Instituto Jorge Sabato, 25 años.
Comisión Nacional de Energía atómica.

Modelización de Materiales 2018

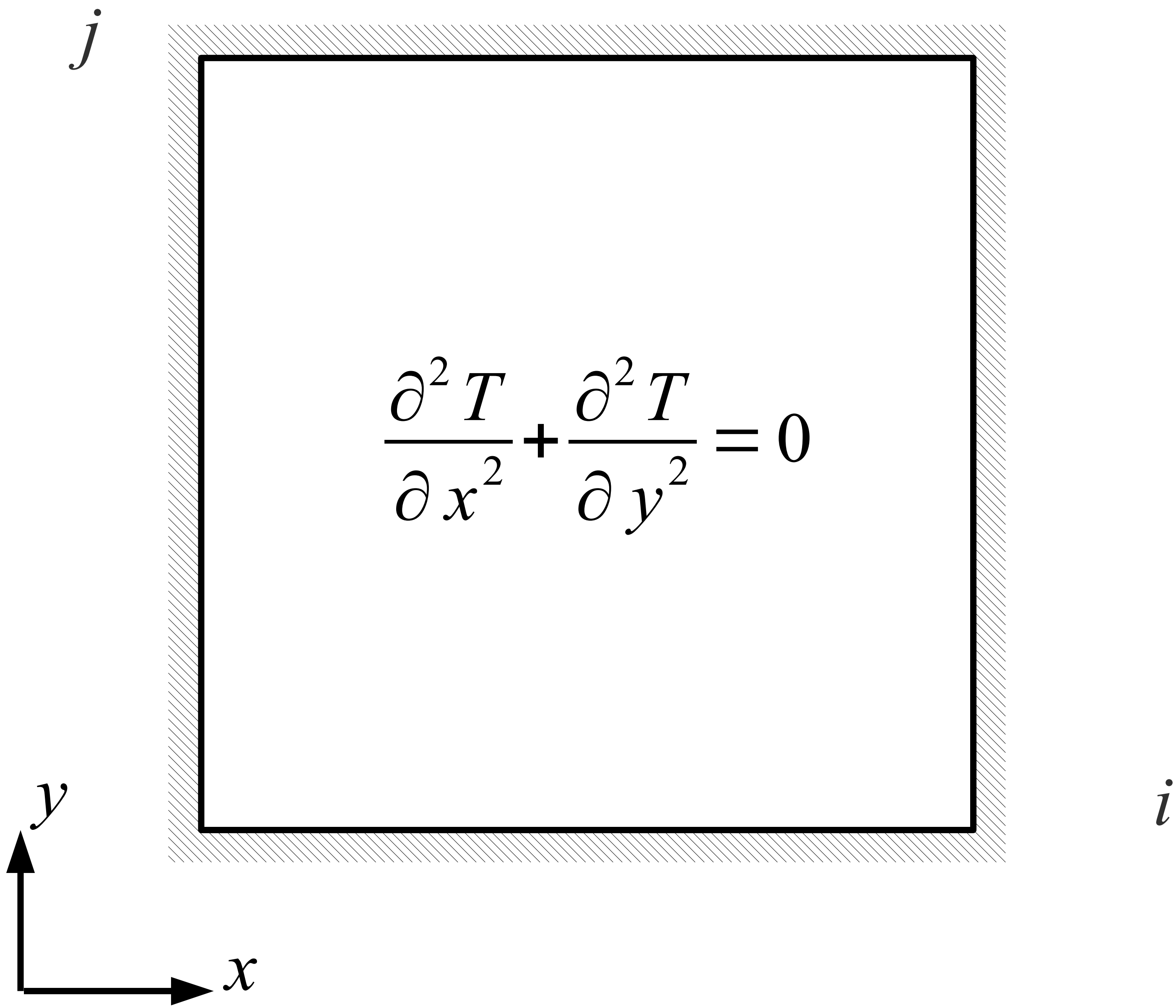
Método de las Diferencias Finitas

Ecuaciones Diferenciales en Derivadas parciales: problema de equilibrio con condiciones de contorno.

Ejemplo: Estado Estacionario de un problema de conductividad térmica.

Presentación del Problema

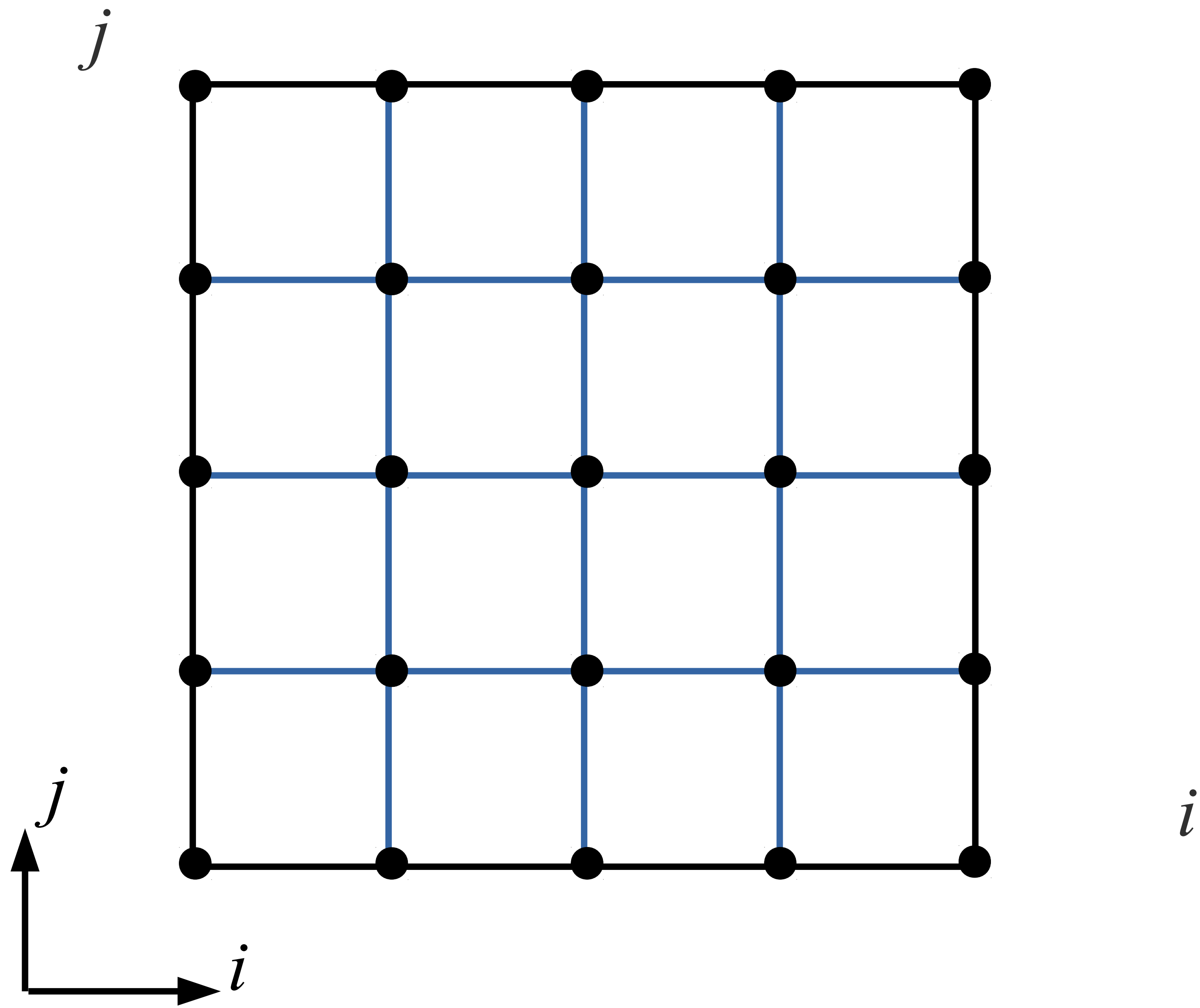
$T_{i\ j}$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

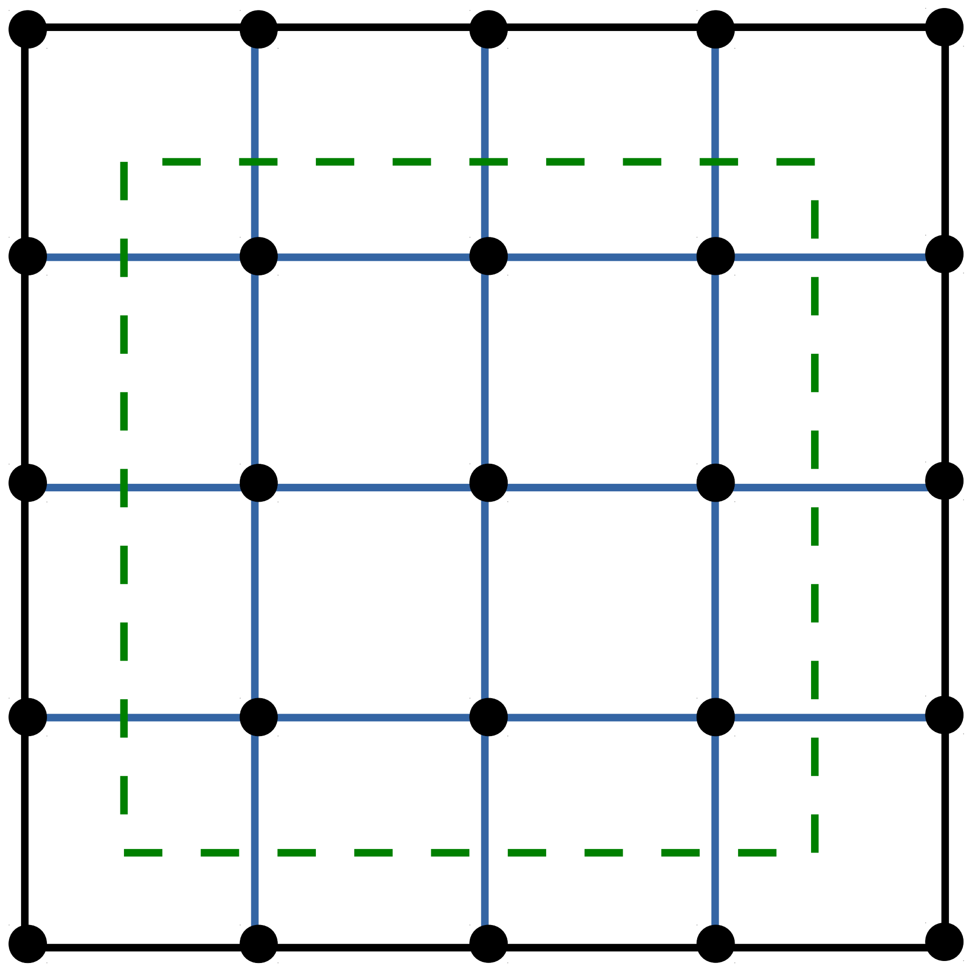
Discretización del Problema

$T_{i\ j}$



Ecuación Matricial

- Temperaturas en nodos internos

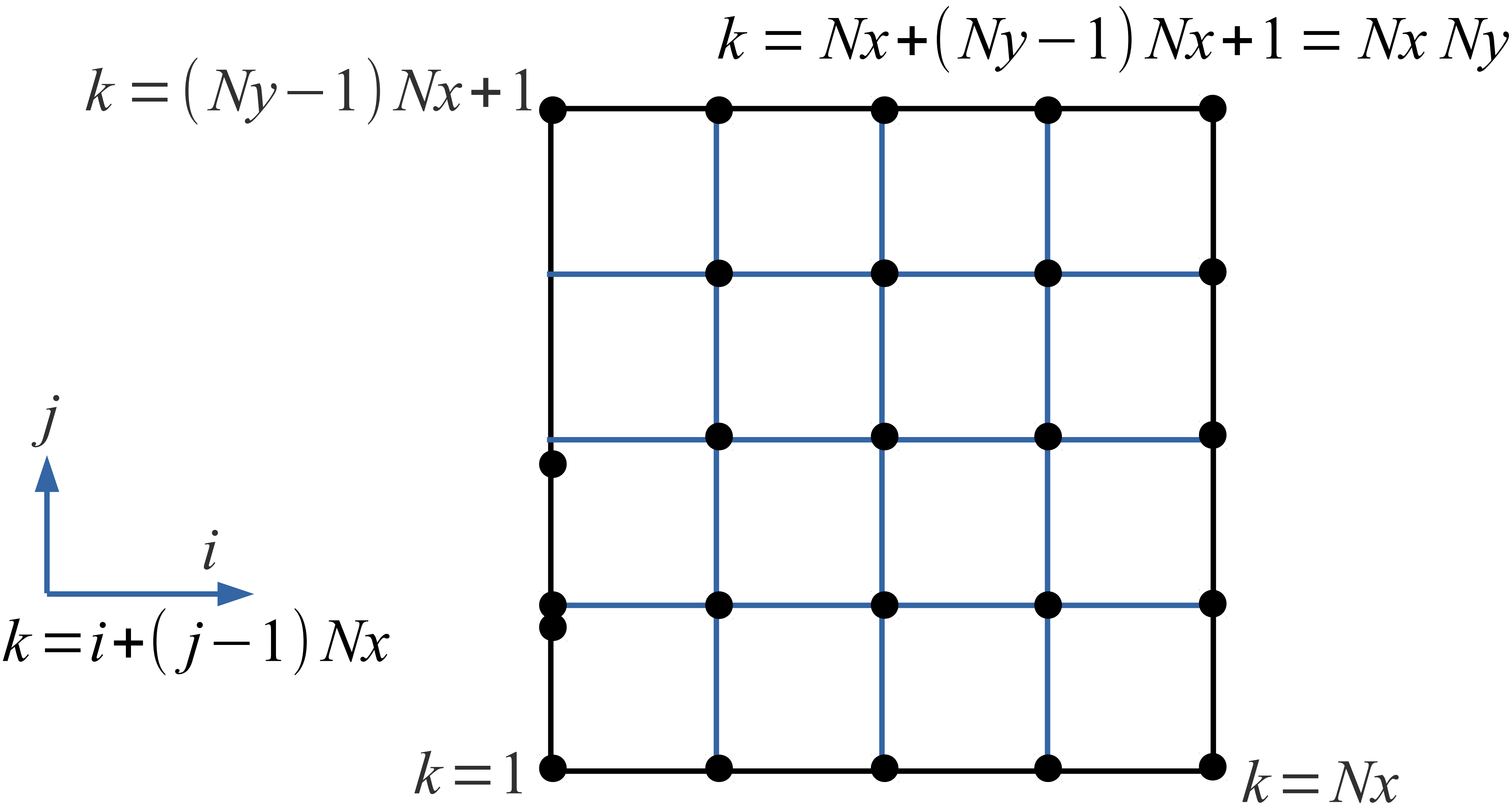


$$T_{i,j} = \boldsymbol{T}_k$$

$$k = i + (j-1)\,Nx$$

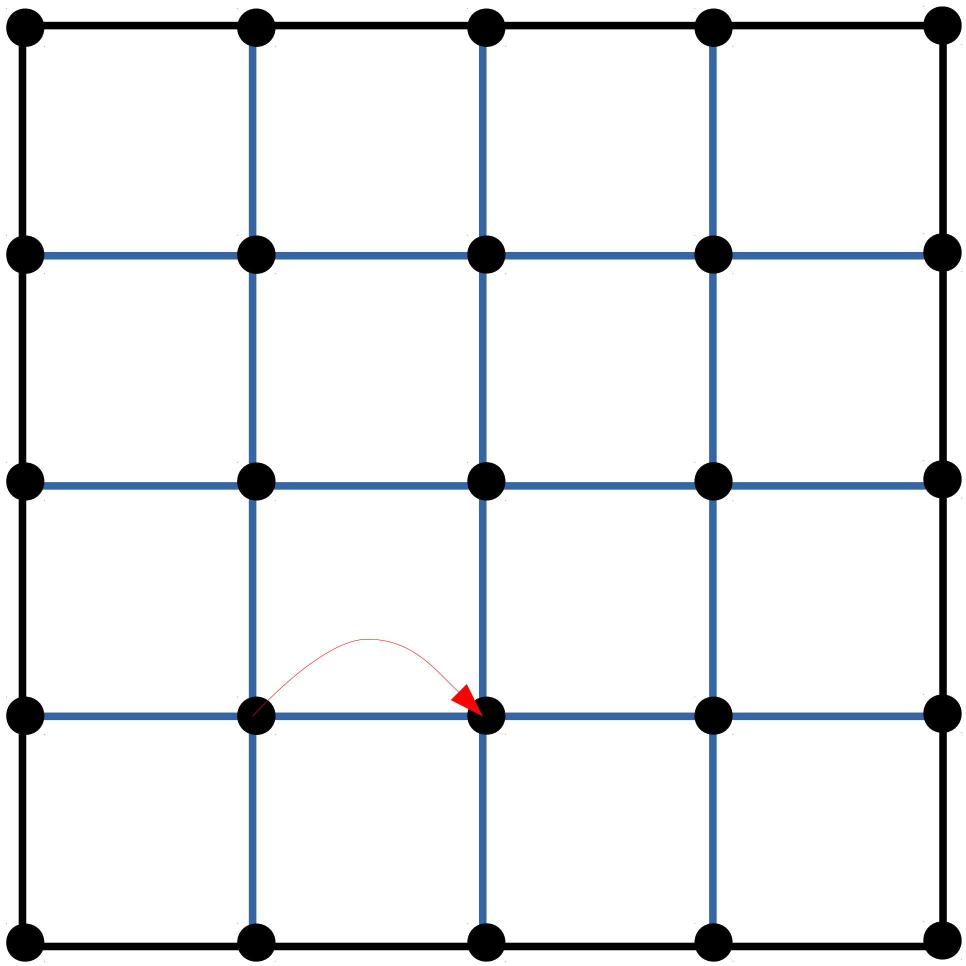
$$T_k = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{NxNy} \end{pmatrix}$$

Ecuación Matricial: Numeración de Nodos

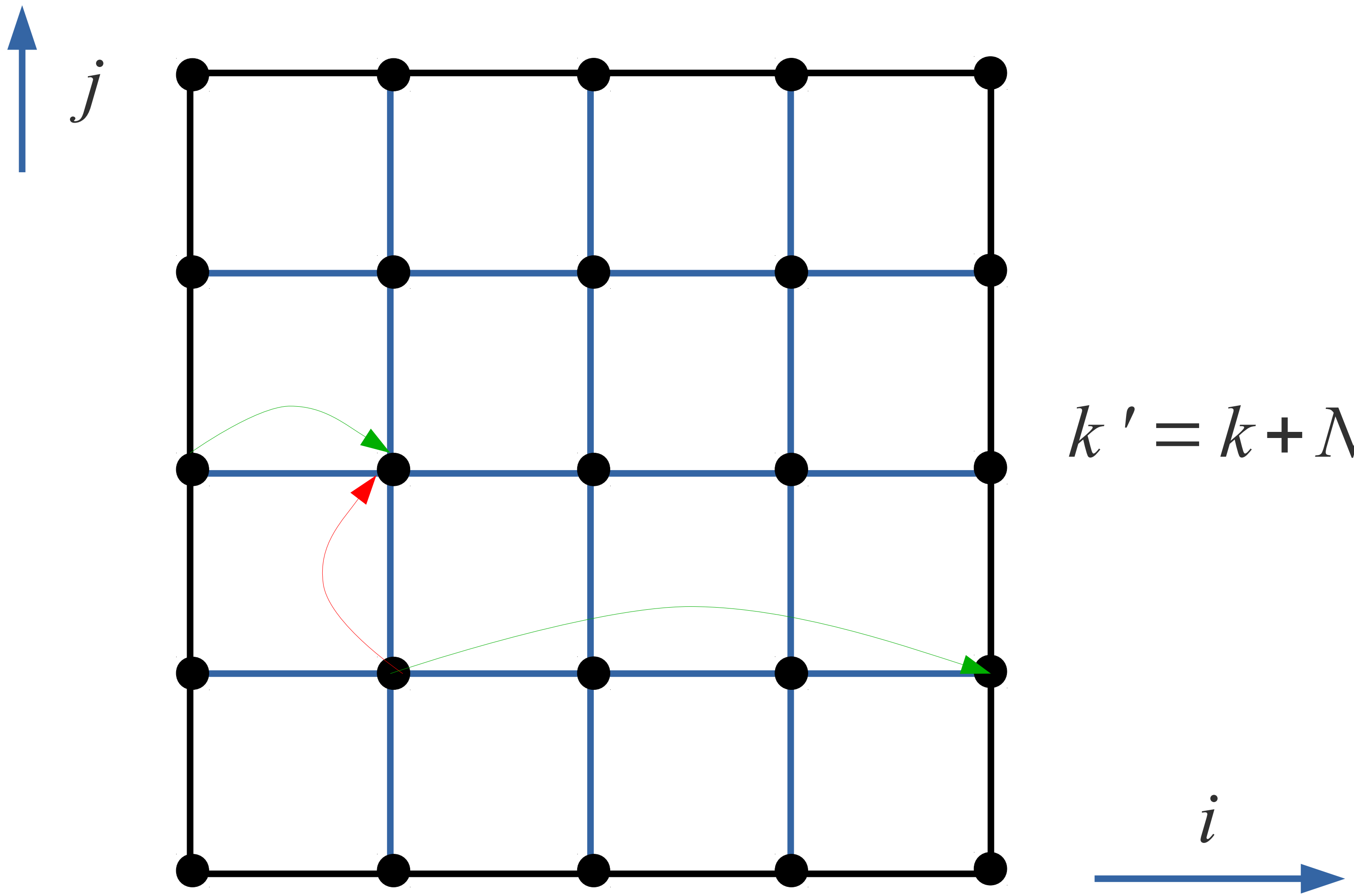


Ecuación Matricial: Numeración de Nodos

$$k' = k + 1$$



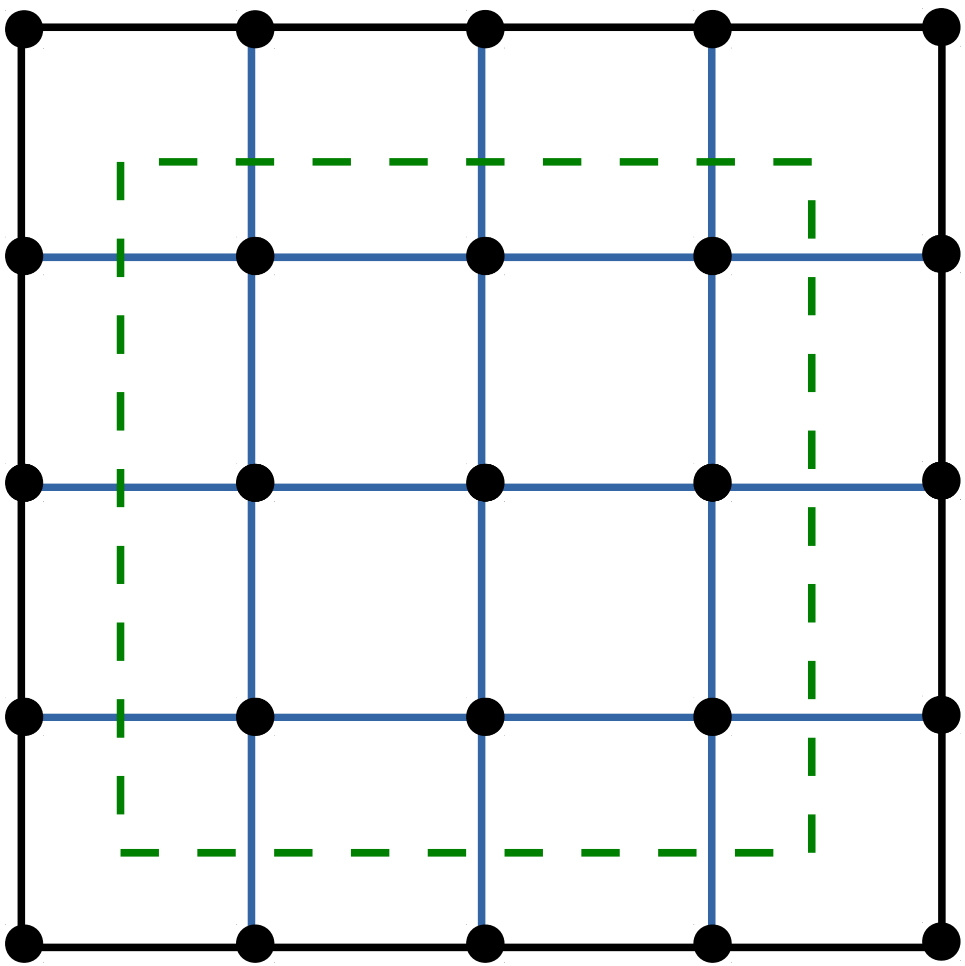
Ecuación Matricial: Numeración de Nodos



$$k' = k + Nx$$

Ecuación General

- Para los nodos Internos:



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

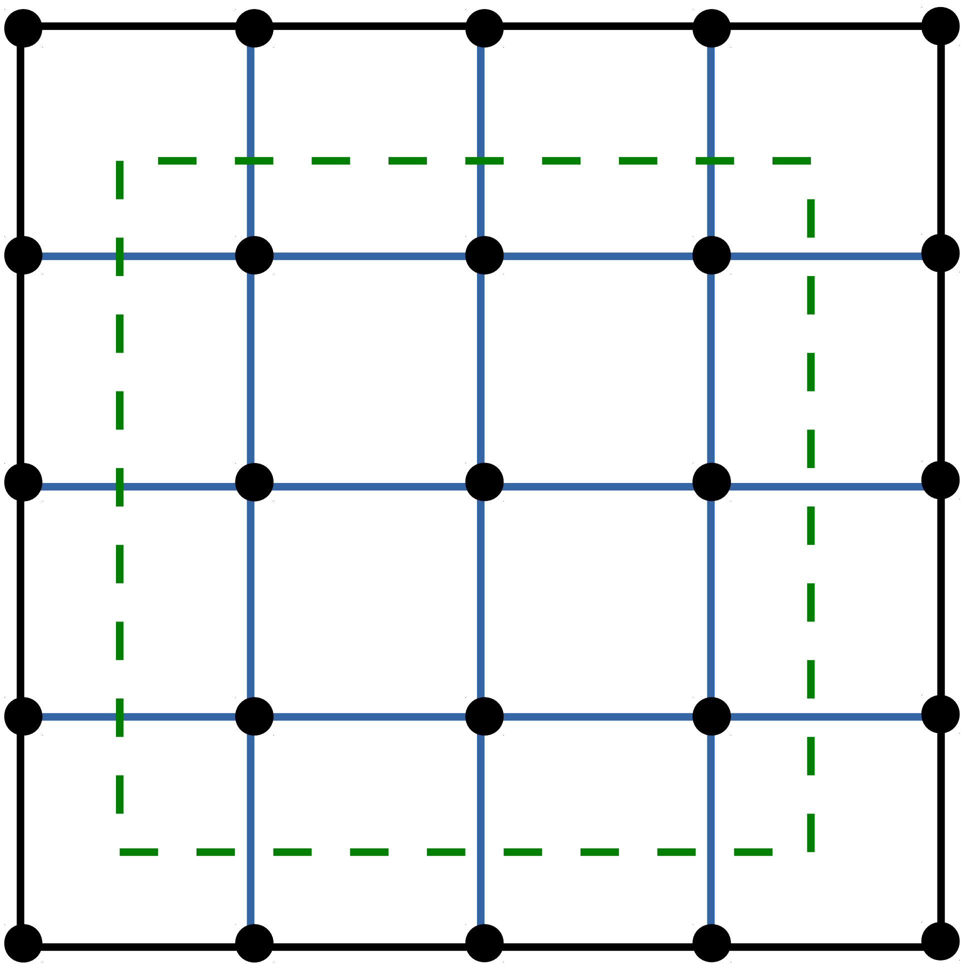
$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{k-1} - 2T_k + T_{k+1}}{dx^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{k-Nx} - 2T_k + T_{k+Nx}}{dy^2}$$

Linealización de la ecuación Diferencial

- Para los nodos Internos

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$



$$\left[M\right]\begin{pmatrix} T_1 \\ \vdots \\ T_k \\ \vdots \\ T_{NxNy} \end{pmatrix} = b$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Linealización de la ecuación Diferencial

- Coeficientes de la Matriz

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$

Fila k-ésima:

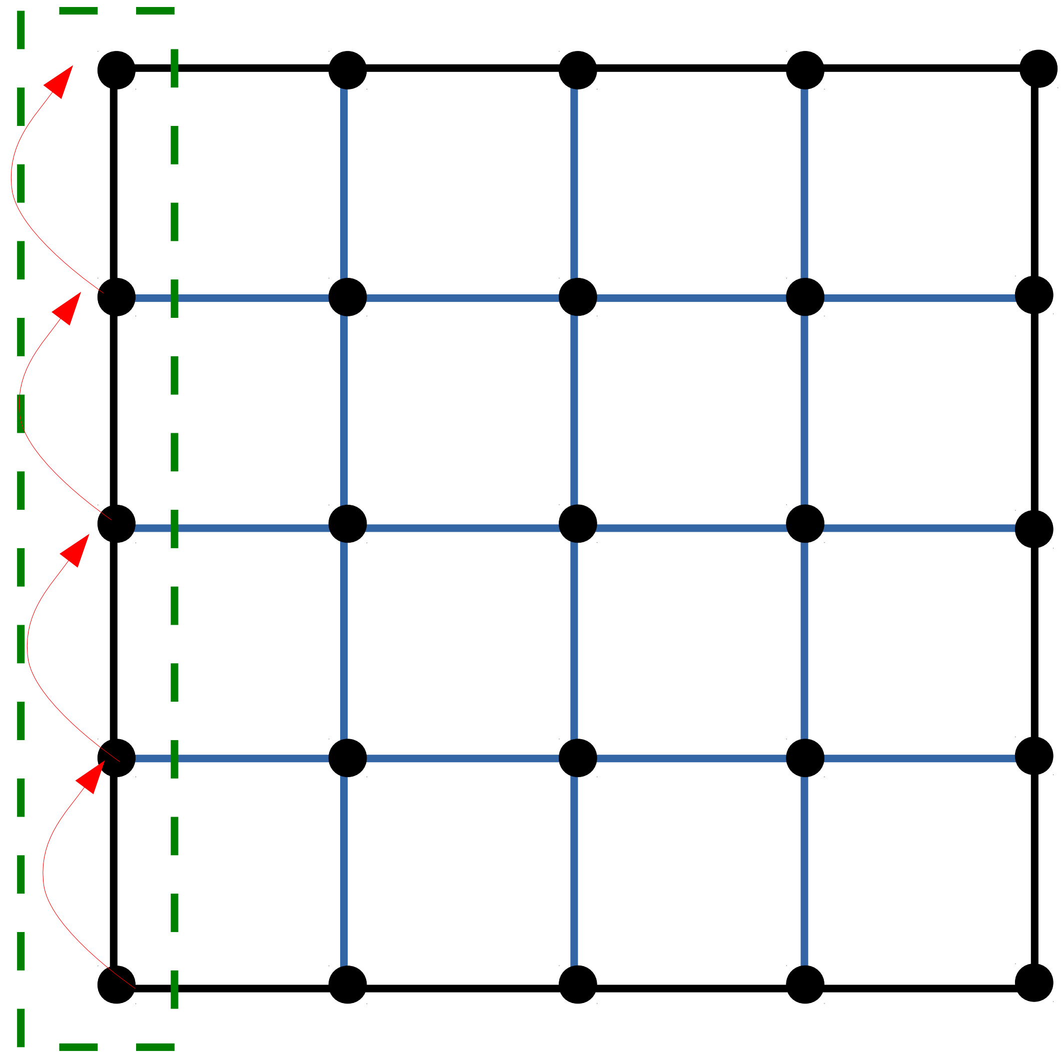
$$M_{k,:} = \left[\cdots \quad \beta^2 \quad \cdots \quad 1 \quad -2\left(1+\beta^2\right) \quad 1 \quad \cdots \quad \beta^2 \quad \cdots\right]$$

$k - Nx$ $\quad$ $k-1$ $\quad$ k $\quad$ $k+1$ $\quad$ $k+Nx$

$$b_k = 0$$

Condiciones de contorno

- Temperatura Fija



$$T_{k_A} = T_A$$

$k_A = ?$ Algun rango ...

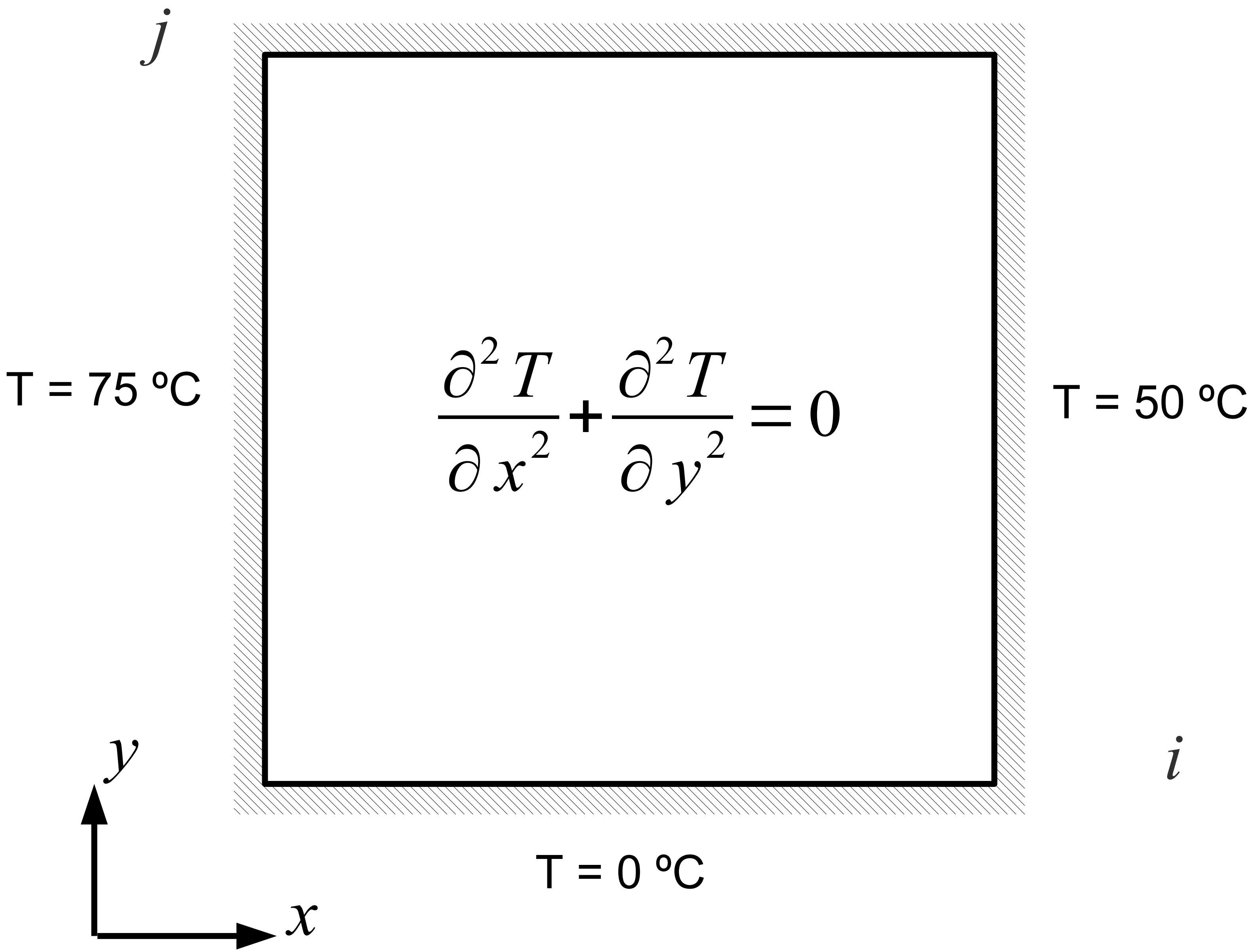
$$M^{fija}_{k_A,:} = \begin{bmatrix} \cdots & 1 & \cdots \end{bmatrix}$$

$$k_A$$

$$b_{k_A} = T_A$$

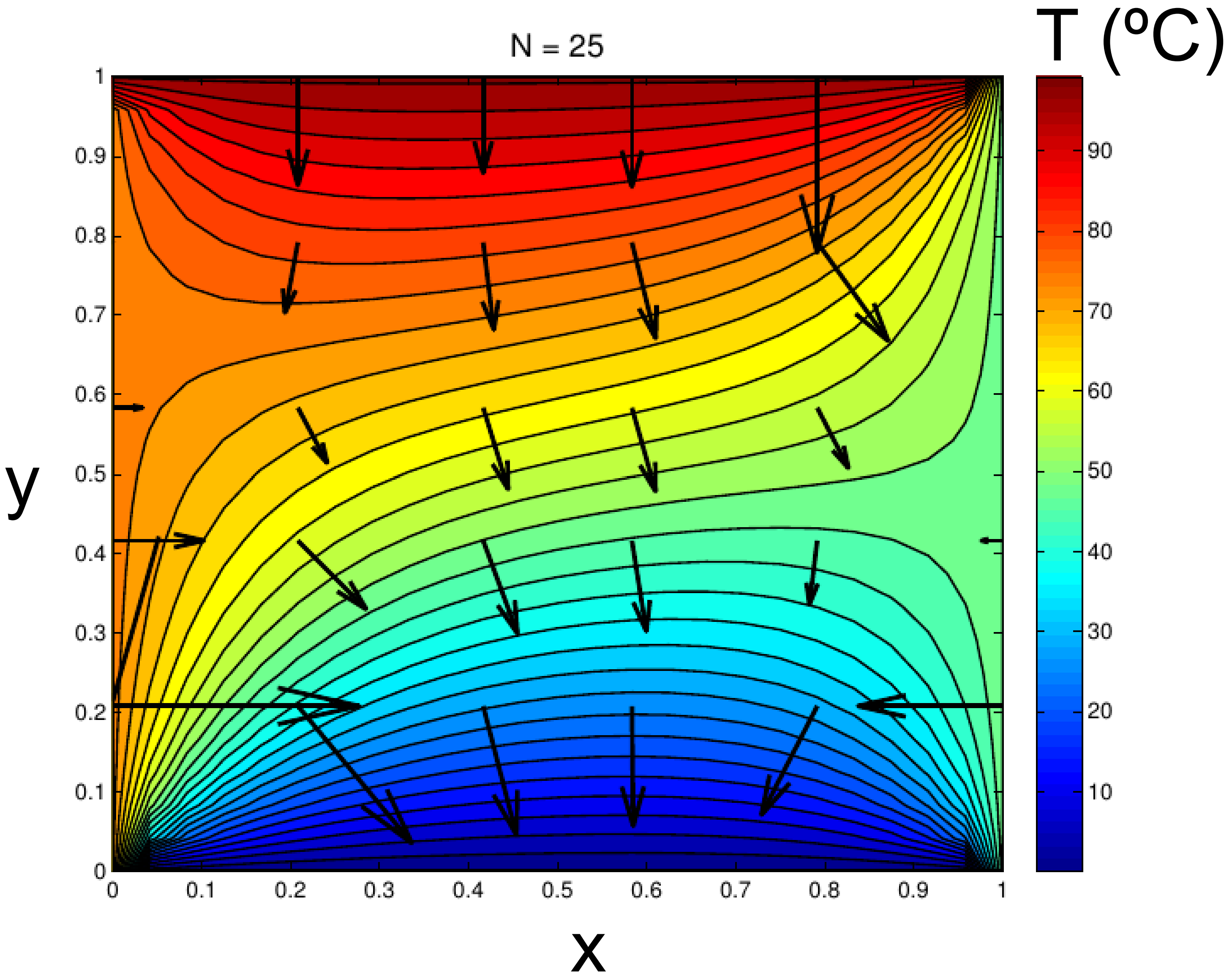
Problema: Primera Aproximación

$T_{i\ j}$



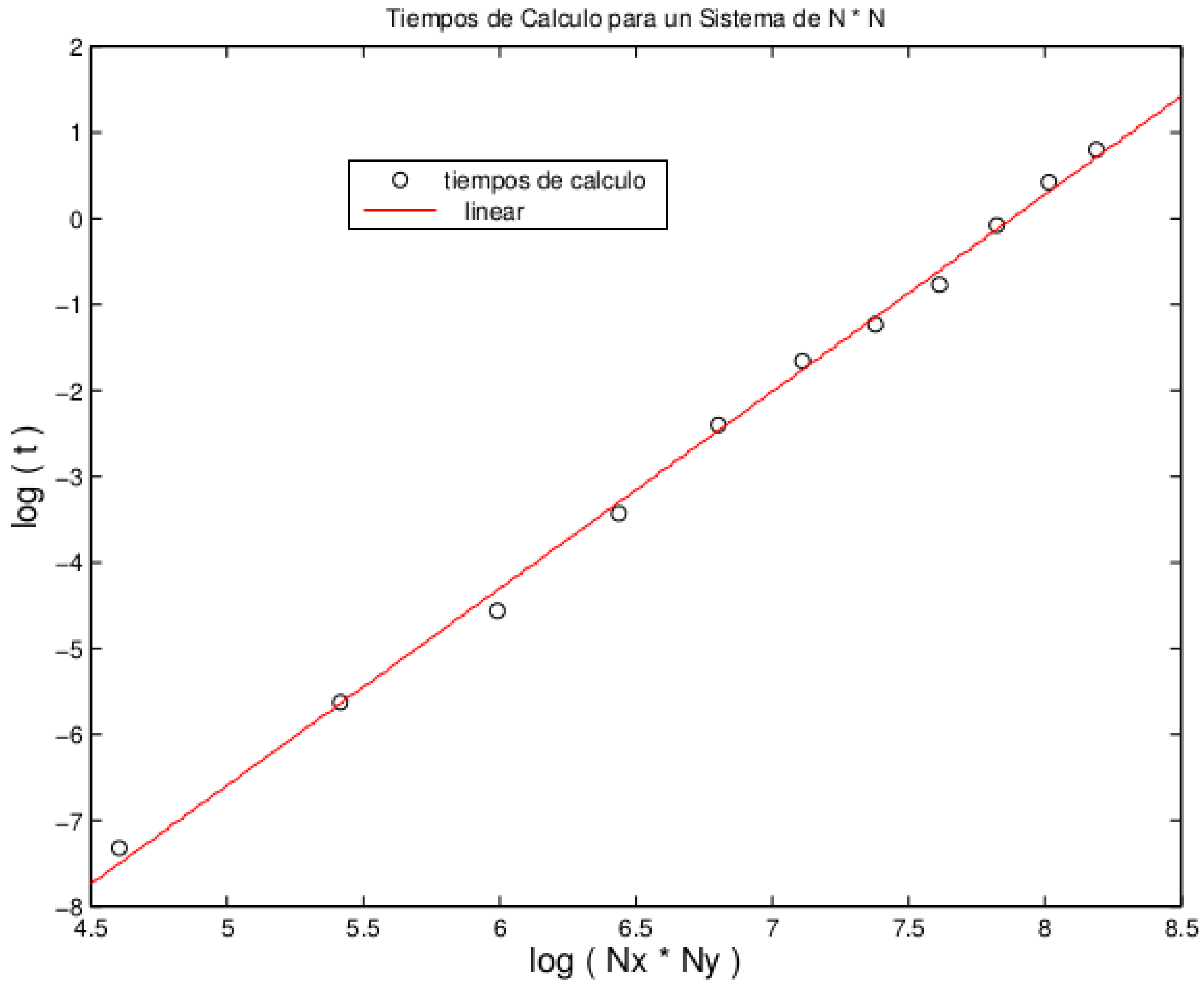
Problema: primera aproximación

- Solución:



Problema: Primera Aproximación

- Solución: Escaleo Temporal



Postproceso: Mapa de temperaturas.

Genera grilla xy para el gráfico.

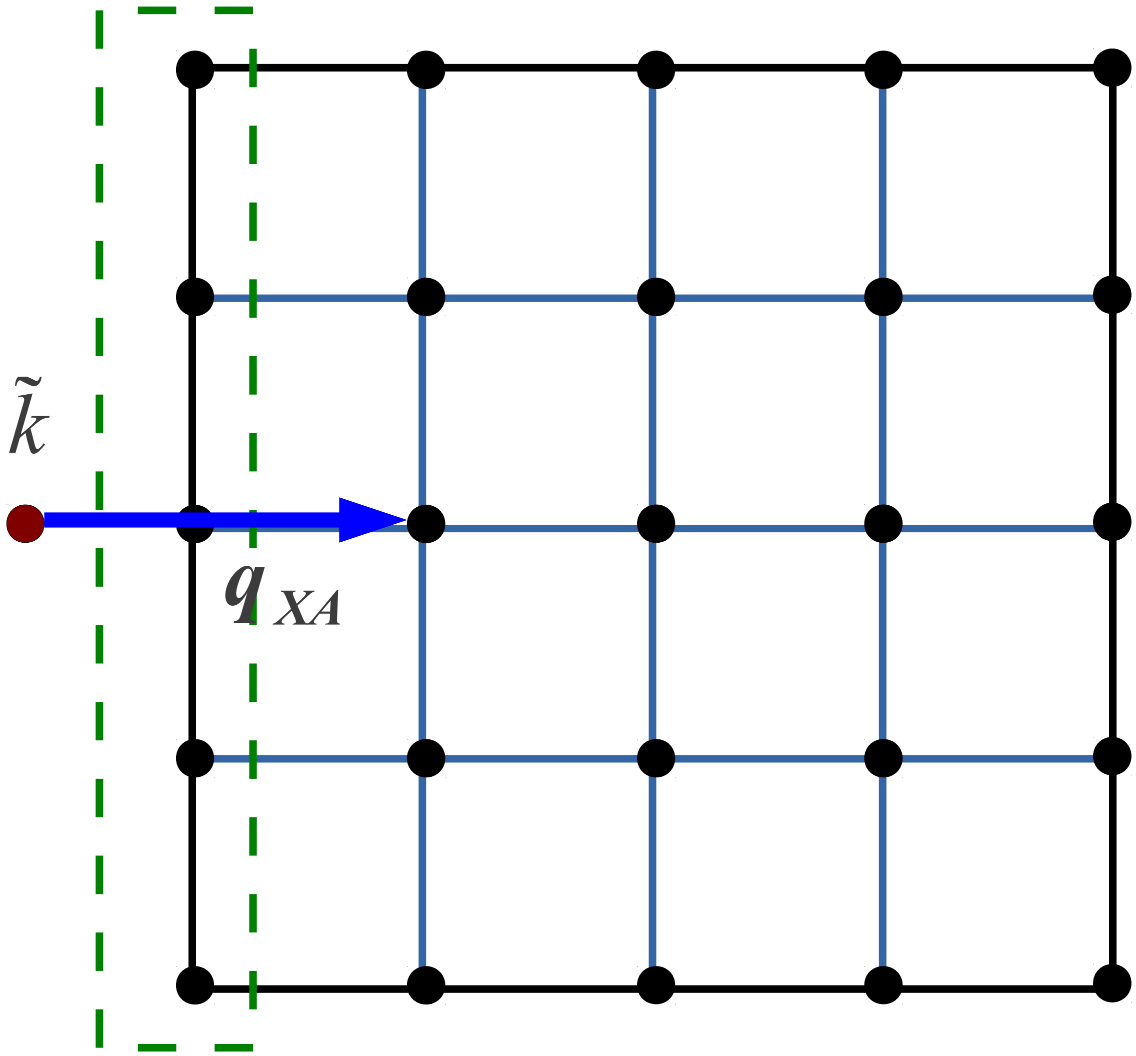
Mapa de colores.

Command Window

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on')
fx >>
```

Condiciones de contorno: Flujo

- Derivada centrada: punto extra



$$Q_x \propto q_{XA} =$$

$$\left.\frac{\partial T}{\partial x}\right|_{k_A} = \frac{T_{k_A+1} - T_{\tilde{k}}}{2\text{dx}}$$

$$k_A = 1 : N_X : (N_Y - 1) N_x + 1$$

Condiciones de contorno: Flujo

- Cambio en los elementos de matriz

$$T_{\tilde{k}} = T_{k_A+1} - 2\mathrm{dxq}_{XA} \qquad k_A = 1 : N_X : (N_Y - 1) N_x$$

Reemplazo en la ecuación general

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$

Reordeno

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\,T_{k+1} + \beta^2 T_{k+N_X} = 2\,dxq_{XA}$$

Condiciones de contorno: Flujo

$$T_{\tilde{k}-1} = T_{k_A+1} - 2\,dxq_{XA} \qquad k_A = 1 : N_X : (N_Y - 1) N_x$$

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\,T_{k+1} + \beta^2 T_{k+N_X} = 2\,dxq_{XA}$$

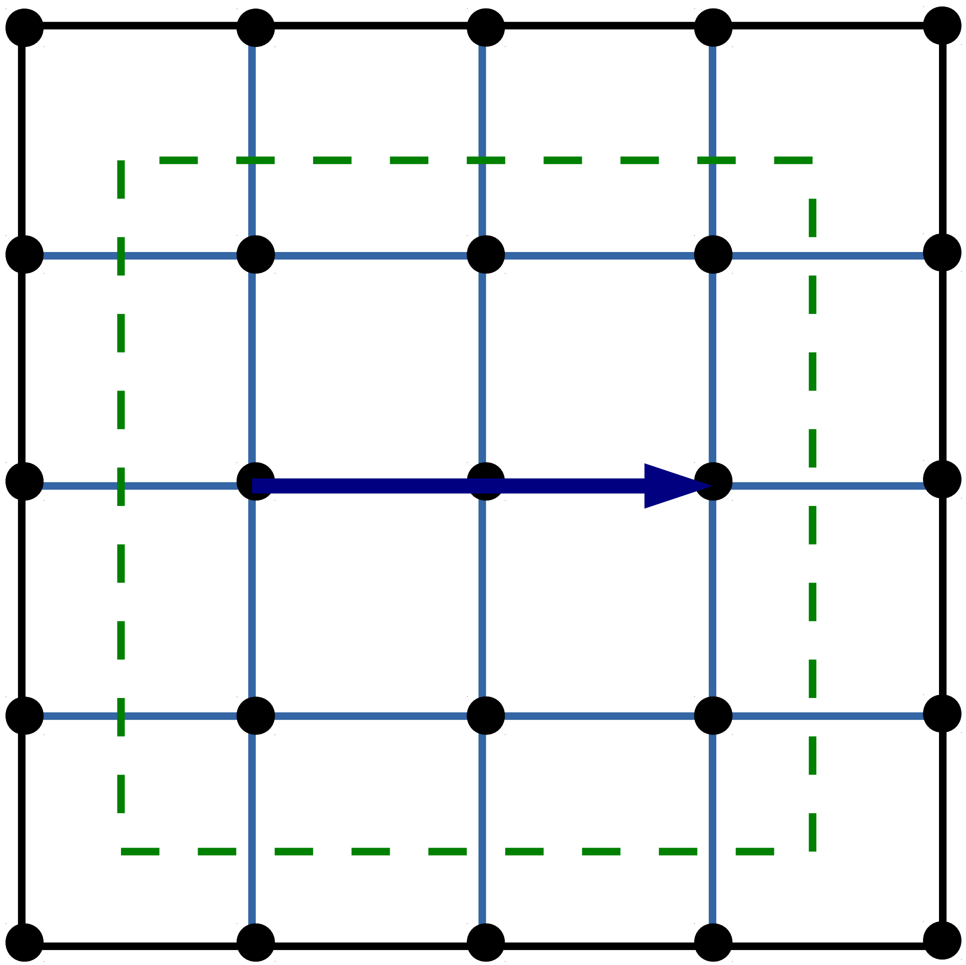
Fila k-ésima :

$$M_{k_A,:} = \left[\cdots \quad \beta^2 \quad \cdots \quad 0 \quad -2\left(1+\beta^2\right) \quad 2 \quad \cdots \quad \beta^2 \quad \cdots\right]$$

$$k - Nx \qquad k-1 \qquad k \qquad k+1 \qquad k+Nx$$

$$b_k = 2\,dxq_{XA}$$

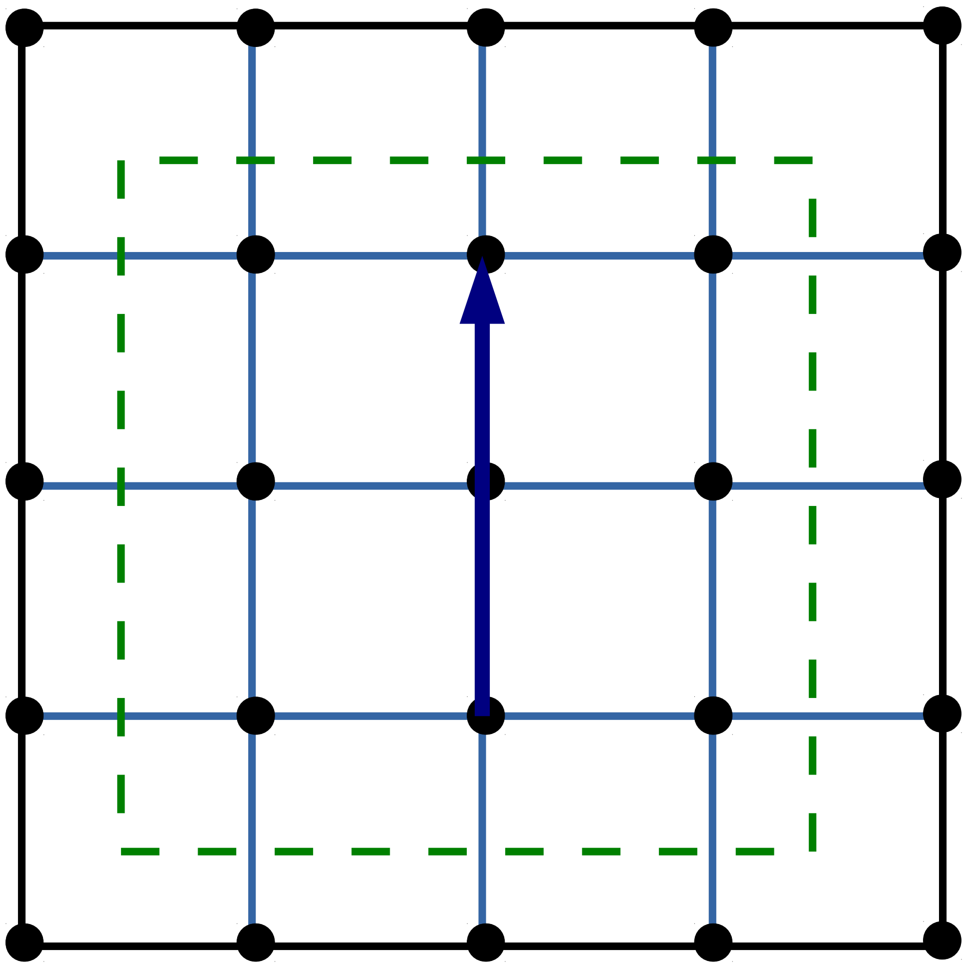
Cálculo de Flujos



$$Q_x \propto q_X =$$

$$\frac{\partial T_k}{\partial x} = \frac{T_{k+1} - T_{k-1}}{2\mathrm{dx}}$$

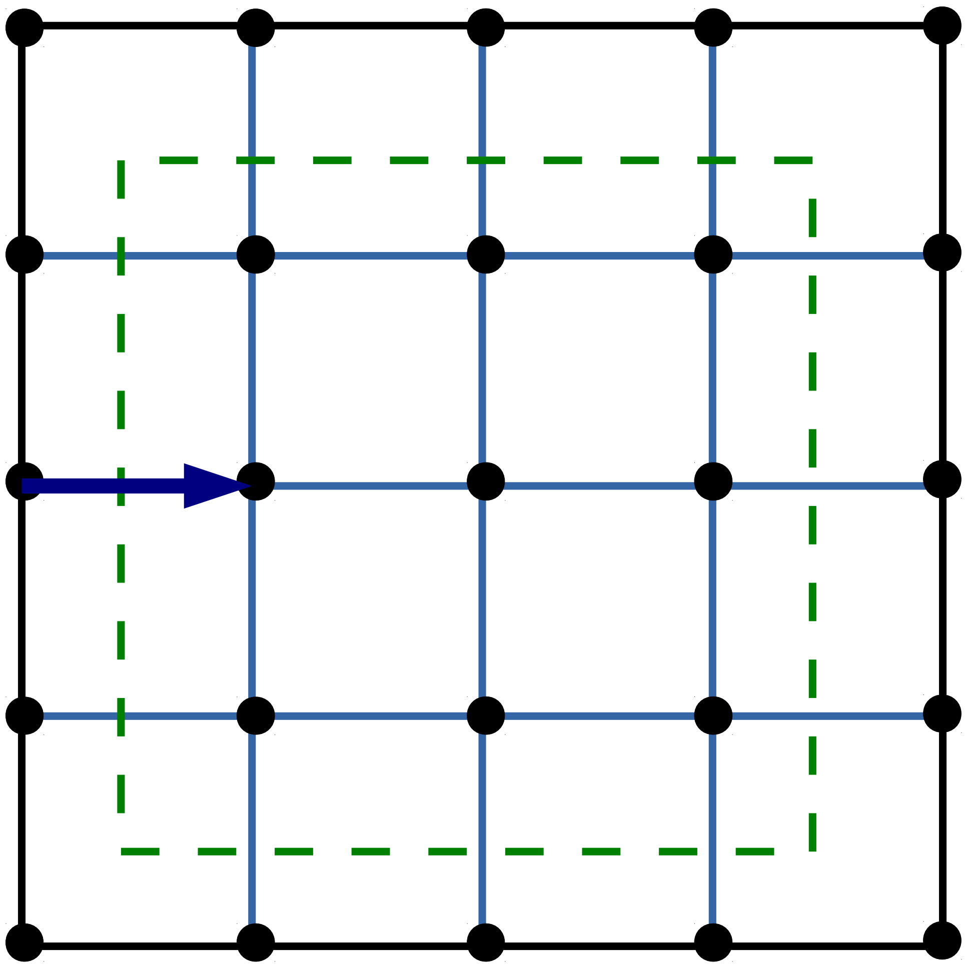
Cálculo de Flujos



$$Q_y \propto q_Y =$$

$$\frac{\partial T_k}{\partial y} = \frac{T_{k+Nx} - T_{k-Nx}}{2\mathrm{dy}}$$

Cálculo de Flujos



$$Q_{yA} \propto q_{YA} =$$

$$\frac{\partial T_{k_A}}{\partial x} = \frac{T_{k_A+1} - T_k}{dx}$$

Resultados: Graficación

Genera grulla xy para el gráfico.

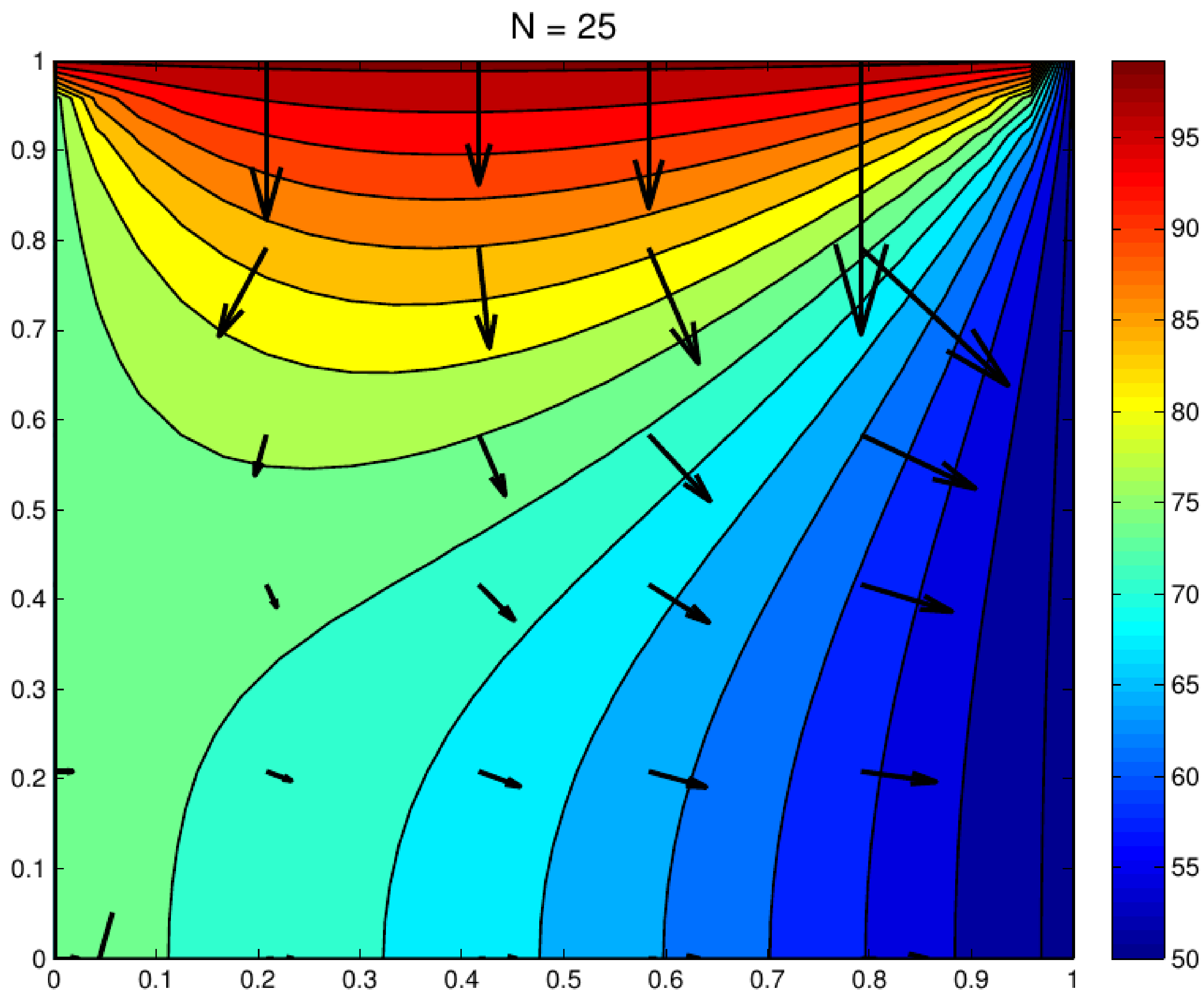
Mapa de colores.

Calcula flujos (propia)

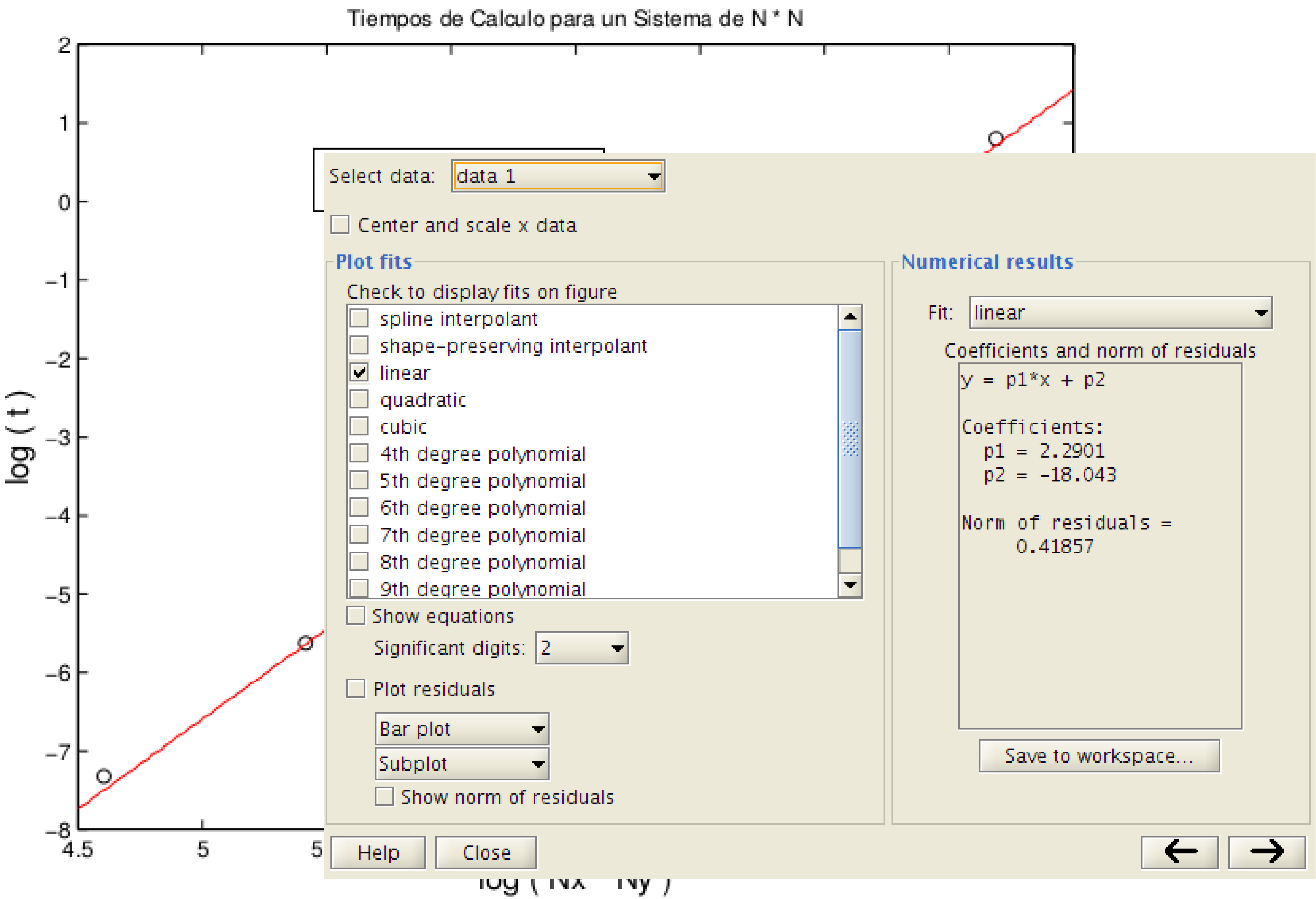
Grafica campo vectorial

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on');
>> [xx,yy,qx,qy]=flujos(N,N,x,y,Tsol);
>> hold on
>> quiver(xx,yy,qx,qy,'color',[0 0 0]);
>>
```

Resultados


N = 25
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
95
90
85
80
75
70
65
60
55
50

Resultados: Escaleo



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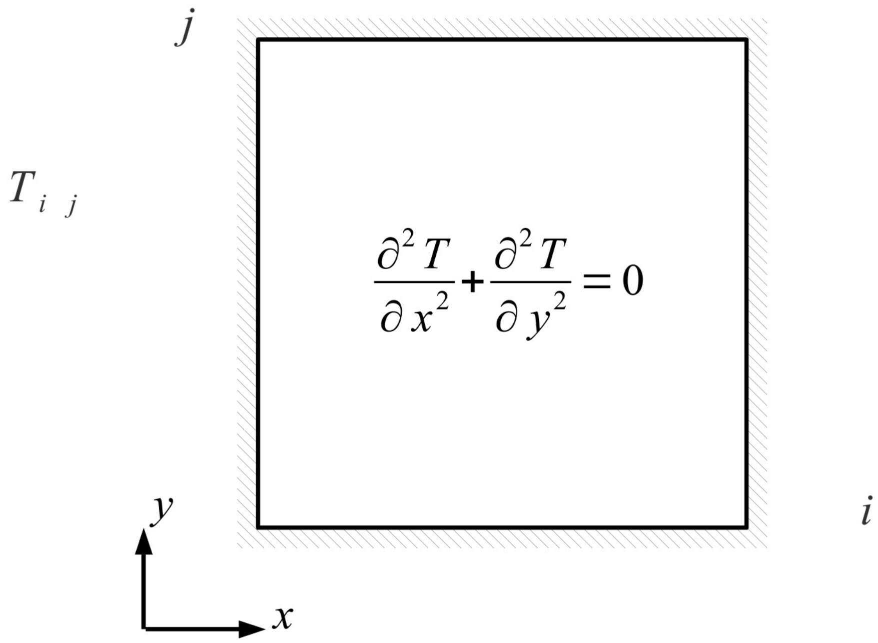
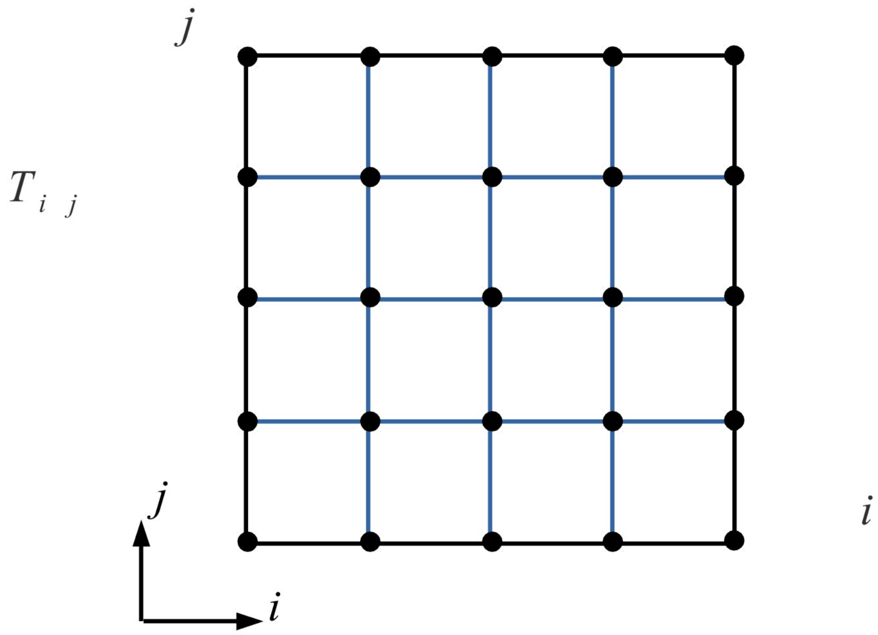


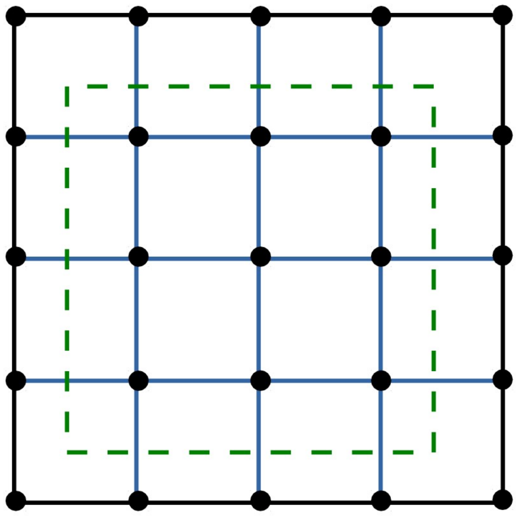
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Discretización del Problema



Ecuación Matricial

- Temperaturas en nodos internos



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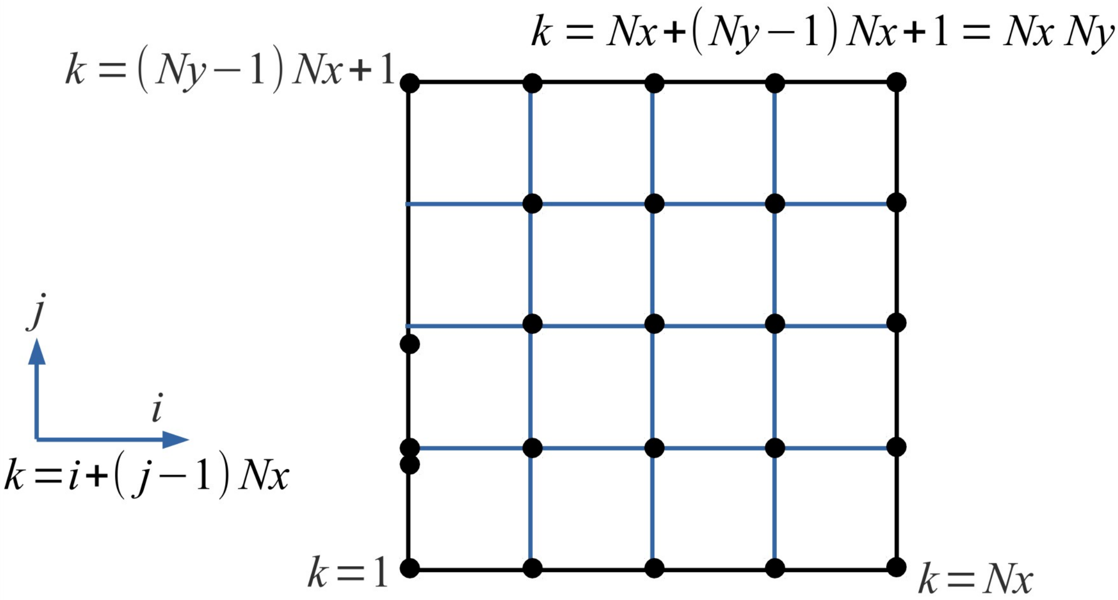
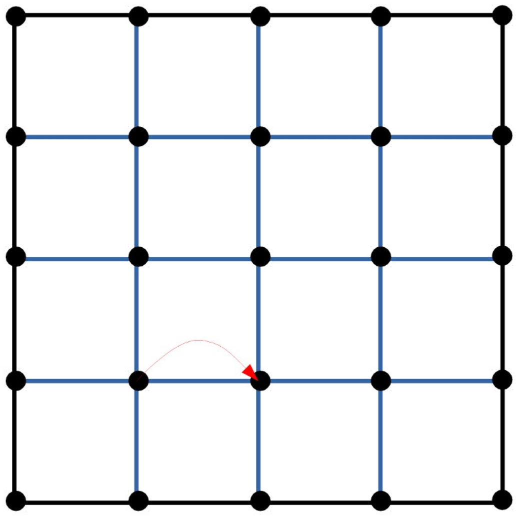
Ecuación Matricial: Numeración de Nodos

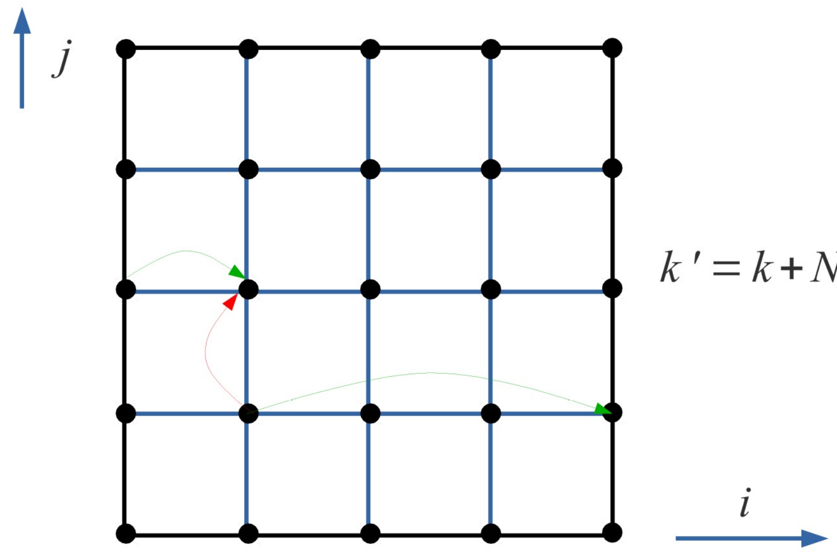


Ecuación Matricial: Numeración de Nodos

$$k' = k + 1$$



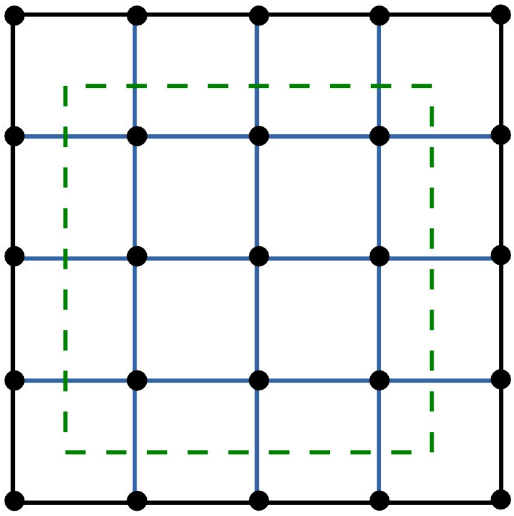
Ecuación Matricial: Numeración de Nodos



$$k' = k + Nx$$

Ecuación General

- Para los nodos Internos:



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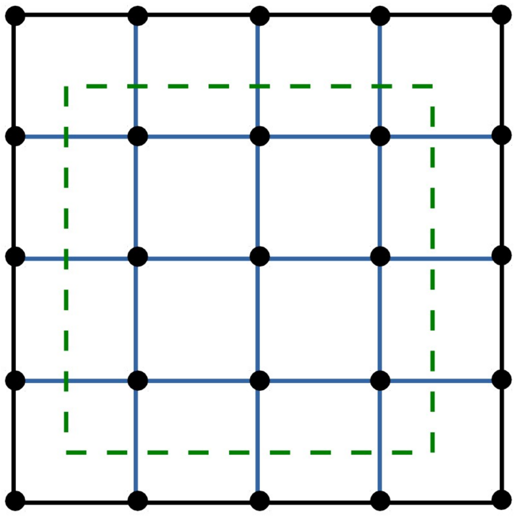
Linealización de la ecuación Diferencial

- Para los nodos Internos

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$



$$\left[M\right]\begin{pmatrix} T_1 \\ \vdots \\ T_k \\ \vdots \\ T_{NxNy} \end{pmatrix} = b$$

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Linealización de la ecuación Diferencial

- Coeficientes de la Matriz

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$

Fila k-ésima:

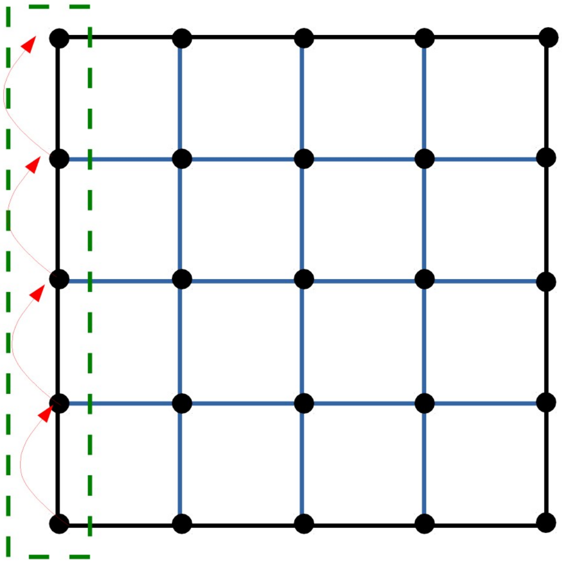
$$M_{k,:} = \begin{bmatrix} \cdots & \beta^2 & \cdots & 1 & -2\left(1+\beta^2\right) & 1 & \cdots & \beta^2 & \cdots \end{bmatrix}$$

$$\begin{matrix} k-Nx & & k-1 & -2\left(1+\beta^2\right) \to k & k+1 & & k+Nx \end{matrix}$$

$$b_k = 0$$

Condiciones de contorno

- Temperatura Fija



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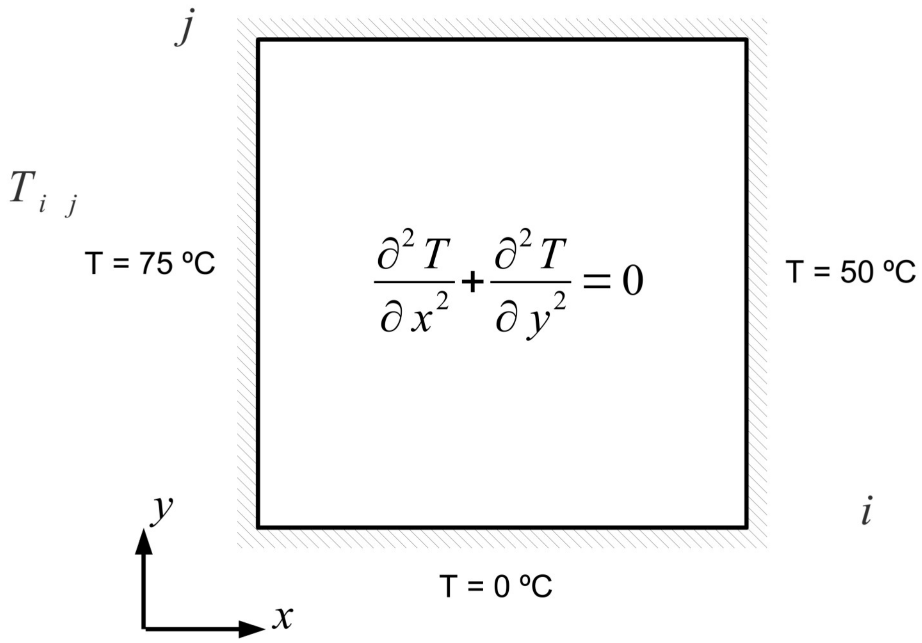
$k_A = ?$ Algun rango ...

$M^{fija}_{k_A,:} = [\ \cdots\ 1\ \cdots\ ]$

k_A

$b_{k_A} = T_A$

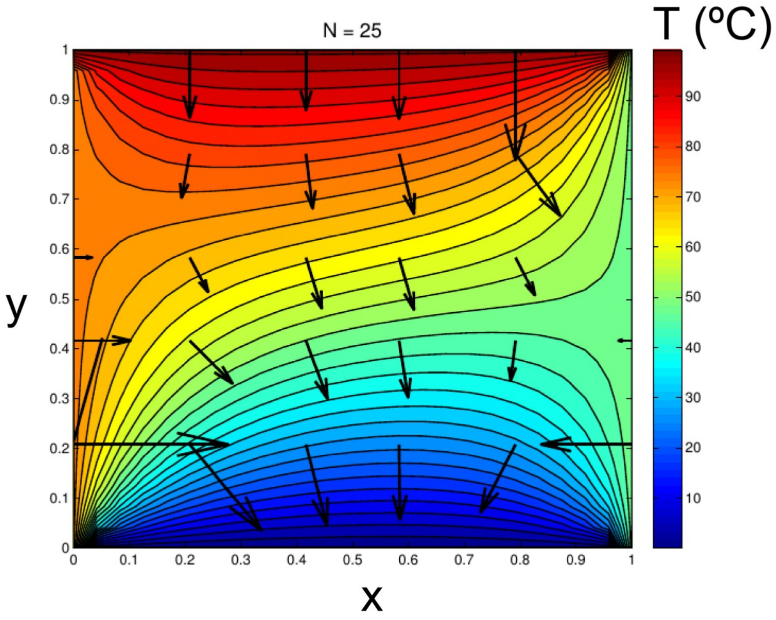
Problema: Primera Aproximación



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

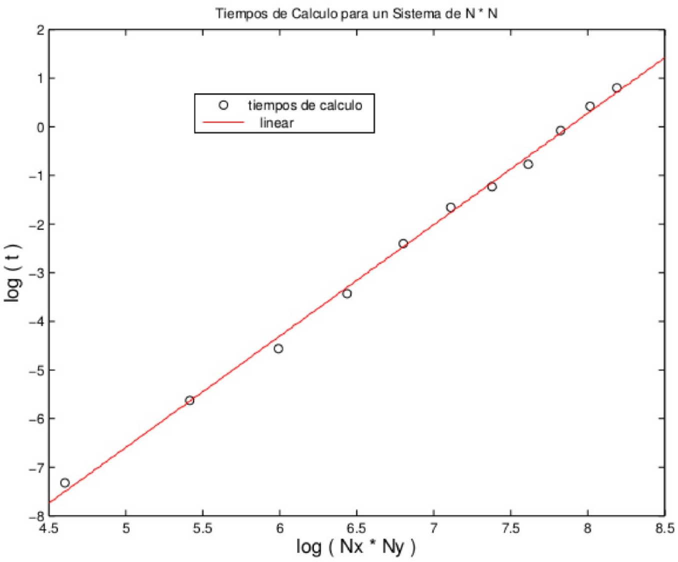
Problema: primera aproximación

- Solución:



Problema: Primera Aproximación

- Solución: Escaleo Temporal



Postproceso: Mapa de temperaturas.

Genera grilla xy para el gráfico.

Mapa de colores.

Command Window

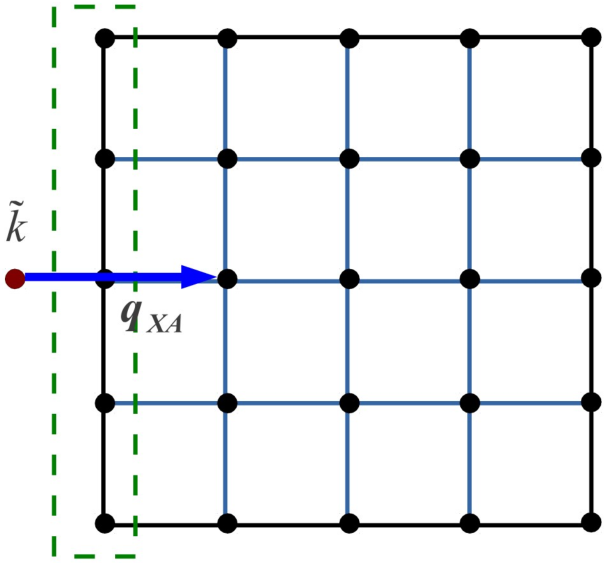
```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on')
>>
```

Command Window

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fil
>>
```

Condiciones de contorno: Flujo

- Derivada centrada: punto extra



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$$k_A = 1 : N_X : (N_Y - 1) N_x + 1$$

Condiciones de contorno: Flujo

- Cambio en los elementos de matriz

$$T_{\tilde{k}} = T_{k_A+1} - 2\mathrm{dxq}_{XA} \qquad k_A = 1 : N_X : (N_Y - 1) N_x$$

Reemplazo en la ecuación general

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$

Reordeno

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\,T_{k+1} + \beta^2 T_{k+N_X} = 2\,dxq_{XA}$$

Condiciones de contorno: Flujo

$$T_{\tilde{k}-1} = T_{k_A+1} - 2\,dxq_{XA} \qquad k_A = 1 : N_X : (N_Y - 1)\,N_x$$

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\,T_{k+1} + \beta^2 T_{k+N_X} = 2\,dxq_{XA}$$

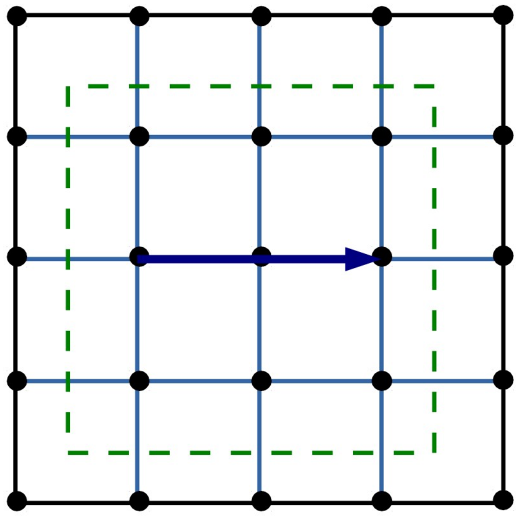
Fila k-ésima :

$$M_{k_A,:} = \left[\cdots \;\; \beta^2 \;\; \cdots \;\; 0 \;\; -2\left(1+\beta^2\right) \;\; 2 \;\; \cdots \;\; \beta^2 \;\; \cdots\right]$$

$$k - Nx \qquad k-1 \qquad k \qquad k+1 \qquad k+Nx$$

$$b_k = 2\,dxq_{XA}$$

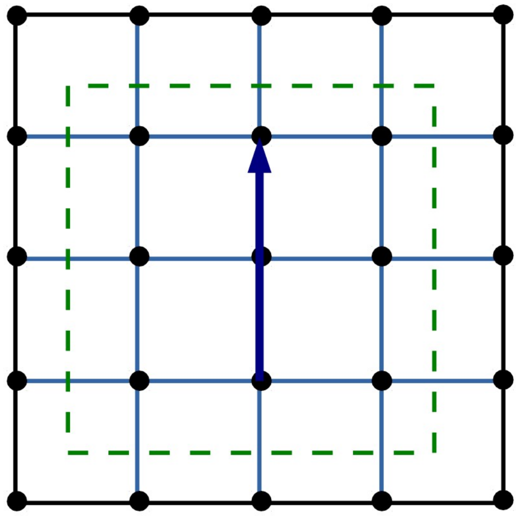
Cálculo de Flujos



$$Q_x \propto q_X =$$

$$\frac{\partial T_k}{\partial x} = \frac{T_{k+1} - T_{k-1}}{2\mathrm{dx}}$$

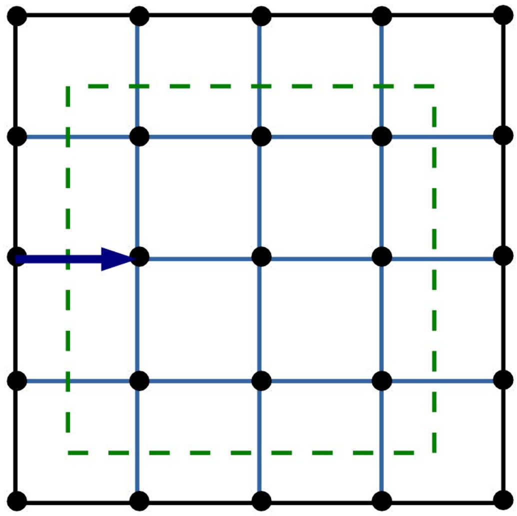
Cálculo de Flujos



$$Q_y \propto q_Y =$$

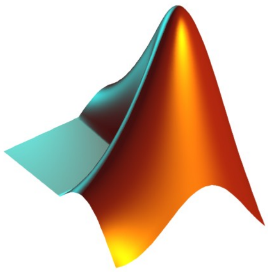
$$\frac{\partial T_k}{\partial y} = \frac{T_{k+Nx} - T_{k-Nx}}{2\mathrm{dy}}$$

Cálculo de Flujos



$$Q_{yA} \propto q_{YA} = \frac{\partial T_{k_A}}{\partial x} = \frac{T_{k_A+1} - T_k}{dx}$$

Resultados: Graficación



Genera grulla xy para el gráfico.

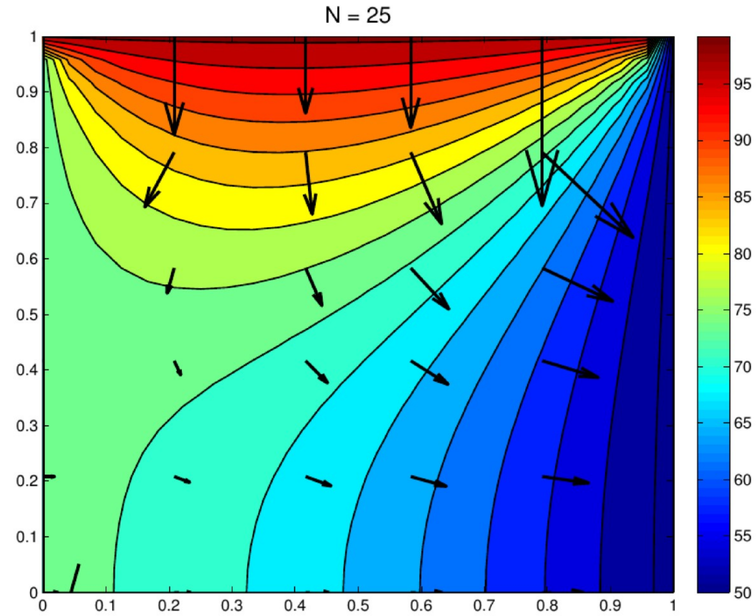
Mapa de colores.

Calcula flujos (propia)

Grafica campo vectorial

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on');
>> [xx,yy,qx,qy]=flujos(N,N,x,y,Tsol);
>> hold on
>> quiver(xx,yy,qx,qy,'color',[0 0 0]);
>>
```

Resultados


N = 25
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
95
90
85
80
75
70
65
60
55
50

Resultados: Escaleo