Math-With Calculator

Marilyn's Hike
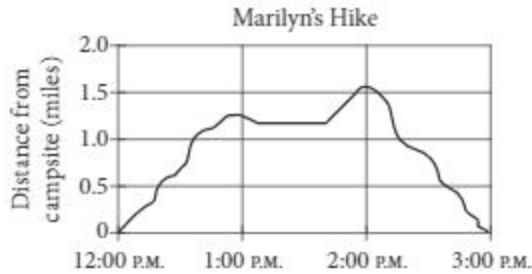

The graph above shows Marilyn's distance from her campsite during a 3-hour hike. She stopped for 30 minutes during her hike to have lunch. Based on the graph, which of the following is closest to the time she finished lunch and continued her hike?

A) 12:40 P.M.

B) 1:10 P.M.

C) 1:40 P.M.

D) 2:00 P.M.

Answer: C

The graph shows Marilyn's distance from her campsite relative to the time of day so the slope indicates Marilyn's change in distance from the campsite during the 3-hour hike.

The portion of the graph representing Marilyn's 30-minute lunch break is the horizontal segment starting around 1:10 P.M. and ending around 1:40 P.M. During this break Marilyn's distance from her campsite remained unchanged, but at around 1:40 P.M. Marilyn's distance began to change indicating she finished lunch and continued her hike.

- Trevor McGee

Problem Solving and Data Analysis

2

	Age		Total
Gender	Under 40	40 or older	
Male	12	2	14
Female	8	3	11
Total	20	5	25

The table above shows the distribution of age and gender for 25 people who entered a contest. If the contest winner will be selected at random, what is the probability that the winner will be either a female under age 40 or a male age 40 or older?

A) $\frac{4}{25}$

B) $\frac{10}{25}$

C) $\frac{11}{25}$

D) $\frac{16}{25}$

Answer: B

The table shows that of the 25 participants, eight are females under age 40 and two are males age 40 or older. Therefore, the probability that the winner will be either a female under age 40 or a male age 40 or older is $\frac{8}{25} + \frac{2}{25} = \frac{10}{25}$, answer B.
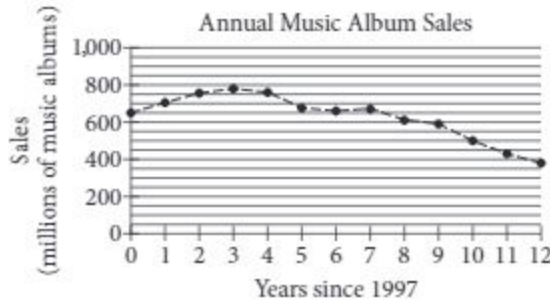
This probability is the fraction 8 females under age 40 over 25 participants plus the fraction 2 males age 40 or older over 25 participants.

- Trevor McGee

Problem Solving and Data Analysis

3

The graph below shows the total number of music album sales, in millions, each year from 1997 through 2009.

Annual Music Album Sales

Based on the graph, which of the following best describes the general trend in music album sales from 1997 through 2009 ?

A) Sales generally increased each year since 1997.
B) Sales generally decreased each year since 1997.
C) Sales increased until 2000 and then generally decreased.
D) Sales generally remained steady from 1997 through 2009.

Answer: C

This graph shows that in 1997 about 650 million albums were sold and that sales increased for the next three years until they peaked at about 875 million albums in 2000.

After 2000, sales generally decreased for the next nine years through 2009 when about 375 million albums were sold.

- Trevor McGee

Problem Solving and Data Analysis

4

n	1	2	3	4
$f(n)$	−2	1	4	7

The table above shows some values of the linear function f. Which of the following defines f ?

A) $f(n) = n - 3$
B) $f(n) = 2n - 4$
C) $f(n) = 3n - 5$
D) $f(n) = 4n - 6$

Answer: C

The best approach to finding which of the following defines f is determining the relationship between n and $f(n)$ which means finding the slope. Slope is $\frac{\Delta f(n)}{\Delta n}$ and according to the table, an increase of 1 n unit results in an increase of 3 $f(n)$ units so the correct function has a slope of 3. Answer C is the only answer with a slope of 3 the graph of $f(n) = 3n - 5$ passes through every point presented in the table.

To further check this answer, one can plug each n value from the table and realize that the values result in the corresponding $f(n)$ values from the table.

- Trevor McGee

Heart of Algebra

5

At Lincoln High School, approximately 7 percent of enrolled juniors and 5 percent of enrolled seniors were inducted into the National Honor Society last year. If there were 562 juniors and 602 seniors enrolled at Lincoln High School last year, which of the following is closest to the total number of juniors and seniors at Lincoln High School last year who were inducted into the National Honor Society?

A) 140
B) 69
C) 39
D) 30

Answer: B

.07 x 562 = 39.34
.05 x 602 = 30.10

39.34+30.10 = 69.44 69

-Harper Robinson
Problem Solving and Data Analysis

6

$$3x^2 - 5x + 2$$
$$5x^2 - 2x - 6$$

Which of the following is the sum of the two polynomials shown above?

A) $8x^2 - 7x - 4$
B) $8x^2 + 7x - 4$
C) $8x^4 - 7x^2 - 4$
D) $8x^4 + 7x^2 - 4$

Answer: A

$(3x^2 - 5x + 2) + (5x^2 - 2x - 6)$
$= (8x^2 - 7x - 4)$

Take each part of the polynomial step by step, including signs:
$$(3x^2) + (5x^2) = 8x^2$$
$$(-5x) + (-2x) = -7x$$
$$(2) + (-6) = -4$$

-Harper Robinson
Passport to Advanced Math

7

If $\frac{3}{5}w = \frac{4}{3}$, what is the value of w ?

A) $\frac{9}{20}$

B) $\frac{4}{5}$

C) $\frac{5}{4}$

D) $\frac{20}{9}$

Answer: D

Multiply both sides by the reciprocal (inverse) of ($\frac{3}{5}$) to isolate the w.

$$\frac{3}{5}w = \frac{4}{3}$$
$$\frac{3}{5}w \left(\frac{5}{3}\right) = \frac{4}{3}\left(\frac{5}{3}\right)$$
$$w = \frac{20}{9}$$

-Harper Robinson

	Heart of Algebra

8

The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation $y = 0.56x + 27.2$, where x represents the number of years since 2000, and y represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation?

A) The total number of students at the school in 2000

B) The average number of students per classroom in 2000

C) The estimated increase in the average number of students per classroom each year

D) The estimated difference between the average number of students per classroom in 2010 and in 2000

Answer: C

Think of it like a linear equation on a graph, with 0.56 being the slope, and 27.2 being the y intercept.

27.2 is the number of students in 2000.
0.56 is the slope, or rate of change in the average number of students (y) each year.
The rate of change is the same as an increase, so 0.56 is the estimated increase in the average number of students per classroom each year.

-Harper Robinson

Heart of Algebra

9

Nate walks 25 meters in 13.7 seconds. If he walks at this same rate, which of the following is closest to the distance he will walk in 4 minutes?

A)   150 meters

B)   450 meters

C)   700 meters

D) 1,400 meters

Answer: B
To find the rate that Nate walks, divide the meters walked by the seconds. (rate= distance/time)
 25 meters per 13.7 seconds =

$\frac{25}{13.7}$ = 1.82 meters per second

Once you have the rate, use d = r x t (distance = rate x time), to find the distance.

Time in seconds:
 4 minutes = 240 seconds

D = r × t
Distance = rate × time
Distance = 1.82 meters per second × 240 seconds

Distance = 426.8 meters, which is closest to 450 meters

-Harper Robinson

	Problem Solving and Data Analysis

Questions 10 and 11 refer to the following information.

Planet	Acceleration due to gravity $\left(\frac{m}{sec^2}\right)$
Mercury	3.6
Venus	8.9
Earth	9.8
Mars	3.8
Jupiter	26.0
Saturn	11.1
Uranus	10.7
Neptune	14.1

The chart above shows approximations of the acceleration due to gravity in meters per second squared $\left(\frac{m}{sec^2}\right)$ for the eight planets in our solar system. The weight of an object on a given planet can be found by using the formula $W = mg$, where W is the weight of the object measured in newtons, m is the mass of the object measured in kilograms, and g is the acceleration due to gravity on the planet measured in $\frac{m}{sec^2}$.

Above (to the right)

10

What is the weight, in newtons, of an object on Mercury with a mass of 90 kilograms?

A) 25
B) 86
C) 101
D) 324

Answer: D

To find the weight of an object on Mercury with a mass of 90 kilograms, plug the information in the equation $W = mg$.
$W = ?$
$m = 90$ kilograms
$g = 3.6$ m/sec² on Mercury, according to the chart

$$W = mg$$
$$W = (90 \text{ kilograms})(3.6 \text{ m/sec}^2)$$
$$W = 324 \text{ newtons}$$

-Harper Robinson
Problem Solving and Data Analysis

11

An object on Earth has a weight of 150 newtons. On which planet would the same object have an approximate weight of 170 newtons?

A) Venus

B) Saturn

C) Uranus

D) Neptune

Answer: B

To find the weight of an object on Earth with a weight of 150 newtons, plug the information in the equation $W = mg$.

$W = 150$ newtons

$m = ?$

$g = 9.8$ m/sec^2 on Earth, according to the chart

$$W = mg$$
$$150 = (m)(9.8 \text{ m/sec}^2)$$
divide both sides by 9.8
$$m = 15.31 \text{ kilograms}$$

To find which planet the same object, with a weight of 15.31 kilograms, will have a weight of 170 newtons, plug the information in the equation $W = mg$.

$W = 170$ newtons

$m = 15.31$ kilograms (found in previous part of the problem)

$g = ?$

$$W = mg$$
$$170 = (15.31 \text{ kilograms})(g)$$
divide both sides by 15.31
$$g = 11.11 \text{ m/sec}^2$$

Now check which planet has a matching acceleration due to gravity.
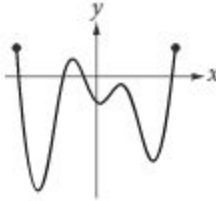
Answer = Saturn

-Harper Robinson
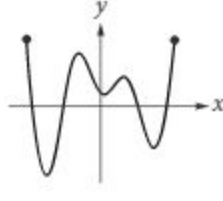
Problem Solving and Data Analysis

12

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?
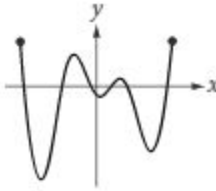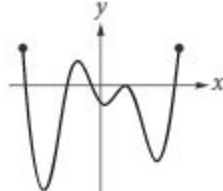
A)

B)

C)

D)

Answer: D

Zeros are x intercepts, where the y value equals zero.

Only function D crosses or touches the x axis only 5 times.

-Harper Robinson
Passport to Advanced Math

13

$$h = -16t^2 + vt + k$$

The equation above gives the height h, in feet, of a ball t seconds after it is thrown straight up with an initial speed of v feet per second from a height of k feet. Which of the following gives v in terms of h, t, and k ?

A) $v = h + k - 16t$

B) $v = \dfrac{h - k + 16}{t}$

C) $v = \dfrac{h + k}{t} - 16t$

D) $v = \dfrac{h - k}{t} + 16t$

Answer: D

Solve for v.

$h = -16t^2 + vt + k$
$h + 16t^2 - k = vt$
$\dfrac{h}{t} + 16t - \dfrac{k}{t} = v$
$\dfrac{h-k}{t} + 16t = v$

-Harper Robinson

Passport to Advanced Math

14

The cost of using a telephone in a hotel meeting room is $0.20 per minute. Which of the following equations represents the total cost c, in dollars, for h hours of phone use?

A) $c = 0.20(60h)$

B) $c = 0.20h + 60$

C) $c = \dfrac{60h}{0.20}$

D) $c = \dfrac{0.20h}{60}$

Answer: A

If it costs $0.20 per <u>minute</u>, then it must cost $0.20 x 60 per <u>hour</u>.

If the cost was $0.20 per <u>minute</u>, the equation would be c=0.20m. However, since it is per <u>hour</u>, you must multiply .20h by 60.
This gives you c= 0.20(60h)

-Harper Robinson

Heart of Algebra

15

In order to determine if treatment X is successful in improving eyesight, a research study was conducted. From a large population of people with poor eyesight, 300 participants were selected at random. Half of the participants were randomly assigned to receive treatment X, and the other half did not receive treatment X. The resulting data showed that participants who received treatment X had significantly improved eyesight as compared to those who did not receive treatment X. Based on the design and results of the study, which of the following is an appropriate conclusion?

A) Treatment X is likely to improve the eyesight of people who have poor eyesight.
B) Treatment X improves eyesight better than all other available treatments.
C) Treatment X will improve the eyesight of anyone who takes it.
D) Treatment X will cause a substantial improvement in eyesight.

Answer: A

The participants who received treatment X had significantly improved eyesight compared to those who did not receive treatment X.

From this you can only conclude that treatment X is likely to improve eyesight, but without further evidence, you cannot make any other conclusions.

-Harper Robinson

Problem Solving and Data Analysis

16
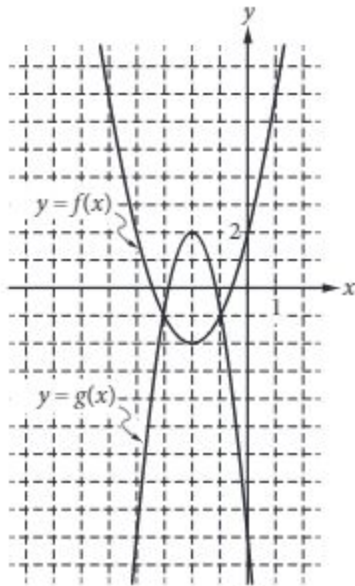
Graphs of the functions f and g are shown in the xy-plane above. For which of the following values of x does $f(x) + g(x) = 0$?

A) -3

B) -2

C) -1

D) 0

Answer: B

For $f(x) + g(x)$ to equal zero, the y values at that point must equal zero.

At x=-2, $f(-2) = -2$, and $g(-2) = 2$.

$-2 + 2 = 0$

-Harper Robinson

Passport to Advanced Math

$$S(P) = \frac{1}{2}P + 40$$
$$D(P) = 220 - P$$

The quantity of a product supplied and the quantity of the product demanded in an economic market are functions of the price of the product. The functions above are the estimated supply and demand functions for a certain product. The function $S(P)$ gives the quantity of the product supplied to the market when the price is P dollars, and the function $D(P)$ gives the quantity of the product demanded by the market when the price is P dollars.

17

How will the quantity of the product supplied to the market change if the price of the product is increased by $10 ?

A) The quantity supplied will decrease by 5 units.
B) The quantity supplied will increase by 5 units.
C) The quantity supplied will increase by 10 units.
D) The quantity supplied will increase by 50 units.

Answer: B

Plug in prices for this solution, that differ by $10. For example, $10 vs. $20.

$$S(P) = \tfrac{1}{2}P + 40$$
$$S(10) = \tfrac{1}{2}(10) + 40 = 45$$
$$S(20) = \tfrac{1}{2}(20) + 40 = 50$$

This quantity increased from 45 to 50, so by 5.

-Harper Robinson
Passport to Advanced Math

18

At what price will the quantity of the product supplied to the market equal the quantity of the product demanded by the market?

A) $90
B) $120
C) $133
D) $155

Answer: B

To find the price at which product supplied equals product demanded, set the two equations equal to each other, then solve for P.

$$S(P) = D(P)$$
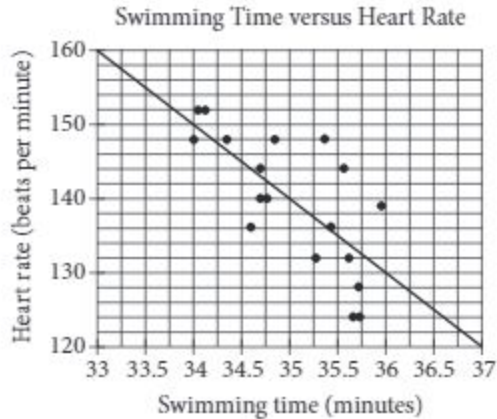$$\tfrac{1}{2}P + 40 = 220 - P$$
Add P to both sides.
$$\tfrac{3}{2}P + 40 = 220$$
$$\tfrac{3}{2}P = 180$$
Multiply both sides by ⅔ to isolate P.
$$P = \$120$$

	-Harper Robinson Heart of Algebra

19
 	
Graphene, which is used in the manufacture of integrated circuits, is so thin that a sheet weighing one ounce can cover up to 7 football fields. If a football field has an area of approximately $1\frac{1}{3}$ acres, about how many acres could 48 ounces of graphene cover?
 	
A) 250
B) 350
C) 450
D) 1,350
	Answer: C
	
	One ounce can cover 7 football fields, and each field is 1 ⅓ acres.
	    1 ⅓ acres x 7 fields = 9 ⅓ acres
	So one ounce can cover 9 ⅓ acres.
	
	48 ounces could then cover 48 x 9 ⅓ acres.
	    48 x 9 ⅓ = 448
	
	    448 is closest to 450.
	
	You can also use a proportion
	$\frac{1 \text{ ounce}}{9 \text{ ⅓ acres}} = \frac{48 \text{ ounces}}{x \text{ acres}}$
	
	Then cross multiply and solve for x.
	
	-Harper Robinson
	
	Problem Solving and Data Analysis

20

Swimming Time versus Heart Rate

Michael swam 2,000 yards on each of eighteen days. The scatterplot above shows his swim time for and corresponding heart rate after each swim. The line of best fit for the data is also shown. For the swim that took 34 minutes, Michael's actual heart rate was about how many beats per minutes less than the rate predicted by the line of best fit?

A) 1

B) 2

C) 3

D) 4

Answer: B
At 34 minutes, his predicted heart rate was 150 beats per minute.
His actual heart rate was 148 beats per minute (each line increases by 2 beats per minute).

150-148 = 2 beats per minute difference

Problem Solving and Data Analysis

21

Of the following four types of savings account plans, which option would yield exponential growth of the money in the account?

A) Each successive year, 2% of the initial savings is added to the value of the account.

B) Each successive year, 1.5% of the initial savings and $100 is added to the value of the account.

C) Each successive year, 1% of the current value is added to the value of the account.

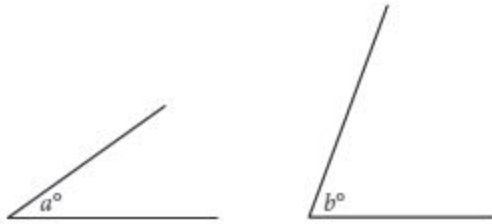D) Each successive year, $100 is added to the value of the account.

Answer: C
Exponential growth means it increases at an increasing rate each year.

A) 2% of the initial value will be the same value each year, so the amount of money will increases at the same rate each year. This is constant growth, not exponential.

B) 1.5% of the initial value will be the same value each year, and $100 is the same each year. This is constant growth, not exponential.

C) 1% of the current value increases each year, since the current value is increasing. This is exponential growth.

D) $100 dollars each year changes the amount of money by the same value each year. This is constant growth, not

	exponential.
	-Harper Robinson
	Problem Solving and Data Analysis

22	Answer: B
The sum of three numbers is 855. One of the numbers, x, is 50% more than the sum of the other two numbers. What is the value of x ?	Let the other two numbers by y and z.
A) 570	$$x + y + z = 855$$
B) 513	$$y + z = 855 - x$$
C) 214	
D) 155	Since x is 50% more than the sum of the other two numbers,
	$$x = 1.5(y+z)$$
	$$\frac{x}{1.5} = y+z$$
	Substitute $\frac{x}{1.5}$ for $y+z$ in the first equation:
	$$\frac{x}{1.5} = 855-x$$
	$$x + \frac{x}{1.5} = 855$$
	multiply by 1.5 to get rid of the denominator
	$$\frac{2x}{3} + x = 855$$
	$$\frac{5x}{3} = 855$$
	multiply by the reciprocal (⅗)
	$$x = 513$$
	-Harper Robinson
	Heart of Algebra

23

Note: Figures not drawn to scale.

The angles shown above are acute and $\sin(a°) = \cos(b°)$. If $a = 4k - 22$ and $b = 6k - 13$, what is the value of k ?

A) 4.5
B) 5.5
C) 12.5
D) 21.5

Answer: C

If $\sin(a) = \cos(a)$, the angles must be complementary according to the complementary angle property of sines and cosines.

$a+b = 90$

$(4k\text{-}22) + (6k\text{-}13) = 90$
$10k\text{-}35 = 90$
$10k = 125$
$k = 12.5$

-Harper Robinson

24

Mr. Kohl has a beaker containing n milliliters of solution to distribute to the students in his chemistry class. If he gives each student 3 milliliters of solution, he will have 5 milliliters left over. In order to give each student 4 milliliters of solution, he will need an additional 21 milliliters. How many students are in the class?

A) 16
B) 21
C) 23
D) 26

Answer: D

$\frac{n-5}{3}$ = # students

$\frac{n+21}{4}$ = # students

Now set these two equations equal to each other

$\frac{n-5}{3} = \frac{n+21}{4}$

Cross multiply and solve for n.

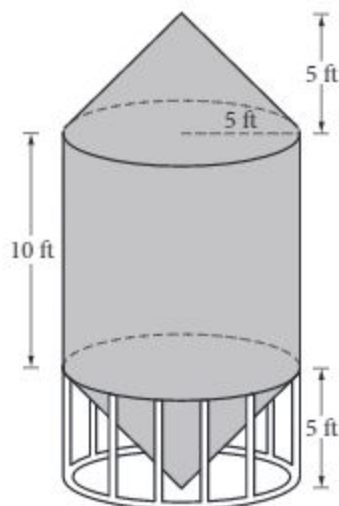$4(n\text{-}5) = 3(n+21)$
$4n\text{-}20 = 3n+63$
$n=83$

Now solve for number of students

$\frac{83-5}{3} = 26$

-Harper Robinson

Heart of Algebra

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25

A grain silo is built from two right circular cones and a right circular cylinder with internal measurements represented by the figure above. Of the following, which is closest to the volume of the grain silo, in cubic feet?

A) 261.8
B) 785.4
C) 916.3
D) 1,047.2

Answer: D

Cone volume: $\frac{1}{3} \pi r^2 h$
Cylinder volume: $\pi r^2 h$

2 Cones:
$\frac{1}{3} \pi r^2 h$
$= \frac{1}{3} \pi (5)^2 (5)$
$= 130.9$
x2 = 261.8

Cylinder:
$\pi r^2 h$
$= \pi (5)^2 10$
$= 785.4$

2 Cones and cylinder
$= 261.8 + 785.4 = 1047.2$

-Harper Robinson

26

In the *xy*-plane, the line determined by the points $(2, k)$ and $(k, 32)$ passes through the origin. Which of the following could be the value of k ?

A) 0
B) 4
C) 8
D) 16

Answer: C

Equation of a line is y=mx+b

If a line passes through the origin, its y intercept is 0, so the equation of the line is y=mx.

Plug in each of the points to this equation to create 2 new equations

$k = m(2) \rightarrow \frac{k}{2} = m$
and
$32 = m(k) \rightarrow \frac{32}{k} = m$

Now set the two equations equal to each other, and solve for k
$\frac{k}{2} = \frac{32}{k}$

	$(k)^2 = 64$ $k = 8$ -Harper Robinson Heart of Algebra

27 A rectangle was altered by increasing its length by 10 percent and decreasing its width by p percent. If these alterations decreased the area of the rectangle by 12 percent, what is the value of p ? A) 12 B) 15 C) 20 D) 22	Answer: C Choose values for the length and width. Example: length is 20, width is 5. $Area\ of\ rectangle = l \times w$ $A = 20 \times 5 = 100$ New area → .12 x 100 = 12 100-12 = 88 New length → .10 x 20 = 2 20+2 = 22 New width → $\frac{p}{100} \times 5 = \frac{5p}{100}$ $5 - \frac{5p}{100}$ $Area\ of\ rectangle = l \times w$ $88 = 22 \times (5 - \frac{5p}{100})$ $4 = 5 - \frac{5p}{100}$ $-1 = -\frac{5p}{100}$ $1 = \frac{5p}{100}$ $100 = 5p$ $p = 20$ -Harper Robinson Problem Solving and Data Analysis

28

In planning maintenance for a city's infrastructure, a civil engineer estimates that, starting from the present, the population of the city will decrease by 10 percent every 20 years. If the present population of the city is 50,000, which of the following expressions represents the engineer's estimate of the population of the city t years from now?

A) $50,000(0.1)^{20t}$

B) $50,000(0.1)^{\frac{t}{20}}$

C) $50,000(0.9)^{20t}$

D) $50,000(0.9)^{\frac{t}{20}}$

Answer: D

For the first part of the equation:
Decreasing by 10 percent
=50,000 - (50,000*.1) each year
now factor out the 50,000
=50,000 (1-.1)
=50,000(.9)

For the second part of the equation:
Say it is 40 years from now. The population should have decreased by 10% twice. t/20 represents how many times the population decreases within these 40 years, or any number of years.

-Harper Robinson

Passport to Advanced Math

29

Gender	Handedness	
	Left	Right
Female		
Male		
Total	18	122

The incomplete table above summarizes the number of left-handed students and right-handed students by gender for the eighth-grade students at Keisel Middle School. There are 5 times as many right-handed female students as there are left-handed female students, and there are 9 times as many right-handed male students as there are left-handed male students. If there is a total of 18 left-handed students and 122 right-handed students in the school, which of the following is closest to the probability that a right-handed student selected at random is female? (Note: Assume that none of the eighth-grade students are both right-handed and left-handed.)

A) 0.410
B) 0.357
C) 0.333
D) 0.250

Answer: A

Let f = the number of left handed female students
Let m = the number of left handed male students

Then, $5f$ = number of right handed female students, and $9m$ = number of right handed male students.

Create 2 equations:
$f + m = 18$
$5f + 9m = 122$

To solve this system of equations, solve for f in the first, and plug it into the second equation.
$$f = 18 - m$$
$$5(18 - m) + 9m = 122$$
$$90 - 5m + 9m = 122$$
$$4m = 32$$
$$m = 8$$

Now plug this into the first equation:
$$f + m = 18$$
$$f + 8 = 18$$
$$f = 10$$

To find the probability that a right-handed student selected at random is female, divide the number of right-handed females by the number of right-handed students.
total number of right-handed females = $5f$ = 50
total number of right-handed students = 122
$50/122 = .4098$

Problem Solving and Data Analysis

30

$$3x + b = 5x - 7$$
$$3y + c = 5y - 7$$

In the equations above, b and c are constants.

If b is c minus $\frac{1}{2}$, which of the following is true?

A) x is y minus $\frac{1}{4}$.

B) x is y minus $\frac{1}{2}$.

C) x is y minus 1.

D) x is y plus $\frac{1}{2}$.

Answer: A

$b = c - \frac{1}{2}$
Choose numbers for b and c.
Example: $b = \frac{1}{2}$, $c = 1$

$3x + \frac{1}{2}$ = $5x - 7$
$3x = 5x - 7\frac{1}{2}$
$-2x = -7\frac{1}{2}$
$x = 3\frac{3}{4}$

$3y + 1 = 5y - 7$
$-2y = -8$
$y = 4$

$4 - 3\frac{3}{4} = \frac{1}{4}$

-Harper Robinson

Heart of Algebra

31

Tickets for a school talent show cost $2 for students and $3 for adults. If Chris spends at least $11 but no more than $14 on x student tickets and 1 adult ticket, what is one possible value of x ?

Answer: 4 or 5

$2x + 3 =$ somewhere between $11 and $14

Plug in numbers and check.

$2(4) + 3 = 11$
$2(5) + 3 = 13$

Or create proportions
$2x + 3 \geq 11$
$x \geq 4$

$2x + 3 \leq 14$
$x \leq 5.5$

$4 \leq x \leq 5.5$ which leaves only $x=4$ and $x=5$

-Harper Robinson

Heart of Algebra

32

Ages of the First 12 United States Presidents
at the Beginning of Their Terms in Office

President	Age (years)	President	Age (years)
Washington	57	Jackson	62
Adams	62	Van Buren	55
Jefferson	58	Harrison	68
Madison	58	Tyler	51
Monroe	59	Polk	50
Adams	58	Taylor	65

The table above lists the ages of the first
12 United States presidents when they began their
terms in office. According to the table, what was the
mean age, in years, of these presidents at the
beginning of their terms? (Round your answer to the
nearest tenth.)

Answer: 58.6

Add up all of the ages, and divide by the number
of presidents.

703/12 = 58.6

-Harper Robinson

Problem Solving and Data Analysis

33

$$(-3x^2 + 5x - 2) - 2(x^2 - 2x - 1)$$

If the expression above is rewritten in the form
$ax^2 + bx + c$, where a, b, and c are constants, what is
the value of b ?

Answer: 9

First, distribute the -2 in the second part of the
equation
$(-3x^2 + 5x - 2) - 2x^2 + 4x + 2$
$-5x^2 + 9x + 0$

$ax^2 + bx + c$
$b = 9$
-Harper Robinson

Passport to Advanced Math

34

In a circle with center O, central angle AOB has a measure of $\dfrac{5\pi}{4}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

Answer: ⅝ or .625

A complete rotation is 360º, which is 2π radians. If central angle is $\frac{5\pi}{4}$ radians, it is

$$\dfrac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}$$ of a rotation.

-Harper Robinson

35

An online store receives customer satisfaction ratings between 0 and 100, inclusive. In the first 10 ratings the store received, the average (arithmetic mean) of the ratings was 75. What is the least value the store can receive for the 11th rating and still be able to have an average of at least 85 for the first 20 ratings?

Answer: 50

Mean $= 75 = \frac{sum\ of\ first\ ten\ ratings}{10}$
So the sum of the first ten ratings $= 750$ (cross multiply)

For the mean of the first 20 to be 85, the sum of the first 20 must be 1700.
Mean $= 85 = \frac{sum\ of\ first\ twenty\ ratings}{20}$
$(85)(20) = 1700$.

The sum of the final 10 ratings must by 1700-750 $= 950$.

The maximum that the ratings could be is 100, so the maximum sum of ratings 12 through 20 is 9 x 100 = 900.

This leaves 950-900 = 50, to be the lowest possible value for the 11th rating.

-Harper Robinson
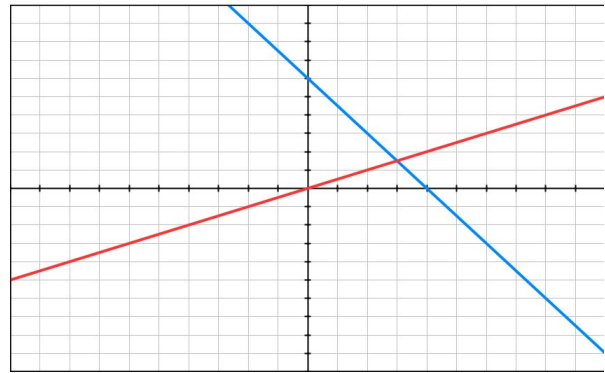
Problem Solving and Data Analysis

36

$$y \leq -15x + 3000$$
$$y \leq 5x$$

In the xy-plane, if a point with coordinates (a, b) lies in the solution set of the system of inequalities above, what is the maximum possible value of b ?

Answer: 750

The red line and everything below it represents $y \leq 5x$.
The blue line and everything below it represents $y \leq -15x + 3000$.

Therefore, the maximum value of b, or y value, is at the intercept of these two lines.

To find the intercept, set the two equations equal to each other to solve for x, then y.

$-15x + 3000 = 5x$
$-20x = -3000$
$x = 150$
$y = 5(150) = 750$

-Harper Robinson
Heart of Algebra

If shoppers enter a store at an average rate of r shoppers per minute and each stays in the store for an average time of T minutes, the average number of shoppers in the store, N, at any one time is given by the formula $N = rT$. This relationship is known as Little's law.

The owner of the Good Deals Store estimates that during business hours, an average of 3 shoppers per minute enter the store and that each of them stays an average of 15 minutes. The store owner uses Little's law to estimate that there are 45 shoppers in the store at any time.

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Little's law can be applied to any part of the store, such as a particular department or the checkout lines. The store owner determines that, during business hours, approximately 84 shoppers per hour make a purchase and each of these shoppers spend an average of 5 minutes in the checkout line. At any time during business hours, about how many shoppers, on average, are waiting in the checkout line to make a purchase at the Good Deals Store?

Answer: 7

The average number of shoppers in the store (or checkout line) is
$N = rT$
84 shoppers enter the checkout line per hour, which is $\frac{84}{60} = 1.4$ shoppers per minute (rate).

Now plug in the rate and the time (5 minutes) into the equation.

$N = 1.4\ (5) = 7$

-Harper Robinson

Problem Solving and Data Analysis

38

The owner of the Good Deals Store opens a new store across town. For the new store, the owner estimates that, during business hours, an average of 90 shoppers per <u>hour</u> enter the store and each of them stays an average of 12 minutes. The average number of shoppers in the new store at any time is what percent less than the average number of shoppers in the original store at any time? (Note: Ignore the percent symbol when entering your answer. For example, if the answer is 42.1%, enter 42.1)

Answer: 60

The average number of shoppers in the new store is $N = rT$
90 shoppers per hour $= 90/60 = 1.5$ shoppers per minute.
Plug this rate and the time given into Little's Law.
$N = (1.5)(12\text{ minutes})$
$N = 18$ shoppers

The original store, according to the original question, had 45 shoppers at anytime.

To find the percent change, use the formula
$\frac{(original-current)}{original}$ x 100$= \frac{(45-18)}{45}$ x 100 $= 60$ percent.

	So the new store has 60% less than the average number of shoppers in the original store at any time. -Harper Robinson Problem Solving and Data Analysis