Assignment 2 (PHYC90010)
To be handed in by the 27th of April (5pm)

Every evening, Andrew visits Reverend Bob's 24-Hour Chapel and Casino in Nevada, bearing x_0 = \$1000 in his wallet, the daily takings from his hydroponic tomato farm. He bets compulsively on every turn of the roulette wheel, once per minute, staking f = 5% of his outstanding capital at every turn and always playing red. He quits when he reaches x_w = \$2000 or sinks below the table minimum, x_m = \$10.

1. Research (theoretically) the odds for colour bets in American roulette. [2]
2. If Andrew tracks his fortune over the course of each evening and constructs a pdf $p(x,t)$ of his capital x at time t (where $t = 0$ corresponds to when he walks in the door), argue that the pdf obeys the Fokker-Planck equation:

$$\frac{\partial p(x,t)}{\partial t} = \frac{f(1-2q)}{\tau}\frac{\partial}{\partial x}\left[xp(x,t)\right] + \frac{f^2}{2\tau}\frac{\partial^2}{\partial x^2}\left[x^2 p(x,t)\right]$$

   where q = 9/19 and τ = 60s. [8]

3. Argue that the probability that Andrew is still sitting at the table after a time t is given by [4]:

$$G\left(x_0,t\right) = \int_{x_m}^{x_w} dx' p\left(x',t|x_0,0\right).$$

4. Show that, if $p(x,t)$ satisfies a homogeneous Fokker-Planck equation of the form

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[A(x)p(x,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\left(B(x)\right)^2 p(x,t)\right],$$

   then $G(x_0,t)$ satisfies the relation [8]

$$\frac{\partial G(x_0,t)}{\partial t} = A(x_0)\frac{\partial G(x_0,t)}{\partial x_0} + \frac{(B(x_0))^2}{2}\frac{\partial^2 G(x_0,t)}{\partial x_0^2}.$$

5. Show *on average*, Andrew sits at the table every evening for a time [5]

$$T(x_0) = \int_0^\infty dt G(x_0,t),$$

   and that $T(x_0)$ satisfies [5]

$$-1 = A(x_0)\frac{dT(x_0)}{dx_0} + \frac{(B(x_0))^2}{2}\frac{d^2T(x_0)}{dx_0^2}$$

with the boundary condition $T(x_m) = T(x_w) = 0$.

6. Evaluate $T(x_0)$ both numerically [by simulating Andrew attending the Casino N times and numerically evaluating the average time Andrew spends at the table] and analytically, expressing your answer in minutes.[8]

Total Marks 40