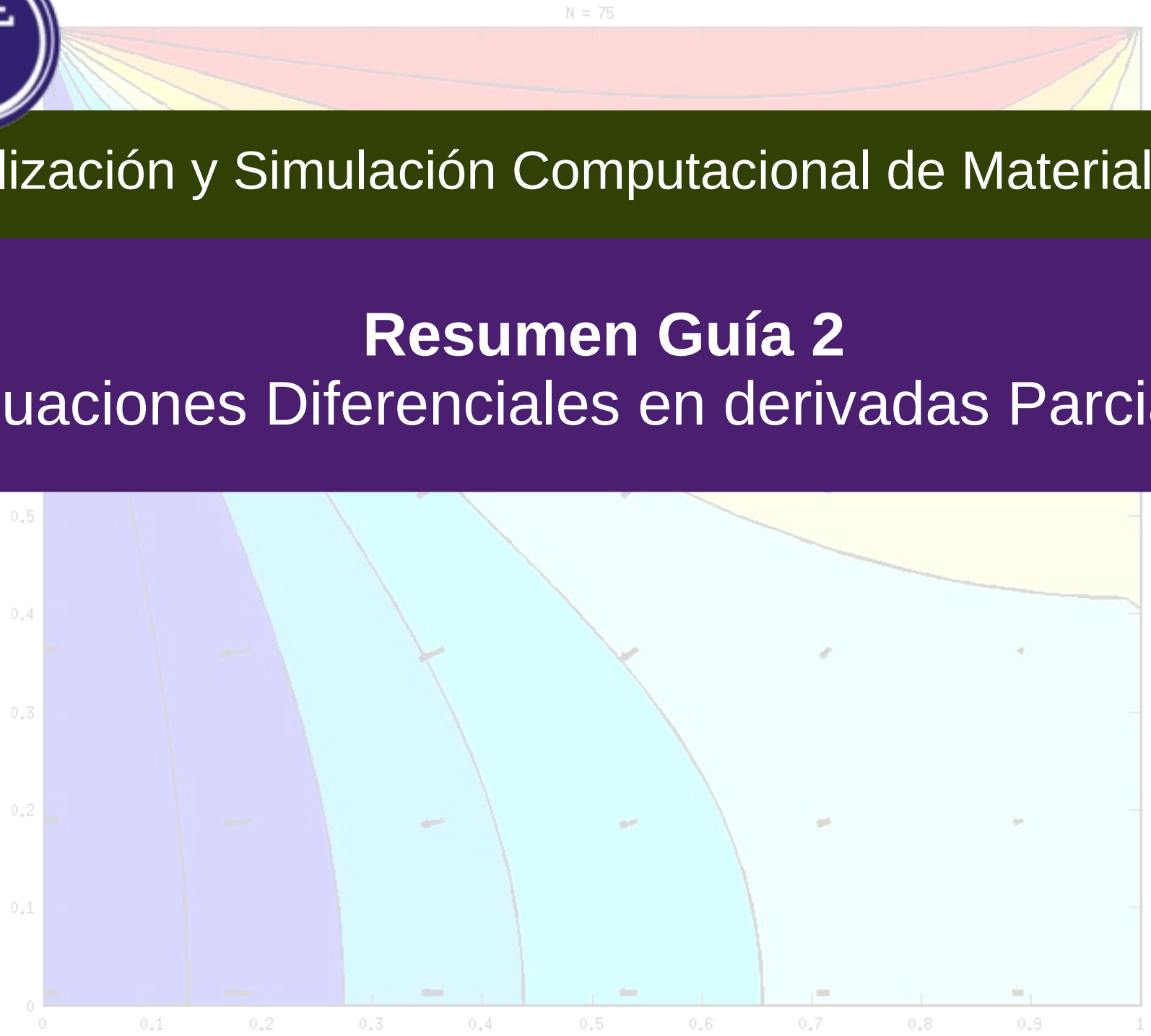
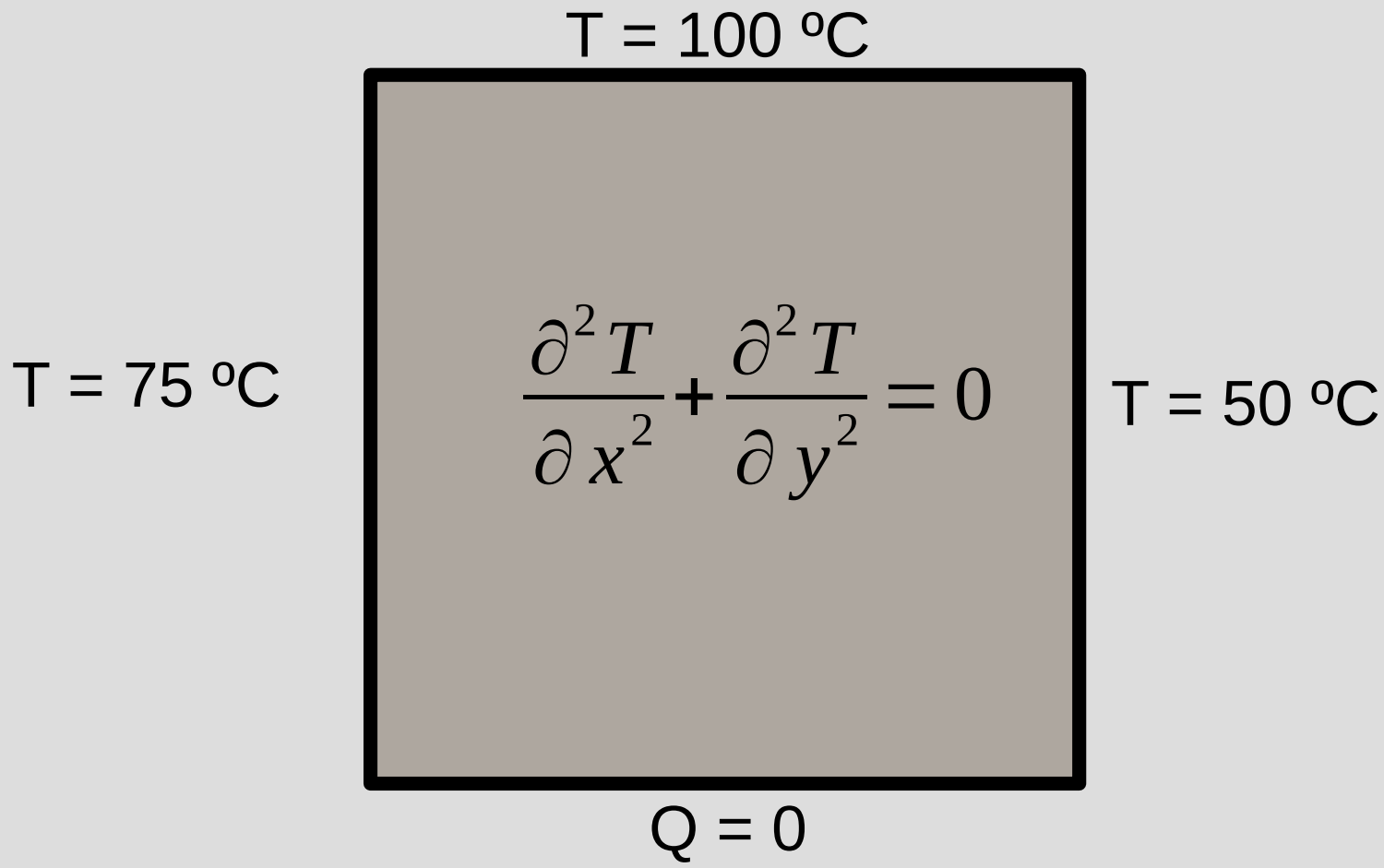
Modelización y Simulación Computacional de Materiales 2017

Resumen Guía 2

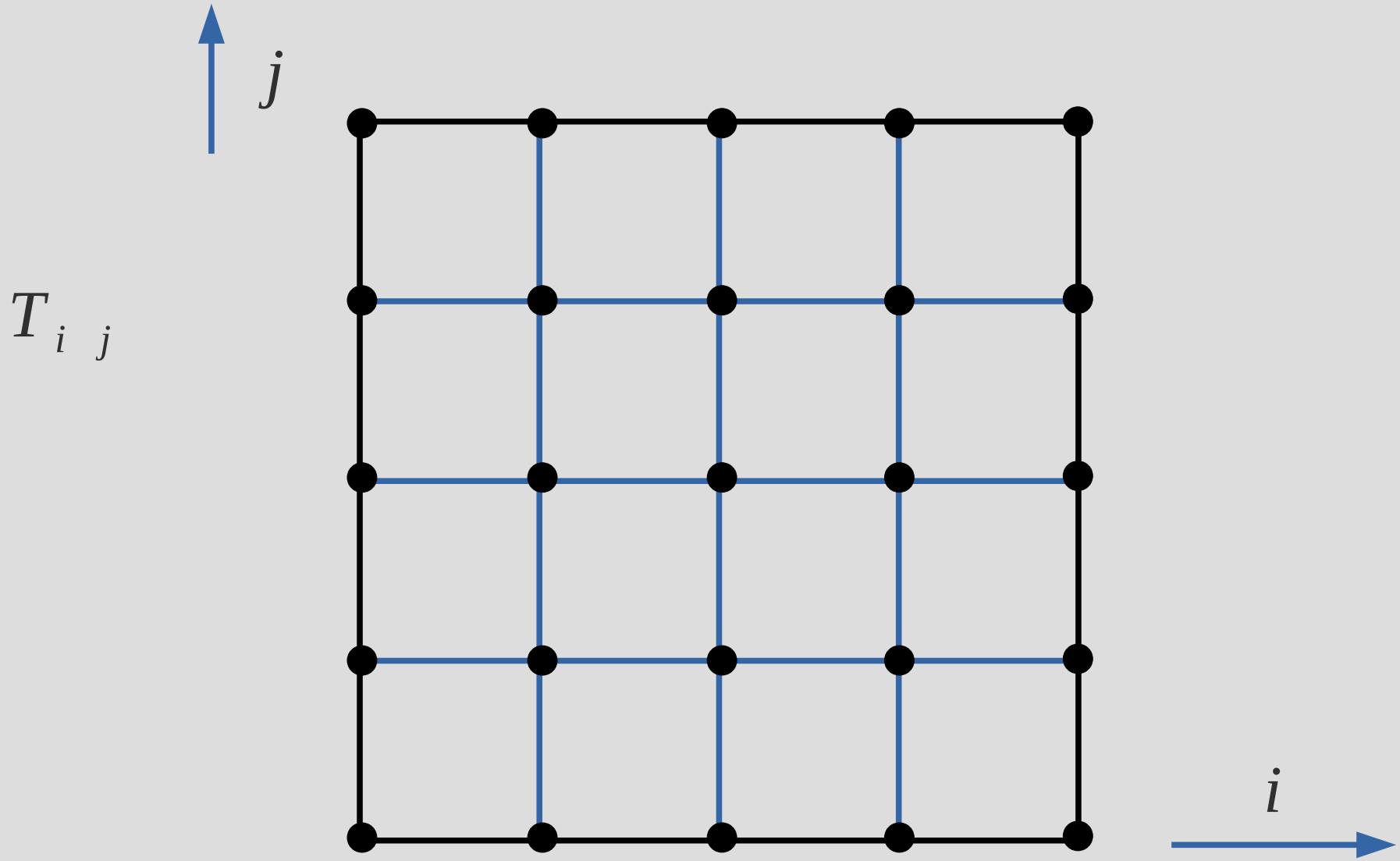
Ecuaciones Diferenciales en derivadas Parciales.



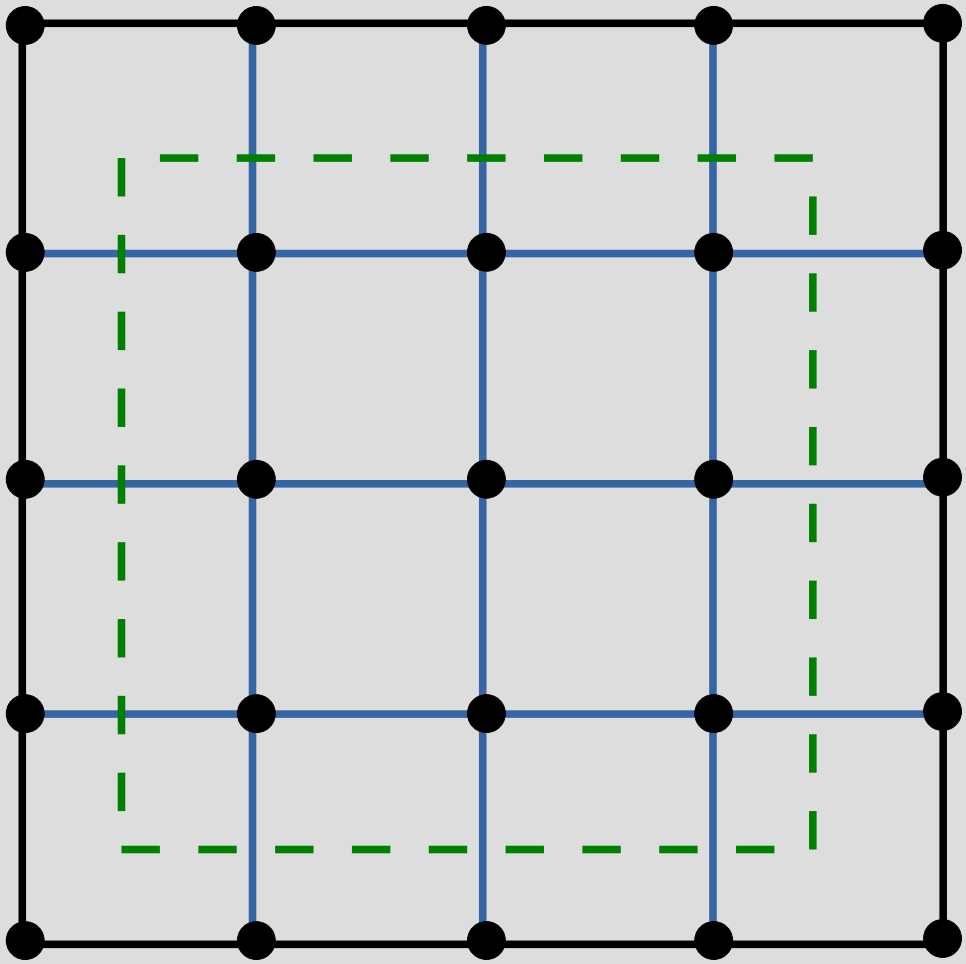
Ejercicio 1


T = 100 ºC
T = 75 ºC
$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$
T = 50 ºC
Q = 0

Ejercicio 1 - Discretización



Ejercicio 1 - Ecuación General

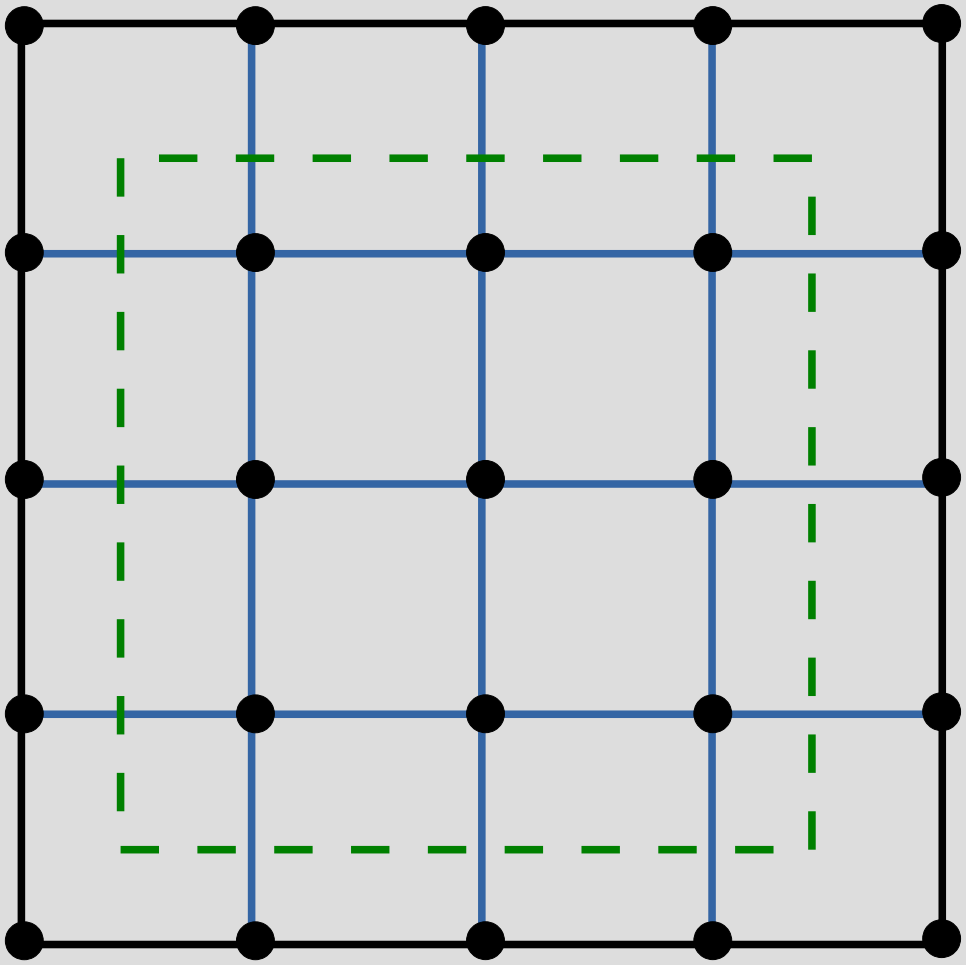


$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{T_{i-1,j}-2T_{i,j}+T_{i+1,j}}{dx^2}$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{T_{i,j-1}-2T_{i,j}+T_{i,j+1}}{dy^2}$$

Ejercicio 1 - Ecuación Matricial

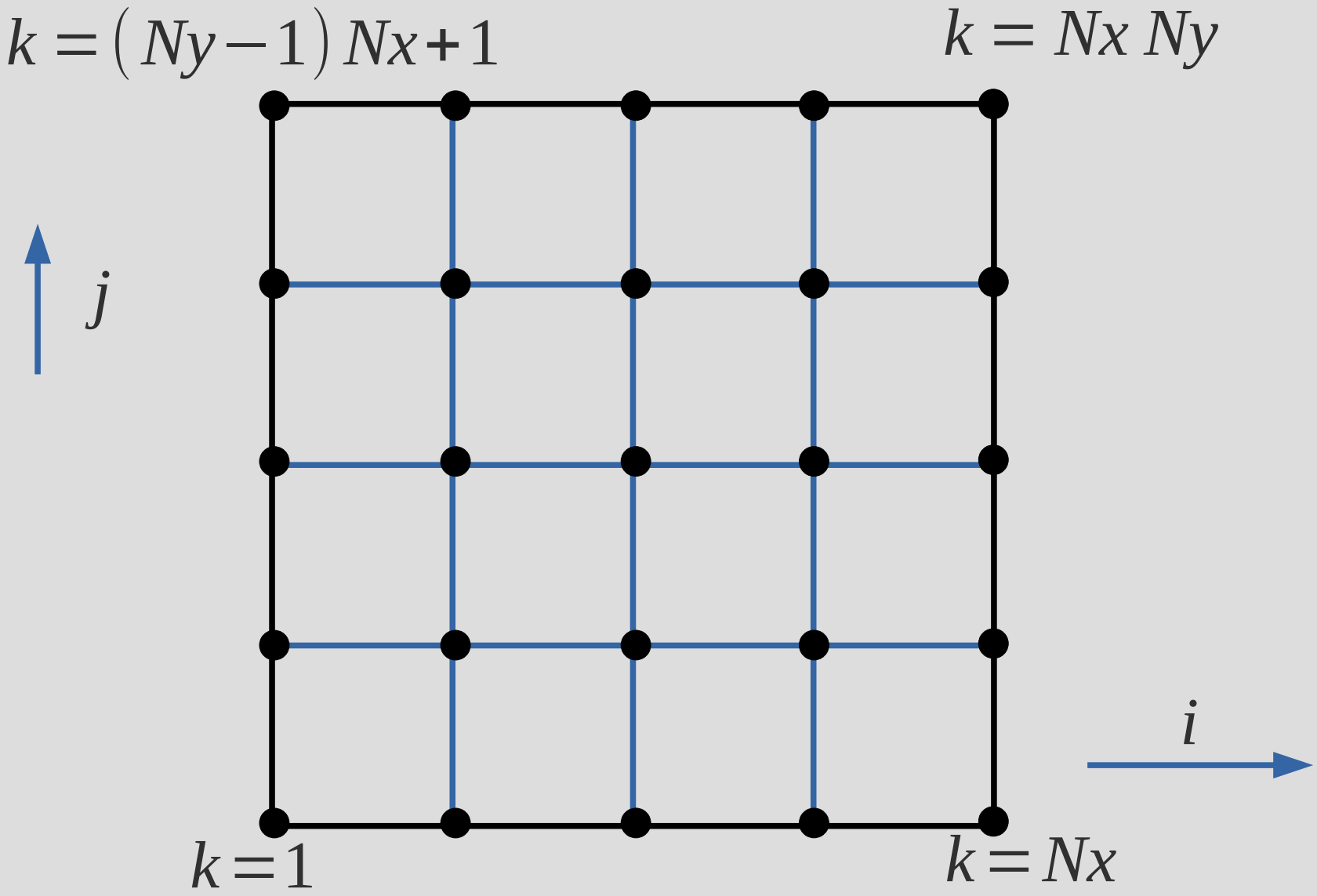


$$T_{i,j} = \boldsymbol{T}_k$$

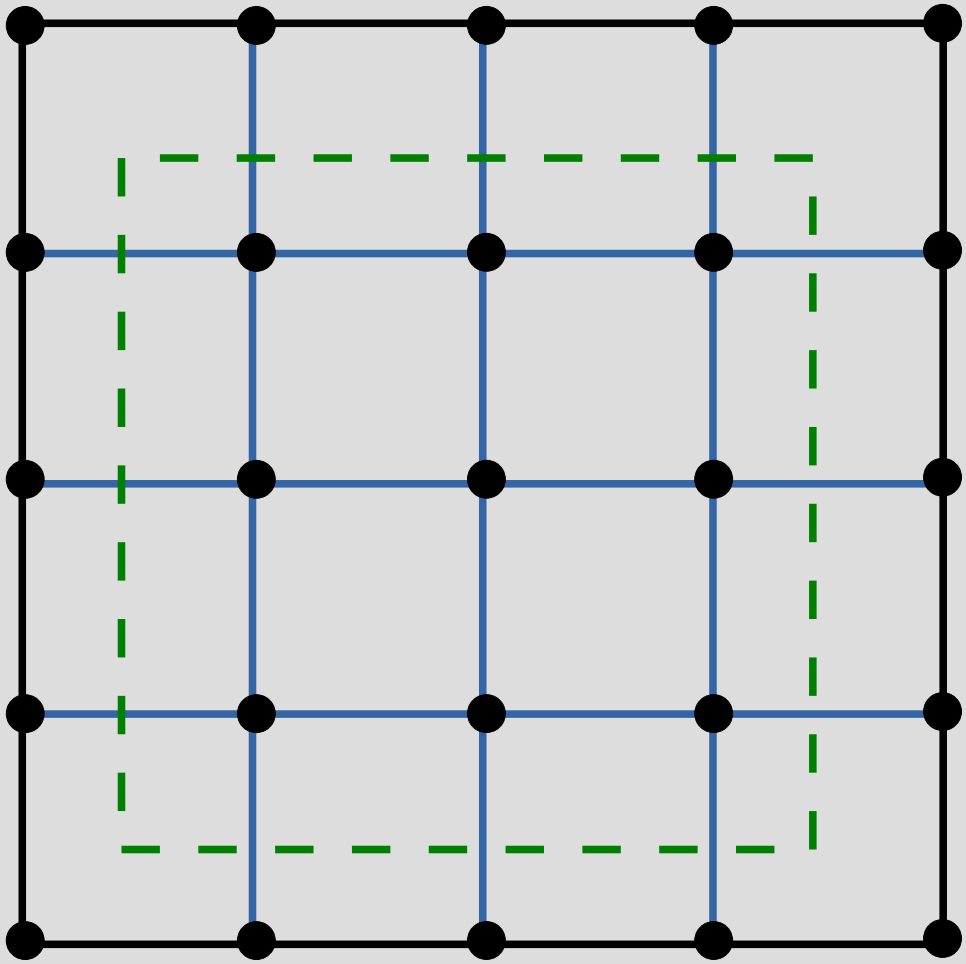
$$k = i + (j-1)\,Nx$$

$$T_k = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{NxNy} \end{pmatrix}$$

Ejercicio 1 - Numeración de Nodos



Ecuación General



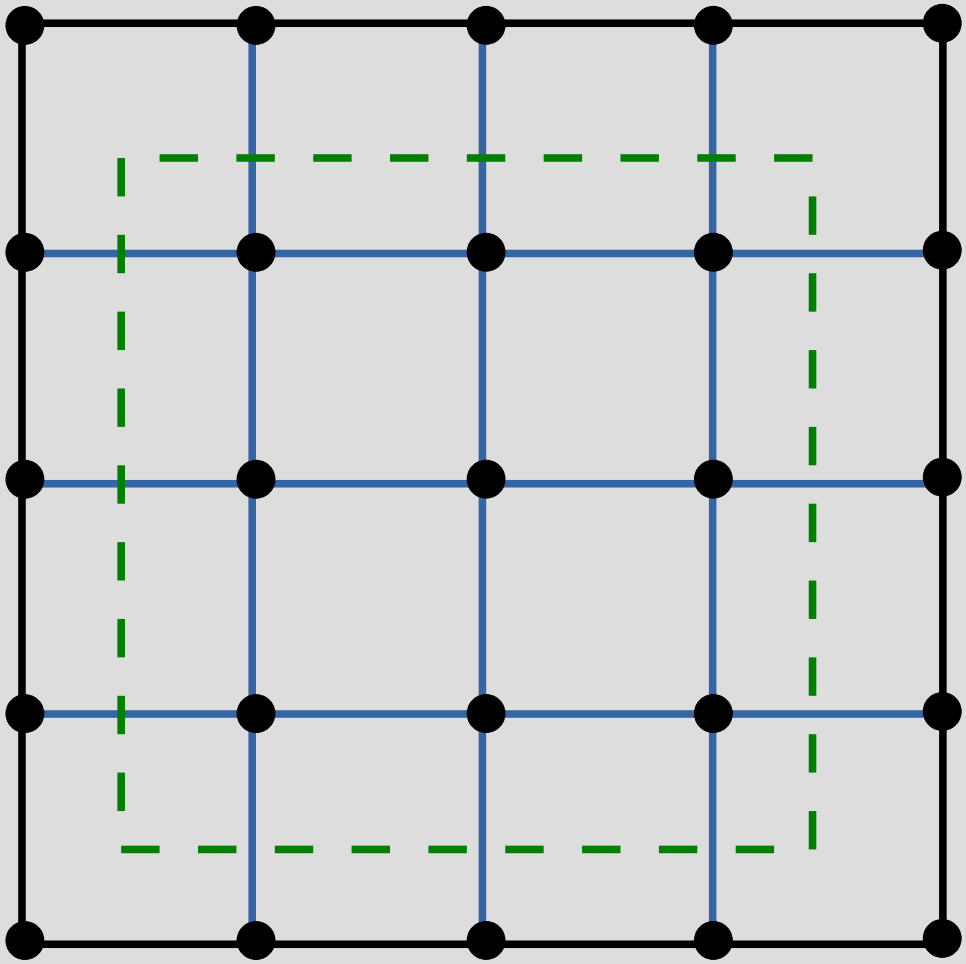
$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{T_{k-1}-2T_k+T_{k+1}}{dx^2}$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{T_{k-Nx}-2T_k+T_{k+Nx}}{dy^2}$$

Ecuación General

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$



$$\left[M \right] \begin{pmatrix} T_1 \\ \vdots \\ T_k \\ \vdots \\ T_{NxNy} \end{pmatrix} = b$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Ecuación General

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$

Fila k-ésima:

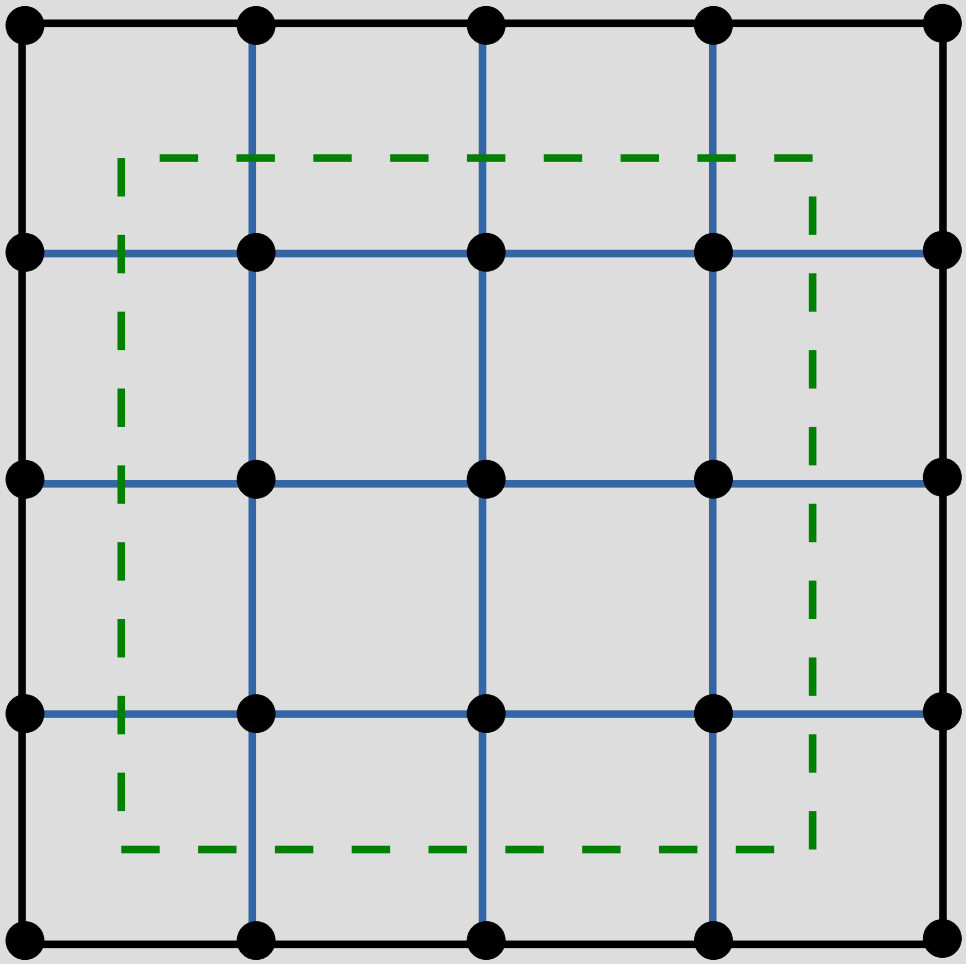
$$M_{k,:} = \left[\cdots \quad \beta^2 \quad \cdots \quad 1 \quad -2\left(1+\beta^2\right) \quad 1 \quad \cdots \quad \beta^2 \quad \cdots \right]$$

$$k-Nx \qquad k-1 \qquad k \qquad k+1 \qquad k+Nx$$

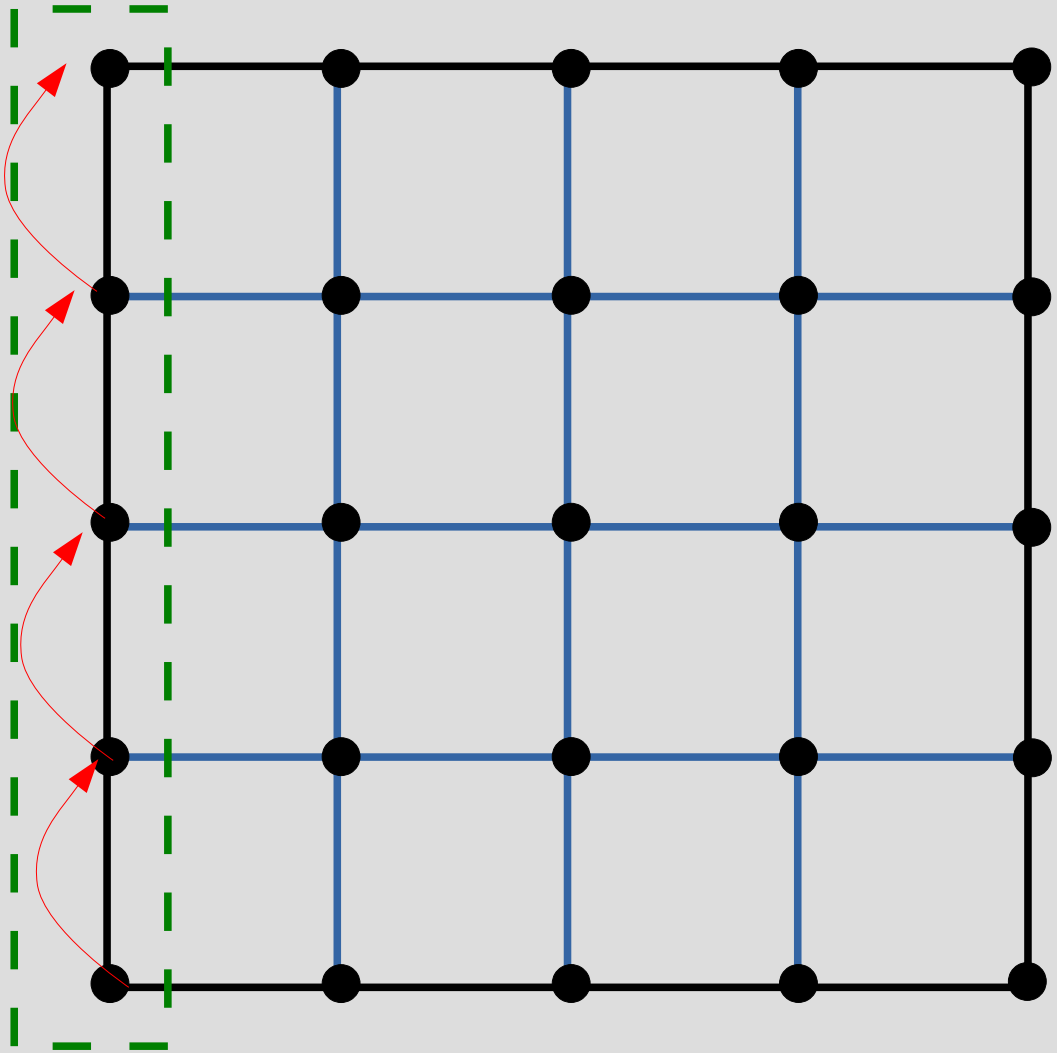
$$b_k = 0$$

Ecuación General - Discretización

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$



Condicion de contorno – Temperatura fija



$$T_{k_A} = T_A$$

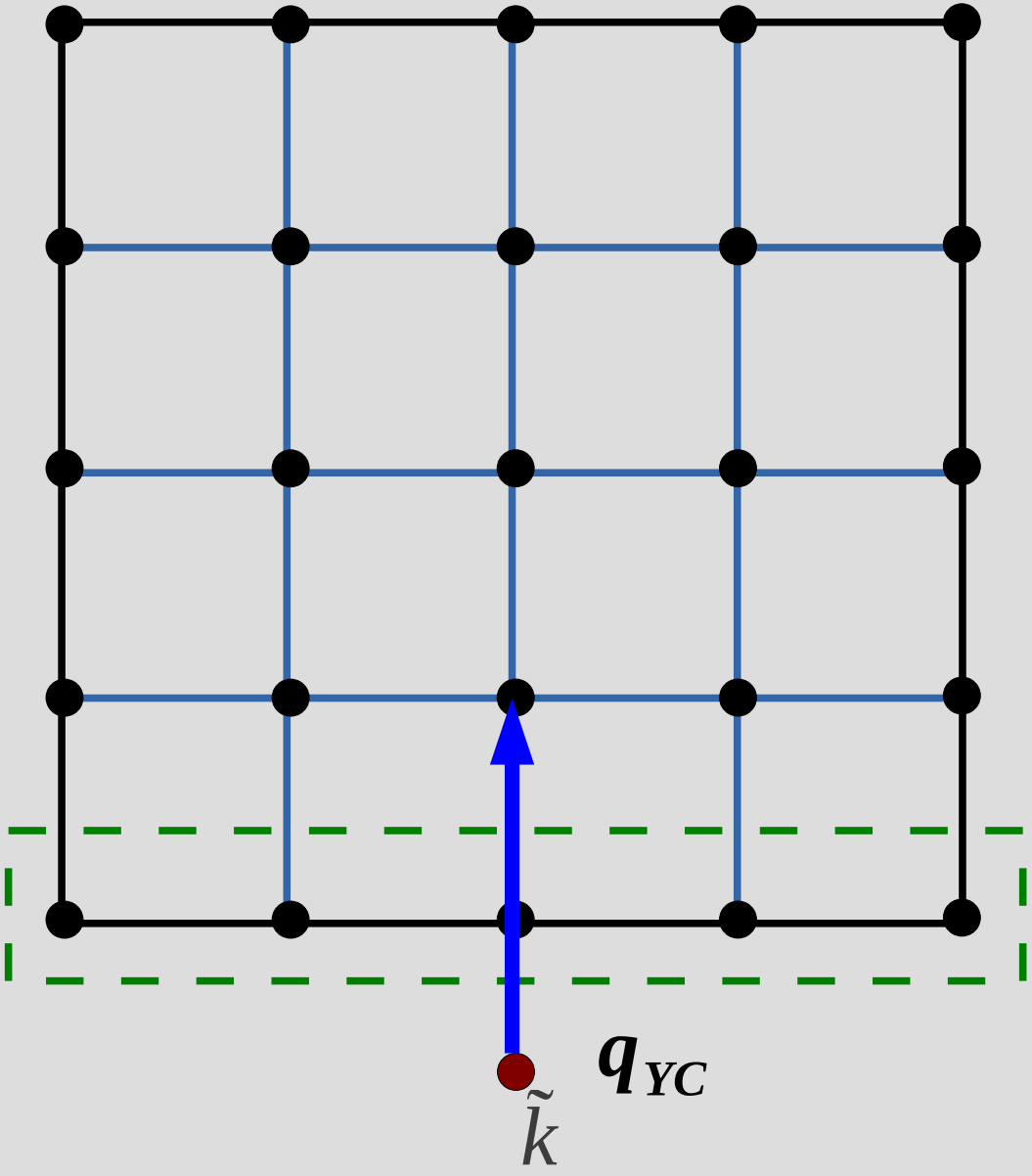
$$k_A = 1 : N_X : (N_Y - 1) N_x$$

$$M^{fija}_{k_A,:} = \begin{bmatrix} \cdots & 1 & \cdots \end{bmatrix}$$

$$k_A$$

$$b_{k_A} = T_A$$

Condicion de contorno – Flujo dado



$$Q_y \propto q_{YC} = \left.\frac{\partial T}{\partial y}\right|_{k_C} = \frac{T_{k_A+N_x} - T_{\tilde{k}}}{2\,dy}$$

$$k_C = 1 : N_X$$

Condicion de contorno – Flujo dado

$$T_{\tilde{k}} = T_{k_C + N_x} - 2\,dx q_{YC} \qquad k_A = 1 : N_X : \left(N_Y - 1\right) N_x$$

Reemplazo en la ecuación general

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$

Reordeno

$$T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + 2\beta^2 T_{k+N_x} = 2\beta^2 dx q_{YC}$$

Condicion de contorno – Flujo dado

$$T_{\tilde{k}-1} = T_{k_A+1} - 2\mathrm{dxq}_{XA} \qquad k_A = 1 : N_X : \left(N_Y - 1\right) N_x$$

$$T_{k-1} - 2\left(1+\beta^2\right)T_k + T_{k+1} + 2\beta^2 T_{k+N_x} = 2\beta^2 dxq_{YC}$$

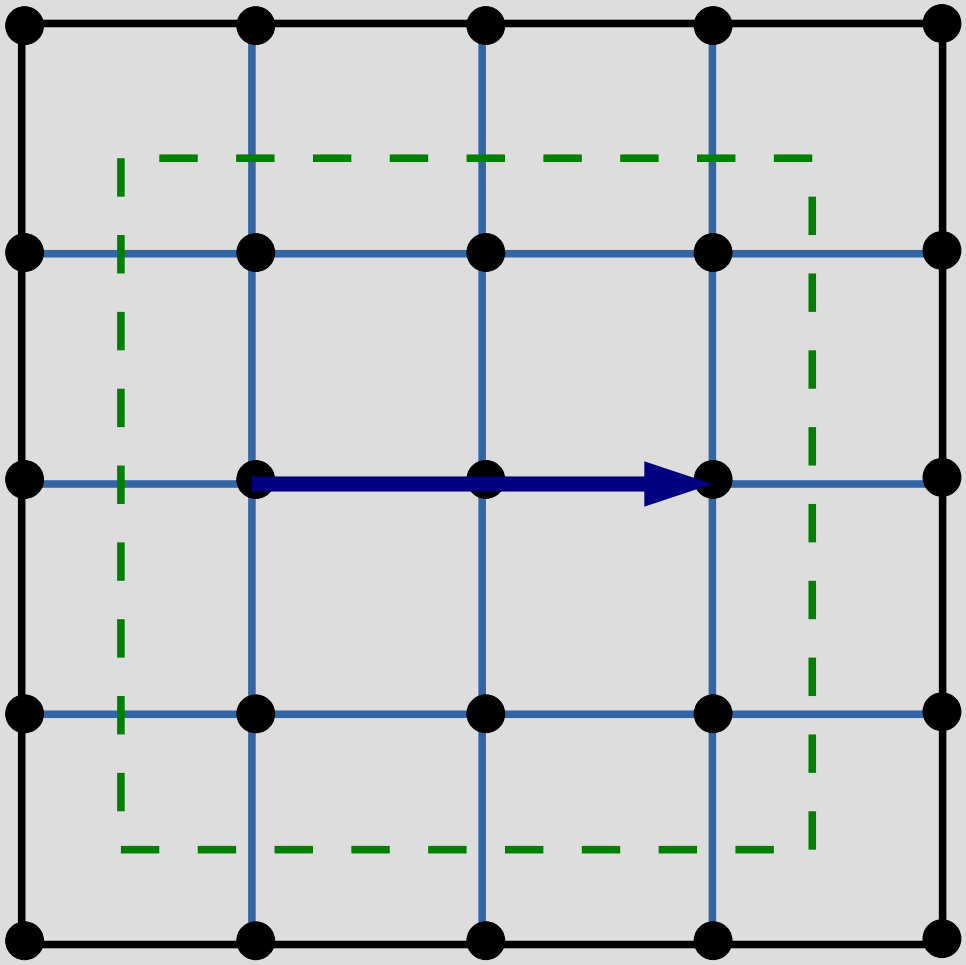
Fila k-ésima :

$$M_{k_A,:} = \left[\cdots \;\; 0 \;\; \cdots \;\; 1 \;\; -2\left(1+\beta^2\right) \;\; 2 \;\; \cdots \;\; \beta^2 \;\; \cdots\right]$$

$$k - Nx \qquad k-1 \qquad k \qquad k+1 \qquad k+Nx$$

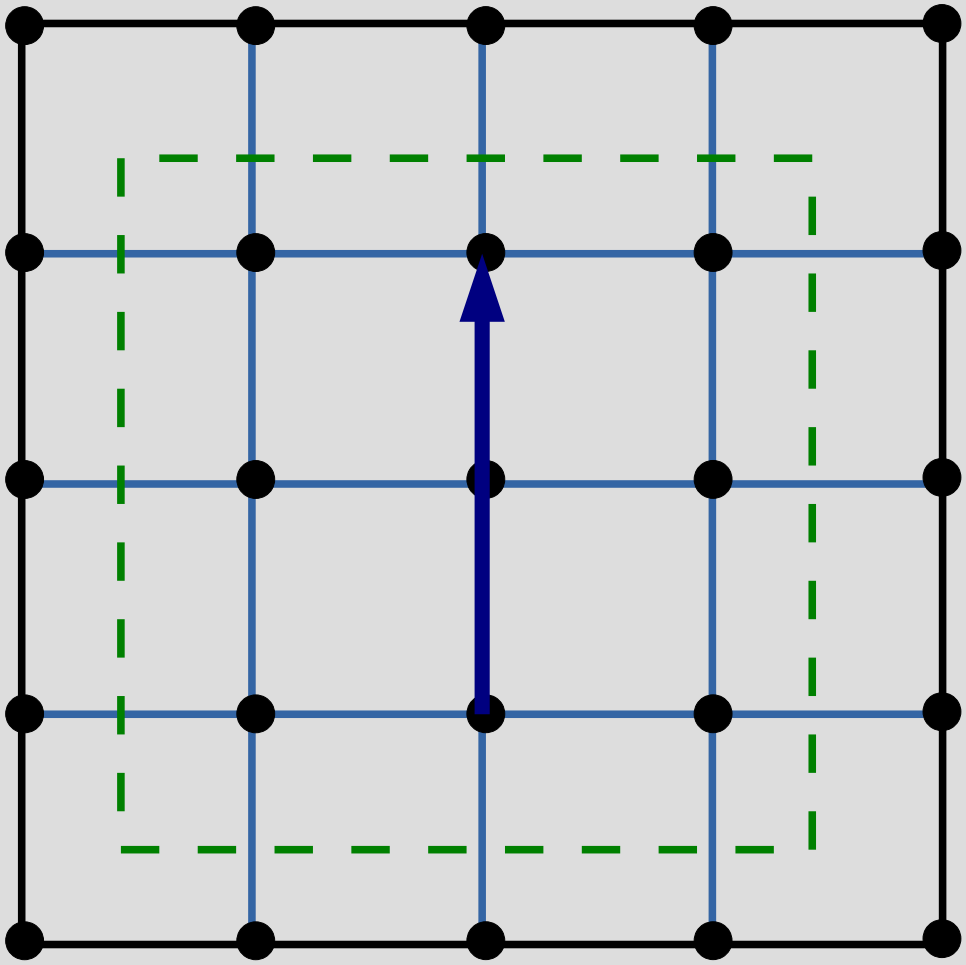
$$b_k = 2\mathrm{dxq}_{XA}$$

Cálculo de flujos



$$Q_x \propto q_X = \frac{\partial T_k}{\partial x} = \frac{T_{k+1} - T_{k-1}}{2\mathrm{dx}}$$

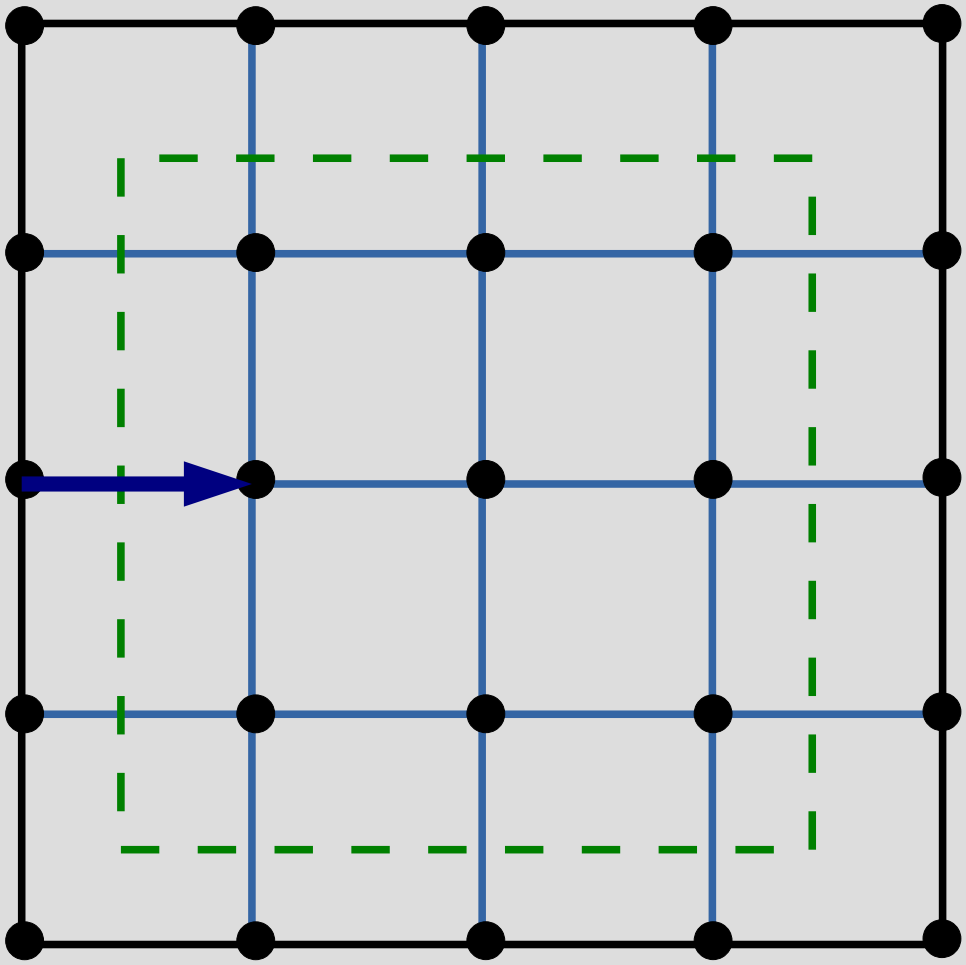
Cálculo de flujos



$$Q_y \propto q_Y =$$

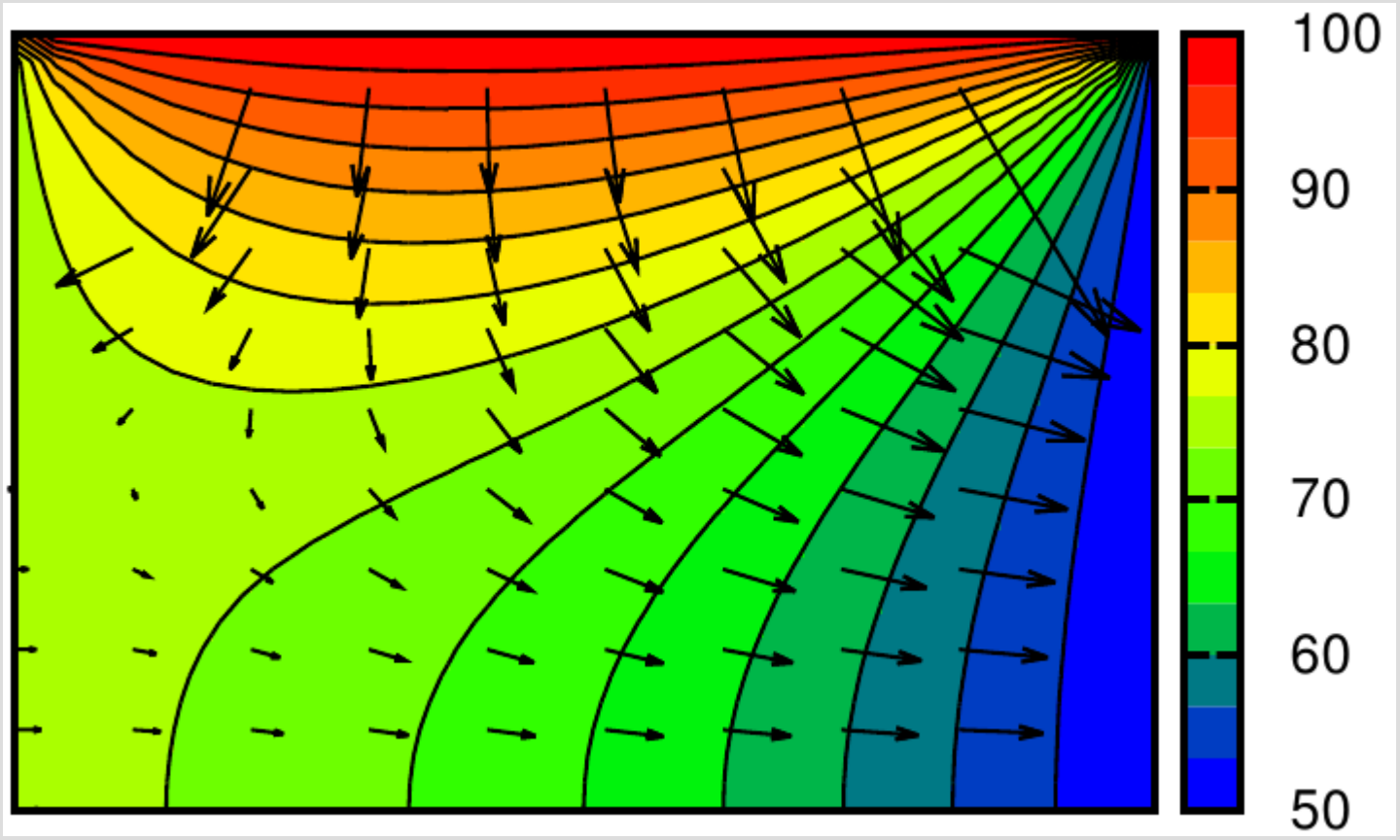
$$\frac{\partial T_k}{\partial y} = \frac{T_{k+Nx} - T_{k-Nx}}{2\mathrm{dy}}$$

Cálculo de flujos

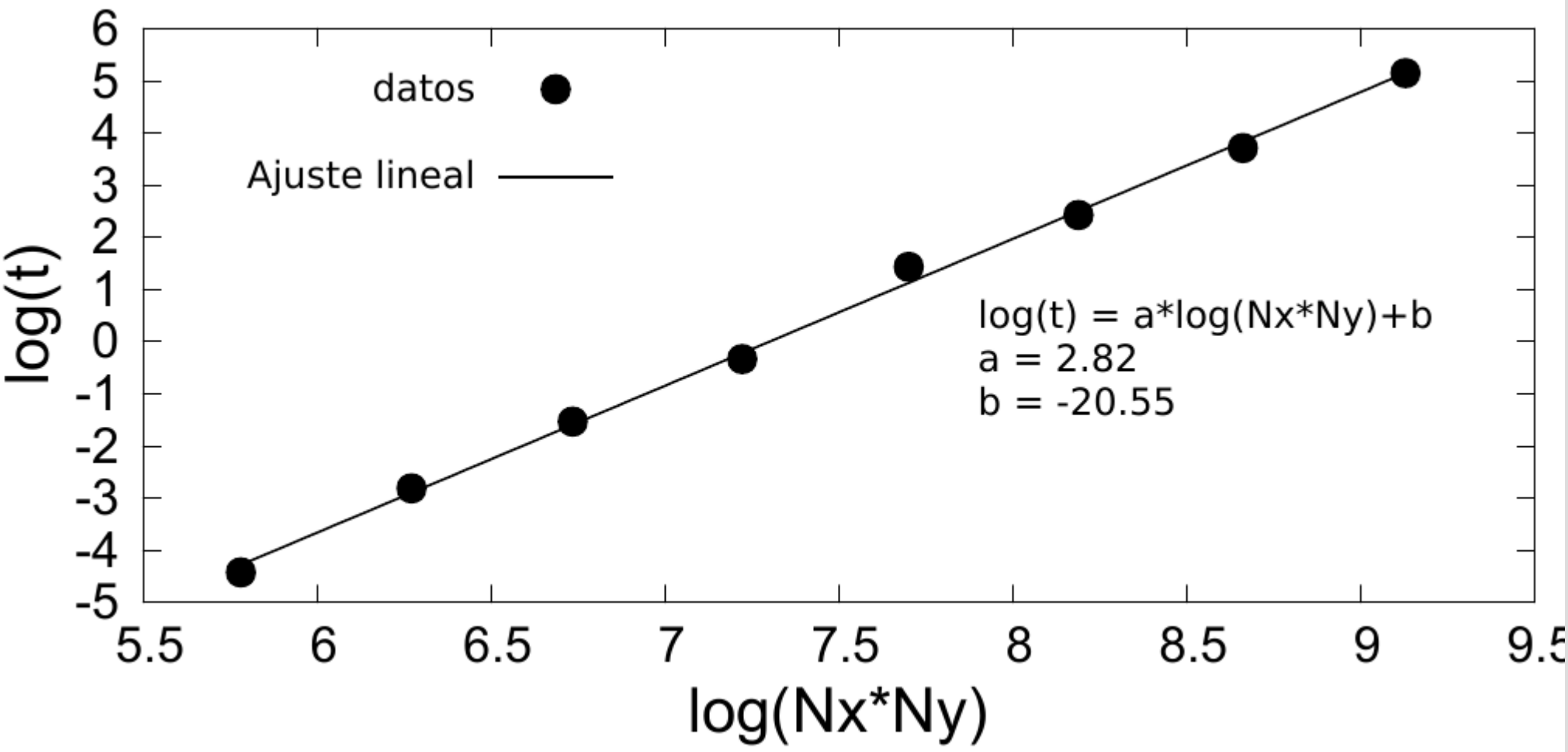


$$Q_{yA} \propto q_{YA} = \frac{\partial T_{k_A}}{\partial x} = \frac{T_{k_A+1} - T_k}{dx}$$

Solución

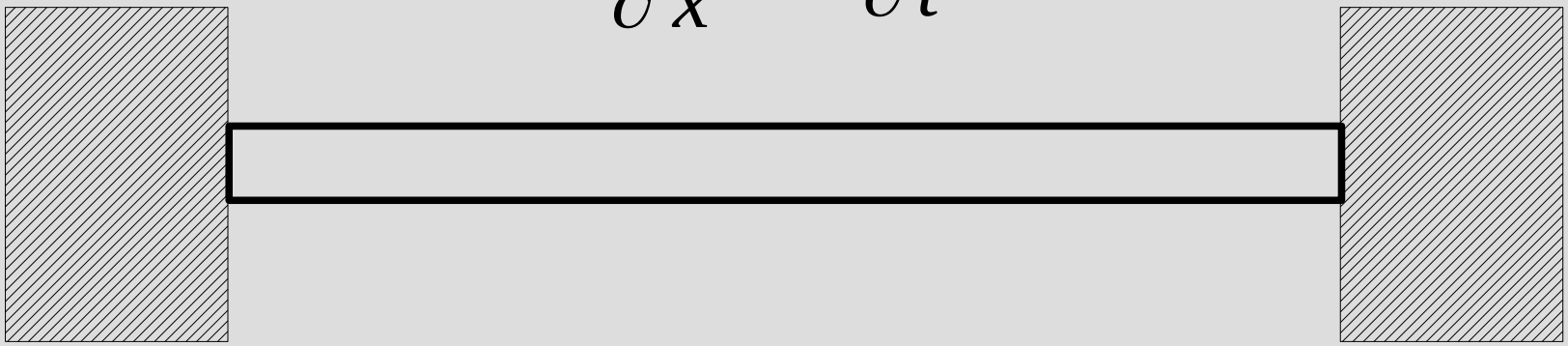


Solución


datos
Ajuste lineal
log(t) = a*log(Nx*Ny)+b
a = 2.82
b = -20.55
log(t)
log(Nx*Ny)
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
5.5
6
6.5
7
7.5
8
8.5
9
9.5

Ejercicio 2

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

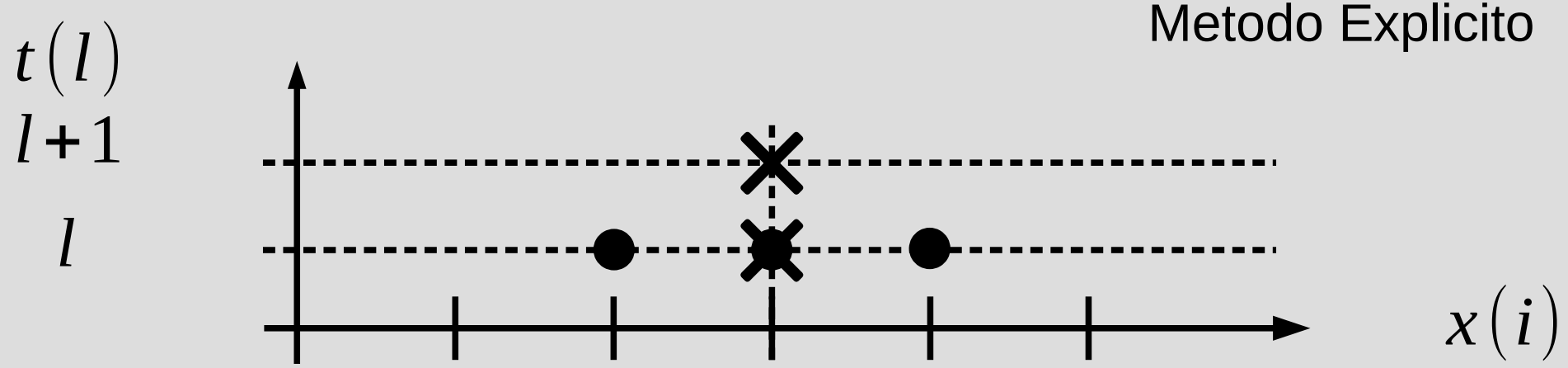


$T_{IZQ} = 100ºC$

$T_{DER} = 50ºC$

Ejercicio 2 - Discretización

Metodo Explicito



$$\frac{\partial^2 T^l}{\partial x^2} = \frac{T^l_{i-1} - 2T^l_i + T^l_{i+1}}{dx^2}$$

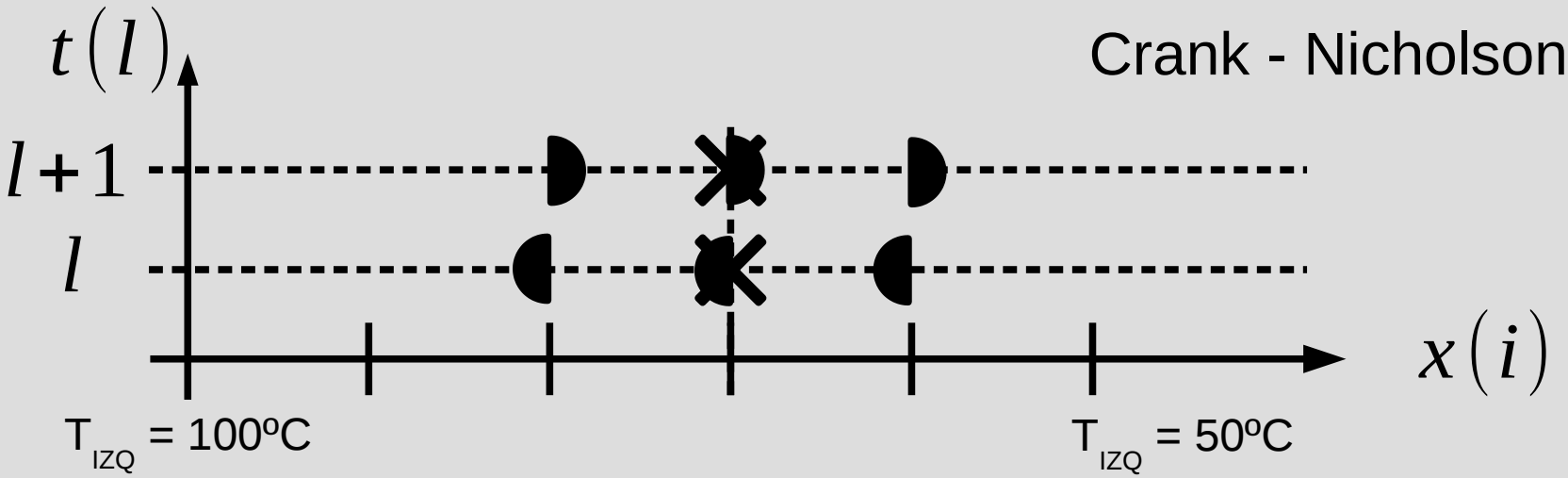
$$\frac{\partial T}{\partial t} = \frac{T^{l+1}_i - T^l_i}{dt}$$

$$T^{l+1}_i = \lambda T^l_{i-1} + (1-2\lambda) T^l_i + \lambda T^l_{i+1}$$

$$\lambda = \frac{k \ dt}{dx^2}$$

Ejercicio 2 - Discretización

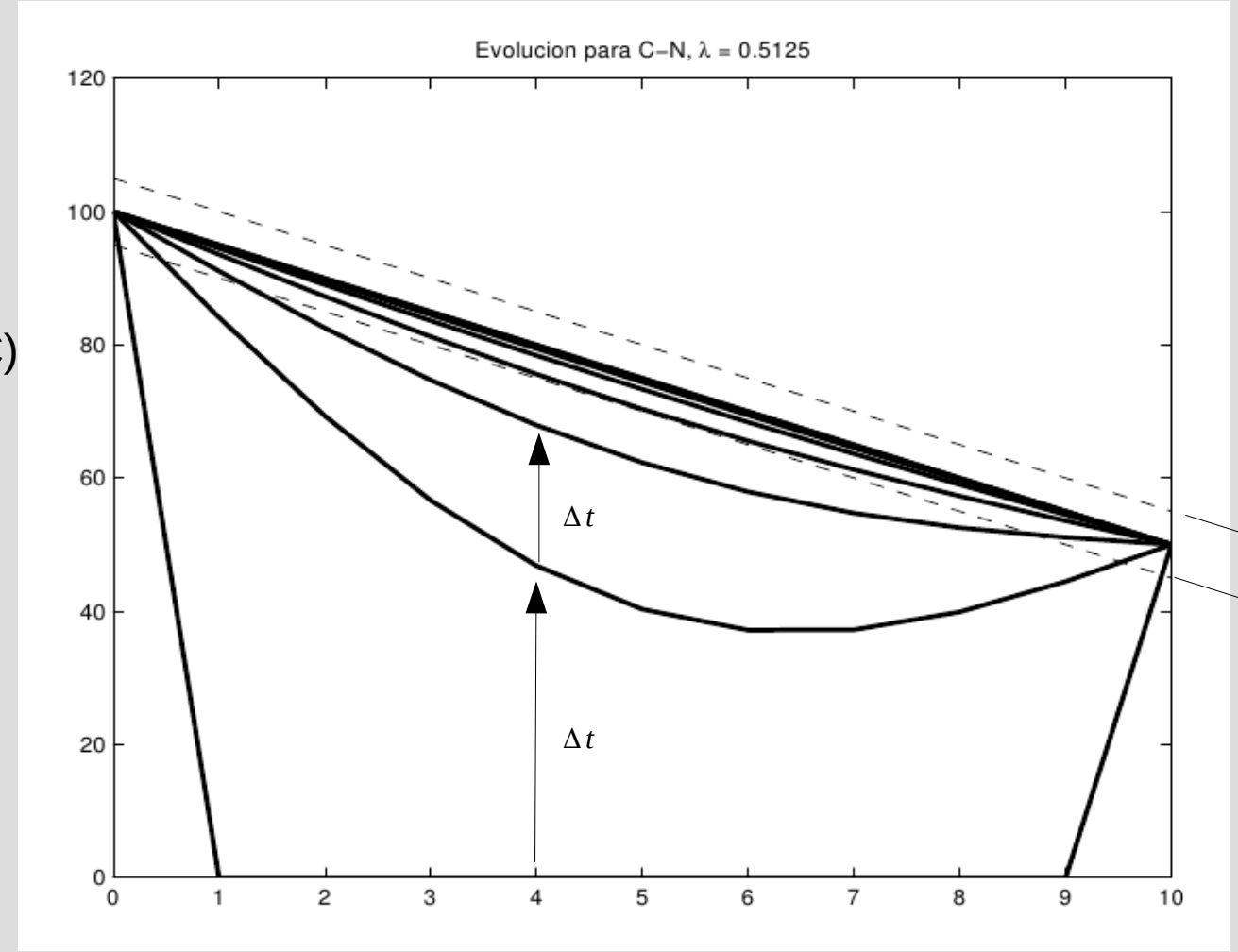
Crank - Nicholson



$$\left.\begin{aligned} \frac{\partial^2 T^l}{\partial x^2} &= \frac{1}{2}\left(\frac{T_{i-1}^{l+1}-2T_i^{l+1}+T_{i+1}^{l+1}}{dx^2}+\frac{T_{i-1}^{l}-2T_i^{l}+T_{i+1}^{l}}{dx^2}\right) \\ \frac{\partial T}{\partial t} &= \frac{T_i^{l+1}-T_i^{l}}{dt} \end{aligned}\right\} \bullet \times$$

$$-\lambda T_{i-1}^{l+1}+2(1+\lambda)T_i^{l+1}-\lambda T_{i+1}^{l+1}=\lambda T_{i-1}^{l}+2(1-\lambda)T_i^{l}+\lambda T_{i+1}^{l}$$

Ejercicio 2 – Estado estacionario y flujos

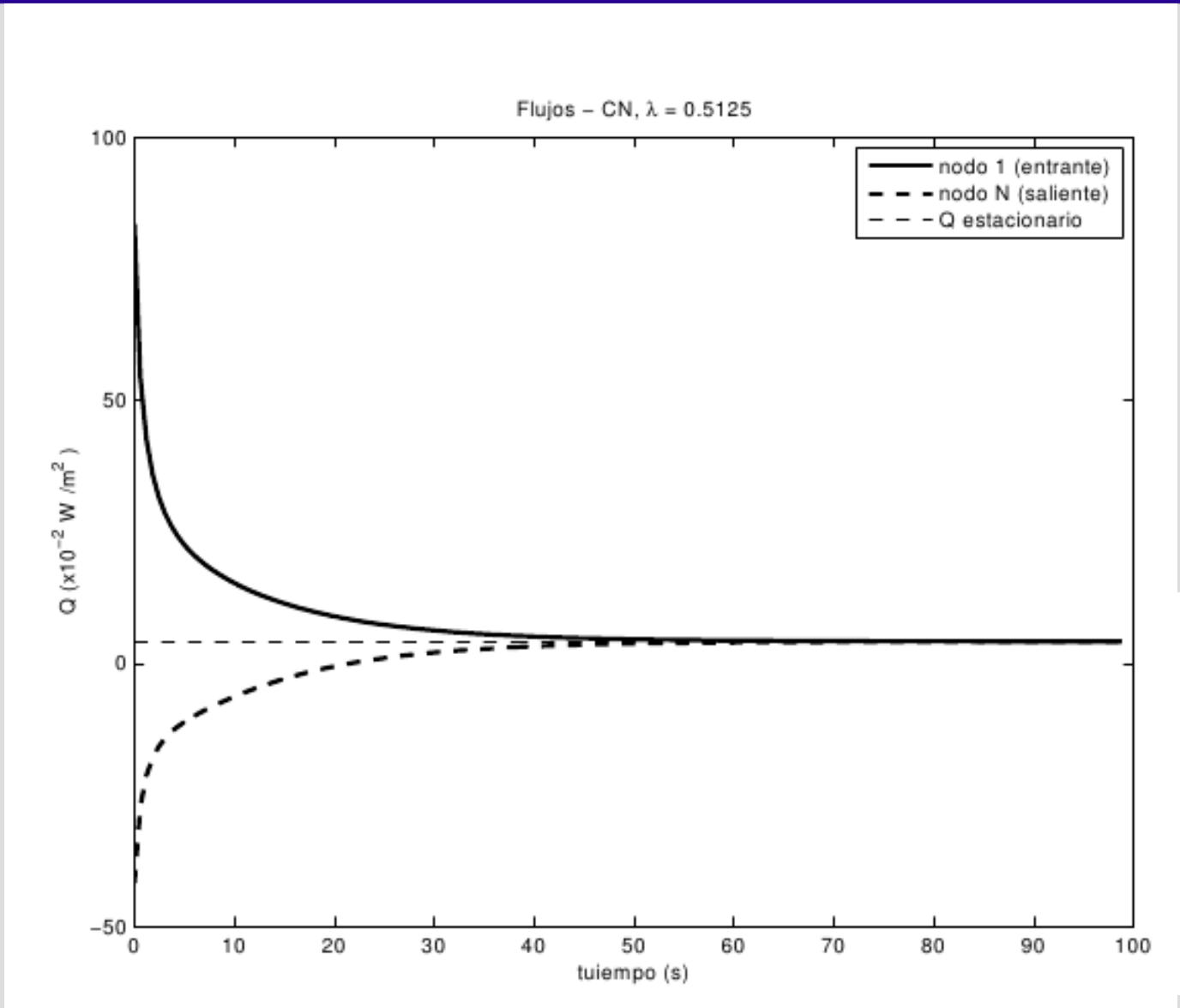


T (ºC)

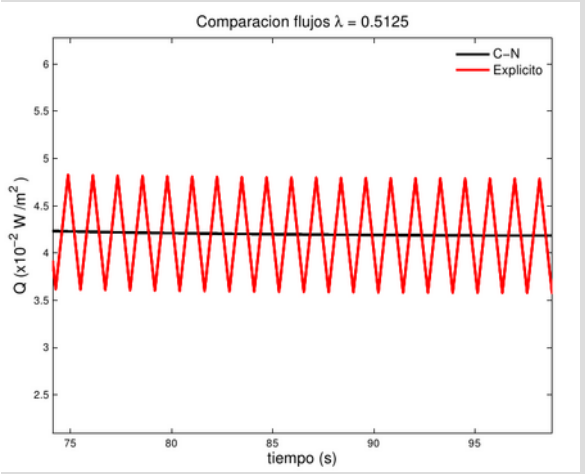
tolerancia

x (cm)

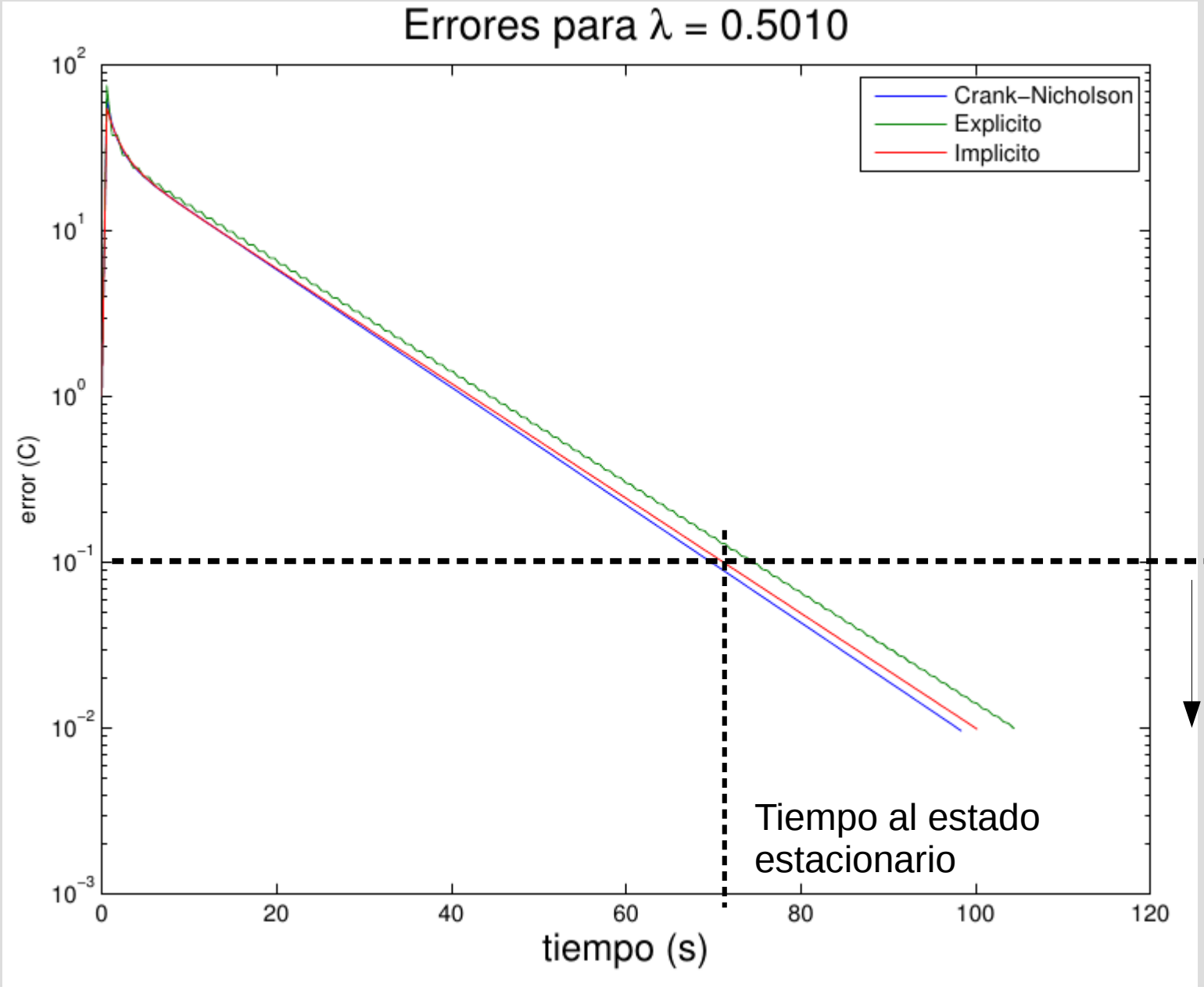
Ejercicio 2 – Estado estacionario y flujos



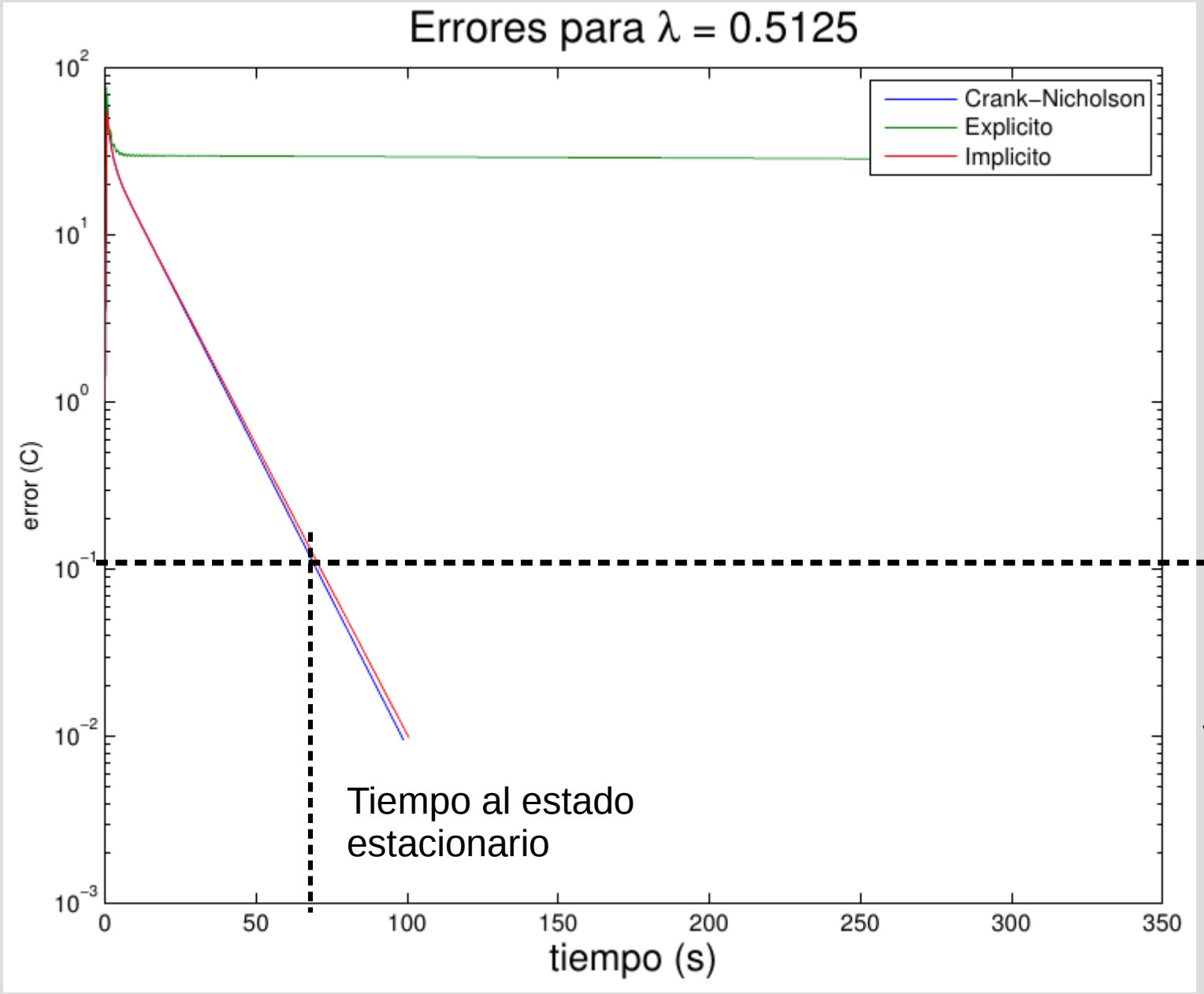
$$Q = -k \times \frac{\partial T}{\partial x}$$



Ejercicio 2 – Errores , estabilidad y convergencia



Ejercicio 2 – Errores , estabilidad y convergencia



Ejercicio 2 – Errores , estabilidad y convergencia