First Summer School on Cognitive Robotics

Sampling-Based Motion Planning

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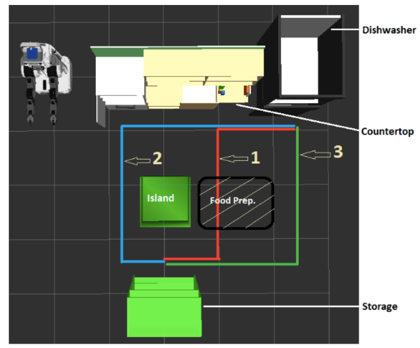
Combined task and motion planning

Motivating example: want all dirty dishes in the kitchen cleaned and stored



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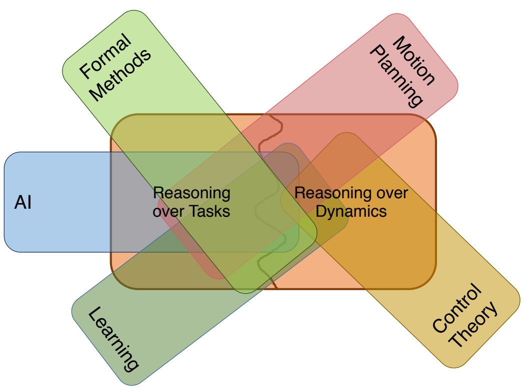

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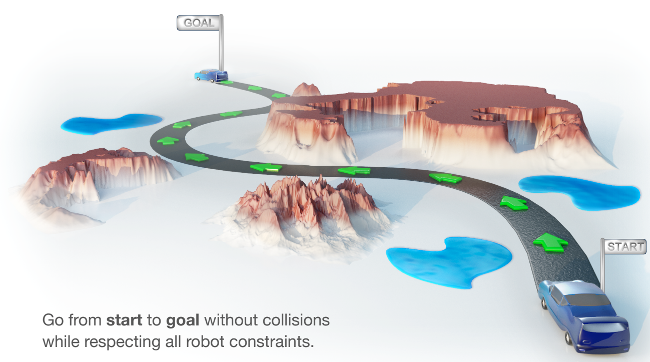
Motion planning: classical setting



Go from **start** to **goal** without collisions
while respecting all robot constraints.

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Motion planning

- Is done in a continuous world and with constrained motions.
- Needs to know robot and world geometry.
- Needs to know robot and world physics.
- Must be accurate and predictive to work in practice.

Some notes:

- More powerful motion planning simplifies the task planner.
- More accurate motion planning simplifies motion execution.
- Motion planning is limited by model accuracy and complexity.

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Motion planner is part of a replanning loop

Bekris et al.

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Motion planning problems are hard

PROBLEM	COMPLEXITY
Geometric Constraints:	
Sofa Mover (3DOF)	$O(n^{2+\epsilon})$ - not implemented [HS96]
Piano Mover (6DOF)	Polynomial – no practical algorithm [SS83]
n Disks in the Plane	NP-Hard [SS83]
n Link Chain in 3D	PSPACE-Complete [HSS87]
Generalized Mover	PSPACE-Complete [Canny88]
Dynamics Constraints:	
Point with Newtonian Dynamics	NP-Hard [DXCR93]
Polygon Dubin's Car (Linear)	Decidable [CPK08]
Nonlinear	Unknown, probably undecidable
Discrete Transitions and Dynamics Constraints:	
Hybrid Systems	Undecidable [Alur et al. 95]

Exact, approximate, and probabilistically complete algorithms

Method	Advantage	Disadvantage
exact	theoretically insightful	impractical
cell decomposition	easy	does not scale easily
control-based	online, very robust	requires good trajectory
potential fields	online, easy	slow or fail
sampling-based	**fast and effective**	**cannot recognize impossible query**

Lecture outline

1. Overview of sampling-based robot motion planning
2. Integrated task and motion planning
3. Overview of the Open Motion Planning Library (OMPL)

Overview of sampling-based robot motion planning

Basic concepts and definitions

- Workspace
- Robot
- State space
- Path
- Planning Instance
- Query/Problem

Workspace

- The **workspace** is the environment that the robot operates in.
- The boundary of the workspace determines the **obstacles**.

Robot

A robot is defined by:

- Geometry
- Parameters or **Degrees of Freedom (DOF)**
  - Different settings for the parameters embed the geometry in different ways into the workspace.

State space

- The parameter space for the robot is called the **state space S**.
- A point in this space is a **state**.

Free state space

- A state is **free** if the corresponding embedding of the robot's geometry lies in the workspace.
- The subspace of free configurations is **free state space S_{free}**.
- **S_{free}** can be very complex even for seemingly simple systems.
- This complexity is the main difficulty in motion planning.

Paths

- A **path** is continuous mapping in S

$$\pi : [0, L] \rightarrow S_{free}$$

- L is the **length** of the path.
- The path is **collision-free** if for all t

$$\pi(t) \in S_{free}$$

Planning instance

A planning instance consists of:

- Robot (S-space and embedding).
- Workspace.
- Constraints.

Query / problem definition

- A problem or **query** is:
  - Given two states, q_0 and q_f, determine if there is a collision-free path between q_0 and q_f.

Probabilistic Roadmap Planner (PRM)

Kavraki, Svestka, Overmars and Latombe, 1996

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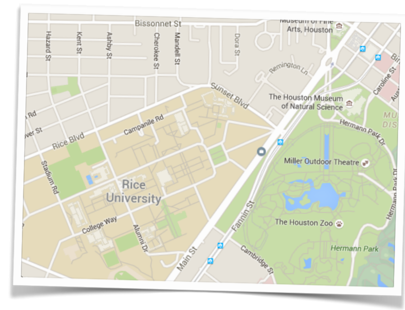
PRM

- Uses random sampling.
- Uses simple local planner.
- Builds a roadmap of the state space.



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PRM

- Illustrate with an easy planning instance/problem set up.
  - Robot is a point in 2D.
  - Robot moves freely.
  - Simple example used for illustration only.
- Isolate primitive techniques.
- Generalize.

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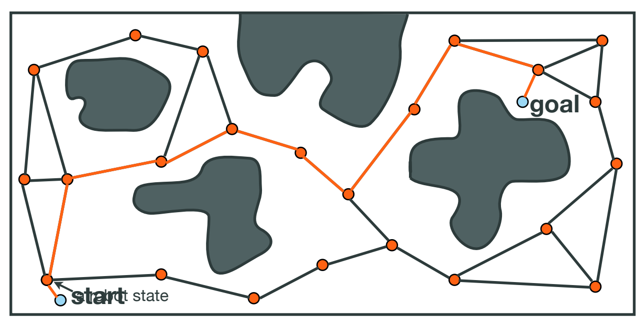
Point robot in 2-D



: nodes: random states
: edges: paths computed by local planner
plan a path: 1. connect start & goal to roadmap
2. perform graph search

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Primitive techniques

- **Select Sample:** (in the example) Uniform sampling to get milestones.
- **Connect:** (in the example) Local planner uses "straight lines."
- **Store in some data structure:** (in the example) A graph.
  - A **roadmap** is finite graph G=(V,E)
  - V is a subset of S_{free}.
  - (s_1,s_2) in E implies that the local planner found a path.

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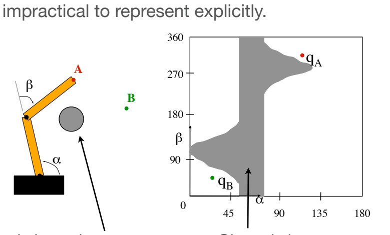
Why use sampling?

- S_{free} is impractical to represent explicitly.



Obstacle in workspace
Obstacle in state space

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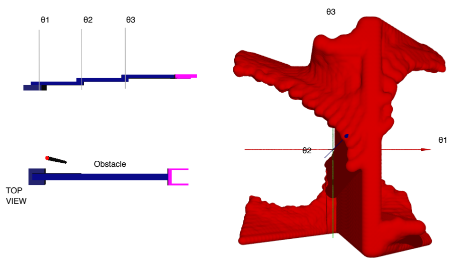
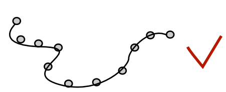


Why use sampling?

- S_{free} is impractical to represent explicitly.
- Sampling can be very efficient.
- Resulting data structure can be very compact.

Connecting samples

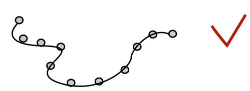
- An example of a simple planner:
  - Computes the straight line path between q1, q2.
  - Checks to see if it is valid.
  - If so, returns SUCCESS and the path.
  - Otherwise, returns FAIL.



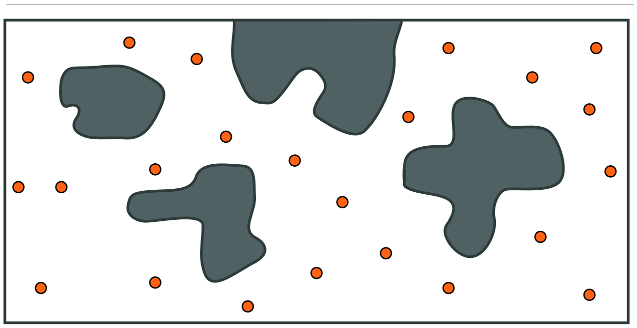
May fail often

State validity checker

- For states
  - Use e.g., collision checking, check any bounds
- For paths
  - State validation along a path is done by recursive refinement.
  - Bounds on clearance are combined with bounds on motion to cover the path with open balls or find a collision.

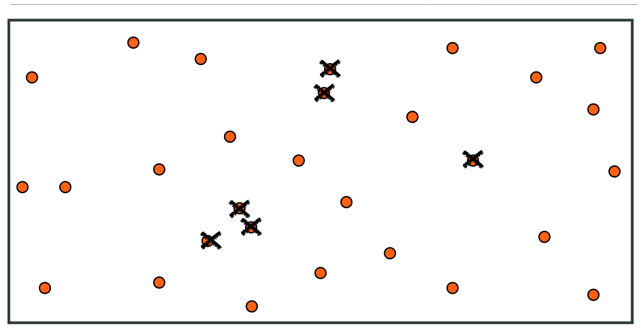


Operation of PRM



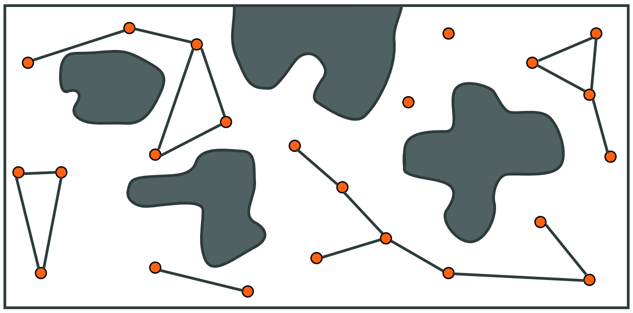
: nodes, random states

Operation of PRM



: states

Operation of PRM



:**edges,** paths computed by local planner

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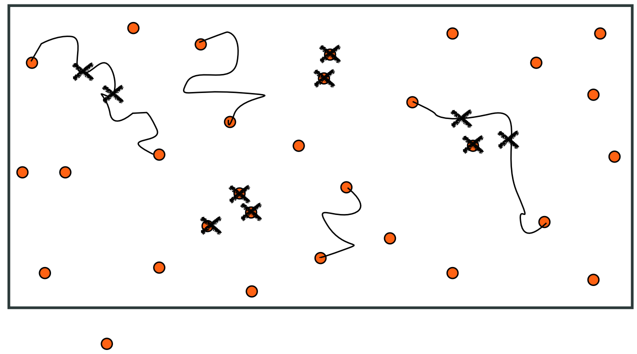
Operation of PRM



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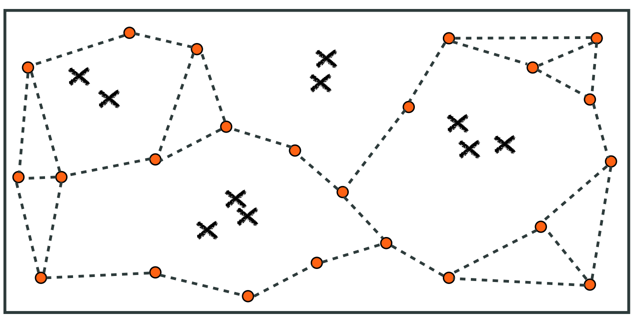
Operation of PRM



feasible path computed by local planner

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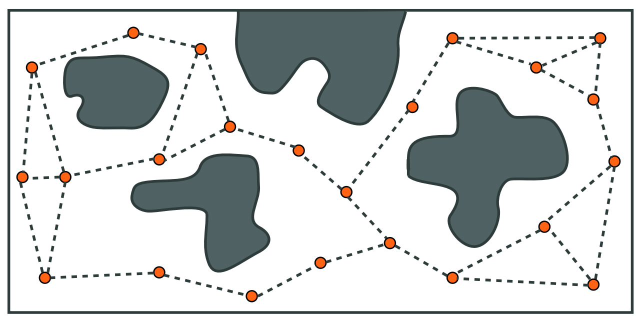
Operation of PRM



feasible path computed by local planner

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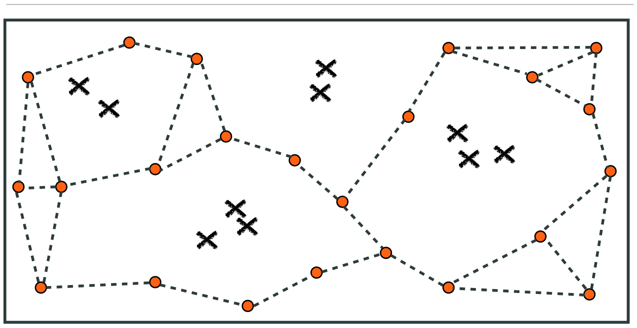
Operation of PRM



feasible path computed by local planner

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Completeness of PRM

- If no path exists, then PRM cannot find the path.
- But… if a path exists, it is possible PRM fails to find it.
- PRM is not complete but instead is **probabilistically complete**.

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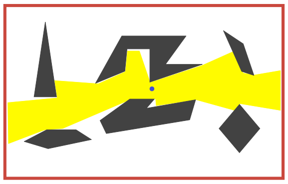
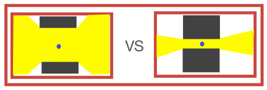
Theoretical analysis of PRM (1/2)



[Kavraki et al 96, 98, 00, 03, 07]

ε-goodness property



- Tradeoff: planner may fail with probability α
- Number of nodes/states:

$$N \approx \frac{1}{\epsilon}[log(\frac{1}{\epsilon}) + log(\frac{4}{\alpha})]$$

- Important: Performance related to properties of the space

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Theoretical analysis of PRM (2/2)

- We sacrifice completeness for speed
- **Probabilistic completeness**
- Novel analysis and performance guarantees

$$Pr(failure) = f(e^{-cN})$$

- How much can the assumptions be relaxed?

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Primitive techniques

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Primitives

- **Select Sample:** Uniform sampling is general but not the most efficient.
  - Optimal selection remains elusive.
- **Connect:** Connect all to all is general but not efficient.
  - Neighbors
  - Notion of “straight line” or other local plan needs to be adapted.
- **Store efficiently**

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Primitives

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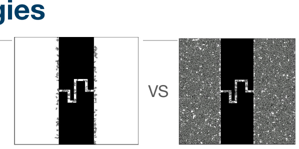
Several sampling strategies

- Gaussian sampling [Overmars et al.]:
  - Places samples close to objects.
  - Distribution is Gaussian around the obstacle boundary.



- Medical-axis sampling [Amato et al.]
- Bridge Test sampling for narrow corridors [Hsu et al.]
- Quasi-Random sampling [LaValle et al.]
- Selective sampling [Kavraki et al.]

One of the most critical parts of the planner [Hsu, Latombe 1998].

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Primitives

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Several connection strategies

- **Nearness:** Try to connect each configuration to a constant number of "nearby" configurations.
  - nearest neighbors by kd-trees, k-NN, k-ANN
  - random neighbors may be helpful
- **Component technique:** Only test edges which reduce the number of connected components in the roadmap.

[Svestka, Overmars, 1996]

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Primitives

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A generic sampling-based tree planner

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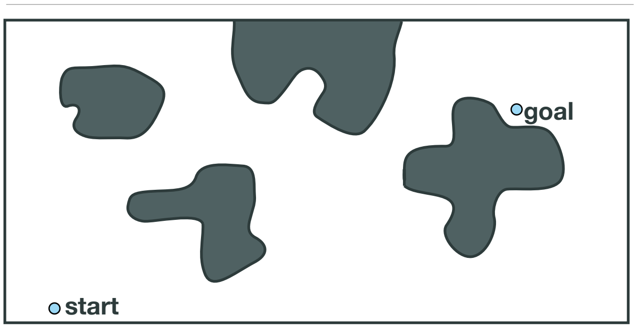
Sampling-based tree planner operation



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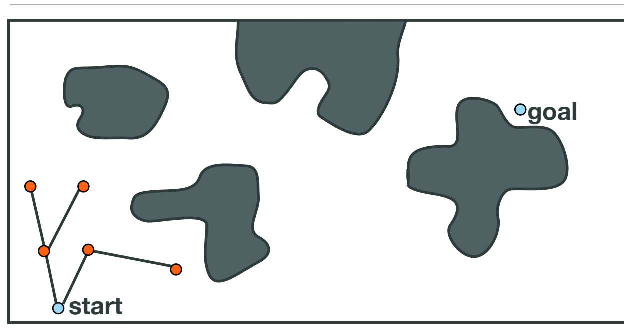
Sampling-based tree planner operation



grow random tree from **start**

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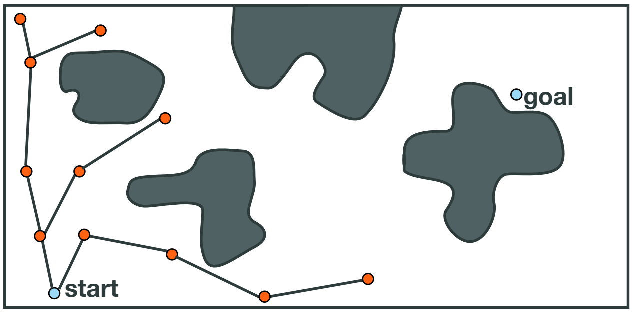
Sampling-based tree planner operation



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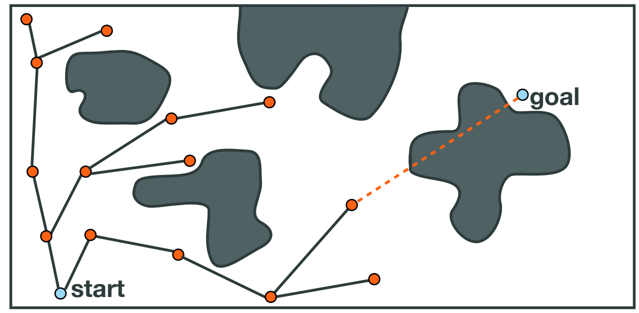
Sampling-based tree planner operation



: occasionally attempt to connect tree to **goal**

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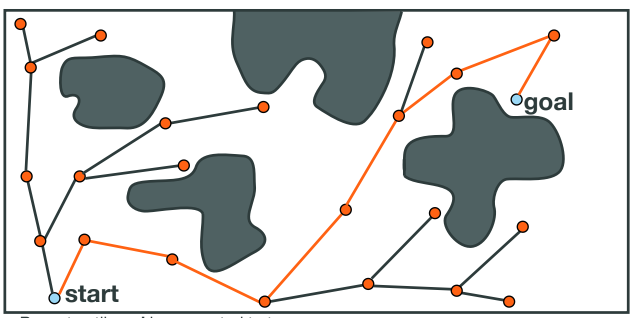
Sampling-based tree planner operation



- Repeat until **goal** is connected to tree.
- Bi-directional trees are possible when considering only geometric constraints.

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Primitives

- **Select Sample**
- **Expand from the sample**
- **Store efficiently**

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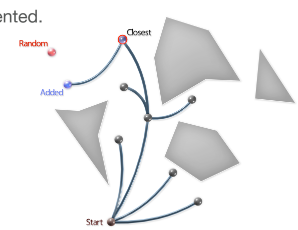
Rapidly Exploring Random Trees (RRT)

- Uses proximity query to guide construction (Voronoi Bias).
- Uses propagation instead of connection.
- Powerful heuristic for single-query planning.
- Bi-directional search can be implemented.



[Lavalle, Kuffer 1999, 2000]

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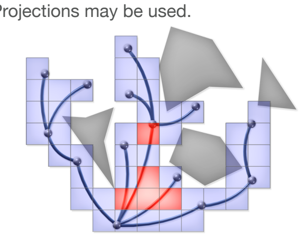
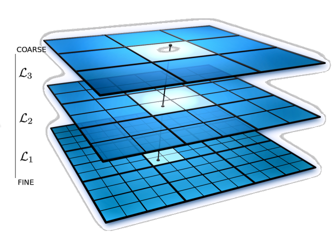
KPIECE

- Keeps track of coverage by using discretization and by distinguishing the boundary from the covered space.
- Keeping track of coverage can be done in a hierarchical fashion.
- Projections may be used.





[Şucan, Kavraki 2008]

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Performance improvements for trees

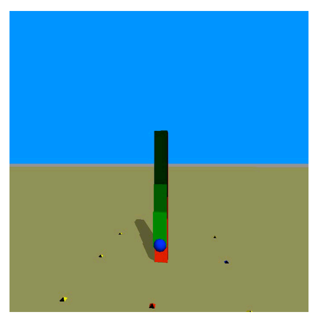
- Bi-directional search.
- Lazy collision checking.
- Goal biasing.
- Accounting for constraint manifolds.
- Employing motion primitives.
- and many others.

Optimal Paths

- Most sampling-based planning algorithms produce **feasible** rather than **optimal** paths.
- Two approaches to achieving optimality:
  - **Local** optimization of path in post-processing step (short-cutting, smoothing, etc.)
  - **Global** optimization: connect to “enough” neighbors, “rewire” tree as nodes are added.
    → provably asymptotic (near-)optimal paths!

Planning with dynamics: Trees offer an advantage

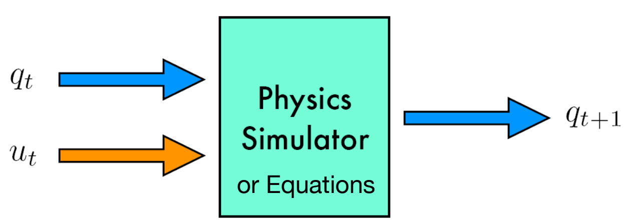
Planning with dynamics



Ladd et al.

Bekris et al.

Physical system planning



$$F(q_t, u_t) = q_{t+1}$$

Space of controls is defined

Physical system planning

Given

1. an initial state $q_0 \in Q$
2. a goal set $G \subset Q$

The discrete physical systems planning problem is to compute a sequence $u_0, \ldots, u_N$ such that:

$$F(q_i, u_i) = q_{i+1}$$

and $q_{N+1} \in G$ is contained in the goal set.

Planning with dynamics

- Adding dynamics is essential to increase physical realism.
- Techniques from control theory can be used to create better paths or reduce differential equation integrations.
- Metrics tend to work poorly.
- Efficient planning for systems with dynamics is still fairly open: sampling-based tree planners offer an advantage.

Primitives

- **Select Sample**
- **Expand from the sample**
- **Store efficiently**

These primitives are combined with various optimizations.

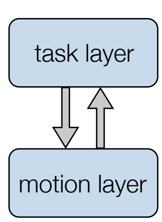
Lecture outline

1. Overview of sampling-based robot motion planning
2. **Integrated task and motion planning**
3. Overview of the Open Motion Planning Library (OMPL)

Integrated task and motion planning

Integrated task and motion planning (TMP)

- **Task Planning**
  - **Discrete** abstraction of the robot/environment
  - **PRO:** efficiently plans in abstractions
  - **CON:** continuous domains hard to abstract
- **Motion Planning**
  - Collision free path in **continuous** space
  - **PRO:** practical planning algorithms
  - **CON:** computational limits on dimensionality / plan length
- **Integrated Task and Motion Planning**
  - Task Planning reduces search space for Motion Planner
  - Motion Planning can help guide abstraction for Task Planner



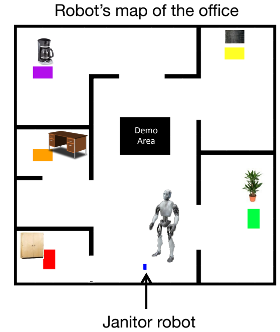
Moving beyond "reach the destination" paradigm

Mission:

"Do the following in any order and always avoid the black demo area

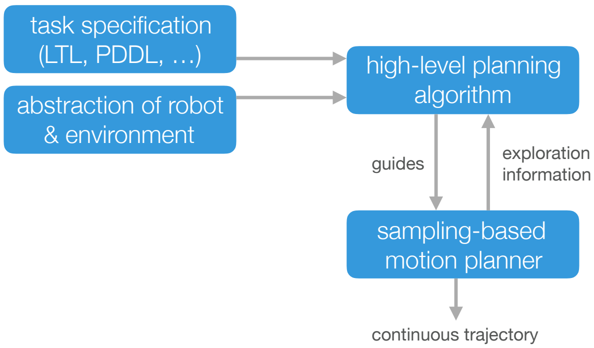
- water the plant
- clean the blackboard
- turn off the coffee maker
- pick up vacuum cleaner from the supply room, then vacuum the orange room and dust its table."

Robot's map of the office



iRobot image: http://i.imgur.com/Y0SITTQ.png

Overview of our approach:
A synergistic framework for task & motion planning



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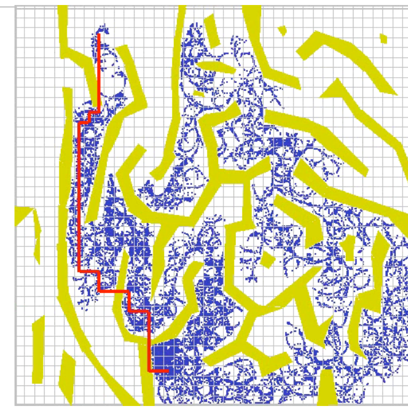
Basic example of synergistic framework:



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Synergistic task & motion planning

Our group has explored two different approaches:

- **Linear Temporal Logic task specifications + complex dynamics:**
  - Use state space abstraction + automaton to construct guides
  - Sampling-based planning around guides, exploration progress feedback
  - ➡ *See LTLPlanner in OMPL*
- **PDDL + Incremental SMT Solvers:**
  - Generate task plans of fixed length
  - Attempt to find corresponding motion plans
  - If no solution found, increase task plan length
  - ➡ *See TMKit at* http://tmkit.kavrakilab.org

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Moving beyond "reach the destination" paradigm

Mission:

"Do the following in any order and always avoid the black demo area

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The framework is powerful

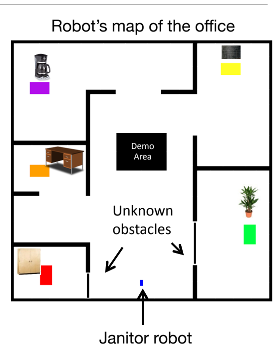
Safe LTL ✓

Hybrid systems ✓

Changes in the environment ✓
(partial satisfaction)

Manipulation ✓

Motion uncertainty ✓



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Lecture outline

1. Overview of sampling-based robot motion planning
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3. **Overview of the Open Motion Planning Library (OMPL)**

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The Open Motion Planning Library

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OMPL in a nutshell

- Common core for sampling-based motion planners
- Includes commonly-used heuristics
- Takes care of many low-level details often skipped in corresponding papers
- Intended for use in:
  - Education
  - Research
  - Industry

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Other related robotics software

ROS MoveIt!

OpenRAVE

v-rep virtual robot experimentation platform

Robotics Library

MORSE the Modular OpenRobots Simulation Engine

VEROSIM SOLUTIONS

All of them support OMPL!

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Abstract interface to core sampling-based motion planning concepts

- state space / control space
- state validator (e.g., collision checker)
- sampler
- goal (problem definition)
- planner
- ...

except robot & workspace...

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Many planners available in OMPL

Planner

geometric planning

KPIECE, BKPIECE, LBKPIECE
PRM, LazyPRM
RRT, RRTConnect, LazyRRT
EST, SBL
PDST
STRIDE

Optimizing planners:
PRM*
RRT*, FMT*, BIT*
T-RRT
SPARS, SPARS-2

planning with controls

KPIECE
RRT
EST
Syclop
PDST
LTLPlanner

Optimizing planner:
SST

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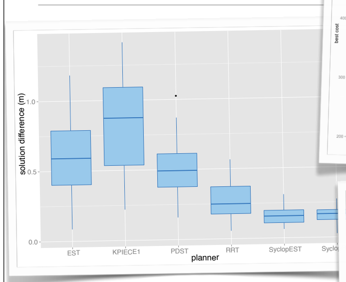
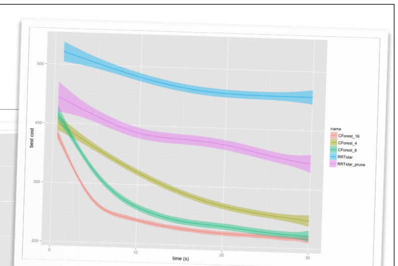
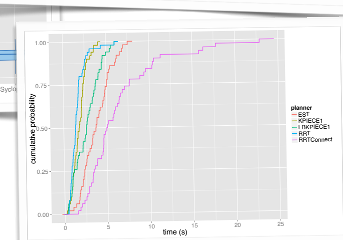
Benchmarking







http://plannerarena.org

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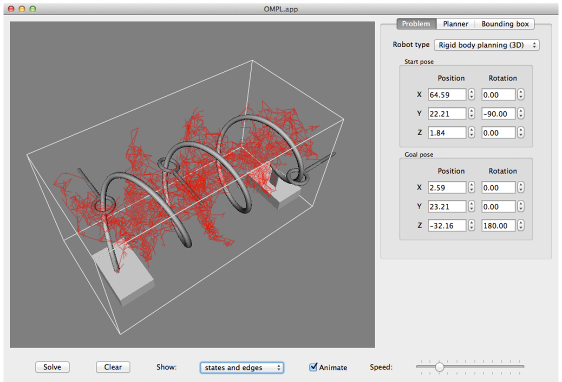
OMPL.app



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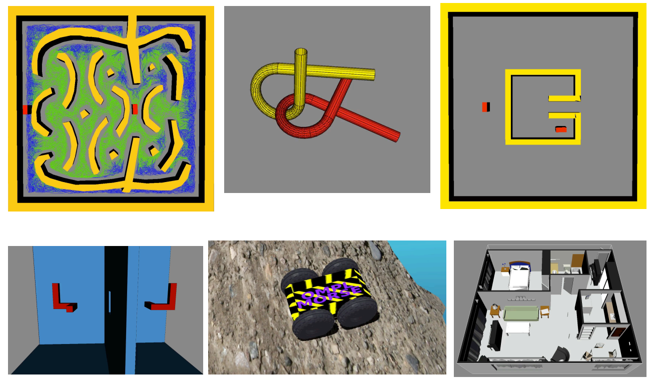
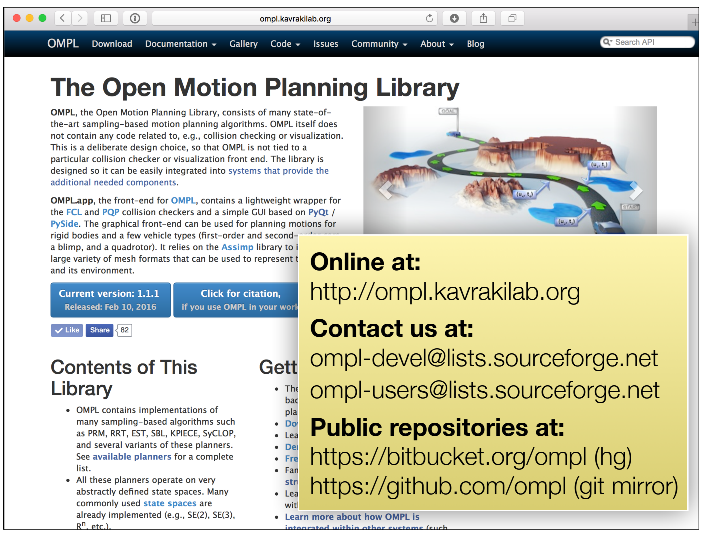
Sample OMPL.app problems



80



Online at:
http://ompl.kavrakilab.org
Contact us at:
ompl-devel@lists.sourceforge.net
ompl-users@lists.sourceforge.net
Public repositories at:
https://bitbucket.org/ompl (hg)
https://github.com/ompl (git mirror)

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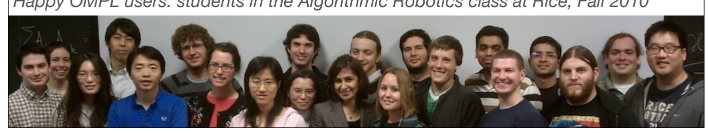
OMPL for education

- Programming assignments centered around OMPL, available upon request.
- Educational assessment.
  Moll et al., *Comp. Sci. Education* 23(4):332–348, 2013
- Already in use in several robotics / motion planning classes.

Happy OMPL users: students in the Algorithmic Robotics class at Rice, Fall 2010



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ROS MoveIt!

- motion planning (using OMPL)
- kinematics
- collision checking integrated with perception
- grasping
- control and navigation for mobile manipulation

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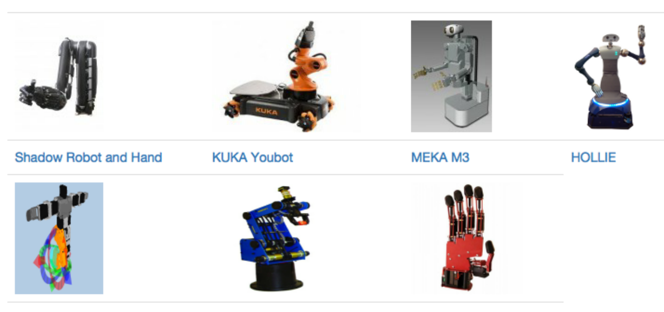
ROS MoveIt!

http://moveit.ros.org/robots

Clopema Robot DLR-Hit Hand iCub REEM-C



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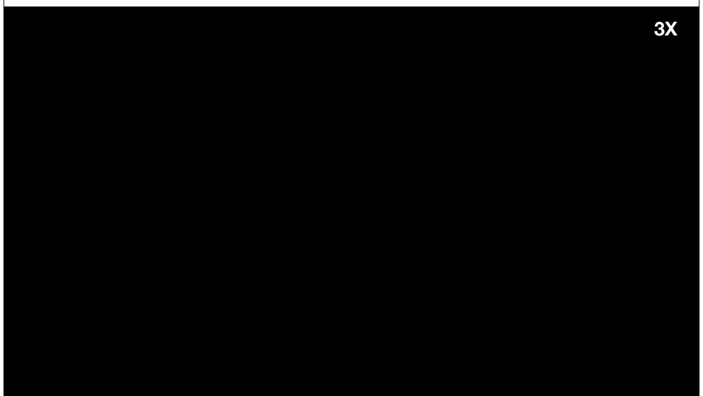
Robonaut 2:
34 degrees of freedom and many constraints



Summary

- Sampling-based motion planning provides effective approach to search high-dimensional, continuous state space of robots.
- Task planning can help guide motion planning,
  motion planning can help refine abstractions used in task planning.
- OMPL provides generic implementations of sampling-based algorithms,
  MoveIt! provides the “glue” to run them on real robots.

THANK YOU!

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