{\theta}{1-\rho}$, since $\eta_{W_P}^{S_P^*} > 1$.

Due to data constraints, most empirical studies focus on the behavior of (very) wealthy donors. With this qualifier in mind, we note that Gordon *et al.* (2007) report large income elasticities for the individual contributions of executives—this would be as high as 5 according to their main estimation (Table 1). Bonica and Rosenthal (2018) instead find wealth elasticities "quite close to zero" for Democrats, and slightly below 1 for Republicans. Given that these studies do not control for the strategic behavior of candidates, they estimate the overall income elasticity of contributions, i.e., $\eta_{y_i}^{s_{P,i}^*}$. The same is true for Ansolabehere *et al.* (2003, p. 122), which considers aggregate itemized contributions. They find an income elasticity slightly above one, and that income growth explains about 80% of the observed increase in aggregate contributions over time.

We are aware of only few papers controlling explicitly for the solicitation effort of candidates and parties when measuring income effects (see for example the discussion of Hassell and Monson 2014 in Section 2). These studies rely on survey data and include different types of donors (small and large). For instance, Brady *et al.* (1999, Table 4) provide evidence that contributions increase much faster with income among those donors that have been solicited than among spontaneous donors.[14] However, they also show that the act of soliciting a donor is strategic, implying that the composition of these subsamples are endogenous. Thus, while there is evidence that both θ and ρ play an important role, we are still short of having a reliable estimate for each of them. Yet, such a decomposition would be valuable to guide policy.

Note that even though donors do not condition their contributions on any characteristic of the election (as assumed in (1)), *in equilibrium* their contributions do depend on such characteristics. It means that even in individual level regressions (as in, e.g., Brady *et al.*

[13] Considering a single "large" donor who contributes a substantial share of total contributions in a race would require the same treatment.

[14] Grant and Rudolph (2002) also find that income affects positively the size of contributions after controling for solicitation.

1999, and Bonica and Rosenthal 2018), one may have to control for relevant characteristics of other donors and of the election, or at least for aggregate contributions to each candidate. Going one step further, Proposition 11 in Appendix 2 shows that even income inequality among P-donors matters.

5 Campaign Finance Laws

Campaign finance laws are, generally speaking, meant to limit the influence of money in politics (see, e.g., Ashworth 2006, Coate 2004a,b). This is because money can distort policy, either by affecting the behavior of politicians while in office (through the trade of policy favors for contributions, i.e.,"quid pro quo", a channel which is absent of our model), or by affecting which politicians are elected to office (this second channel being central to our setup). Since contributions are increasing in income, candidates and policies supported by richer donors become favored electorally. Limiting this advantage of rich donors is often referred to as the "equalization argument" (Vanberg 2008). The challenge when designing campaign finance laws is to limit the distortions brought about by money, without unduly restricting the expression of genuine preferences via contributions (a key "free speech" argument against strict campaign finance regulation).[15]

In this section, we assess the potential consequences –intended and unintended– of common campaign finance regulations with a focus on that trade-off between the equalization and free speech arguments. The analysis also identifies a novel instrument, a combination of taxes and subsidies, that we show to be superior to regulations more commonly discussed.

To perform this analysis, we focus on a slightly simplified version of the model: since the details of the equilibrium do not vary with the convexity of the cost function, τ, we set $\tau = 1$ for simplicity. Second, the previous results apply to any contribution function

[15] The debate about campaign finance in the United States, as reflected in U.S. Supreme Court decisions, has been largely framed in terms of issues of 'freedom of speech'. In the Buckley v. Valeo decision, a majority held that limits on campaign spending and individual contributions in the Federal Election Campaign Act of 1971 were unconstitutional because they violated the First Amendment provision on freedom of speech, the argument being that a restriction on spending "necessarily reduces the quantity of expression". Arguments in favor of restrictions have also relied on such considerations. In Austin v. Michigan Chamber of Commerce (1990) the court had upheld previous limits on corporate spending, writing "Corporate wealth can unfairly influence elections." Analogously, Justice Stevens, in the minority dissent in Citizens United, reiterated the "unfair influence" argument, writing that "unregulated expenditures will give corporations 'unfair influence' in the electoral process and distort public debate in ways that undermine rather than advance the interests of listeners."

$\sigma(\cdot) = \lambda(v_P, y_i)(e_{P,i})^\rho/\rho$. From now on, we focus on the Cobb-Douglas functional form:

$$\sigma(e_{P,i}|v_P, y_i) = v_P\ y_i\ \frac{e_{P,i}^\rho}{\rho}. \tag{6}$$

which normalizes to 1 the elasticity θ of contributions with respect to income. (We note that our main results directly generalize to different elasticities). When the parameters v_P and y_i are fixed, we write $\sigma_{P,i}(e_{P,i})$ for $\sigma(e_{P,i}|v_P, y_i)$. When there is no risk of confusion, we omit the argument $e_{P,i}$ for the sake of readability.

5.1 Welfare Benchmark

As detailed in Appendix 4, (6) is consistent with donors maximizing a standard utility function subject to a budget constraint whereby they allocate a budget y_i to either political contributions $s_{P,i}$ or private consumption $c_i = y_i - s_{P,i}$:

$$\max_{s_{P,i}} U_i(s_{P,i}|S_P, S_{-P}, e_{P,i}) = v_P\ \pi_P(S_P, S_{-P}) + v_P\frac{e_{P,i}^\rho}{\rho}\ln(s_{P,i}) + \left(1 - v_P\frac{e_{P,i}^\rho}{\rho}\right)\ln(y_i - s_{P,i}). \tag{7}$$

The first term in (7) says that a P-donor's utility increases by v_P when her preferred party wins the election, which happens with probability $\pi_P(\cdot)$. Small donors treat that probability as given: this term is therefore a constant in this optimization problem. The second term is her consumption utility from contributing: it increases in v_P and in the party's targeted effort $e_{P,i}$. Finally, each dollar of contribution has an opportunity cost in terms of private consumption. When there is no regulation, the result of the unconstrained maximization process is to contribute $v_P y_i \frac{e_{P,i}^\rho}{\rho}$, which is the same as $\sigma_{P,i}$ in (6). Replacing $s_{P,i}$ by $\sigma_{P,i}$ in (7) thus gives us the indirect utility of donor i.

This utility function highlights two potential sources of inefficiencies. First, individual contributions produce both positive and negative externalities, which are similar to the voting externalities in Borgers (2004) and Ghosal and Lockwood (2009). A contribution for, say, party A, generates a positive externality on all other A-donors (since it increases the probability that A wins) and a negative externality on all B-donors (since it decreases the probability that B wins).

Second, when the party marginally increases $e_{P,i}$, it impacts the donor's indirect utility by $[\ln(\sigma_{P,i}) - \ln(y_i - \sigma_{P,i})]\, v_P e_{P,i}^{\rho-1}$, which is negative whenever $\sigma_{P,i} < y_i/2$. That is, for realistic contribution levels, the parties' hunting for contributions hurts donors through foregone private consumption.[16] While interesting in and of itself, determining the socially optimal size of the contributions pool from a consumption value standpoint is not central to this paper. We thus concentrate on the electoral impact of contributions, that is on the two externalities that operate through $\pi_P(\cdot)$.

Since there are n_P donors of type P, their total valuation of having their party winning is $n_P v_P$. The net social value of electing A over B is $n_A v_A - n_B v_B$, which may be positive or negative. When positive (negative), this *net externality* of a contribution for A is positive (negative). Ex post, the Social Welfare optimum is thus:

$$\begin{aligned} \pi_A^{SW} &= 1 \text{ if } n_A v_A - n_B v_B > 0 \\ &= 0 \text{ if } n_A v_A - n_B v_B < 0. \end{aligned}$$

A campaign finance regulation is thus welfare improving ex post if it increases the probability that A wins when $n_A v_A > n_B v_B$, and decreases it when $n_A v_A < n_B v_B$. The gain from moving (closer) to the social optimum can be large or small, depending on the magnitude of $n_A v_A - n_B v_B$. However, legislation must be designed ex ante, under a veil of ignorance. From an ex ante standpoint, we must allow this net externality to be either positive or negative, large or small in magnitude.

Formally, we consider an a priori symmetric situation, in which $n_A = n_B$, $v_A = v_B$ and income distributions are identical. Hence, no party has a systematic ex ante advantage: $\mathsf{E}[W_A] = \mathsf{E}[W_B]$. Without loss of generality, we normalize both $n_A v_A$ and $n_B v_B$ to 1 in this a priori situation. Starting from there, A and B are equally likely to be the socially preferred party: either party may benefit from (i) a popularity boost that multiplies all P-donor numbers $n_{P,i}$ by a factor $n > 1$ (in that case, $n_P v_P = n$), and (ii) a preference intensity boost that multiplies v_P by a factor $v > 1$ (in that case, $n_P v_P = v$). If a party benefits from both shocks at once, then $n_P v_P = nv > \max\{n, v\} > 1$. $n_A v_A - n_B v_B$ can then assume 4 different, equally likely, values:

$$nv - 1 > n - v, v - n > 1 - nv.$$

Finally, either A or B-donor incomes get multiplied by a factor $y > 1$. This only impacts

[16] To be clear, the value of this cutoff depends on the fact that, with the utility function in (7), donors do not value contributing when there is no fundraising effort. Moreover, different utility functions would yield different cutoffs. For instance, with $v_P\ \pi_P(\cdot) + v_P \frac{e_{P,i}}{\rho} \ln(\sigma_{P,i}) + \ln(y_i - \sigma_{P,i})$, the cutoff is at $\sigma_{P,i}$ larger or smaller than 1. The overall message is that there is no general argument for the consumption externality to be systematically positive or negative.

W_A or W_B, and benefits either A or B independently of the realization of $n_A v_A - n_B v_B$. W_A/W_B can thus take 8 possible realizations, each realization being equally likely.

We will distinguish between the cases in which these income shocks are small or large. When small, the value of W_A/W_B mainly depends on the net externality, that is, on the n and v shocks. Still, as seen in Section 4.4, parties' winning probabilities need not correspond to the social optimum: we may for instance have $n_A v_A - n_B v_B > 0$ and $\pi_A^* < 1/2$. When the magnitude of the income shocks, y, is large instead, the value of W_A/W_B mainly depends on the realization of the income shock. In that case, the probability that A wins only marginally depends on n and v. Having richer donors is what it takes to win the election.

To capture the fundamental difference between small and large income shocks in a tractable way, Case 1 focuses on arbitrarily small income shocks:

$$y \underset{>}{\to} 1. \tag{8}$$

Case 2 focuses on arbitrarily large income shocks:

$$y \to \infty. \tag{9}$$

5.2 Case 1: Aggregate Income Shocks Are Negligible

We start from the two parties having similar support, and some non-degenerate income distribution within each party (that it, $\mathbf{n}_A = \mathbf{n}_B$, $v_A = v_B$). This allows for high-income and low-income donors within each party, but without a systematic advantage for either party.

5.2.1 Caps on Individual Contributions

One of the most common campaign finance regulations is to cap individual contributions. Intuitively, such caps mainly restrict the contributions by the highest-income donors. In that sense, they reduce the influence of income on equilibrium outcomes. However, as the next two propositions will show, the optimal cap can either be very high (the social planner should minimize the fraction of constrained donors) or very low (the social planner should maximize the fraction of constrained donors), depending on the source of a party's advantage.

Denote the cap on individual contributions by $\bar{s}$. The contribution schedule of donor of type i thus becomes:

$$\min\left(\bar{s};\ \sigma_{P,i}\left(\cdot\right)\right).$$

Define $\bar{s}_0$ the highest possible cap such that all donors (including those with income y_0) are constrained, that is, any higher cap will mean that those who contribute the least are unconstrained. At the other extreme, $\bar{s}_I$ is the lowest possible cap such that no contribution is constrained, that is, any lower cap will constrain those who contribute the most.

The next two propositions explore the welfare consequences of an individual cap:

Proposition 5 *Consider the case in which party advantage is driven by the number of donors, whereas preference intensities are fixed: $n > 1$ and $v = 1$. For $y \to 1$:*
(1) Caps higher than $\bar{s}_I$, that is, when no donor is constrained, are welfare minimizing;
(2) Caps no larger than $\bar{s}_0$, that is when all donors are constrained, are welfare maximizing.
(3) Depending on the shape of the income distribution, varying the cap between these levels can have a non-monotonic impact on welfare.

Hence, when the main source of uncertainty is popularity, *i.e.* shocks to the number of donors supporting each party, a tight individual cap dominates. Regulation should be tight because of the classic underdog effect that prevails when the only shock is on n_A/n_B (see Section 4.4). Ceteris paribus, with $n_A > n_B$, A exerts lower individual solicitation efforts than B: $e^*_{A,i} \leq e^*_{B,i}$. Hence, an uncapped A-donor contributes less than a B-donor with the same income level. The cap therefore constrains more B-donors than A-donors. If the popularity shock favored B, then the effect would be symmetric: independently of the direction of the shock, a tight cap on individual contributions is best, because it mainly hurts the socially undesirable party.

However, a problem is that the effect of tightening the cap need not be monotonic, as illustrated in Figure 1.[17] That is, starting from some intermediate cap, one cannot tell whether welfare would be marginally increased or decreased by a marginal tightening of the cap. To understand why, note that the simulations behind this figure consider

[17] The simulation behind Figure 1 builds on a two-group income distribution with $y_0 = 1$ and $y_1 = 2$; while we set $\theta = v_P = 1$, $\rho = 1/2$, and $\gamma = 2$. The number of low- and high-income donors are: $n_{A,0} = 18 > n_{B,0} = 9$ and $n_{A,0} = 6 > n_{B,1} = 3$. That is, both income classes are a priori willing to contribute the same amount (this proxies actual values in the 2015-16 US presidential elections), but there are twice as many A- as B-donors.

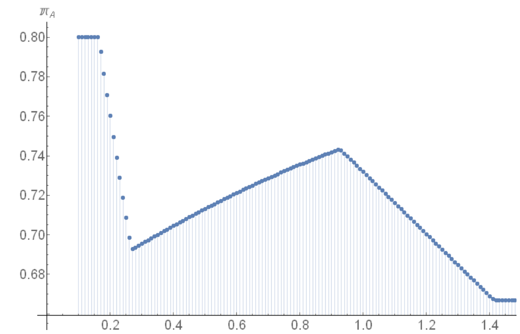

Figure 1: Simulated effect of an individual contribution cap (horizontal axis) on the probability of A winning the election (vertical axis) when $n_A = 2n_B$ and the other parameters are identical across donor groups.

two groups of donors: in the unrestricted equilibrium, about half of the total amount of contributions comes in the shape of relatively large contributions, and the other half as relatively small contributions. This proxies what we typically observe in actual data, where there is a huge number of very small contributions, and a second peak at higher levels (typically bunched at legal limits).

Because of the underdog effect, A-donors are initially less constrained by the cap than B-donors. Thus, when the cap is initially high, a tightening contributes to addressing the underdog effect. But the effect reverses when the cap is initially intermediate: large B-contributions are already constrained. Further tightening the cap now constrains A-donors, thereby magnifying the underdog effect. It is only when all donors are capped that the underdog effect is neutralized, driving the probability of winning to 0.8 in Figure 1.

This result was for the case where parties are only hit by popularity shocks. The complementary case is that of *preference intensity shocks*:

Proposition 6 *Consider the case in which preference intensities are uncertain, whereas popularity is fixed: $v > 1$ and $n = 1$. For $y \to 1$, the effects of a cap are the* opposite *of those in Proposition 5:*

(1) Caps higher than $\bar{s}_I$, that is, when no donor is constrained, are welfare maximizing;

(2) Caps no larger than $\bar{s}_0$, that is when all donors are constrained, are welfare minimizing.

(3) Varying the cap between these levels can have a non-monotonic impact on welfare.

The essence of this result is that, ceteris paribus, a donor contributes more when her preference intensity increases. Caps mute this valuable signal, and are therefore welfare-

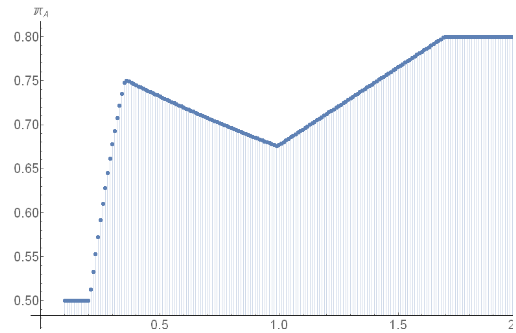

Figure 2: Simulated effect of an individual contribution cap (horizontal axis) on the probability of A winning the election (vertical axis) when $v_A = 2v_B$ but the number of donors is identical across parties.

reducing. The proposition shows that this simple logic applies to any income distribution, provided it is symmetric across parties.

Technically, the mechanism of the proof is similar to that of the previous proposition, with the difference that, if A-donors have a more intense preference, $v_A/v_B = v > 1$, it is A- rather than B-donors who are the first to be constrained. Hence, the direction of all the effects identified in the previous proposition are reversed, including the direction of non-monotonicities (see Figure 2).[18]

The empirical literature on the effects of caps on individual contributions finds seemingly contradictory evidence on the effects of individual caps. Stratmann and Aparicio-Castillo (2006) find that, for elections to US state Assemblies between 1980 and 2001, caps on individual contributions led to closer elections.[19] Lott (2006) finds the opposite result for elections to US state Senates (upper house) from 1984 to 2002: caps led to more lopsided elections.[20]

Propositions 5 and 6 suggest one way these findings might be reconciled. First, empirical studies inevitably focus on the effects of "local" changes in caps on contributions. But, Propositions 5 and 6 show that such local effects need not be monotonic. Estimates as in Stratmann and Aparicio-Castillo (2006) and Lott (2006) may thus have opposite

[18] This numerical example builds on the same two income classes as in the previous simulation: $y_{P,0} = 1$ and $y_{P,1} = 3$, $\forall P$. The number of donors are now identical: $n_{P,0} = 18$ and $n_{P,1} = 6$, $\forall P$, but preference intensities differ $v_A = 2 > v_B = 1$. The other parameters remain $\theta = v_P = 1$, $\rho = 1/2$, and $\gamma = 2$. Thus A's donors again have twice the preference intensity of B's.

[19] They also find that both the share and the absolute level of total contributions going to the incumbent decrease significantly. This is also in line with the result in Proposition 6.

[20] Similarly, Bonneau and Cann (2011) find that, in US state supreme court elections from 1990 to 2004, campaign finance restrictions (more broadly defined) hurt challengers more than incumbents.

signs simply because changes in the specific cap affect different parts of the distribution of donors. Second, these propositions also highlight how the effects of caps on individual contributions change sign depending on which is the main source of support for the candidates. Instead of the conclusion in the literature that the effect of caps is uncertain, our model identifies the need to empirically disentangle the possible direct and indirect effects of a cap, and to separately control for the number and the distribution of individual donations.

5.2.2 Caps on Total Spending

Another type of cap observed in many countries (Ohman 2012) constrains a party's *total* campaign spending. In our model, a party's campaign spending is equal to total contributions by its supporters. Trivially, a cap on total contributions so tight that it is binding for both parties would make their total contributions identical, and bring their winning probabilities to 1/2. The other case is when the cap only constrains one of the two parties, *i.e.* the frontrunner. This case is in line with the evidence in Bekkouche *et al.* (2020), who show that candidates and parties in British and French elections were affected quite differently by total spending caps. We find that:

Proposition 7 *For $y \to 1$, a cap on total contributions affecting both parties is socially undesirable. A cap on total contributions affecting only the frontrunner necessarily reduces social welfare when either $v > n$ or $v < n^{1-\rho}$.*

To understand these results, one must first remember that, in the absence of any cap, the contribution ratio of a party, say A, is strictly increasing in W_A/W_B. A cap on total contributions amounts to weakening the relationship between the contribution ratio and the donors' contribution potential. When both parties are capped, their total contributions are identical; the link between contributions and preferences is broken. When only the leading party is capped, its probability of winning is reduced. The question is when this reduction may be welfare improving. We have seen that contributions may display a strong underdog effect in a party's popularity, but not in the intensity of donor preferences. A cap may thus be beneficial when each of the two shocks favors a different party: if the socially preferred party is the one with the largest donor base, but the frontrunner is the one with the highest preference intensity, then the cap enhances the odds of the socially preferred party. However, this is only one possible realization of the shocks. The other realizations are (i) when both shocks favor the same party, and (ii) when the frontrunner is favored by the popularity shock but not by the preference intensity shock. In those cases, an aggregate cap affecting only the frontrunner necessarily reduces welfare. The ex ante effect of a cap thus depends on the comparison of the gains and losses in those different cases. Instead of performing such a comparison, the proposition identifies a sufficient condition for the cap to be welfare reducing in all situations. It suffices that the underdog effect is not too strong.

The proof of the proposition also shows that a cap only affecting the frontrunner induces the other party to increase its effort level. It is easy to produce numerical examples such that this "crowding-in" effect on total contributions to B is so strong that the sum of contributions to A and B actually increase when the cap $\bar{S}$ is tightened. Going back to the consumption motive discussed in Section 5.1, this escalation in the size of campaign spending may also be welfare decreasing.

The empirical literature confirms that stricter spending limits typically make elections more competitive (see, e.g., Milligan and Rekkas 2008, Avis *et al.* 2017, and Fouirnaies 2018). Fouirnaies (2018) also finds that tighter spending limits in British House of Commons elections stimulated entry by new candidates, which may be interpreted as an increase in the budget of small candidates. Yet, in that instance, the average effect was still to decrease total spending.[21]@ALLAN: CHECK THE FOOTNOTE!!! !!!

5.2.3 Campaign Subsidies and Taxes

Our results so far indicate that, even in the absence of aggregate income shocks, caps are a fairly ineffective tool of campaign regulation. An alternative is to rely on subsidies and taxes on contributions. Consider a uniform *matching subsidy or tax*, where for each dollar of contributions, the government adds (or subtracts in the case of a tax on contributions) m dollars.[22] Given total contributions S_P by the donors, the total contributions available to party P's campaigns become:

$$\tilde{S}_P = (1 + m)\, S_P. \tag{10}$$

[21] Drazen, Limao, and Stratmann (2007) find an analogous counterintuitive effect of individual contribution caps on PAC activity both theoretically and empirically.

[22] Ashworth (2006) considers a situation that complements our analysis. In his model, incumbents may have an unfair advantage in fundraising, and matching subsidies are then a way to correct the situation. Yet, as he shows, welfare effects may be less than straightforward even in such a situation.

We find that:

Proposition 8 *For any y, a uniform matching subsidy/tax m has no impact on total campaigning efforts E_A^* and E_B^*. They therefore have no impact on equilibrium donor contributions (S_A^* and S_B^*), nor on the outcome of the election, and hence on social welfare.*

A simple intuition behind the proposition is that, since a proportional subsidy (or tax) does not modify the parties' relative positions, it should not in itself affect the election outcome. Perhaps more surprisingly, such taxes and subsidies do not modify fundraising effort, and hence gross contributions. The reason is that, while one more unit of fundraising effort levies $(1+m)$ more (or less if $m<0$) revenue, the total revenue of the opponent is also multiplied by $(1+m)$. The larger budget of the opponent implies that the marginal return of one additional dollar on the probability of winning is reduced. These two effects exactly compensate each other. While having exactly zero impact on total efforts may be related to the specific form of our contest success function, the mechanism behind Proposition 8 clarifies why a matching subsidy should generally not have a major effect on small donors.

Such a prediction is in line with the extremely limited impact of similar regulations in the U.S.: "The federal government offered a tax credit (and, briefly, a tax deduction) for small political contributions from 1972 to 1986. This tax credit program enjoyed modest success but did not bring about large increases in small-dollar contributions. Several states currently maintain political contribution incentive programs [... but state] programs structured similarly to the federal tax credit program have largely replicated the federal experience" (Cmar 2005, pp. 106-7). This leads him to conclude that (p. 107) "A political contribution incentive program will be successful at bringing in new small donors only if potential recipients of contributions actively solicit those donors and encourage them to participate in the incentive program."

Of course, a matching subsidy that only applies to contributions below a certain level would generally have an effect on the electoral outcome.[23] As Cmar (2005) and Malbin et al. (2012) suggest, it would induce candidates to focus more of their solicitation efforts on donors who make contributions sufficiently small to be matched. The eventual effect

[23] In New York City campaigns, for example, donations up to $175 from New York City residents are matched at a rate of 6:1. Malbin et al. (2012) argue that this program increased both the share of small contributions, and the number of small donors. In 2013, small donations and matching funds accounted for 71 percent of the individual contributions in the city's elections. See https://nyccfb.info/program/impact-of-public-funds

of such regulations can thus go either way, depending on which candidate has the greater support among those who contribute below the threshold. In any case, such a matching subsidy would increase the importance of the smallest of small donors, but would not systematically bring the expected outcome of the election closer to the social optimum.

5.3 Case 2: Income Shocks Dominate

Case 1 abstracted from aggregate income shocks. When such shocks are large, in the absence of campaign regulation, the party most likely to win becomes the one with the richest donors. For $y \to \infty$, independently of the realizations of $n_A v_A$ and $n_B v_B$, party A wins almost surely if its donors are richer, and loses almost surely in the opposite case. On average, either party wins with probability 1/2. That is, aggregate income shocks just add noise to the signals coming from the popularity and preference intensity shocks.

This section focuses on individual caps and a novel tax and subsidy scheme that emerges from the model. Starting with the former, we find:

Proposition 9 *For $y \to \infty$, tight caps on individual contributions ($\bar{s}$ close to $\bar{s}_0$) welfare dominate lax caps ($\bar{s}$ close to $\bar{s}_I$) when $n > 1 = v$, whereas for $n = 1 < v$, there exists a cap $\bar{s} \in (\bar{s}_0, \bar{s}_I)$ that dominates both tight and lax caps.*

The intuition of this proposition is simple. When all donors are constrained by the cap, they all contribute the same amount to their preferred party. The total amount contributed to a party is thus proportional to the number of donors of that party. This means that any difference in income between the supporters of the two parties has no impact on the electoral outcome. This is socially desirable to the extent that income differences add noise from an ex ante standpoint. The problem is that such a tight cap also eliminates any impact of differences in preferences intensity. It is thus not welfare improving when the largest shocks are on the donors' intensity of preferences.

The challenge is to find an instrument that eliminates the noise brought by income shocks but without preventing donors to express the intensity of their preferences. Our model suggests that a matching tax/subsidy contingent on income, $\tau(y_i)$, would help in achieving this distinction, that is, in equalizing the voice of donors with different incomes but still allowing differences in preference intensity to be expressed. With that matching tax/subsidy, for any contribution s, the funds available to party P are $(1 - \tau(y_i))\, s$.

Proposition 10 *For any y and any constant k, a matching tax/subsidy $\tau(y_i) = 1 - k/y_i$ removes the effect of income and income inequalities from equilibrium contributions, and hence from the candidates' winning probabilities. Such a subsidy/tax is socially preferred to laissez-faire.*

First, note that for any value of k, donors with the lowest income may have to be subsidized ($\lim_{y_i \to 0} 1 - k/y_i < 0$), whereas the richest donors may have to be taxed ($\lim_{y_i \to \infty} 1 - k/y_i = 1$). Second, note that this "optimal" contingent subsidy/tax generalizes to any constant income elasticity θ such that $\sigma_{P,i} = (y_i)^{\theta} v_P \ (e_{P,i})^{\rho} / \rho$. For $\theta \neq 1$, the optimal tax and subsidy schedule is $\tau(y_i) = 1 - k/(y_i)^{\theta}$. However the latter requires having a good estimate of θ, which is typically difficult to obtain (see Section 4.5). These remarks notwithstanding, Proposition 10 makes a case for substituting caps and uniform matching subsidies/taxes with an income-contingent tax-and-subsidy scheme. In other words, it proposes that a proper campaign finance incentive scheme should be made an integral part of the personal income tax. Donations to parties or candidates would be deductible for low levels of income and subject to a tax at high income levels.

Though such a tax-and-subsidy may seem distant from what is observed in current campaign finance regulations across countries, a regulation broadly mimicking such a policy was actually in place in the U.S. between 1972 and 1986 (Cmar 2005). It is still in line with existing tax laws, for example in the U.S., in the following sense. Suppose campaign contributions were deductible from income tax liabilities (including perhaps a subsidy as in the footnote 23), but where the allowed deduction was a decreasing function of income. In the United States, for example, allowed itemized deductions as a whole fall with income for high income taxpayers, with deductions in specific categories differentially limited by income. Suppose further that an income-adjusted deductibility specifically for political contributions as described in the sentence above were combined with an increase in tax rates overall. The net effect would be akin to a tax on campaign contributions that increases with income and with the size of the contribution. For examples of similar tax incentives in various U.S. States, see Cmar (2005) and Magleby *et al.* (2018).

There is of course the question of the political feasibility of adopting such a change. Any proposal framed as a tax on contributions that increases with income would have little prospect of being adopted in the U.S. In contrast, deductibility of contributions that gets phased out as income increases should be far more politically viable. For a thorough discussion of the feasibility and implementability of such a federal tax (or tax credit), see Rosenberg (2002) and Cmar (2005).

6 Conclusions

Small contributions are now viewed as increasingly important to political campaigns, both in the United States and in other countries. Given their small size, they cannot be seen as motivated by the desire to influence policy, nor realistically by a desire to impact election outcomes. Yet, these motives are typically the focus of the academic literature and of legal scholars, who are mainly concerned with the risk of large donors dominating policy. A separate literature on small contributions has focused on a consumption motive, identified for example by Ansolabehere *et al.* (2003). This paper proposes a simple formal model to study small donors' contributions that expands on this approach by studying the interaction between donors and candidates (or parties). Our model gives a crucial role to the solicitation of campaign contributions, so that the decisions of fundraisers shape contributions made by small donors. Since resources for fundraising are not unlimited, fundraisers must decide how to target their costly efforts across potential donors. Hence, though donors do not make strategic decisions themselves, the equilibrium outcomes are "as if" they did.

The model produces a number of results. We identify how strategic fundraisers should target their efforts and the implications for equilibrium contributions. These include: a positive income effect, but with a larger income elasticity than a simple consumption-motive model would imply; a closeness effect, whereby contributions increase when the support (in terms of willingness to contribute) for the two candidates is more even; and bandwagon and underdog effects, whereby the relative total contributions for the two parties respond asymmetrically to a party's *popularity* (the number of its donors) as opposed to the donors' preference intensity. As discussed in the paper, these results are consistent with the empirical literature on political contributions, suggesting the empirical relevance of our model. We also study implications for campaign finance laws, specifically caps on donors and parties, and subsidies to or taxes on contributions. To perform this analysis, we use a Benthamite social welfare benchmark that depends on the number of donors for each party and their preference intensity. We detail how different regulations correct or amplify the externalities produced by campaign contributions. We find a number of novel results on the effects of campaign finance laws.

Our model abstracts from large donors (*i.e.*, donors whose contributions are driven by an investment motive). An avenue for future research would be to combine the lessons from our approach to small donors with those of the literature on large donors. For example, a key result of the latter literature is that lobbying and influence buying distorts policies away from the social optimum, hence from the small donors' preferred policy, to favor special interests. This creates an obvious link between the two types of donors: the more favors are obtained by special interests, the less small donors should be willing to contribute. This results in a novel endogenous cost to the exchange of favors: the more reactive are small donors, the lower the incentive to either buy or sell influence. A pernicious effect of individual caps in that regard is that they mute the small donors' reaction, and may thus stimulate the exchange of favor. In short, we hope our model not only provides insights into the empirically crucial issue of small contributions in campaign financing, but also opens the way to studying how small and large donors may interact.

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7 Appendix

Appendix 1. Equilibrium Analysis

In this Appendix, we detail the intermediate steps and the proofs for the equilibrium analysis of Section 4. For the sake of readability, we repeat verbatim the lemmas and propositions that are in the core text.

This is a game in which each party needs to choose a vector of $I + 1$ effort levels to maximize (3) in the paper. We derive the equilibrium in three steps: first, Lemma 1 (in the core text) derives the optimal allocation of effort across donors for a fixed total effort $\bar{E}_P$. This lemma allows us to reduce the dimensionality of the problem from $2 \times (I + 1)$ control variables to just two: E_A and E_B. Second, Proposition 1 (in the core text) identifies a sufficient condition for the existence of a (unique) pure strategy Nash equilibrium. Third, we characterize the equilibrium: the effort targeted at each individual, the two total effort levels, total contributions received by each party, and probabilities of victory.

As a preliminary step, note that:

Lemma A.1 $\frac{\partial \pi_P}{\partial S_P} = \frac{\gamma}{S_P} \pi_A (1 - \pi_A)$.

That is, the marginal return of contributions are increasing in the closeness of the election $\pi_A (1 - \pi_A)$ and decreasing in total contributions S_P. Of course, both are equilibrium objects, which need to be derived from the parties' choice of effort.

To derive the equilibrium choice of effort vectors $(\mathbf{e}_A^*, \mathbf{e}_B^*)$, we begin by checking how a party should allocate its effort levels across its donors to maximize its contributions for a given level of total effort (this is the dual problem of minimizing total cost of effort for a given level of contributions):

Lemma 1. *For any given total efforts $\bar{E}_P$ and $\bar{E}_{-P}$, party P maximizes its payoff by setting $e_{P,i} = \frac{(\lambda_{P,i})^{\frac{1}{1-\rho}}}{W_P} \bar{E}_P$, with $W_P = \sum_j n_{P,j} \; (\lambda_{P,j})^{\frac{1}{1-\rho}}$.*

Proof. The maximization problem is:

$$\max_{e_{P,i}} S_P \text{ s.t. } \sum_i n_{P,i} \; e_{P,i} \leq \bar{E}_P$$

Denote the multiplier of the constraint by μ. The first order conditions are then:

$$n_{P,i} \; \lambda_{P,i} e_{P,i}^{\rho - 1} / \rho = \mu \; n_{P,i} \Longleftrightarrow e_{P,i} = (\lambda_{P,i} / \rho\mu)^{\frac{1}{1-\rho}} \;\; \forall i.$$

Substituting into the constraint, we obtain:

$$\begin{aligned}\sum_j n_{P,j}\ \left(\lambda_{P,j}/\rho\mu\right)^{\frac{1}{1-\rho}} &= \bar{E}_P \Longleftrightarrow \mu = \left(\frac{\sum_j n_{P,j}\ \left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}}{\rho\bar{E}_P}\right)^{1-\rho} \\ &\Longrightarrow \\ e_{P,i}\left(\bar{E}_P|\mathbf{n}_P,\mathbf{y},v_P\right) &= \frac{\left(\lambda_{P,i}\right)^{\frac{1}{1-\rho}}}{\sum_j n_{P,j}\ \left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}}\bar{E}_P.\end{aligned}$$

■

Lemma 1 allows us to express total contributions as a function of total effort:

$$\begin{aligned}S\left(E_P|W_P\right) &= \sum_i n_{P,i}\ \lambda_{P,i}\frac{e_{P,i}^{\rho}}{\rho} = \frac{E_P^{\rho}}{\rho}\sum_i n_{P,i}\ \lambda_{P,i}\frac{\left(\lambda_{P,i}\right)^{\frac{\rho}{1-\rho}}}{W_P^{\rho}} = \frac{E_P^{\rho}}{\rho}\frac{\sum_i n_{P,i}\ \lambda_{P,i}^{\frac{1}{1-\rho}}}{W_P^{\rho}} \\ &= E_P^{\rho}W_P^{1-\rho}/\rho. \qquad (11)\end{aligned}$$

This allocation of effort reflects that, ceteris paribus, is is cheaper to raise an extra dollar from a donor with a higher $\lambda_{P,i}$. But, in equilibrium, marginal costs must be equalized.

We are now in position to identify conditions under which a pure strategy equilibrium exists and is unique:

Proposition 1. *When $\gamma\rho < \tau$, a unique equilibrium exists. This equilibrium is in pure strategies.*

Proof. We prove that, for $\gamma\rho < \tau$, the parties' first order condition for a local extremum implies that the second order condition for a maximum is automatically satisfied. This in turn implies that this extremum is unique, and results in a global maximum.

As a preliminary step, note that a party's best response must lie in the compact set $E_P \in \left[0,\tau^{1/\tau}\right]$. Indeed, for $E_P = 0$, $\pi_P - \frac{E_P^{\tau}}{\tau} \geq 0$, and for $E_P \geq \tau^{1/\tau}$, we have $\pi_P - \frac{E_P^{\tau}}{\tau} \leq \pi_P - 1 \leq 0$. Hence, party P's objective function must reach a global maximum for some value of $E_P \in \left[0,\tau^{1/\tau}\right]$.

To derive the first order conditions, we start by substituting $S\left(\mathbf{e}_P|\mathbf{n}_P,\mathbf{y},v_P\right)$ with $S\left(E_P|W_P\right)$ from (11) in the objective function of party P, which becomes:

$$\max_{E_P}\frac{1}{1+\left(\frac{E_{-P}^{\rho}W_{-P}^{1-\rho}}{E_P^{\rho}W_P^{1-\rho}}\right)^{\gamma}} - \frac{E_P^{\tau}}{\tau}.$$

By Lemma A.1, the first order condition with respect to E_P becomes:

$$\begin{aligned}\frac{\gamma\rho}{E_P}\pi_A\left(1-\pi_A\right) - E_P^{\tau-1} &= 0 \\ \gamma\rho\pi_A\left(1-\pi_A\right) &= E_P^{\tau}. \qquad (12)\end{aligned}$$

Next, we verify that the second order condition for a maximum is reached whenever the FOC (12) is satisfied. Differentiating (12) yields:

$$\gamma\rho\frac{\partial\pi_P}{\partial E_P}\left(1-2\pi_P\right) - \tau E_P^{\tau-1}.$$

Note that this SOC is automatically satisfied for $\pi_P \geq 1/2$, i.e. whenever a party is leading. Hence, we only need to verify that it is also satisfied for the party that trails behind. By Lemma A.1, for $\pi_P < 1/2$, the SOC requires that:

$$\begin{aligned} \gamma\rho \frac{\gamma\rho}{E_P} \pi_A (1-\pi_A)(1-2\pi_P) - \tau E_P^{\tau-1} &< 0 \\ (\gamma\rho)^2 \pi_A (1-\pi_A)(1-2\pi_P) - \tau E_P^{\tau} &< 0. \end{aligned} \tag{13}$$

Clearly, this condition is not necessarily satisfied globally, since the probability of winning is not globally concave. But the second order condition only needs to be satisfied at the point where the FOC is satisfied. For any $\pi_A \in (0,1)$, replacing E_P^τ by the value found in (12) implies that (13) is verified iff:

$$\gamma\rho (1-2\pi_P) - \tau < 0. \tag{14}$$

The first term being strictly decreasing in π_P, (14) is larger or equal to $\gamma\rho - \tau$, which itself is necessarily negative for $\gamma\rho < \tau$.

This proves that (i) the global maximum must be reached on a compact set and (ii) whenever the FOC is satisfied, it corresponds to a local maximum. It immediately follows that this maximum must be unique, and hence global: indeed, since the objective function is continuously differentiable in E_P, the existence of another (higher) local maximum would imply that there exists a local minimum between these two maxima. But at that minimum, the FOC must be satisfied (by Fermat's Theorem) and the SOC must not, by the definition of a minimum. This contradicts (14), hence there cannot be a local minimum, in turn implying a unique, hence global, maximum. ■

We can fully characterize such a pure strategy equilibrium:

Proposition 2. *When $\gamma\rho < \tau$, in equilibrium, winning probabilities are:*

$$\pi_P^* = \left(1 + (W_{-P}/W_P)^{(1-\rho)\gamma}\right)^{-1}.$$

Proof. When $\gamma\rho < \tau$, it follows from (12), that $E_A^* = E_B^*$ in equilibrium. Using (11), this implies that

$$\frac{S_A^*}{S_B^*} = \left(\frac{W_A}{W_B}\right)^{1-\rho}. \tag{15}$$

Using (2), it immediately follows that equilibrium winning probabilities are:

$$\pi_P^* = \left(1 + (W_{-P}/W_P)^{(1-\rho)\gamma}\right)^{-1},$$

which are only a function of the parameters of the model. ■

Proposition 2, allows us to derive the equilibrium election closeness, $\omega = \pi_A^* \pi_B^*$, as a function of the parameters of the model:

$$\begin{aligned}
\omega &= \left(1+(W_B/W_A)^{(1-\rho)\gamma}\right)^{-1}\left(1-\left(\left(1+(W_B/W_A)^{(1-\rho)\gamma}\right)^{-1}\right)\right) \\
&= \left(\left(\frac{W_A}{W_B}\right)^{\gamma(\rho-1)}+1\right)^{-2}\left(\frac{W_A}{W_B}\right)^{\gamma(\rho-1)}.
\end{aligned}$$

This is a real number between 0 and 1/4, with a maximum in $W_A = W_B$.

From the FOC (12) , we then have:

$$\begin{aligned}
E_P^* &= \left(\gamma\rho\pi_A^*\left(1-\pi_A^*\right)\right)^{\frac{1}{\tau}} \\
&= (\gamma\rho\omega)^{\frac{1}{\tau}}, \qquad (16)
\end{aligned}$$

where we write E_P^* for $E\left(\mathbf{e}_P^*|\mathbf{n}_P\right)$. This implies that $E_A^* = E_B^*$ for any values of the model's parameters, which may appear counterintuitive at first. From (11), a party enjoying a higher donor support W_P should benefit from a higher marginal return to effort in terms of funds raised from an extra unit of effort. However, what matters is the return in terms of probability of winning. Lemma 1 showed that a party with a larger budget S_P experiences a lower effect of the marginal dollar on the probability of winning. In equilibrium, these two effects exactly offset each other.

Plugging (16) in (11) , we obtain the equilibrium total contribution levels as a function of the parameters of the model, summarized by W_A and W_B. In contrast with total efforts, equilibrium contributions are *not* equal:

$$S_P^* = (\gamma\rho\omega)^{\frac{\rho}{\tau}}\frac{W_P^{1-\rho}}{\rho}.$$

Appendix 2: Proofs and Additional Results for Sections 4.3-4.5

Proof of Proposition 3. By assumption, W_A and W_B are constants. Hence, so are E_A^* and E_B^*, by Lemma 2. By Lemma 1, we have that:

$$e_{P,i}^* = \frac{(\lambda_{P,i})^{\frac{1}{1-\rho}}}{W_P}E_P^*,$$

implying that the equilibrium contribution by donor i is:

$$s_{P,i}^* = \lambda_{P,i}\left(\frac{(\lambda_{P,i})^{\frac{1}{1-\rho}}}{W_P}E_P^*\right)^{\rho}/\rho = (\lambda_{P,i})^{\frac{1}{1-\rho}}\left(\frac{E_P^*}{W_P}\right)^{\rho}/\rho,$$

where we write $s^*_{P,i}$ for $\sigma\left(e^*_{P,i}|v_P, y_i\right)$. Thus, the equilibrium elasticity of her contribution with respect to her income is:

$$\begin{aligned}
\frac{\partial s^*_{P,i}}{\partial y_i} \times \frac{y_i}{s^*_{P,i}} &= \frac{1}{1-\rho} \left(\lambda_{P,i}\right)^{\frac{\rho}{1-\rho}} \frac{\partial \lambda_{P,i}}{\partial y_i} \left(\frac{E^*_P}{W_P}\right)^{\rho} \times \frac{y_i}{\left(\lambda_{P,i}\right)^{\frac{1}{1-\rho}} \left(\frac{E^*_P}{W_P}\right)^{\rho}} \\
&= \frac{1}{1-\rho} \frac{\partial \lambda_{P,i}}{\partial y_i} \frac{y_i}{\lambda_{P,i}} = \frac{\theta_{P,i}}{1-\rho}.
\end{aligned}$$

■

Proof of Proposition 4. We know that, in equilibrium, total contributions are:

$$S^*_P = (\gamma\rho\omega)^{\frac{\rho}{\tau}} \frac{W_P^{1-\rho}}{\rho}, \text{ with } \omega = \left(1 + \left(\frac{W_B}{W_A}\right)^{(1-\rho)\gamma}\right)^{-2} \left(\frac{W_B}{W_A}\right)^{(1-\rho)\gamma}.$$

Taking logarithms yields:

$$\ln S^*_P = (1-\rho)\ln W_P + \frac{\rho}{\tau} \times (\ln\omega + \ln(\gamma\rho)) - \rho.$$

Let η^Z_X denote the elasticity of some variable Z with respect to another variable X. We have:

$$\begin{aligned}
\eta^{S^*_P}_{W_P} &= \frac{d\ln S^*_P}{d\ln W_P} = (1-\rho) + \frac{\rho}{\tau}\frac{d\ln\omega}{d\ln W_P}, \text{ with} \\
&\quad \eta^{\omega}_{W_A} = \tfrac{d\ln\omega}{d\ln W_A} = \tfrac{\partial\omega}{\partial W_A}\tfrac{W_A}{\omega} = \gamma(1-\rho)\tfrac{1-\left(\frac{W_A}{W_B}\right)^{\gamma(1-\rho)}}{1+\left(\frac{W_A}{W_B}\right)^{\gamma(1-\rho)}} \quad (<0) \\
&\quad \eta^{\omega}_{W_B} = \tfrac{d\ln\omega}{d\ln W_B} = \tfrac{\partial\omega}{\partial W_B}\tfrac{W_B}{\omega} = -\tfrac{d\ln\omega}{d\ln W_A} \quad (>0),
\end{aligned}$$

and thus $\eta^{S^*_B}_{W_B} > 1-\rho > \eta^{S^*_A}_{W_A}$.

Noting that $s^*_{P,i} = \lambda_{P,i} \times \left(\frac{(\lambda_{P,i})^{\frac{1}{1-\rho}}}{W_P} E^*_P\right)^{\rho} / \rho = (\lambda_{P,i})^{\frac{1}{1-\rho}} \left(\frac{E^*_P}{W_P}\right)^{\rho} / \rho$ and that $S^*_P = (E^*_P)^{\rho}\, W_P^{1-\rho}/\rho$, we have

$$s^*_{P,i} = (\lambda_{P,i})^{\frac{1}{1-\rho}} \frac{S^*_P}{W_P}.$$

Taking logarithms and then differentiating with respect to $\ln y_{P,i}$:

$$\begin{aligned}
\ln s^*_{P,i} &= \frac{1}{1-\rho}\ln\lambda_{P,i} + \ln S^*_P - \ln W_P, \text{ and:} \\
\eta^{s^*_{P,i}}_{y_{P,i}} &= \frac{d\ln s^*_{P,i}}{d\ln y_{P,i}} = \frac{\theta_{P,i}}{1-\rho} + \left(\frac{d\ln S^*_P}{d\ln W_P} - 1\right)\frac{d\ln W_P}{d\ln y_{P,i}} \\
&= \frac{\theta_{P,i}}{1-\rho} + \left(\eta^{S^*_P}_{W_P} - 1\right)\eta^{W_P}_{y_{P,i}}.
\end{aligned}$$

$\eta^{s^*_{P,i}}_{y_{P,i}}$ is necessarily positive because (1) $\eta^{S^*_P}_{W_P} > 0$ by Lemma A.2 below, (2) $\eta^{W_P}_{y_{P,i}}$ is necessarily strictly smaller than $\frac{\theta_{P,i}}{1-\rho}$, since:

$$\begin{aligned}
\frac{\partial W_P}{\partial y_i}\frac{y_i}{W_P} &= \frac{\partial \sum_j n_{P,j}\ \left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}}{\partial y_i}\frac{y_i}{W_P} \\
&= n_{P,i}\frac{1}{1-\rho}\left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}-1}\frac{\partial \lambda_{P,i}}{\partial y_i}\frac{y_i}{W_P} \\
&= \frac{1}{1-\rho}\frac{n_{P,i}\left(\lambda_{P,i}\right)^{\frac{1}{1-\rho}}}{W_P}\frac{\partial \lambda_{P,i}}{\partial y_i}\frac{y_i}{\lambda_{P,i}} \\
&= \frac{\theta_{P,i}}{1-\rho}\frac{n_{P,i}\left(\lambda_{P,i}\right)^{\frac{1}{1-\rho}}}{W_P}, \qquad (17)
\end{aligned}$$

where the second fraction on the RHS of (17) is the share of the income group y_i in W_P, which is necessarily smaller than 1. ■

Additional Results

We know that $S_A^* > S_B^*$ iff $W_A > W_B$. Letting A denote the party who is ahead and B the party who is behind , we have:

Lemma A.2 *S_P^* is increasing in W_P, $\forall P \in \{A, B\}$. For any $S_A^* > S_B^*$, S_A^* is increasing in W_B, whereas S_B^* is decreasing in W_A.*

Proof. From (5) and the definition of ω, we have:

$$\left(S_A^*, S_B^*\right) = \left(\left(\gamma\rho\omega\right)^{\frac{\rho}{\tau}} W_A^{1-\rho}, \left(\gamma\rho\omega\right)^{\frac{\rho}{\tau}} W_B^{1-\rho}\right).$$

Taking derivatives and simplifying yields:

$$\frac{\partial S_A^*}{\partial W_A} > 0 \Leftrightarrow \left(\tau+\gamma\rho\right) W_A^{\rho\gamma} W_B^{\gamma} + \left(\tau-\gamma\rho\right) W_A^{\gamma} W_B^{\rho\gamma} > 0 \text{ and } \frac{\partial S_A^*}{\partial W_B} > 0 \Leftrightarrow W_A > W_B.$$

The latter is always satisfied. $\frac{\partial S_A^*}{\partial W_A}$ is necessarily positive when an equilibrium exists, since the condition for existence in Proposition 1 is that $\gamma\rho < \tau$, in which case all terms in $\left(\gamma\rho+\tau\right) W_A^{\rho\gamma} W_B^{\gamma} +$ $\left(\tau-\gamma\rho\right) W_A^{\gamma} W_B^{\rho\gamma}$ are positive.

The same steps demonstrate that $\frac{\partial S_B^*}{\partial W_B} > 0 > \frac{\partial S_B^*}{\partial W_A}$. ■

Proposition 11 *If and only if $\left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}$ is convex in y_j, a mean-preserving spread:*
(1) of the A-donors' income distribution increases S_A^* and decreases S_B^*.
(2) of the B-donors' income distribution increases both S_A^* and S_B^*.

Proof. First, we show that a Mean Preserving Spread (MPS) of the income distribution increases $\sum_j n_{P,j}\ \left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}$ –and therefore W_P– iff $\left(\lambda_{P,j}\right)^{\frac{1}{1-\rho}}$ is convex in y_j. This is a particular case of Rothschild and Stiglitz (1970, Section 2): take two income distributions F and G with associated densities f and g. Let:

$$\gamma_i = g\left(y_i\right) - f\left(y_i\right).$$

By the definition of CDFs it must be that $\sum_i \gamma_i = 0$. We will say that G is a *single MPS* of F if $\gamma_i = 0$ for all but four income levels $y_a < y_b < y_c < y_d$ such that:

$$\gamma_a = -\gamma_b > 0 \text{ and } \gamma_d = -\gamma_c > 0,$$
$$\gamma_a y_a + \gamma_b y_b + \gamma_c y_c + \gamma_d y_d = 0.$$

The first of these conditions imposes a transfer of probability mass from the center (y_b and y_c) to the tails (y_a and y_d), whereas the second imposes that the mean is preserved. By Theorem 1 and Lemma 1 in Rothschild and Stiglitz (1970), any MPS can be approximated by a sequence of single MPS.

The mean preserving condition can be expressed as:

$$\gamma_a (y_a - y_b) + \gamma_d (y_d - y_c) = 0.$$

By definition, we have that $h'(y)$ is strictly increasing in y for any strictly convex function $h(y)$. Hence,

$$\gamma_a (h(y_a) - h(y_b)) + \gamma_d (h(y_d) - h(y_c)) \simeq \gamma_a (y_a - y_b) h'(y_a) + \gamma_d (y_d - y_c) h'(y_c),$$

which is strictly larger than:

$$[\gamma_a (y_b - y_a) + \gamma_d (y_d - y_c)] \times h'(y_a) = 0.$$

Conversely, the inequality reverses for strictly concave functions. This proves that any MPS of the income distribution among P-donors strictly increases W_P iff $(\lambda_{P,j})^{\frac{1}{1-\rho}}$ is convex in y_j.

It remains to prove the effects on S_P^* and S_{-P}^*. This is an application of Lemma A.2: an increase in W_A increases S_A^* and decreases S_B^*. An increase in W_B increases both S_A^* and S_B^*. ■

Appendix 3. Proofs for Section 5

Proof of Proposition 5. With $v = 1$, we have that the welfare maximizing probability that A wins is $\pi_A^{SW} = 1$ iff $n_A/n_B = n$ and is $\pi_A^{SW} = 0$ iff $n_A/n_B = 1/n$. This is true for any $n > 1$. The proof focuses on the realization of the shock $n_A/n_B = n$. By symmetry, the results are identical when party labels are swapped.

First, by (5), for $\bar{s} \geq \bar{s}_I$: $\frac{S_A^*}{S_B^*} = \left(\frac{W_A}{W_B}\right)^{1-\rho} = n^{1-\rho}$.

For lower caps, donor I is necessarily constrained. The two steps below establish that, for any cap $\bar{s}$ tighter than $\bar{s}_I$, the ratio of equilibrium contributions under a cap $\bar{s}$, which we denote $S_A(\bar{s})/S_B(\bar{s})$, must be strictly larger than S_A^*/S_B^*. That is, in comparison to no cap, A's advantage is reinforced with any binding cap. Hence, a binding cap is welfare improving compared to no cap. Further, $S_A(\bar{s}_0)/S_B(\bar{s}_0) \geq S_A(\bar{s})/S_B(\bar{s})$, $\forall \bar{s} \geq \bar{s}_0$: a tight cap welfare dominates any other cap.

Step 1: Take the derivative of the payoff function (3) with respect to $e_{P,i}$ given some pair of total contributions (S_A,S_B). By Lemma A.1, it is: $\frac{\gamma}{S_P}\pi_A (1-\pi_A) v_P (y_i)^\theta (e_{P,i})^{\rho-1}$, which must be equal to $c'(e_{P,i}) = 1$ for the FOC to be satisfied. Hence the FOC for $e_{P,i}$ becomes:

$$\frac{\gamma}{S_P}\pi_A(1-\pi_A)\,v_P(y_i)^\theta(e_{P,i})^{\rho-1} = 1. \tag{18}$$

While this condition depends on S_P and π_A, which are endogenous to the level of efforts, we can compare different equilibrium efforts. Indeed, with $v_A/v_B = 1$, (18) implies for instance that, when donors are unconstrained:

$$\left(\frac{e_{A,i}}{e_{B,i}}\right)^{1-\rho} = \frac{S_B}{S_A}. \tag{19}$$

Similarly, comparing two unconstrained donors y_0 and y_i within a same party P yields:

$$\frac{e_{P,i}}{e_{P,0}} = \left(\frac{y_i}{y_0}\right)^{\frac{\theta}{1-\rho}}. \tag{20}$$

That is, among unconstrained donors, effort levels must be increasing in donor income at the same rate as in the unconstrained equilibrium. Hence, knowing $e_{P,0}$ is sufficient to determine the entire vector of effort levels that satisfy (18). It is also pinpoints the income level above which the cap starts to be binding. It is the value $\bar{y}$ that solves the condition:

$$\sigma\left(\left(\frac{\bar{y}}{y_0}\right)^{\frac{\theta}{1-\rho}} e_{P,0}|v_P,\bar{y}\right) = \bar{s}, \tag{21}$$

where the left-hand side is the value of $\sigma(e_{P,i}|v_P, y_i)$ at that point. Denote that solution $\bar{y}(e_{P,0},\bar{s})$. For income levels above that threshold, donors are constrained, which reverses the relationship between effort and income. For constrained donors, the equilibrium level of effort must be:

$$e_{P,i}(\bar{s}) = \left(\frac{\bar{s}}{y_i\ v_P}\right)^{1/\rho},\ \forall y_i \geq \bar{y}(e_{P,0},\bar{s}).$$

It follows that, as a party increases $e_{P,0}$ (and hence total effort), each individual effort level increases for unconstrained donors, *i.e.* $y_i < \bar{y}(e_{P,0},\bar{s})$, and remains constant for the higher income groups, $y_i \geq \bar{y}(e_{P,0},\bar{s})$. This implies that the share of capped donors increases in total effort since $\bar{y}(e_{P,0},\bar{s})$ is decreasing in $e_{P,0}$. Conversely, if the cap is raised, the share of capped donors decreases for any given level of effort since $\bar{y}(e_{P,0},\bar{s})$ is increasing in $\bar{s}$.

Denoting by $S_P(\bar{s})$, the total contributions flowing to P in the constrained equilibrium when the cap is $\bar{s}$ (hence $S_P(\bar{s}) = S_P^*$ for any $\bar{s} \geq \bar{s}_I$), we find that $\frac{S_B(\bar{s})}{S_A(\bar{s})}$ must be less than 1: indeed, from (19), $\frac{S_B}{S_A} \geq 1$ would imply $\frac{e_{A,0}}{e_{B,0}} \geq 1$. By (20), this would in turn result in higher individual contributions for A. With $n_A > n_B$, this necessarily implies $\frac{S_B}{S_A} < 1$, a contradiction. Conversely, $\frac{S_B}{S_A} < 1$ implies $\frac{e_{A,0}}{e_{B,0}} < 1$, and hence that all unconstrained A-donors will contribute strictly less than B-donors with the same income for any $\bar{s} > \bar{s}_0$.

Step 2: Let $e_{P,i}(\bar{s})$ denote the equilibrium level of effort when the cap is $\bar{s}$ and let $s_{P,i}(\bar{s}) = \sigma(e_{P,i}(\bar{s})|v_P, y_i)$ be the equilibrium level of contributions by a donor i. We show that there are potentially three subsets of donors: for a given cap $\bar{s} \in (\bar{s}_0, \bar{s}_I)$,

(1) there is a (possibly empty) set of income levels y_i such that neither A- nor B-donors are capped: $s_{A,i}(\bar{s}) < s_{B,i}(\bar{s}) < \bar{s}$

(2) there is a (non-empty) set of income levels y_i such that A-donors are uncapped and B-donors

are capped: $s_{A,i}(\bar{s}) < s_{B,i}(\bar{s}) = \bar{s}$
(3) there is a (possibly empty) set of income levels y_i such that both A- and B-donors are capped, $s_{A,i}(\bar{s}) = s_{B,i}(\bar{s}) = \bar{s}$.

Parts (1) and (2) imply that $\forall \bar{s} \in (\bar{s}_0, \bar{s}_I)$, $\pi_A(\bar{s})$ must be smaller than $\pi_A(\bar{s}_0)$. Since $n_A > n_B$, the cap $\bar{s}_0$ delivers the highest level of expected welfare. Next, from (2), proportionately more B-donors than A-donors are capped when $\bar{s}_I > \bar{s} > s_0$. Hence, their joint contribution capacity is reduced more than A's. Since this amounts to letting W_B/W_A decrease through a reduction in top B incomes, by Proposition 2, $\pi_A(\bar{s})$ increases above π_A^*. That is, $\pi_A(\bar{s}) > \pi_A(\bar{s}_I)$ for any $\bar{s} < \bar{s}_I$. Since $\pi_A^{SW} = 1$, expected welfare is lowest in $\bar{s}_I$. The proof of non-monotonicity is provided by the example in the main text. ■

Proof of Proposition 6. With $v > 1$, we have that the welfare maximizing level of π_A, is $\pi_A^{SW} = 1$ iff $v_A/v_B = v$ and is $\pi_A^{SW} = 0$ iff $v_A/v_B = 1/v$. This is true for any $v > 1$. The proof focuses on the realization of the shock $v_A/v_B = v$. By symmetry, the results are identical when party labels are swapped.

First, by (5), for $\bar{s} \geq \bar{s}_I$: $\frac{S_A(\bar{s})}{S_B(\bar{s})} = \frac{S_A^*}{S_B^*} = \left(\frac{W_A}{W_B}\right)^{1-\rho} = v$.

Next, we establish that, for all caps $\bar{s}$ tighter than s_I, we will have $S_A(\bar{s})/S_B(\bar{s}) < S_A^*/S_B^*$, that is A's advantage is hindered by any binding cap, in comparison to no cap. Thus, any binding cap is welfare reducing. Further, $S_A(\bar{s}_0)/S_B(\bar{s}_0) \leq S_A(\bar{s})/S_B(\bar{s})$: a tight cap is welfare dominated by any other cap.

Step 1: $1 \leq S_A(\bar{s})/S_B(\bar{s})$. The FOC with respect to the effort allocated to any unconstrained donor with income y_i is the same as in (18). With $v_A/v_B = v$, this implies that:

$$\left(\frac{e_{P,i}}{e_{P,0}}\right)^{1-\rho} = v\frac{S_B}{S_A}. \tag{22}$$

Comparing two unconstrained donors y_0 and y_i within a same party P still yields:

$$\frac{e_{P,i}}{e_{P,0}} = \left(\frac{y_i}{y_0}\right)^{\frac{\theta}{1-\rho}}, \tag{23}$$

and hence $\bar{y}(e_{P,0}, \bar{s})$ is defined in the same way as in (21). From (22), $\frac{S_B}{S_A} \geq 1$ would imply $\frac{e_{A,0}}{e_{B,0}} \geq 1$. By (23), we then have $\frac{e_{A,i}}{e_{B,i}} \geq 1$, $\forall i$. By $\sigma(e_{P,i}|v_P, y_i) = v_P\ y_i\ \frac{e_{P,i}^{\rho}}{\rho}$ and $v > 1$, this implies $s_{A,i}(\bar{s}) \geq s_{B,i}(\bar{s})$ for any given income level y_i, which implies $\frac{S_B}{S_A} < 1$, a contradiction. Hence, $\frac{S_B}{S_A} < 1$, implying that $\pi_A(\bar{s}) > 1/2$ for any $\bar{s} > \bar{s}_0$ whereas $\pi_A(\bar{s}_0) = 1/2$.

Step 2: Next, we show that $s_{A,i}(\bar{s}) \geq s_{B,i}(\bar{s})$ in equilibrium. We prove this by contradiction: by (22), $s_{A,i}(\bar{s})$ could only be smaller than $s_{B,i}(\bar{s})$ if $\left(\frac{S_B(\bar{s})}{S_A(\bar{s})}\right)^{\rho} \leq 1/v$. However, this would in turn require that $\frac{S_A(\bar{s})}{S_B(\bar{s})} \geq v$, which could only happen if individual A-donors contribute more than B-donors; hence the contradiction. As a consequence, any cap $\bar{s} < \bar{s}_I$ constrains more A than B donors.

Step 3: $S_A(\bar{s})/S_B(\bar{s}) \leq v$. To prove this step, note that, by Step 2, a necessary condition for $S_A(\bar{s})/S_B(\bar{s}) > v$ is that A collects sufficiently more than B from uncapped donors: for them,

$\frac{s_{A,i}(\bar{s})}{s_{B,i}(\bar{s})}$ should be strictly larger than v. Given $\sigma\left(e_{P,i}|v_P, y_i\right) = v_P\ y_i\ \frac{e_{P,i}^{\rho}}{\rho}$, (22) implies that:

$$\frac{s_{A,i}\left(\bar{s}\right)}{s_{B,i}\left(\bar{s}\right)} = v^{\frac{1}{1-\rho}}\left(\frac{S_B\left(\bar{s}\right)}{S_A\left(\bar{s}\right)}\right)^{\frac{\rho}{1-\rho}}.$$

This contribution ratio is only larger than v if:

$$v^{\frac{1}{1-\rho}}\left(\frac{S_B\left(\bar{s}\right)}{S_A\left(\bar{s}\right)}\right)^{\frac{\rho}{1-\rho}} > v \Leftrightarrow \frac{S_A\left(\bar{s}\right)}{S_B\left(\bar{s}\right)} < v,$$

which proves the claim: even when uncapped A donors contribute much more than B, $S_A\left(\bar{s}\right)/S_B\left(\bar{s}\right) < S_A\left(\bar{s}_I\right)/S_B\left(\bar{s}_I\right)$. Hence, A's probability of winning is lower with $\bar{s} < \bar{s}_I$ than under the lax cap, which is welfare reducing. ■

Proof of Proposition 7. First, note that if both parties are capped, then their probability of winning is 1/2 independently of donor preferences. This is necessarily worse than laissez-faire, which aligns election results with the realization of the popularity and preference intensity shocks.

By Lemma A.2, any reduction in S_A, the total contributions of the frontrunner, whether it is the result of a drop in W_A or of a legal constraint, must increase solicitation effort by the trailing party, here B, and hence its contributions S_B. The cap thus necessarily reduces the probability that the frontrunner wins.

Thus, a necessary condition for a cap to be welfare improving is that the ranking of parties under laissez-faire differs from the socially preferred ranking when the popularity and preference intensity shocks each favor a different party, that is $W_A > W_B$ (A is favored under laissez-faire) but $n_A v_A < n_B v_B$ (B is socially desirable). From the results in Section 4.4, this may only happen if A benefits from the preference intensity shock and B from the popularity shock:

$$\begin{aligned} W_A &= v^{\frac{1}{1-\rho}} > n = W_B \Leftrightarrow v > n^{1-\rho} \\ \text{but} &: n_A v_A = v < n = n_B v_B \Leftrightarrow v < n. \end{aligned}$$

This results in the following *necessary* condition for the cap to be possibly welfare *improving*:

$$n > v > n^{1-\rho}.$$

Hence, a *sufficient* condition for the cap to be welfare *reducing* is that either $v > n$ or $v < n^{1-\rho}$.
■

Proof of Proposition 8. As before, an effort $e_{P,i}$ results in a pre-matching subsidy contribution:

$$\sigma_{P,i}\left(e_{P,i}|v_P, y_i\right) = \lambda_{P,i} e_{P,i}^{\rho}.$$

From Lemma 1, for a given level of total effort $\bar{E}_P$, we have:

$$e_{P,i}\left(\bar{E}_P\right) = \frac{\left(\lambda_{P,i}\right)^{\frac{1}{1-\rho}}}{W_P}\bar{E}_P.$$

It follows that total pre- and post-matching subsidy contributions are respectively:

$$S_P\left(\bar{E}_P|W_P\right) = \bar{E}_P^{\rho} W_P^{1-\rho}$$

and

$$\begin{aligned}\tilde{S}_P\left(\bar{E}_P|W_P\right) &= \sum_i n_{P,i}\ \lambda_{P,i}\left(1+m\right) e_{P,i}^{\rho} = \left(1+m\right)\bar{E}_P^{\rho}\sum_i n_{P,i}\ \lambda_{P,i}\frac{\left(\lambda_{P,i}\right)^{\frac{\rho}{1-\rho}}}{W_P^{\rho}} \\ &= \left(1+m\right)\bar{E}_P^{\rho} W_P^{1-\rho}.\end{aligned}$$

Plugging this into P's optimization problem obtains:

$$\max_{E_P} \frac{1}{1+\left(\frac{(1+m)E_{-P}^{\rho}W_{-P}^{1-\rho}}{(1+m)E_P^{\rho}W_P^{1-\rho}}\right)^{\gamma}} - \frac{E_P^{\tau}}{\tau} = \frac{1}{1+\left(\frac{E_{-P}^{\rho}W_{-P}^{1-\rho}}{E_P^{\rho}W_P^{1-\rho}}\right)^{\gamma}} - \frac{E_P^{\tau}}{\tau},$$

which is thus independent of m. Hence, both E_A^* and E_B^* must also be independent of m: equilibrium efforts and post-matching subsidy contributions are identical to those in Proposition 2. ■

Proof of Proposition 9. From an ex ante standpoint, one must consider $2^3 = 8$ possible outcomes: either party may benefit from the income shock, from the popularity shock, and from the preference intensity shock. Assume the income shock benefits A. By assumption here, y is so large that in the absence of a cap, the equilibrium contributions of A-donors become much higher than that of B-donors (the shock y benefiting A increases W_A/W_B by orders of magnitude).

Let $\bar{s}_I$ denote the highest individual contribution of any A-donor across all possible realizations of the popularity and preference intensity shocks. Conversely, $\bar{s}_0$ is the contribution of a B-donor with income y_0 when all the shocks are in favor of A, and all the other donors are capped at $\bar{s}_0$.

Now, consider the electoral outcome when $\bar{s}$ is close to $\bar{s}_I$, i.e. the cap does not constrain any donor. For $y \to \infty$, the party benefitting from the income shock (A in the above case) wins almost for sure, independently of $n_A v_A$ and $n_B v_B$. From a welfare standpoint, victory is a pure lottery.

Contrast this laissez-faire situation with the effect of a tight cap. For $\bar{s} = \bar{s}_0$, $S_A/S_B = n_A/n_B$. Hence $\pi_A\left(\bar{s}_0\right) \lesseqgtr 1/2$ iff $n_A/n_B \lesseqgtr 1$. This implies that expected welfare may only increase in comparison with the laissez-faire benchmark in the presence of a popularity shock, but the outcome remains a pure lottery in the presence of preference intensity shocks only ($n = 1 < v$).

Finally, consider an intermediate cap, high enough that most donors remain uncapped unless they are hit by a positive income shock. In contrast, the cap is low enough that (almost) all donors are capped when hit by the income shock (for $y \to \infty$, the unconstrained contributions of high-income donors are always the highest). Still when A is the party with the positive income shock, we have:

$$\begin{aligned}S_A\left(\bar{s}\right) &= n_A \bar{s} \\ S_B\left(\bar{s}\right) &\in \left\{S_{11}, S_{n1}, S_{1v}, S_{nv}\right\},\end{aligned}$$

with $S_{11} < S_{n1}, S_{1v} < S_{nv}$, denoting the equilibrium level of contributions when B has, respectively, the lowest popularity and lowest preference intensity (S_{11}); the highest popularity but lowest preference intensity (S_{n1}), etc. With $\bar{s}$ being finite, $S_A(\bar{s})$ is constant and finite. Hence:

$$1 > \pi_A(S_{11}) > \pi_A(S_{n1}), \pi_A(S_{1v}) > \pi_A(S_{nv}) \geq 1/2,$$

where $\pi_A(S_{11})$ is a shortcut for $\pi_A(n_A\bar{s}, S_{11})$, and similar shortcuts are used for $\pi_A(S_{n1})$, $\pi_A(S_{1v}) > \pi_A(S_{nv})$. Thus, the party with the positive income shock still wins with probability above 1/2, but there must exist a cap $\bar{s}$ such that (i) this probability is brought to a level significantly lower than 1 and (ii) the ranking of probabilities broadly aligns with the preference shocks. The drop in $\pi_A(S_{11})$ below 1 is necessarily welfare reducing, whereas that in $\pi_A(S_{nv})$ is welfare increasing.

Comparing expected welfare under laisser-faire and under the cap $\bar{s}$, we need to verify whether welfare increasing effects dominate, *i.e.* whether:

$$\begin{aligned}\frac{1}{4}(nv+n+v+1) \quad < \quad & \frac{1}{4}(\pi_A(S_{11})nv + (1-\pi_A(S_{11})) + \pi_A(S_{n1})n + (1-\pi_A(S_{n1}))v + ... \\ & ... + \pi_A(S_{1v})v + (1-\pi_A(S_{1v}))n + \pi_A(S_{nv}) + (1-\pi_A(S_{nv}))nv),\end{aligned}$$

where the left-hand side reads as follows: under laissez-faire, party A wins with probability about 1, independently of preference shocks. There are four equally likely outcomes: first, A benefits both from the popularity and preference intensity shock. In that case, welfare is nv (instead of 1 if B were to win). Second, A benefits from the popularity shock, but it is B that benefits from the preference intensity shock. In that case, welfare is n (instead of v if B were to win). Third, A benefits from the preference intensity shock, and B from the popularity shock. Welfare is v (instead of n). Finally, if B benefitted from both shocks, welfare would be 1 (instead of nv).

The right-hand side are the corresponding value of expected welfare under the cap $\bar{s}$: instead of A winning with probability 1, A wins respectively with probabilities $\pi_A(S_{11})$, $\pi_A(S_{n1})$, $\pi_A(S_{1v})$ and $\pi_A(S_{nv})$, and B wins with the complementary probabilities. Rearranging terms yields:

$$\frac{nv+1-\pi_A(S_{11})nv-(1-\pi_A(S_{11}))-\pi_A(S_{nv})-(1-\pi_A(S_{nv}))nv}{4} < \frac{-n-v+\pi_A(S_{n1})n+(1-\pi_A(S_{n1}))v+\pi_A(S_{1v})v+(1-\pi_A(S_{1v}))n}{4},$$

$$\text{or: } \frac{(\pi_A(S_{11}) - \pi_A(S_{nv}))(1-nv)}{4} < \frac{(\pi_A(S_{1v}) - \pi_A(S_{n1}))(v-n)}{4},$$

where the last inequality is necessarily satisfied. Indeed, $\pi_A(S_{11}) - \pi_A(S_{nv}) > \pi_A(S_{1v}) - \pi_A(S_{n1})$ and $nv - 1 > \max\{0, v-n, n-v\}$. Hence, expected welfare with $\bar{s}$ must be strictly higher than with $\bar{s}_I$. For $n = 1 < v$, we have seen that expected welfare with the tight cap $\bar{s}_0$ was equal to the one with the lax cap $\bar{s}_I$. Hence, the intermediate cap dominates both $\bar{s}_I$ and $\bar{s}_0$. ■

Proof of Proposition 10. Since the proof applies to any income elasticity θ, let donor i's pre-tax/subsidy donation for any effort $e_{P,i}$ be:

$$\sigma_{P,i}(e_{P,i}) = (y_i)^\theta v_P \ (e_{P,i})^\rho / \rho.$$

This results in a net contribution of:

$$\sigma_{P,i}(e_{P,i}) = (1-\tau(y_i))(y_i)^\theta v_P \ (e_{P,i})^\rho / \rho.$$

We need to design the tax so as to ensure that contributions are independent of income. This requires that:

$$(1-\tau(y_i))(y_i)^{\theta}=k,$$

where k is a positive constant. Solving for $\tau(y_i)$:

$$\tau(y_i)=1-\frac{k}{(y_i)^{\theta}}.$$

Following the same argument as in the last step of the proof of Proposition 9, since the probabilities of winning then align with social preferences, social welfare may only increase. ■

Appendix 4: Microfoundations of Contribution Strategies

Our theory of small campaign contributions relies on several important premises: small donors (i) derive consumption utility from the act of contributing, (ii) this consumption utility is modulated by the fundraising effort from their party, and (iii) are not affected by the electoral and influence motives. We formalize these premises through assumption (1), which captures in a reduced-form and tractable way the link between contributions and fundraising effort. As we discussed in the core of the text, this specification makes three implicit assumptions: (i) small donors increase their contributions when their party solicits them; (ii) small donors *only* contribute in response to their party soliciting their contributions; (iii) party solicitation efforts and the donor's contribution potential are complementary: $\partial^2\sigma/\partial\lambda_{P,i}\partial e_{P,i}>0$.

The third implicit assumption draws inspiration from models of advertising such as Stigler and Becker (1977) and, in particular, Becker and Murphy's (1993) treatment of "advertisements and the goods advertised as complements in stable meta-utility functions". In particular, assumption (1) should capture the outcome of the donors' maximizing some utility function $U_i(c_i, s_{P,i}|S_A, S_B, e_{P,i})$ that depends on the consumption of "other goods" c_i and on the "advertised good"—here the small donor's contributions, $s_{P,i}$.[24] Like in Becker and Murphy (1993), independently of whether the party's fundraising effort $e_{P,i}$ is a "good" ($\partial U_i/\partial e_{P,i}>0$) or a "bad" ($\partial U_i/\partial e_{P,i}<0$), the marginal utility of contributing for the donor's preferred party is increasing in $e_{P,i}$: $\partial^2 U_i/\partial s_{P,i}\partial e_{P,i}>0$. As a result, the equilibrium contribution by i is strictly increasing in the party's effort $e_{P,i}$.

Such a utility function can take many different forms. Focusing on additively separable utility functions, consider a donor i who supports party P and has a budget y_i to allocate between c_i and

[24] As we detail below, this notation clarifies that donor i takes the total contribution to the two parties, S_A, S_B, and hence the expected outcome of the election as given.

$s_{P,i}$. Her utility function is:

$$\begin{aligned} U_i\left(c_i, s_{P,i} | S_A, S_B, e_{P,i}\right) &= w\left(s_{P,i} | e_{P,i}, v_P\right) + x\left(c_i\right) + v_P\ \pi_P\left(S_A, S_B\right), \\ &\quad \text{s.t. } c_i + s_{P,i} = y_i. \\ &= w\left(s_{P,i} | e_{P,i}, v_P\right) + x\left(y_i - s_{P,i}\right) + v_P\ \pi_P\left(S_A, S_B\right). \end{aligned} \tag{24}$$

The first term in (24) captures the small donor's consumption utility from contributing. In line with the discussion above, it increases both in v_P, the intensity of her political preference for party P, and in the party's targeted effort $e_{P,i}$. Moreover, both arguments are complements with contributions $s_{P,i}$: $\partial^2 w / \partial s_{P,i} \partial e_{P,i} > 0$ and $\partial^2 w / \partial s_{P,i} \partial v_P > 0$. The second term is her utility of consuming the other goods, c_i. We impose that the functions x and w satisfy Inada conditions and the standard assumptions of non-satiation and concavity ($x', w' > 0 > x'', w''$). The third term is a constant in the framework of the donor's optimization problem. It represents the donor's political preference: her utility increases by v_P when her preferred party wins the election, which happens with probability π_P. Key to our approach, small donors consider themselves as too small to have any meaningful influence on the outcome of the election, i.e., on π_P. Hence, each donor bases her decision of how to allocate her budget between contributions $s_{P,i}$ and consumption c_i in a decision-theoretic way, as opposed to a game-theoretic way. Note that y_i does not directly affect the consumption utility of contributions. That is, we do not make any assumption of complementarity between $s_{P,i}$ and y_i.

Maximizing (24) with respect to c_i and $s_{P,i}$, the first order condition becomes:

$$\frac{\partial w\left(s^*_{P,i} | e_{P,i}, v_P\right)}{\partial s_{P,i}} = -\frac{\partial x\left(y_i - s^*_{P,i}\right)}{\partial s_{P,i}}.$$

Since both w and x are strictly concave, it is straightforward to show that $s^*_{P,i}$ is strictly increasing in $e_{P,i}$, v_P, and y_i.

Making more specific assumptions on w and x, let $w(\cdot) = \alpha / (1-\alpha) \log\left(s_{P,i}\right)$ and $x(\cdot) = \log\left(c_i\right)$, with $\alpha = v_P \frac{e^\rho_{P,i}}{\rho}$, $0 < \rho < 1$ and v_P small enough to ensure that the fraction is strictly positive in the relevant range of $e_{P,i}$. In that case, the FOC becomes:

$$\frac{v_P \frac{e^\rho_{P,i}}{\rho}}{1 - v_P \frac{e^\rho_{P,i}}{\rho}} \frac{1}{s^*_{P,i}} = \frac{1}{y_i - s^*_{P,i}} \Leftrightarrow s^*_{P,i} = v_P y_i \frac{e^\rho_{P,i}}{\rho},$$

which is a specific version of our initial assumption (1): $\sigma_{P,i}\left(e_{P,i} | v_P, y_i\right) = \lambda_{P,i}\left(v_P, y_i\right) \frac{e^\rho_{P,i}}{\rho}$. The main advantage of this specification is that it captures all the important elements we have discussed, and it proves particularly tractable and yet flexible. The Cobb-Douglas nature of the latter specification produces the tractable solution we hypothesize in (1), and it parametrizes the elasticity of contributions with respect to effort as a constant, ρ. Since an important part of the empirical literature we discuss aims at estimating such an elasticity, assuming iso-elasticity allows us to directly connect this literature to our model.