Mathematical Analysis Notes

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Chapter 1

Banach Spaces

Metric: For a given set $\mathbf{X}$, $d : \mathbf{X} \times \mathbf{X} \to \mathbb{R}$ is a metric if,

1. $\forall x, y \in \mathbf{X}$, $d(x, y) \geq 0$
2. $\forall x, y \in \mathbf{X}$, $d(x, y) = d(y, x)$
3. $\forall x, y, z \in \mathbf{X}$, $d(x, z) \leq d(x, y) + d(y, z)$

Metric Space: Let $\mathbf{X}$ b a set and $d : \mathbf{X} \times \mathbf{X} \to \mathbb{R}$ be a metric, then $(\mathbf{X}, d)$ is a metric space.

A metric space is complete if every Cauchy sequence converges to a point in it.

Normed Linear Space: Let $\mathbf{X}$ be a vector space and $\|\cdot\|$ a norm such that $\forall \mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\lambda \in \mathbb{R}$ or $\mathbb{C}$,

1. $\|\lambda \mathbf{x}\| = |\lambda| \|\mathbf{x}\|$
2. $\|\mathbf{x}\| = 0 \leftrightarrow \mathbf{x} = 0$
3. $\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$

then $(\mathbf{X}, \|\cdot\|)$ is a NLS.

Example:

$$C([a, b]) = \left\{ f : [a, b] \to \mathbb{F}, \text{ where f is cont. and } \sup_{[a,b]} |f(x)| < \infty \right\}$$