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Modelización de Materiales 2019

Método de las Diferencias Finitas

Ejemplo: Estado Estacionario de un problema de conductividad térmica.

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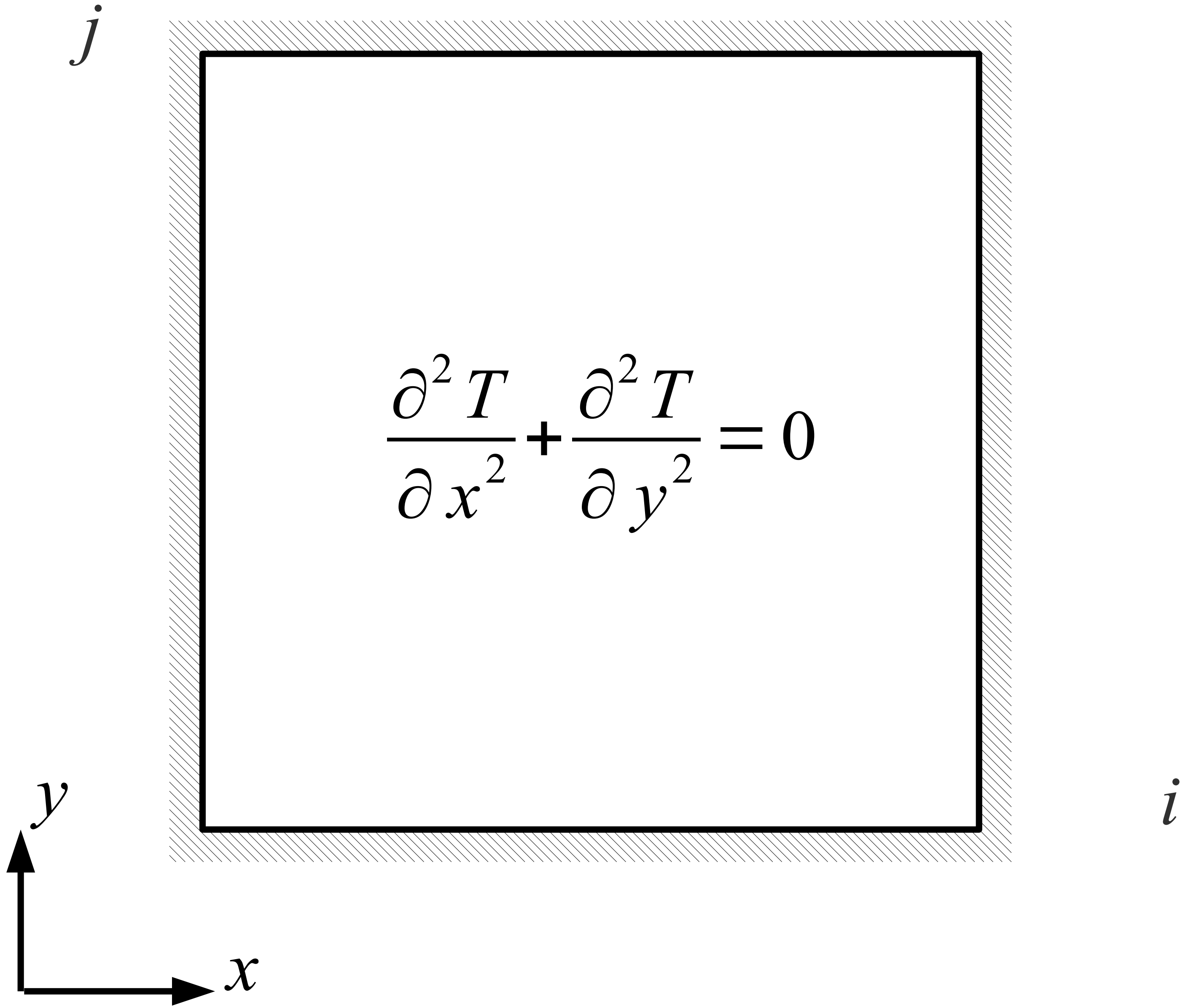
www.tandar.cnea.gov.ar/~weht/Modelizacion

https://mdforti.github.io/Modelizacion/

Presentación del Problema

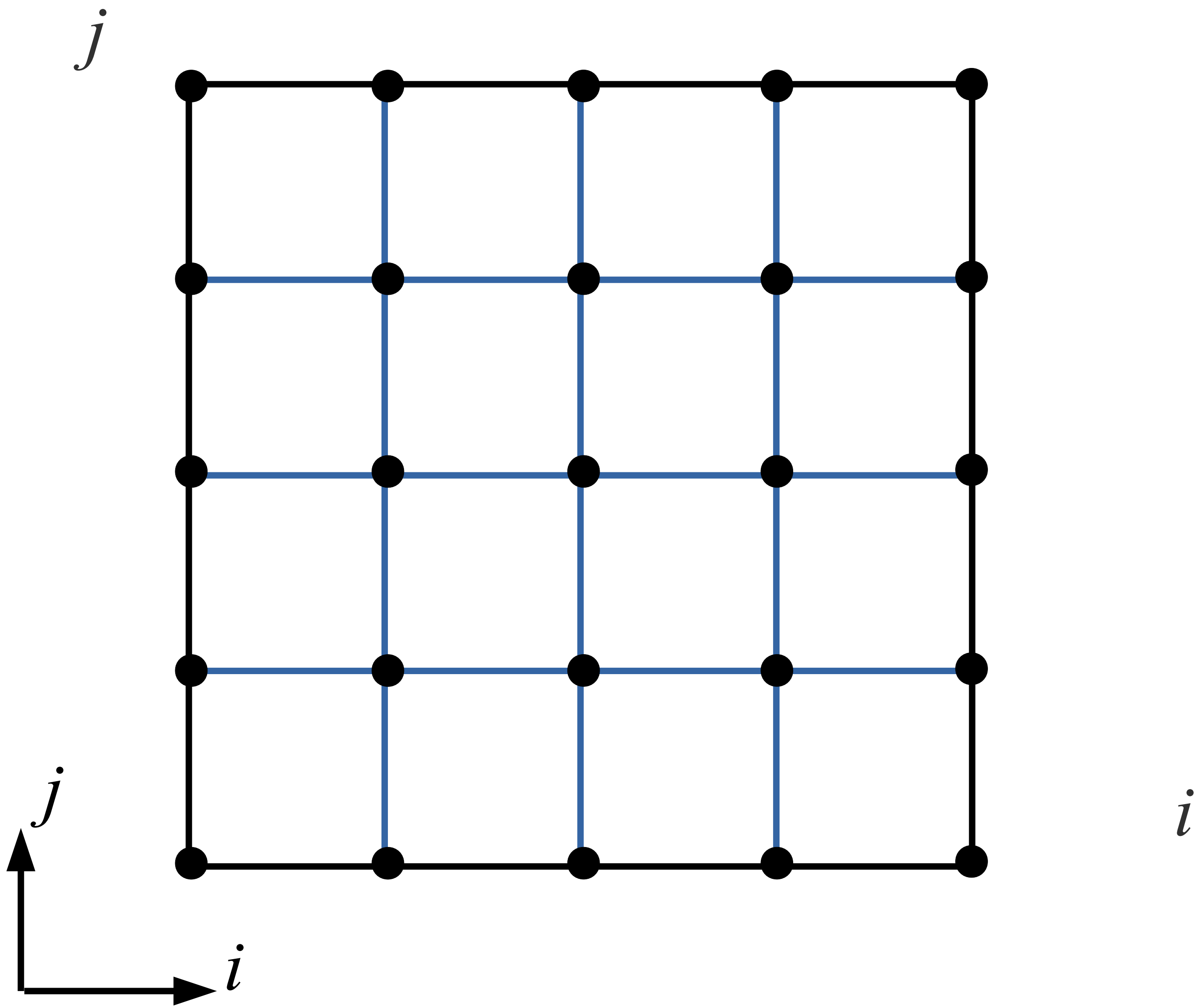
$T_{i\ j}$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

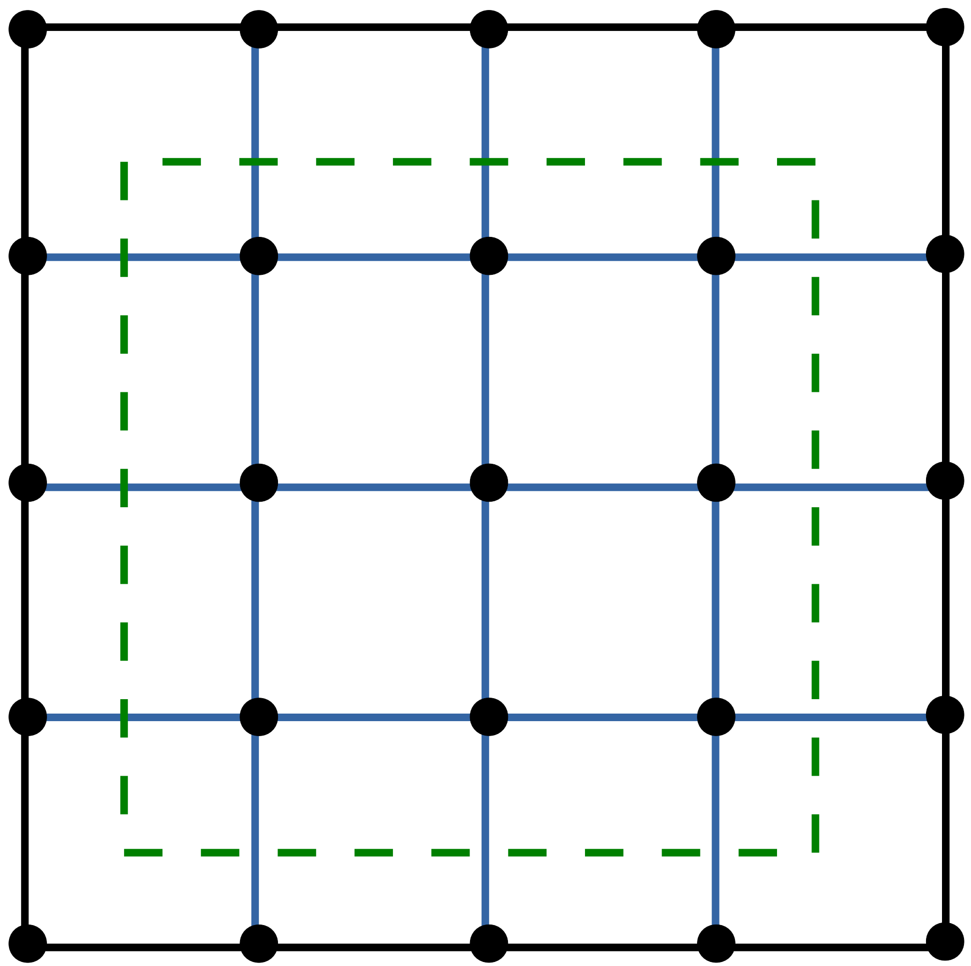
Discretización del Problema

$T_{i\ \ j}$



Ecuación Matricial

- Temperaturas en nodos internos



$$T_{i,j} = \boldsymbol{T}_k$$

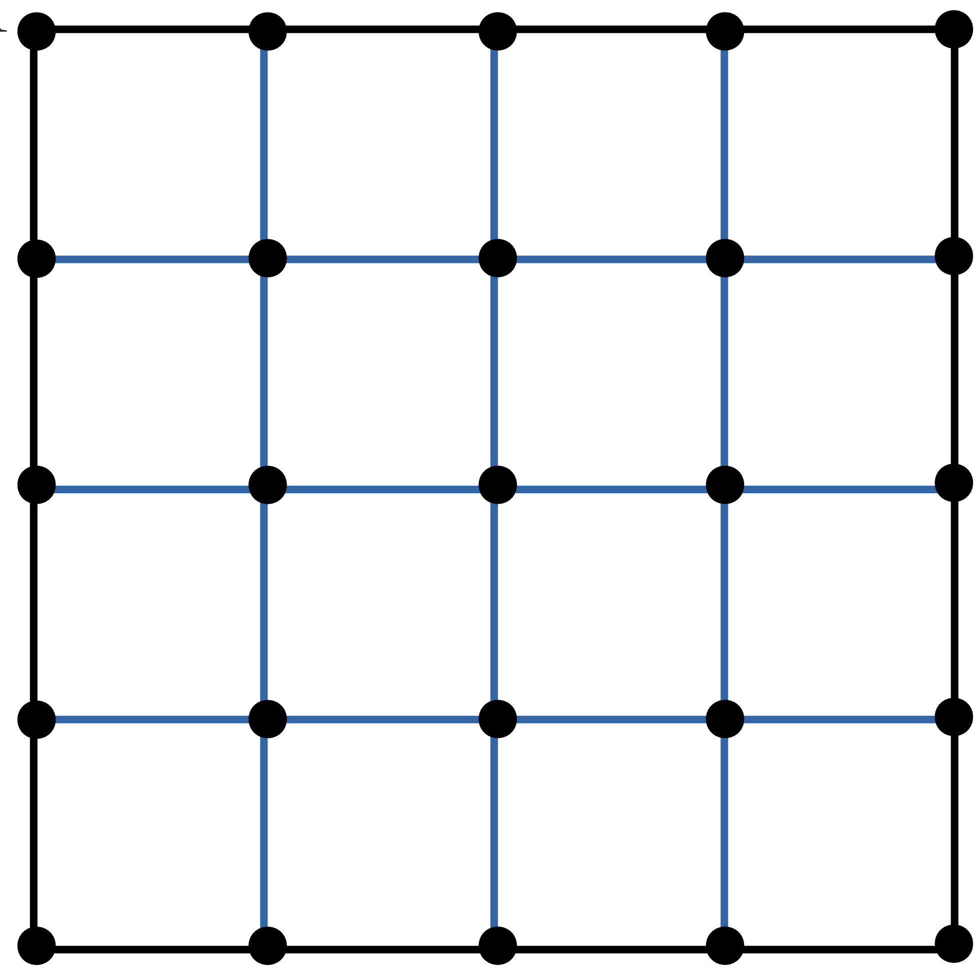
$$k = i + (j-1)Nx$$

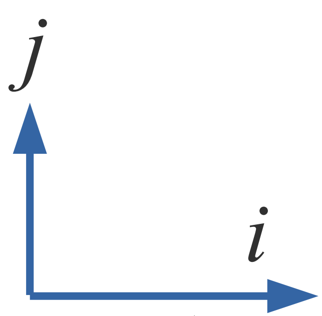
$$T_k = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{NxNy} \end{pmatrix}$$

Ecuación Matricial: Numeración de Nodos

$$k = Nx + (Ny-1)\,Nx + 1 = Nx\,Ny$$

$$k = (Ny-1)\,Nx + 1$$





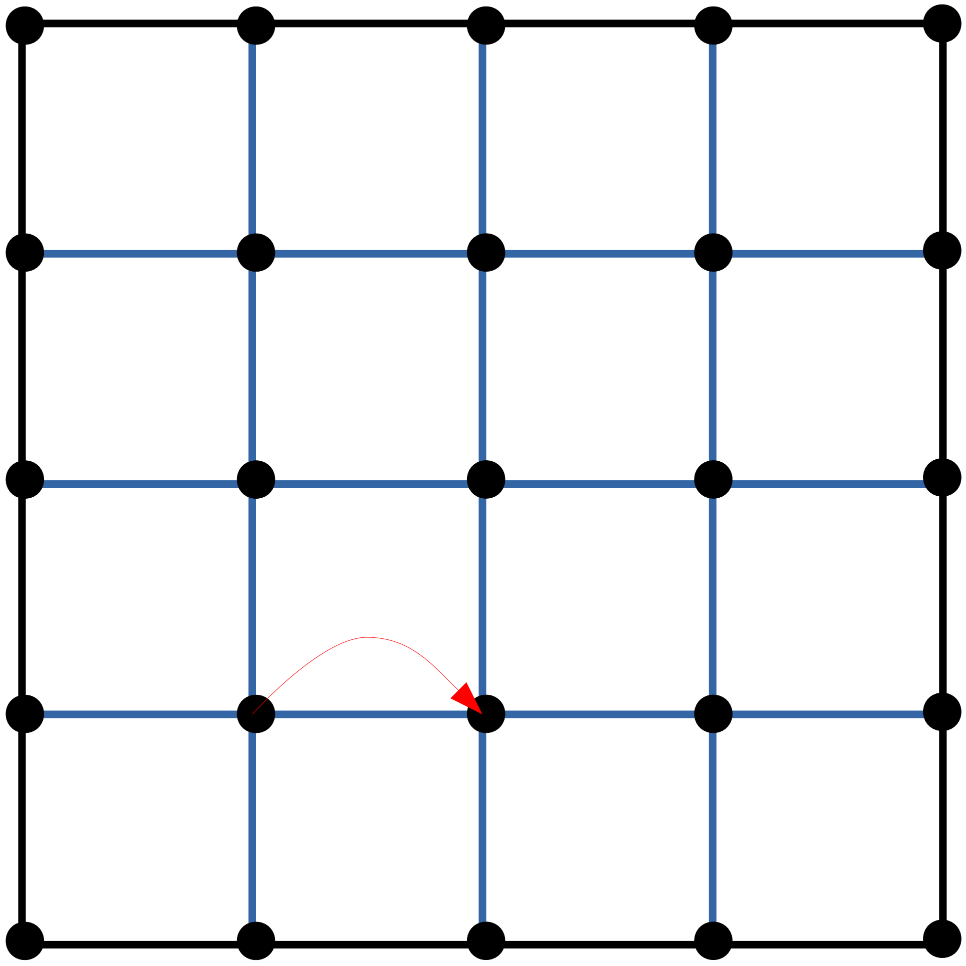
$$k = i + (j-1)\,Nx$$

$$k = 1$$

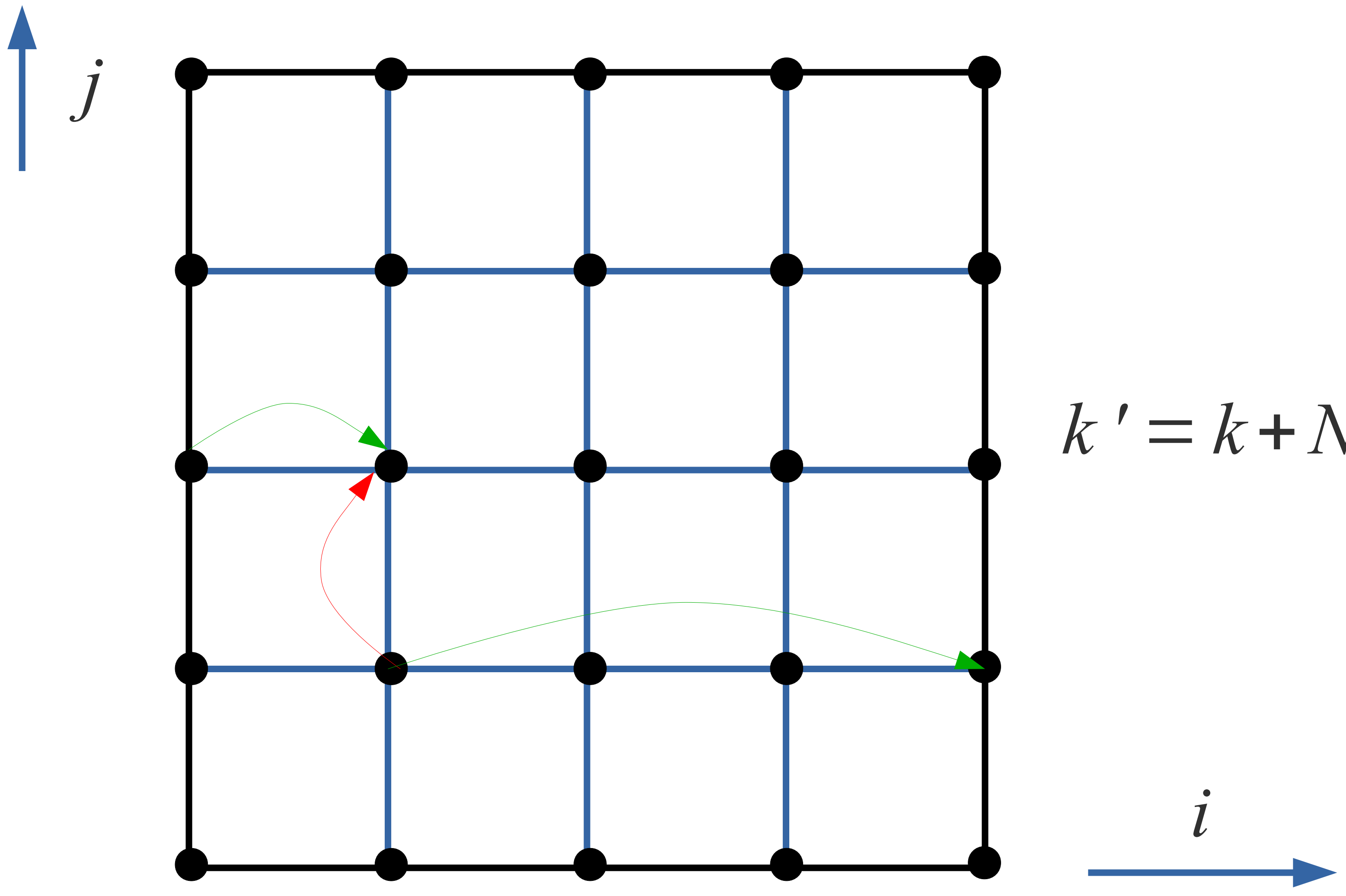
$$k = Nx$$

Ecuación Matricial: Numeración de Nodos

$$k' = k + 1$$



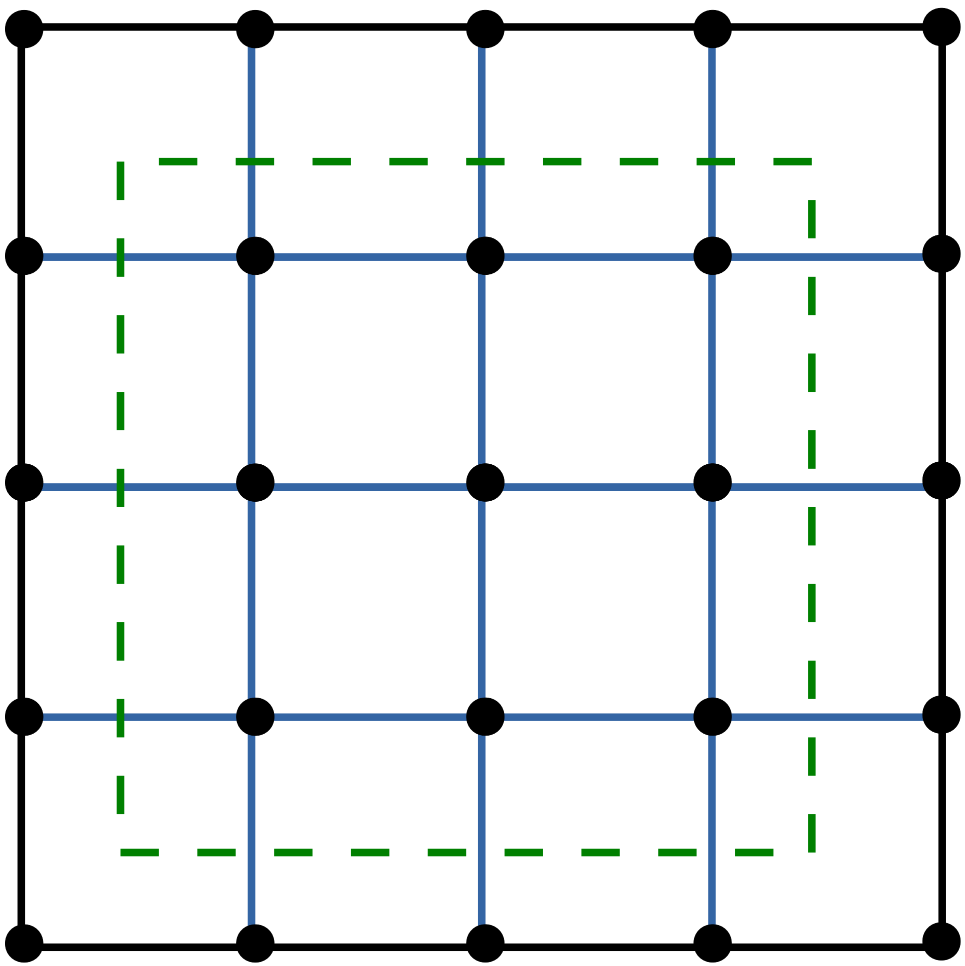
Ecuación Matricial: Numeración de Nodos



$$k' = k + Nx$$

Ecuación General

- Para los nodos Internos:



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

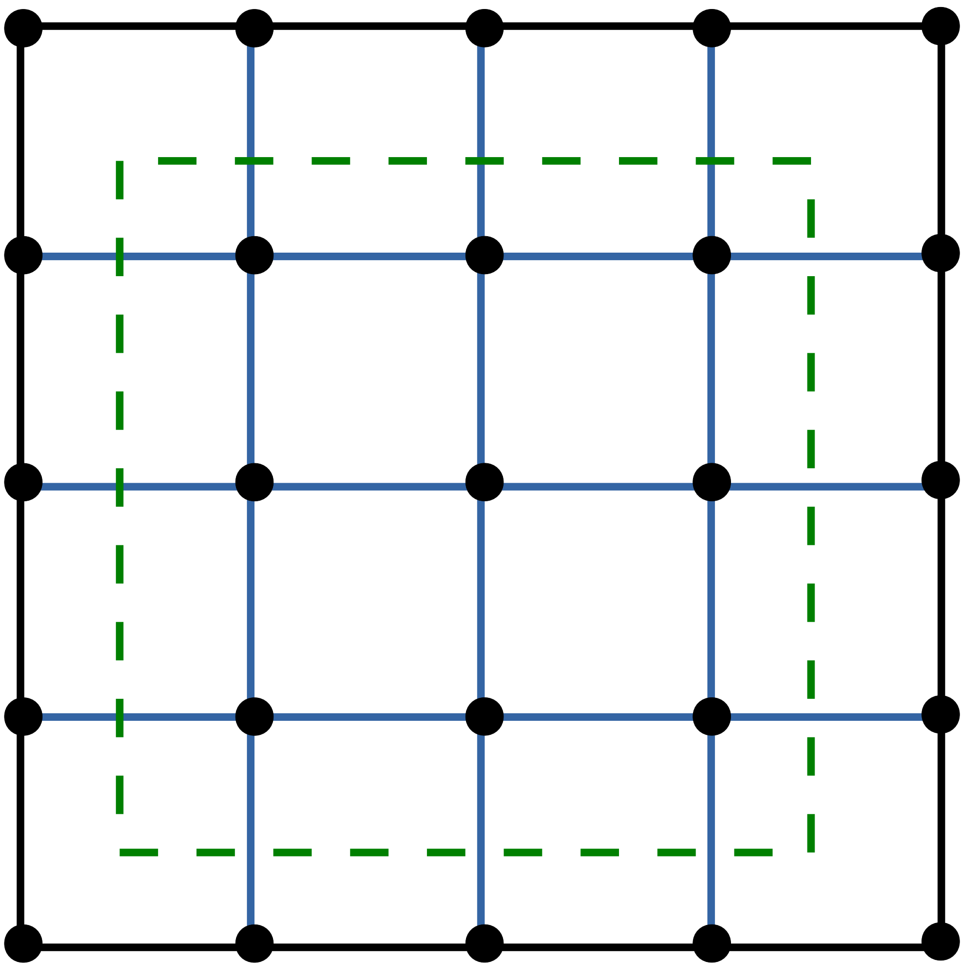
$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{k-1} - 2T_k + T_{k+1}}{dx^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{k-Nx} - 2T_k + T_{k+Nx}}{dy^2}$$

Linealización de la ecuación Diferencial

- Para los nodos Internos

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$



$$\left[M\right]\begin{pmatrix} T_1 \\ \vdots \\ T_k \\ \vdots \\ T_{NxNy} \end{pmatrix} = b$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Linealización de la ecuación Diferencial

- Coeficientes de la Matriz

$$\beta^2 T_{k-N_x} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_x} = 0$$

Fila k-ésima:

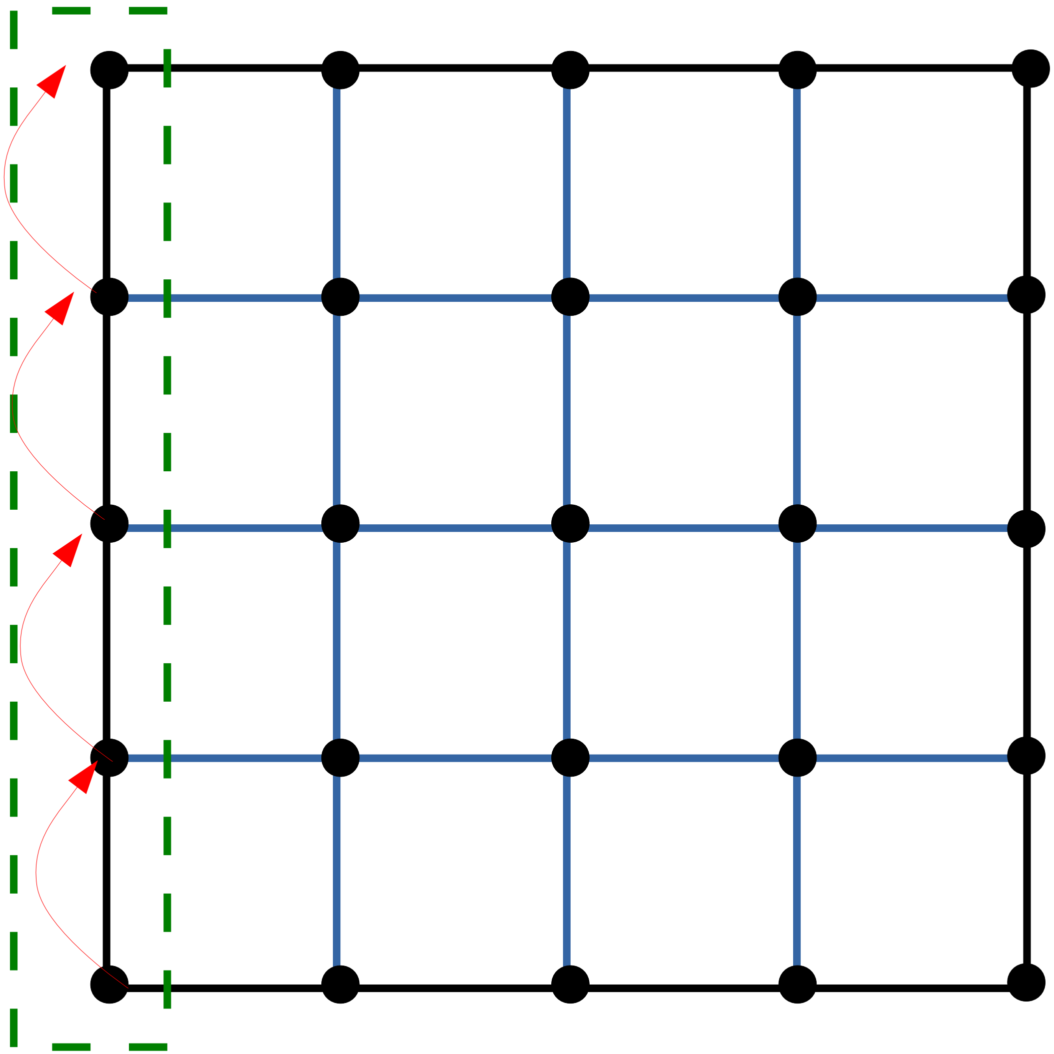
$$M_{k,:} = \left[\cdots \quad \beta^2 \quad \cdots \quad 1 \quad -2\left(1+\beta^2\right) \quad 1 \quad \cdots \quad \beta^2 \quad \cdots\right]$$

$k - Nx$ $\qquad k-1$ $\qquad k$ $\qquad k+1$ $\qquad k+Nx$

$$b_k = 0$$

Condiciones de contorno

- Temperatura Fija



$$T_{k_A} = T_A$$

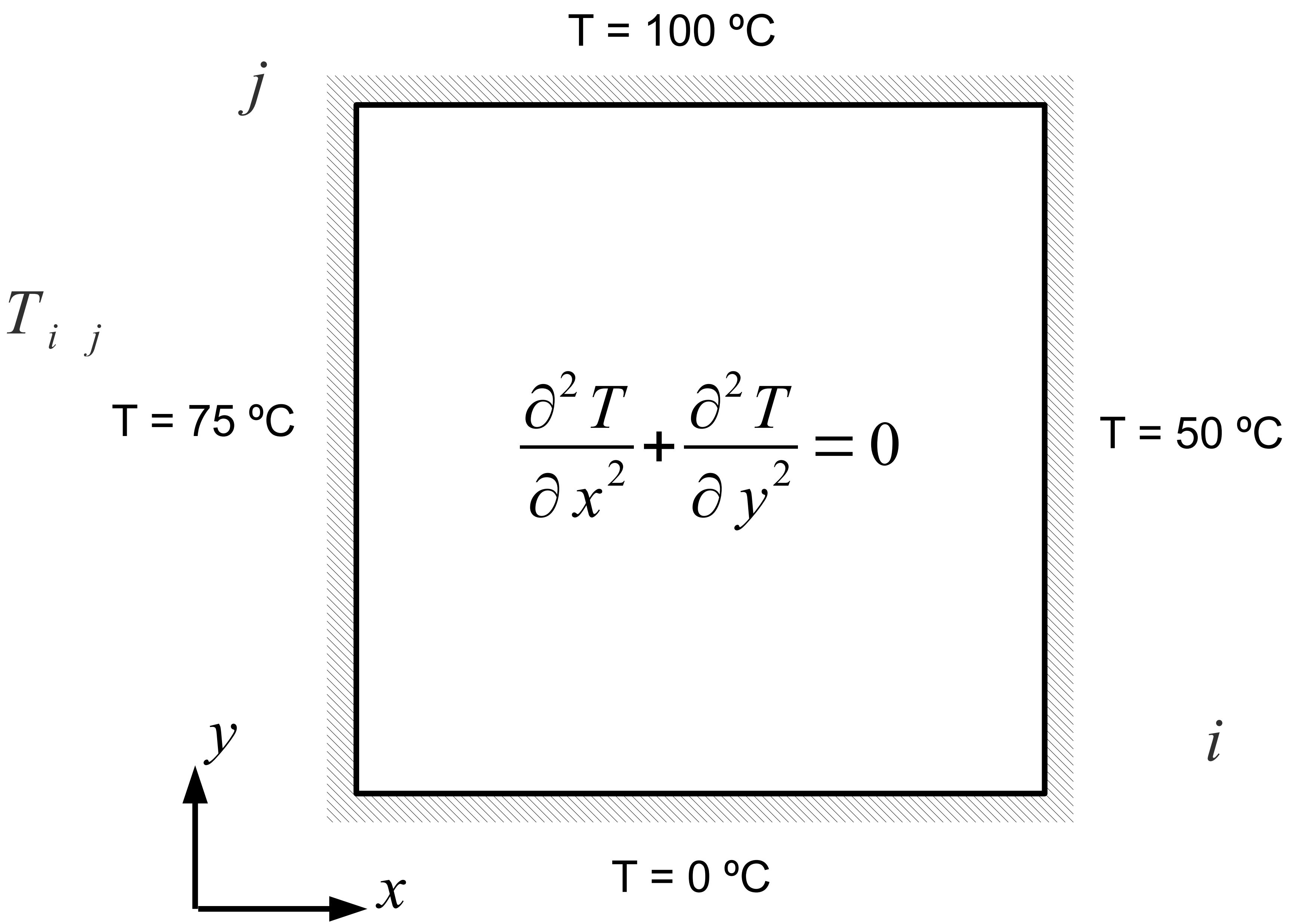
$k_A = ?$ Algun rango ...

$$M^{fija}_{k_A,:} = \begin{bmatrix} \cdots & 1 & \cdots \end{bmatrix}$$

$$k_A$$

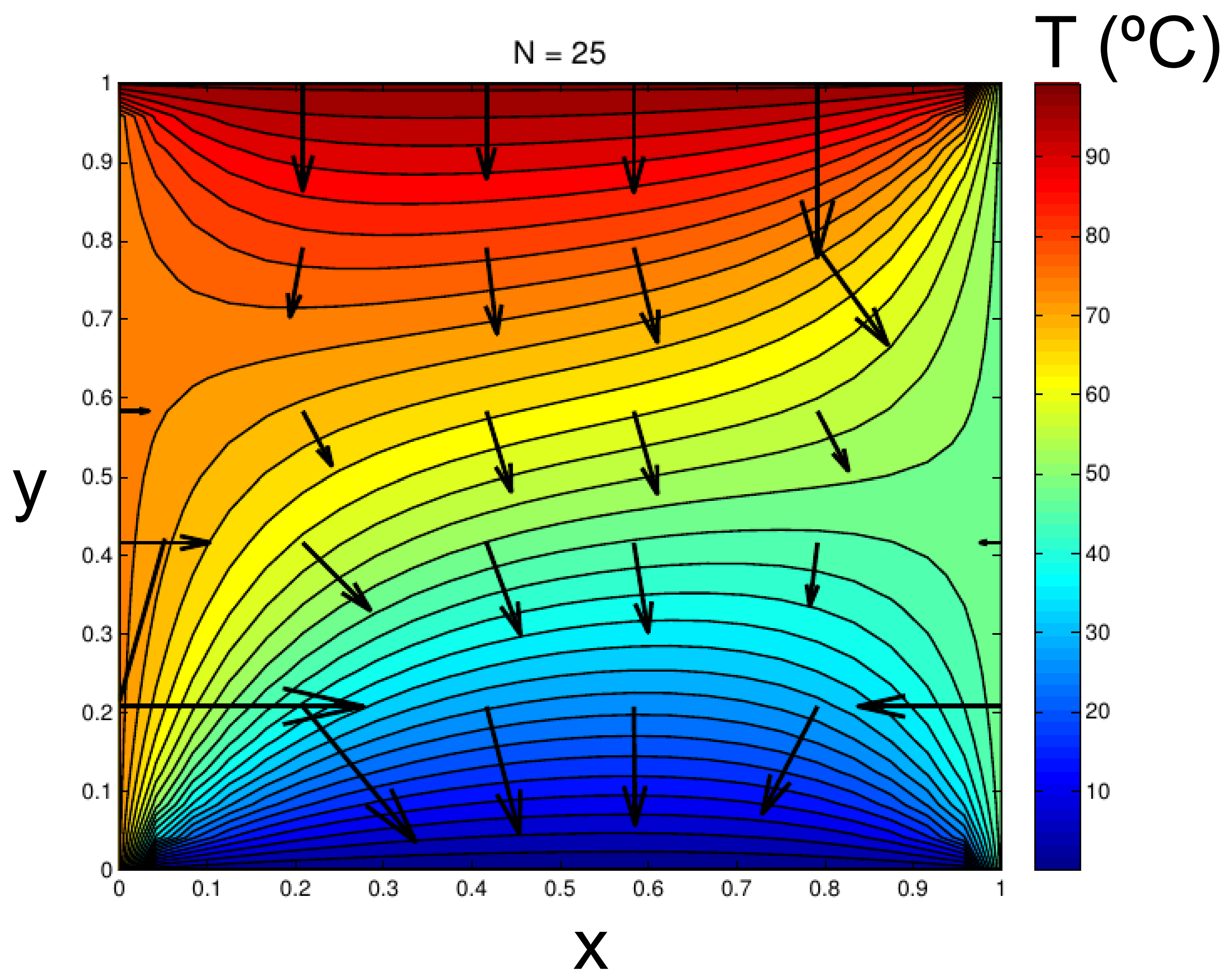
$$b_{k_A} = T_A$$

Problema: Primera Aproximación



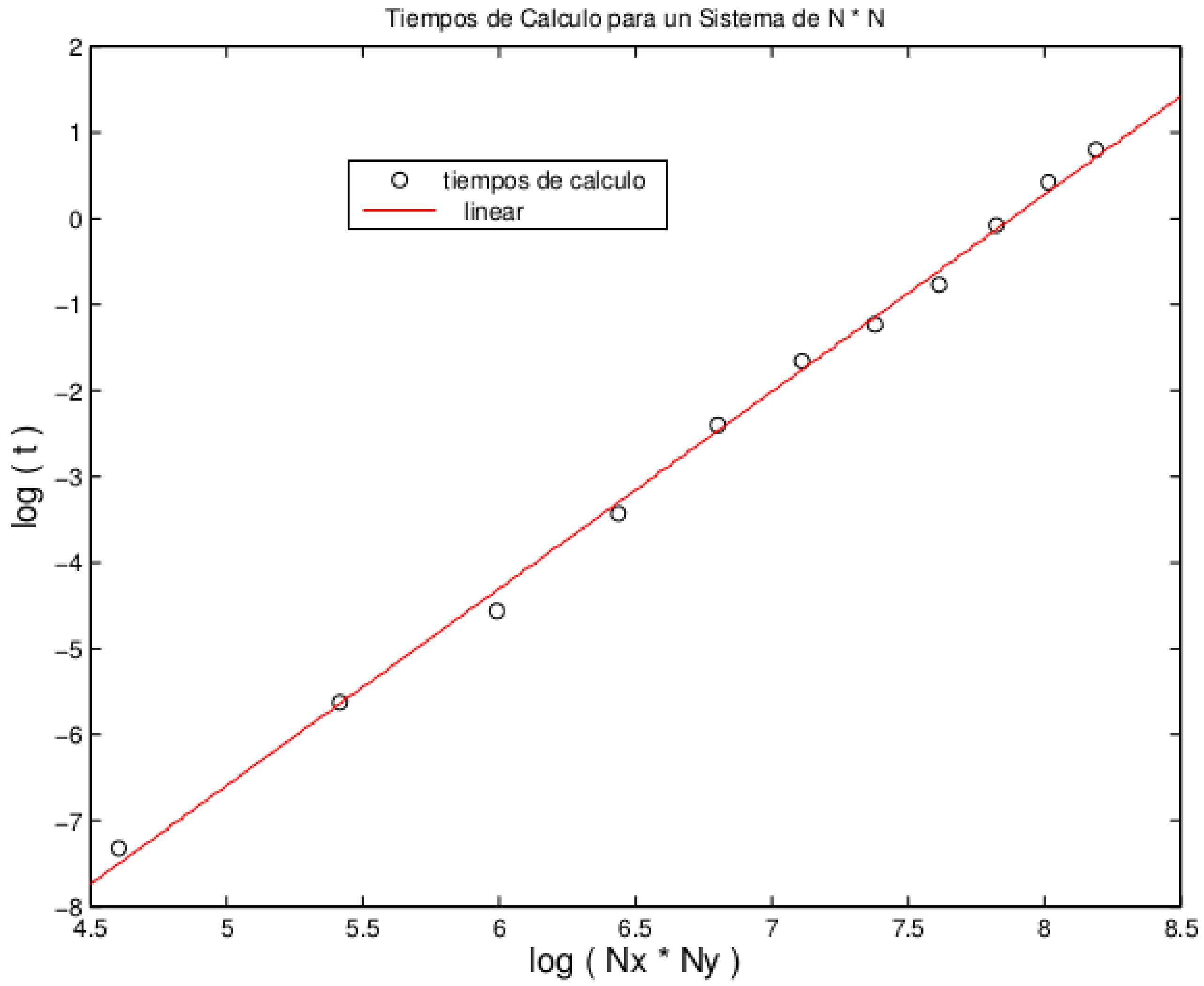
Problema: primera aproximación

- Solución:



Problema: Primera Aproximación

- Solución: Escaleo Temporal



Postproceso: Mapa de temperaturas.

Genera grilla xy para el gráfico.

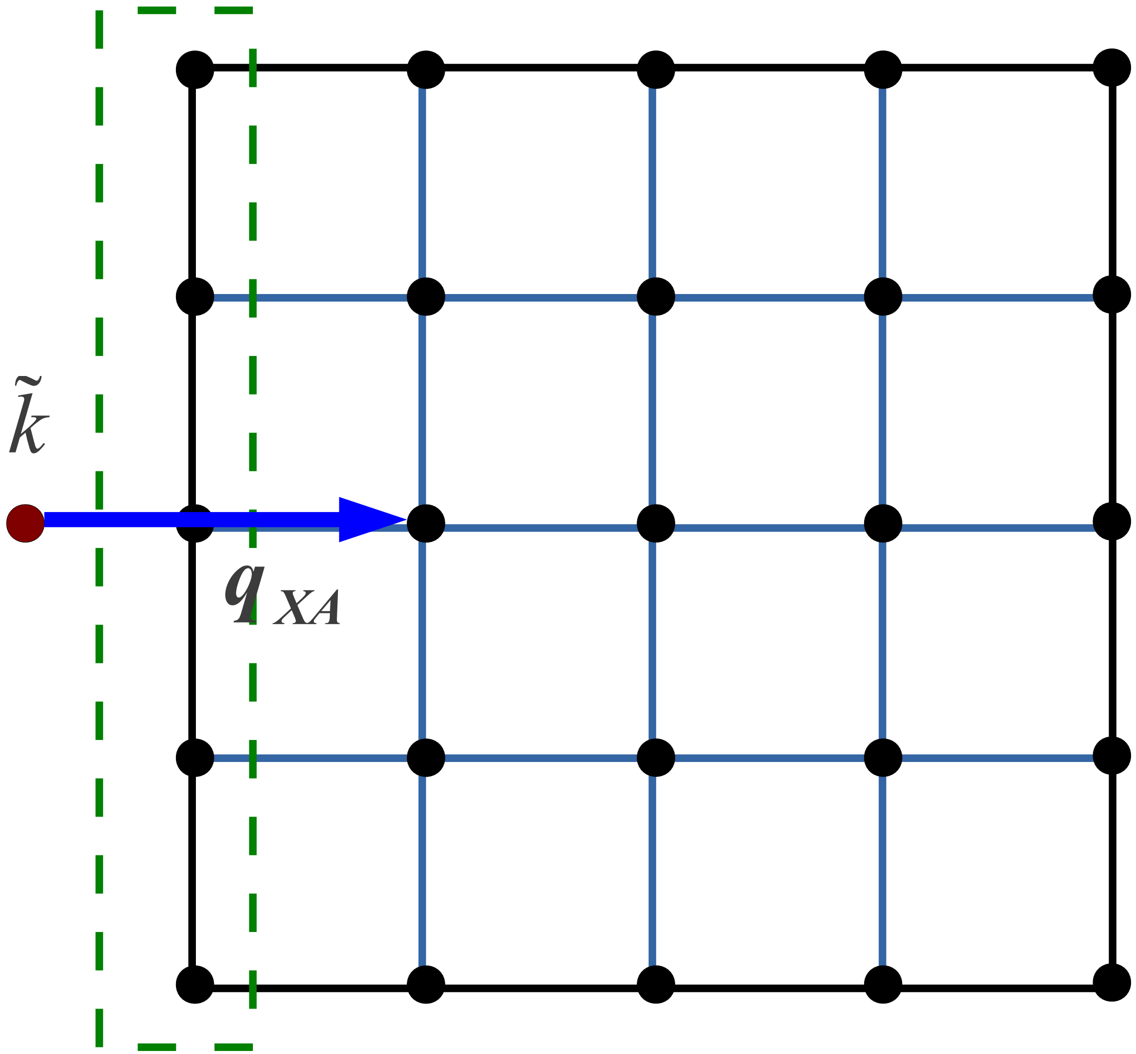
Mapa de colores.

Command Window

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on')
>>
```

Condiciones de contorno: Flujo

- Derivada centrada: punto extra



$$Q_x \propto q_{XA} =$$

$$\left.\frac{\partial T}{\partial x}\right|_{k_A} = \frac{T_{k_A+1} - T_{\tilde{k}}}{2\mathrm{dx}}$$

$$k_A = 1 : N_X : (N_Y - 1) N_x + 1$$

Condiciones de contorno: Flujo

- Cambio en los elementos de matriz

$$T_{\tilde{k}} = T_{k_A+1} - 2\mathrm{dxq}_{XA} \qquad k_A = 1 : N_X : (N_Y - 1) N_x$$

Reemplazo en la ecuación general

$$\beta^2 T_{k-N_X} + T_{k-1} - 2\left(1+\beta^2\right) T_k + T_{k+1} + \beta^2 T_{k+N_X} = 0$$

Reordeno

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2T_{k+1} + \beta^2 T_{k+N_X} = 2dxq_{XA}$$

Condiciones de contorno: Flujo

$$T_{\tilde{k}-1} = T_{k_A+1} - 2\,dxq_{XA} \qquad k_A = 1 : N_X : (N_Y - 1)\,N_x$$

$$\beta^2 T_{k-N_X} - 2\left(1+\beta^2\right) T_k + 2\,T_{k+1} + \beta^2 T_{k+N_X} = 2\,dxq_{XA}$$

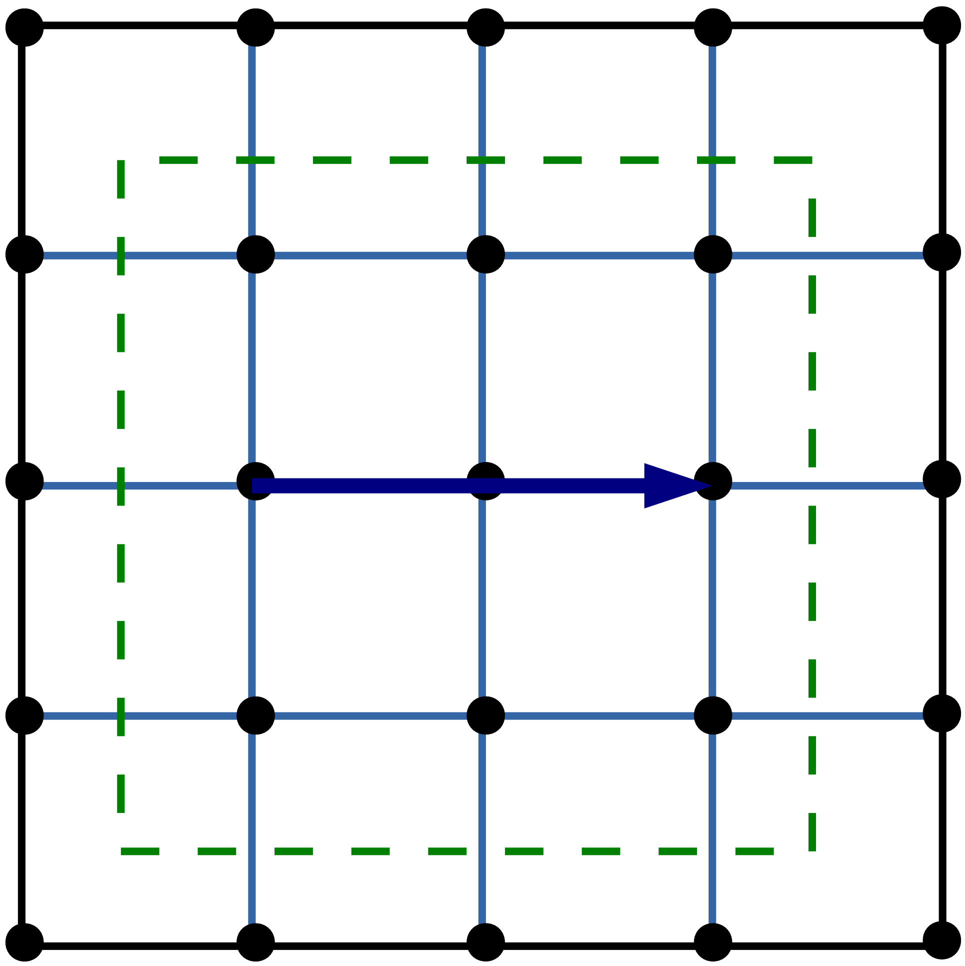
Fila k-ésima :

$$M_{k_A,:} = \begin{bmatrix} \cdots & \beta^2 & \cdots & 0 & -2\left(1+\beta^2\right) & 2 & \cdots & \beta^2 & \cdots \end{bmatrix}$$

$k - Nx$ $\quad k - 1$ $\quad k$ $\quad k + 1$ $\quad k + Nx$

$$b_k = 2\,dxq_{XA}$$

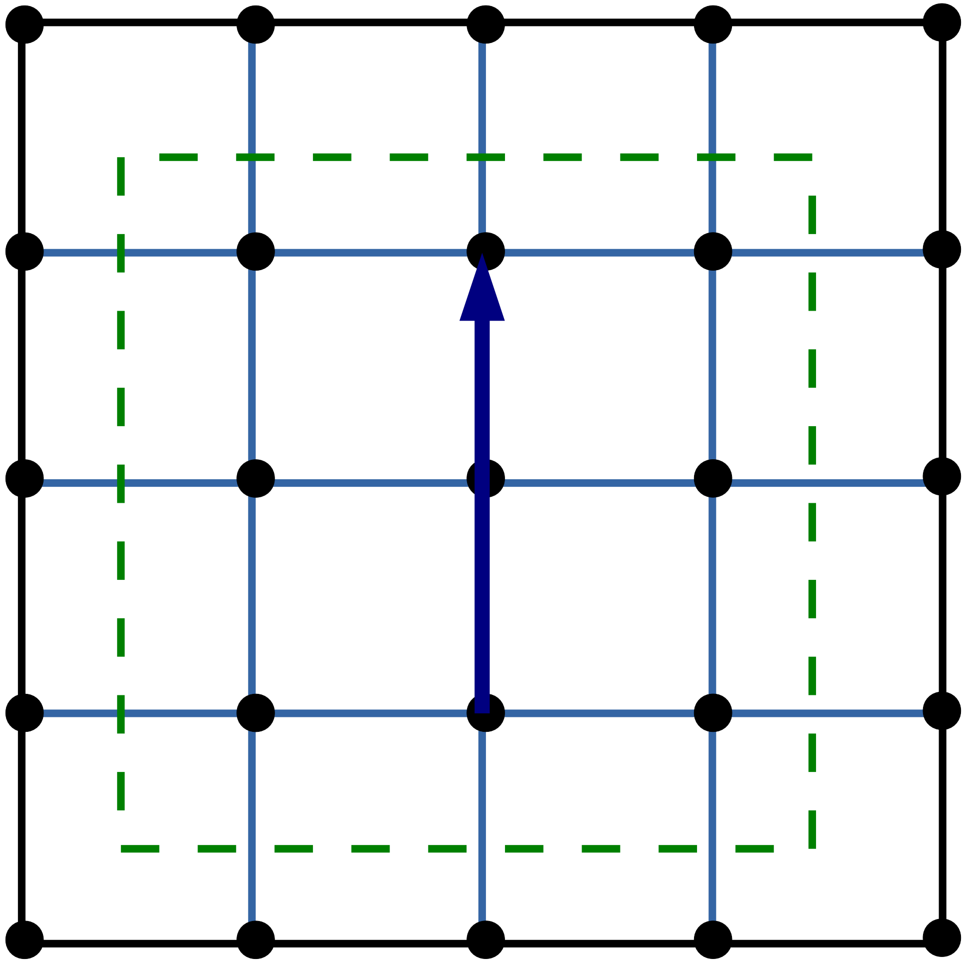
Cálculo de Flujos



$$Q_x \propto q_X =$$

$$\frac{\partial T_k}{\partial x} = \frac{T_{k+1} - T_{k-1}}{2\mathrm{dx}}$$

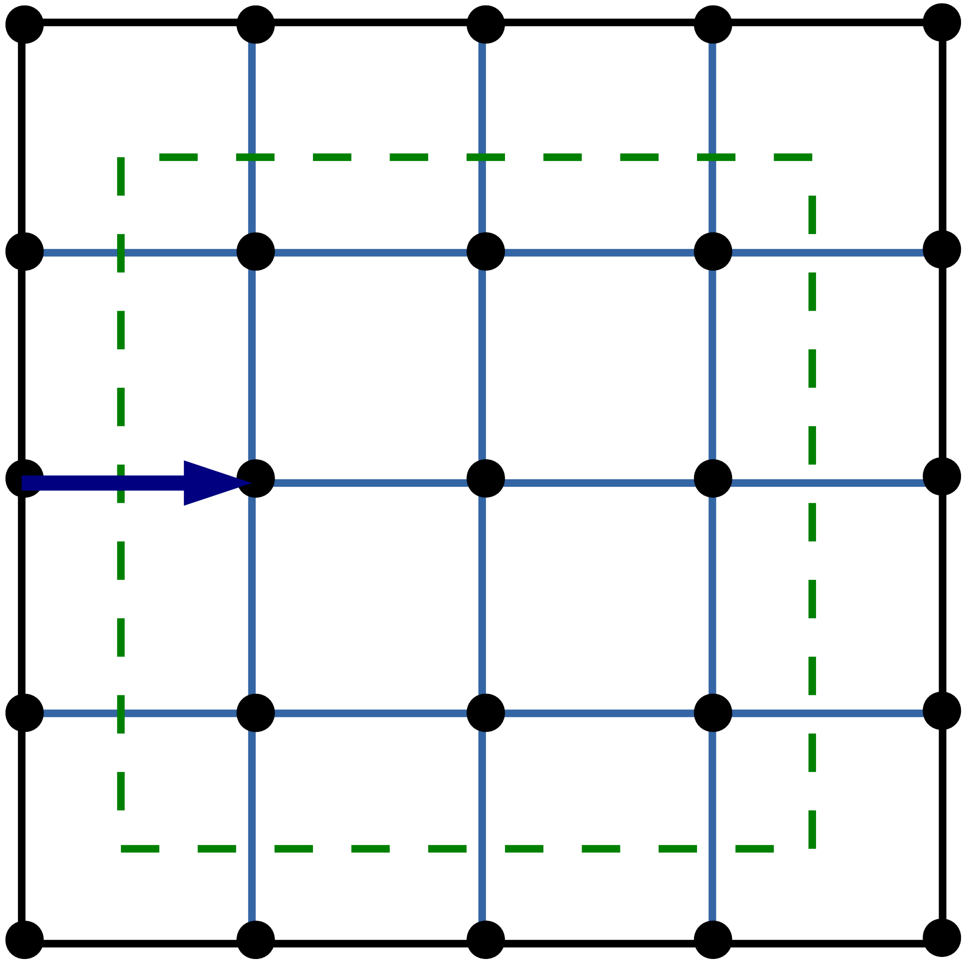
Cálculo de Flujos



$$Q_y \propto q_Y =$$

$$\frac{\partial T_k}{\partial y} = \frac{T_{k+Nx} - T_{k-Nx}}{2\mathrm{dy}}$$

Cálculo de Flujos



$$Q_{yA} \propto q_{YA} =$$

$$\frac{\partial T_{k_A}}{\partial x} = \frac{T_{k_A+1} - T_k}{dx}$$

Resultados: Graficación

Genera grulla xy para el gráfico.

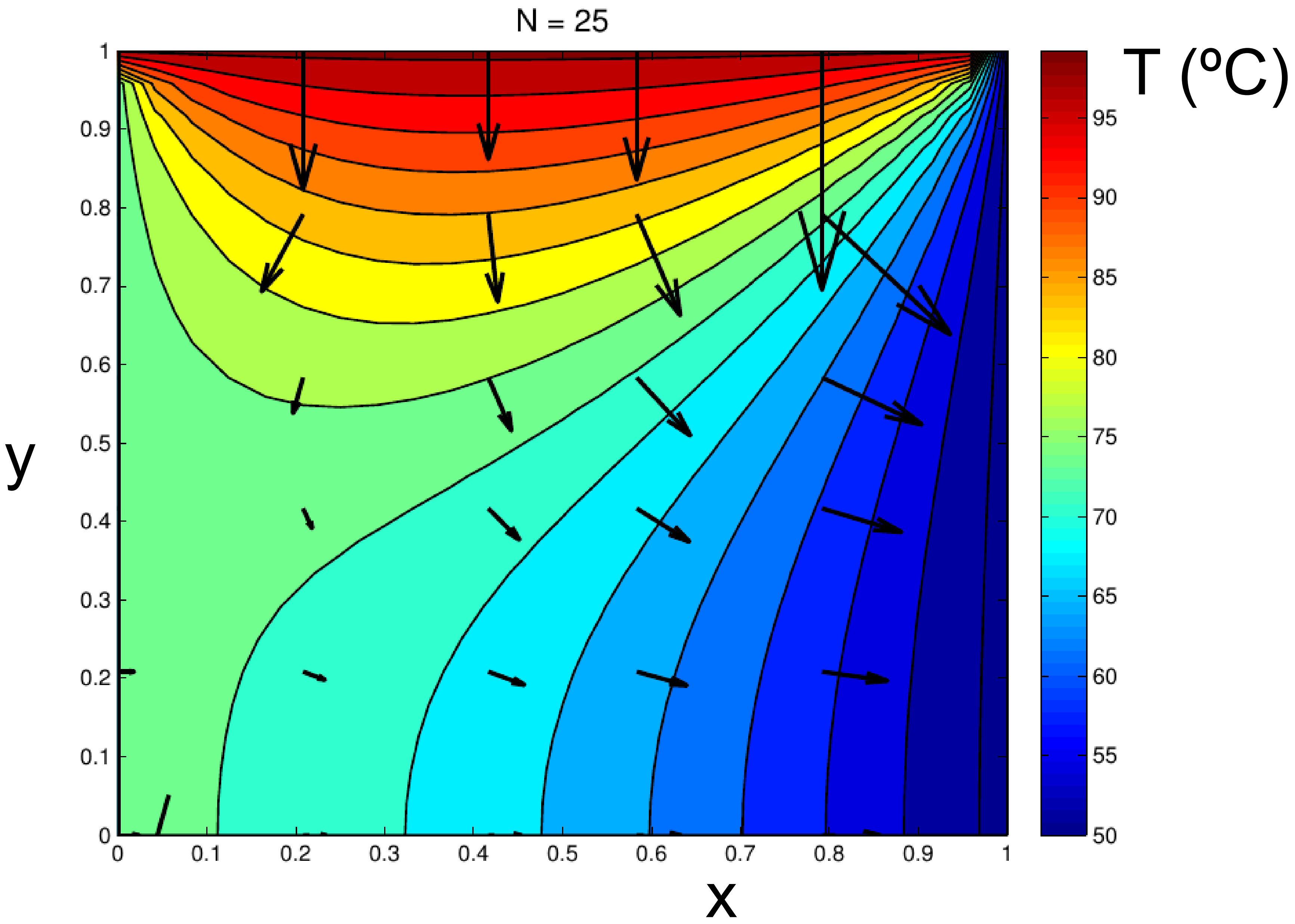
Mapa de colores.

Calcula flujos (propia)

Grafica campo vectorial

```
>> [X,Y]=meshgrid(x,y);
>> contour(X,Y,Tsol,'Fill','on');
>> [xx,yy,qx,qy]=flujos(N,N,x,y,Tsol);
>> hold on
>> quiver(xx,yy,qx,qy,'color',[0 0 0]);
>>
```

Resultados



Resultados: Escaleo