Ecuaciones Diferenciales en Derivadas Parciales: Método de las Diferencias Finitas

Ecuación Diferencial de la difusión del Calor.

Ec de Fourier:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \quad (1)$$

Discretización de la ecuación diferencial

Dado el nodo de posición (x_i , y_j) , la temperatura T_{ij} cumple:

$$\beta^2 T_{k-N_x}+T_{k-1}-2(1+\beta^2)T_k+T_{k+1}+\beta^2 T_{k+N_x}=0 \quad (2)$$

Donde se usa $k=i+jNx$

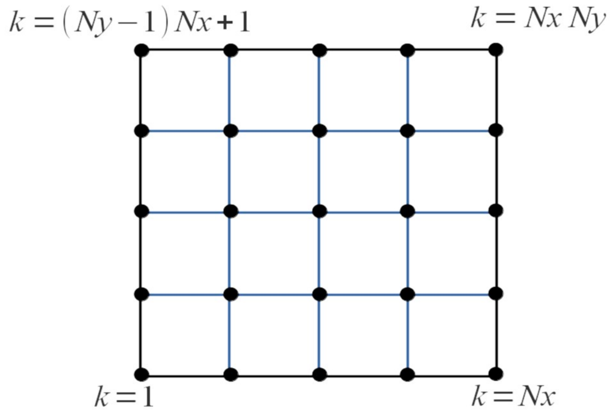

Figura 1: Numeración de los nodos

Condiciones de contorno:

Borde A (vertical izquierdo)

$$k_A=1:N_X:(N_Y-1)N_x \quad (3)$$

Si T fija:

$$T_k = T_A \tag{4}$$

Si flujo fijo,

$$q_{XA} = \frac{\partial T}{\partial x} = \frac{T_{k+1} - T_{\tilde{k}-1}}{2\,dx} \quad \Rightarrow \quad T_{\tilde{k}-1} = T_{k+1} - 2\mathrm{dxq}_{XA}$$

$$\beta^2 T_{k-N_X} - 2\mathrm{dxq}_{XA} - 2\left(1+\beta^2\right) T_k + 2\mathrm{T}_{k+1} + \beta^2 T_{k+N_X} = 0 \quad \Rightarrow$$

$$\boxed{\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\mathrm{T}_{k+1} + \beta^2 T_{k+N_X} = 2\mathrm{dxq}_{XA}} \tag{5}$$

Borde B (vertical derecho):

$$k_B = N_X : N_X : N_X N_Y \tag{6}$$

Si T fija;

$$T_k = T_B \tag{7}$$

Si flujo fijo, ahora es

$$q_{XB} = \frac{\partial T}{\partial x} = \frac{T_{\tilde{k}+1} - T_{k-1}}{2\mathrm{dx}} \quad \Rightarrow \quad T_{\tilde{k}+1} = T_{k-1} + 2\mathrm{dxq}_{XB}$$

$$\boxed{\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\mathrm{T}_{k-1} + \beta^2 T_{k+N_X} = -2\mathrm{dxq}_{XB}} \tag{8}$$

Borde C (horizontal inferior):

$$k_C = 1 : N_X \tag{9}$$

Si T fija,

$$T_k = T_C \tag{10}$$

si flujo fijo,

$$q_{YC}=\frac{\partial T}{\partial y}=\frac{T_{k+N_X}-T_{k-N_X}}{2\mathrm{dy}} \quad \Rightarrow \quad T_{\tilde{k}-N_X}=T_{k+N_X}-2\mathrm{dxq}_{YC}$$

$$\beta^2\left(T_{k+N_X}-2\mathrm{dxq}_{YC}\right)+T_{k-1}-2\left(1+\beta^2\right)T_k+T_{k+1}+\beta^2 T_{k+N_X}=0$$

$$\boxed{T_{k-1}-2\left(1+\beta^2\right)T_k+T_{k+1}+2\beta^2 T_{k+N_X}=2\beta^2 dxq_{YC}} \tag{11}$$

Borde D (horizontal):

$$k_D=\left(N_Y-1\right)N_X+1:N_X N_Y \tag{12}$$

Si T fija,

$$T_k=T_D \tag{13}$$

Si flujo fijo:

$$q_{YC}=\frac{\partial T}{\partial y}=\frac{T_{k+N_X}-T_{k-N_X}}{2\mathrm{dy}} \quad \Rightarrow \quad T_{\tilde{k}+N_X}=T_{k-N_X}+2\mathrm{dxq}_{YC}$$

$$\boxed{2\beta^2 T_{k-N_X}+T_{k-1}-2\left(1+\beta^2\right)T_k+T_{k+1}=-2\beta^2 dyq_{YD}} \tag{14}$$