Instituto Jorge Sabato, 25 años
Comisión Nacional de Energía Atómica

Modelización de Materiales 2018

Resumen de la guía 2

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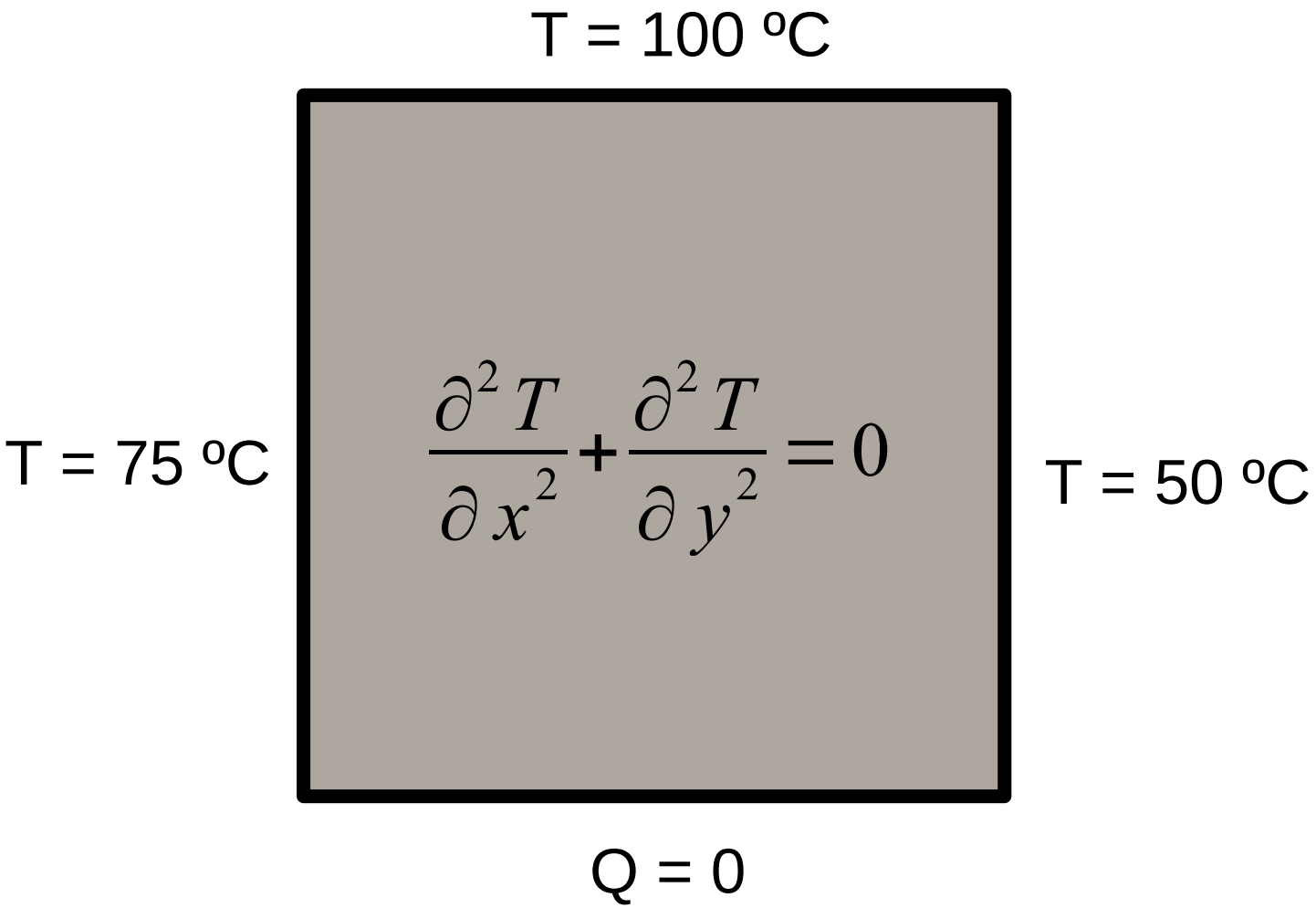
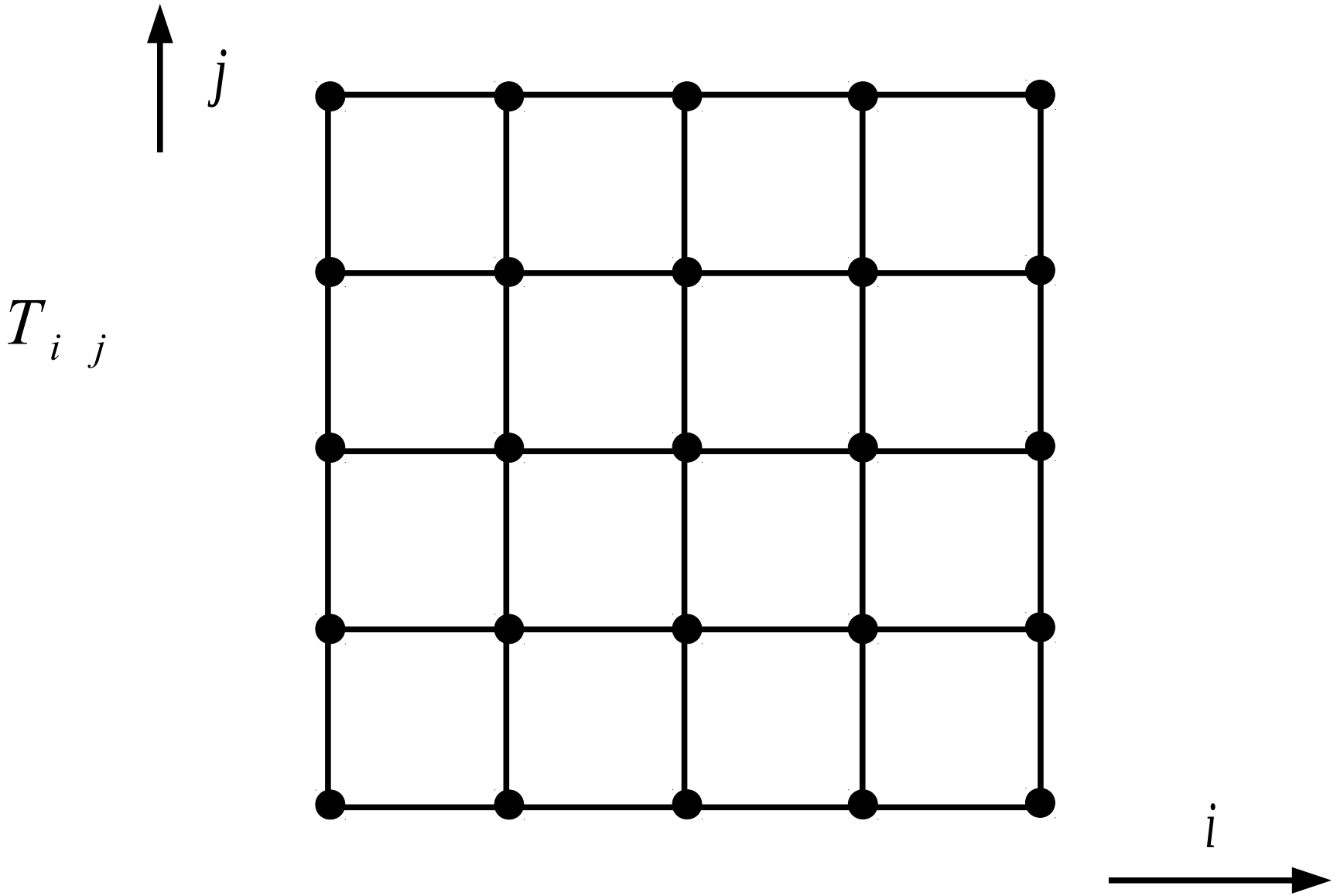
https://mdforti.github.io/Modelizacion/

Ejercicio 1

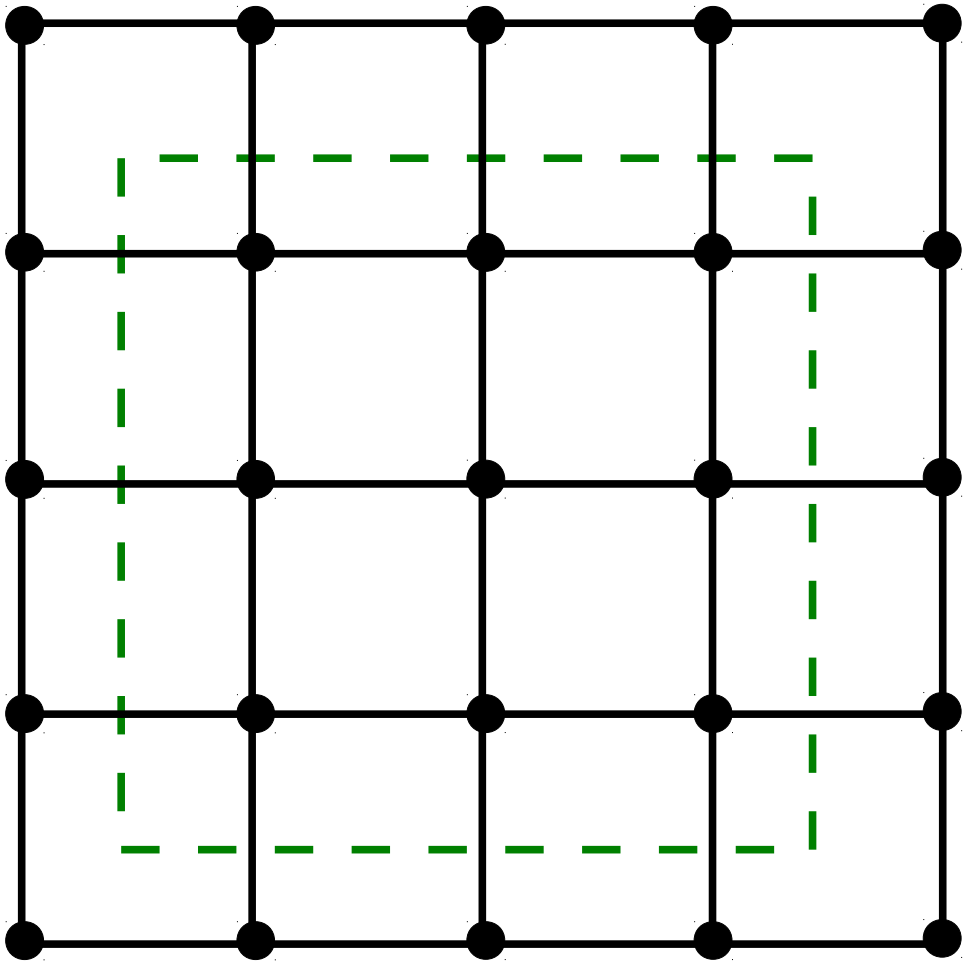
Presentación del Problema



Discretización del recinto



Ecuación General

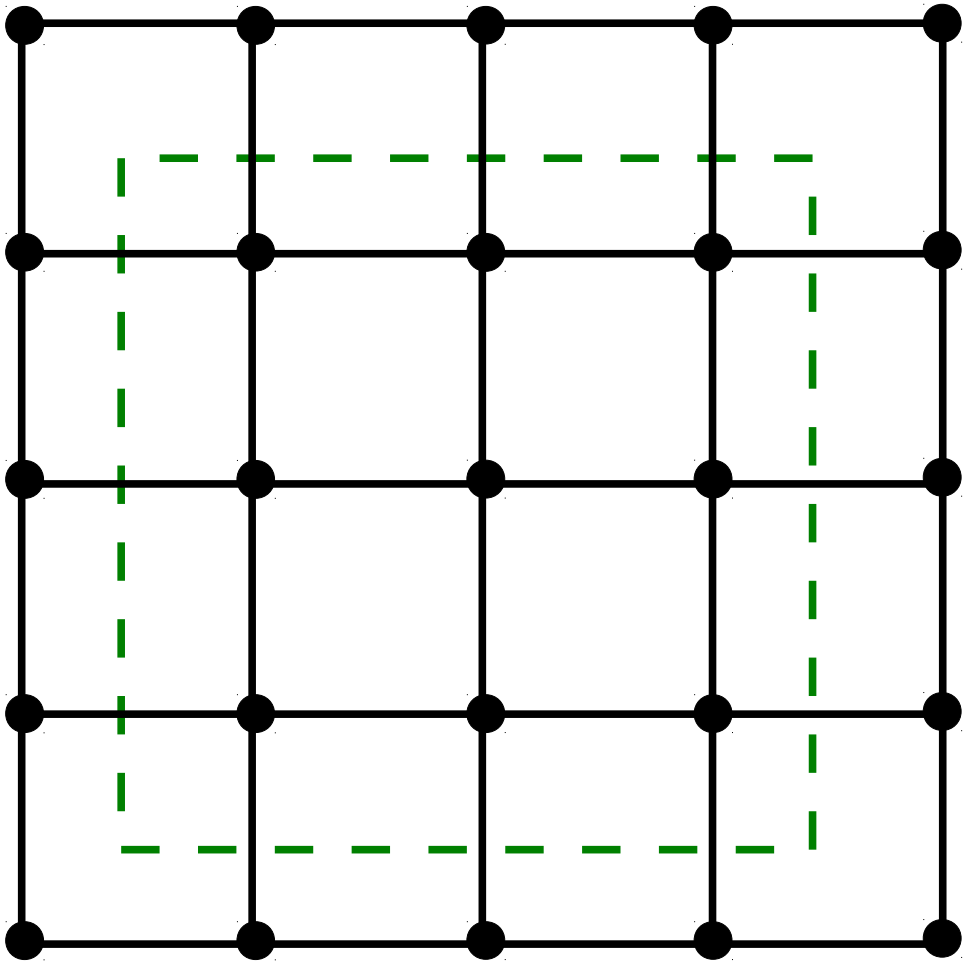


$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{dx^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{dy^2}$$

Ecuación Matricial



$$T_{i,j} = \boldsymbol{T}_k$$

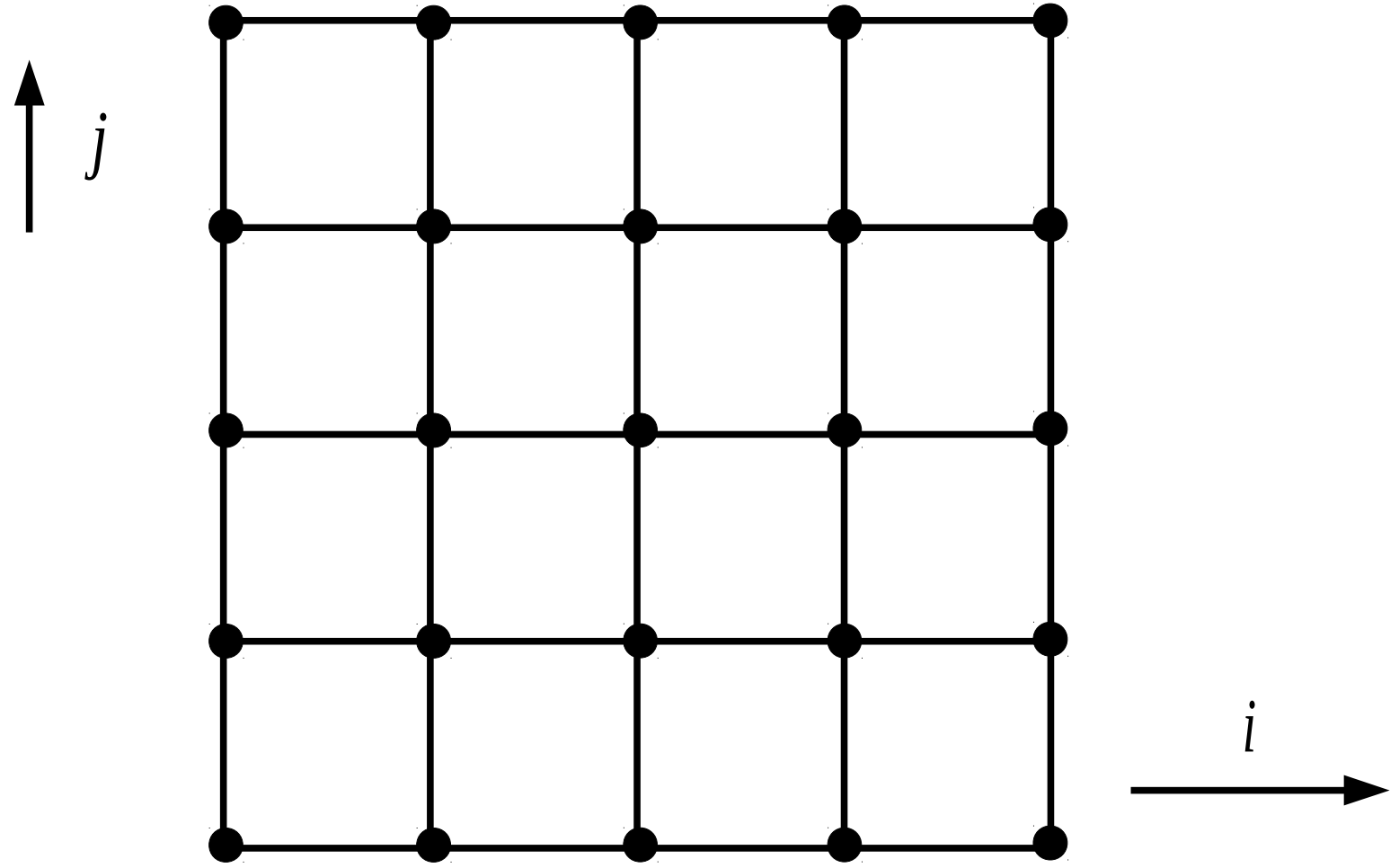
$$k = i + (j-1)\,Nx$$

$$T_k = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{NxNy} \end{pmatrix}$$

Numeración de Nodos

$k = (Ny - 1)\, Nx + 1$

$k = Nx\, Ny$

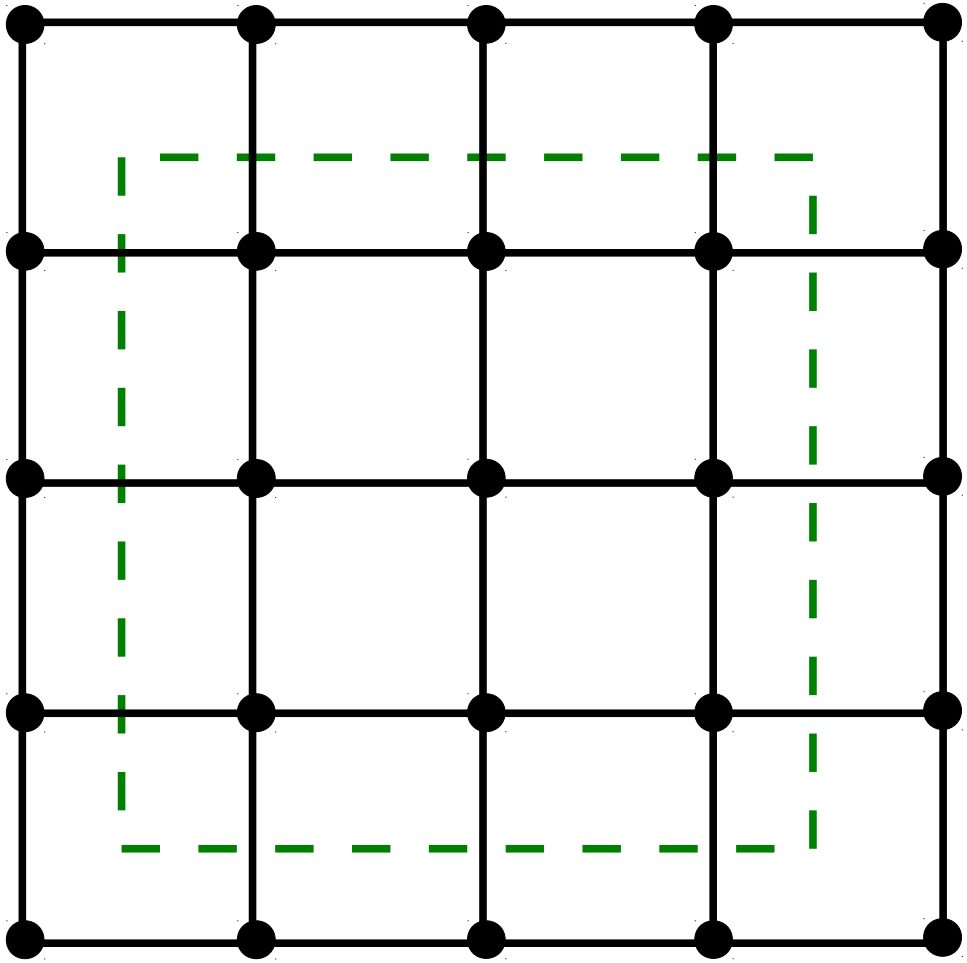


$k = 1$

$k = Nx$

Ecuación Lineal

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$



$$\left[M \right] \begin{pmatrix} T_1 \\ \vdots \\ T_k \\ \vdots \\ T_{NxNy} \end{pmatrix} = b$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Elementos de Matriz

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$

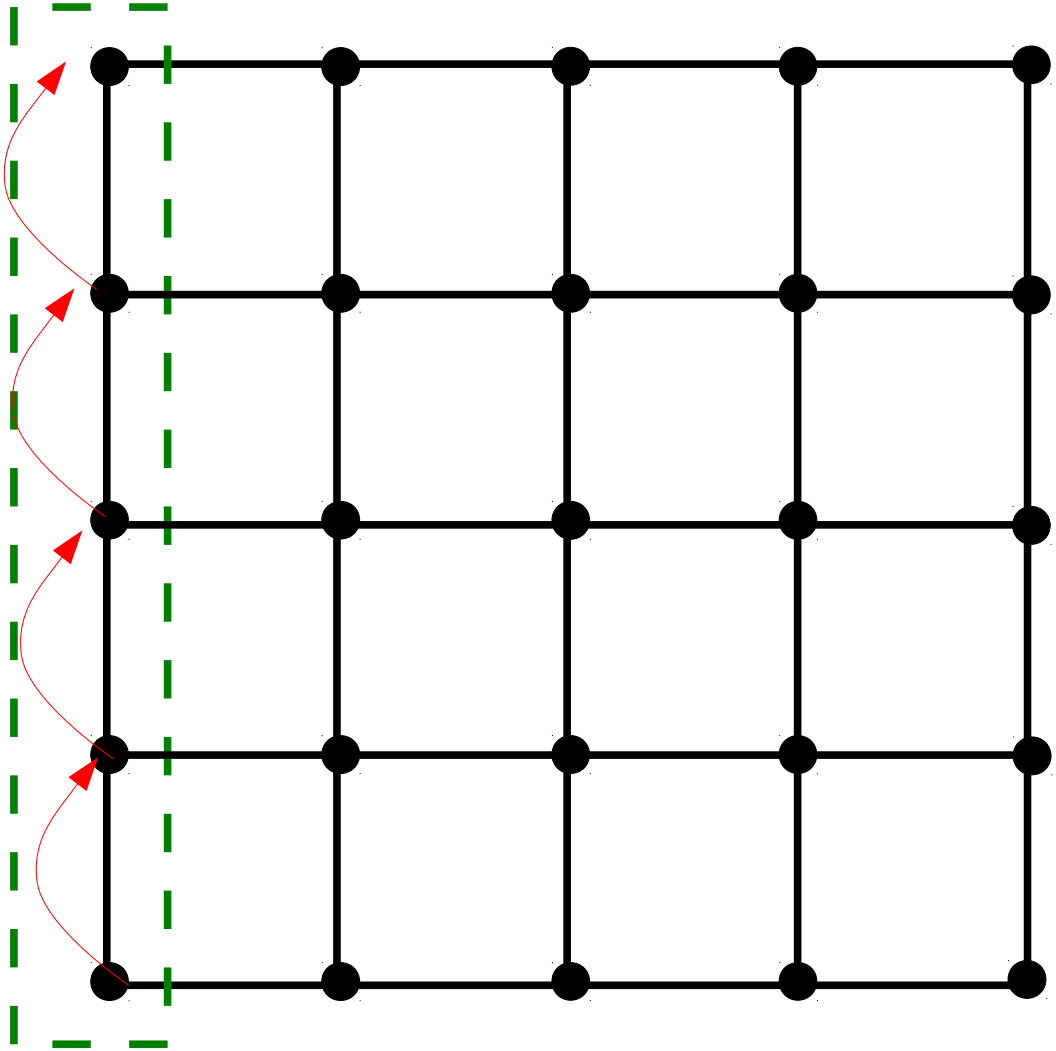
Fila k-ésima:

$$M_{k,:} = \left[\cdots \quad \beta^2 \quad \cdots \quad 1 \quad -2\left(1+\beta^2\right) \quad 1 \quad \cdots \quad \beta^2 \quad \cdots \right]$$

$k-Nx$ $\quad$ $k-1$ $\quad$ k $\quad$ $k+1$ $\quad$ $k+Nx$

$$b_k = 0$$

Condiciones de Contorno: Temperatura fija



$$T_{k_A} = T_A$$

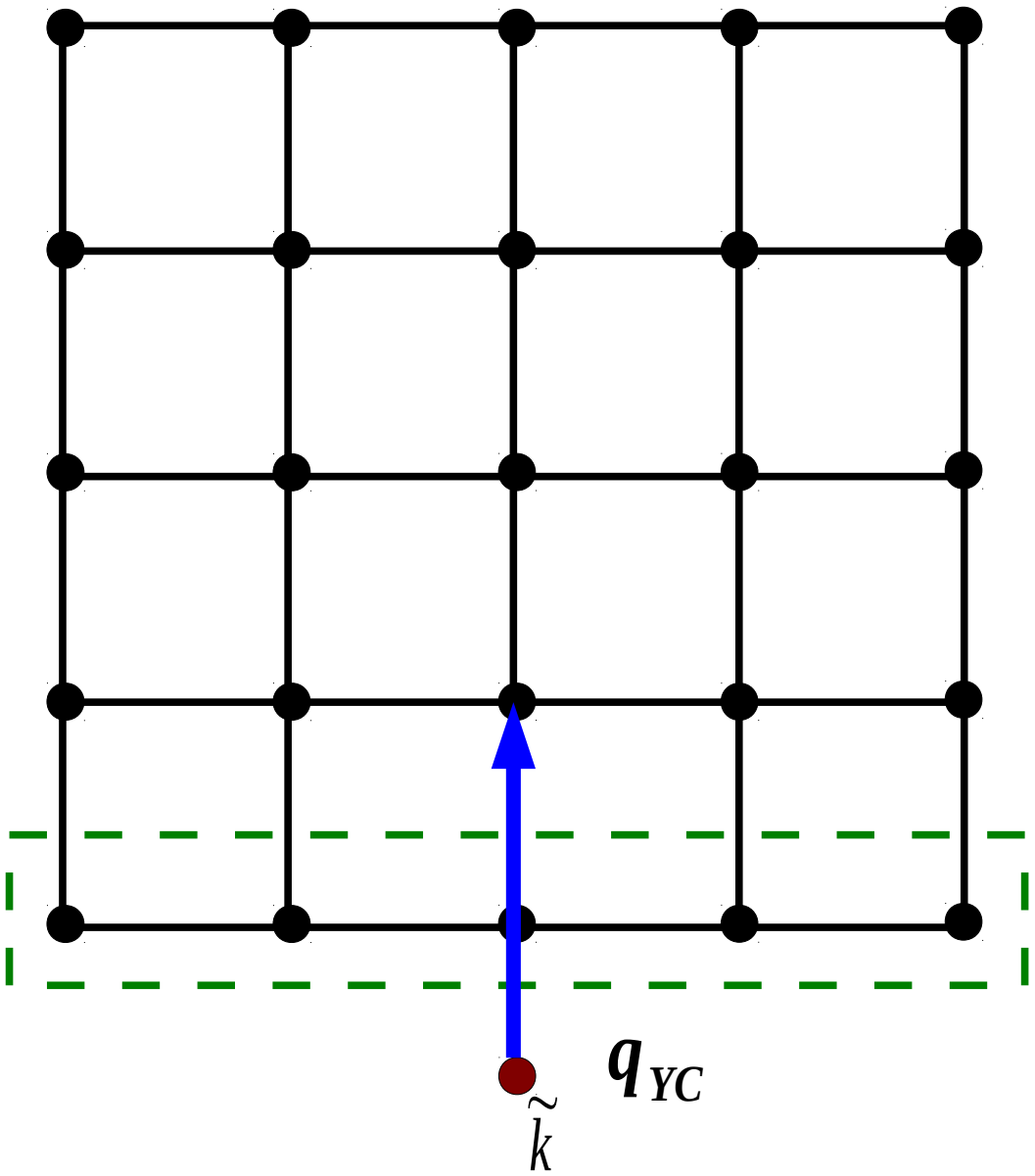
$$k_A = 1 : N_X : \left(N_Y - 1\right) N_x$$

$$M^{fija}_{k_A,:} = \left[\; \cdots \; 1 \; \cdots \; \right]$$

$$k_A$$

$$b_{k_A} = T_A$$

Condiciones de Contorno: Flujo dado.



$$Q_y \propto q_{YC} = \left.\frac{\partial T}{\partial y}\right|_{k_C} = \frac{T_{k_A+N_x} - T_{\tilde{k}}}{2\,dy}$$

$$k_C = 1 : N_X$$

Condiciones de Contorno: Flujo dado

Elementos de Matriz

$$T_{\tilde{k}} = T_{k_C + N_x} - 2\,dx q_{YC} \quad , \quad k_C = 1 : N_X$$

Reemplazo en la ecuación general

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$

Reordenando

$$T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + 2\beta^2 T_{k+N_X} = 2\beta^2\, dx q_{YC}$$

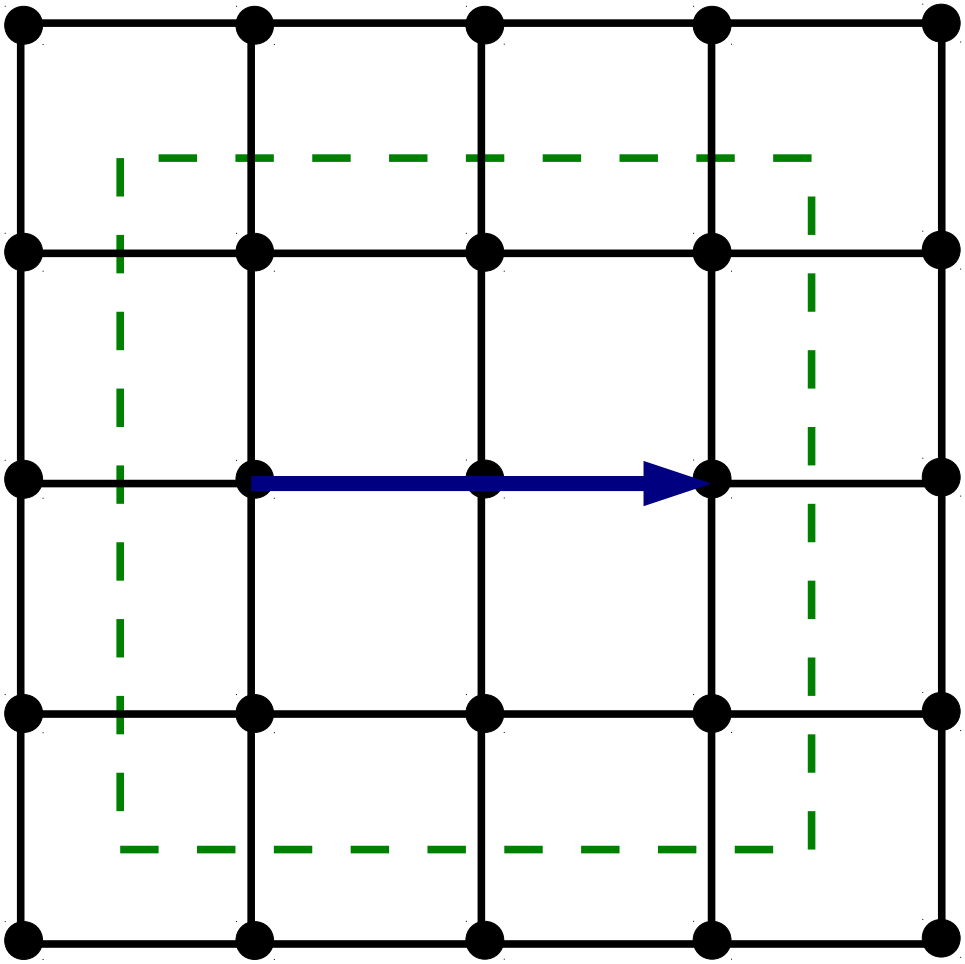
Fila k-ésima

$$M_{k_A,:} = \left[\cdots \; 0 \; \cdots \; 1 \; -2\left(1+\beta^2\right) \; 2 \; \cdots \; \beta^2 \; \cdots\right]$$

$k - Nx$ $\quad$ $k-1$ $\quad$ k $\quad$ $k+1$ $\quad$ $k+Nx$

$$b_k = 2\,dx q_{XA}$$

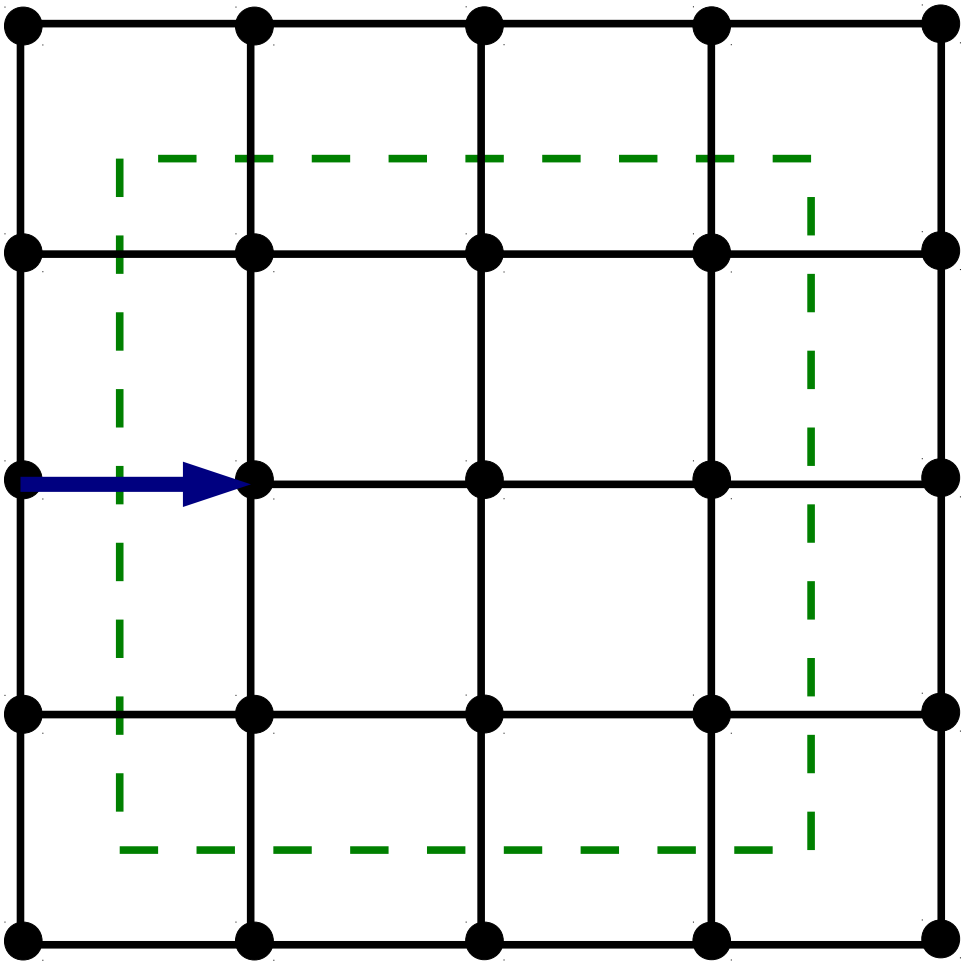
Cálculo de flujos en nodos internos



$$Q_x \propto q_X =$$

$$\frac{\partial T_k}{\partial x} = \frac{T_{k+1} - T_{k-1}}{2\,dx}$$

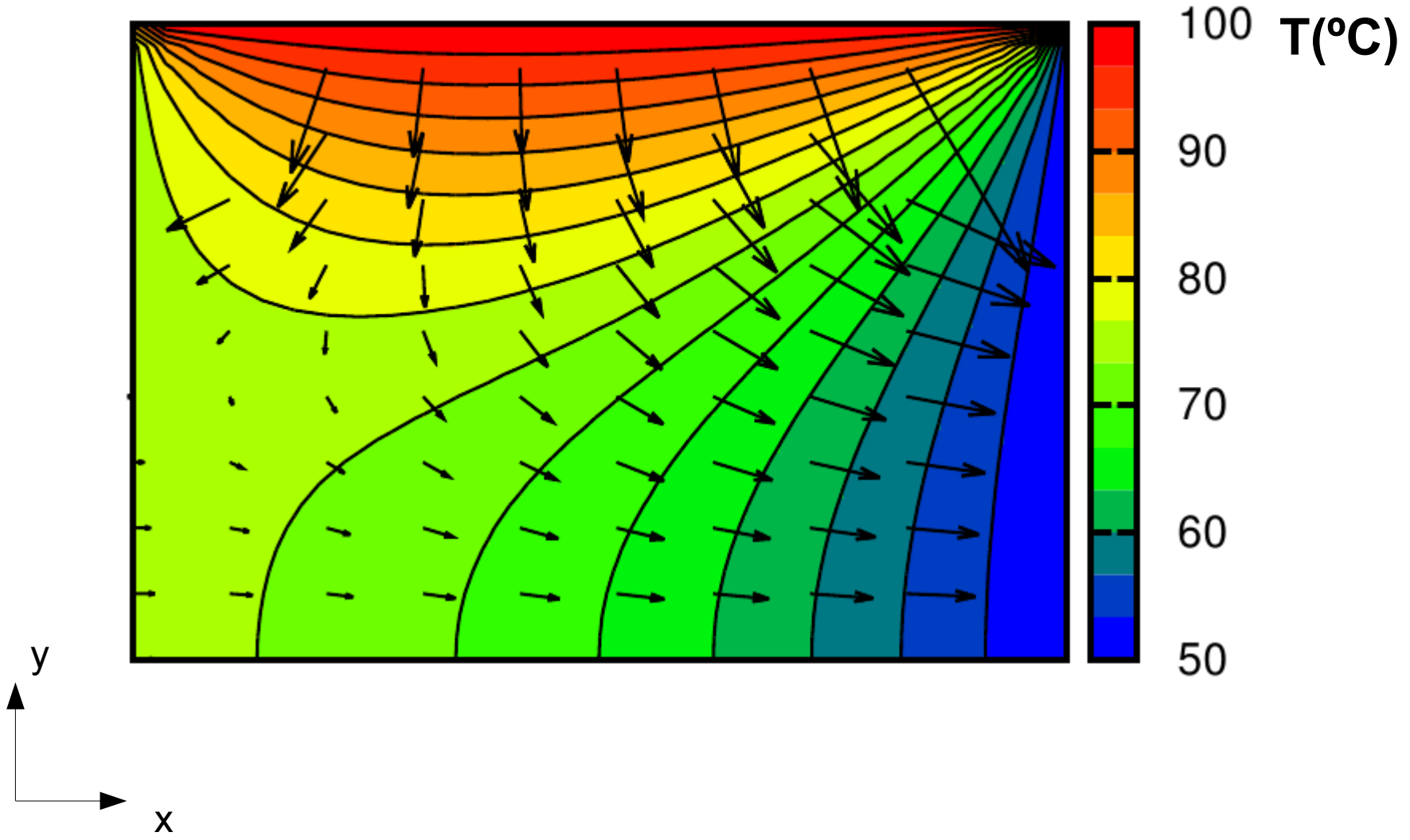
Cálculo de flujos en los bordes



$$Q_{yA} \propto q_{YA} = \frac{\partial T_{k_A}}{\partial x} = \frac{T_{k_A+1} - T_k}{dx}$$

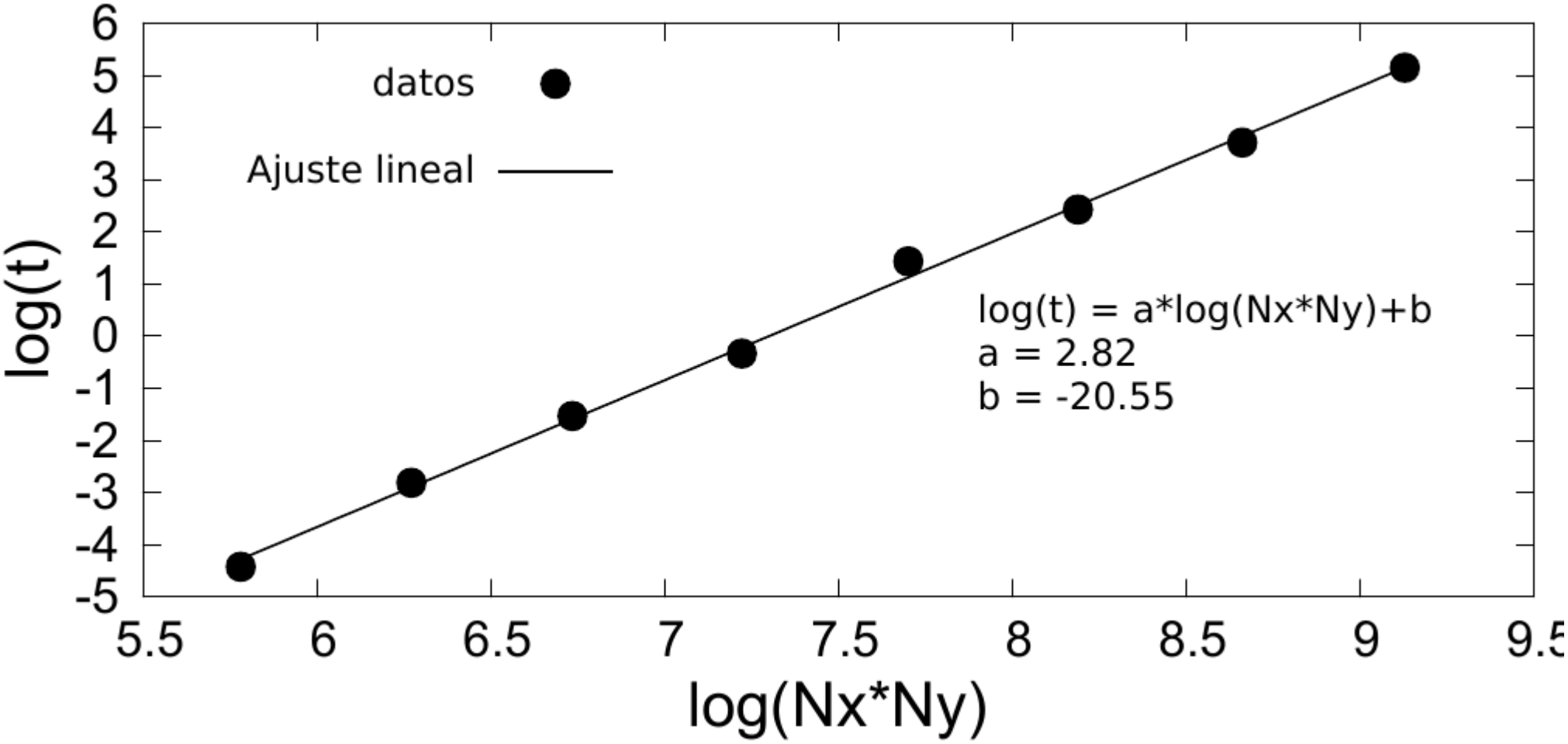
Resultados

Mapa de temperaturas



Resultados

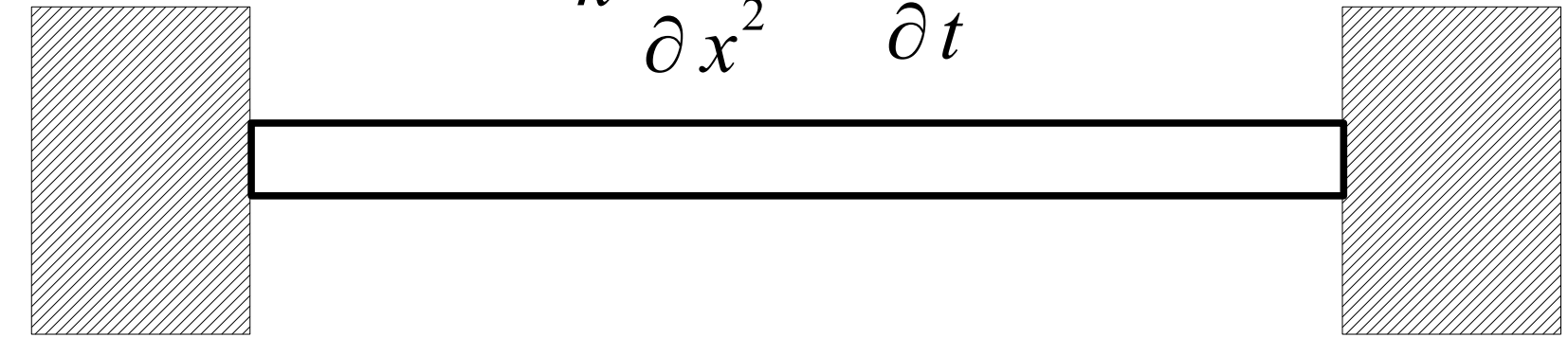
Tiempo de cálculo



Ejercicio 2

Presentación del Problema

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

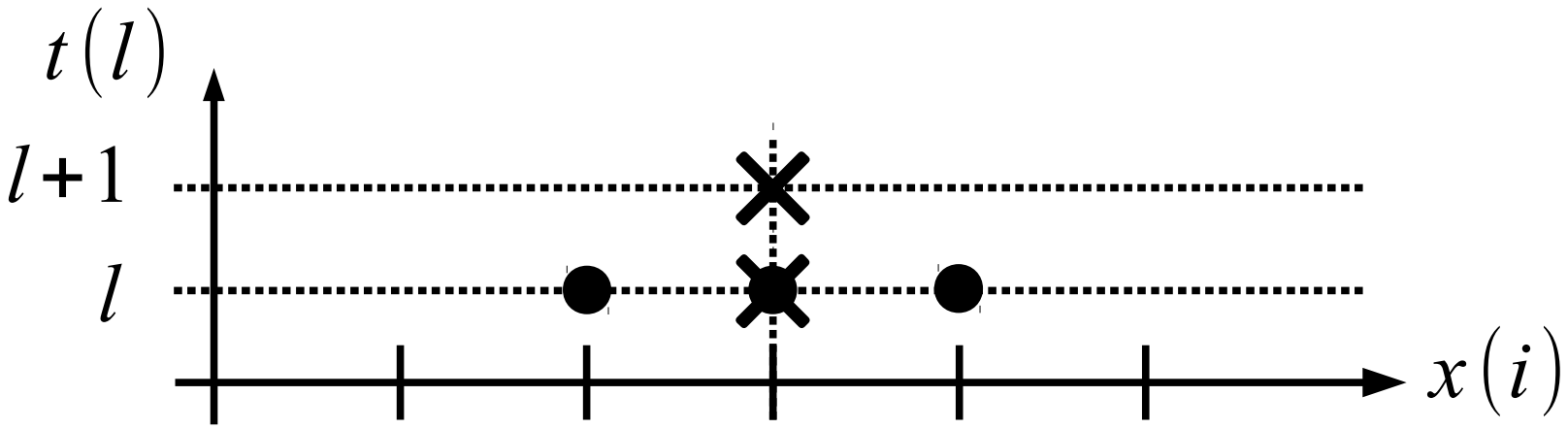


T_{IZQ} = 100ºC

T_{DER} = 50ºC

Discretización e integración Temporal

Método Explícito



T_{IZQ} = 100ºC

T_{DER}=50ºC

$$\frac{\partial^2 T^l}{\partial x^2} = \frac{T^l_{i-1} - 2\,T^l_i + T^l_{i+1}}{dx^2}$$

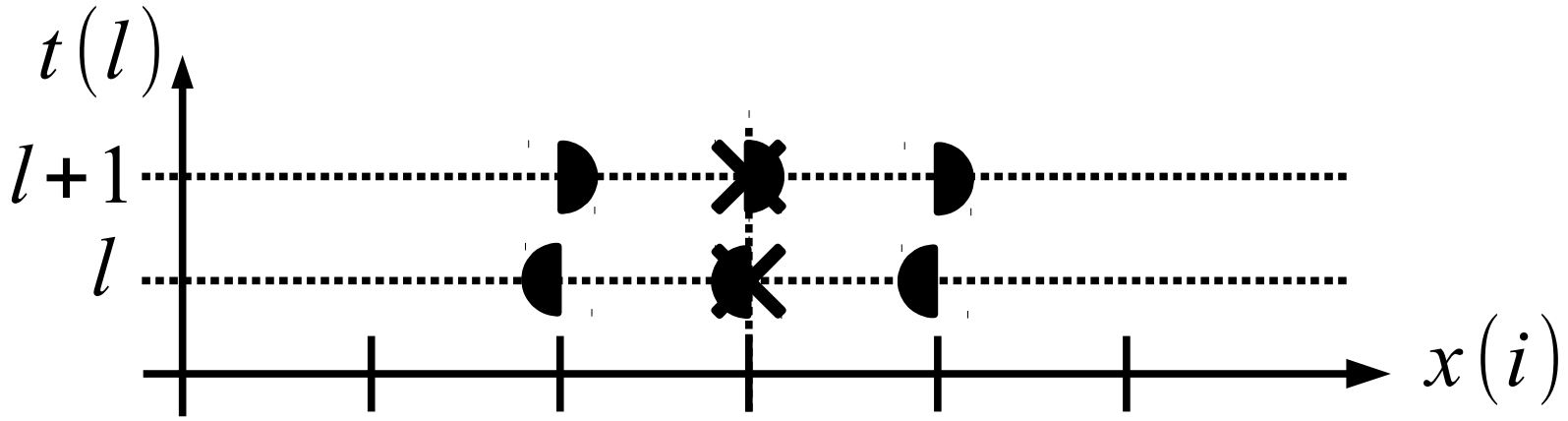
$$\frac{\partial T}{\partial t} = \frac{T^{l+1}_i - T^l_i}{d\,t}$$

$$T^{l+1}_i = \lambda\,T^l_{i-1} + (1-2\lambda)\,T^l_i + \lambda\,T^l_{i+1}$$

$$\lambda = \frac{k\ \ dt}{dx^2}$$

Discretización e integración Temporal

Método Implícito / Cranck-Nicholson



T_{IZQ} = 100ºC

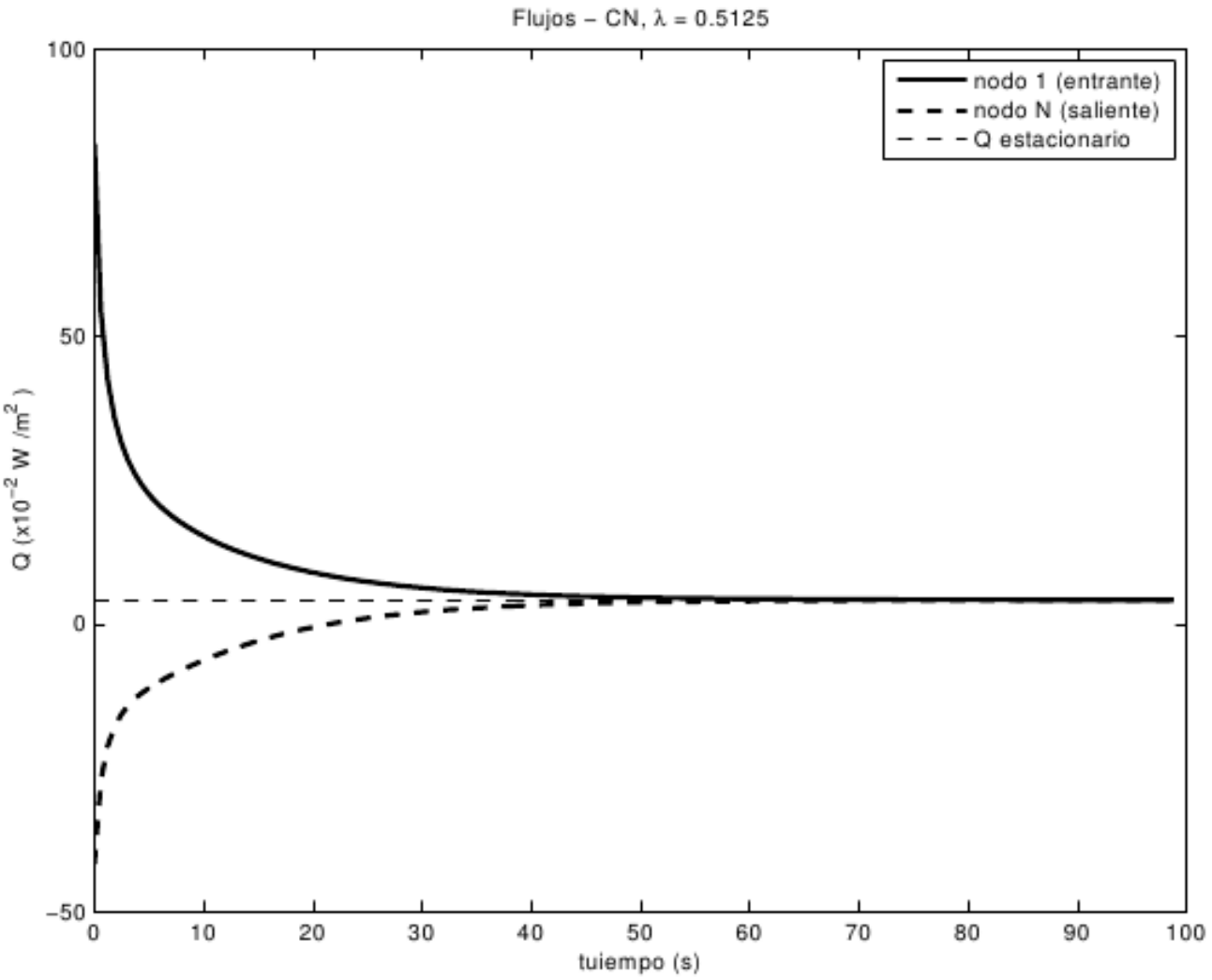
T_{DER}=50ºC

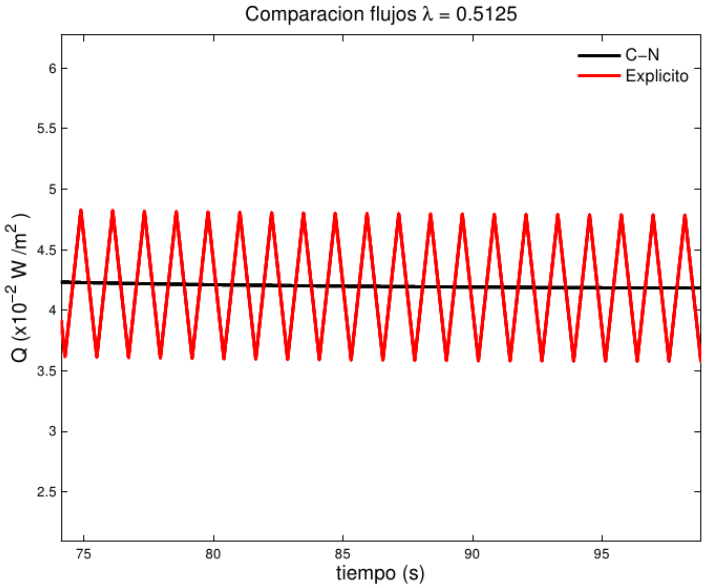
$$\left.\begin{aligned}\frac{\partial^2 T^l}{\partial x^2} &= \frac{1}{2}\left(\frac{T_{i-1}^{l+1}-2T_i^{l+1}+T_{i+1}^{l+1}}{dx^2}+\frac{T_{i-1}^{l}-2T_i^{l}+T_{i+1}^{l}}{dx^2}\right)\\ \frac{\partial T}{\partial t} &= \frac{T_i^{l+1}-T_i^{l}}{dt}\end{aligned}\right\} \times$$

$$-\lambda T_{i-1}^{l+1}+2(1+\lambda)T_i^{l+1}-\lambda T_{i+1}^{l+1}=\lambda T_{i-1}^{l}+2(1-\lambda)T_i^{l}+\lambda T_{i+1}^{l}$$

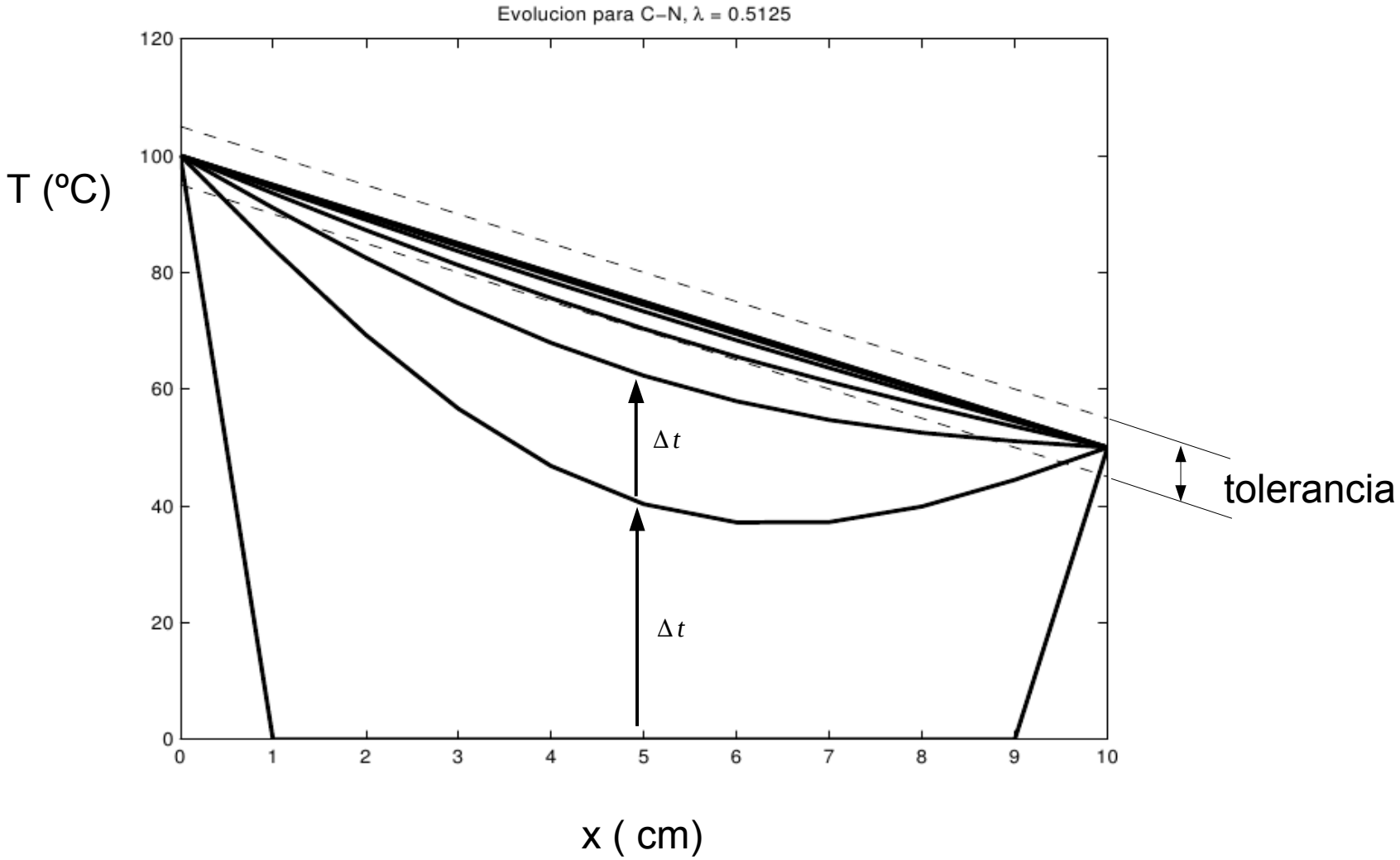
Resultados: Estado Estacionario y Flujos

$$Q = -k \times \frac{\partial T}{\partial x}$$



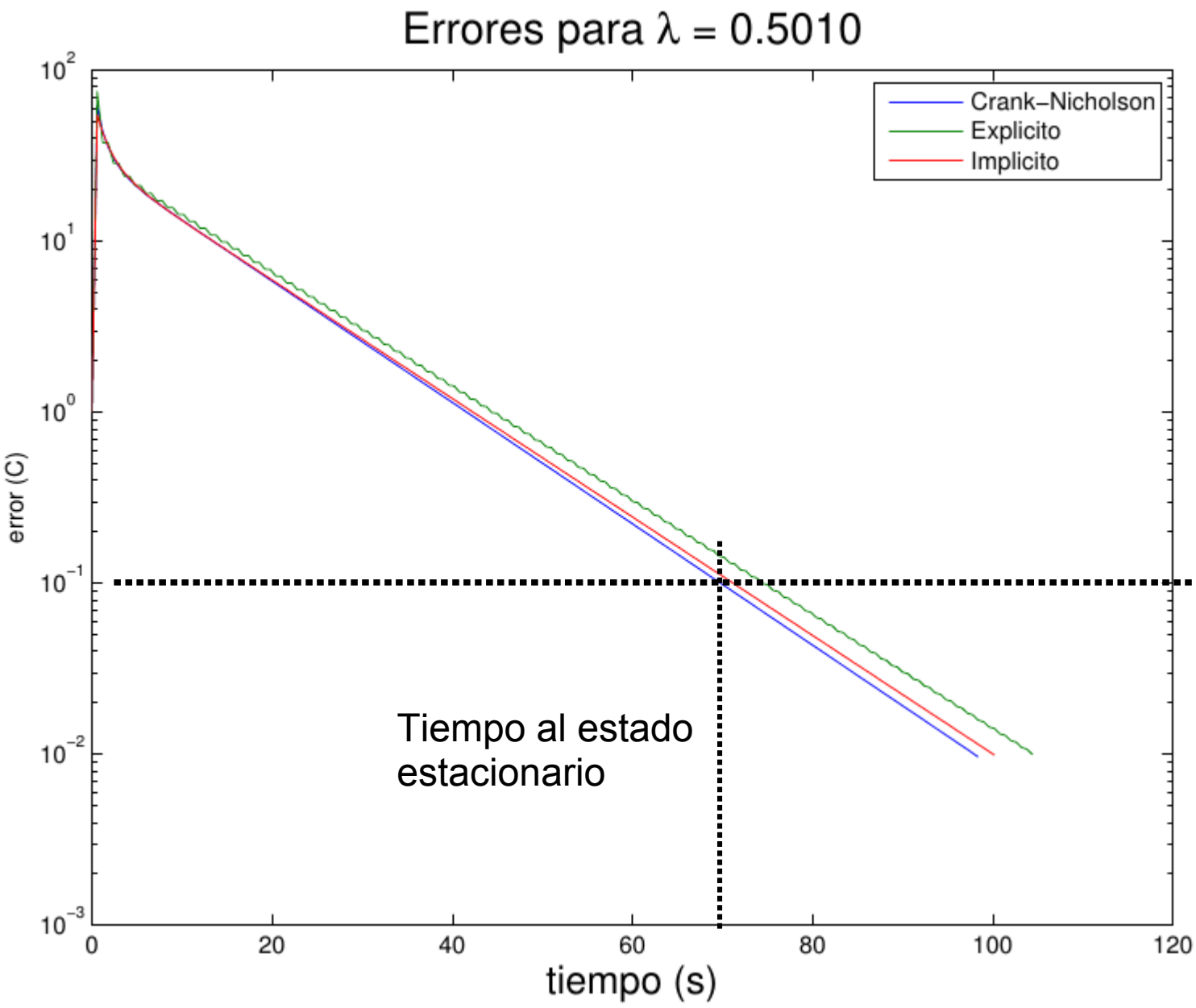


Resultados: Estado Estacionario y Perfiles de temperaturas



Resultados: Errores, Estabilidad y Convergencia

dt pequeño



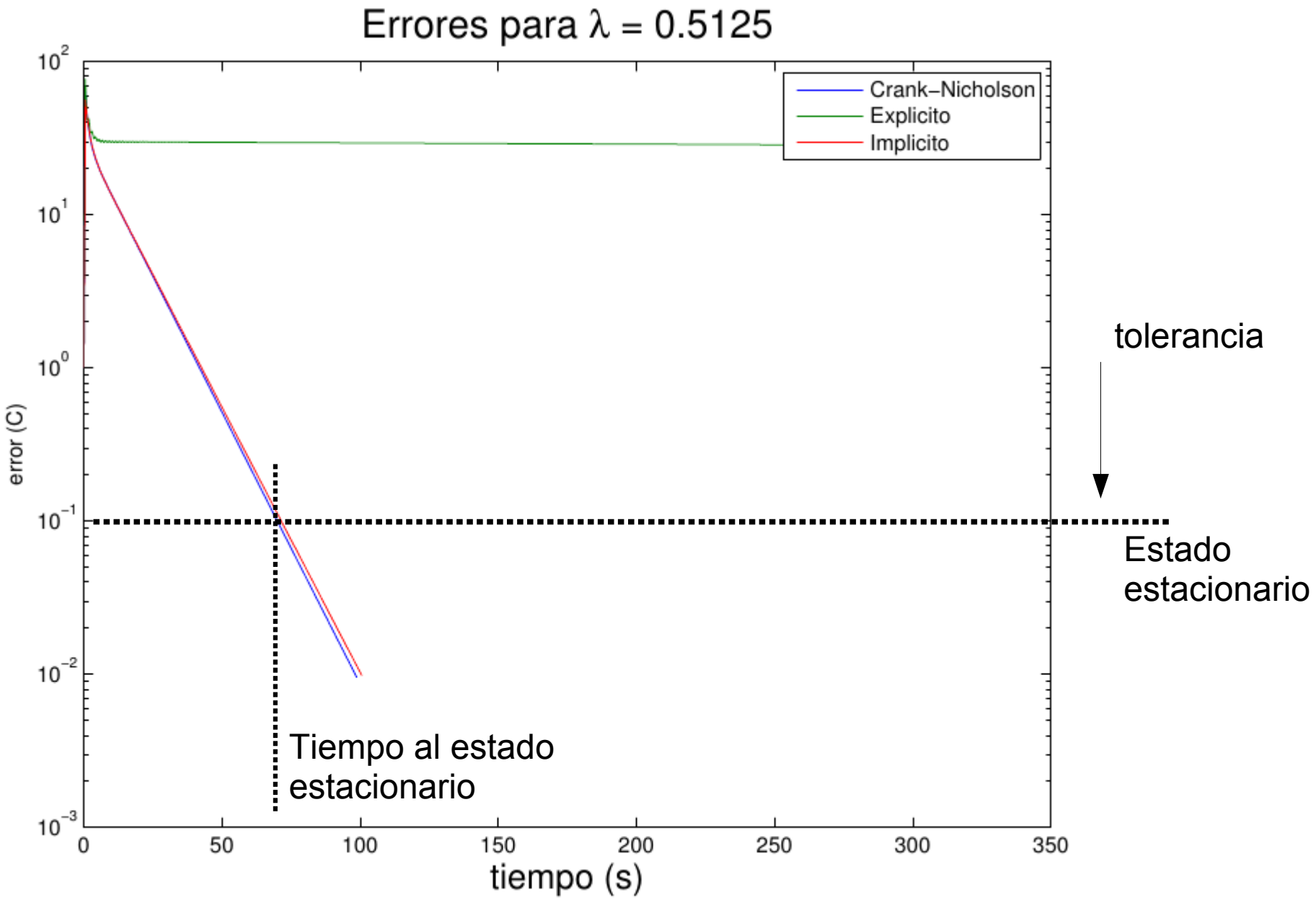
tolerancia

Estado estacionario

Tiempo al estado estacionario

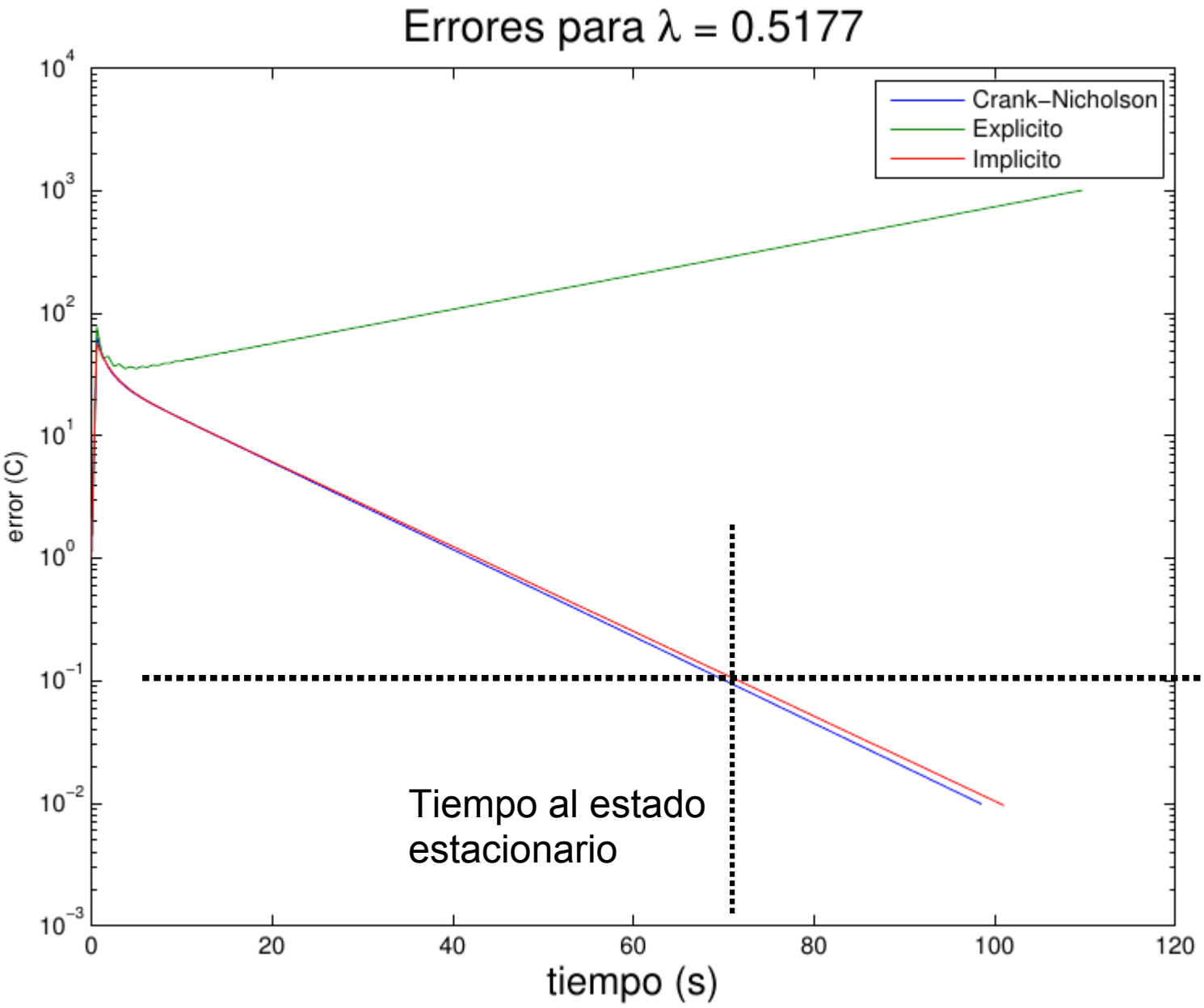
Resultados: Errores, Estabilidad y Convergencia

dt crítico



Resultados: Errores, Estabilidad y Convergencia

dt grande



tolerancia

Estado estacionario

Tiempo al estado estacionario