Assignment 4 (PHYC90010)
To be handed in by the 31st of May (5pm)

The dynamics of a spruce budworm (caterpillar) population is often modelled by the following equation

$$\frac{\partial u}{\partial t} = \left[ru\left(1 - \frac{u}{k}\right)\right] - \frac{u^2}{1+u^2} + D\nabla^2 u$$

where *u* describes the number of budworms per unit area of the forest. The first term, in big square brackets, on the right-hand-side describes the dynamics of the local density of budworms without predators, with *r* being the birth rate and *k*, the capacitor, reflects the competition, between the budworms, for food resources. The second term describes the effect of predation (mostly birds), which saturates at large numbers of budworms (birds can only eat so much). The last term represents the dispersal of budworms (in the butterfly stage - caterpillars cannot move from tree to tree).

(a) Show that there is a trivial uniform unstable stationary solution for $u=0$. [3]

(b) Elucidate why, depending on the values of *k* and *r*, there may be either 1 or 3 additional uniform stationary solutions. [8]

(c) Find the regimes for *k* and *r* where there is only one additional uniform stationary solution. [10]

(d) In the presence of diffusion does the linear stability of the $u=0$ equilibrium state change (assuming an infinite forest)? [6]

(e) Suppose that the forest is finite and, for simplicity, is a square with an area $L\times L$.

i. Argue why the boundary conditions $u(0,y) = u(L,y) = u(x,0) = u(x,L) = 0$ are appropriate. [3]

ii. Why is this boundary condition only compatible with the uniform state $u = 0$? [2]

iii. Given that this state is unstable determine the minimum size, *L*, of the system that can support a nonzero population. [8]

Total Marks 40