

Math- With Calculator

<p>1</p> <p>A musician has a new song available for downloading or streaming. The musician earns \$0.09 each time the song is downloaded and \$0.002 each time the song is streamed. Which of the following expressions represents the amount, in dollars, that the musician earns if the song is downloaded d times and streamed s times?</p> <p>A) $0.002d + 0.09s$ B) $0.002d - 0.09s$ C) $0.09d + 0.002s$ D) $0.09d - 0.002s$</p>	<p>Answer: C</p> <ul style="list-style-type: none"> - Multiply d or s with their respective income values - Add the “d” and “s” values <p>Answer by David Painter Heart of Algebra</p>
<p>2</p> <p>A quality control manager at a factory selects 7 lightbulbs at random for inspection out of every 400 lightbulbs produced. At this rate, how many lightbulbs will be inspected if the factory produces 20,000 lightbulbs?</p> <p>A) 300 B) 350 C) 400 D) 450</p>	<p>Answer: B</p> <p>How many times will a group of 7 lightbulbs be inspected?</p> <p>Answer: Divide the total number of lightbulbs produced by the number of lightbulbs between two successive inspections:</p> $\frac{20,000 \text{ total}}{400} = 50$ <p>So now we know a group of 7 lightbulbs will be inspected 50 times. To determine how many total lightbulbs that is, multiply 50 with the number of bulbs randomly selected during each inspection:</p> $50(7) = 350$ <p>Answer by David Painter Problem Solving and Data Analysis</p>

<p>3</p> $\ell = 24 + 3.5m$ <p>One end of a spring is attached to a ceiling. When an object of mass m kilograms is attached to the other end of the spring, the spring stretches to a length of ℓ centimeters as shown in the equation above. What is m when ℓ is 73?</p> <p>A) 14 B) 27.7 C) 73 D) 279.5</p>	<p>Answer: A</p> <p>Plug in:</p> $73 = 24 + 3.5m$ $49 = 3.5m$ $m = 14$ <p>Answer by David Painter Heart of Algebra</p>
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<p>Questions 4 and 5 refer to the following information.</p> <p>The amount of money a performer earns is directly proportional to the number of people attending the performance. The performer earns \$120 at a performance where 8 people attend.</p> <p>4</p> <p>How much money will the performer earn when 20 people attend a performance?</p> <p>A) \$960 B) \$480 C) \$300 D) \$240</p>	<p>Answer: C</p> <p>Find the scalar factor between 20 and 8. In other words, how many times greater is 20 than 8?</p> $\frac{20}{8} = 2.5$ <p>So for a 20 person audience, the performer makes 2.5 times more money than for an 8 person audience. Therefore, multiply \$120 with 2.5:</p> $\$120(2.5) = \300 <p>Answer by David Painter Problem Solving and Data Analysis</p>
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<p>Questions 4 and 5 refer to the following information.</p> <p>The amount of money a performer earns is directly proportional to the number of people attending the performance. The performer earns \$120 at a performance where 8 people attend.</p>	<p>Answer: C</p> <p>Profit = Income - Expenses</p> $\text{Profit} = \$120 - (.43)(\$120)$ $\text{Profit} = \$120 - \51.60
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5

The performer uses 43% of the money earned to pay the costs involved in putting on each performance. The rest of the money earned is the performer's profit. What is the profit the performer makes at a performance where 8 people attend?

- A) \$51.60
- B) \$57.00
- C) \$68.40
- D) \$77.00

Profit = \$68.40

Note: Remember that 43% can be written in decimal form as 0.43

Answer by David Painter
Problem Solving and Data Analysis

6

When 4 times the number x is added to 12, the result is 8. What number results when 2 times x is added to 7?

- A) -1
- B) 5
- C) 8
- D) 9

Answer: B

a) Solve for x :

$$4x + 12 = 8$$

$$4x = -4$$

$$x = -1$$

b) Plug in x :

$$2x + 7 = ?$$

$$2(-1) + 7 = 5$$

Answer by David Painter
Heart of Algebra

7

$$y = x^2 - 6x + 8$$

The equation above represents a parabola in the xy -plane. Which of the following equivalent forms of the equation displays the x -intercepts of the parabola as constants or coefficients?

- A) $y - 8 = x^2 - 6x$
- B) $y + 1 = (x - 3)^2$
- C) $y = x(x - 6) + 8$
- D) $y = (x - 2)(x - 4)$

Answer: D

- Factor right side of equation to determine x -intercepts
 - We can make a factoring table to make this easier

Multiply (two outer terms)	Add (inner term)
8	-6

- What two numbers multiply to equal 8, and add to equal -6?
 - Answer: -4 and -2

	<p>therefore...</p> $y = x^2 - 6x + 8$ <p><i>can be factored into</i></p> $y = (x - 2)(x - 4)$ <p>Answer by David Painter Passport to Advanced Math</p>
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<p>8</p> <p>In a video game, each player starts the game with k points and loses 2 points each time a task is not completed. If a player who gains no additional points and fails to complete 100 tasks has a score of 200 points, what is the value of k ?</p> <p>A) 0 B) 150 C) 250 D) 400</p>	<p>Answer: D</p> <p>Score = $k - 2(100)$ $200 = k - 200$ $k = 400$</p> <p>Answer by David Painter Heart of Algebra</p>
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<p>9</p> <p>A worker uses a forklift to move boxes that weigh either 40 pounds or 65 pounds each. Let x be the number of 40-pound boxes and y be the number of 65-pound boxes. The forklift can carry up to either 45 boxes or a weight of 2,400 pounds. Which of the following systems of inequalities represents this relationship?</p> <p>A) $\begin{cases} 40x + 65y \leq 2,400 \\ x + y \leq 45 \end{cases}$</p> <p>B) $\begin{cases} \frac{x}{40} + \frac{y}{65} \leq 2,400 \\ x + y \leq 45 \end{cases}$</p> <p>C) $\begin{cases} 40x + 65y \leq 45 \\ x + y \leq 2,400 \end{cases}$</p> <p>D) $\begin{cases} x + y \leq 2,400 \\ 40x + 65y \leq 2,400 \end{cases}$</p>	<p>Answer: A</p> <p>a) First inequality: Multiply "x" and "y" by their respective weights, and set that value less than or equal to (cannot exceed!) 2,400 lbs.</p> <p>b) Second inequality: Add "x" and "y" (number of boxes in their respective weight categories) together and set that value less than or equal to 45, the greatest number of boxes the forklift can carry</p> <p>Answer by David Painter Heart of Algebra</p>
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10

A function f satisfies $f(2) = 3$ and $f(3) = 5$. A function g satisfies $g(3) = 2$ and $g(5) = 6$. What is the value of $f(g(3))$?

- A) 2
- B) 3
- C) 5
- D) 6

Answer: B

$$g(3) = 2, \text{ so}$$

$$f(g(3)) = f(2) = 3$$

Answer by David Painter
Passport to Advanced Math

11

Number of hours Tony plans to read the novel per day	3
Number of parts in the novel	8
Number of chapters in the novel	239
Number of words Tony reads per minute	250
Number of pages in the novel	1,078
Number of words in the novel	349,168

Tony is planning to read a novel. The table above shows information about the novel, Tony's reading speed, and the amount of time he plans to spend reading the novel each day. If Tony reads at the rates given in the table, which of the following is closest to the number of days it would take Tony to read the entire novel?

- A) 6
- B) 8
- C) 23
- D) 324

Answer: B

Three hours = 180 minutes

180 minutes read per day

$$349,168 / 250 = 1,396.672$$

$$1,396.672 / 180 = 7.76$$

7.76 up rounds to 8

By Nicole D'Onofrio

Problem Solving and Data Analysis

12

On January 1, 2000, there were 175,000 tons of trash in a landfill that had a capacity of 325,000 tons. Each year since then, the amount of trash in the landfill increased by 7,500 tons. If y represents the time, in years, after January 1, 2000, which of the following inequalities describes the set of years where the landfill is at or above capacity?

- A) $325,000 - 7,500 \leq y$
- B) $325,000 \leq 7,500y$
- C) $150,000 \geq 7,500y$
- D) $175,000 + 7,500y \geq 325,000$

Answer: D

The capacity of the landfill is 325,000 tons, so it goes to the right side of the equation. The amount on the other side must be equal or greater than 325,000 tons because the words “at or above” are used. The amount of trash to start with is 175,000 which goes on the left side of the equation. 7,500 tons of trash are added per year so that is represented as $7,500y$ and this goes on the left side of the equation

By Nicole D’Onofrio

Heart of Analysis

13

A researcher conducted a survey to determine whether people in a certain large town prefer watching sports on television to attending the sporting event. The researcher asked 117 people who visited a local restaurant on a Saturday, and 7 people refused to respond. Which of the following factors makes it least likely that a reliable conclusion can be drawn about the sports-watching preferences of all people in the town?

- A) Sample size
- B) Population size
- C) The number of people who refused to respond
- D) Where the survey was given

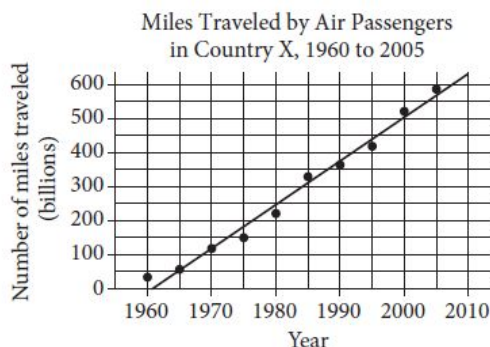
Answer: D

Where the survey is completed could be linked to preference for watching sports on television versus attending the sporting event. People who go out to a restaurant on Saturday are the ones who are not at Saturday sporting events. Those who might prefer attending a sporting event are likely already at one and being excluded from the survey, making the results unrepresentative.

By Nicole D’Onofrio

Problem Solving and Data Analysis

14



According to the line of best fit in the scatterplot above, which of the following best approximates the year in which the number of miles traveled by air passengers in Country X was estimated to be 550 billion?

- A) 1997
- B) 2000
- C) 2003
- D) 2008

580,000,000 mi. / (24 hrs. per day) (365 days per yr.)

580,000,000 mi. / 8760 hrs. per yr.

66,210 mi. per hr.

Answer: C Approximately 66,000 mi. per hr.

By Nicole D'Onofrio

Problem Solving and Data Analysis

15

The distance traveled by Earth in one orbit around the Sun is about 580,000,000 miles. Earth makes one complete orbit around the Sun in one year. Of the following, which is closest to the average speed of Earth, in miles per hour, as it orbits the Sun?

- A) 66,000
- B) 93,000
- C) 210,000
- D) 420,000

Answer: A

580,000,000 mi. / (24 hrs. per day) (365 days per yr.)

580,000,000 mi. / 8760 hrs. per yr.

66,210 mi. per hr.

Approximately 66,000 mi. per hr.

By Nicole D'Onofrio

Problem Solving and Data Analysis

16

Results on the Bar Exam of Law School Graduates

	Passed bar exam	Did not pass bar exam
Took review course	18	82
Did not take review course	7	93

The table above summarizes the results of 200 law school graduates who took the bar exam. If one of the surveyed graduates who passed the bar exam is chosen at random for an interview, what is the probability that the person chosen did not take the review course?

- A) $\frac{18}{25}$
- B) $\frac{7}{25}$
- C) $\frac{25}{200}$
- D) $\frac{7}{200}$

Answer: B

25 (18 + 7) people passed the bar exam

7 of those 25 people who passed the bar did not take the review class

7 / 25

By Nicole D'Onofrio
Problem Solving and Data Analysis

17

The atomic weight of an unknown element, in atomic mass units (amu), is approximately 20% less than that of calcium. The atomic weight of calcium is 40 amu. Which of the following best approximates the atomic weight, in amu, of the unknown element?

- A) 8
- B) 20
- C) 32
- D) 48

Answer: C

$$20\% \rightarrow .2$$

$$(40 \text{ amu}) \times (.2) = 8$$

$$(40 \text{ amu} - 8 \text{ amu}) = (32 \text{ amu})$$

OR

$$100\% - 20\% = 80\%$$

$$80\% \rightarrow .8$$

$$(40 \text{ amu}) \times (.8) = 32$$

By Nicole D'Onofrio
Problem Solving and Data Analysis

18

A survey was taken of the value of homes in a county, and it was found that the mean home value was \$165,000 and the median home value was \$125,000. Which of the following situations could explain the difference between the mean and median home values in the county?

- A) The homes have values that are close to each other.
- B) There are a few homes that are valued much less than the rest.
- C) There are a few homes that are valued much more than the rest.
- D) Many of the homes have values between \$125,000 and \$165,000.

Answer: C

The middle number of the set is 125,000.
The mean is higher because there are a few homes that have high value, which pulls the mean upwards.

By Nicole D'Onofrio

Problem Solving and Data Analysis

Questions 19 and 20 refer to the following information.

A sociologist chose 300 students at random from each of two schools and asked each student how many siblings he or she has. The results are shown in the table below.

Students' Sibling Survey

Number of siblings	Lincoln School	Washington School
0	120	140
1	80	110
2	60	30
3	30	10
4	10	10

There are a total of 2,400 students at Lincoln School and 3,300 students at Washington School.

19

What is the median number of siblings for all the students surveyed?

- A) 0
- B) 1
- C) 2
- D) 3

Answer: B

Students' Sibling Survey			
Number of siblings	Lincoln School	Washington School	Combined Schools
0	120	140	260
1	80	110	190
2	60	30	90
3	30	10	40
4	10	10	20

There are 600 people total so the median occurs at the 300th reported number of siblings. Count 300 down or up the combined total column. The 300th reported number of siblings occurs in the 1 row.

By Nicole D'Onofrio

Problem Solving and Data Analysis

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There are a total of 2,400 students at Lincoln School and 3,300 students at Washington School.

20

Based on the survey data, which of the following most accurately compares the expected total number of students with 4 siblings at the two schools?

- A) The total number of students with 4 siblings is expected to be equal at the two schools.
- B) The total number of students with 4 siblings at Lincoln School is expected to be 30 more than at Washington School.
- C) The total number of students with 4 siblings at Washington School is expected to be 30 more than at Lincoln School.
- D) The total number of students with 4 siblings at Washington School is expected to be 900 more than at Lincoln School.

Answer: C

The middle number of the set is 125,000.
The mean is higher because there are a few homes that have high value, which pulls the mean upwards.

By Nicole D'Onofrio

Problem Solving and Data Analysis

21

A project manager estimates that a project will take x hours to complete, where $x > 100$. The goal is for the estimate to be within 10 hours of the time it will actually take to complete the project. If the manager meets the goal and it takes y hours to complete the project, which of the following inequalities represents the relationship between the estimated time and the actual completion time?

- A) $x + y < 10$
- B) $y > x + 10$
- C) $y < x - 10$
- D) $-10 < y - x < 10$

Answer: D

(the amount of hours it actually takes) - (the goal amount of hours)

$y - x$

The difference should be no more than ten or less than 10

$-10 < y - x < 10$

By Nicole D'Onofrio
Heart of Algebra

Questions 22 and 23 refer to the following information.

$$I = \frac{P}{4\pi r^2}$$

At a large distance r from a radio antenna, the intensity of the radio signal I is related to the power of the signal P by the formula above.

22

Which of the following expresses the square of the distance from the radio antenna in terms of the intensity of the radio signal and the power of the signal?

- A) $r^2 = \frac{IP}{4\pi}$
- B) $r^2 = \frac{P}{4\pi I}$
- C) $r^2 = \frac{4\pi I}{P}$
- D) $r^2 = \frac{I}{4\pi P}$

Answer: B

$$I = P / 4\pi(r^2)$$

$$I4\pi(r^2) = P$$

$$(r^2) = P / (I4\pi)$$

By Nicole D'Onofrio
Passport to Advanced Math

Questions 22 and 23 refer to the following information.

$$I = \frac{P}{4\pi r^2}$$

At a large distance r from a radio antenna, the intensity of the radio signal I is related to the power of the signal P by the formula above.

23

For the same signal emitted by a radio antenna, Observer A measures its intensity to be 16 times the intensity measured by Observer B. The distance of Observer A from the radio antenna is what fraction of the distance of Observer B from the radio antenna?

- A) $\frac{1}{4}$
- B) $\frac{1}{16}$
- C) $\frac{1}{64}$
- D) $\frac{1}{256}$

Let $I = \frac{p}{4\pi r^2}$ represent the intensity felt by observer B. So, if observer A feels an intensity 16 times that of observer B, then observer A feels $16I = \frac{p}{4\pi r^2} = \frac{4p}{\pi r^2}$. In order to see what fraction of the distance observer A is with respect to observer B, you must substitute a value of r into B that, when r in A is equal to 1, makes the two equations equal to one another. So, $\frac{4p}{\pi} = \frac{p}{4\pi r^2}$ occurs only when $r = \frac{1}{4}$.

Answer: A

-John Cavaliere

Passport to Advanced Math

24

$$x^2 + y^2 + 4x - 2y = -1$$

The equation of a circle in the xy -plane is shown above. What is the radius of the circle?

- A) 2
- B) 3
- C) 4
- D) 9

This problem requires you to do factoring by grouping.

$x^2 + 4x + y^2 - 2y = -1$ First, pair your variables $(x + 2)^2 - 4 + (y - 1)^2 - 1 = -1$ perform factoring by grouping.

$(x + 2)^2 + (y - 1)^2 = 4$ separate equation.

4 is equal to the radius of squared. So, the radius is 2.

Answer: A

-John Cavaliere

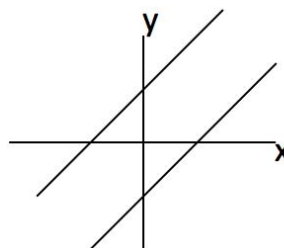
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The graph of the linear function f has intercepts at $(a, 0)$ and $(0, b)$ in the xy -plane. If $a + b = 0$ and $a \neq b$, which of the following is true about the slope of the graph of f ?

- A) It is positive.
- B) It is negative.
- C) It equals zero.
- D) It is undefined.

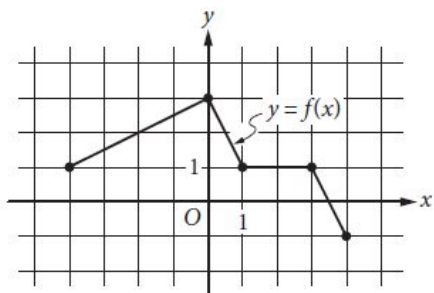
Answer: A

As a and b must add to 0, a and b must be opposites of each other. So, in any case where a and b are opposites of each other, a line with positive slope exists.



-John Cavaliere

26



The complete graph of the function f is shown in the xy -plane above. Which of the following are equal to 1?

- I. $f(-4)$
- II. $f\left(\frac{3}{2}\right)$
- III. $f(3)$

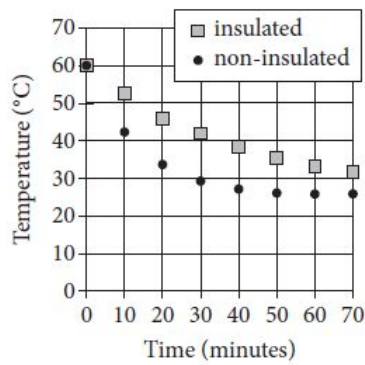
- A) III only
- B) I and III only
- C) II and III only
- D) I, II, and III

Answer: D

As this problem is graph based, the only knowledge that you must know is that you are looking for what x values have a corresponding y-value equal to 1. By testing all the points provided, you will see that all the given points have a corresponding y-value equal to 1.

-John Cavaliere

27



Two samples of water of equal mass are heated to 60 degrees Celsius (°C). One sample is poured into an insulated container, and the other sample is poured into a non-insulated container. The samples are then left for 70 minutes to cool in a room having a temperature of 25°C. The graph above shows the temperature of each sample at 10-minute intervals. Which of the following statements correctly compares the average rates at which the temperatures of the two samples change?

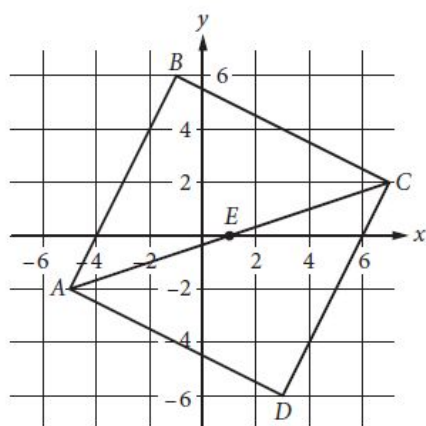
- A) In every 10-minute interval, the magnitude of the rate of change of temperature of the insulated sample is greater than that of the non-insulated sample.
- B) In every 10-minute interval, the magnitude of the rate of change of temperature of the non-insulated sample is greater than that of the insulated sample.
- C) In the intervals from 0 to 10 minutes and from 10 to 20 minutes, the rates of change of temperature of the insulated sample are of greater magnitude, whereas in the intervals from 40 to 50 minutes and from 50 to 60 minutes, the rates of change of temperature of the non-insulated sample are of greater magnitude.
- D) In the intervals from 0 to 10 minutes and from 10 to 20 minutes, the rates of change of temperature of the non-insulated sample are of greater magnitude, whereas in the intervals from 40 to 50 minutes and from 50 to 60 minutes, the rates of change of temperature of the insulated sample are of greater magnitude.

A is not the correct answer because in the first 10 min interval, the non-insulated cup has a higher magnitude of change in temperature. B is not correct because in the 60-70 minute interval, the non-insulated cup has a lower rate of change than the insulated cup. C is not correct as from 1-20 minutes, the non-insulated cup has a higher magnitude of change. So, the answer must be D.

Answer: D

-John Cavaliere

28



First, find the slope between the points C and E.

$$\frac{0-2}{1-7} = 3$$

The slope of this line is 3. As the slope from E to D is just a 90 degree rotation of C to E, the slope of E to D is negative the slope of C and E, or -3. This rules out answers C and D. By testing A, you will see that $y = -3x - 1$ will not intersect B and D. So, equation B must be the correct answer.

Answer: B

-John Cavaliere

In the xy -plane above, $ABCD$ is a square and point E is the center of the square. The coordinates of points C and E are $(7, 2)$ and $(1, 0)$, respectively. Which of the following is an equation of the line that passes through points B and D ?

- A) $y = -3x - 1$
- B) $y = -3(x - 1)$
- C) $y = -\frac{1}{3}x + 4$
- D) $y = -\frac{1}{3}x - 1$

Heart of Algebra

29

$$y = 3$$

$$y = ax^2 + b$$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have exactly two real solutions?

- A) $a = -2, b = 2$
- B) $a = -2, b = 4$
- C) $a = 2, b = 4$
- D) $a = 4, b = 3$

Answer: B

$$y = 3 \text{ and } y = ax^2 + b$$

$$\text{so } 3 = ax^2 + b$$

$$ax^2 + b - 3 = 0$$

plug in the answer choices

$$\text{A) } -2x^2 + 2 - 3 = 0$$

$$2x^2 = -1$$

no solution since we'll end up with $\sqrt{\text{negative number}}$

$$\text{C) } 2x^2 + 4 - 3 = 0$$

$$2x^2 = -1$$

no solution since we'll end up with $\sqrt{\text{negative number}}$

$$\text{D) } 4x^2 + 3 - 3 = 0$$

$$4x^2 = 0$$

only one solution $x = 0$

$$\text{B) } -2x^2 + 4 - 3 = 0$$

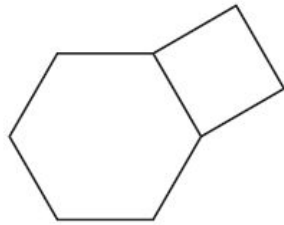
$$2x^2 = 1$$

solution is $\pm \sqrt{\frac{1}{2}}$

this is the correct answer

-written by Elise Favia

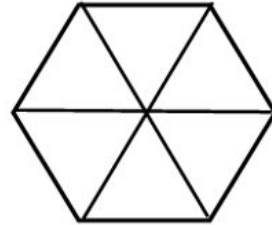
30



The figure above shows a regular hexagon with sides of length a and a square with sides of length a . If the area of the hexagon is $384\sqrt{3}$ square inches, what is the area, in square inches, of the square?

- A) 256
- B) 192
- C) $64\sqrt{3}$
- D) $16\sqrt{3}$

Answer: A



$$\frac{384\sqrt{3}}{6} = 64\sqrt{3}$$

$$\text{Area of Triangle} = \frac{\sqrt{3}}{4} \text{base}^2 = 64\sqrt{3}$$

$$64 \times 4 = \text{base}^2$$

$$\text{base}^2 = \text{area of square}$$

$$\text{area of square} = 64 \times 4 = 256$$

31

A coastal geologist estimates that a certain country's beaches are eroding at a rate of 1.5 feet per year. According to the geologist's estimate, how long will it take, in years, for the country's beaches to erode by 21 feet?

Answer: 14

This question is basically asking how many times 1.5 goes into 21. So, basic division is all that is necessary.

$$\frac{21}{1.5} = 14$$

So, the answer is 14.

-John Cavaliere

32

If h hours and 30 minutes is equal to 450 minutes, what is the value of h ?

So, there are 60 minutes in an hour. So,

$$h + 30 = 450$$

$$h = 420$$

h is equal to 420 minutes. But, we want hours. So, divide 420 by 60.

$$\frac{420}{60} = 7.$$

$$\text{So, } h = 7.$$

-John Cavaliere

Problem Solving and Data Analysis

33

In the xy -plane, the point $(3, 6)$ lies on the graph of the function $f(x) = 3x^2 - bx + 12$. What is the value of b ?

$$f(x) = 3x^2 - bx + 12 \text{ passing through } (3, 6).$$

So,

$$6 = 3(3)^2 - b(3) + 12$$

$$6 = 27 - b(3) + 12$$

$$6 = 39 - b(3)$$

$$-33 = -3b$$

$$b = 11$$

-John Cavaliere

Passport to Advanced Math

34

In one semester, Doug and Laura spent a combined 250 hours in the tutoring lab. If Doug spent 40 more hours in the lab than Laura did, how many hours did Laura spend in the lab?

If Laura spent x hours in the lab, and

Doug spent 40 more hours than Laura,

then Doug spent $x + 40$ hours in the lab.

If they spent a total of 250 hours in the lab,

the amount of lab time can be modeled by

$$250 = x + (x + 40) \text{ or } 250 = 2x + 40$$

Then, solve for x .

$$210 = 2x \text{ so } x = 105.$$

-John Cavaliere

Heart of Algebra

35

$$a = 18t + 15$$

Jane made an initial deposit to a savings account. Each week thereafter she deposited a fixed amount to the account. The equation above models the amount a , in dollars, that Jane has deposited after t weekly deposits. According to the model, how many dollars was Jane's initial deposit? (Disregard the \$ sign when gridding your answer.)

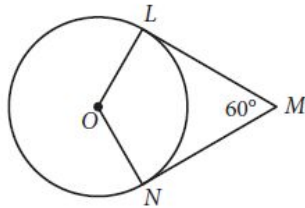
Answer: 15

If the equation $a = 18t + 15$ models the amount of money that Jane deposits after t deposits, then her starting deposit can be found by setting $t = 0$. So, if $a + 18(0) + 15$ then $a = 15$.

-John Cavaliere

Heart of Algebra

36



In the figure above, point O is the center of the circle, line segments LM and MN are tangent to the circle at points L and N , respectively, and the segments intersect at point M as shown. If the circumference of the circle is 96, what is the length of minor arc \widehat{LN} ?

As all closed 4 sided objects must have an interior angle total of 360 degrees, you can subtract from 360 the known angle values to find the value of O . As L and N are both tangent to the circle, they form a 90 degree angle with O . So, $360 - 90 - 90 - 60 = 120$. As 120 degrees is a third of a circle, the arc length of L to N is a third of the total circumference.

$$96 \times \frac{1}{3} = 32.$$

-John Cavaliere

Questions 37 and 38 refer to the following information.

A botanist is cultivating a rare species of plant in a controlled environment and currently has 3000 of these plants. The population of this species that the botanist expects to grow next year, $N_{\text{next year}}$, can be estimated from the number of plants this year, $N_{\text{this year}}$, by the equation below.

$$N_{\text{next year}} = N_{\text{this year}} + 0.2 \left(N_{\text{this year}} \right) \left(1 - \frac{N_{\text{this year}}}{K} \right)$$

The constant K in this formula is the number of plants the environment is able to support.

37

According to the formula, what will be the number of plants two years from now if $K = 4000$? (Round your answer to the nearest whole number.)

This problem is basically straight substitution. However, you must remember to do substitution twice as the question asks for "two years from now."

$$N = 3000 + .2(3000) \left(1 - \frac{3000}{4000} \right) = 3150$$

$$N_2 = 3150 + .2(3150) \left(1 - \frac{3150}{4000} \right) = 3284$$

Answer=3284

-John Cavaliere

Problem Solving and Data Analysis

Questions 37 and 38 refer to the following information.

A botanist is cultivating a rare species of plant in a controlled environment and currently has 3000 of these plants. The population of this species that the botanist expects to grow next year, $N_{\text{next year}}$, can be estimated from the number of plants this year, $N_{\text{this year}}$, by the equation below.

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The constant K in this formula is the number of plants the environment is able to support.

38

The botanist would like to increase the number of plants that the environment can support so that the population of the species will increase more rapidly. If the botanist's goal is that the number of plants will increase from 3000 this year to 3360 next year, how many plants must the modified environment support?

This is just substitution with a rearranging of variables in order to solve for x .

$$3360 = 3000 + .2(3000) \left(1 - \frac{3000}{x} \right)$$

$$360 = .2(3000) \left(1 - \frac{3000}{x} \right)$$

$$.6 = 1 - \frac{3000}{x}$$

$$\frac{3000}{x} = .4$$

$$x = 7500$$

-John Cavaliere

Problem Solving and Data Analysis