COLLEGE READINESS INITIATIVE

Math-With Calculator

1

The monthly membership fee for an online television and movie service is $9.80. The cost of viewing television shows online is included in the membership fee, but there is an additional fee of $1.50 to rent each movie online. For one month, Jill's membership and movie rental fees were $12.80. How many movies did Jill rent online that month?

A) 1
B) 2
C) 3
D) 4

Answer: B

$cost = member\,fee + rent\,fee * movies\,rented$
$12.8 = 9.8 + 1.5(movies\,rented)$
$12.8 - 9.8 = 1.5(movies\,rented)$
$3 = 1.5(movies\,rented)$
$\frac{3}{1.5} = movies\,rented$
$movies\,rented = 2$

written by Elise Favia

Heart of Algebra

2

One of the requirements for becoming a court reporter is the ability to type 225 words per minute. Donald can currently type 180 words per minute, and believes that with practice he can increase his typing speed by 5 words per minute each month. Which of the following represents the number of words per minute that Donald believes he will be able to type m months from now?

A) $5 + 180m$
B) $225 + 5m$
C) $180 + 5m$
D) $180 - 5m$

Answer: C

He can currently type 180 words per minute. Each month he gains 5 words, so after m months, he will have gained $5m$

In total, this is $180 + 5m$

Heart of Algebra

3

If a 3-pound pizza is sliced in half and each half is sliced into thirds, what is the weight, in ounces, of each of the slices? (1 pound = 16 ounces)

A) 4
B) 6
C) 8
D) 16

Answer: C

Our original slice produces 2 pieces.
Our second cut produces $2 * 3 = 6$ pieces

The pizza weighs $3 * 16$ ounces.
Each slice then weighs $\frac{3*16}{6} = \frac{16}{2} = 8$ ounces.

-written by Elise Favia

	Problem Solving and Data Analysis

4 Nick surveyed a random sample of the freshman class of his high school to determine whether the Fall Festival should be held in October or November. Of the 90 students surveyed, 25.6% preferred October. Based on this information, about how many students in the entire 225-person class would be expected to prefer having the Fall Festival in October? A) 50 B) 60 C) 75 D) 80	Answer: B $.256 * 225 = 57.6 \approx 60$ Problem Solving and Data Analysis

5 The density of an object is equal to the mass of the object divided by the volume of the object. What is the volume, in milliliters, of an object with a mass of 24 grams and a density of 3 grams per milliliter? A) 0.125 B) 8 C) 21 D) 72	Answer: B $d = \frac{m}{v}$ $dv = m$ $v = \frac{m}{d} = \frac{24}{3} = 8$ written by Elise Favia Problem Solving and Data Analysis

6 Last week Raul worked 11 more hours than Angelica. If they worked a combined total of 59 hours, how many hours did Angelica work last week? A) 24 B) 35 C) 40 D) 48	Answer: A Let a = *hours Angelica worked* $59 = a + (a + 11)$ $59 = 2a + 11$ $2a = 59 - 11 = 48$ $a = \frac{48}{2} = 24$ written by Elise Favia Heart of Algebra

7

Movies with Greatest Ticket Sales in 2012

MPAA rating	Type of movie				
	Action	Animated	Comedy	Drama	Total
PG	2	7	0	2	11
PG-13	10	0	4	8	22
R	6	0	5	6	17
Total	18	7	9	16	50

The table above represents the 50 movies that had the greatest ticket sales in 2012, categorized by movie type and Motion Picture Association of America (MPAA) rating. What proportion of the movies are comedies with a PG-13 rating?

A) $\frac{2}{25}$

B) $\frac{9}{50}$

C) $\frac{2}{11}$

D) $\frac{11}{25}$

Answer: A

There are 50 total movies.
There are 4 Comedies with PG-13 rating
$\frac{4}{50} = \frac{2}{25}$

Problem Solving and Data Analysis

8

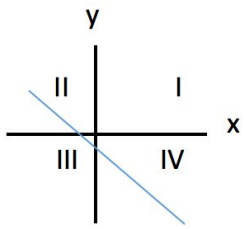
Line ℓ in the xy-plane contains points from each of Quadrants II, III, and IV, but no points from Quadrant I. Which of the following must be true?

A) The slope of line ℓ is undefined.

B) The slope of line ℓ is zero.

C) The slope of line ℓ is positive.

D) The slope of line ℓ is negative.



Drawing out the situation is helpful. As you can see from the diagram above, any straight line originating from the second quadrant, passing through the third, and then ending up in the fourth, will have a negative slope.
Answer: D
-John Cavaliere

	Heart of Algebra

9

Number of Registered Voters
in the United States in 2012, in Thousands

	Age, in years					
Region	18 to 24	25 to 44	45 to 64	65 to 74	75 and older	Total
Northeast	2,713	8,159	10,986	3,342	2,775	27,975
Midwest	3,453	11,237	13,865	4,221	3,350	36,126
South	5,210	18,072	21,346	7,272	4,969	56,869
West	3,390	10,428	11,598	3,785	2,986	32,187
Total	14,766	47,896	57,795	18,620	14,080	153,157

The table above shows the number of registered voters in 2012, in thousands, in four geographic regions and five age groups. Based on the table, if a registered voter who was 18 to 44 years old in 2012 is chosen at random, which of the following is closest to the probability that the registered voter was from the Midwest region?

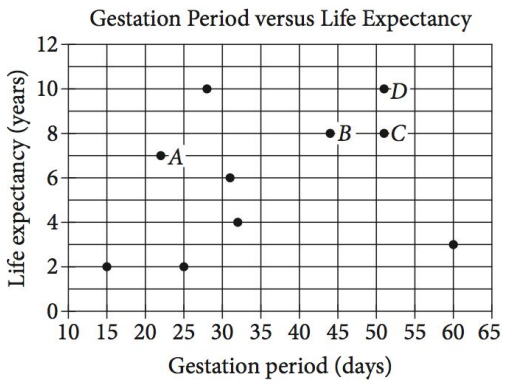
A) 0.10
B) 0.25
C) 0.40
D) 0.75

Answer: B

According to the table, there are 14,766 registered voters age 18 to 44.
Of these, 3,453 are from the Midwest.
$probability = \frac{Midwest}{Total} = \frac{3,453}{14,766} \approx .234$
Closest answer choice is .25.
-Elise Favia

Problem Solving and Data Analysis

Questions 10 and 11 refer to the following information.



A curator at a wildlife society created the scatterplot above to examine the relationship between the gestation period and life expectancy of 10 species of animals.

10

What is the life expectancy, in years, of the animal that has the longest gestation period?

A) 3
B) 4
C) 8
D) 10

Answer: A

The longest gestation period (x axis) is 60 days.

This point is at (60,3), meaning the life expectancy is 3.

Problem Solving and Data Analysis

11

Of the labeled points, which represents the animal for which the ratio of life expectancy to gestation period is greatest?

A) *A*
B) *B*
C) *C*
D) *D*

Answer: A

The labeled points are at approximately:
A (22, 7), B (44, 8), C (51, 8), D (51, 10)

We want the ratio $\frac{life\ expectancy}{gestation} = \frac{y}{x}$ to be the greatest.
Divide each y by x.
A $\frac{7}{22} \approx .32$
B $\frac{8}{44} \approx .18$
C $\frac{8}{51} \approx .16$
D $\frac{10}{51} \approx .2$

A has the greatest ratio

-Elise Favia
Problem Solving and Data Analysis

12

In the xy-plane, the graph of function f has x-intercepts at -3, -1, and 1. Which of the following could define f ?

A) $f(x) = (x-3)(x-1)(x+1)$

B) $f(x) = (x-3)(x-1)^2$

C) $f(x) = (x-1)(x+1)(x+3)$

D) $f(x) = (x+1)^2(x+3)$

Answer: C

for every x-intercept x_0, $(x - x_0)$ must be a factor of the function f

so $(x-(-3))$, $(x-(-1))$, and $(x-1)$ must be factors of f

$f(x) = (x+3)(x+1)(x-1)$

written by Elise Favia

Passport to Advanced Math

13

The population of mosquitoes in a swamp is estimated over the course of twenty weeks, as shown in the table.

Time (weeks)	Population
0	100
5	1,000
10	10,000
15	100,000
20	1,000,000

Which of the following best describes the relationship between time and the estimated population of mosquitoes during the twenty weeks?

A) Increasing linear

B) Decreasing linear

C) Exponential growth

D) Exponential decay

Answer: C

A linear function is one that can be written in the form $y = mx + b$, which is not the case here.

An exponential function is one that can be written in the form $y = x_0(a)^{cx}$. In this case, our formula appears to be $y = 100(10)^{x/5}$. Notice that $10 > 1$. This means that our result is growing.

So we have an exponential growth.
-Elise Favia

Problem Solving and Data Analysis

14

$$1{,}000\left(1+\frac{r}{1{,}200}\right)^{12}$$

The expression above gives the amount of money, in dollars, generated in a year by a \$1,000 deposit in a bank account that pays an annual interest rate of r%, compounded monthly. Which of the following expressions shows how much additional money is generated at an interest rate of 5% than at an interest rate of 3% ?

A) $1{,}000\left(1+\frac{5-3}{1{,}200}\right)^{12}$

B) $1{,}000\left(1+\frac{\frac{5}{3}}{1{,}200}\right)^{12}$

C) $\frac{1{,}000\left(1+\frac{5}{1{,}200}\right)^{12}}{1{,}000\left(1+\frac{3}{1{,}200}\right)^{12}}$

D) $1{,}000\left(1+\frac{5}{1{,}200}\right)^{12}-1{,}000\left(1+\frac{3}{1{,}200}\right)^{12}$

Answer: D

We want to compare the values produced by the function when $r=5$ and $r=3$

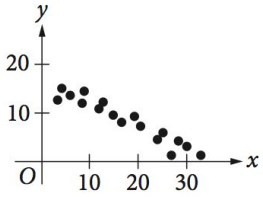
We do so by subtracting the value produced by $r=3$ from $r=5$. In other words, D.

Problem Solving and Data Analysis

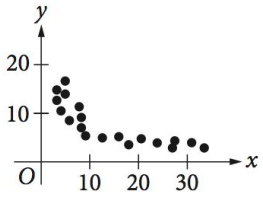
15

Which of the following scatterplots shows a relationship that is appropriately modeled with the equation $y = ax^b$, where a is positive and b is negative?

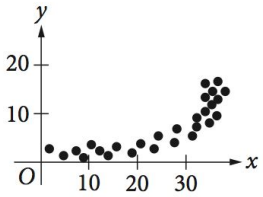
A)



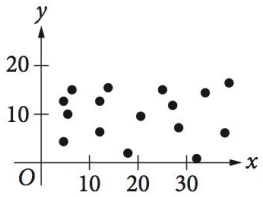
B)



C)



D)



Answer: B

If b is negative that indicates exponential decay and therefore the values are the largest close to the y axis (at low x values).

-Liam Mulcahy

Problem Solving and Data Analysis

Questions 16 and 17 refer to the following information.

Mr. Martinson is building a concrete patio in his backyard and deciding where to buy the materials and rent the tools needed for the project. The table below shows the materials' cost and daily rental costs for three different stores.

Store	Materials' Cost, M (dollars)	Rental cost of wheelbarrow, W (dollars per day)	Rental cost of concrete mixer, K (dollars per day)
A	750	15	65
B	600	25	80
C	700	20	70

The total cost, y, for buying the materials and renting the tools in terms of the number of days, x, is given by $y = M + (W + K)x$.

16

For what number of days, x, will the total cost of buying the materials and renting the tools from Store B be less than or equal to the total cost of buying the materials and renting the tools from Store A ?

A) $x \leq 6$

B) $x \geq 6$

C) $x \leq 7.3$

D) $x \geq 7.3$

First, model the cost of each situation with a linear equation. The equation for store A is $y = 750 + 80x$ and the equation for B is $y = 600 + 105x$. Set these two equations equal to one another in order to get the x value at which they intersect. The two equations intersect at $x = 6$. Because equation B has a higher slope and a lower y-intercept than store A, it is more effective to use store B for values where $x \leq 6$. Therefore, A is the correct answer.
Answer: A
-John Cavaliere

Heart of Algebra

17

If the relationship between the total cost, y, of buying the materials and renting the tools at Store C and the number of days, x, for which the tools are rented is graphed in the xy-plane, what does the slope of the line represent?

A) The total cost of the project

B) The total cost of the materials

C) The total daily cost of the project

D) The total daily rental costs of the tools

According to the equation given by the problem, the price of renting tools from each store is given by the equation $y = M + (W + K)x$, where W and K are rental costs per day. As W and K vary based upon the amount of time they are rented, it is a variable cost, and thus represents the slope of the equation.
Answer: D
-John Cavaliere

Heart of Algebra

18

Jim has identical drinking glasses each in the shape of a right circular cylinder with internal diameter of 3 inches. He pours milk from a gallon jug into each glass until it is full. If the height of milk in each glass is about 6 inches, what is the largest number of full milk glasses that he can pour from one gallon of milk? (Note: There are 231 cubic inches in 1 gallon.)

A) 2
B) 4
C) 5
D) 6

Answer: C

The volume of a cylinder is
$V = \pi r^2 h$

$r = d/2 = 3/2$
$V = \pi(\frac{3}{2})^2(6) = \frac{27}{2}\pi$
This is the amount one glass can hold

$glasses = \frac{gallon}{volume} = \frac{231}{\frac{27}{2}\pi} = \frac{462}{27\pi} \approx 5.45$
so we can fill 5 glasses completely

-written by Elise Favia

19

If $3p - 2 \geq 1$, what is the least possible value of $3p + 2$?

A) 5
B) 3
C) 2
D) 1

Answer: A

$3p - 2 \geq 1$
$3p - 2 + 4 \geq 1 + 4$
$3p + 2 \geq 5$
so the least possible value of $3p + 2 = 5$

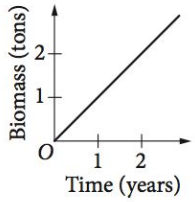
written by Elise Favia

Heart of Algebra

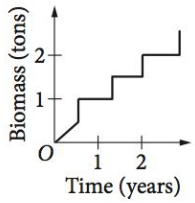
20

The mass of living organisms in a lake is defined to be the biomass of the lake. If the biomass in a lake doubles each year, which of the following graphs could model the biomass in the lake as a function of time? (Note: In each graph below, O represents $(0, 0)$.)

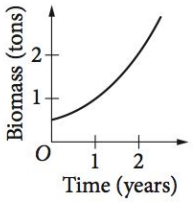
A)



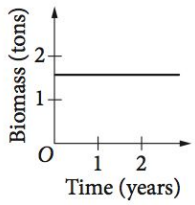
B)



C)



D)



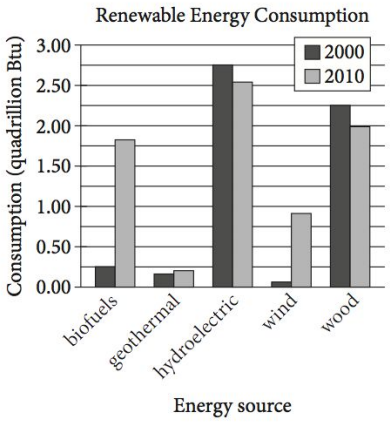
Answer: C

If the biomass doubles each year, we are looking for an exponential function particularly $y = 2^x$.

The only graph that increases exponentially is C.

Problem Solving and Data Analysis

Questions 21 and 22 refer to the following information.



The bar graph above shows renewable energy consumption in quadrillions of British thermal units (Btu) in the United States, by energy source, for several energy sources in the years 2000 and 2010.

21

In a scatterplot of this data, where renewable energy consumption in the year 2000 is plotted along the x-axis and renewable energy consumption in the year 2010 is plotted along the y-axis for each of the given energy sources, how many data points would be above the line $y = x$?

A) 1
B) 2
C) 3
D) 4

Answer: C

To be above the line $y = x$,
$y > x$.

y represents the consumption for 2010, so we are looking for the energy sources that have a greater consumption in 2010 than 2000.

These are: biofuels, geothermal, and wind.

So there are 3 data points above the line $y = x$.

-Elise Favia
Problem Solving and Data Analysis

22

Of the following, which best approximates the percent decrease in consumption of wood power in the United States from 2000 to 2010 ?

A) 6%
B) 11%
C) 21%
D) 26%

Answer: B

In 2000, wood consumption was 2.25
In 2010, wood consumption was 2.00

$decrease = 2.25 - 2 = .25$
$\%\ decrease = \frac{decrease}{consumption\ in\ 2000} = \frac{.25}{2.25} \approx .11$
or 11%

-Elise Favia
Problem Solving and Data Analysis

23

The tables below give the distribution of high temperatures in degrees Fahrenheit (°F) for City A and City B over the same 21 days in March.

City A

Temperature (°F)	Frequency
80	3
79	14
78	2
77	1
76	1

City B

Temperature (°F)	Frequency
80	6
79	3
78	2
77	4
76	6

Which of the following is true about the data shown for these 21 days?

A) The standard deviation of temperatures in City A is larger.

B) The standard deviation of temperatures in City B is larger.

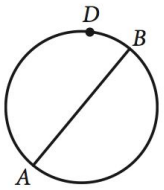
C) The standard deviation of temperatures in City A is the same as that of City B.

D) The standard deviation of temperatures in these cities cannot be calculated with the data provided.

In simplest terms, standard deviation is a measure of how spread out a set of numbers are. Because city B's data set is more spread out, with less frequency centering around the average temperature, city B's standard deviation is larger.
Answer: B
-John Cavaliere

Problem Solving and Data Analysis

24



In the circle above, segment AB is a diameter. If the length of arc $\overset{\frown}{ADB}$ is 8π, what is the length of the radius of the circle?

A) 2
B) 4
C) 8
D) 16

Answer: C

AB divides the circle in half

This means the specified arc length is half the circumference of the circle

$circumference = 2\pi r$
$arc\ length = \frac{2\pi r}{2} = \pi r$
$8\pi = \pi r$
$r = 8$

-Written by Elise Favia

25

$$f(x) = 2x^3 + 6x^2 + 4x$$
$$g(x) = x^2 + 3x + 2$$

The polynomials $f(x)$ and $g(x)$ are defined above. Which of the following polynomials is divisible by $2x + 3$?

A) $h(x) = f(x) + g(x)$
B) $p(x) = f(x) + 3g(x)$
C) $r(x) = 2f(x) + 3g(x)$
D) $s(x) = 3f(x) + 2g(x)$

The only way to do this problem is to write out the division. Find out the values of $h(x), p(x), r(s), and\ s(x)$, and then divide each equation by $2x + 3$. An equation is divisible by 2x+3 if the division does not yield a remainder. The only equation that follows this is $p(x)$.
Answer: B
-John Cavaliere

Passport to Advanced Math

26

Let x and y be numbers such that $-y < x < y$. Which of the following must be true?

I. $|x| < y$
II. $x > 0$
III. $y > 0$

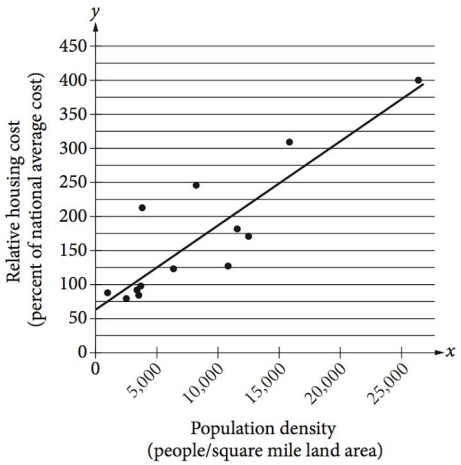
A) I only
B) I and II only
C) I and III only
D) I, II, and III

The easiest way to answer this problem is by assigning an appropriate value for y.
Let y=2, -y=-2, and let x be any value in between.
No matter what value x takes, $|x| < 2$ and y must be greater than zero. But, there's no reason why x can't be a negative value. So, only statements 1 and 3 must be correct, so the answer is C.
Answer: C
-John Cavaliere

Heart of Algebra

27

The relative housing cost for a US city is defined to be the ratio $\frac{\text{average housing cost for the city}}{\text{national average housing cost}}$, expressed as a percent.



The scatterplot above shows the relative housing cost and the population density for several large US cities in the year 2005. The line of best fit is also shown and has equation $y = 0.0125x + 61$. Which of the following best explains how the number 61 in the equation relates to the scatterplot?

A) In 2005, the lowest housing cost in the United States was about \$61 per month.

B) In 2005, the lowest housing cost in the United States was about 61% of the highest housing cost.

C) In 2005, even in cities with low population densities, housing costs were never below 61% of the national average.

D) In 2005, even in cities with low population densities, housing costs were likely at least 61% of the national average.

In order to answer this question, you must first understand what the equation represents. The equation outlines a situation where the larger a population, x, the higher relative housing costs became. So, the y-intercept represents the relative housing costs when there is a population of zero people. Once you understand that, answers A and B can be eliminated. The reason why D is the answer over C is because the model created is
just a sample, making C too certain of a statement given limited knowledge.
Answer: D
-John Cavaliere

Problem Solving and Data Analysis

28

$$f(x) = (x + 6)(x - 4)$$

Which of the following is an equivalent form of the function f above in which the minimum value of f appears as a constant or coefficient?

A) $f(x) = x^2 - 24$

B) $f(x) = x^2 + 2x - 24$

C) $f(x) = (x - 1)^2 - 21$

D) $f(x) = (x + 1)^2 - 25$

The most important thing to note is that this problem is not just asking you to find an equivalent expression, but is asking you to find the equivalent expression where the constant is also the minimum. If the two x-intercepts of the equation are -6 and 4, then the x value of the vertex is -1, making the y value of the vertex -25. So, D is the answer as the constant value of the expression is -25, which is also the minimum value of the equation.
Answer: D
-John Cavaliere

Passport to Advanced Math

29

If x is the average (arithmetic mean) of m and 9, y is the average of $2m$ and 15, and z is the average of $3m$ and 18, what is the average of x, y, and z in terms of m ?

A) $m + 6$

B) $m + 7$

C) $2m + 14$

D) $3m + 21$

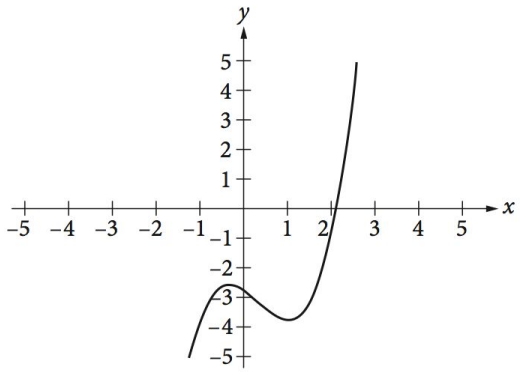
Answer: B

$x = \frac{m+9}{2}$
$y = \frac{2m+15}{2}$
$z = \frac{3m+18}{2}$
$x + y + z = \frac{m+9+2m+15+3m+18}{2} = \frac{6m+42}{2} = 3m + 21$
$avg\ (x + y + z) = \frac{x+y+z}{3} = \frac{3m+21}{3} = m + 7$

-Elise Favia

Heart of Algebra

30



The function $f(x) = x^3 - x^2 - x - \frac{11}{4}$ is graphed in the xy-plane above. If k is a constant such that the equation $f(x) = k$ has three real solutions, which of the following could be the value of k ?

A) 2
B) 0
C) −2
D) −3

This question is asking you to find a horizontal line that will intersect three times with the given function. To find this, try out each solution. A and B will only intersect the equation once, while C intersects the equation only twice. Answer D, $k = -3$, intersects the equation 3 times, and is thusly the correct answer.
Answer: D
-John Cavaliere

Passport to Advanced Math

31

A partially filled pool contains 600 gallons of water. A hose is turned on, and water flows into the pool at the rate of 8 gallons per minute. How many gallons of water will be in the pool after 70 minutes?

Answer: 1160

Let m = *number of minutes*
pool water = $600 + 8m$
we also have $m = 70$
pool water = $600 + 8(70) = 600 + 560 = 1160$

-Elise Favia
Problem Solving and Data Analysis

32

The normal systolic blood pressure P, in millimeters of mercury, for an adult male x years old can be modeled by the equation $P = \frac{x + 220}{2}$. According to the model, for every increase of 1 year in age, by how many millimeters of mercury will the normal systolic blood pressure for an adult male increase?

Answer: ½ or 0.5

$P = \frac{x+220}{2} = \frac{x}{2} + \frac{220}{2}$
If $x = 1$, $\frac{x}{2} = \frac{1}{2}$

Heart of Algebra

33

The *pes*, a Roman measure of length, is approximately equal to 11.65 inches. It is also equivalent to 16 smaller Roman units called digits. Based on these relationships, 75 Roman digits is equivalent to how many feet, to the nearest hundredth? (12 inches = 1 foot)

Answer: 4.55

convert digits to *pes*:
$75\ digits = \frac{75\ digits}{16\ digits\ per\ pes} = 4.6875\ pes$

convert *pes* to inches:
$4.6875\ pes = 4.6875\ pes * 11.65\ inches\ per\ pes$
$= 54.609375\ inches$

convert inches to feet:
$54.609375\ inches = \frac{54.609375\ inches}{12\ inches\ per\ foot} \approx 4.55\ feet$

-Elise Favia
Problem Solving and Data Analysis

34

In a study of bat migration habits, 240 male bats and 160 female bats have been tagged. If 100 more female bats are tagged, how many more male bats must be tagged so that $\frac{3}{5}$ of the total number of bats in the study are male?

Let the number of new males = x
$total = 240 + 160 + 100 + x = 500 + x$
$male = 240 + x$

we want $\frac{male}{total} = \frac{3}{5}$
$\frac{3}{5} = \frac{240+x}{500+x}$
$3(500 + x) = 5(240 + x)$
$1500 + 3x = 1200 + 5x$
$1500 - 1200 = 5x - 3x$
$300 = 2x$
$x = 150$

-Elise Favia
Heart of Algebra

35

$$q = \frac{1}{2}nv^2$$

The dynamic pressure q generated by a fluid moving with velocity v can be found using the formula above, where n is the constant density of the fluid. An aeronautical engineer uses the formula to find the dynamic pressure of a fluid moving with velocity v and the same fluid moving with velocity $1.5v$. What is the ratio of the dynamic pressure of the faster fluid to the dynamic pressure of the slower fluid?

Answer: 9/4 or 2.25

$q = \frac{1}{2}nv^2$

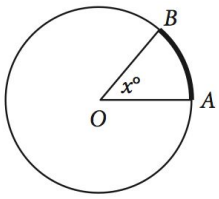
For the faster velocity, we have:
$q = \frac{1}{2}n(1.5v)^2 = \frac{1}{2}nv^2(1.5)^2$

the ratio is
$\frac{faster}{slower} = \frac{\frac{1}{2}nv^2(1.5)^2}{\frac{1}{2}nv^2} = (1.5)^2 = 2.25$

-Elise Favia

Passport to Advanced Math

36



Note: Figure not drawn to scale.

In the figure above, the circle has center O and has radius 10. If the length of arc $\overset{\frown}{AB}$ (shown in bold) is between 5 and 6, what is one possible integer value of x ?

Answer: 29, 30, 31, 32, 33, or 34

The percent of the circumference the arc length takes is the same as the percent of the circle (360°) the angle is.

Let's find the circumference
$C = 2\pi r = 2\pi(10) = 20\pi$

Find the fraction of the circumference the arc length takes up for both 5 and 6.
These are $\frac{5}{20\pi}$ and $\frac{6}{20\pi}$.

Multiply by 360, the number of degrees in a circle.
$(\frac{5}{20\pi})(360) \approx 28.6$
$(\frac{6}{20\pi})(360) \approx 34.4$

The number we choose must be greater than 28.6 and less than 34.4
So we can choose any whole number between 29 and 34.

Questions 37 and 38 refer to the following information.

The stock price of one share in a certain company is worth \$360 today. A stock analyst believes that the stock will lose 28 percent of its value each week for the next three weeks. The analyst uses the equation $V = 360(r)^t$ to model the value, V, of the stock after t weeks.

37

What value should the analyst use for r ?

Answer: 0.72

The stock will lose 28 percent.
That means each week it will be worth
$100 - 28 = 72$ percent of its previous value.
This is the same as $\frac{72}{100} = .72$

-Elise Favia
Passport to Advanced Math

38

To the nearest dollar, what does the analyst believe the value of the stock will be at the end of three weeks? (Note: Disregard the \$ sign when gridding your answer.)

Answer: 134

$V = 360(r)^t$
$V = 360(.72)^t$
$t = 3$ at the end of three weeks
$V = 360(.72)^3 \approx 134.37 \approx 134$

-Elise Favia
Passport to Advanced Math