MSC 301: Introduction to Physical Oceanography
Fall 2018 · Homework 05

*Carefully answer the following questions showing all work, including all units, and using complete English sentences when appropriate. If you use a programming language, please turn in a **well commented** code that shows and explains your step-by-step approach to the problem.*

1 Vorticity

a. Define vorticity, relative vorticity, and planetary vorticity. (One or two sentences each is fine.) (3 points)

***Vorticity** is the tendency of a fluid to rotate. **Relative vorticity** is rotation of a fluid due to shear in a velocity field. Planetary vorticity is spin of a fluid due to the rotation of the Earth.*

b. Write the equation for potential vorticity. Carefully define all terms and variables. If something can be expanded, be sure to expand and define its terms as well. Be sure to include units. (10 points)

***Potential vorticity**, Q, is given by:*

$$Q = \frac{\zeta + f}{H} \tag{1}$$

where:
ζ = *relative vorticity* = $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ *(units: 1/s)*
x = *northward distance (m)*
y = *eastward distance (m)*
u = *eastward velocity component (m/s)*
v = *northward velocity component (m/s)*
f = *planetary vorticity (a.k.a. Coriolis parameter)* = $2\Omega\sin(\varphi)$ *(1/s), where:*
Ω = *rate of rotation of Earth* = $7.2921\text{x}10^{-5}$ *rad/s*
φ = *latitude (degrees)*
H = *height of the fluid parcel (m)*

c. In one or two sentences, explain what is meant by the conservation of potential vorticity. (2 points)

***Conservation of potential vorticity** (PV) means that PV does not change in time:*

$$\frac{D}{Dt}\left(\frac{\zeta + f}{H}\right) = 0 \tag{2}$$

d. Consider a particle in an ocean current moving from west to east. Can the particle's trajectory veer north while conserving potential vorticity? Can its trajectory veer to the south while conserving potential vorticity? Justify your answers in a couple well-articulated sentences or with a clear, illustrative sketch. (5 points)

For a particle moving from west to east, we first assume that relative vorticity is initially 0, that is, its motion exhibits no rotation. If the particle trajectory veers to the north, planetary vorticity, f, increases. By the right hand rule (RHR), we note that relative vorticity also increases in this case. If we assume the height of the particle H to be constant, than the particle cannot move north while conserving potential vorticity. If the particle were to veer south, f will decrease and, by the RHR, relative vorticity also decreases. Again, PV cannot be conserved if H is constant.

e. Repeat (d) for an ocean current moving from east to west. (5 points)

As in part (d), we assume that relative vorticity is initially 0 as the particle moves from east to west. If the particle trajectory veers to the north, planetary vorticity, f, increases and relative vorticity decreases (by RHR). In this case, f and PV balance each other out, so the particle can move north while conserving PV (again assuming H is constant). If the particle veers south, f decreases while relative vorticity increase. PV can again be conserved in this case.

2 Application

In homework 3, we calculated geostrophic velocities across a loop current eddy.

a. Estimate the planetary vorticity at the center of this eddy. (3 points)

$$f = 2\Omega \sin\varphi = 2(7.2921\text{x}10^{-5} rad/s) \sin\left(25.5^\circ\left(\frac{\pi}{180} rad/^\circ\right)\right) \approx 6.3\text{x}10^{-5}\ \text{sec}^{-1}$$

b. Use the geostrophic velocities to estimate the relative vorticity of the loop current eddy (Hint: Use two perpendicular transects across the eddy.) (10 points)

See sample code for workflow and explanation. $\zeta \approx -9.6\text{x}10^{-6}\ \text{sec}^{-1}$

c. Assume that the eddy is vertically homogenous and can be represented by a cylinder extending to a depth of 1000 m. Calculate the potential vorticity (PV). What does the sign of your answer indicate? Does it agree with your expectations? (2 points)

$$Q = \frac{\zeta + f}{H} = \frac{-9.6\text{x}10^{-6}\ \text{s}^{-1} + 6.3\text{x}10^{-5}\ \text{s}^{-1}}{1000\ \text{m}} \approx 5.3\text{x}10^{-8}\ \text{m}^{-1}\text{s}^{-1} \tag{3}$$

d. Loop current eddies move from east to west before meeting their demise in the "eddy graveyard" of the western Gulf of Mexico. What will happen to this eddy's vorticity as it approaches the Texas/Mexico coast? (A complete answer will reference all terms in the PV equation!) (5 points)

As the eddy moves westward, planetary vorticity will be unchanged because f is a function of latitude only. In other words, $df/dx = 0$. As the eddy encroaches upon the continental shelf of the western Gulf of Mexico basin, its depth H must decrease as the depth of the water column decreases. According to the conservation of potential vorticity, we would expect relative vorticity to also decrease in response, meaning the eddy's "spin" will slow down. This, together with the effects of friction, ultimately contributes to the eddy's decay and eventual death.