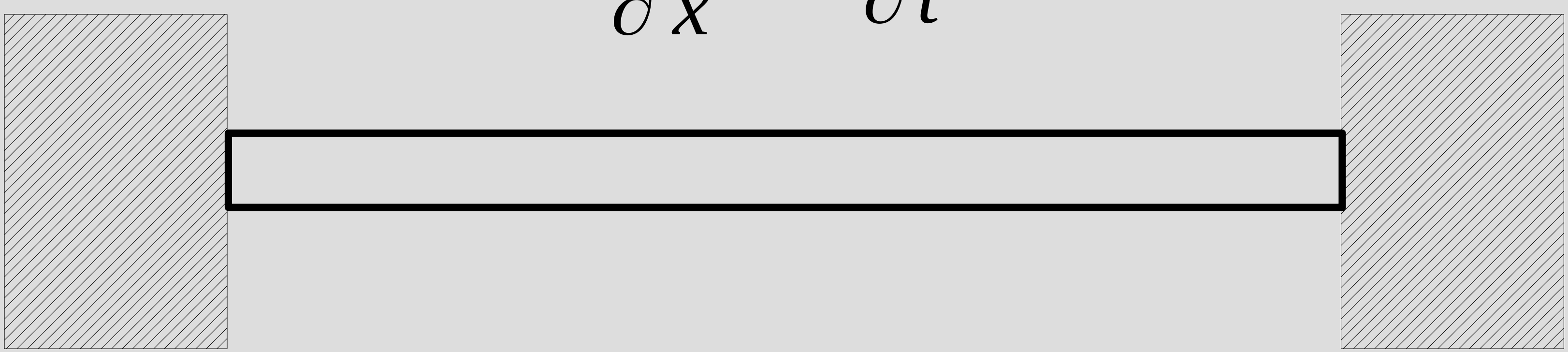
Ejercicio 2

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$



T_{IZQ} = 100ºC

T_{DER} = 50ºC

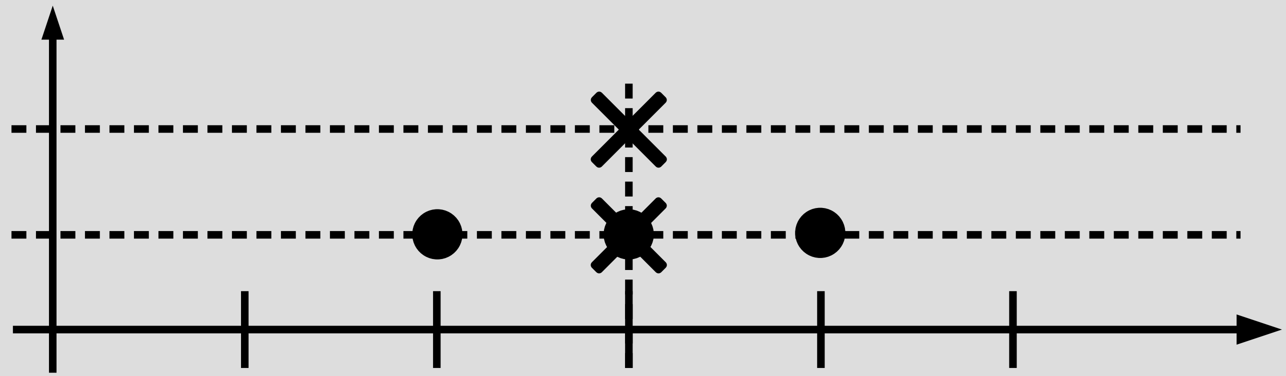
Ejercicio 2 - Discretización

Metodo Explicito

$t(l)$

$l+1$

l



$x(i)$

T_{IZQ} = 100ºC

T_{IZQ} = 50ºC

$$\frac{\partial^2 T^l}{\partial x^2} = \frac{T^l_{i-1} - 2T^l_i + T^l_{i+1}}{dx^2} \quad \bullet$$

$$\frac{\partial T}{\partial t} = \frac{T^{l+1}_i - T^l_i}{dt} \quad \times$$

$$T^{l+1}_i = \lambda T^l_{i-1} + (1-2\lambda) T^l_i + \lambda T^l_{i+1}$$

$$\lambda = \frac{k \quad dt}{dx^2}$$

Ejercicio 2 – Matricialización Método explícito

$$T_i^{l+1} = \lambda T_{i-1}^{l} + (1-2\lambda) T_i^{l} + \lambda T_{i+1}^{l}$$

Condiciones de contrno:

$$T_1^{l=1\ldots N_1} = T_A$$

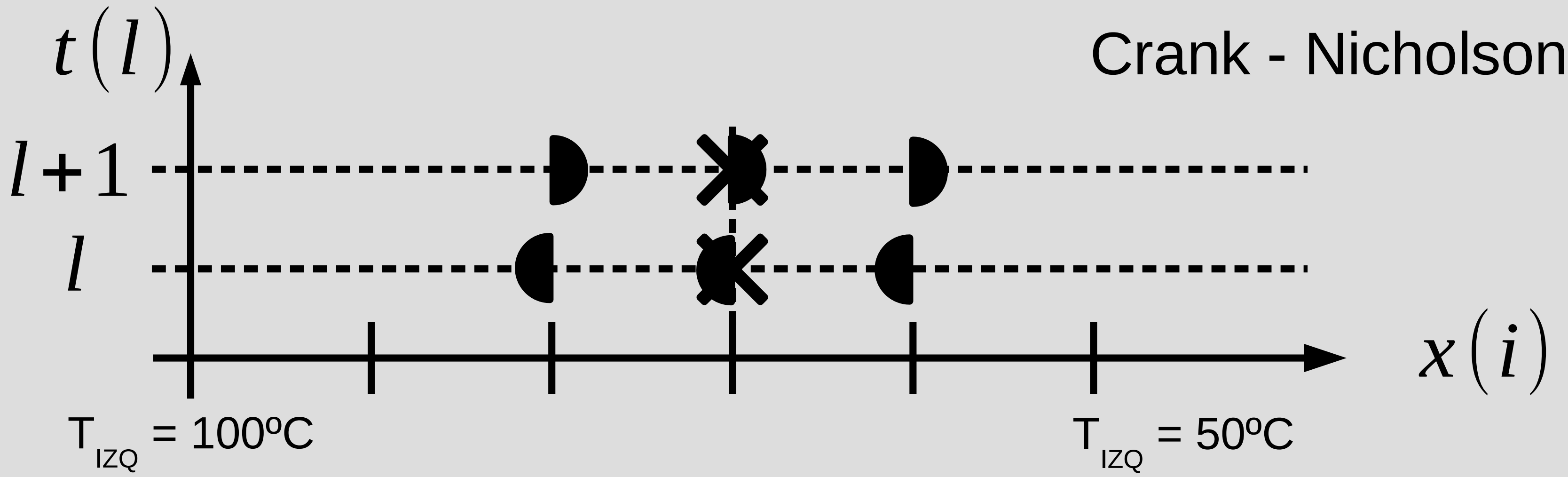
$$T_N^{l=1\ldots N_1} = T_B$$

Si T es un vector columna, se puede reescribir como operación matricial:

$$\begin{pmatrix} T_1 \\ \vdots \\ T_N \end{pmatrix}^{l+1} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ \lambda & 1-2\lambda & \lambda & 0 & \cdots \\ & & & \cdots & \\ \cdots & \lambda & 1-2\lambda & \lambda & 0 \\ \cdots & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T_1 \\ \vdots \\ T_N \end{pmatrix}^{l}$$

Ejercicio 2 - Discretización

Crank - Nicholson



$$\left.\begin{aligned}
&\frac{\partial^2 T^l}{\partial x^2} = \frac{1}{2}\left(\frac{T_{i-1}^{l+1} - 2T_i^{l+1} + T_{i+1}^{l+1}}{dx^2} + \frac{T_{i-1}^{l} - 2T_i^{l} + T_{i+1}^{l}}{dx^2}\right) \quad \bullet \\
&\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{dt} \quad \times
\end{aligned}\right\}$$

$$-\lambda T_{i-1}^{l+1} + 2(1+\lambda) T_i^{l+1} - \lambda T_{i+1}^{l+1} = \lambda T_{i-1}^{l} + 2(1-\lambda) T_i^{l} + \lambda T_{i+1}^{l}$$

Ejercicio 2 – Matricialización Método Crank Nicholson

$$-\lambda T_{i-1}^{l+1}+2(1+\lambda)T_{i}^{l+1}-\lambda T_{i+1}^{l+1}=\lambda T_{i-1}^{l}+2(1-\lambda)T_{i}^{l}+\lambda T_{i+1}^{l}$$

Condiciones de controno

$$T_{1}^{l=1\ldots N_1}=T_A$$

$$T_{N}^{l=1\ldots N_1}=T_B$$

$$A=\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ -\lambda & 2(1+\lambda) & -\lambda & 0 & \cdots \\ & & \cdots & & \\ \cdots & -\lambda & 2(1+\lambda) & -\lambda & 0 \\ \cdots & 0 & 0 & 0 & 1 \end{pmatrix}$$

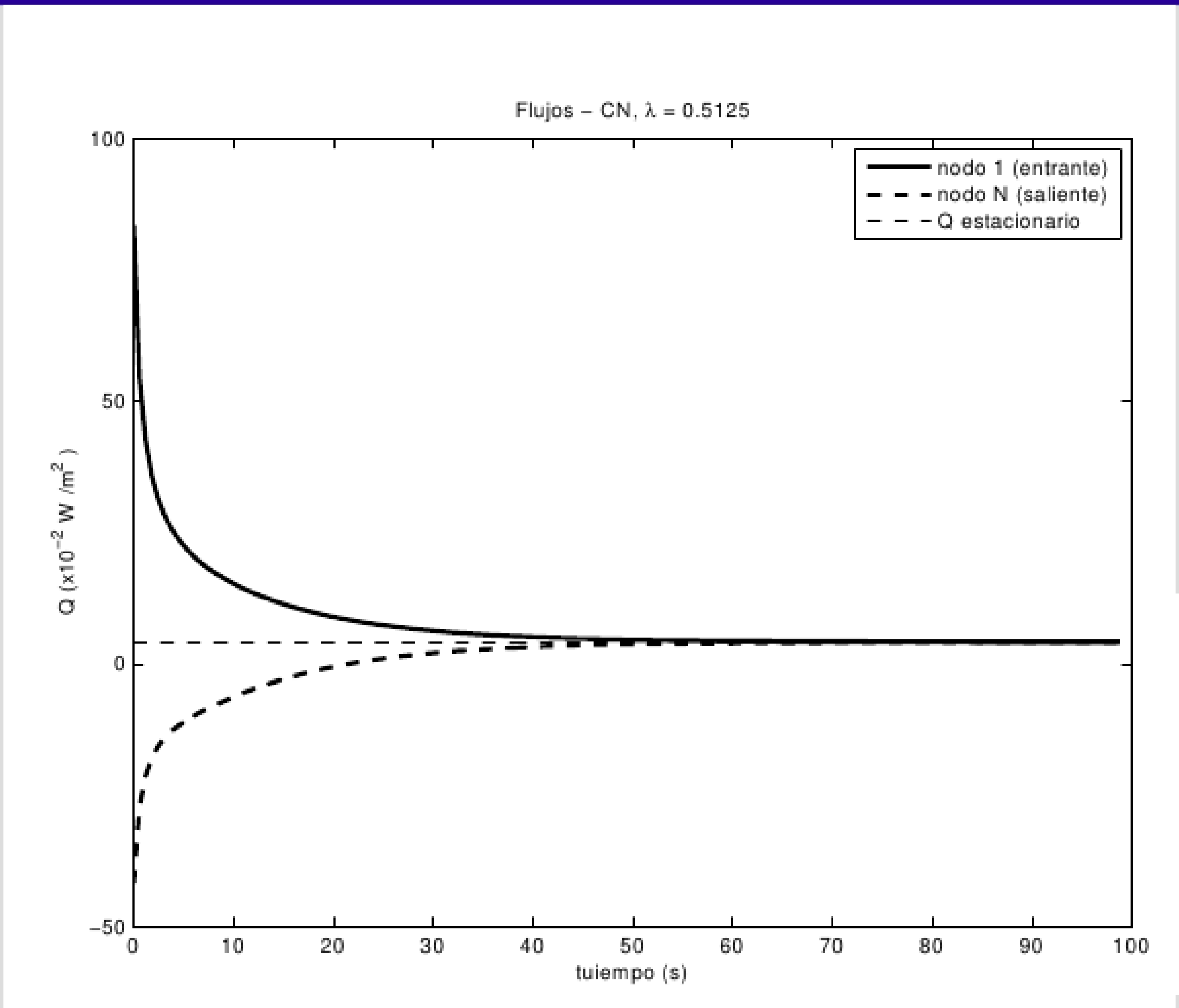
$$B=\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ \lambda & 2(1-\lambda) & \lambda & 0 & \cdots \\ & & \cdots & & \\ \cdots & \lambda & 2(1-\lambda) & \lambda & 0 \\ \cdots & 0 & 0 & 0 & 1 \end{pmatrix}$$

En este caso tenemos dos matrices, a demas resolver a l+1 implica resolver el sistema lineal:

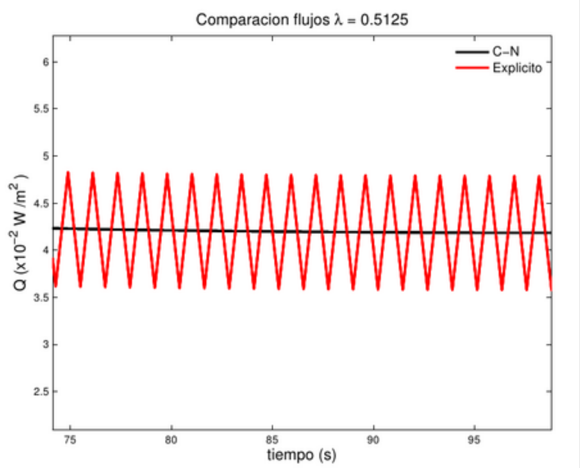
$$A T^{l+1}=B T^{l}$$

$$T^{l+1}=\left(A^{-1}B\right)T^{l}$$

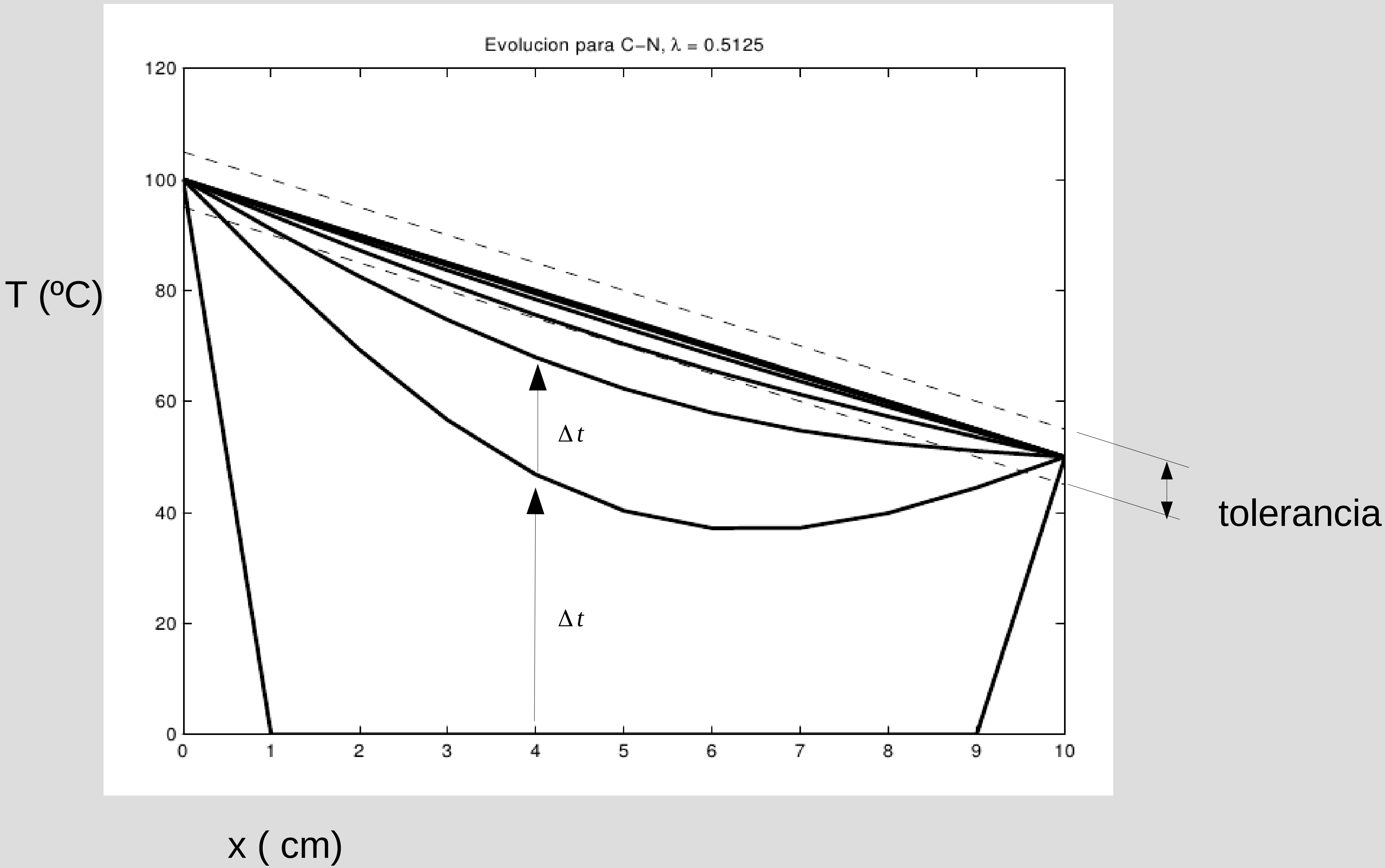
Ejercicio 2 – Estado estacionario y flujos



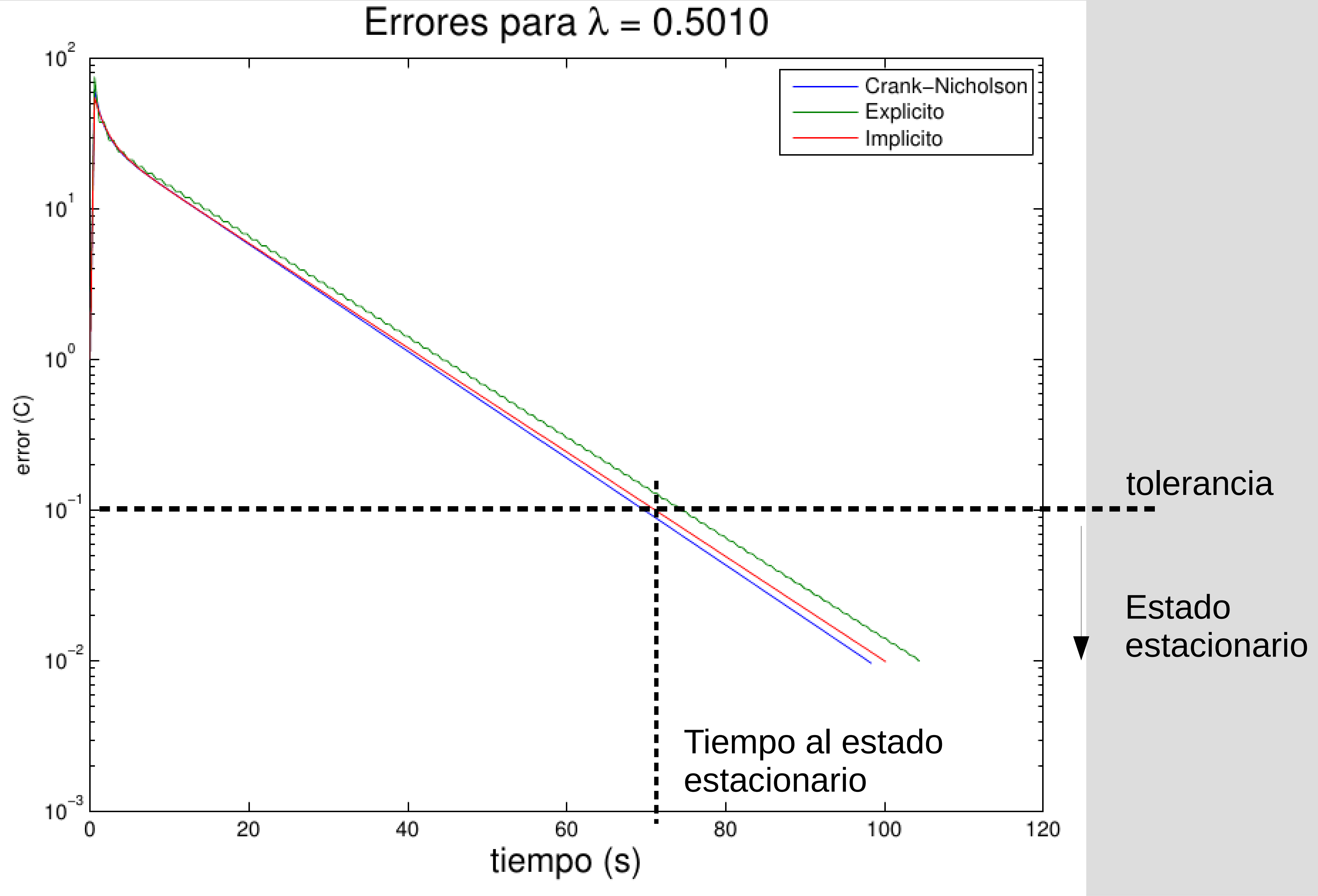
$$Q = -k \times \frac{\partial T}{\partial x}$$



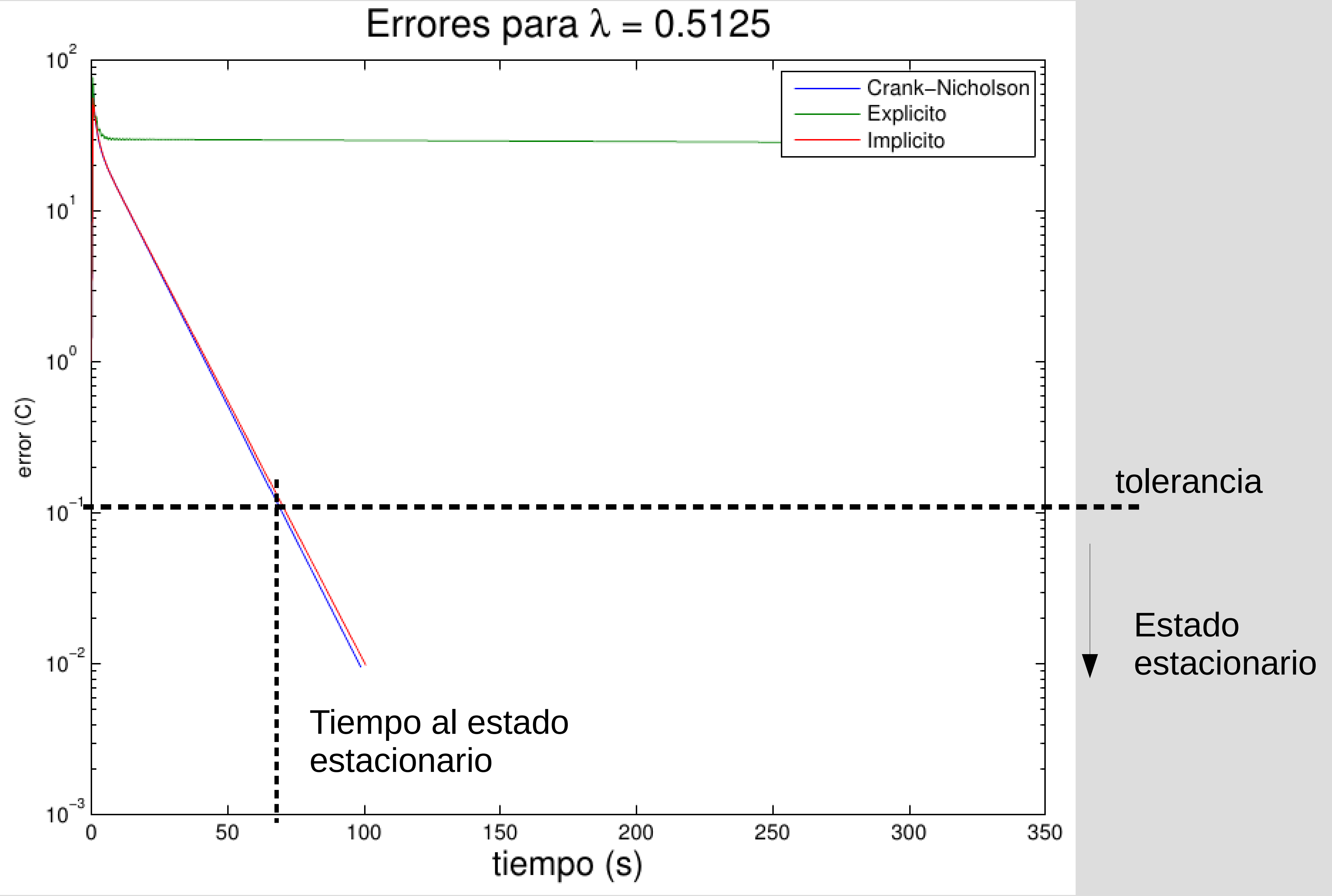
Ejercicio 2 – Estado estacionario y flujos



Ejercicio 2 – Errores , estabilidad y convergencia



Ejercicio 2 – Errores , estabilidad y convergencia



Ejercicio 2 – Errores , estabilidad y convergencia