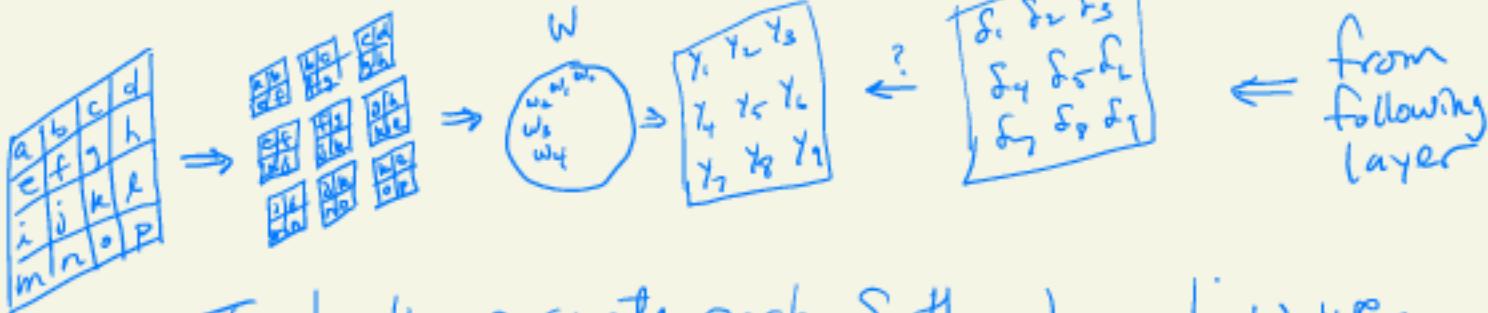
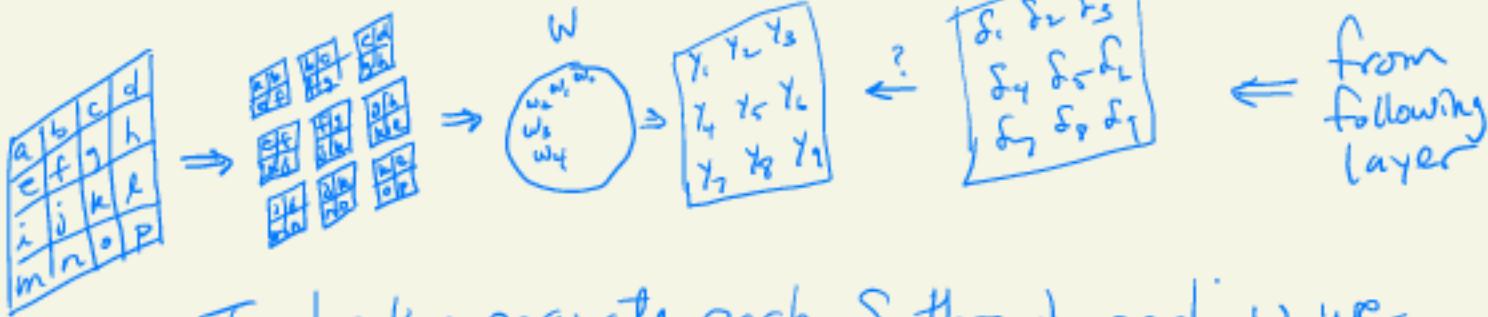


Back propagation
through a Convolutional
Layer



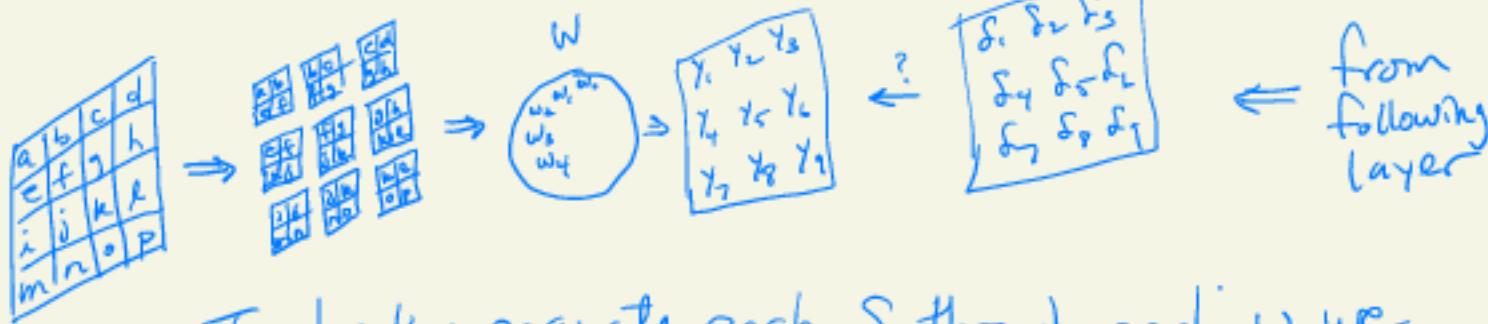
To back-propagate each δ through each w , we need to know which δ s are dependent on each input a, b, \dots, p .



To back-propagate each s through each w , we need to know which s 's are dependent on each input a, b, \dots, p .

$$\begin{array}{c}
 \begin{matrix} a \\ b \\ e \\ f \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_1 \\
 \begin{matrix} c \\ fg \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_2 \\
 \begin{matrix} cd \\ gh \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_3 \\
 \begin{matrix} cf \\ ij \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_4 \\
 \begin{matrix} fg \\ jk \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_5 \\
 \begin{matrix} gh \\ kl \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_6 \\
 \begin{matrix} ij \\ mn \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_7 \\
 \begin{matrix} kl \\ op \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_8 \\
 \begin{matrix} kl \\ op \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_9
 \end{array}$$

Start with input a . Where does it appear?

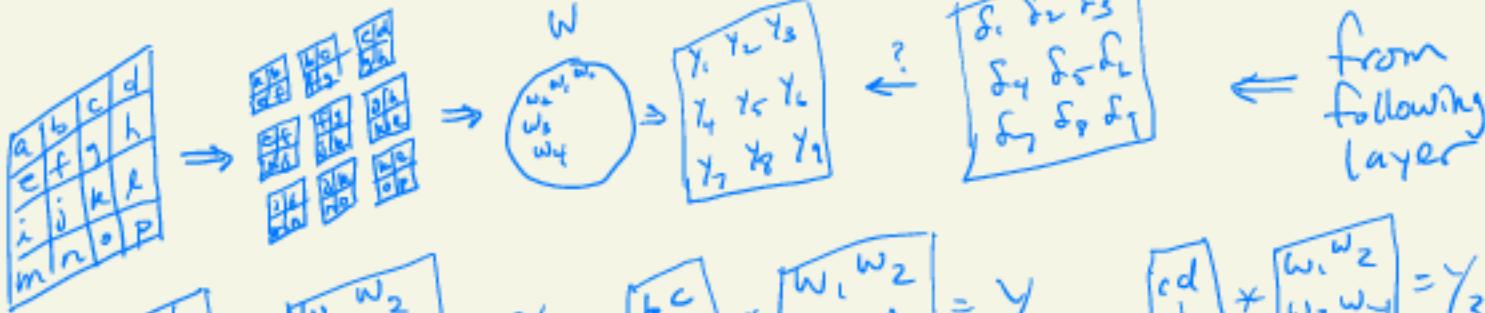


To back-propagate each δ through each w , we need to know which δ s are dependent on each input a, b, \dots

$$\begin{array}{c}
 \boxed{ab} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_1 \\
 \boxed{ef} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_4 \\
 \boxed{ij} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_7 \\
 \hline
 \boxed{bc} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_2 \\
 \boxed{fg} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_5 \\
 \boxed{ik} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_8 \\
 \hline
 \boxed{gh} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_3 \\
 \boxed{hl} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_6 \\
 \boxed{op} * \boxed{\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix}} = y_9
 \end{array}$$

Start with input a . Where does it appear?

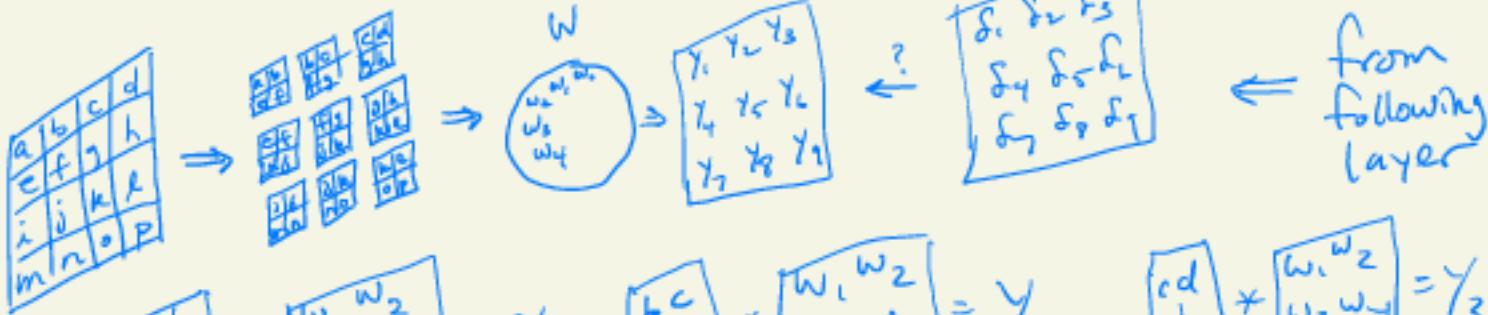
Can we construct a matrix operation that will multiply only a and w_1 ?



$$\begin{array}{l}
 \begin{array}{c}
 \begin{array}{|c|c|} \hline a & b \\ \hline e & f \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_1 \\
 \begin{array}{|c|c|} \hline c & d \\ \hline g & h \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_2 \\
 \begin{array}{|c|c|} \hline i & j \\ \hline o & p \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_3
 \end{array} \\
 \begin{array}{c}
 \begin{array}{|c|c|} \hline a & b \\ \hline e & f \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_4 \\
 \begin{array}{|c|c|} \hline c & d \\ \hline g & h \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_5 \\
 \begin{array}{|c|c|} \hline i & j \\ \hline o & p \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_6
 \end{array} \\
 \begin{array}{c}
 \begin{array}{|c|c|} \hline k & l \\ \hline n & o \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_7 \\
 \begin{array}{|c|c|} \hline m & n \\ \hline r & s \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_8 \\
 \begin{array}{|c|c|} \hline k & l \\ \hline o & p \\ \hline \end{array} + \begin{array}{|c|c|} \hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} = y_9
 \end{array}
 \end{array}$$

W "flipped"

$$\begin{array}{c}
 \begin{array}{|c|c|} \hline w_3 & w_4 \\ \hline w_2 & w_1 \\ \hline \end{array} * \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & s_1 \\ \hline 0 & s_2 \\ \hline 0 & s_3 \\ \hline 0 & s_4 \\ \hline 0 & s_5 \\ \hline 0 & s_6 \\ \hline 0 & s_7 \\ \hline 0 & s_8 \\ \hline 0 & s_9 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} = 0
 \end{array}$$



$$\begin{matrix} a \\ b \\ e \\ f \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_1$$

$$\begin{matrix} b \\ c \\ f \\ g \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_2$$

$$\begin{matrix} c \\ d \\ g \\ h \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_3$$

$$\begin{matrix} c \\ f \\ i \\ j \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_4$$

$$\begin{matrix} f \\ g \\ i \\ k \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_5$$

$$\begin{matrix} g \\ h \\ k \\ l \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_6$$

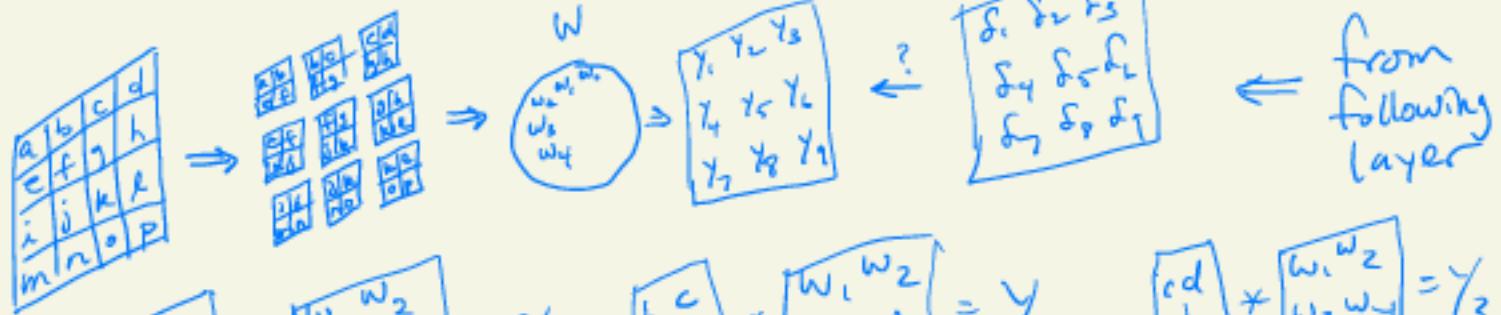
$$\begin{matrix} i \\ j \\ m \\ n \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_7$$

$$\begin{matrix} i \\ j \\ k \\ l \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_8$$

$$\begin{matrix} k \\ l \\ o \\ p \end{matrix} * \begin{matrix} w_1, w_2 \\ w_3, w_4 \end{matrix} = y_9$$

$$\begin{matrix} w_3 & w_4 \\ w_2 & w_1 \end{matrix} * \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & s_1 & s_2 \\ 0 & s_4 & s_5 & s_6 \\ 0 & s_7 & s_8 & s_9 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Will this idea work for input b?



$$\begin{bmatrix} ab \\ ef \\ fg \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_1$$

$$\begin{bmatrix} c \\ fg \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_2$$

$$\begin{bmatrix} cd \\ gh \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_3$$

$$\begin{bmatrix} ef \\ ij \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_4$$

$$\begin{bmatrix} fg \\ ik \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_5$$

$$\begin{bmatrix} gh \\ kl \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_6$$

$$\begin{bmatrix} ij \\ mn \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_7$$

$$\begin{bmatrix} kl \\ op \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_8$$

$$\begin{bmatrix} kl \\ op \end{bmatrix} * \begin{bmatrix} w_1, w_2 \\ w_3, w_4 \end{bmatrix} = y_9$$

$$a \leftarrow w_1, \delta_1$$

$$b \leftarrow w_1 \delta_2 + w_2 \delta_1$$

$$c \leftarrow w_1 \delta_3 + w_2 \delta_2$$

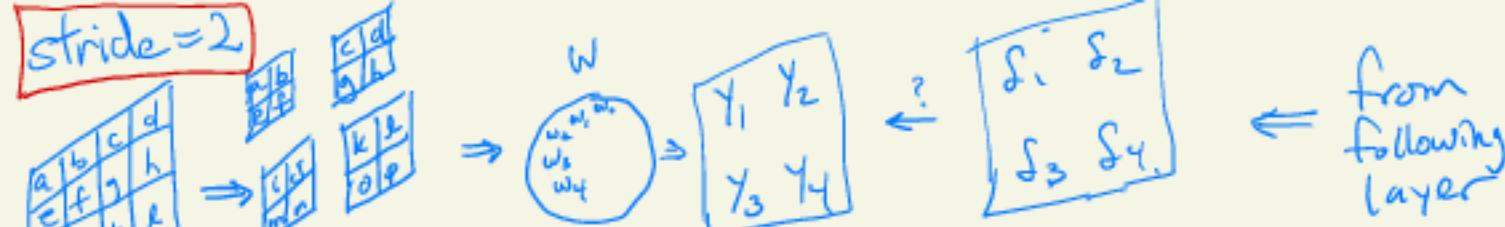
$$d \leftarrow w_2 \delta_3$$

$$\begin{bmatrix} w_3 & w_4 \\ w_2 & w_1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_1 & \delta_2 & \delta_3 \\ 0 & \delta_4 & \delta_5 & \delta_6 \\ 0 & \delta_7 & \delta_8 & \delta_9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_1 \delta_2 & \delta_3 & 0 \\ 0 & \delta_2 \delta_3 & \delta_4 & 0 \\ 0 & \delta_3 \delta_4 & \delta_5 & 0 \\ 0 & \delta_4 \delta_5 & \delta_6 & 0 \\ 0 & \delta_5 \delta_6 & \delta_7 & 0 \\ 0 & \delta_6 \delta_7 & \delta_8 & 0 \\ 0 & \delta_7 \delta_8 & \delta_9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_1 \delta_2 \delta_3 & 0 & 0 \\ 0 & \delta_2 \delta_3 \delta_4 & 0 & 0 \\ 0 & \delta_3 \delta_4 \delta_5 & 0 & 0 \\ 0 & \delta_4 \delta_5 \delta_6 & 0 & 0 \\ 0 & \delta_5 \delta_6 \delta_7 & 0 & 0 \\ 0 & \delta_6 \delta_7 \delta_8 & 0 & 0 \\ 0 & \delta_7 \delta_8 \delta_9 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_1 \delta_2 \delta_3 & 0 & 0 \\ 0 & \delta_2 \delta_3 \delta_4 & 0 & 0 \\ 0 & \delta_3 \delta_4 \delta_5 & 0 & 0 \\ 0 & \delta_4 \delta_5 \delta_6 & 0 & 0 \\ 0 & \delta_5 \delta_6 \delta_7 & 0 & 0 \\ 0 & \delta_6 \delta_7 \delta_8 & 0 & 0 \\ 0 & \delta_7 \delta_8 \delta_9 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$\begin{bmatrix} a & b \\ e & f \end{bmatrix} * \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} = Y_1$$

$$\begin{bmatrix} c & d \\ g & h \end{bmatrix} * \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} = Y_2$$

$$\begin{bmatrix} i & j \\ m & n \end{bmatrix} * \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} = Y_3$$

$$\begin{bmatrix} k & l \\ o & p \end{bmatrix} * \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} = Y_4$$

$$a \leftarrow w_1 s_1$$

$$\begin{bmatrix} w_3 & w_4 \\ w_2 & w_1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & s_2 \\ 0 & 0 & 0 & 0 \\ 0 & s_3 & 0 & s_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b \leftarrow w_2 s_1$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & s_2 \\ 0 & 0 & 0 & 0 \\ 0 & s_3 & 0 & s_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c \leftarrow w_1 s_2$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & s_2 \\ 0 & 0 & 0 & 0 \\ 0 & s_3 & 0 & s_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d \leftarrow w_2 s_2$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & s_2 \\ 0 & 0 & 0 & 0 \\ 0 & s_3 & 0 & s_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$