Classical Planning@Robotics: Methods and Challenges

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Planning@Robotics

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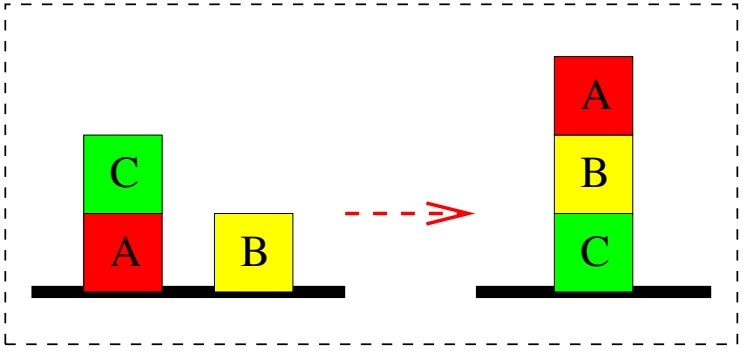

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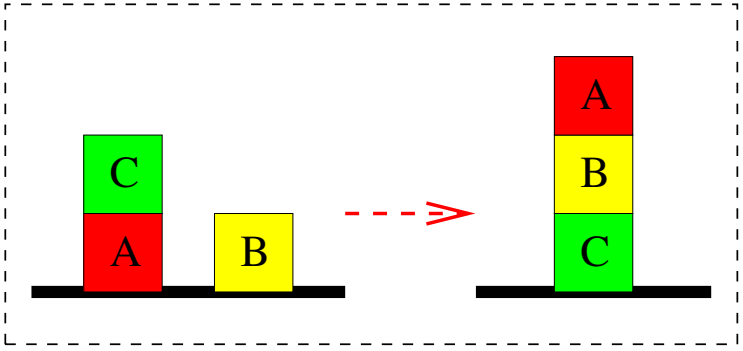
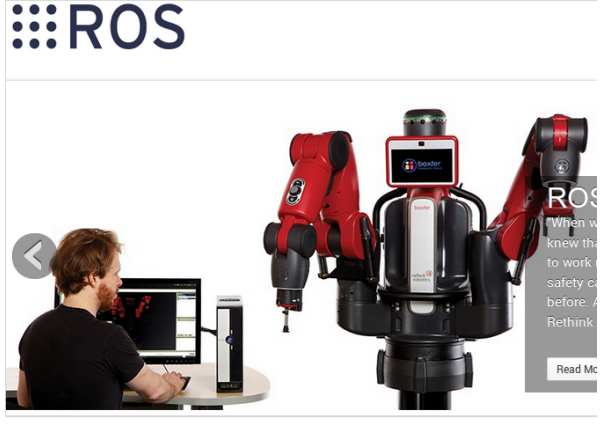
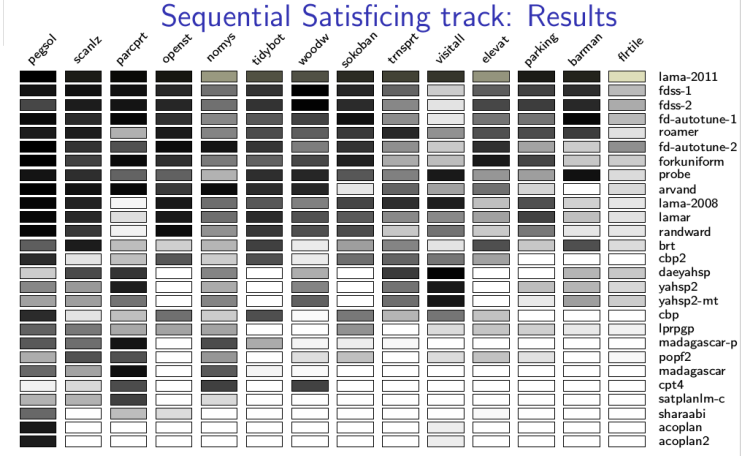


2017





. . . and it’s finally becoming a reality.

Agenda

(A Canonical) Classical Planning Language

Definition. *A planning task is a 4-tuple $\Pi = (V, A, I, G)$ where:*

- *V is a set of state variables, each $v \in V$ with a finite domain D_v.*
- *A is a set of actions; each $a \in A$ is a triple (pre_a, eff_a, c_a), of precondition and effect (partial assignments), and the action's cost $c_a \in \mathbb{R}_0^+$.*
- *Initial state I (complete assignment), goal G (partial assignment).*

$\rightarrow$ Solution ("Plan"): Action sequence mapping I into s s.t. $s \models G$.

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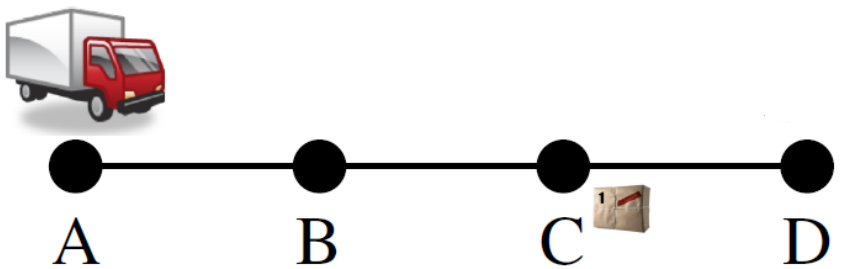

- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$.

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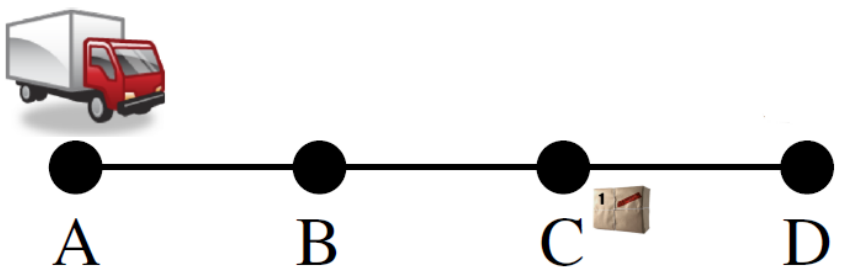

- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$.
- Initial state I: $truck = A$, $pack1 = C$.
- Goal G: $truck = A$, $pack1 = D$.

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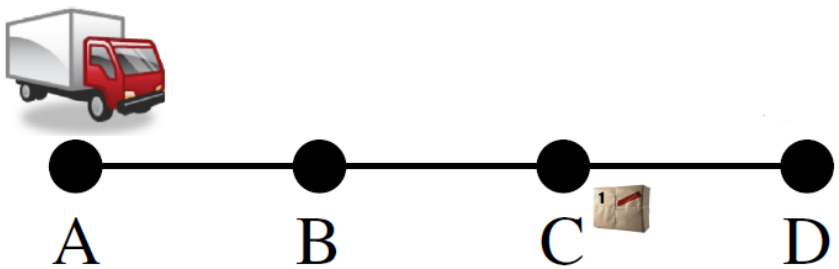

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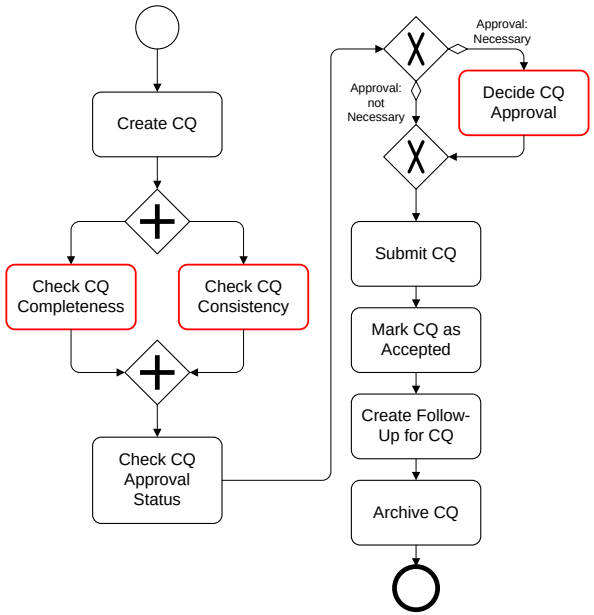
- Initial state I: $truck = A$, $pack1 = C$.
- Goal G: $truck = A$, $pack1 = D$.
- Actions A (unit costs): $drive(x, y)$, $load(p, x)$, $unload(p, x)$. E.g.: $load(pack1, x)$ precondition $truck = x, pack1 = x$, effect $pack1 = T$.

Semantic BPM@SAP

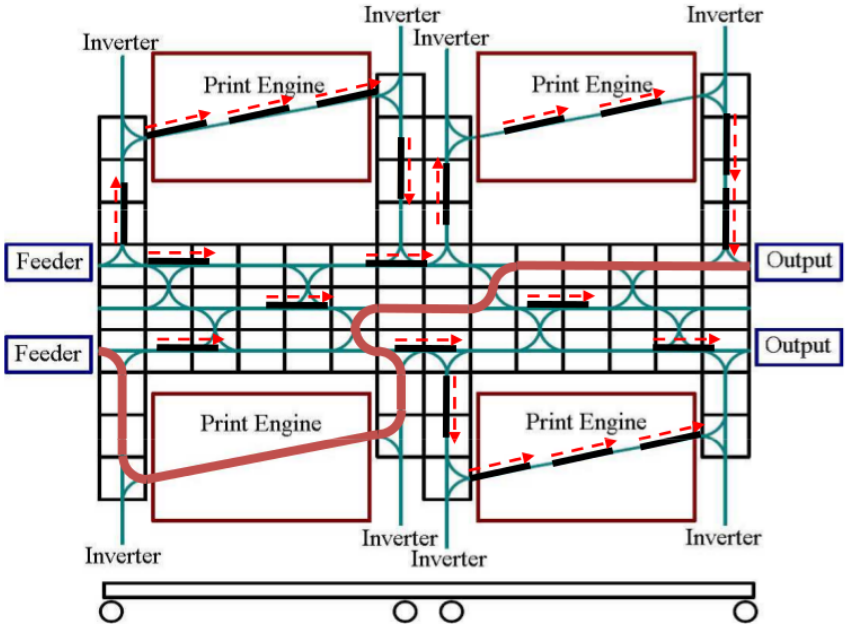
[Hoffmann *et al.* (2012)]

Action name	precondition	effect
Check CQ Completeness	CQ.archiving:notArchived	CQ.completeness:complete OR CQ.completeness:notComplete
Check CQ Consistency	CQ.archiving:notArchived	CQ.consistency:consistent OR CQ.consistency:notConsistent
Check CQ Approval Status	CQ.archiving:notArchived AND CQ.approval:notChecked AND CQ.completeness:complete AND CQ.consistency:consistent	CQ.approval:necessary OR CQ.approval:notNecessary
Decide CQ Approval	CQ.archiving:notArchived AND CQ.approval:necessary	CQ.approval:granted OR CQ.approval:notGranted
Submit CQ	CQ.archiving:notArchived AND (CQ.approval:notNecessary OR CQ.approval:granted)	CQ.submission:submitted
Mark CQ as Accepted	CQ.archiving:notArchived AND CQ.submission:submitted	CQ.acceptance:accepted
Create Follow-Up for CQ	CQ.archiving:notArchived AND CQ.acceptance:accepted	CQ.followUp:documentCreated
Archive CQ	CQ.archiving:notArchived	CQ.archiving:archived



- **Planning model:** SAP-scale model of activities on Business Objects; desired process endpoint.
- **Solution:** Process template leading to this point.
- **Key advantage:** Eases process creation. Automation wrpto activities/objects (i. e., arbitrary models thereof).

Controlling Modular Printers@Xerox [Ruml *et al.* (2011)]



- **Planning model:** Modular printer components, printer configuration, current jobs.
- **Solution:** Optimal schedule of current jobs.
- **Key advantage:** Automation wrpto possible configurations: A controller for a system whose exact design is not known at software-design time.

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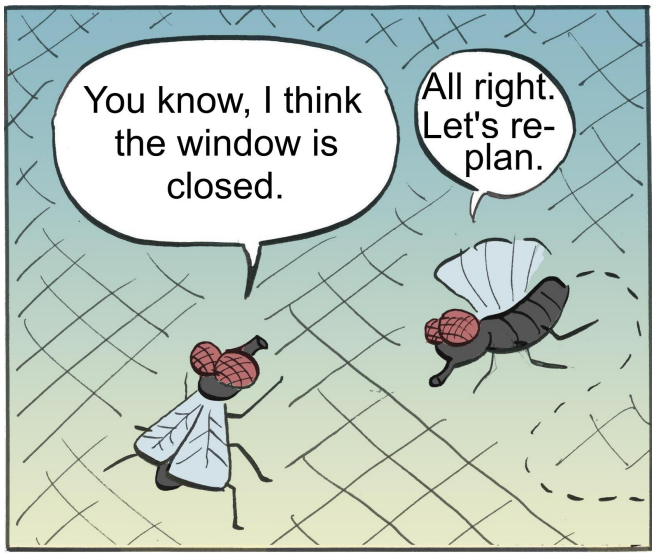
VS.



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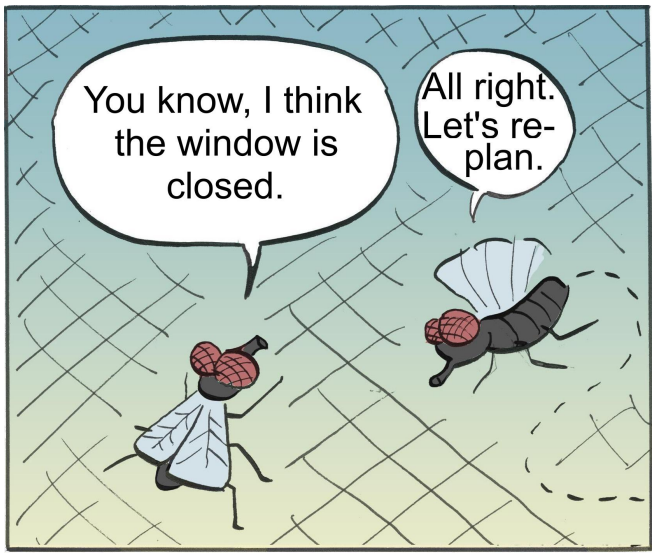

Plan-Based Control:

- Planning model allows to capture arbitrary states, actions, goals.
- Reacting to arbitrary state/goal change (within fixed model): Re-planning.
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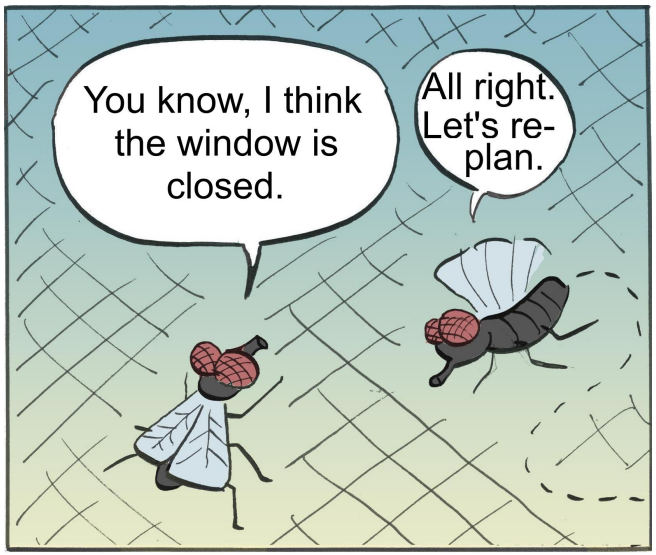

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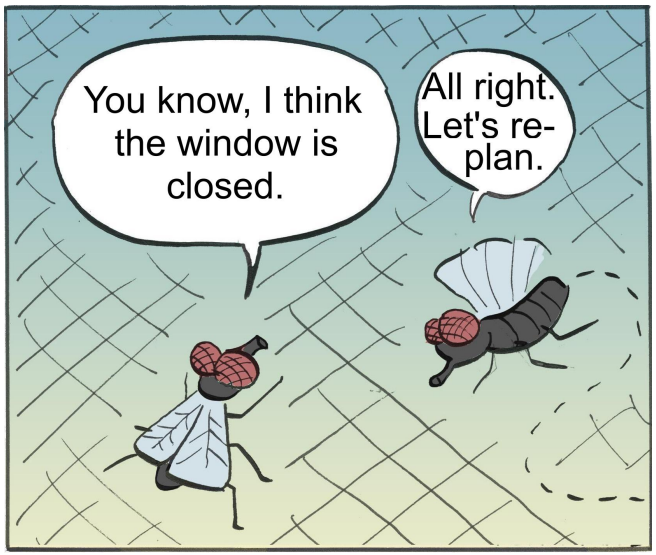

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→ Long-term autonomy; dynamic environments (unexpected events).

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So why bother?

- Fast re-planning. (Classical planning already is **PSPACE**-complete, never mind the more accurate variants.)
- "Your need" actually depends on your system architecture/what the "Activity Planning" component is being used for.
- Transfer of algorithmic ideas to richer planning problems.

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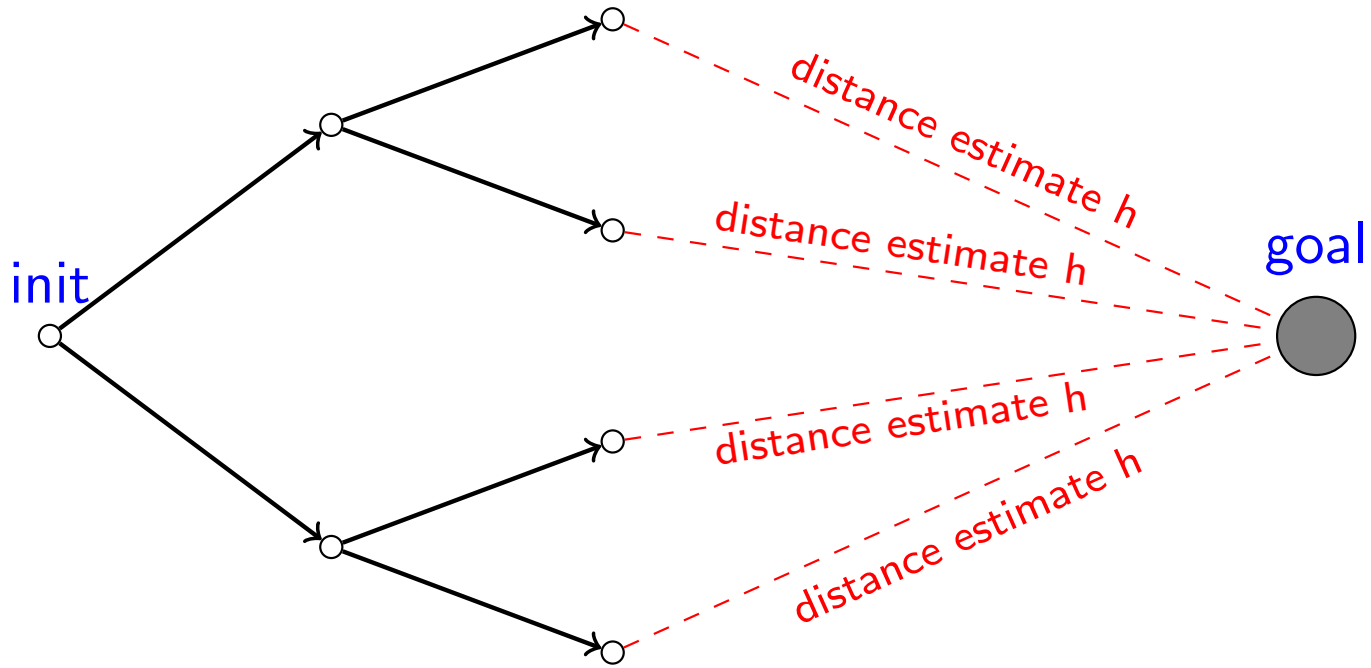
→ Classical planning is very restricted, but provides fast mechanisms for flexible reactive high-level planning. (And is a good frame for basic algorithms research.)

A Successful Family of Solvers: Heuristic Search



→ Forward state space search. Heuristic function h maps states s to an estimate $h(s)$ of goal distance.

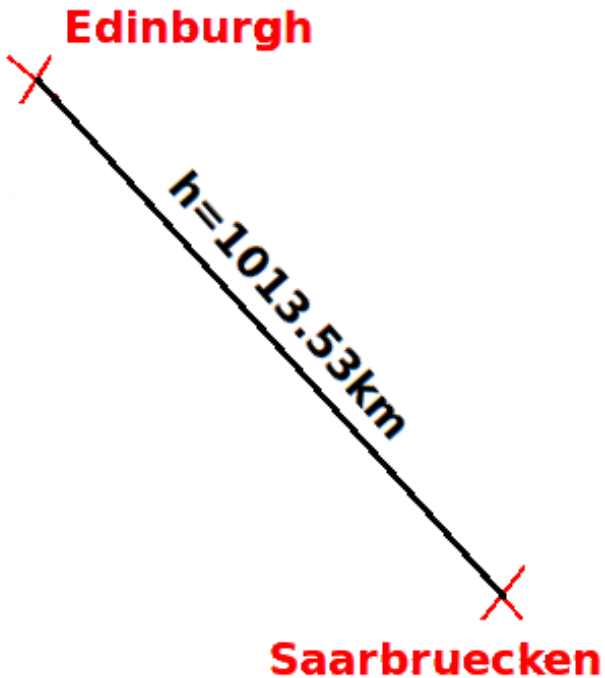
Heuristic Functions

Problem: Find a route from Saarbruecken To Edinburgh.

Heuristic Functions

Relaxed Problem: Throw away the map.

Heuristic Functions



Heuristic function: Straight line distance.

Agenda

1. Classical Planning: What? Why?
2. The Delete Relaxation
3. Beyond the Delete Relaxation
4. Proving Things about the Space of Plans
5. Future Topics in Robotics

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Pretending Things Can Only Get Better

Delete Relaxation = "What was once true remains true forever."

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Real world: (before)

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A Little More Formally

Real planning:

Definition (State Transitions). *Let $\Pi = (V, A, I, G)$ be a planning task, s a state in Π, and $a \in A$. We say that a is applicable in s if $pre_a \subseteq s$. In that case, the outcome state s' of applying a to s is:*

$$s'(v) := \begin{cases} eff_a(v) & eff_a \text{ is defined on } v \\ s(v) & \text{otherwise} \end{cases}$$

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Delete-relaxed planning:

Definition (Relaxed State Transitions). *Let $\Pi = (V, A, I, G)$ be a planning task. A relaxed state in Π is any set s^+ of variable/value pairs. Let s^+ be a relaxed state, and $a \in A$. We say that a is applicable in s^+ if $pre_a \subseteq s^+$. In that case, the outcome state $s^{+\prime}$ of applying a to s^+ is $s^{+\prime} := s^+ \cup eff_a$.*

$\rightarrow$ Under the delete relaxation, state variables accummulate their values, rather than switching between them.

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→ In the "Logistics" Example, e. g., we don't need to drive the truck back to A.

And Now for Something Completely Different

The Relaxed Dompteur

(Using a propositional-logic like notation for Boolean variables:)

- $V = \{alive, haveTiger, tamedTiger, haveJump\}$; all Boolean.
- Initial state I: $alive$.
- Goal G: $alive, haveJump$.
- Actions A:

  $getTiger$: pre $alive$; eff $haveTiger$

  $tameTiger$: pre $alive$, $haveTiger$; eff $tamedTiger$

  $jumpTamedTiger$: pre $alive$, $tamedTiger$; eff $haveJump$

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$\rightarrow$ Relaxed plan for this task? $\langle getTiger, jumpTiger \rangle$

$\langle getTiger, tameTiger, jumpTamedTiger \rangle$ works as well, but the previous relaxed plan is "better" ...

h^+: The Ideal Delete Relaxation Heuristic

Definition (h^+). *Let $\Pi = (V, A, I, G)$ be a planning task, with state set S. The optimal delete relaxation heuristic h^+ for Π is the function $h^+ : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$ where $h^+(s)$ is the cost of an optimal relaxed plan for s.*

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Good news: (Notation: h^* is the perfect heuristic)

Proposition (h^+ is Admissible). *Let Π be a planning task, with state set S. Then, for every $s \in S$, $h^+(s) \leq h^*(s)$.*

Proof. Every real plan for a state s in Π also is a relaxed plan for $s^+ := s$ in Π.

Example: h^+ in TSP

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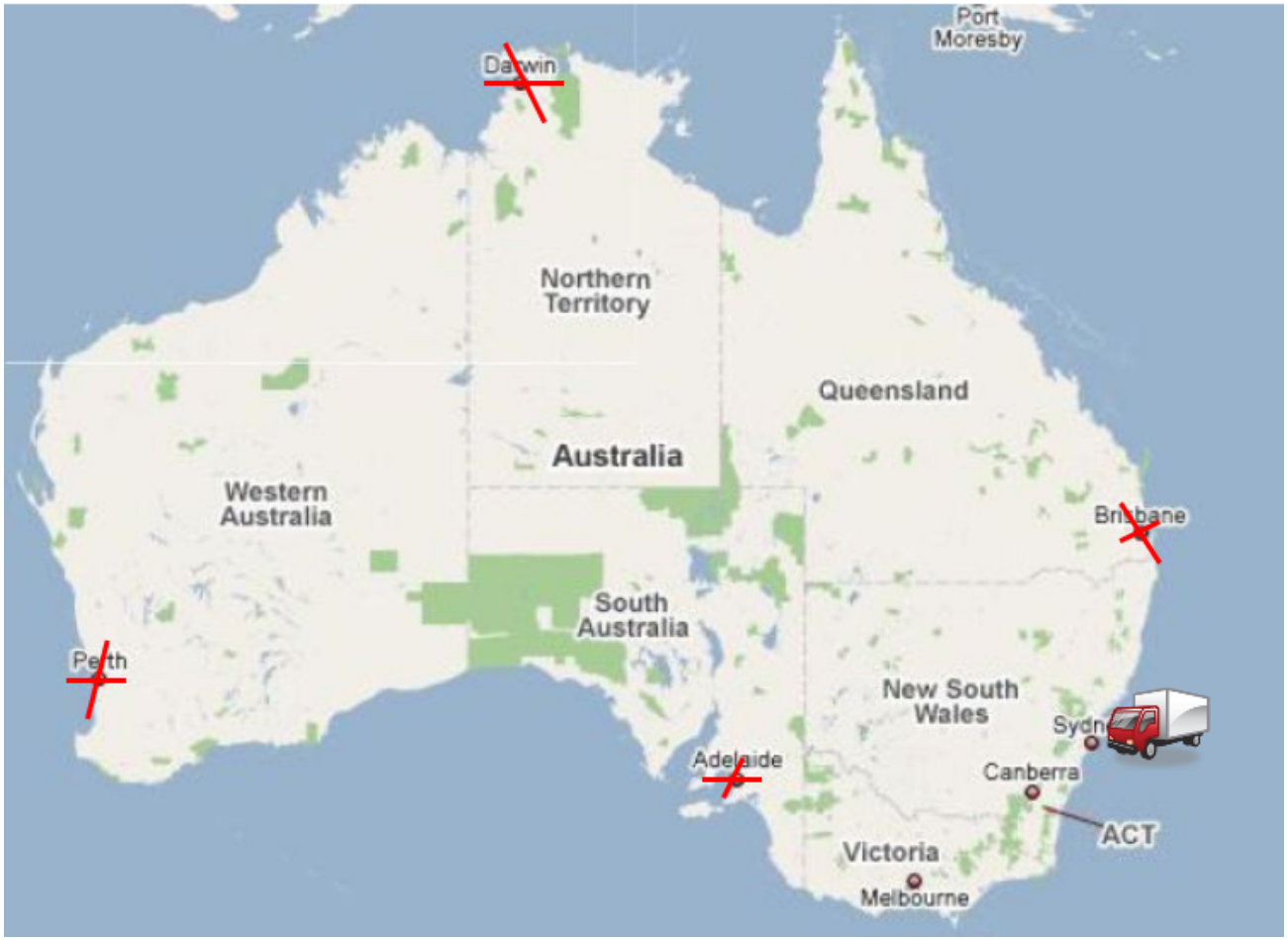

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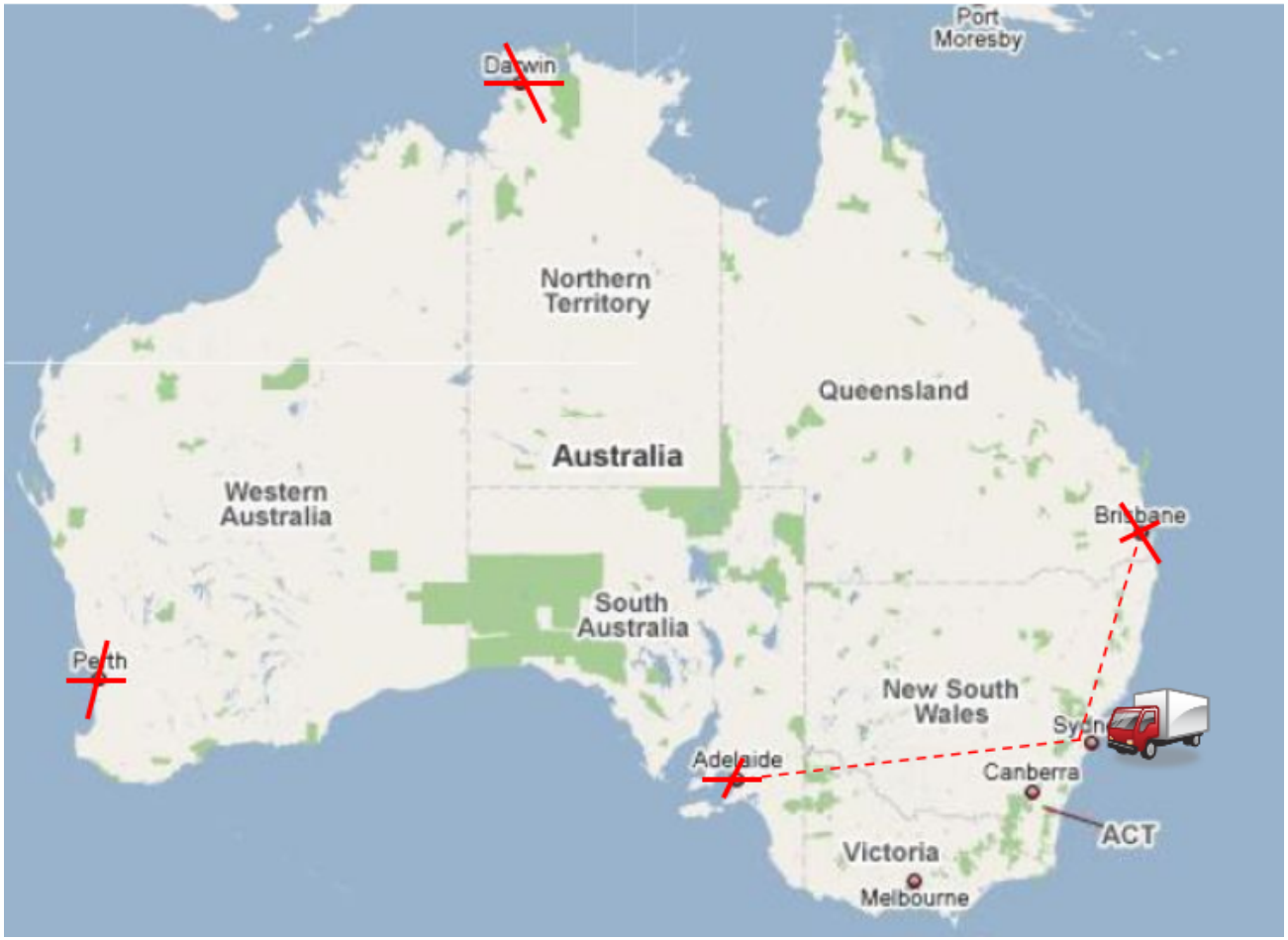

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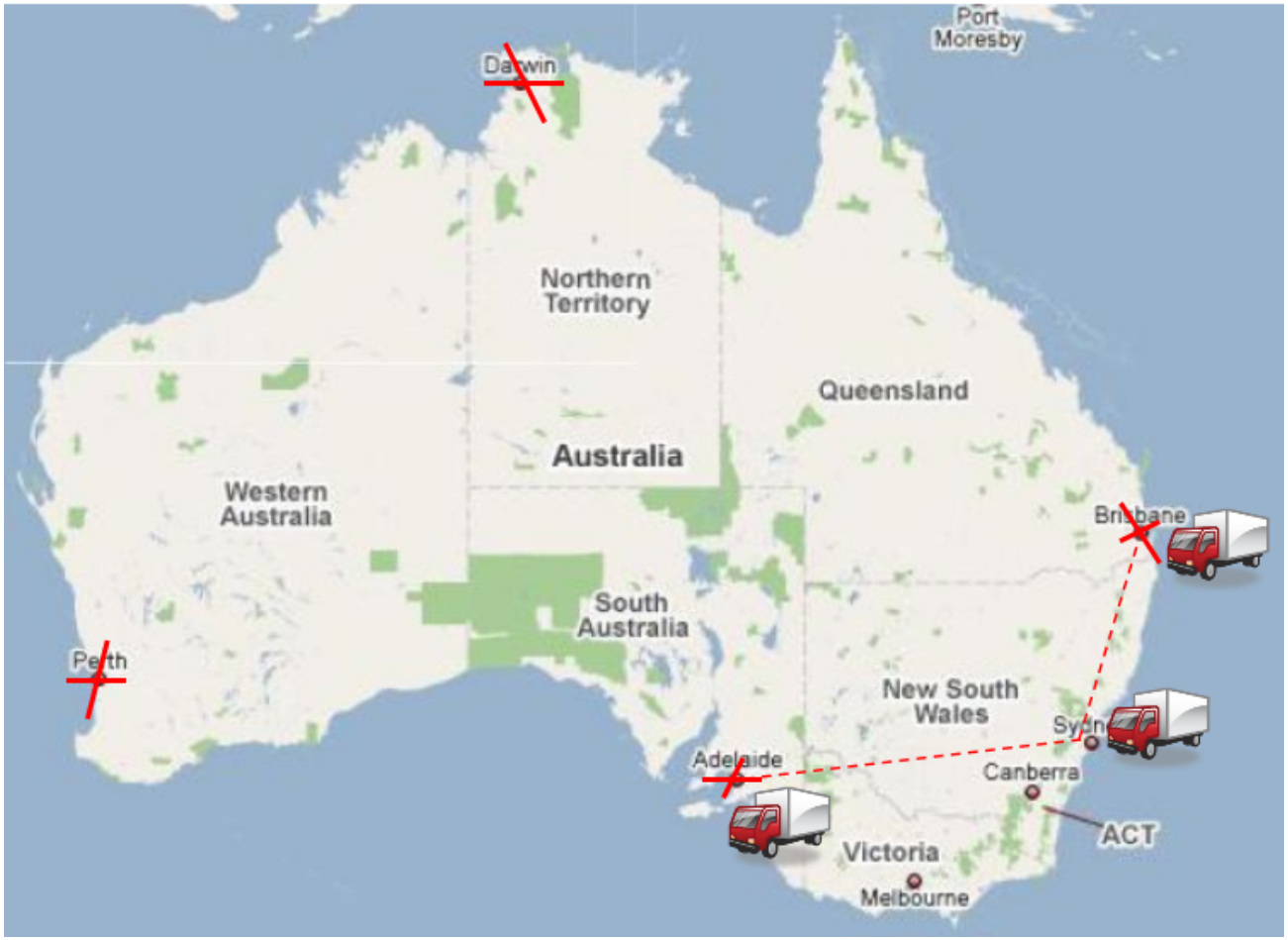

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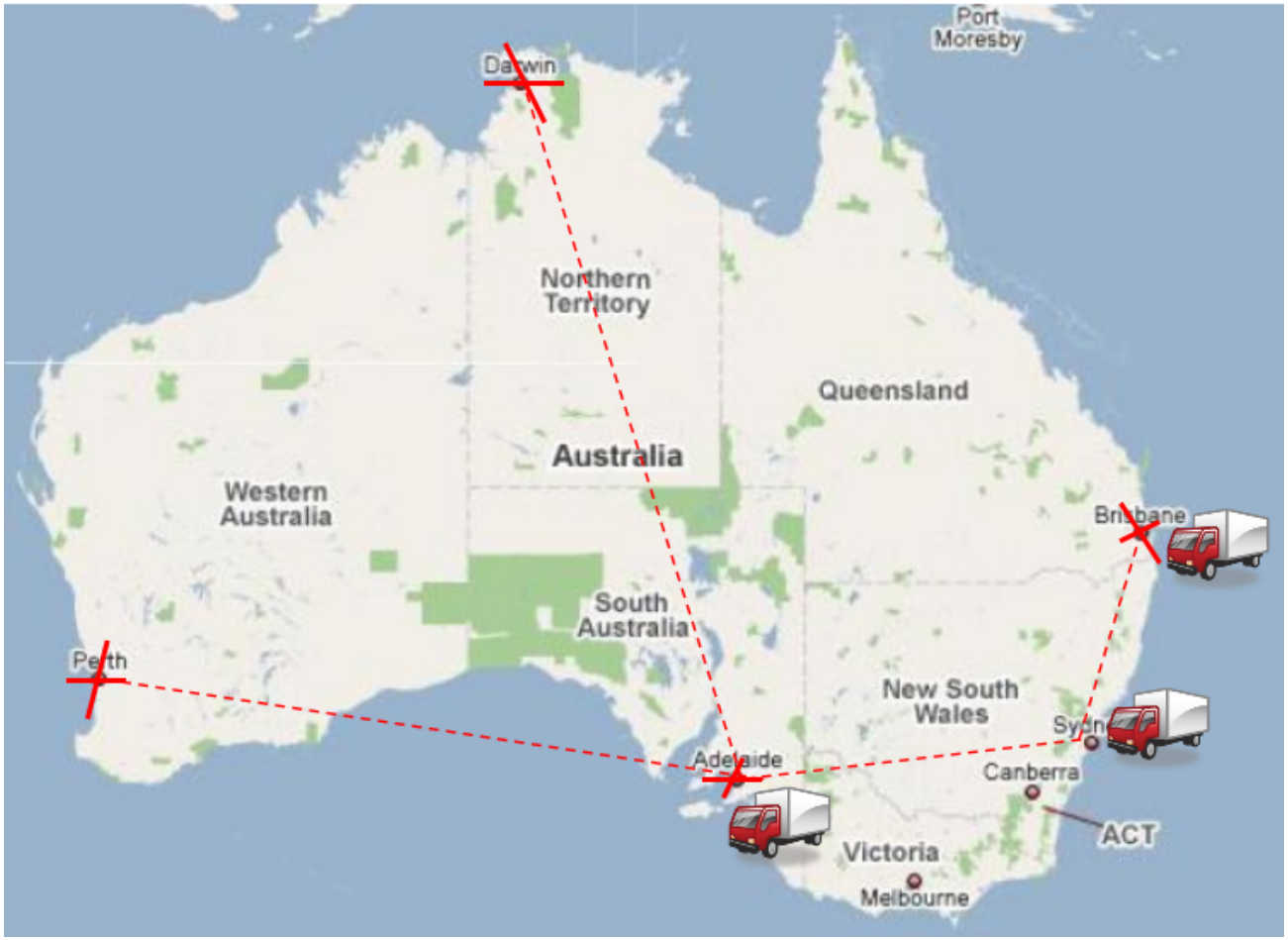

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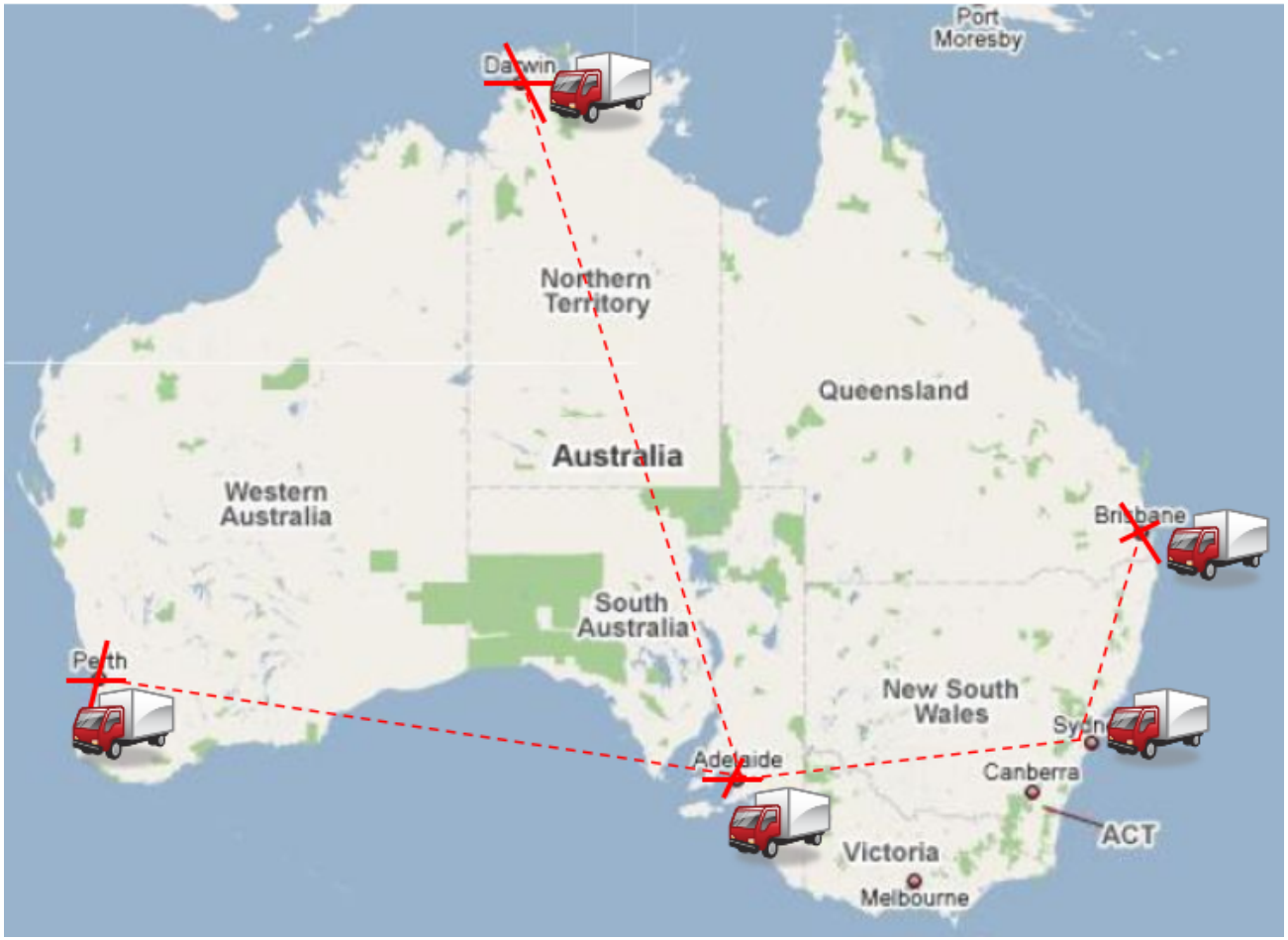

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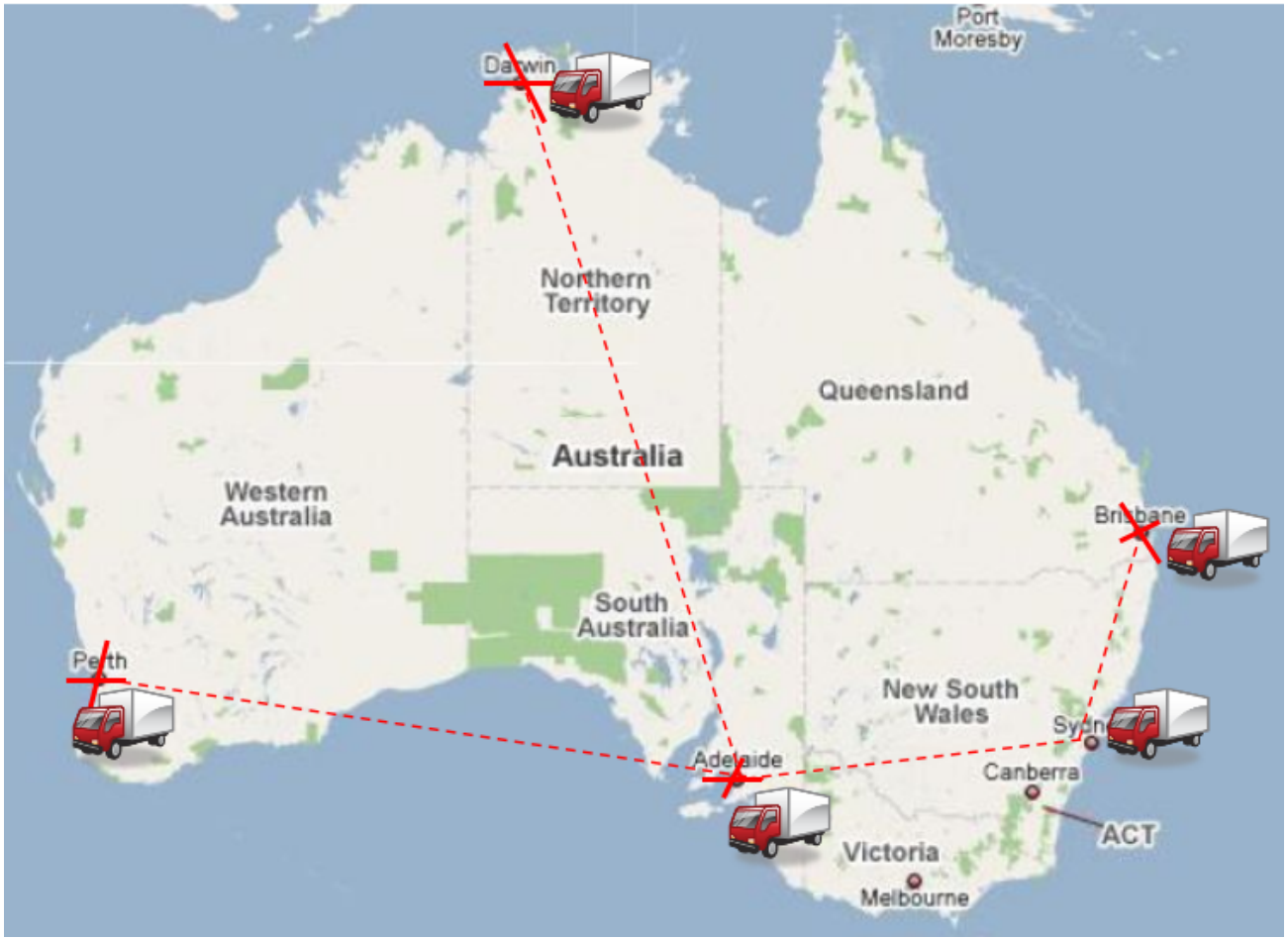

$h^+(\text{TSP}) = \text{Minimum Spanning Tree}$

h^+: The Ideal Delete Relaxation Heuristic

Definition (h^+). *Let $\Pi = (V, A, I, G)$ be a planning task, with state set S. The optimal delete relaxation heuristic h^+ for Π is the function $h^+ : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$ where $h^+(s)$ is the cost of an optimal relaxed plan for s.*

Good news: (Notation: h^* is the perfect heuristic)

Proposition (h^+ is Admissible). *Let Π be a planning task, with state set S. Then, for every $s \in S$, $h^+(s) \leq h^*(s)$.*

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→ (Quote Patrik Haslum) "So we approximate . . ."

Approximating h^+, Take I: h^1

Notation (Regression): $regr(g, a) = (g \setminus \mathit{eff}_a) \cup \mathit{pre}_a$ if (i) $\mathit{eff}_a \cap g \neq \emptyset$ and (ii) eff_a does not contradict g on any variable; else, $regr(g, a)$ is undefined.

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$$h^1(s,g) = \begin{cases} 0 & g \subseteq s \\ min_{a \in A, regr(g,a) \text{ is defined}} c_a + h^1(s, regr(g,a)) & |g| = 1 \\ \max_{p \in g} h^1(s, \{p\}) & |g| > 1 \end{cases}$$

→ Estimate the cost of sets g of subgoal facts (variable/value pairs) by the cost of the most costly fact $p \in g$.

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$\rightarrow$ Estimate the cost of sets g of subgoal facts (variable/value pairs) by the cost of the most costly fact $p \in g$.

$\rightarrow$ This is indeed a delete relaxation heuristic:

Proposition (h^1 estimates h^+). *Let Π be a planning task, with state set S. Then, for every $s \in S$, $h^1(s) \leq h^+(s)$.*

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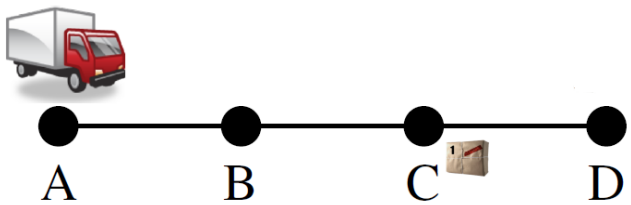
Computing h^1: Forward Fixpoint on Singleton Subgoals

Example "Logistics":



- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$. Short: t, p.
- Initial state I: $truck = A$, $pack1 = C$.
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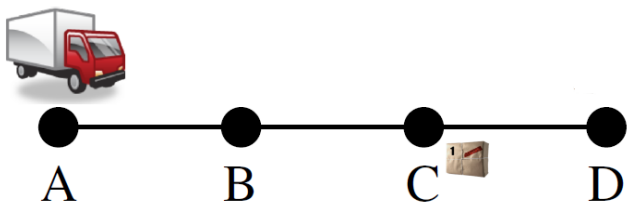

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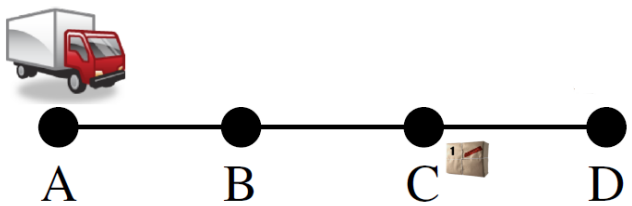

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0	0	∞	∞	∞	∞	∞	∞	0	∞

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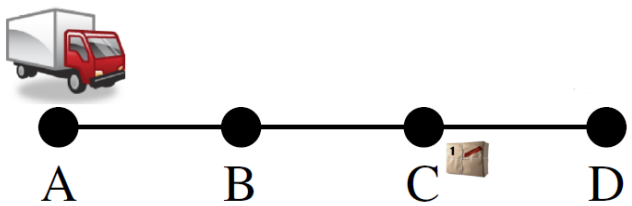

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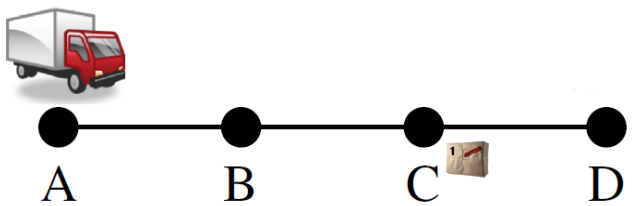

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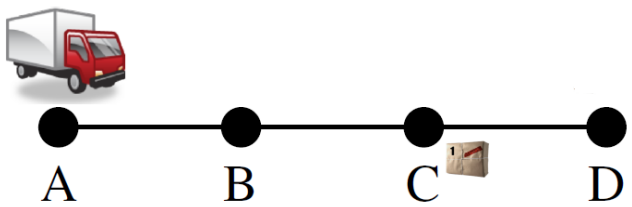

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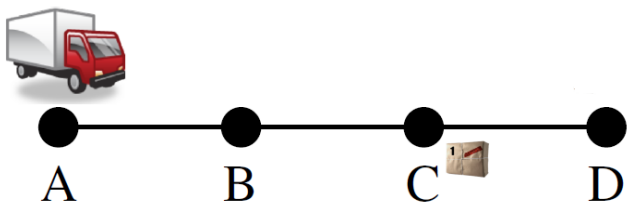

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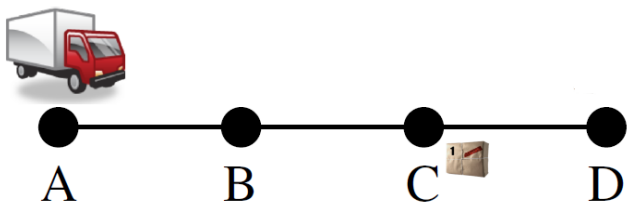

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0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
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5	0	1	2	3	3	4	4	0	4

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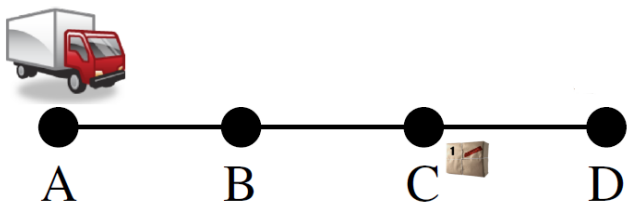

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$\rightarrow$ So $h^1(I) = 4$.

Computing h^1: Forward Fixpoint on Singleton Subgoals

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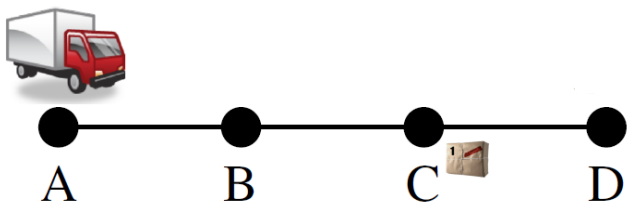

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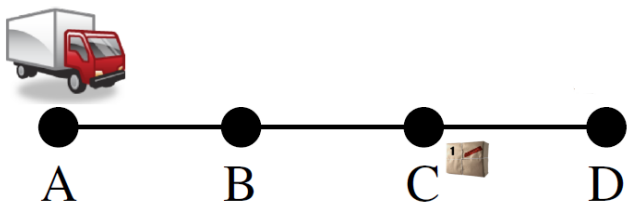
4	0	1	2	3	3	4	4	0	4
5	0	1	2	3	3	4	4	0	4

$\rightarrow$ So $h^1(I) = 4$. And for 100 packages with init C and goal D?

Computing h^1: Forward Fixpoint on Singleton Subgoals

Example "Logistics":



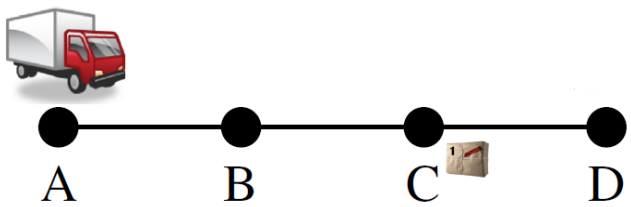
- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$. Short: t, p.
- Initial state I: $truck = A$, $pack1 = C$.
- Goal G: $truck = A$, $pack1 = D$.
- Actions A (unit costs): $drive(x, y)$, $load(p, x)$, $unload(p, x)$. Short: $dr(x, y), lo(x), ul(x)$.

i	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	4	0	4
5	0	1	2	3	3	4	4	0	4

$\rightarrow$ So $h^1(I) = 4$. And for 100 packages with init C and goal D? Still $h^1(I) = 4$.

Computing h^1: Forward Fixpoint on Singleton Subgoals

Example "Logistics":



- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$. Short: t, p.
- Initial state I: $truck = A$, $pack1 = C$.
- Goal G: $truck = A$, $pack1 = D$.
- Actions A (unit costs): $drive(x, y)$, $load(p, x)$, $unload(p, x)$. Short: $dr(x, y), lo(x), ul(x)$.

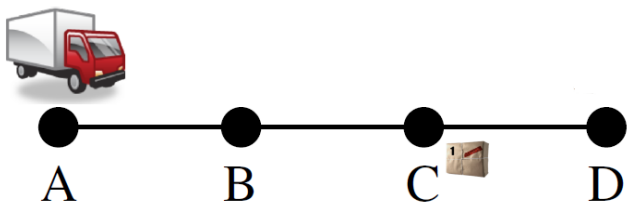
i	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	4	0	4
5	0	1	2	3	3	4	4	0	4

$\rightarrow$ So $h^1(I) = 4$. And for 100 packages with init C and goal D? Still $h^1(I) = 4$.

$\rightarrow$ h^1 is admissible, but is typically VERY uninformed.

Computing h^1: Forward Fixpoint on Singleton Subgoals

Example "Logistics":



- State variables V: $truck : \{A, B, C, D\}$; $pack1 : \{A, B, C, D, T\}$. Short: t, p.
- Initial state I: $truck = A$, $pack1 = C$.
- Goal G: $truck = A$, $pack1 = D$.
- Actions A (unit costs): $drive(x, y)$, $load(p, x)$, $unload(p, x)$. Short: $dr(x, y), lo(x), ul(x)$.

i	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	4	0	4
5	0	1	2	3	3	4	4	0	4

$\rightarrow$ So $h^1(I) = 4$. And for 100 packages with init C and goal D? Still $h^1(I) = 4$.

$\rightarrow$ h^1 is admissible, but is typically VERY uninformed. To the rescue: h^{FF}.

Approximating h^+, Take II: h^{FF}

Relaxed Plan Extraction for state s and **best-supporter function** bs

if $h^1(s) = \infty$ **then return** $h^{\mathsf{FF}}(s) := \infty$ **endif**
$Open := G \setminus s$
$Closed := \emptyset$
$RPlan := \emptyset$
while $Open \neq \emptyset$ **do**:
    select $p \in Open$
    $Open := Open \setminus \{p\}$; $Closed := Closed \cup \{p\}$;
    $RPlan := RPlan \cup \{bs(p)\}$; $Open := Open \cup (pre_{bs(p)} \setminus (s \cup Closed))$
endwhile
return $h^{\mathsf{FF}}(s) := |RPlan|$

$\rightarrow$ Starting with the top-level goal facts $p \in G$, iteratively close open singleton subgoal-facts p by selecting the best supporter $bs(p)$ for p.

Approximating h^+, Take II: h^{FF}

Relaxed Plan Extraction for state s and **best-supporter function** bs

if $h^1(s) = \infty$ **then return** $h^{\mathsf{FF}}(s) := \infty$ **endif**
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    $Open := Open \setminus \{p\}$; $Closed := Closed \cup \{p\}$;
    $RPlan := RPlan \cup \{bs(p)\}$; $Open := Open \cup (pre_{bs(p)} \setminus (s \cup Closed))$
endwhile
return $h^{\mathsf{FF}}(s) := |RPlan|$

$\rightarrow$ Starting with the top-level goal facts $p \in G$, iteratively close open singleton subgoal-facts p by selecting the best supporter $bs(p)$ for p.

$\rightarrow$ Err, so where do we get bs from?

Approximating h^+, Take II: h^{FF}

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if $h^1(s) = \infty$ **then return** $h^{\mathsf{FF}}(s) := \infty$ **endif**
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    $Open := Open \setminus \{p\}$; $Closed := Closed \cup \{p\}$;
    $RPlan := RPlan \cup \{bs(p)\}$; $Open := Open \cup (pre_{bs(p)} \setminus (s \cup Closed))$
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$\rightarrow$ Starting with the top-level goal facts $p \in G$, iteratively close open singleton subgoal-facts p by selecting the best supporter $bs(p)$ for p.

$\rightarrow$ Err, so where do we get bs from? From h^1.

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs_s^1 : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^1(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Best-Supporter Function from h^1

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Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Best-Supporter Function from h^1

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Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \rightarrow ul(D) \rightarrow t = D, p = T;$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \rightarrow ul(D) \rightarrow t = D, p = T; t = D \rightarrow dr(C, D) \rightarrow t = C;$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \rightarrow ul(D) \rightarrow t = D, p = T; t = D \rightarrow dr(C, D) \rightarrow t = C; t = C \rightarrow dr(B, C) \rightarrow t = B;$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \rightarrow ul(D) \rightarrow t = D, p = T; t = D \rightarrow dr(C, D) \rightarrow t = C; t = C \rightarrow dr(B, C) \rightarrow t = B;$
$t = B \rightarrow dr(A, B) \rightarrow t = A;$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \to ul(D) \to t = D, p = T; t = D \to dr(C, D) \to t = C; t = C \to dr(B, C) \to t = B;$
$t = B \to dr(A, B) \to t = A; p = T \to lo(C) \to t = C, p = C.$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \to ul(D) \to t = D, p = T; t = D \to dr(C, D) \to t = C; t = C \to dr(B, C) \to t = B;$
$t = B \to dr(A, B) \to t = A; p = T \to lo(C) \to t = C, p = C.$

$\to$ So $h^{\mathsf{FF}}(I) = 5$.

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \to ul(D) \to t = D, p = T; t = D \to dr(C, D) \to t = C; t = C \to dr(B, C) \to t = B;$
$t = B \to dr(A, B) \to t = A; p = T \to lo(C) \to t = C, p = C.$

$\to$ So $h^{\mathsf{FF}}(I) = 5$. And for 100 packages with init C and goal D?

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \to ul(D) \to t = D, p = T; t = D \to dr(C, D) \to t = C; t = C \to dr(B, C) \to t = B;$
$t = B \to dr(A, B) \to t = A; p = T \to lo(C) \to t = C, p = C.$

$\to$ So $h^{\mathsf{FF}}(I) = 5$. And for 100 packages with init C and goal D?
$h^{\mathsf{FF}}(I) = 103$

Best-Supporter Function from h^1

Definition (Best-Supporters from h^1). *Let $\Pi = (V, A, I, G)$ be a planning task, and let s be a state. Denote $P := \{(v = d) \mid v \in V, d \in D_v\}$. The h^1 supporter function $bs^1_s : \{p \in P \mid 0 < h^1(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs^1_s(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^1(s, pre_a)$.*

Example "Logistics":

Heuristic values:

	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
h^1	0	1	2	3	3	4	4	0	4

Yield best-supporter function:

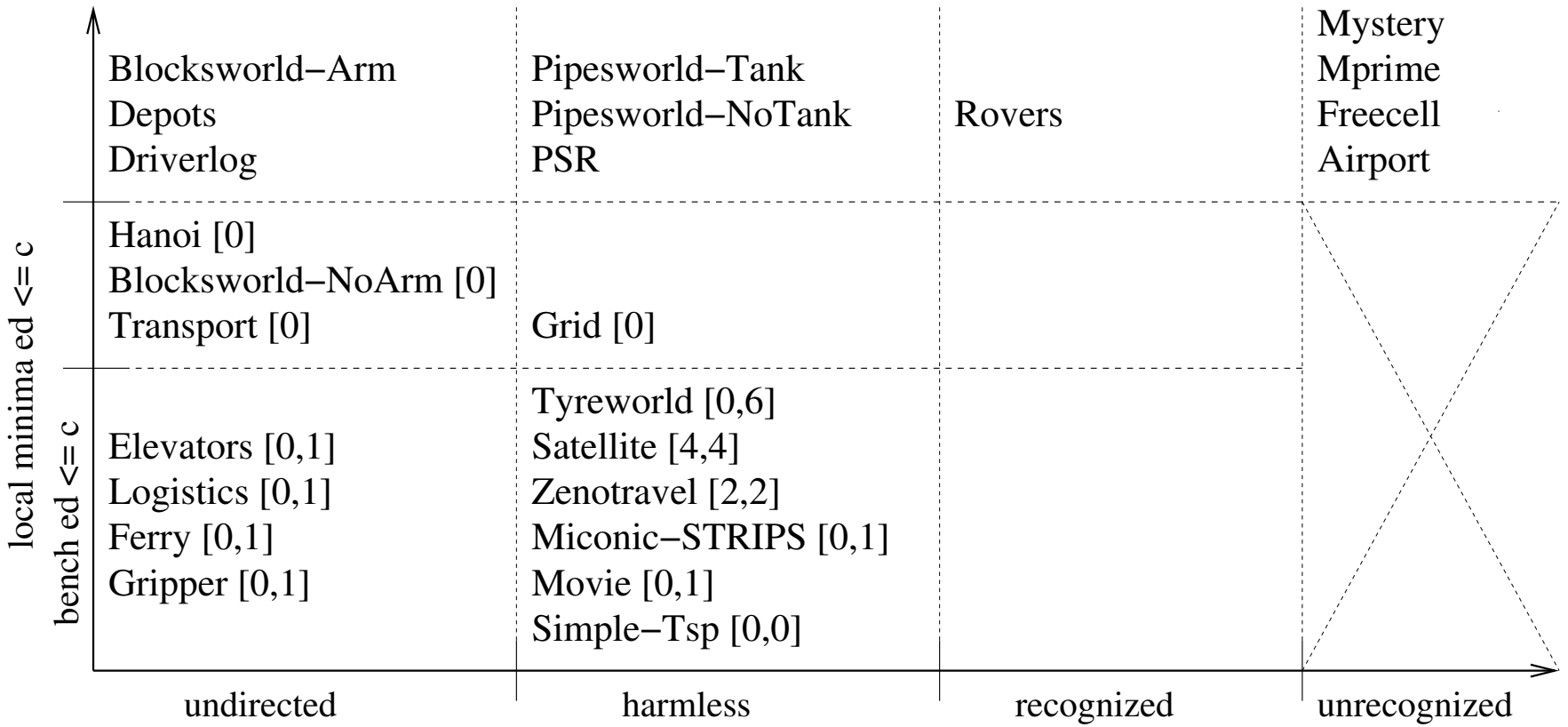
	$t = A$	$t = B$	$t = C$	$t = D$	$p = T$	$p = A$	$p = B$	$p = C$	$p = D$
bs^1	–	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	–	$ul(D)$

Yield relaxed plan extraction:

$p = D \to ul(D) \to t = D, p = T; t = D \to dr(C, D) \to t = C; t = C \to dr(B, C) \to t = B;$
$t = B \to dr(A, B) \to t = A; p = T \to lo(C) \to t = C, p = C.$

$\to$ So $h^{\mathsf{FF}}(I) = 5$. And for 100 packages with init C and goal D?
$h^{\mathsf{FF}}(I) = 103 = h^+(I) \gg h^1(I) = 4$ (compare slide 18).

Proved Quality of h^+ (Largely Inherited by h^{FF}) [Hoffmann (2005)]



Legend: x-axis: 4 classes regarding dead ends, each domain in "highest" class of any of its instances. y-axis: Does there exist a constant bound on exit distance from bench states and/or local minimum states in the domain? [lm,bench] where both exist, [lm] where only the latter exists; lm=0 means no local minima at all. Bottom right crossed out since unrecognized dead ends imply infinite exit distance.

h^1 vs. h^+ vs. h^{FF}: Properties

Proposition ($h^1 \leq h^+ \leq h^{\mathsf{FF}}$). *Let Π be a planning task, with state set S. Then, for every $s \in S$, $h^1(s) \leq h^+(s) \leq h^{\mathsf{FF}}(s)$. There exist Π and s where $h^{\mathsf{FF}}(s) > h^*(s)$.*

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Proposition (Agreement on ∞). *Let Π be a planning task, with state set S. Then, for every $s \in S$, $h^1(s) = \infty \Leftrightarrow h^+(s) = \infty \Leftrightarrow h^{\mathsf{FF}}(s) = \infty$.*

Proof. $h^{\mathsf{FF}}(s) = \infty$ iff $h^1(s) = \infty$ by definition. h^1 returns $\infty \Rightarrow$ some subgoal fact cannot be achieved $\Rightarrow h^+(s) = \infty$; done with $h^1 \leq h^+$.

$\rightarrow$ For dead-end detection, all heuristics so far are the same. (More later!)

Historical/Literature Notes

- Critical path heuristics root in Graphplan [Blum and Furst (1997)], and can be defined and computed over arbitrary-size subgoals [Haslum and Geffner (2000)], and even over arbitrary choices of atomic subgoals [Hoffmann and Fickert (2015)]. (We'll get back to the latter later.)

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- Nevertheless, the delete relaxation has many pitfalls. To the rescue: See next.

Agenda

Partial Delete Relaxation

Delete Relaxation:

Relaxed world: (after)

Partial Delete Relaxation

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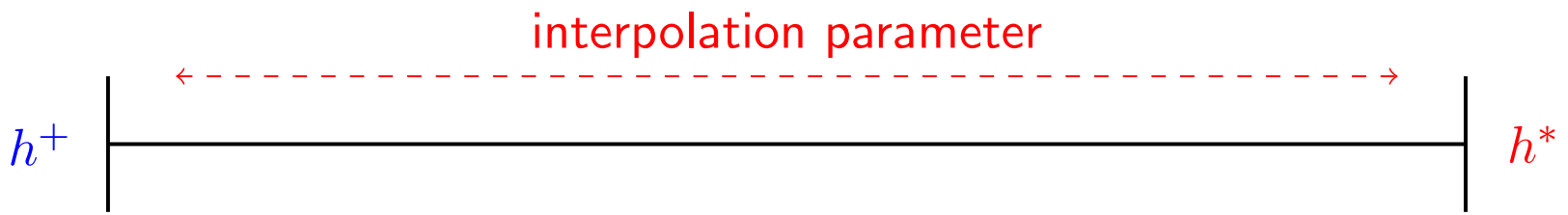
Partial Delete Relaxation:



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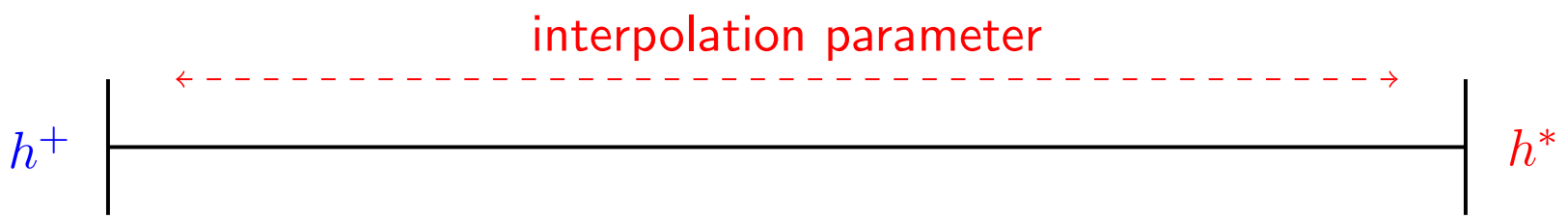

Why?

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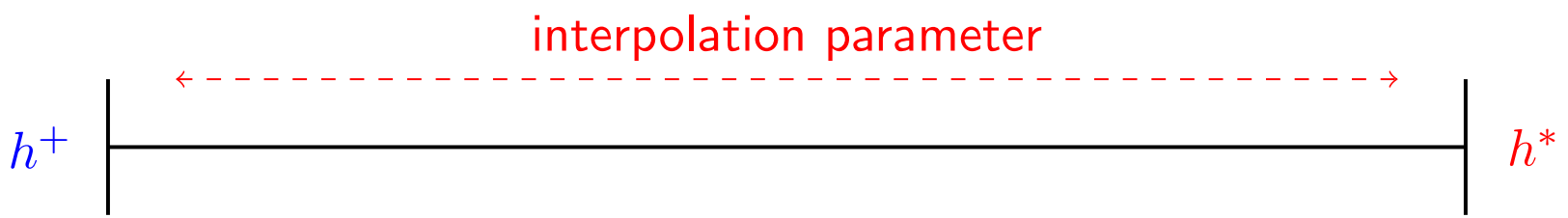

Why? Resource consumption!

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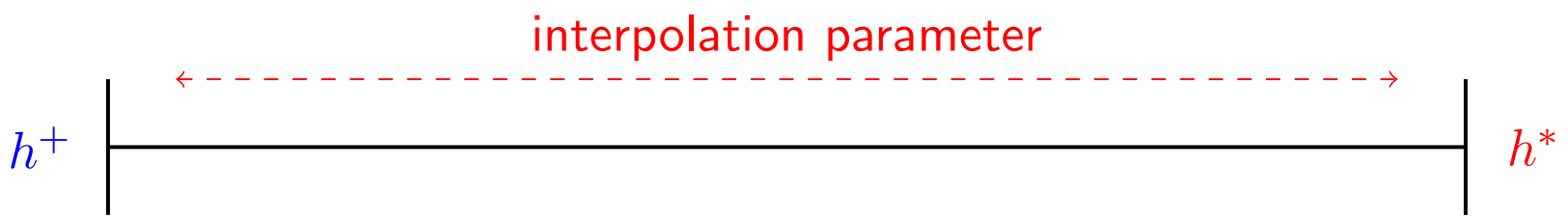
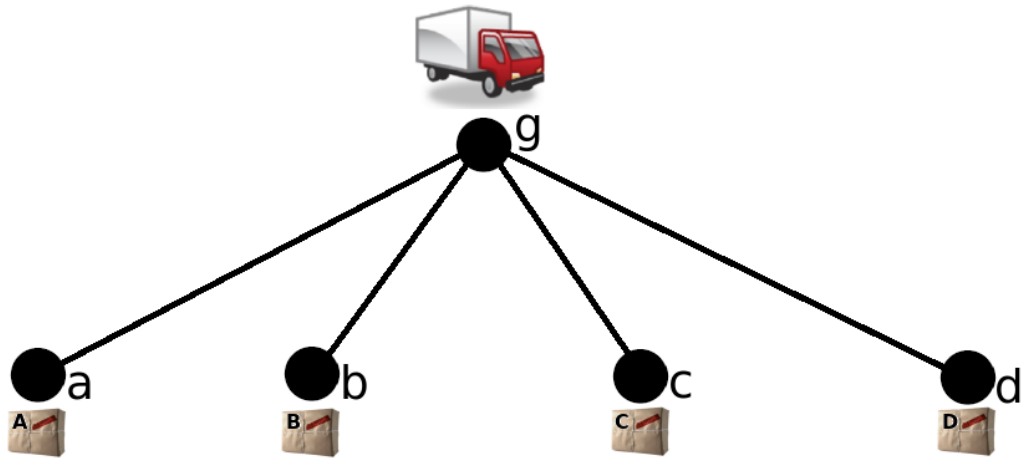


Why? Resource consumption! Moving to-and-fro!

Moving To-and-Fro: Example "Star-Shape Logistics"



- State variables: $v_T : \{g, a, b, c, d\}$; $v_A, v_B, v_C, v_D : \{t, g, a, b, c, d\}$.
- Initial state: $v_T = g, v_A = a, v_B = b, v_C = c, v_D = d$.
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- Actions (unit costs): $drive(x, y), load(x, y), unload(x, y)$.
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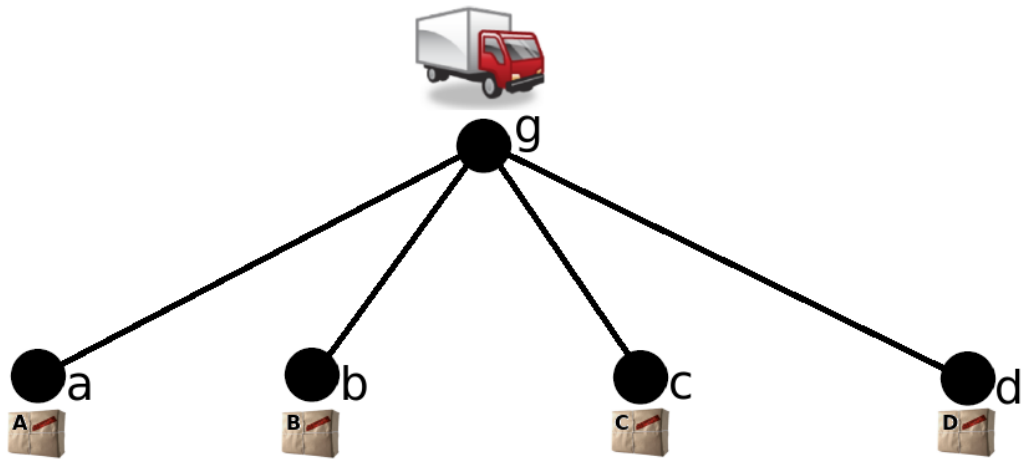

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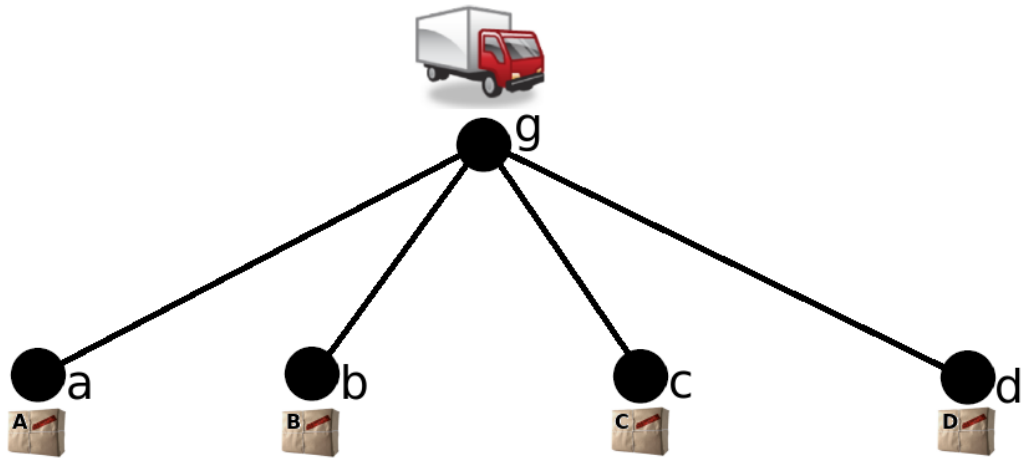

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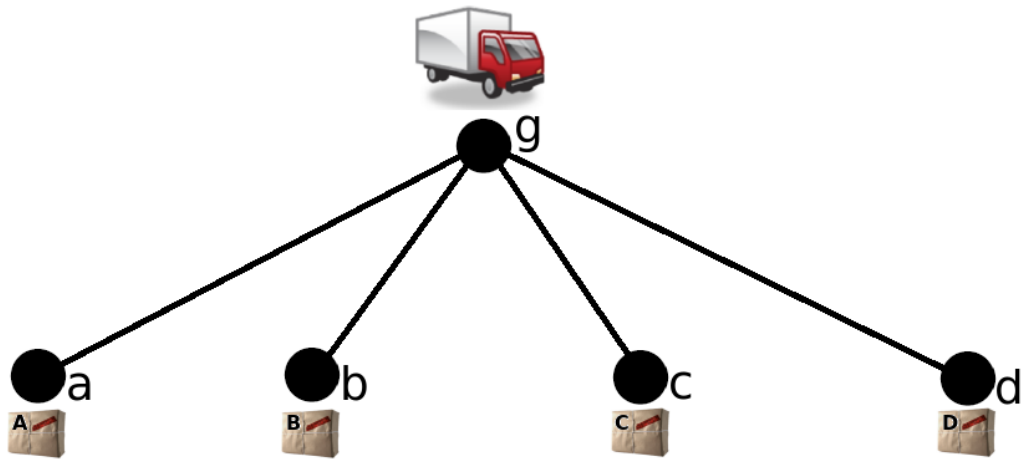

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$\rightarrow$ And with 100 star-leaf locations & packages?

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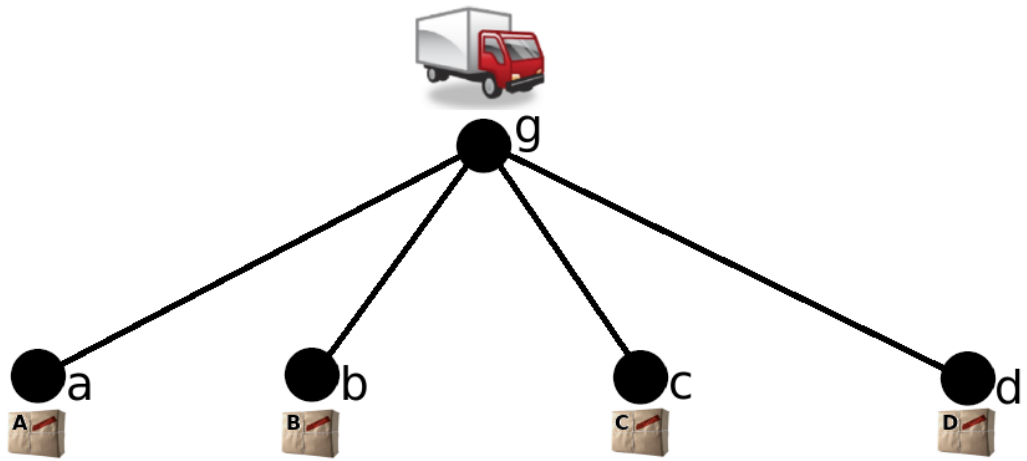

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$\rightarrow$ And with 100 star-leaf locations & packages? $h^+ = 300 \ll 400 = h^*$.

Red-Black Planning

Definition (Red-Black State Transitions). *Let $\Pi = (V, A, I, G)$ be a planning task, and let $V^{\mathsf{B}} \cup V^{\mathsf{R}} = V$ be disjoint. A red-black state s^{RB} in Π is a set of variable/value pairs that assigns each $v \in V$ a subset of its domain, where $|s^{RB}(v)| = 1$ for $v \in V^{\mathsf{B}}$. The outcome state $s^{RB'}$ of applying a to s^{RB} is:*

$$s^{RB'}(v) := \begin{cases} eff_a(v) & v \in V^{\mathsf{B}}, eff_a \text{ is defined on } v \\ s^{RB}(v) \cup eff_a(v) & v \in V^{\mathsf{R}}, eff_a \text{ is defined on } v \\ s^{RB}(v) & \text{otherwise} \end{cases}$$

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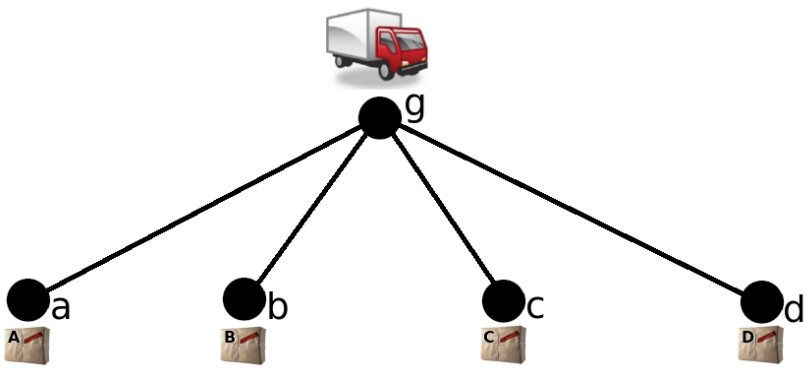
b-it bots@Work, 2nd prize 2016 RoboCup@Work, uses Mercury [Domshlak *et al.* (2015)], which uses a red-black plan heuristic.

Red-Black Planning in Star-Shape Logistics



Relaxed plan:

1. Initial state: $\{v_T = g, \dots\}$.
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   $\{v_T = g, v_T = a, \dots\}$.
3. Apply $drive(g, b)$:
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4. $\dots$

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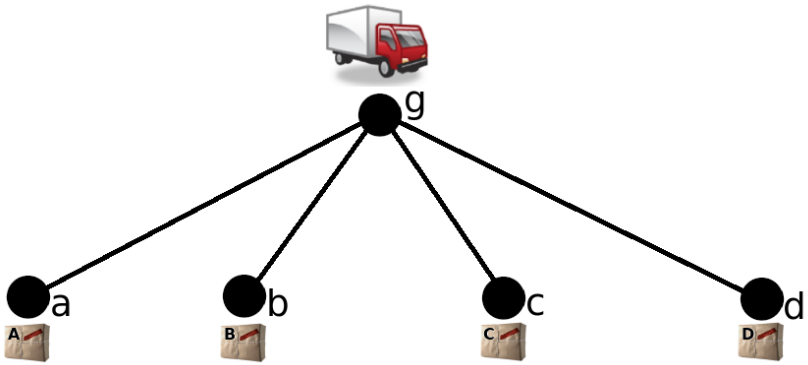

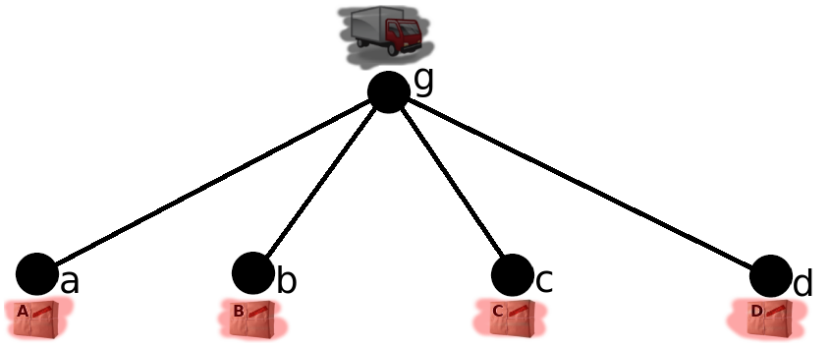

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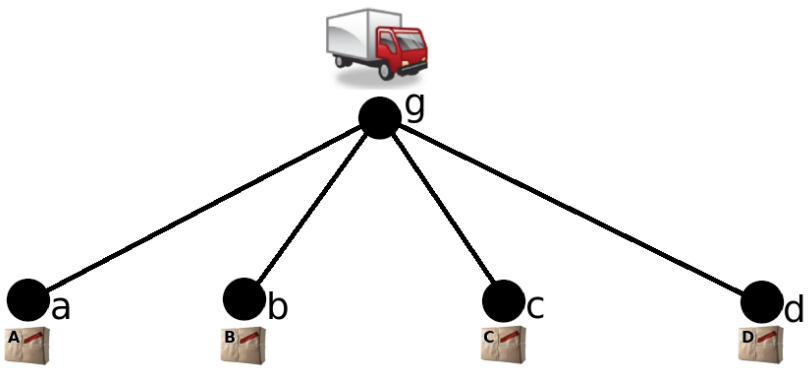

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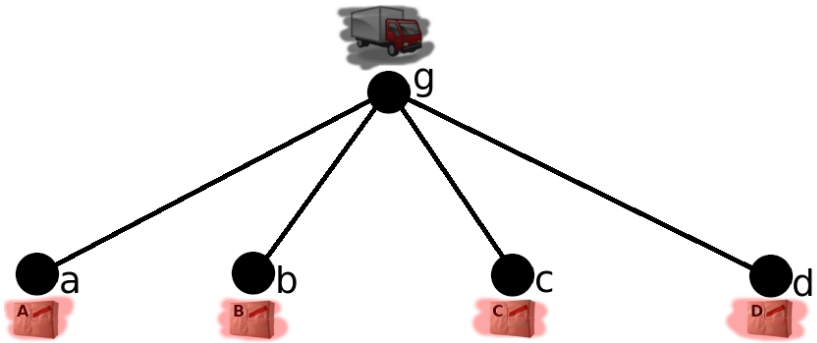

Red-black plan:

1. Initial state: $\{v_T = g, \dots\}$.
2. Apply $drive(g, a)$: $\{v_T = a, \dots\}$.
3. Apply $drive(g, b)$: Not applicable!

$\rightarrow$ It's easy to see that any optimal red-black plan is a real plan here. In particular, $h^{*\mathsf{RB}}(I) = h^*(I)$.

Basic Observations about Red-Black Planning

Method: Given a planning task (V, A, I, G), choose a subset $V^{\mathsf{R}} \subseteq V$ of variables. The red-black relaxation of Π then is the red-black planning task $\Pi^{\mathsf{RB}} = (V^{\mathsf{B}} := V \setminus V^{\mathsf{R}}, V^{\mathsf{R}}, A, I, G)$.

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$\rightarrow$ *How many variables do we have to paint red in order to obtain a tractable (polynomial-time solvable) planning problem?*

Questionnaire

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What if, in Star-Shape Logistics, instead of the truck we paint the packages black?

(A): $h^{*\mathsf{RB}} = h^*$

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$\rightarrow$ (B): Yes, see (A).

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(A): $h^{*\mathrm{RB}} = h^*$

(B): $h^{*\mathrm{RB}} = h^+$

(C): We can't paint the packages black

(D): Honestly, I don't care what color the packages have

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$\rightarrow$ (D): "I see a package and I want to paint it black ..." More seriously: In fact, it doesn't matter (to the heuristic value) what color the packages have: see (A). And that's actually the case for *any* causal graph leaf variables, which are "pure clients" and don't need to move to-and-fro (see [Katz *et al.* (2013)] for details).

"How Many Variables do We Have to Paint Red" = All??

Theorem (Hardness for a Single Black Variable). *The problem of deciding, given a red-black planning task* $\Pi^{\mathsf{RB}} = (V^{\mathsf{B}}, V^{\mathsf{R}}, A, I, G)$ *where* $|V^{\mathsf{B}}| = 1$*, whether* Π^{RB} *is solvable, is* **NP***-complete.*

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$\rightarrow$ We cannot "cheat" because the black "index variable" v_0 forces us to assign each v_i exactly once!

To the Rescue, Part I: The Black Causal Graph

Don't despair: The theorem just stated holds *unless we impose any additional restrictions on the planning task*.

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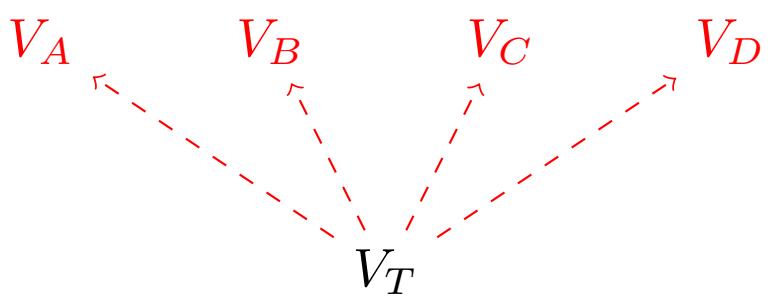

→ Relevant for us here: There are no arcs between black variables.

To the Rescue, Part II: Invertible Variables

Definition (Domain Transition Graph). *Let $\Pi = (V, A, I, G)$ be a planning task, and let $v \in V$. The domain transition graph (DTG) of v is the arc-labeled directed graph with vertices D_v, and, for every $d, d' \in D_v$ and $a \in A$ where either (i) $pre_a(v) = d$ and $eff_a(v) = d'$ or (ii) $pre_a(v)$ is not defined and $eff_a(v) = d'$, an arc $d \xrightarrow{a} d'$. We refer to $d \xrightarrow{a} d'$ as a value transition of v. We write $d \xrightarrow{a}_{\varphi} d'$ where $\varphi = pre_a \setminus \{v = d\}$ is the outside condition.*

Let $d \rightarrow_{\varphi} d'$ be a value transition of v. We say that $d \rightarrow_{\varphi} d'$ is invertible if there exists a value transition $d' \rightarrow_{\varphi'} d$ where $\varphi' \subseteq \varphi$.

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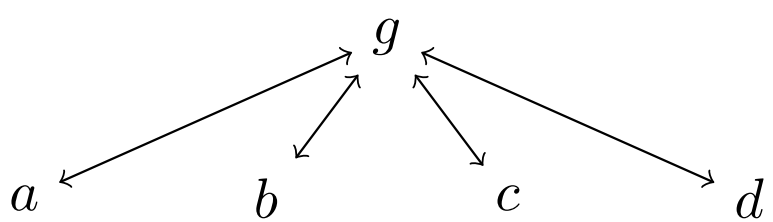

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- **Usage:** On any state s encountered during search, generate a red-black plan for s and take its cost as the heuristic value. (= "In h^{FF}, replace relaxed plan by red-black plan.")

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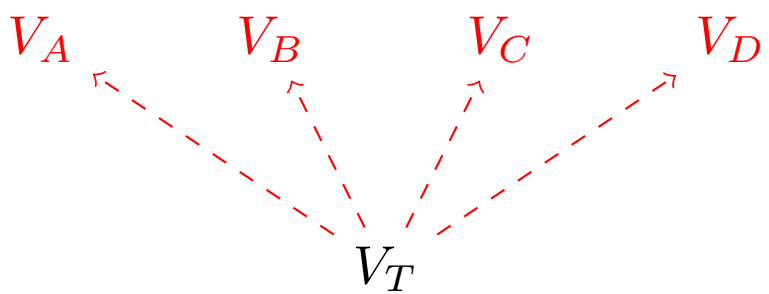

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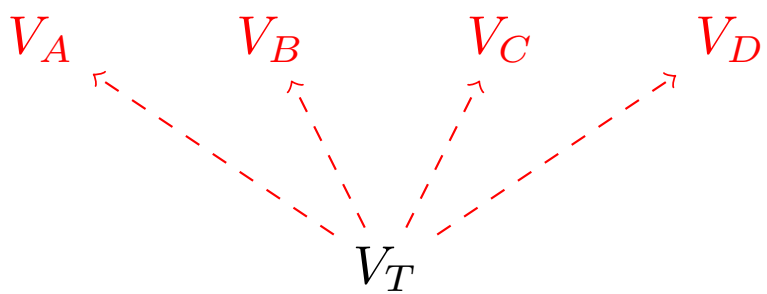


→ We can leave "service variables" – like the robot positions! – black, and paint red only the "client variables". In fact, the SMS Theorem can be extended to allow arbitrary acyclic dependencies over the black variables.

(Which is what's inside Mercury, cf. slide 27.)

Relaxed Plan Repair

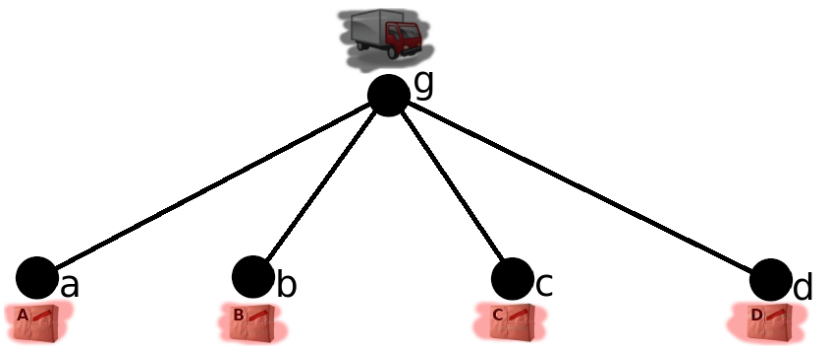
(no details here; see [Domshlak *et al.* (2015)])

// $\Pi = (V, A, c, I, G)$, relaxed plan $\vec{a}^+ = \langle a_1, \ldots, a_n \rangle$, black and red variables V^{B}, V^{R}
$\vec{a} := \langle a_1 \rangle$; $s := appl(I, a_1)$ *// "appl": red-black semantics (slide **??**)*
for $i = 2$ to n **do** *// Repair black action preconditions*
  if $pre_{a_i}(V^{\mathsf{B}}) \not\subseteq s$ **then**
    $\vec{a}^{\mathsf{B}} :=$ Achieve$(s, pre_{a_i}(V^{\mathsf{B}}))$; $\vec{a} := \vec{a} \circ \vec{a}^{\mathsf{B}}$; $s := appl(s, \vec{a}^{\mathsf{B}})$
  endif
  $\vec{a} := \vec{a} \circ \langle a_i \rangle$; $s := appl(s, a_i)$
endfor
if $G(V^{\mathsf{B}}) \not\subseteq s$ **then** *// Repair black goals*
  $\vec{a}^{\mathsf{B}} :=$ Achieve$(s, G(V^{\mathsf{B}}))$; $\vec{a} := \vec{a} \circ \vec{a}^{\mathsf{B}}$
endif
return $\vec{a}$

Procedure: Achieve(s, g)
$\vec{a}^{\mathsf{B}} := \langle\rangle$
for $v \in V^{\mathsf{B}}$ s.t. $g(v)$ is defined **do** *// Move black variables into place separately*
  $\vec{a}^{\mathsf{B}} := \vec{a}^{\mathsf{B}} \circ$ invert path used by $\vec{a}$ from $I(v)$ to $s(v)$ *// Black variables are invertible*
  $\vec{a}^{\mathsf{B}} := \vec{a}^{\mathsf{B}} \circ$ path used by $\vec{a}^+$ from $I(v)$ to $g(v)$
endfor
return $\vec{a}^{\mathsf{B}}$

Relaxed Plan Repair in Star-Shape Logistics

[**Note:** In the illustration, the packages move. This is just for simplicity of illustration: as these variables are red, actually the packages accumulate their positions.]



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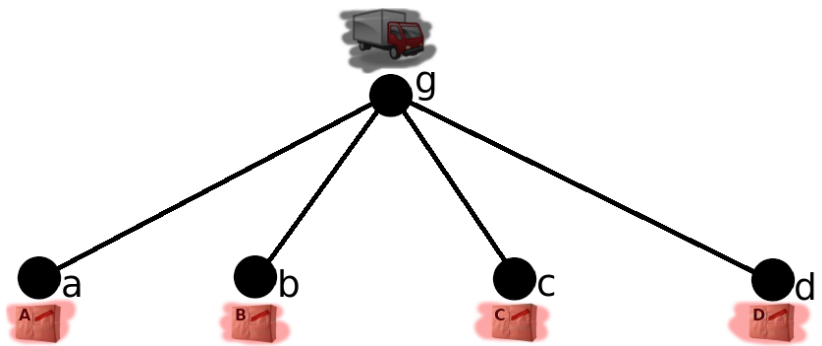

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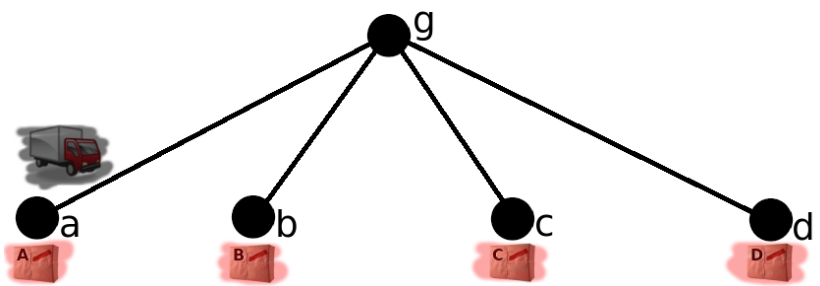

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- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$

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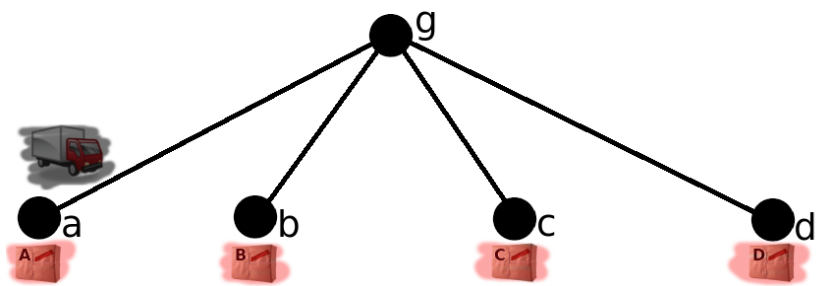

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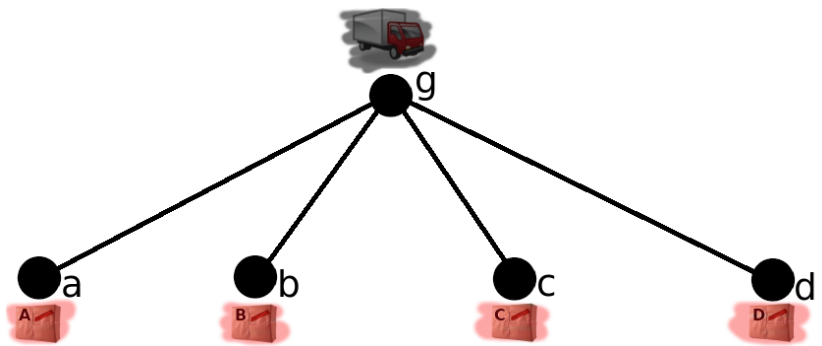
Relaxed Plan Remainder: $drive(g,b)$, $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$

Relaxed Plan Repair in Star-Shape Logistics

[**Note:** In the illustration, the packages move. This is just for simplicity of illustration: as these variables are red, actually the packages accumulate their positions.]



Relaxed Plan Repair:

- Relaxed plan $\vec{a}^+ = \langle a_1, \ldots, a_n \rangle$.
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- For any black v, if $s(v) \neq z$ precondition of a_2: Move v to value z.
- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,b)$, $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$

Relaxed Plan Repair in Star-Shape Logistics

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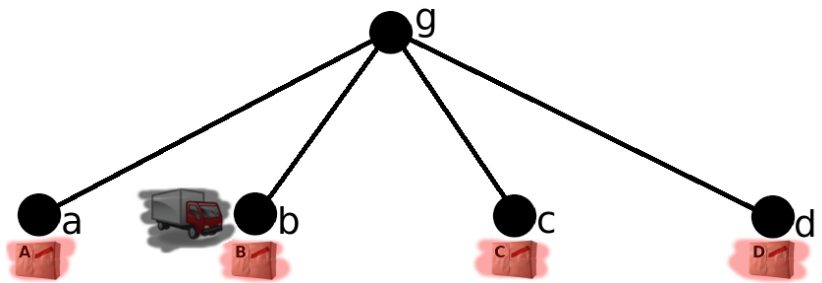

Relaxed Plan Repair:

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,b)$, $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$

Relaxed Plan Repair in Star-Shape Logistics

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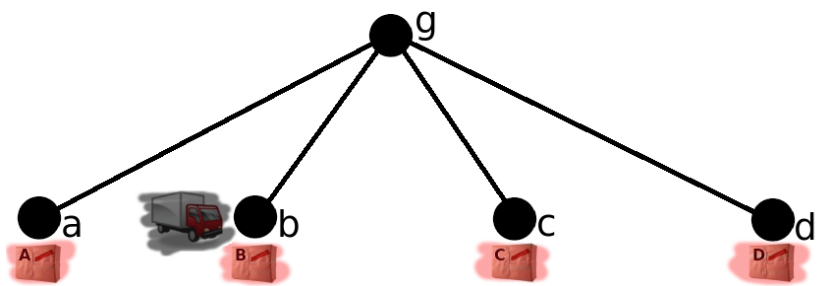

Relaxed Plan Repair:

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
- $a_3 = drive(g,c)$: Move v_T back to g. Apply a_3, giving $v_T = c$, $v_x = \ldots$

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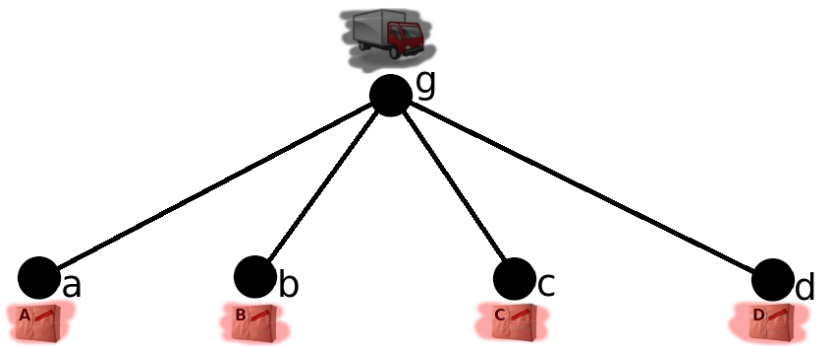

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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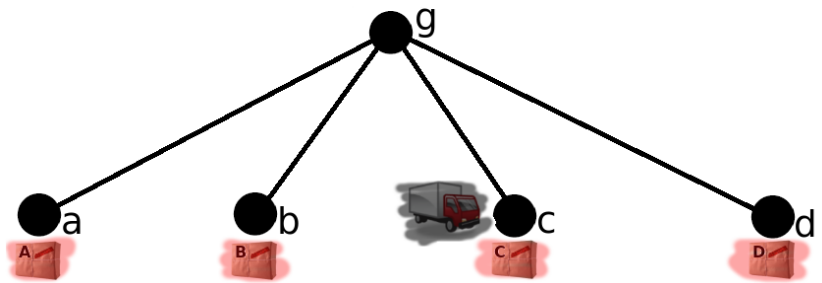

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,c)$, $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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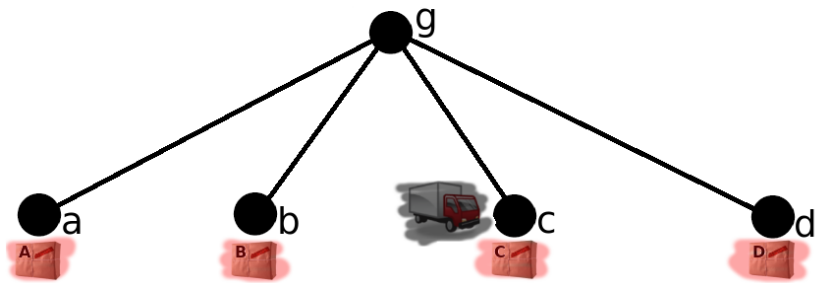

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $drive(g,d)$, $load(A,a)$, $load(B,b)$, $load(C,c)$, $load(D,d)$, $unload(A,g)$, $unload(B,g)$, $unload(C,g)$, $unload(D,g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g,b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
- $a_3 = drive(g,c)$: Move v_T back to g. Apply a_3, giving $v_T = c$, $v_x = \ldots$
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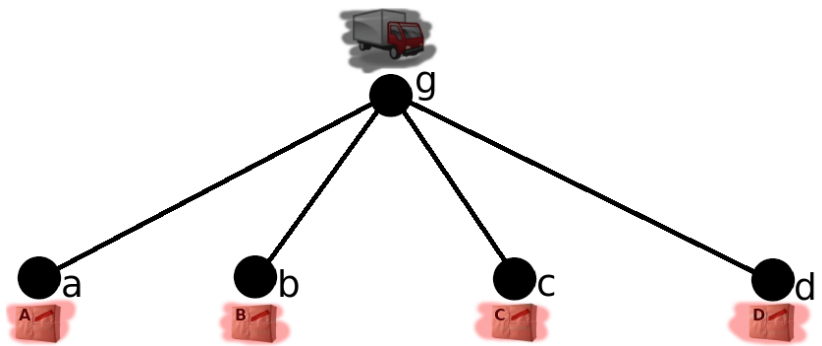

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- Proceed with $a_3, \ldots, a_n$ and the goal.

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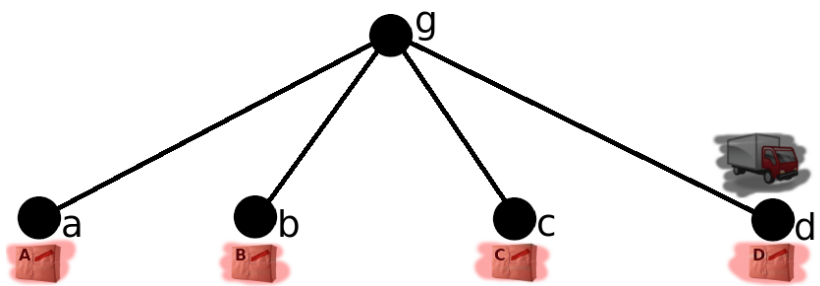

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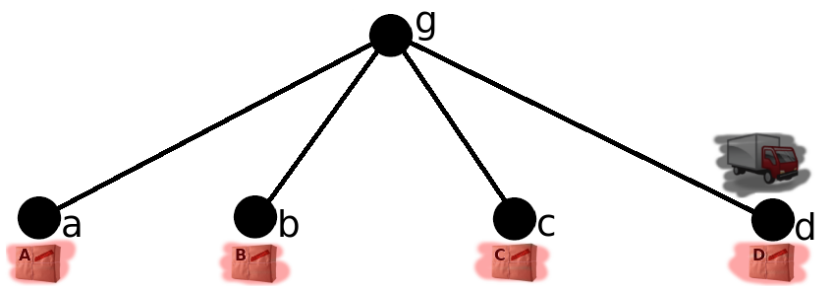

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

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- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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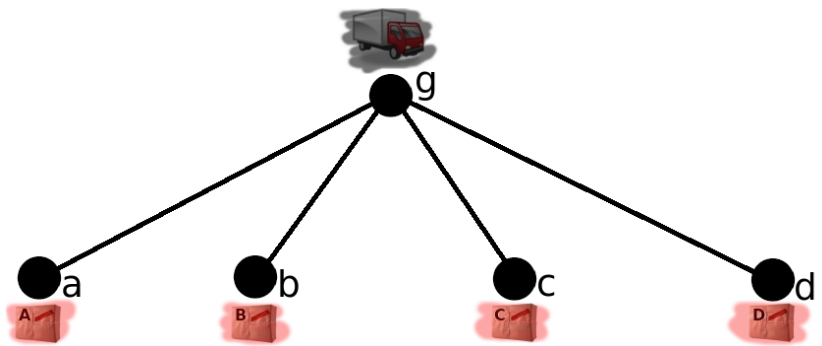

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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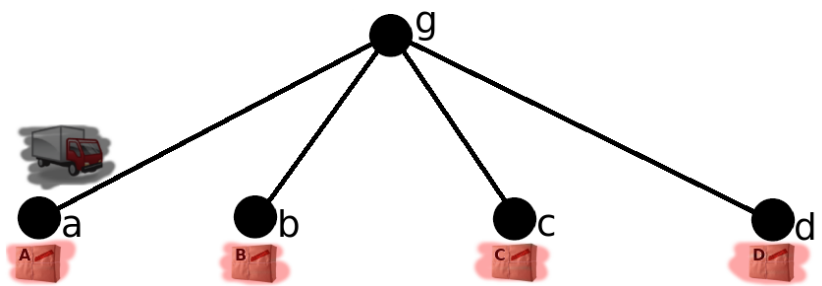

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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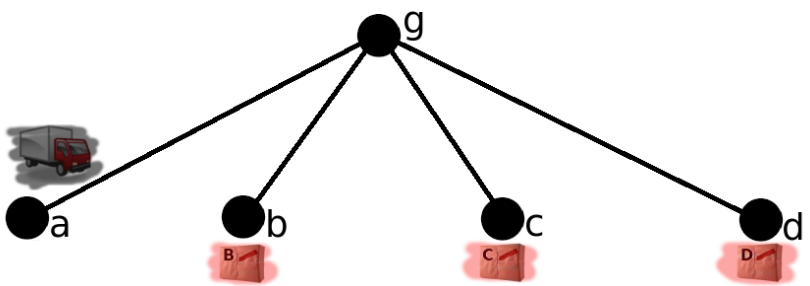

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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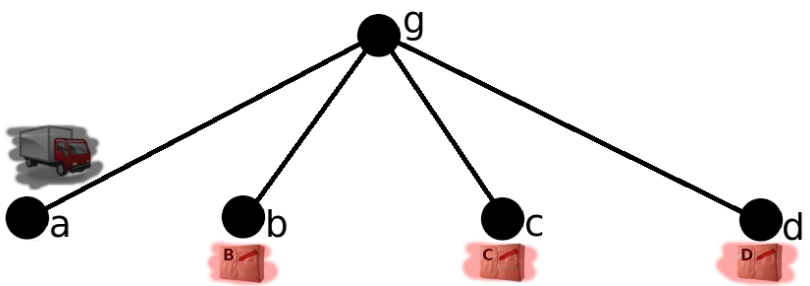

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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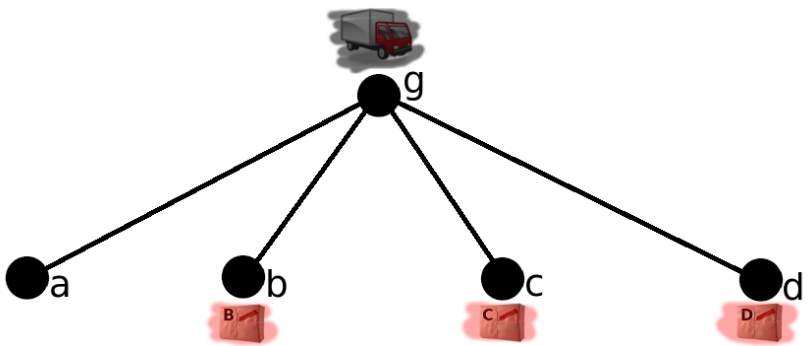

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

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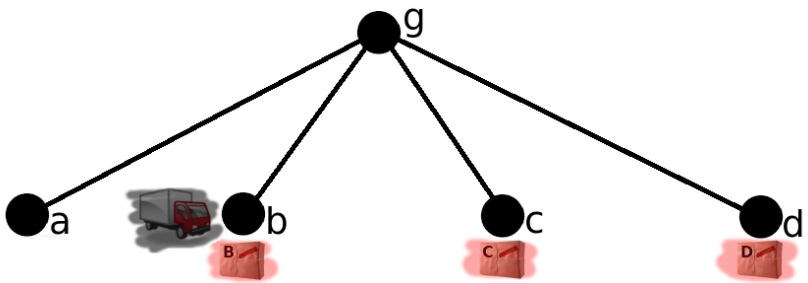

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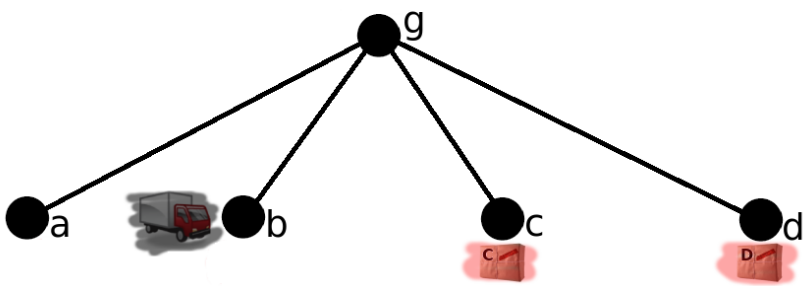

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- $s :=$ red-black outcome of a_2.
- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

- After a_1 moving v_T to a: $v_T = a$, $v_x = \ldots$
- $a_2 = drive(g, b)$: Move v_T back to g. Apply a_2, giving $v_T = b$, $v_x = \ldots$
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- $\ldots$

Relaxed Plan Repair in Star-Shape Logistics

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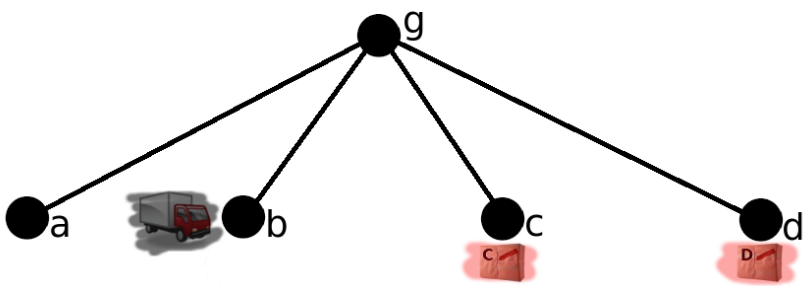

Relaxed Plan Repair:

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- Proceed with $a_3, \ldots, a_n$ and the goal.

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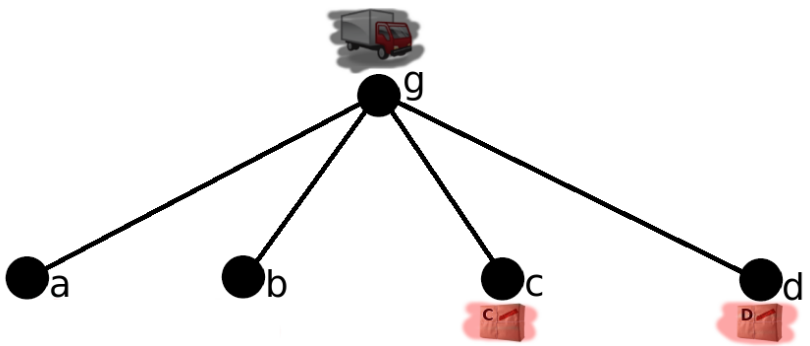

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Relaxed Plan Remainder: $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

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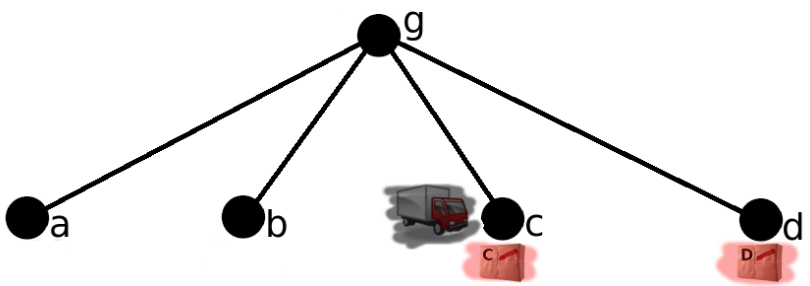

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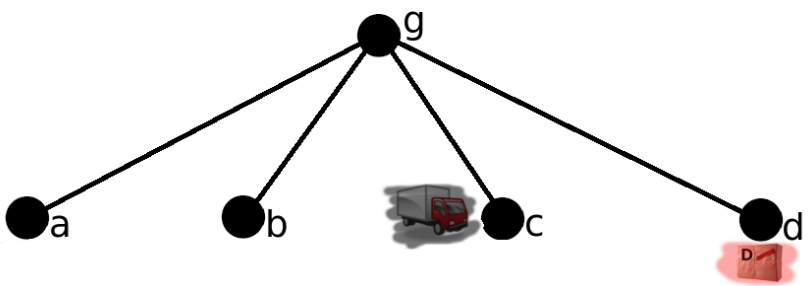

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- Proceed with $a_3, \ldots, a_n$ and the goal.

Relaxed Plan Remainder: $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

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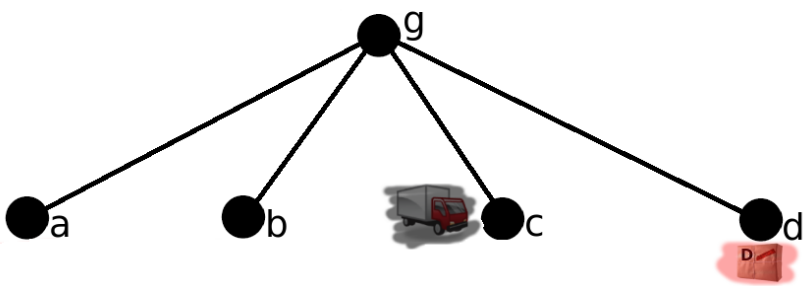

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- Proceed with $a_3, \ldots, a_n$ and the goal.

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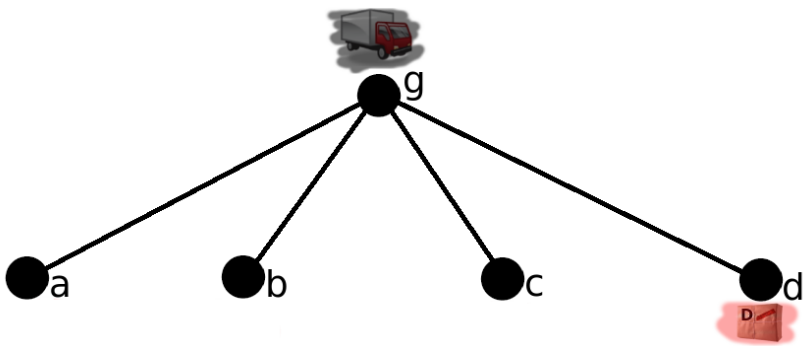

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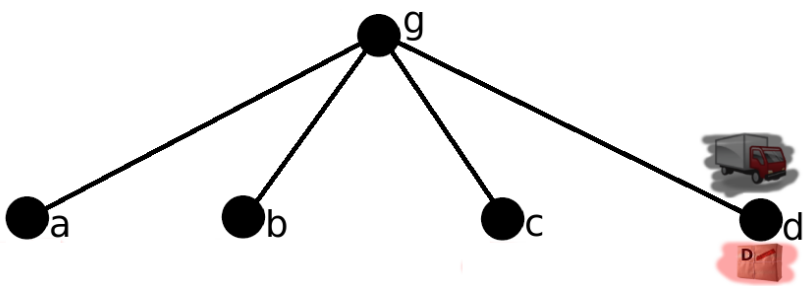

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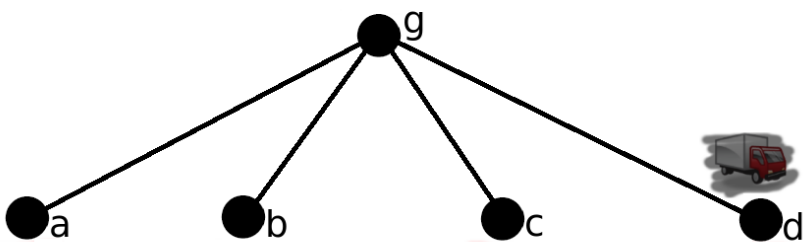

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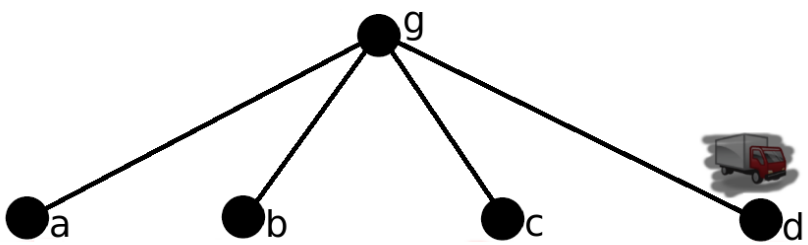

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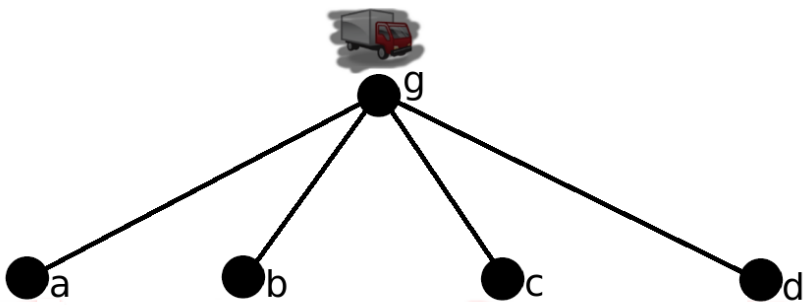

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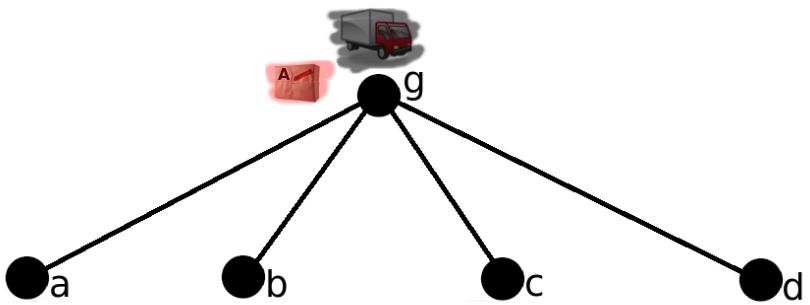

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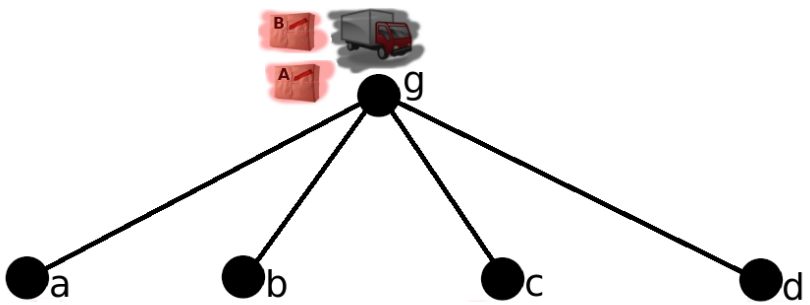

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Relaxed Plan Remainder: $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

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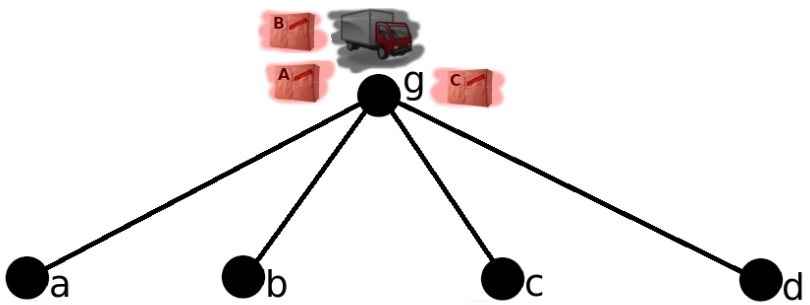

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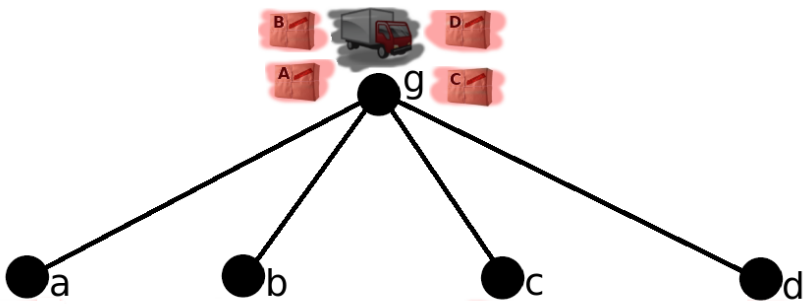

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Questionnaire

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Does Relaxed Plan Repair yield an accurate heuristic function?

(A): Yes (B): No

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Historical/Literature Notes

- The weaknesses of the delete relaxation were recognized early, and many proposals have been made to address them (e. g. [Do and Kambhampati (2001); Fox and Long (2001); Gerevini *et al.* (2003); Helmert (2004); Keyder and Geffner (2008); Helmert and Geffner (2008); Baier and Botea (2009)]).

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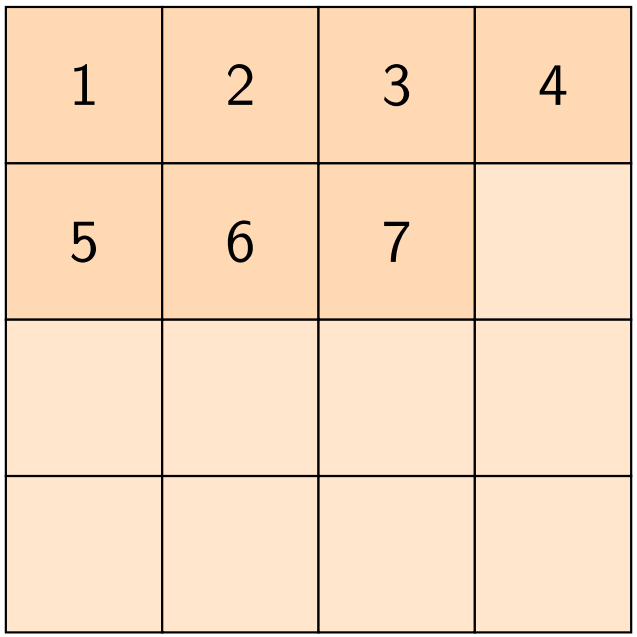
- Pre-merging can be simulated by explicit conjunctions; other than that, the methods are complementary [Hoffmann *et al.* (2014b)].

Agenda

Highlight Nr. 1: Summing Arbitrary Admissible Heuristics



→ Is the sum of the two heuristics admissible?

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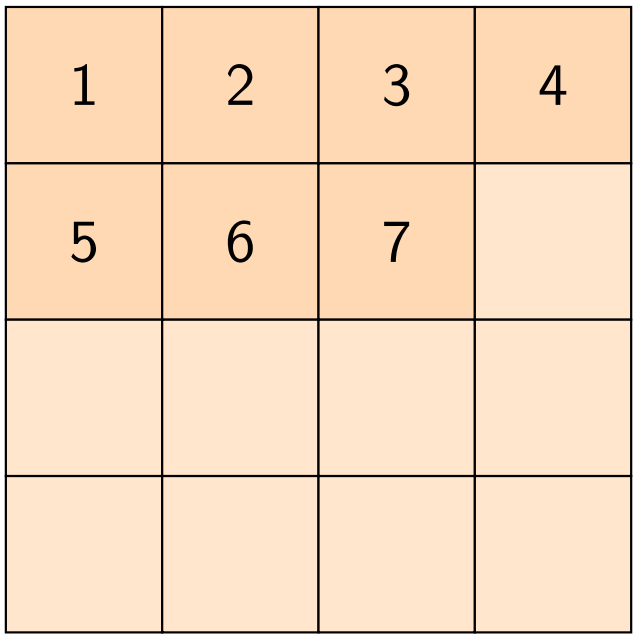

→ Is the sum of the two heuristics admissible? Yes, because each move is accounted for by only one of the two abstractions.

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			8
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$\rightarrow$ Is the sum of the two heuristics admissible? No, because the same moves of tile 3 may be counted by both abstractions.

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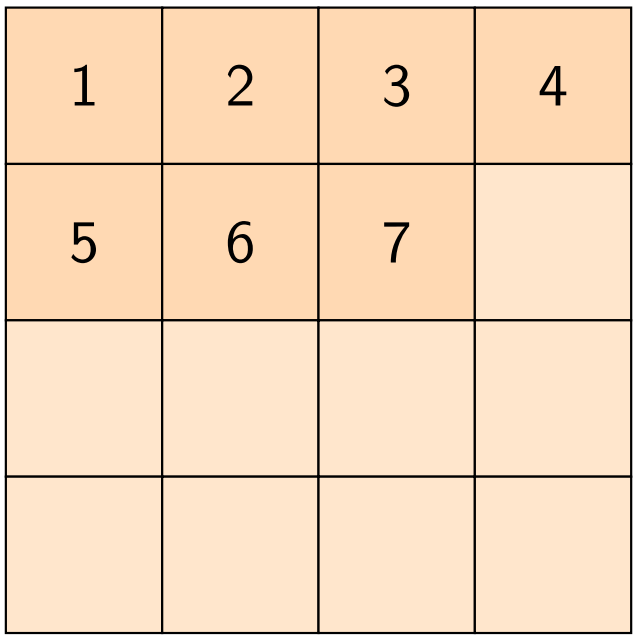

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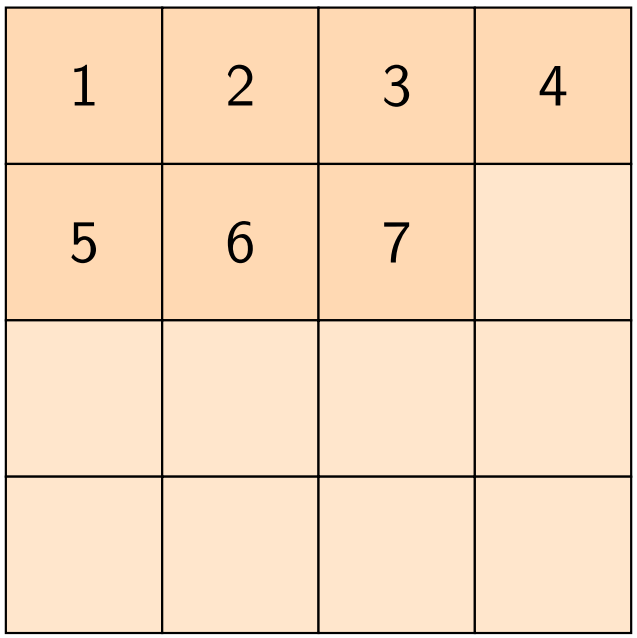

→ Is the sum of the two heuristics admissible? No, because the same moves of tile 3 may be counted by both abstractions.

→ But what if, on each side, we count only 0.5 for each move of tile 3? Then yes, because "duplicate moves" will be accounted for as cost 1.

Cost Partitioning

Definition (Cost Partitioning). *Let Π be a planning task with actions A and cost function c. An ensemble of functions $c_1, \ldots, c_n : A \mapsto \mathbb{R}_0^+$ is a cost partitioning for Π if, for all $a \in A$, $\sum_{i=1}^{n} c_i(a) \leq c(a)$.*

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$\rightarrow$ Cost partitionings distribute the cost of each action across a set of otherwise identical planning tasks. This technique can be used to admissibly combine arbitrary admissible heuristic functions.

- First proposed by [Katz and Domshlak (2008)], then investigated widely in planning [e. g. Karpas *et al.* (2011); Pommerening *et al.* (2015); Seipp *et al.* (2017)].
- For landmark heuristics and abstraction heuristics (see next), an optimal cost partitioning – one that maximizes the value of the partitioned sum in a given state s – is computable in polynomial time, via an LP encoding [Katz and Domshlak (2008); Karpas and Domshlak (2009); Katz and Domshlak (2010)]!
- The well-known and wide-spread concept of independent heuristics [Korf and Felner (2002); Felner *et al.* (2004)] is the special case of 0/1 cost partitionings!

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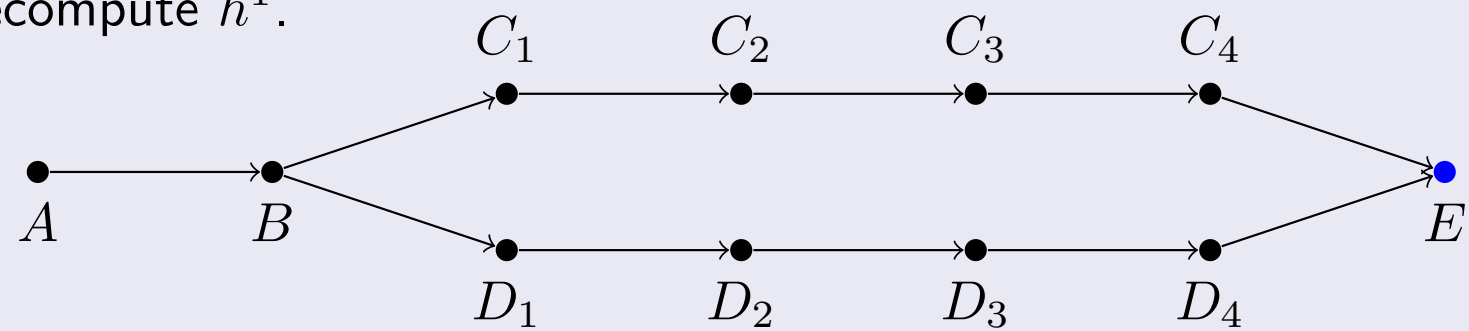

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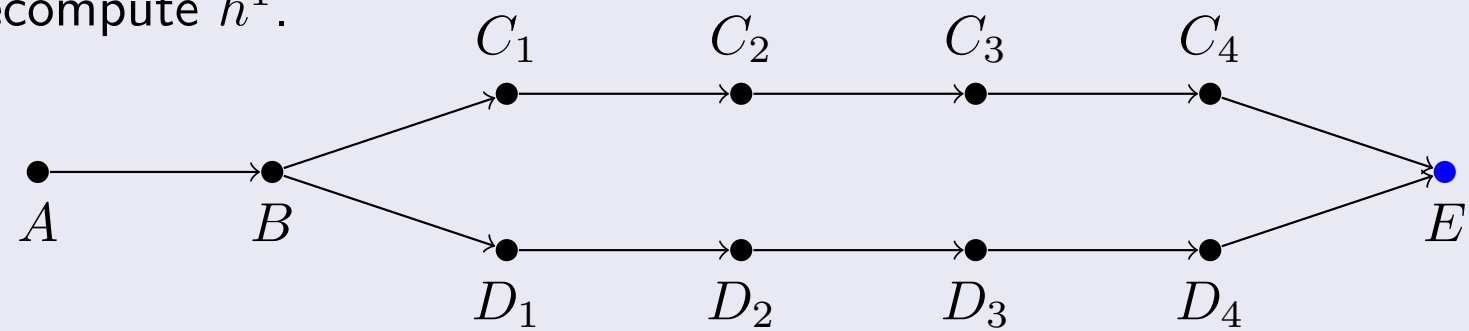

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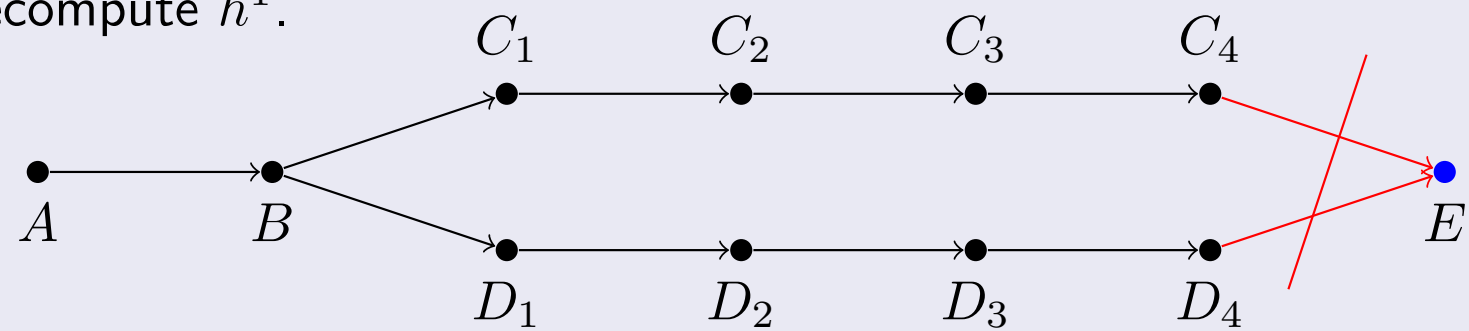

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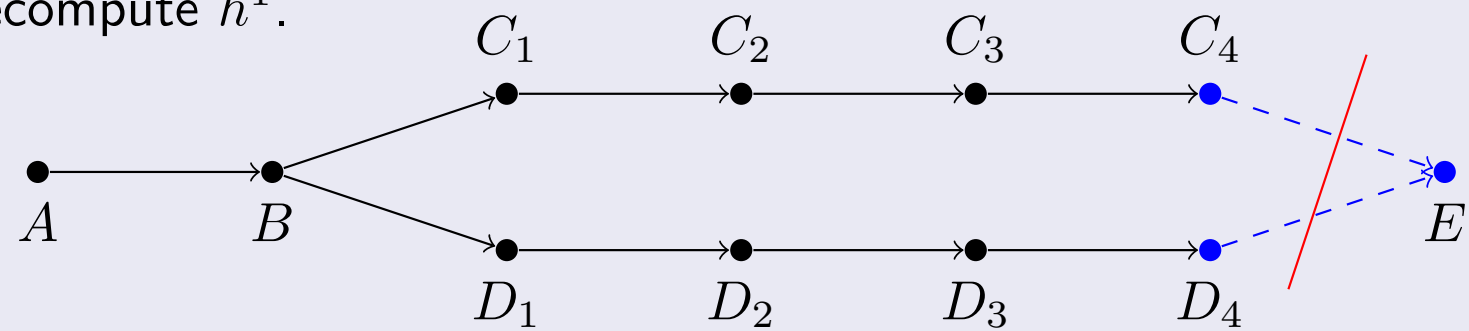

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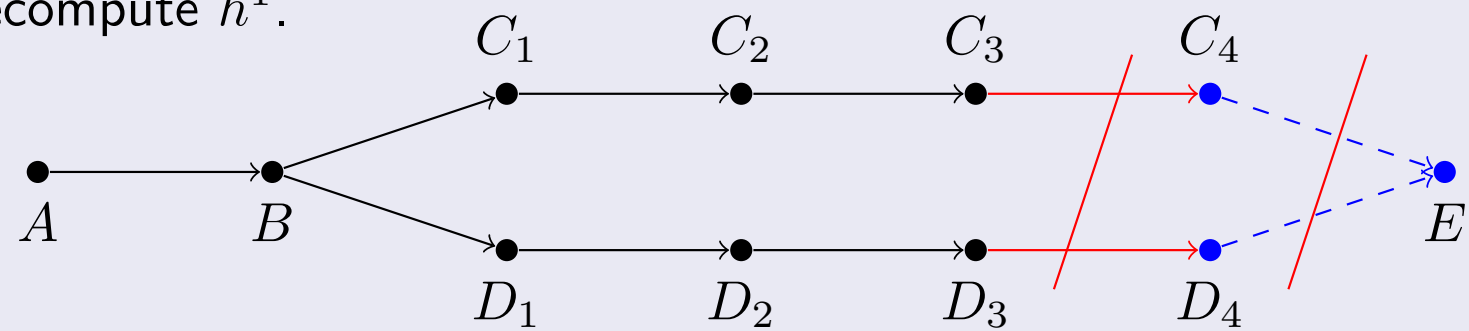

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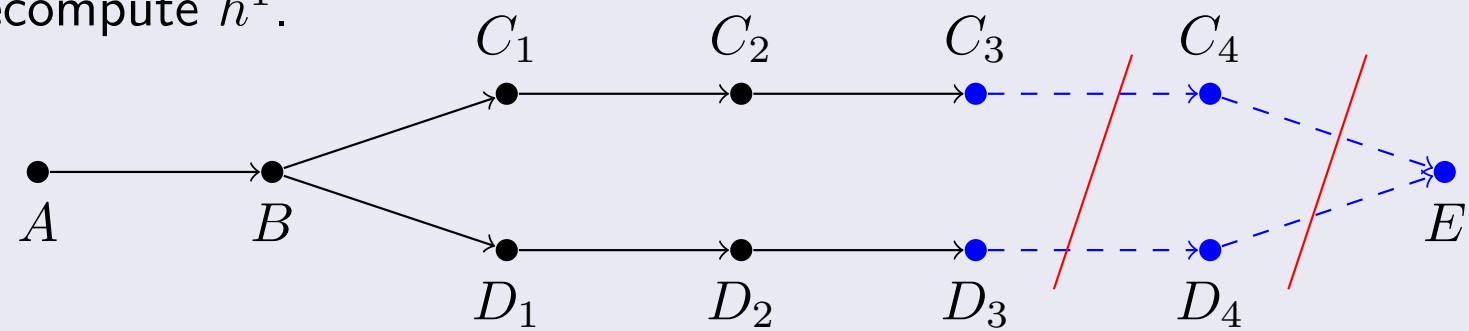

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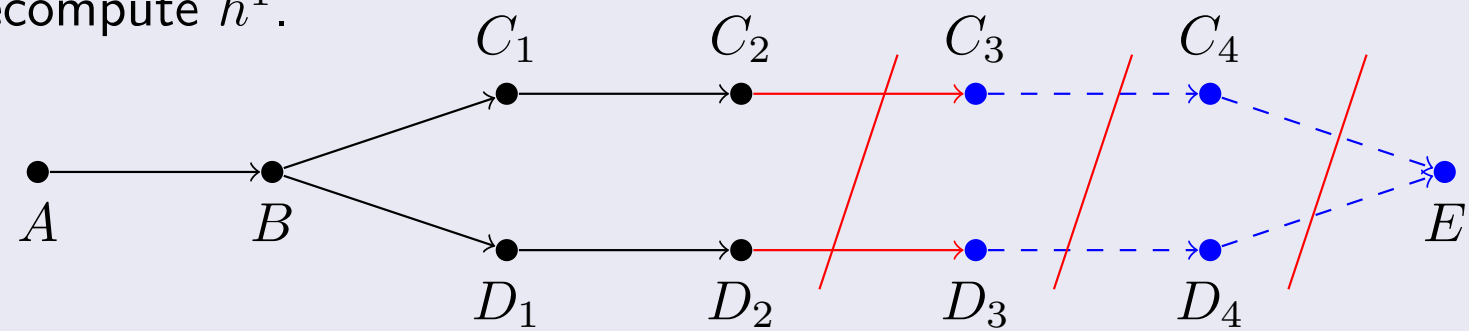

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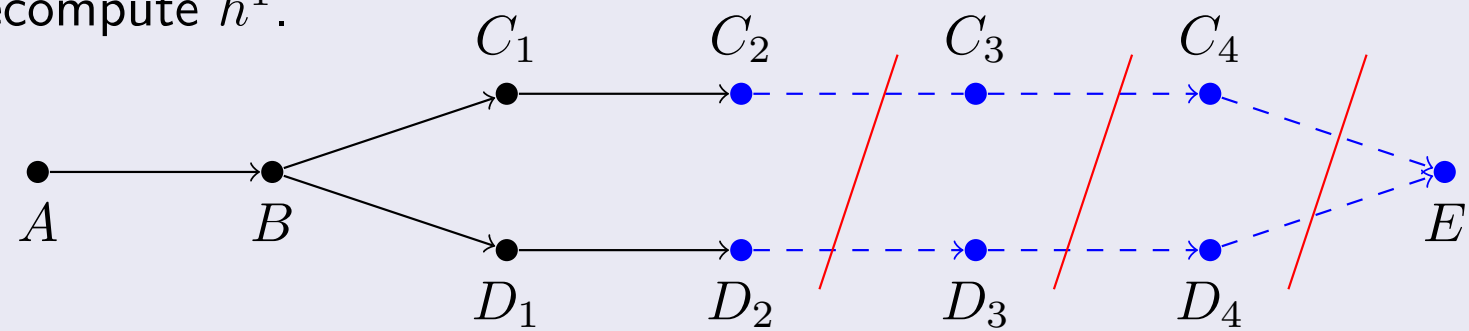

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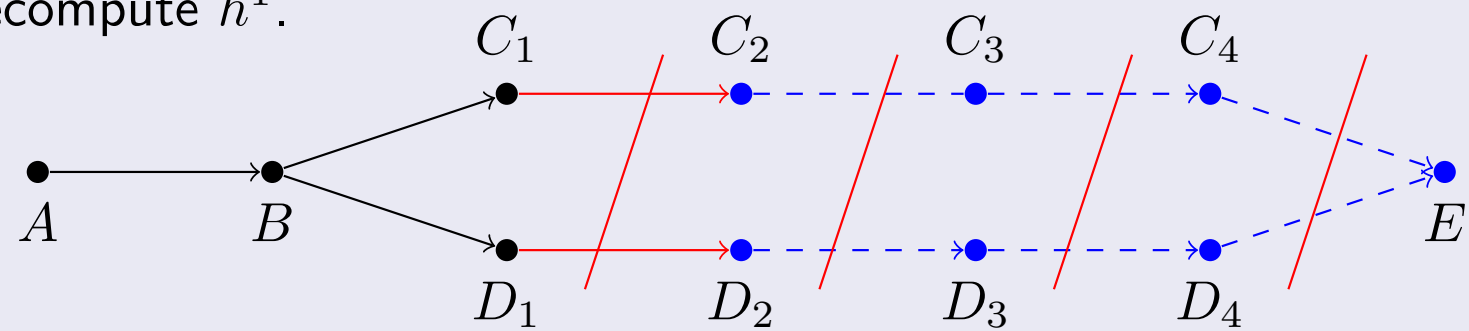

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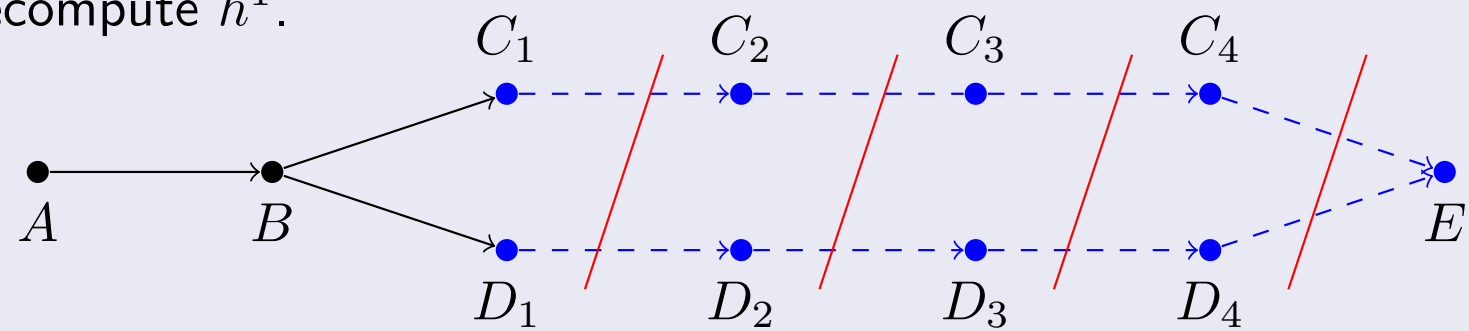

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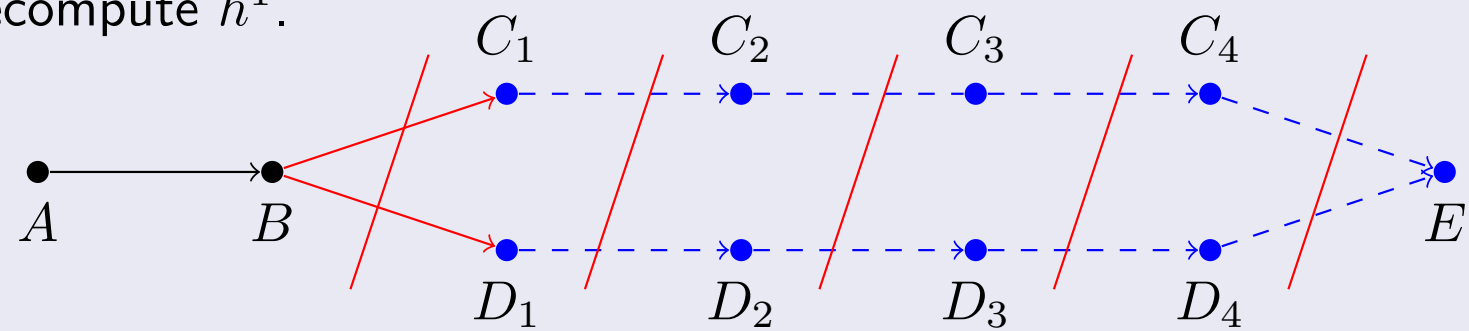

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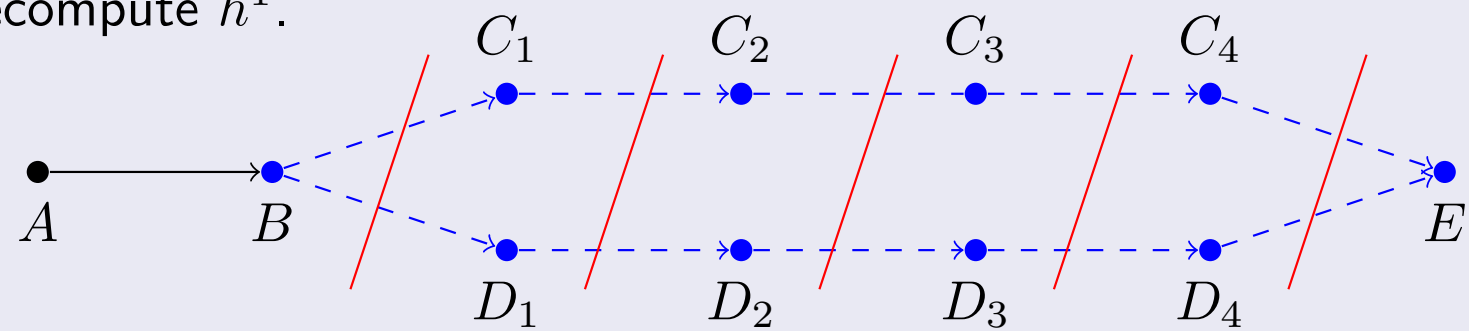

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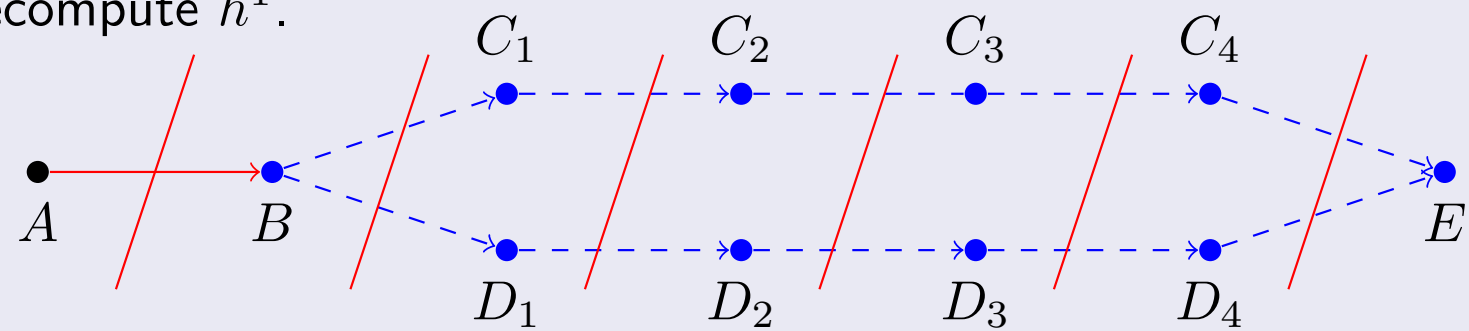

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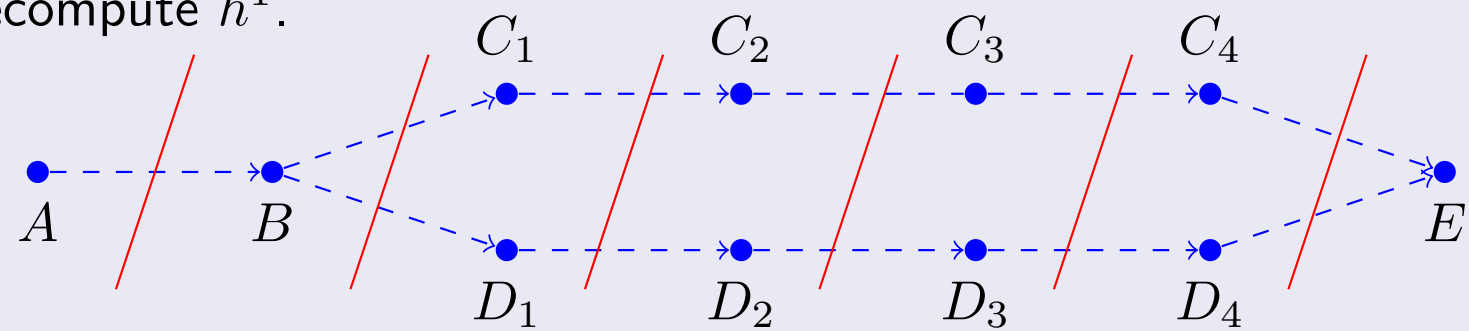


- Each cut is a landmark: a set of actions at least one of which must be used on every plan [Hoffmann *et al.* (2004); Karpas and Domshlak (2009)].
- Given a landmark C, c as above is a lower bound on cost-to-goal.
- Reducing the cost in each step iteratively yields a cost partitioning over these lower bounds.

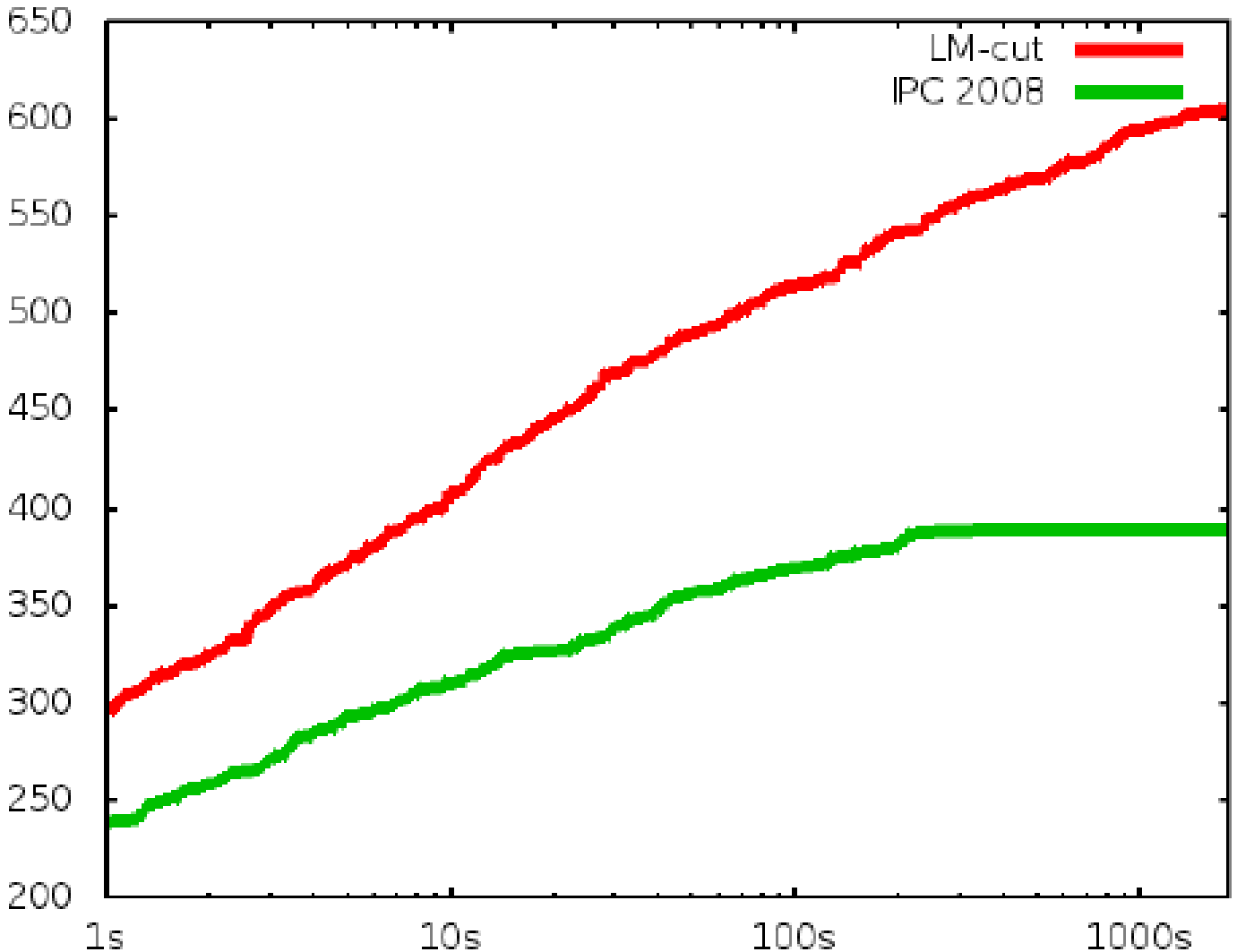
Almost h^+!

(table verbatim from [Helmert and Domshlak (2009)])

Domain	h^+	$h^{\max}$	HBG	CFLS	h^{LA}	$h^{\text{LM-cut}}$
Airport (37)	114.38	36.68	n/a	110.49	108.97	**114.38**
Blocks (35)	17.37	7.54	16.86	12.00	**17.37**	**17.37**
Gripper (20)	47.00	2.00	n/a	**47.00**	**47.00**	**47.00**
Logistics-2000 (26)	35.12	5.85	31.42	33.81	35.00	**35.12**
Miconic-STRIPS (150)	50.47	2.99	5.11	32.00	**50.47**	**50.47**
Pathways (5)	15.60	5.80	5.80	9.00	7.60	**15.60**
PSR-Small (50)	3.14	1.46	2.78	2.46	**3.14**	**3.14**
TPP (18)	32.17	6.39	n/a	n/a	17.61	**32.17**
Depot (10)	20.90	4.70	14.80	17.40	17.50	**20.50**
Driverlog (14)	15.50	4.71	10.71	12.00	13.43	**15.00**
Grid (2)	15.00	10.50	10.50	11.50	11.50	**14.00**
Logistics-1998 (10)	27.90	5.30	n/a	22.10	23.50	**27.70**
MPrime (24)	5.42	3.54	n/a	4.38	3.42	**4.92**
Rovers (14)	18.21	3.71	12.21	11.43	11.64	**18.00**
Satellite (9)	17.11	3.00	4.22	9.33	15.89	**16.89**
Zenotravel (13)	11.62	2.85	9.08	9.46	11.00	**11.54**
FreeCell (6)	8.33	3.00	7.00	3.33	**7.67**	7.17
Mystery (18)	6.44	3.56	n/a	4.72	3.67	**5.39**
Openstacks (5)	21.00	4.00	12.00	17.00	**21.00**	17.20
Pipesworld-NoTankage (18)	10.28	4.33	n/a	4.50	7.17	**8.28**
Pipesworld-Tankage (11)	8.36	3.91	n/a	3.91	6.27	**6.82**
Trucks (10)	21.70	4.00	**20.50**	9.90	14.60	**20.50**
avg. additive error compared to h^+		27.99	17.37	8.05	1.94	**0.28**
avg. relative error compared to h^+		68.5%	40.9%	25.2%	9.5%	**2.5%**

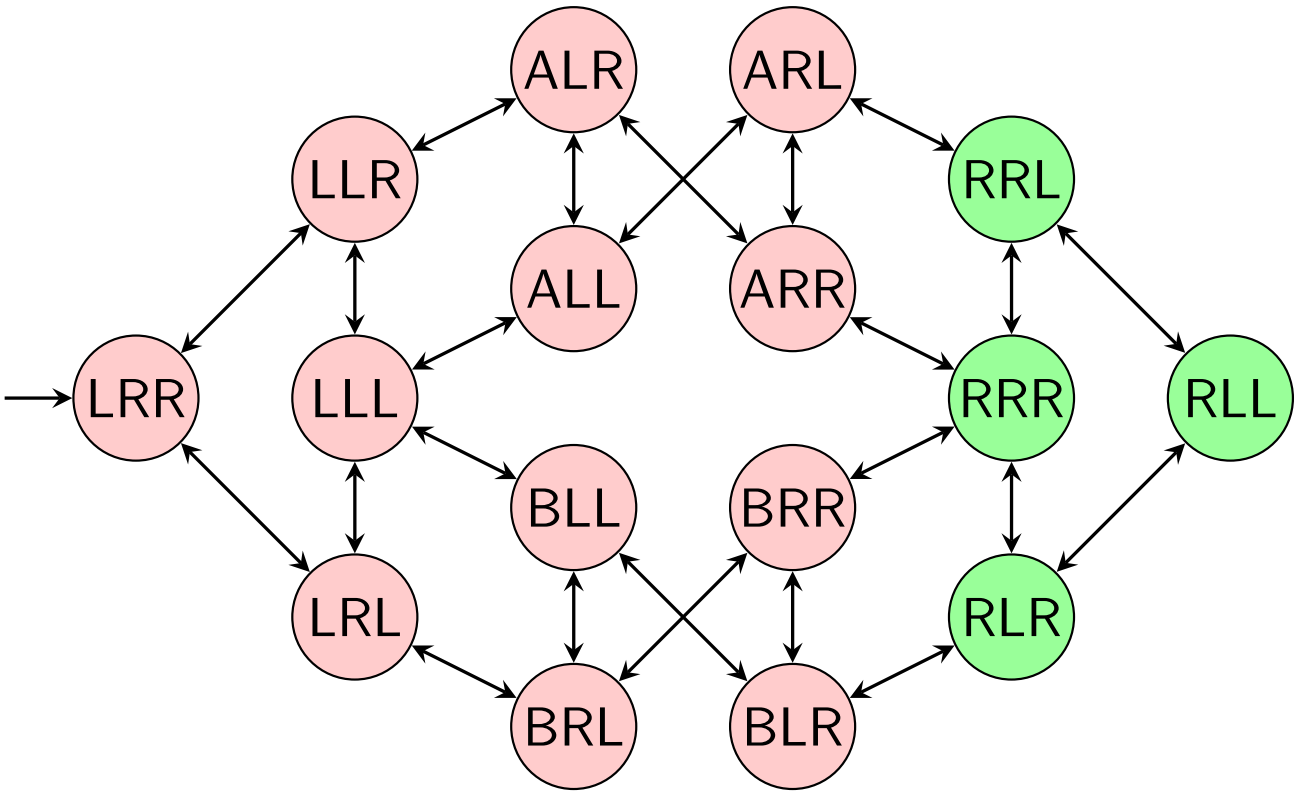
Huge Performance Improvement!

(data courtesy of Malte Helmert)



Highlight Nr. 3: Abstraction Heuristics

Concrete state space:



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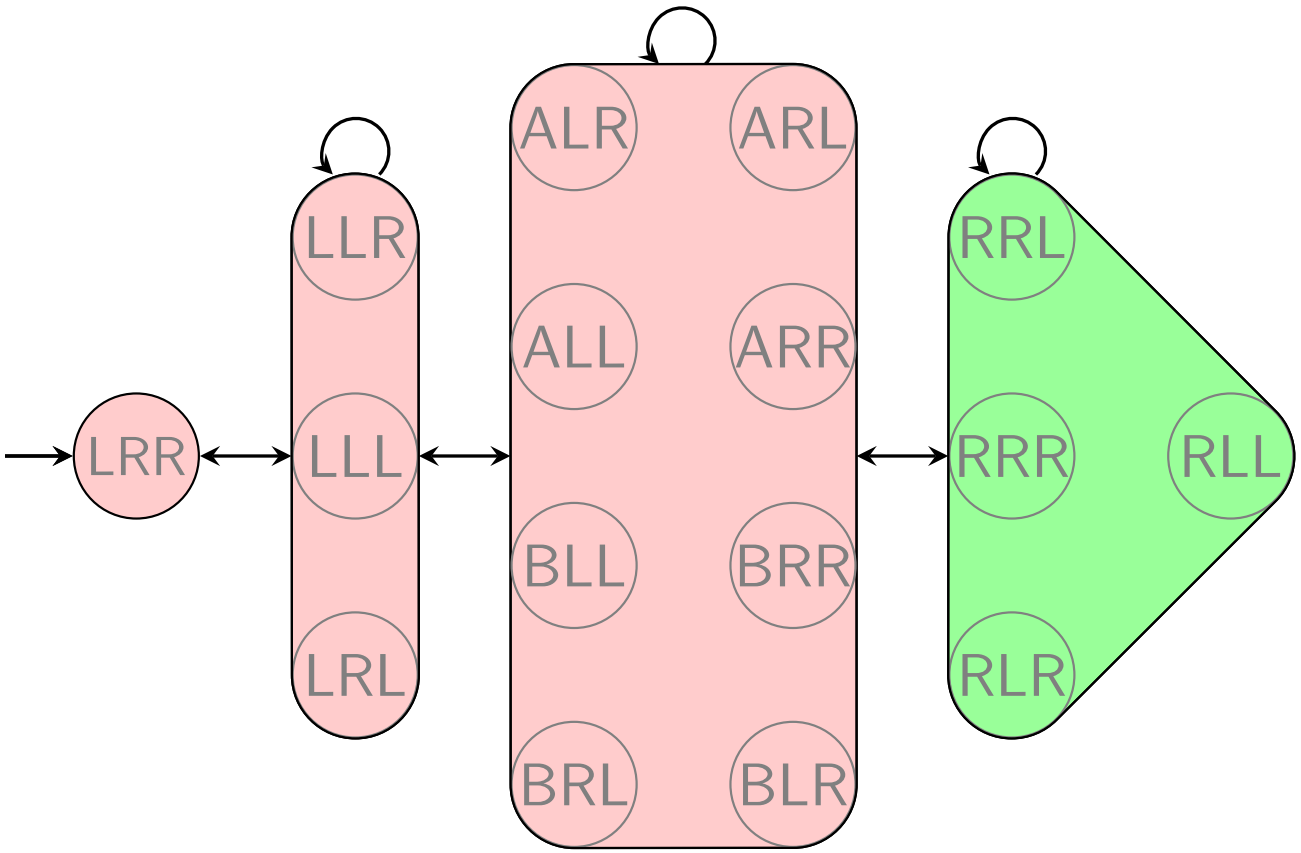
Abstract state space:



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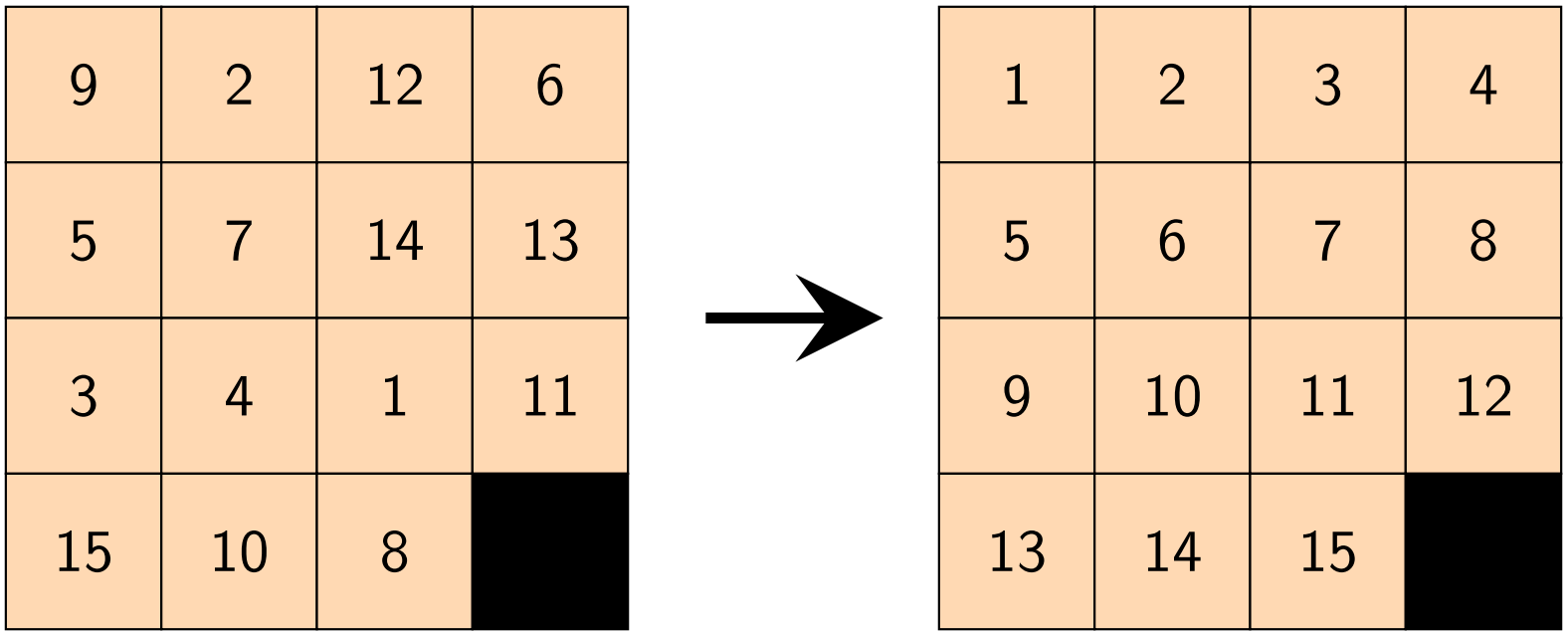

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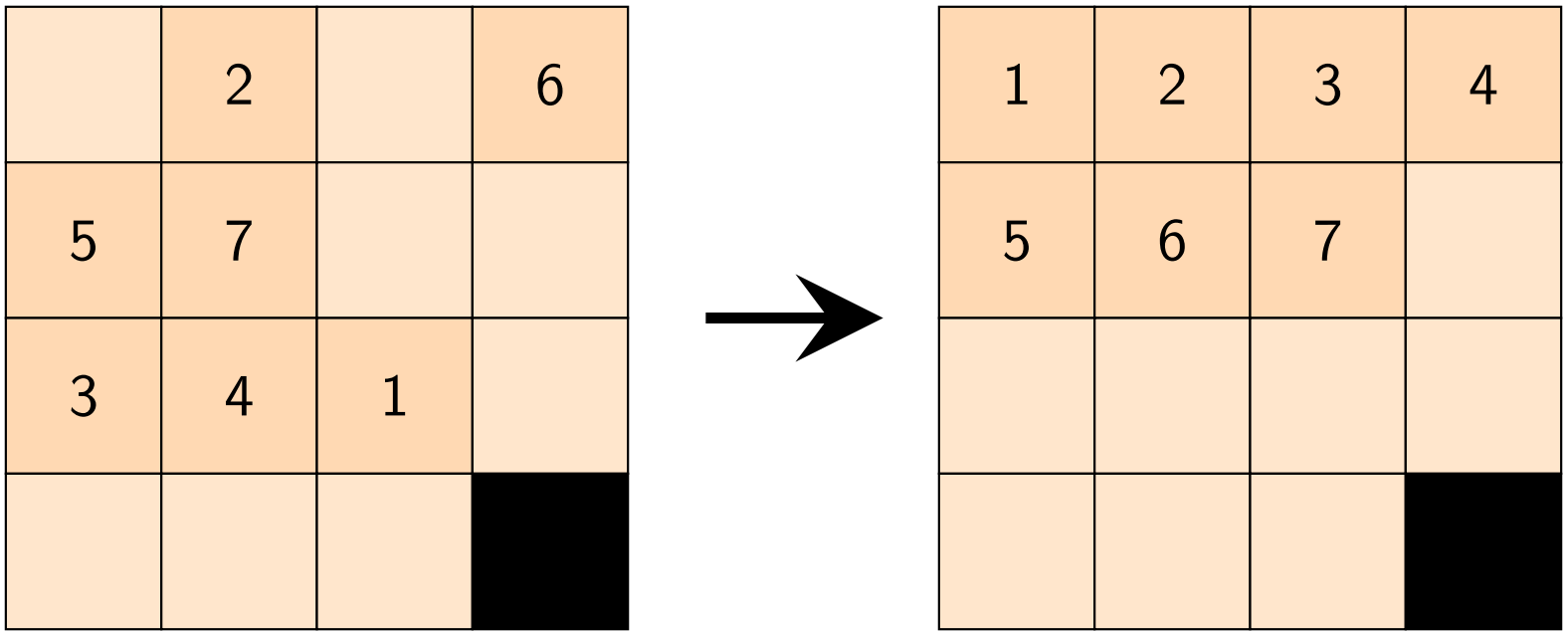

$h :=$ **Solution in abstract state space of projected puzzle**

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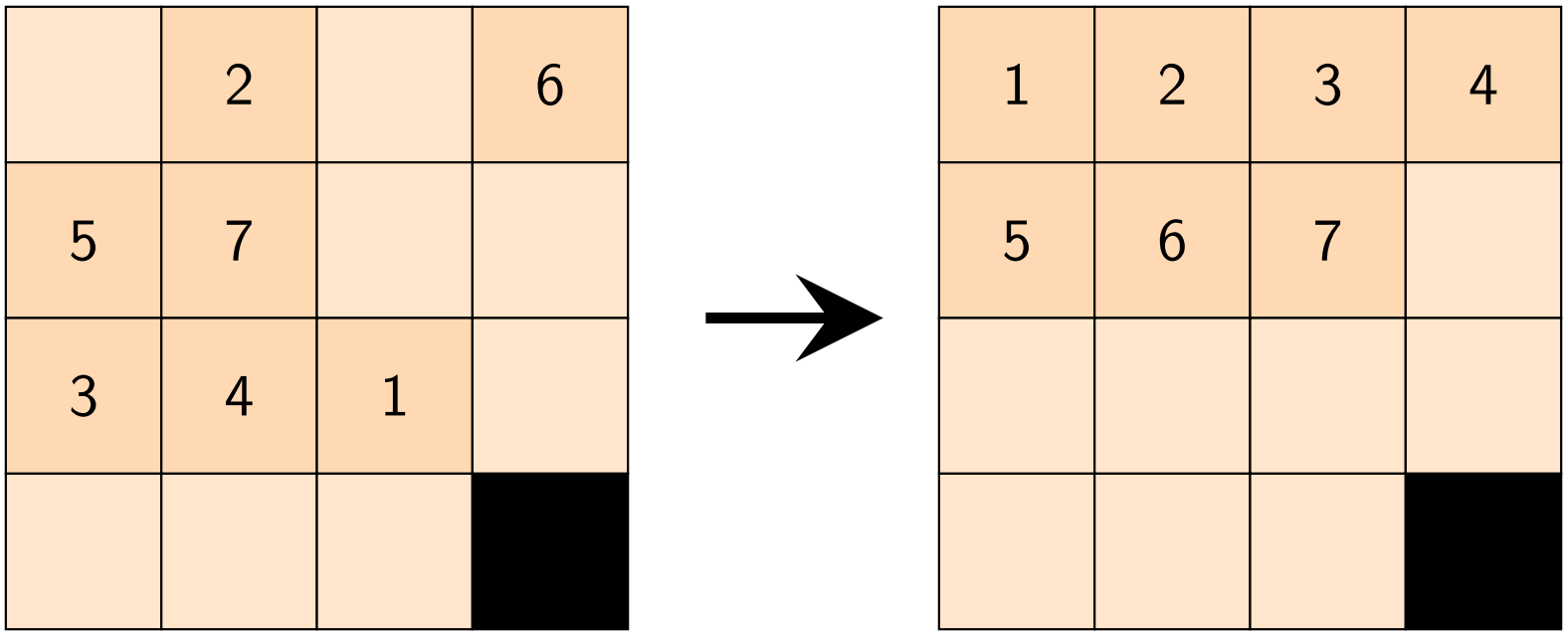

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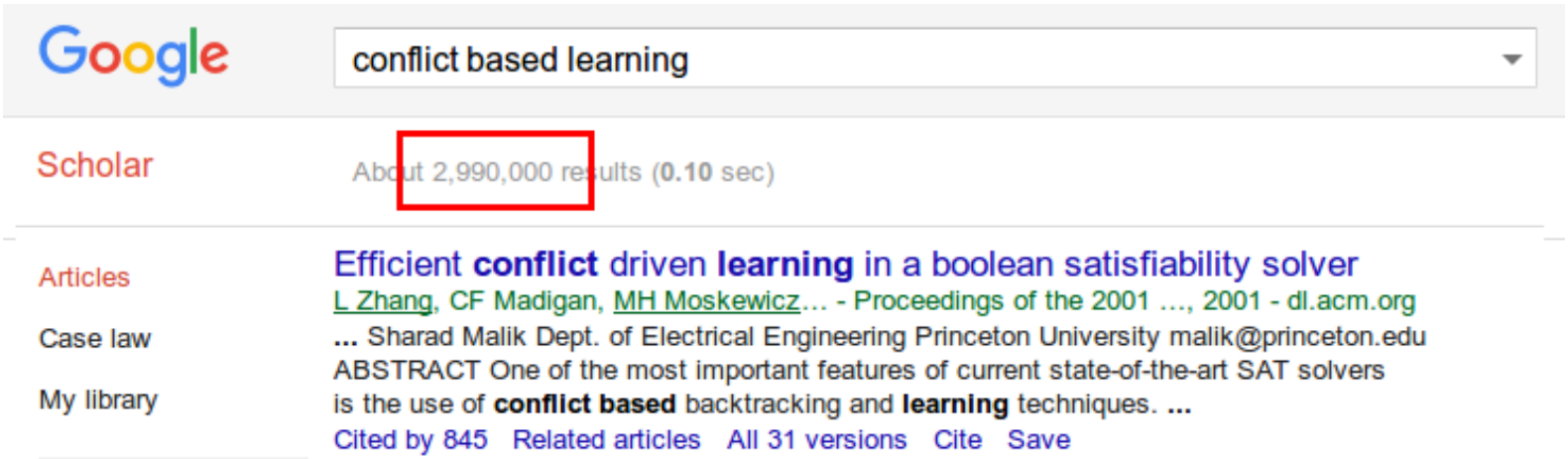
→ Lots of methods for doing this effectively! PDBs project onto part of the problem [e. g. Culberson and Schaeffer (1998); Edelkamp (2001); Pommerening *et al.* (2013)]; merge-and-shrink can compactly compute much more general abstractions [e. g. Dräger *et al.* (2009); Helmert *et al.* (2014)]; Cartesian abstraction allows effective abstraction-refinement operations [e. g. Seipp and Helmert (2013)]; saturated cost partitioning allots each abstraction only the minimum cost function needed for the lower bound [e. g. Seipp *et al.* (2017)].

Highlight Nr. 4: State-Space Conflict Based Learning

Conflict Based Learning:



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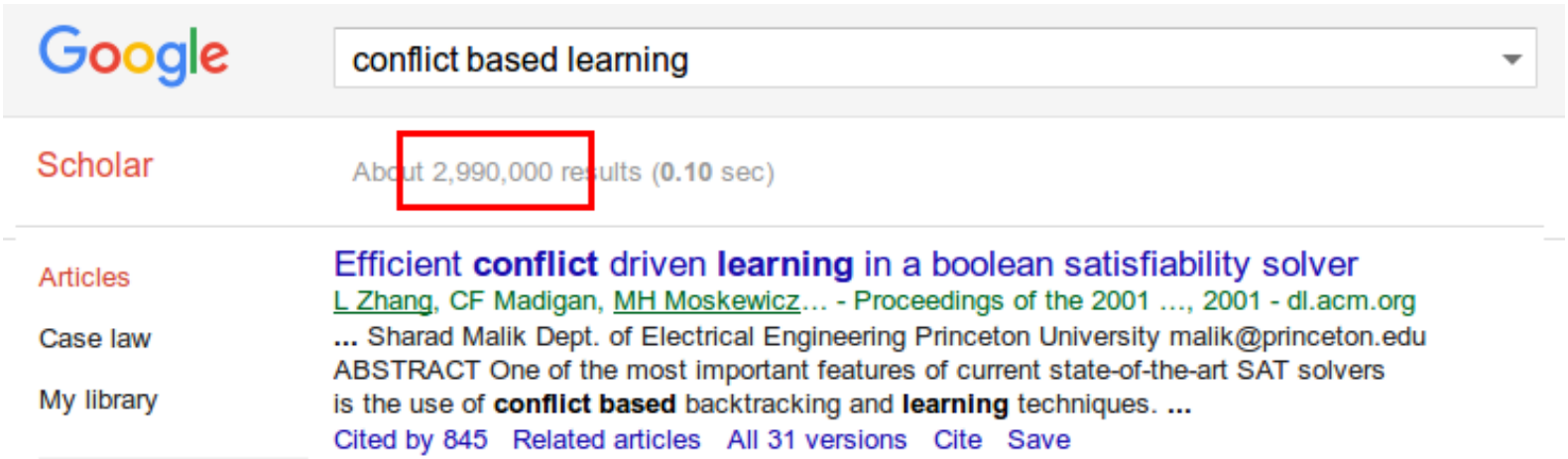

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- Conflict-based learning is ubiquitous in constraint reasoning.
- Planning/reachability checking: Limited to bounded-length reachability (which is a form of constraint reasoning, easily encoded into e. g. SAT).

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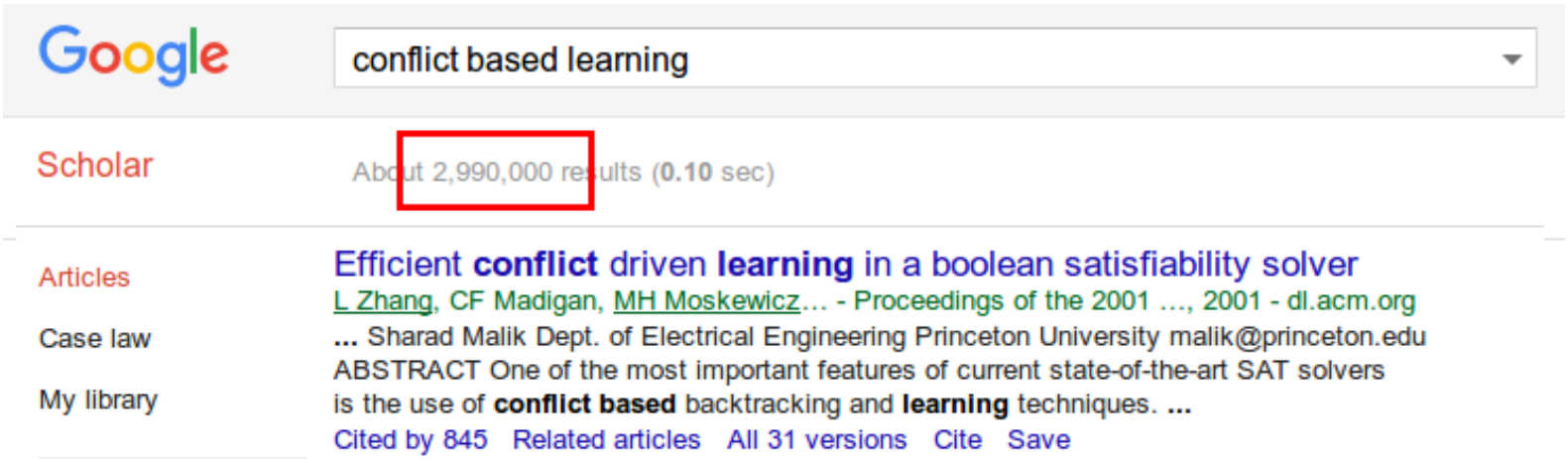

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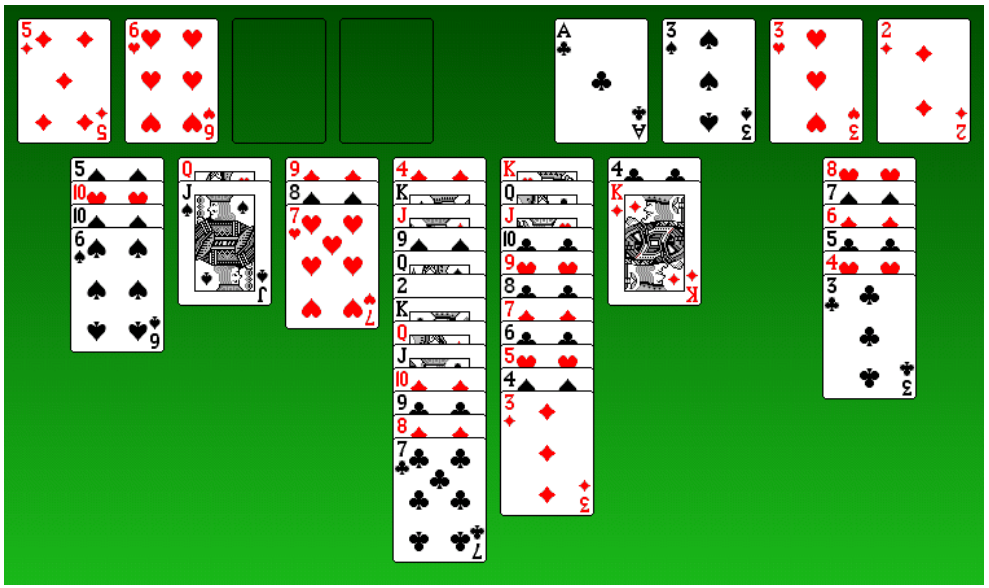
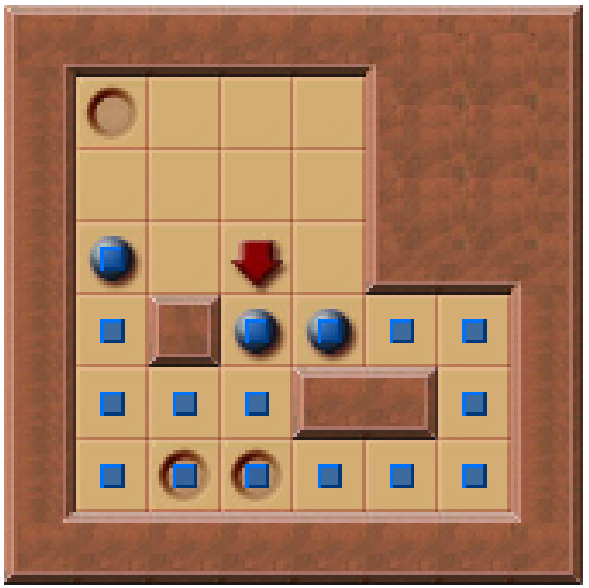
→ Can we learn from conflicts in unbounded-length state space search?

State Space Conflicts & Learning





→ Conflict = *dead-end* state from which the goal is unreachable.

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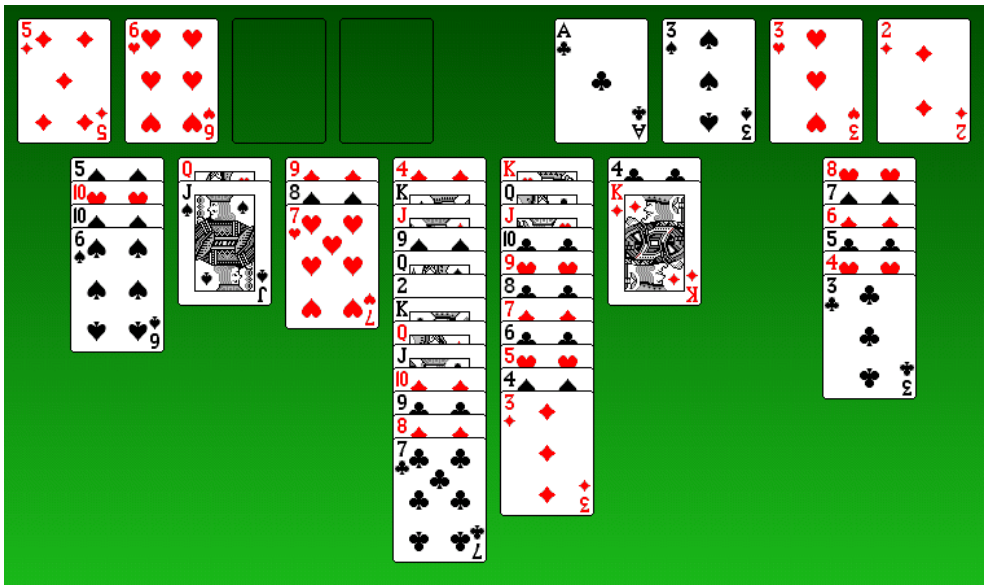

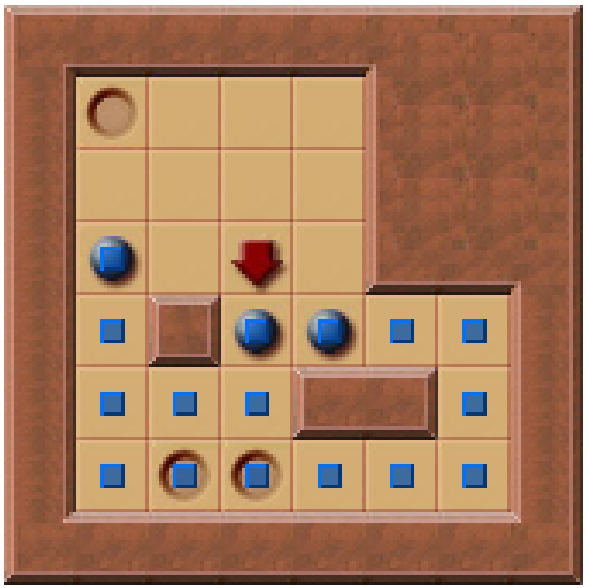

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- Run Tarjan's algorithm; learn upon backtracking out of SCC.
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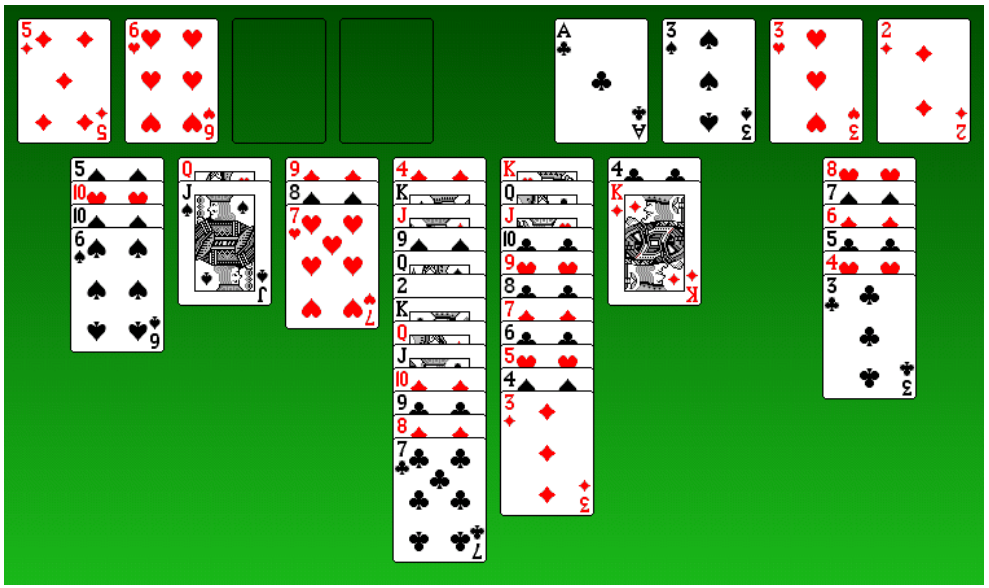
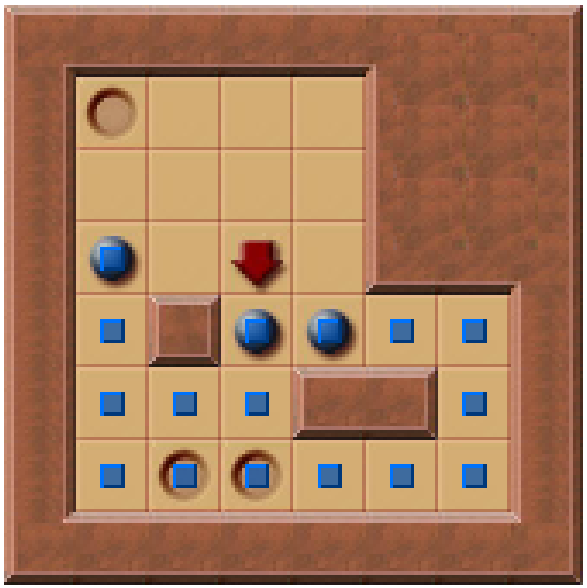
State Space Conflicts & Learning





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Idea:

- Run Tarjan's algorithm; learn upon backtracking out of SCC.
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- When learning on s, refine Δ so that it detects s.
- So far: Δ based on atomic conjunctions [Steinmetz and Hoffmann (2017b)], Δ based on traps [Lipovetzky *et al.* (2016); Steinmetz and Hoffmann (2017a)].

Search Space Reduction (on Resource-Constrained Planning)

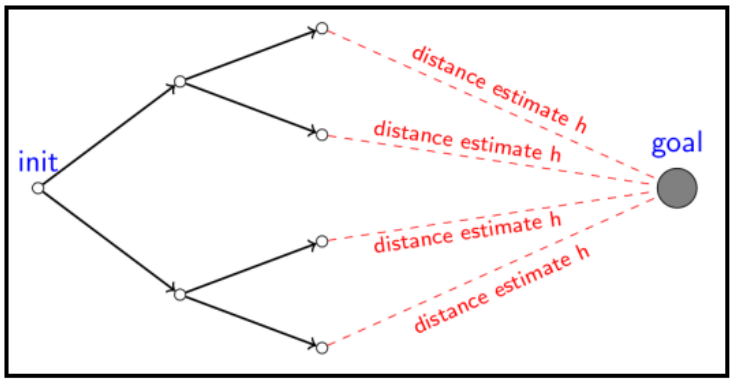
vs. merge-and-shrink abstraction Δ^{MS} [Hoffmann et al. (2014a)]:

	Number of Tasks Solved or Proved Unsolvable											
	Transport (30)				Rovers (30)				TPP (5)			
	h^{FF}	DFS		h^{FF}	h^{FF}	DFS		h^{FF}	h^{FF}	DFS		h^{FF}
W	(Δ^1)	Δ^1	Δ^C	Δ^{MS}	(Δ^1)	Δ^1	Δ^C	Δ^{MS}	(Δ^1)	Δ^1	Δ^C	Δ^{MS}
0.5	25	25	**30**	**30**	3	5	**30**	29	4	4	**5**	**5**
0.6	15	15	**30**	**30**	2	2	**30**	25	1	1	**5**	**5**
0.7	5	7	**29**	**29**	0	0	**30**	23	0	0	2	**3**
0.8	0	0	**26**	**26**	0	0	**29**	21	0	0	**1**	**1**
0.9	0	0	14	**24**	0	0	**24**	13	0	0	0	0
1.0	2	1	12	**20**	0	0	**23**	6	0	**1**	**1**	0
1.1	3	0	12	**20**	0	0	**23**	5	0	0	**2**	1
1.2	5	4	14	**21**	0	0	**18**	1	0	2	**3**	2
1.3	6	5	13	**23**	0	0	**20**	2	2	2	**5**	4
1.4	12	3	17	**24**	0	1	**19**	4	3	3	**5**	4
$\sum$	73	60	197	**247**	5	8	**246**	129	10	13	**29**	25

→ Search space reduction factors (min/geomean/max) for learning vs. no-learning, on unsolvable tasks: Transport 6.7/436.5/37561.5; Rovers 65.0/1286.6/69668.1; TPP 190.0/711.9/1900.5.

Agenda

Planning under a Strict Time Limit



VS.

→ Robots usually have little opportunity to "stand around and think".

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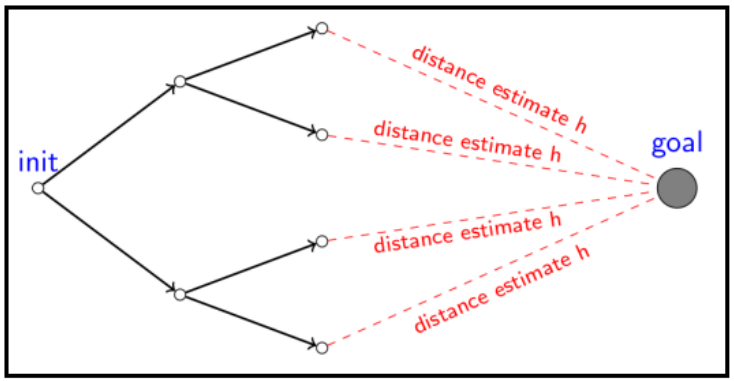

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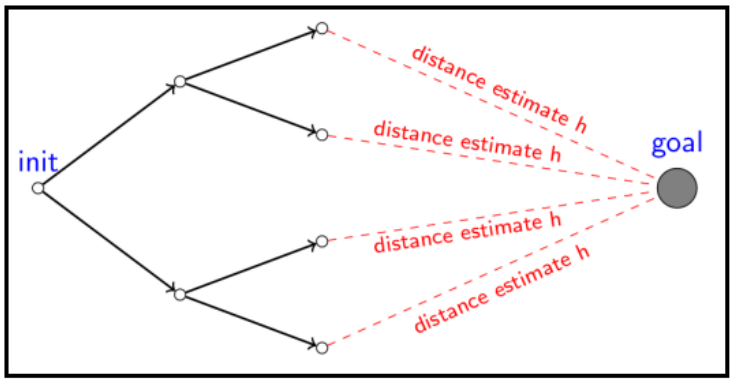

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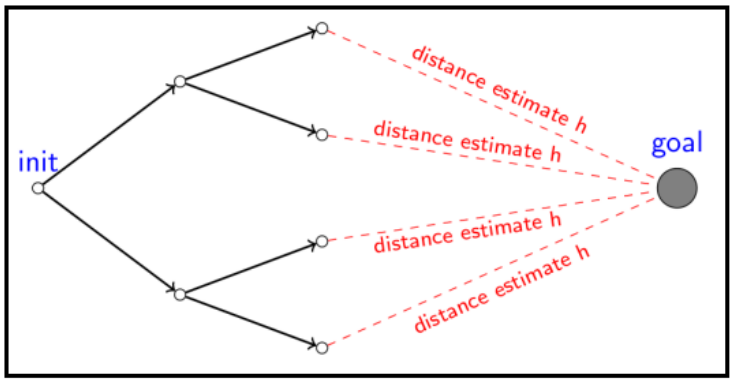

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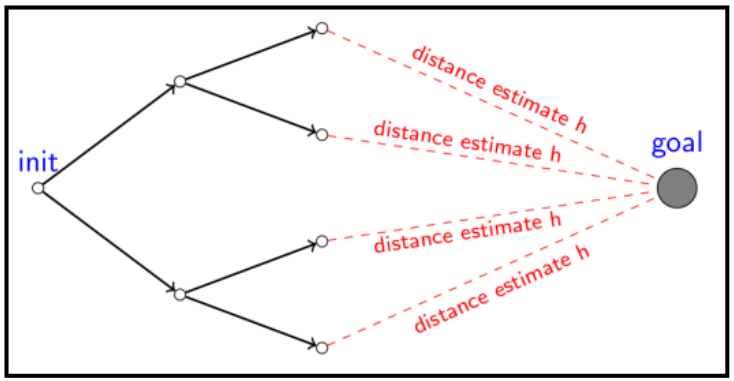

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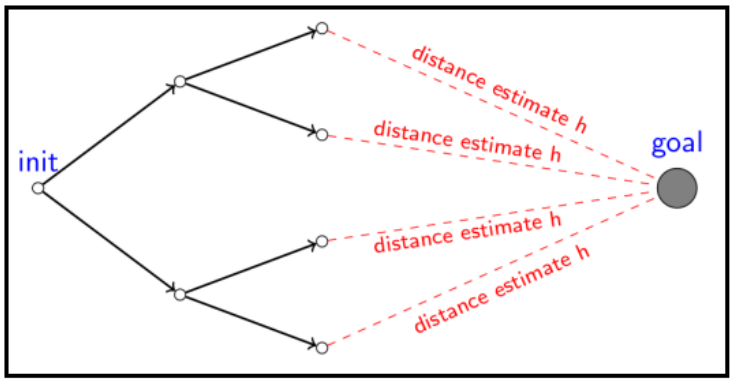

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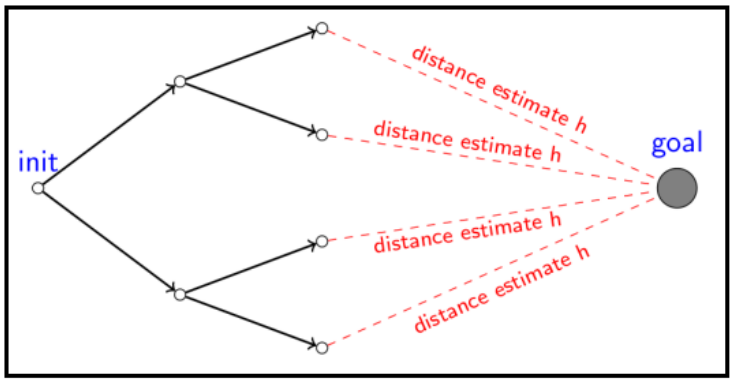

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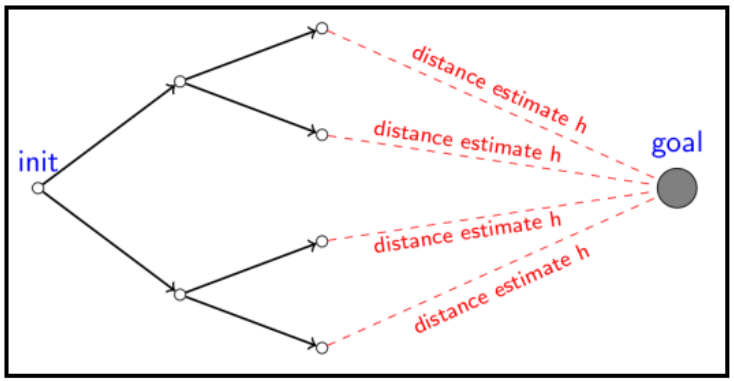

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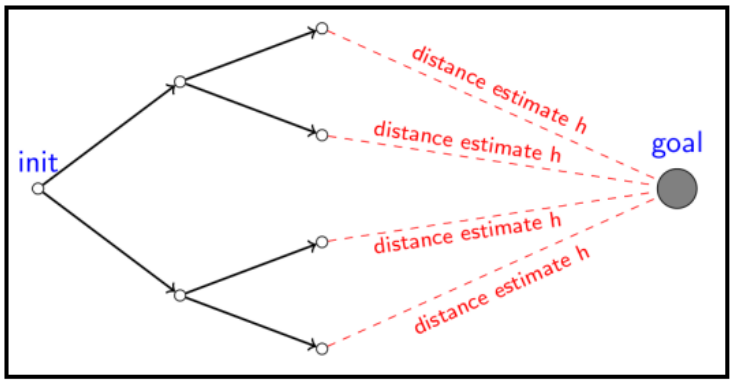
→ Preparation at design-time? E.g. synergies with (partial/approximate) Verification; transfer of ideas from route planning [e. g. Geisberger *et al.* (2012)].

Model Assessment/Certification



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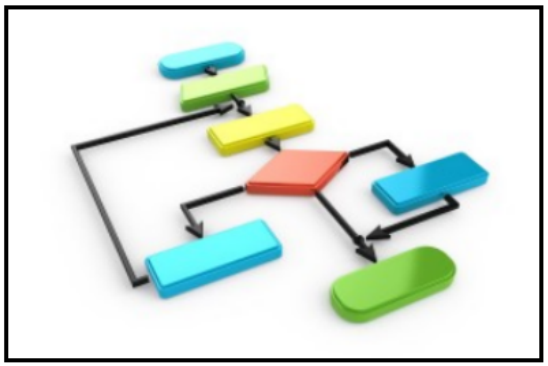

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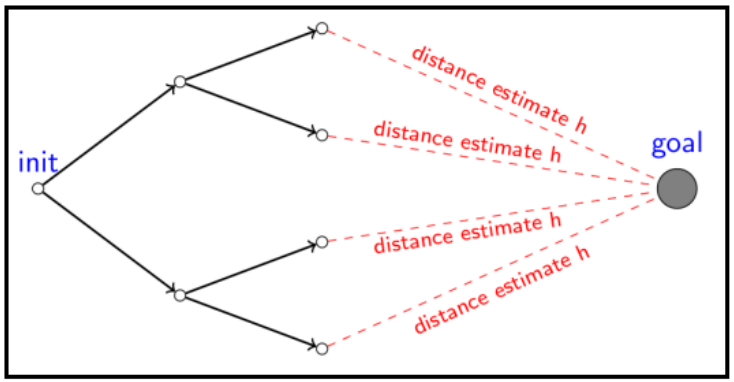

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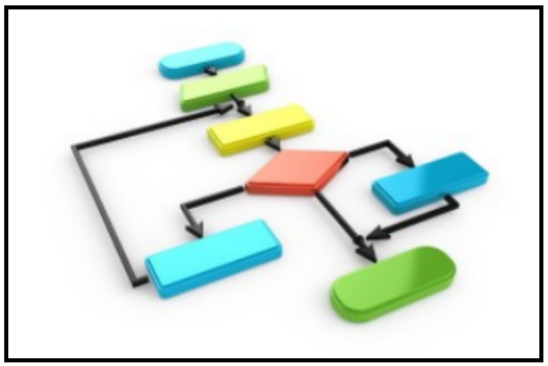

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- Support for modeling/model understanding in planning: knowledge engineering sub-community [e. g. Simpson *et al.* (2007); Vaquero *et al.* (2013)].
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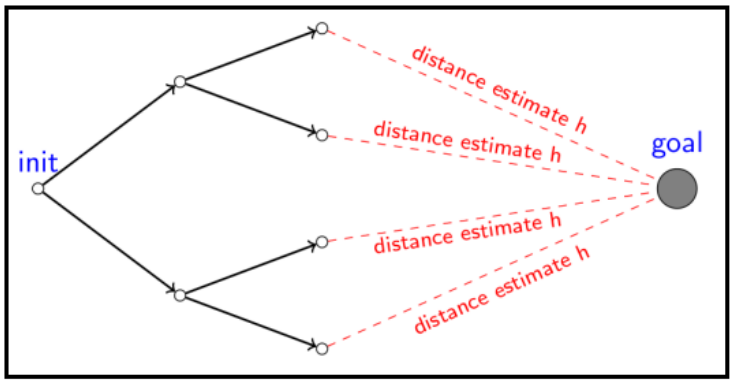

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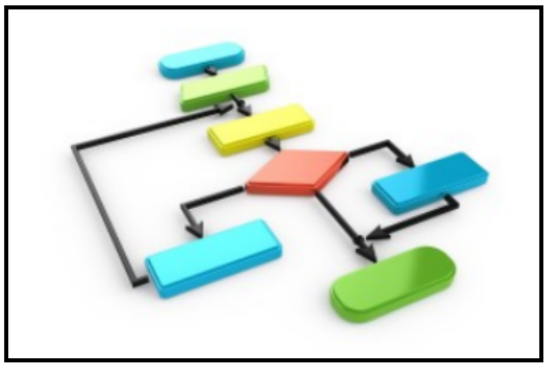

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- But this is chronically under-addressed and way more needs to be done.

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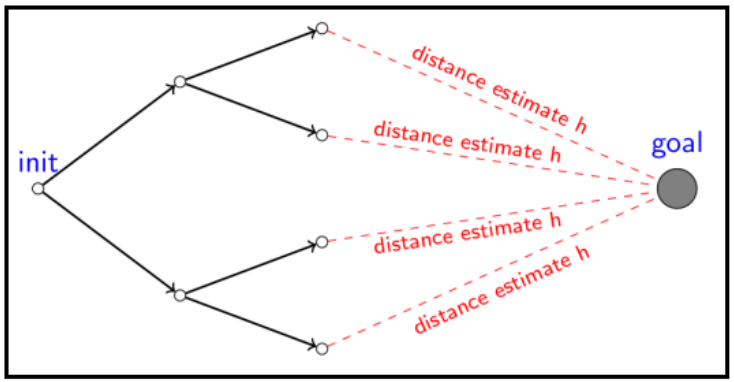

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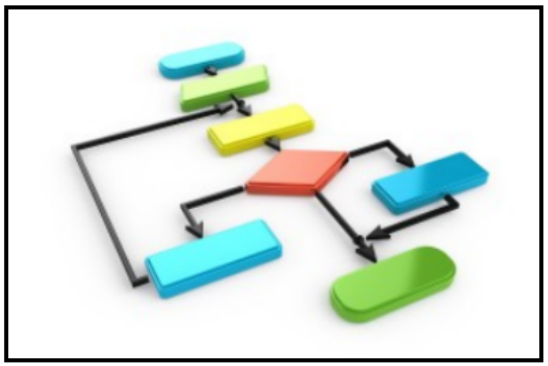
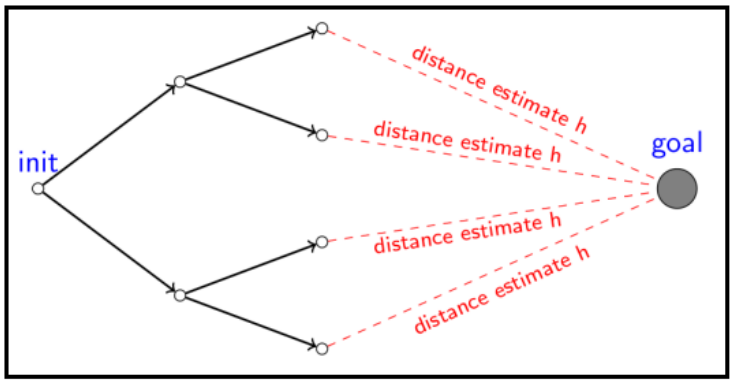


→ Explicit workflows are way easier to understand, assess, and trust than a complex planning machinery working on a general & flexible model.

- Support for modeling/model understanding in planning: knowledge engineering sub-community [e. g. Simpson *et al.* (2007); Vaquero *et al.* (2013)].
- Knowledge engineering competitions, see `http://www.icaps-conference.org/index.php/Main/Competitions`.
- But this is chronically under-addressed and way more needs to be done.

→ Talk to Tiago Vaquero here at the school!

Model Assessment/Certification



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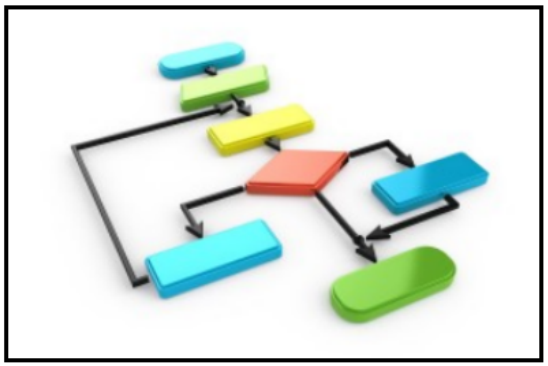

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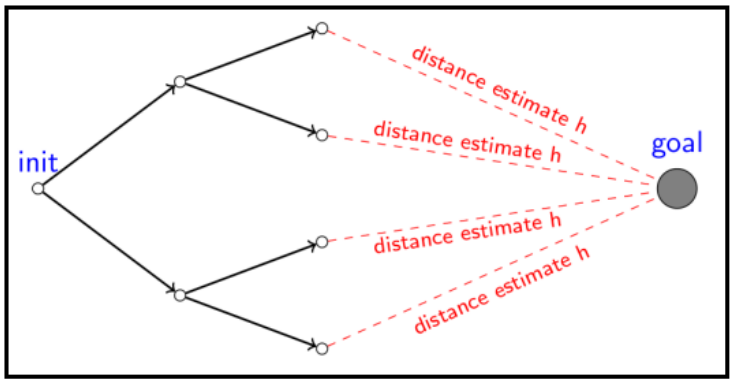
→ Verification to assure that there won't be any "fatal plans"? How will the planner react to rare situations? Systematic testing, validation, certification processes?

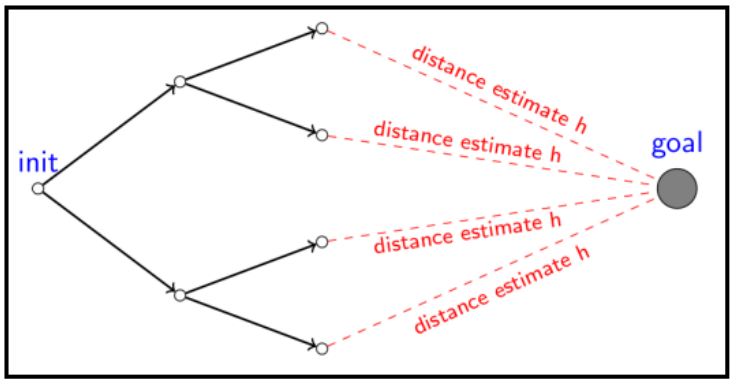
Explanation



VS.

→ If you want to interact with people, you need to be able to explain yourself.

Explanation



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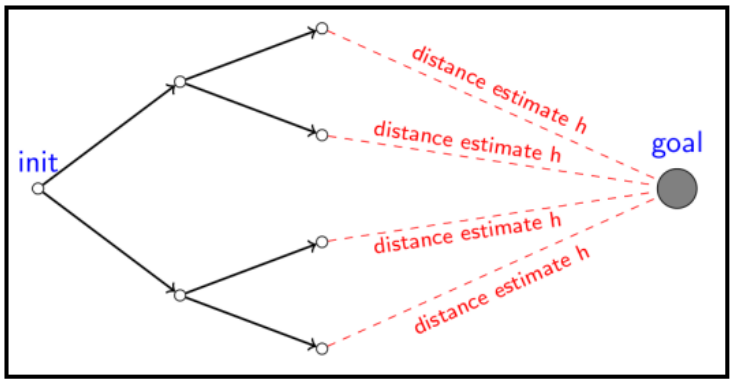

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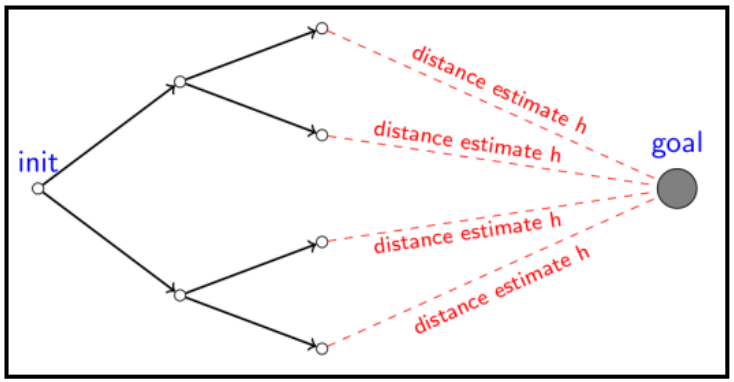

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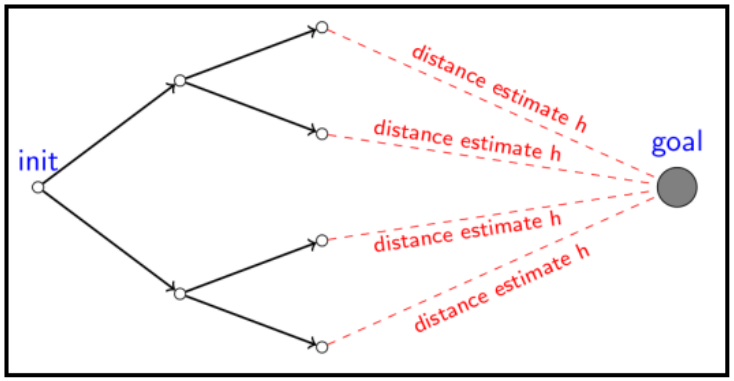
→ What about explaining the plan decision itself, "why this plan"? E.g. for simplicity, given state s and applicable actions A vs. B, why A and not B?

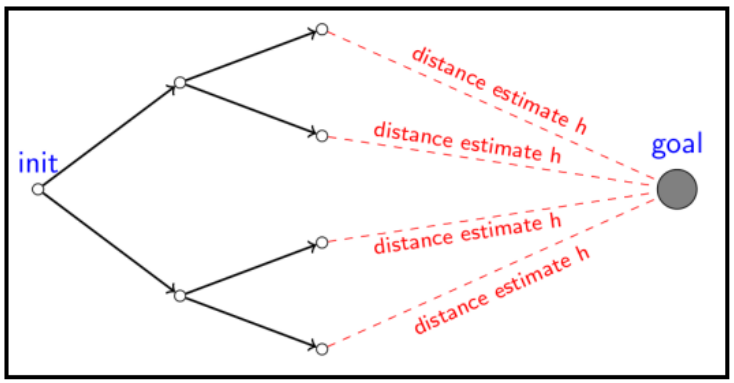
Deep Learning



VS.

→ Everybody's going crazy about it, so we must do it as well!

Deep Learning



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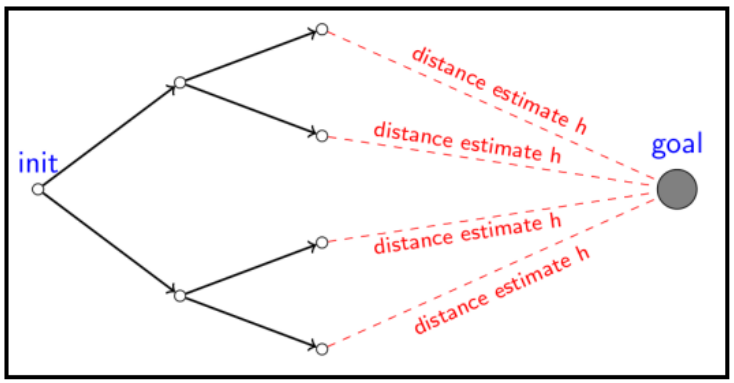
- AlphaGo definitely shows that "search + learning" can be key, and that NN can represent very informative search guidance.
- Same thing we've been doing in classical planning all along, yet using NN instead of model-based approximations for search guidance.

Deep Learning



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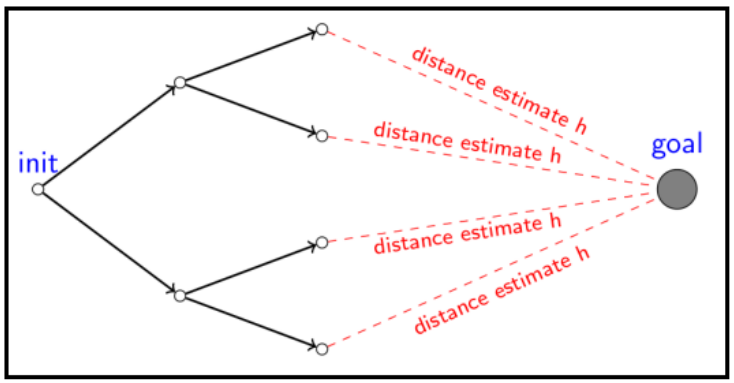

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→ Using models & planning to generate robot-NN training data in simulations? Using model-based simulation to gain more confidence in NN decisions?

Last Slide

Thanks for your attention. Questions?

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