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A Sunlike Star From Birth to Death

A simple simulation on Python

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ABSTRACT

Context. A course of study on star formation and life-cycle, which delved into the physics governing the properties of a star
Aims. To simulate a sun-like star in python and plot the variation of its properties
Methods. By solving the differential equations for mass, density, pressure, and luminosity variations, along with the temperature gradients for convective and radiative transfer, and the equation of state
Results. The plots describing the variations of the star's properties and what they mean for the life and death of the star

Key words. star formation – stellar evolution

1. Introduction

1.1. Structure of a star

A star can be modeled as layers of thin spherical shells, assuming that the properties of the material throughout each shell are constant. These properties are described by 6 equations which are discussed in detail in section 2.

At the core of the star is a hydrogen-burning furnace. The heat and pressure here are so high that it can fuse hydrogen into helium, thus sustaining the star for billions of years. The fusion creates an outward pressure that balances gravity and keeps the star in equilibrium.

Beyond the core, there are zones of convective and radiative transfer. In smaller stars, convective transfer dominates, and there may be radiative transfer nearer to the core. For the sun, there is a small outer layer of convective transfer, and the remaining is radiative. Radiative transfer is dominant in stars bigger than the sun, and those with 2 or more times the mass of the sun will have convective transfer nearer to the core.

1.2. Star formation

Stars are formed from interstellar gas clouds. Bubbles of higher density arise within each cloud. If these bubbles are small, they will simply dissipate. If the bubbles are large enough, the gravitational potential energy of an average atom will exceed its kinetic energy, thus allowing the bubble to collapse into a protostar with a very hot and dense center.

An envelope of gas surrounds the protostar. The protostar keeps pulling gas from its surroundings, increasing its mass and temperature. Once the temperature and pressure become high enough, the core begins to fuse hydrogen into helium. The kinetic energy released from this fusion creates an outward pressure that counters the gravitational force. After some time, the force from this outward pressure becomes equal and opposite to the gravitational force, and the star reaches hydrostatic equilibrium, becoming stable.

1.3. Life and death of a single star

Once stable, a star will maintain its size, mass, and luminosity for as long as fusion occurs in its core. At this stage, the star is called a main sequence star. It is an uneventful phase. Various changes occur once the star runs out of hydrogen, and thus fusion stops.

Once fusion stops, there is no longer enough outward pressure to balance gravity, causing the core of the star to start shrinking. However, even though fusion stops at the core, some fusion takes place in the shells close to the core. The energy generated by this shell burning causes the outer layers to expand and cool, while the core continues to shrink and heat up. The star at this stage is called a red giant.

The outer layers will eventually be ejected away from the star, creating a planetary nebula. Only the hot, dense core of the star remains, which is very small and is called a white dwarf. This is the final stage of a star, and it slowly cools down and becomes dimmer.

1.4. Sunlike stars

Sunlike stars are dominated by radiative transfer, with only a thin outer layer of convective transfer. They stay stable in the main sequence for about 10 billion years. Our sun is 4.5 billion years old.

During the red giant phase, the core of these stars reaches a temperature high enough to start a sudden burst of helium fusion known as a helium flash. This produces carbon and oxygen and stabilizes the core. The star continues as a red giant, eventually creating a planetary nebula and then finishing as a white dwarf.

1.5. Massive stars

Stars that are more than about 12 times the mass of the sun will lead a much more interesting life than sunlike stars. They stay in the main sequence for a much shorter time, only millions of years. This is because of the higher temperature and pressure inside their cores, leading to a higher rate of fusion.

The high temperatures and pressures allow the star to burn heavier elements, from carbon and oxygen, to neon, to magnesium, to silicon, and finally, iron. Iron cannot be fused further, thus no further energy is produced at the core. This causes the core to massively collapse in on itself within seconds. The result is a supernova explosion which releases all these elements into space. The remnant of the supernova will be a neutron star or a black hole, depending on the mass of the original star.

2. Modeling method

2.1. Equations of stellar structure

In order to understand the internal structure and evolution of stars, we utilize computer programs that simulate their behavior based on fundamental physical principles. A computer program that is designed to calculate the properties of a stellar model. It breaks down a star into thin spherical shells and computes the properties within each shell.

To calculate these values, we need differential equations for four fundamental properties — Mass, Pressure, Temperature, and Luminosity of a star. All as a function of the radius from the center.

For Mass Distribution:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \tag{1}$$

Where:
M_r: Mass interior to the shell
r: Radius
ρ: Density

For Maintaining Hydrostatic Equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\rho(r)g(r) \tag{2}$$

Where:
P: Pressure
r: Radius
g: Gravity
ρ: Density

For Luminosity Distribution:

$$\frac{dL}{dr} = 4\pi r^2 \rho\epsilon \tag{3}$$

Where:
L: Luminosity
r: Radius
ρ: Density
ϵ: Energy generation function

And one of the following, depending on whether energy is **transported by radiation**:

$$\frac{dT}{dr} = -\frac{3\kappa\rho L_r}{64\pi\sigma T^3 r^2} \tag{4}$$

Where:
T: Temperature
r: Radius
L_r: Luminosity
κ: Opacity
σ: Stefan-Boltzmann constant
ρ: Density

or **Convection:**

$$\frac{dT}{dr} = -\frac{m_{\text{avg}}}{k}\left(1 - \frac{1}{\gamma}\right)\frac{GM_r}{r^2} \tag{5}$$

Where:
T: Temperature
r: Radius
m_{avg}: Average particle mass
k: Boltzmann constant
γ: Ratio of specific heats
G: Gravitational Constant
M_r: Mass interior to the shell

When we build a stellar model, we break the star up into thin little spherical shells and assume that the properties are constant throughout each shell. Suppose that we have run our model successfully up to shell number n, and so we know the values in this shell for,

- M_r: mass interior to this shell
- P: pressure in this shell
- L_r: luminosity interior to this shell
- T: temperature of this shell

Now, these aren't the only quantities which affect the nature of the material in this shell. There are a bunch of others, such as

- ρ: density
- κ: opacity
- ϵ: energy generation rate

To determine the values of these other quantities, we need some "equations of state" which we get from the Ideal gas Law. To determine the values of these other quantities, we need some "equations of state" which we get from the Ideal gas Law.

$$\rho = \frac{m}{k}\frac{P}{T} \tag{6}$$

2.2. Solving differential equations

Modeling requires an iterative process of adjusting input parameters and examining the results to ensure they are physically meaningful. The properties of each shell, such as mass, pressure, temperature, density, opacity, and energy generation rate, are determined using equations of state and other relationships that describe the behavior of stellar materials.

Once we know all the relevant properties of the gas in shell number (**n**), then we can apply the differential equations to compute the properties in the next shell number (**n+1**). For example, to compute the mass interior to shell **50**, we could use the very simple Euler's method,

$$M_{(r,\mathbf{50})} = M_{(r,\mathbf{49})} + \left.\frac{dM_r}{dr}\right|_{\mathbf{49}} \tag{7}$$

However, a technique such as Euler's method is a simple forward approximation to estimate the next value of the dependent variable. More advanced methods like the "fourth-order Runge-Kutta" method are diversely used to achieve the desired level of accuracy and stability in the calculations.

While Euler's method uses a simple forward approximation to estimate the next value of the dependent variable, Runge-Kutta methods employ multiple intermediate steps and a weighted average to refine this estimation. By incorporating these additional calculations, Runge-Kutta methods can better capture the behavior of the solution curve. The method simply follows,

$$k_1 = hf(x_n, y_n), \tag{8}$$

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}), \tag{9}$$

$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}), \tag{10}$$

$$k_4 = hf(x_n + h, y_n + k_3), \tag{11}$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{12}$$

The main loop of the program iterates through the shells of the star, computing the properties of each shell based on the properties of the previous shell and the differential equations derived for the stellar structure.

The process continues until certain ending conditions are met, at which point the program outputs a set of diagnostic messages and writes the properties of the calculated stellar model to a file. The program may not always produce viable models, and careful examination and modification of the inputs may be necessary to obtain sensible results.

Table 1. First 15 rows of data of Stellar Parameters from the generated table.

r	M	L_r	T	P	ρ	κ
1. 35E+09	1.00E+00	2.53E+32	1.40E+07	1.45E+17	7.72E+01	1.40E+00
2. 06E+09	9.99E-01	2.83E+32	1.39E+07	1.43E+17	7.66E+01	1.42E+00
3. 77E+09	9.97E-01	3.40E+32	1.38E+07	1.40E+17	7.57E+01	1.44E+00
4. 48E+09	9.93E-01	4.28E+32	1.37E+07	1.36E+17	7.45E+01	1.46E+00
5. 19E+09	9.88E-01	5.51E+32	1.35E+07	1.32E+17	7.31E+01	1.49E+00
6. 90E+09	9.82E-01	7.06E+32	1.33E+07	1.27E+17	7.15E+01	1.53E+00
7. 61E+09	9.73E-01	8.90E+32	1.31E+07	1.22E+17	6.97E+01	1.58E+00
8. 32E+09	9.62E-01	1.10E+33	1.28E+07	1.16E+17	6.77E+01	1.63E+00
9. 03E+09	9.49E-01	1.32E+33	1.25E+07	1.10E+17	6.55E+01	1.70E+00
10. 75E+09	9.33E-01	1.54E+33	1.22E+07	1.03E+17	6.30E+01	1.76E+00
11. 46E+09	9.15E-01	1.77E+33	1.19E+07	9.65E+16	6.04E+01	1.83E+00
12. 17E+09	8.94E-01	1.99E+33	1.16E+07	8.96E+16	5.76E+01	1.91E+00
13. 88E+09	8.71E-01	2.19E+33	1.13E+07	8.28E+16	5.46E+01	1.99E+00
14. 06E+10	8.46E-01	2.38E+33	1.10E+07	7.60E+16	5.16E+01	2.07E+00
15. 13E+10	8.19E-01	2.55E+33	1.07E+07	6.95E+16	4.86E+01	2.16E+00

Table 2. Last 15 rows of data of Stellar Parameters from the generated table.

r	M	L_r	T	P	ρ	κ
410. 01E+10	2.58E-09	3.29E+33	4.55E+04	2.16E+06	3.53E-07	6.60E+01
411. 01E+10	1.90E-09	3.29E+33	4.24E+04	1.56E+06	2.74E-07	6.89E+01
412. 02E+10	1.36E-09	3.29E+33	3.93E+04	1.10E+06	2.09E-07	7.21E+01
413. 03E+10	9.63E-10	3.29E+33	3.62E+04	7.58E+05	1.55E-07	7.57E+01
414. 03E+10	6.68E-10	3.29E+33	3.32E+04	5.04E+05	1.13E-07	7.99E+01
415. 04E+10	4.17E-10	3.29E+33	2.98E+04	3.07E+05	7.64E-08	8.48E+01
416. 05E+10	2.47E-10	3.29E+33	2.65E+04	1.76E+05	4.94E-08	9.06E+01
417. 06E+10	1.37E-10	3.29E+33	2.31E+04	9.41E+04	3.02E-08	9.75E+01
418. 06E+10	6.92E-11	3.29E+33	1.98E+04	4.55E+04	1.70E-08	1.06E+02
419. 07E+10	3.11E-11	3.29E+33	1.65E+04	1.92E+04	8.61E-09	1.17E+02
420. 08E+10	1.18E-11	3.29E+33	1.32E+04	6.64E+03	3.72E-09	1.30E+02
421. 08E+10	3.40E-12	3.29E+33	9.88E+03	1.66E+03	1.24E-09	1.48E+02
422. 09E+10	6.08E-13	3.29E+33	6.58E+03	2.26E+02	2.52E-10	1.72E+02
423. 10E+10	3.94E-14	3.29E+33	3.29E+03	7.97E+00	1.75E-11	2.30E+02
424. 11E+10	0.00E+00	3.29E+33	0.00E+00	0.00E+00	0.00E+00	0.00E+00

3. Our star

We set starting parameters in our computer program, such as the star's surface and core temperatures, to begin the simulation. These circumstances usually include necessary elements such as temperature, pressure, and density, which act as the foundation for our computations.

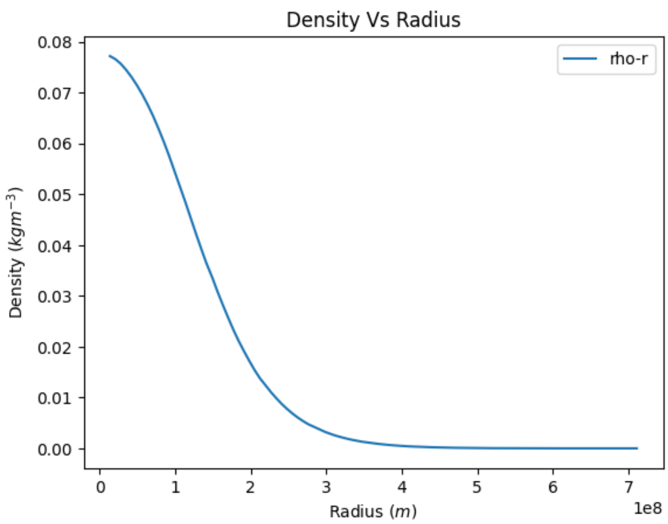
Then comes the part of the Integration. The Runge-Kutta method of numerical integration forms the basis of the procedure. In order to do this, the differential equation system that controls stellar behavior must be methodically solved. The technique modifies a number of state variables, including temperature, mass, and brightness, as we move through each phase.

Through the iterative application of the Runge-Kutta method, we are able to accurately mimic the star's dynamic evolution throughout time. We finally analyze and visualize the variations of key stellar parameters like mass, density, temperature, pressure, luminosity, opacity a function of radius throughout the star's life cycle.

3.1. Surface and central conditions

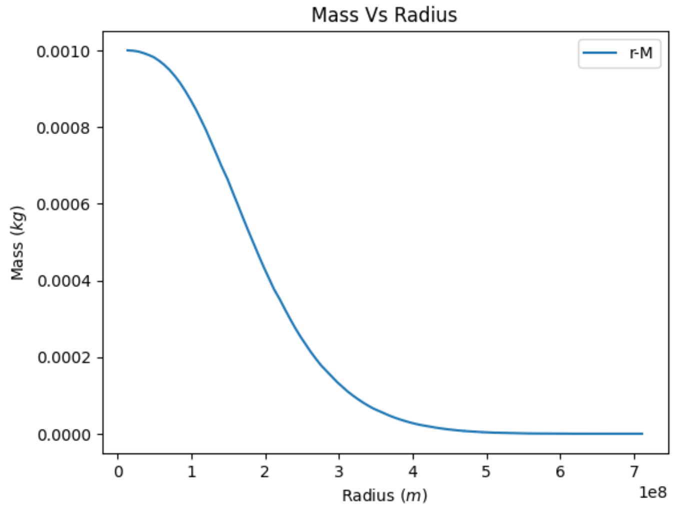
The following parameters were given to run the function (the function takes input in terms of solar properties):
Mass = 1 solar mass
Luminosity = 0.860710 solar luminosity
Temperature = 5500.2 Kelvin
Hydrogen mass fraction = 0.70
Metals mass fraction = 0.008

3.2. Mass and density variations



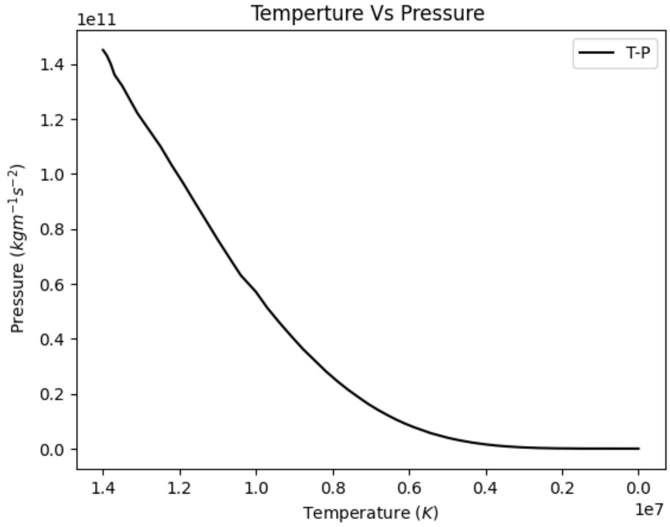
The graph illustrates a clear, exponential decrease in density as the radius increases. Initially, near the centre of the star (radius approaching zero), the density is at its maximum, indicating the concentration of mass in the core. As the radius increases, the density decreases sharply until it approaches a near-constant low value past approximately $3x10^8 meters$.

This pattern of density distribution is typical for stars, where the core contains most of the mass, and the outer layers are less dense. The steep curve at the beginning followed by a flattening indicates that the density changes most dramatically near the center, stabilizing as the distance from the core increases.



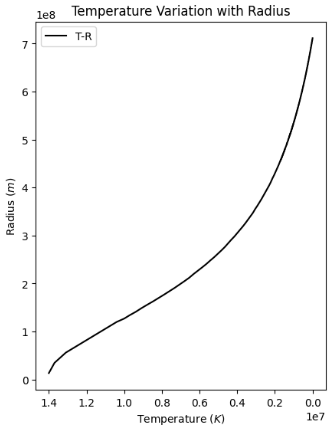
In the case of the Mass vs Radius plot, it shows a rapid decrease in mass as the radius increases, starting from a peal near zero radii. As the radius extends, the mass drops steeply and then plateaus, approaching nearly zero as the radius approaches $7x10^8$ meters. This trend indicates that the bulk if the mass is concentrated near the centre of the object, with significantly less mass distributed in the outer regions. Also, the shape of the curve suggests a significant decrease in density away from the core, which is typical for many types of celestial bodies where central regions are denser and outer layers are more sparse.

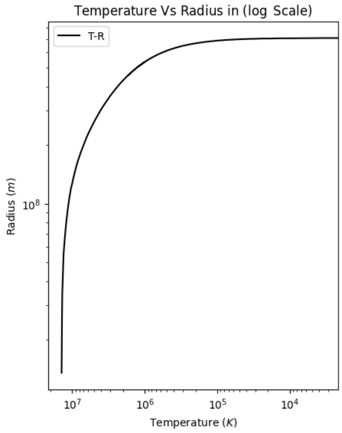
3.3. Temperature and pressure variations



The plot above shows the relationship between temperature (in Kevin) and pressure (in $kg/m^3/s^2$). The x-axis represents the temperature and the y-axis measures pressure.

The curve shows a negative correlation, as the temperature decreases, the pressure similarly decreases. Initially, at high temperatures close to 10 million kelvin, the pressure is at its maximum. As the temperature lowers, the pressure follows a steep decline, becoming more pronounced as the temperature approaches zero. This suggests that the internal pressure of a star like the Sun decreases significantly as it cools, highlighting the critical interplay between these two fundamental parameters in stellar physics. The smooth, continuous decline in the plot underlines the consistent behaviour of pressure in response to changes in temperature within the stellar environment.

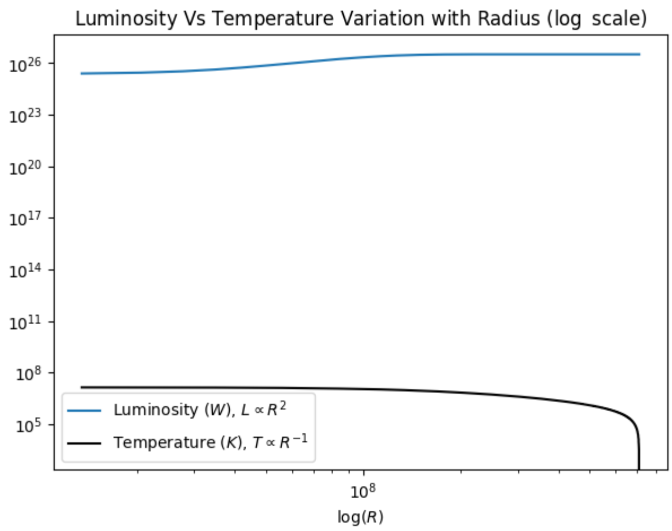




The figures above compare the relationship between temperature and radius in two different scales: linear and logarithmic. Linear scale plot analysis: The temperature decreases from nearly 10 million kelvin to nearly zero while the radius increases from zero to $7x10^8$ meters. the graph reveals as the temperature decreases, the radius increases significantly.

Logarithmic scale: The plot shows that the radius changes primarily occur at lower temperatures. Suggesting that the most considerable expansion in radius occurs at lower temperatures.

3.4. Luminosity and Temperature variations

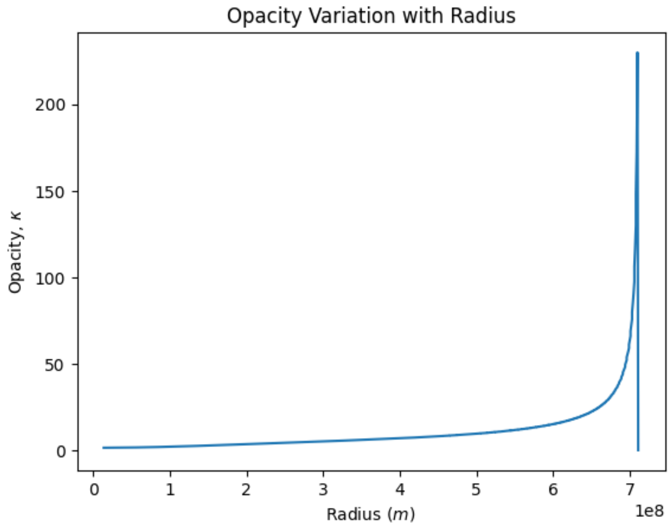


This plot depicts the relationship between the radius (in meters) of the sun-like star with its luminosity (in Watts) and Temperature (in K). The x-axis scales the radius in log scale, while the y-axis measures luminosity temperature. The **temperature** curve is intentionally plotted in **Black** other than the default color(blue) in "Colab.Python" environment to avoid any false temperature color profile of the star.

Here we see the demonstration of exponential growth in luminosity as a function of radius, with a rapid increase initially. Where, temperature growth follows the inversely proportional relation with radius.

This behaviour illustrates a key physical principle of stellar structure, indicating that a star's luminosity becomes increasingly sensitive to changes in radius up to a certain point, after which it stabilizes. This plateau suggests a limit to the efficiency with which the star can convert mass and radius increase into luminous output, a critical insight for understanding the life cycle and stability of stars similar to the sun.

3.5. Opacity variation



The plot portrays the relationship between the opacity k and the radius.

This graph features an interesting behavior of opacity as a function of the stellar radius, Initially, the opacity remains nearly constant and very low across the majority of the star's radius, suggesting high transparency in these regions. However, as the radius approaches its maximum, there is a dramatic and sharp increase in opacity, soaring to its highest values.
This steep rise in opacity at the outer layers of the star indicates regions where light and radiation face significant absorption and scattering, possibly due to changes in temperature, composition, or state of the stellar material. This outer layer is critical for understanding stellar light curves and energy output.

4. The birth and death of our star

4.1. The birth of our star

A star comparable to our Sun originates predominantly from molecular hydrogen and commences its development within a serene, extensive molecular cloud. These vast clouds, spanning hundreds of light-years, represent the densest and chilliest regions of the interstellar medium. Scattered amidst these clouds are denser pockets teeming with intricate molecules, dust particles, and opaque, enigmatic dark matter. Cosmic rays infiltrate these clouds, while magnetic fields intricately weave through them. Under certain circumstances, such as random turbulent motions or the gravitational influence of neighboring galactic structures, these dense regions may further intensify.

As the density within the cloud escalates, specific regions cross a critical threshold, initiating a gravitational collapse. Various external factors, including shockwaves from supernovae, collisions with other clouds, or the gravitational pull from passing stars, can trigger this collapse. Consequently, the cloud fragment becomes denser and hotter as additional material is drawn inward by gravity. Each of these smaller cloud fragments has the potential to give rise to a new star or even a star system. Angular momentum causes the collapsing cloud to spin, eventually forming a rotating disk of material encircling the nascent star's core.

The fragment continues its collapse until internal pressure and temperature reach a threshold where it can withstand further gravitational contraction. At this stage, the term "protostar" is applied to this emerging star, signifying a delicate balance between outward pressure from gas and radiation and inward gravitational forces. Although the protostar radiates brilliantly in infrared and submillimeter wavelengths, its visibility in visible light remains obscured by its cocoon of gas and dust

As the protostar evolves, it sheds its surrounding material envelope through vigorous ejections facilitated by strong jets and stellar winds. Concurrently, gravitational compression persists, leading to a rise in internal temperature. Despite these transformations, the protostar remains distinct from a genuine star as it has yet to initiate hydrogen fusion within its core. During this phase, although the protostar's surface temperature remains relatively stable, its brightness diminishes, resulting in a near-vertical trajectory on the Hertzsprung-Russell (HR) diagram.

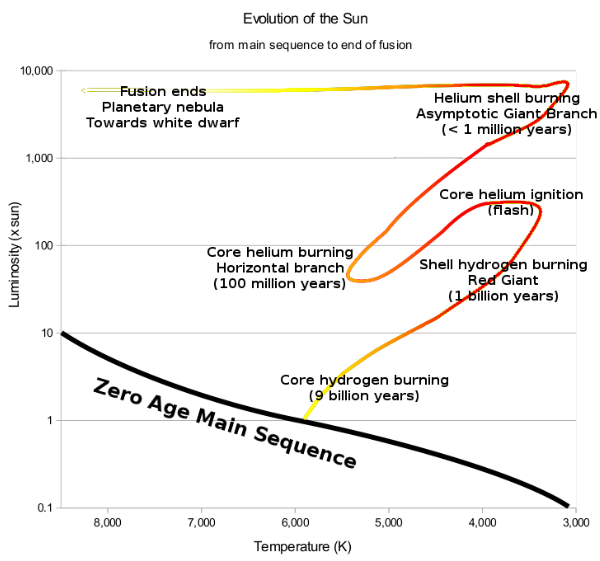
Transitioning into the T Tauri stage, named after the variable young star prototype, the protostar displays characteristic features such as prominent dark patches on its surface, intense stellar winds, and erratic fluctuations in brightness. During this period, the protostar continues to contract, potentially hosting a circumstellar disk from which planets may eventually form. Additionally, strong magnetic fields are believed to manifest around the star at this juncture.

Over millions of years of contraction and evolution, the protostar's core temperature escalates to approximately 15 million Kelvin, initiating the proton-proton chain reaction responsible for converting hydrogen into helium. This marks the star's entry into the main sequence phase, where it will persist for the majority of its lifespan. Maintaining hydrostatic equilibrium, the outward pressure generated by nuclear fusion balances the inward force of gravity, rendering the star stable and relatively unchanged on human timescales. The star's position on the main sequence is directly linked to its mass, with a lifespan akin to that of the Sun estimated to be around 10 billion years. Throughout this phase, the star's core fuses approximately 600 million tonnes of hydrogen per second, sustaining the condition of hydrostatic equilibrium.

4.2. The death and afterlife

Subsequently, as the star depletes its hydrogen fuel, its core undergoes gravitational compression, leading to a rise in temperature. Hydrogen fusion commences in a shell surrounding the inert helium core, while the star's outer layers thicken, causing it to depart from the main sequence and ascend the subgiant branch of the HR diagram. The enhanced energy production in the shell contributes to the star's increased luminosity, with its radius expanding to several times its initial size.

Progressing along the red giant branch, the core's contraction intensifies, heating the hydrogen shell, and causing the star's outer envelope to significantly expand and cool. Consequently, the star shines hundreds of times brighter than during its main sequence phase, with its radius potentially expanding up to 100 times its original size. The star's surface temperature decreases during this phase, imparting a reddish hue, hence earning it the designation of a red giant.



The triple-alpha process, crucial for the conversion of helium into carbon and oxygen, is initiated within the star's core when conditions of high temperature and density are met. For stars less massive than approximately 2.5 solar masses, this ignition occurs explosively, known as a helium flash, owing to the degenerate nature of the core. Following this intense event, the outer layers of the star undergo shrinkage, while the core expands and experiences a slight cooling phase. Subsequently, the star transitions to a horizontal branch on the Hertzsprung-Russell (HR) diagram as it settles into a steady phase of helium burning within its core.

Once the helium in the core is depleted, the star reverts to the red giant phase, initiating hydrogen fusion in a shell surrounding the carbon-oxygen core and helium shell. This phase is termed the asymptotic giant branch (AGB), distinguished by its extreme luminosity and rapid mass loss due to powerful stellar winds. AGB stars often expel complex chemicals and dust into space, enriching the interstellar medium.

Continuing through the AGB phase, the star loses mass via stellar winds, ultimately shedding its outer layers entirely. This shedding exposes the hot core, illuminating the expelled material and forming a planetary nebula. Although planetary nebulae have relatively short lifetimes of tens of thousands of years on an astronomical timescale, they are visually striking objects that contribute significantly to the enrichment of the interstellar medium with heavier elements.

Upon the fading of the planetary nebula, the final remnant of the star is a compact, Earth-sized object primarily composed of carbon and oxygen, known as a white dwarf. The white dwarf emits light solely from the thermal energy retained after nuclear fusion ceases. Over billions of years, the white dwarf gradually cools and darkens, eventually transforming into a black dwarf. However, due to the universe's young age, no black dwarfs have yet formed.