Handling Rational Expressions

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Rational Expressions and Rational Numbers are hidden throughout the Math SAT Section. This guide is designed to help you recognize problems that involve Rational Expressions and help you develop a plan of attack!

Below is a great example of a scary rational expression that is actually easy.

Example 1:

$$m = \frac{\left(\frac{r}{1,200}\right)\left(1 + \frac{r}{1,200}\right)^N}{\left(1 + \frac{r}{1,200}\right)^N - 1} P$$

The formula above gives the monthly payment m needed to pay off a loan of P dollars at r percent annual interest over N months. Which of the following gives P in terms of $m, r,$ and N ?

A) $P = \dfrac{\left(\frac{r}{1,200}\right)\left(1 + \frac{r}{1,200}\right)^N}{\left(1 + \frac{r}{1,200}\right)^N - 1} m$

B) $P = \dfrac{\left(1 + \frac{r}{1,200}\right)^N - 1}{\left(\frac{r}{1,200}\right)\left(1 + \frac{r}{1,200}\right)^N} m$

C) $P = \left(\dfrac{r}{1,200}\right) m$

D) $P = \left(\dfrac{1,200}{r}\right) m$

Let's first look at this related example:

$$m = \frac{3}{2}p$$

Solving for p:

$$p = \frac{2}{3}m$$

What if we thought of our original question this way?

$$m = \frac{\left(\frac{r}{1200}\right)\left(1 + \frac{r}{1200}\right)^N}{\left(1 + \frac{r}{1200}\right)^N - 1} P$$

Flip it!

$$m \frac{\left(1 + \frac{r}{1200}\right)^N - 1}{\left(\frac{r}{1200}\right)\left(1 + \frac{r}{1200}\right)^N} = P$$

If you know what to look for, that is the easiest question on the test!

Example 2:

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If $\frac{a}{b} = 2$, what is the value of $\frac{4b}{a}$?

A) 0
B) 1
C) 2
D) 4

If $\frac{a}{b} = \frac{2}{1}$ then what is $\frac{b}{a}$?

$$\frac{a}{b} = \frac{2}{1}$$

Taking the reciprocal of both sides- "putting 1 over both sides"

$$\frac{1}{\frac{a}{b}} = \frac{1}{\frac{2}{1}}$$

So:

$$\frac{b}{a} = \frac{1}{2}$$

And finally: $4\left(\frac{b}{a}\right) = 4\left(\frac{1}{2}\right) = 2$

Example 3:

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If $x > 3$, which of the following is equivalent

to $\dfrac{1}{\dfrac{1}{x+2} + \dfrac{1}{x+3}}$?

A) $\dfrac{2x+5}{x^2+5x+6}$

B) $\dfrac{x^2+5x+6}{2x+5}$

C) $2x+5$

D) x^2+5x+6

This one is slightly different than the other questions. For this question, we must focus on the denominator first.

$$\frac{1}{x+2} + \frac{1}{x+3}$$

Cross multiplying:

$$\frac{x+3+x+2}{(x+2)(x+3)} = \frac{2x+5}{x^2+5x+6}$$

Remember that what we just simplified was in the denominator of the original expression.

$$\frac{1}{\dfrac{2x+5}{x^2+5x+6}} = \frac{x^2+5x+6}{2x+5}$$