

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/261586932>

A formal theory of social power

Article in *Journal of Mathematical Sociology* · August 2010

DOI: 10.1080/0022250X.1986.9990008

CITATIONS

183

READS

1,362

1 author:



Noah Friedkin

University of California, Santa Barbara

75 PUBLICATIONS 4,999 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Models of opinion formation: system-theoretic analysis [View project](#)



MURI QUANTA [View project](#)

A FORMAL THEORY OF SOCIAL POWER

NOAH E. FRIEDKIN

University of California, Santa Barbara

This paper builds on French's (1956) Formal Theory of Social Power. In the theory, a population's power structure is formally related to its structure of influential communications which, in turn, is formally related to its pattern and prevalence of interpersonal agreements. The theory's predictions include the following about the members of a population: (1) the expected influence of each member in determining other members' opinions on an issue; (2) the probability of consensus on an issue in the population or in any given subset (dyad or cluster) of the members; and (3) the probability that any given proportion of the members (e.g., a majority) will be in agreement on an issue. The theory overcomes well-recognized limitations of French's seminal effort. Its predictions rest (1) on a micro-level process of opinion change and (2) on macro-level variations in the pattern and strengths of the ties that comprise a power structure.

INTRODUCTION

Thirty years have elapsed since the publication of French's Formal Theory of Social Power. At one time, the theory was viewed as offering a promising approach to understanding effects of social structure (conceived as a network of interpersonal ties) on cognitive structure (conceived as a pattern of interpersonal agreements). While the paper is still regularly cited, sociologists do not now view French's theory as

An earlier version of this paper was presented at the Annual Meeting of the American Sociological Association, Washington, D.C., 1985. For their helpful comments on this work, I am indebted to members of the social network groups of the University of California at Irvine and Santa Barbara.

providing an adequate account of the relationship between social and cognitive structures. I share this view, but believe that French's approach warrants further development. The most general assertions of the approach are (1) that settled opinions develop mainly as a consequence of interpersonal influences, (2) that an explanation of members' opinions must rest on a description of the process by which their opinions are influenced, (3) that an explanation of members' opinions also must rest on a description of the social structural context in which this process of opinion change operates, and (4) that the process of opinion change generates different patterns of interpersonal agreement as a result of differences in social structural context.

The limitation of French's theory is that it does not adequately predict effects of social structural context on the pattern of interpersonal agreements. It is possible that the source of the theory's inadequacy is its description of the process of opinion change; however, at present, no more accurate model has been proposed (Hunter, Danes and Cohen 1984). My work in this paper is motivated by the belief that French's general approach is a good one and that the inadequacy of his theory stems, not so much from the model of opinion change involved in it, but from an incomplete conceptualization of the social structural context in which the process of opinion change occurs.

According to French's theory, a social structural context is comprised of (1) a stable power structure describing (in terms of an adjacency matrix) dyadic-level opportunities for influential communication and (2) a stable influence structure describing (in terms of a matrix of coefficients) the effect of influential communication when such communication occurs in a particular dyad. My contribution to this formulation boils down to the suggestion that power structures are more fruitfully conceptualized as consisting of interpersonal relationships that either entail influential communication or not with *probabilities* that are stable over some period of time. This probabilistic conceptualization of power structures subsumes French's original formulation since opportunities for influential communication exist wherever there is a positive probability of influential communication. I show that this conceptualization overcomes those objections to French's theory that revolve on the theory's inability to discriminate effects of detailed variations in social structure on the pattern of interpersonal agreements.

A number of "network" terms occur in the paper with which not all readers may be familiar. The definitions of these terms are located in the Appendix.

BACKGROUND

French (1956) formulated a model of how persons' opinions are affected by the opinions of other persons with whom they are in direct communication. At each point in time, the members of a population simultaneously change their opinions to a value that is the *mean* of their own opinion and the opinions of those members who have directly communicated their opinions to them (each member's opinion having been represented by a single real number). For instance, if members j and k communicate their opinions to i , then the opinion of i after one unit of time is the mean

$$\frac{1}{3}(m_i + m_j + m_k).$$

where m_i , m_j , and m_k are, respectively, the opinions of i , j , and k . In effect, at time t all persons move to positions that minimize the sum of the squared distances among their own and other influential opinions at $t-1$. French deduced that the opinions of the members would converge over time to a single opinion, depending on the *structure* of influential communications in the population.

French's deductions about the effect of communication structure on consensus were consistent with the hypothesis, widely held among social psychologists, that the occurrence of shared opinions in a population is more likely the more cohesive the structure of interpersonal communications among the population's members. Drawing on the work of Harary, Norman, and Cartwright (1965), French rigorously operationalized the concept of cohesion in terms of an ordinal scale of connectedness in the pattern of communications: strong, unilateral, weak, and disconnected. French deduced that consensus *must* arise only in populations whose members are strongly or unilaterally connected. The combination of (1) a rigorous operationalization of the concept of cohesion, (2) a formal model of opinion change, and (3) deductions that supported previous findings on the effects of cohesion on group opinions proved extremely attractive to social psychologists. A further attraction of the theory was its demonstration that effects of communication structure (a macro-level phenomenon) might be deduced from a micro-level model of the process of opinion change.

Subsequent work by Harary (1959) and DeGroot (1974) revealed that French's model of opinion change is a special case of a more general model. The description of the general model need not occupy us at the

moment (it will be provided at a later point in the paper). The main conclusions of the general model include those forwarded by French: strongly or unilaterally connected structures of influential communication will result in consensus. In addition, Harary deduced that consensus must arise in some *weakly connected* communication structures (i.e., those with a single strongly connected point basis). Since all unilateral structures contain a single strongly connected point basis, Harary's result subsumes French's deduction about unilateral structures. Abelson (1964) has analyzed more elaborate forms of the model. He, for example, permits rates of communication to vary among pairs of members. His main conclusion is same as those generated previously: if interpersonal communication entails some degree of positive effect of one member on another whenever communication occurs, then consensus will arise in strongly connected, in unilaterally connected, and in those weakly connected structures that contain a single strongly connected point basis.

French's theory is no longer viewed as useful in light of recent work on the relationship between social and cognitive structures. The main approach of this recent work has been (1) to obtain information on the opinions (views, beliefs, convictions, persuasions, sentiments) and the face-to-face relationships (e.g., friendships) occurring among the members of a population, (2) to construct a measure of structural proximity or distance based on some attribute of the pattern of the face-to-face relationships, and (3) to cluster the members so that same-cluster members are more proximate than different-cluster members (see Alba 1981, Burt 1980, Marsden and Laumann 1984). Repeatedly, it has been found that same-cluster members tend to be more similar in their opinions than different-cluster members: the populations appear to be internally differentiated, often markedly so, with respect to their members' opinions. Because many of the surveyed social structures are conceptualized as structures of *influential communication* and fall within that class of structures for which French's theory predicts consensus, the accumulated evidence appears to disconfirm his theory.

However, the disconfirming evidence is based on social structures that do not strictly indicate the actual occurrence of influential communication on any particular issue. The structures are more accurately conceptualized as indicating lines of *potential* interpersonal influence — the occurrence of positive probabilities of influential communication on any given issue. In terms of French's theory, they are *power* structures rather than *influence* structures.

French does not rigorously specify the connection between power

and influence structures, apart from noting that a power structure constrains the pattern of influential communications that can occur on an issue. I elaborate his theory in this area. My development of the theory also involves a set of restrictions on the process of opinion change that differs from the set of restrictions entertained by French or Harary. While the differences in these restrictions are not theoretically trivial, they are less important to the conclusions of the present paper than the analysis of power structures.

THE THEORY

Opinion Change in Structures of Influential Communication

Consider the model shown in Equation 1. It describes the process of opinion change that occurs among the members of a population about a particular matter:

$$m_{i(t+1)} = \sum_{j=1}^N w_{ij} m_{j(t)} \text{ for all } i (i = 1, N) \quad (1)$$

where N is the number of members of the population, t is the point in time at which the process starts, $m_{i(t)}$ is the opinion of member i on a given issue at time t , and w_{ij} is the weight member i accords to the opinion of member j (i.e., the effect of member j 's opinion on member i 's opinion).

The model deals strictly with instances of influential communication: the possibilities of influence without direct communication and direct communication without influence are ignored (cf. Friedkin 1983). The model assumes that the members of the population simultaneously revise their opinions at each point in time from the start of the process. It assumes that the revised opinion of each member is a linear combination of the opinions, at the immediately prior point in time, of those persons who have directly communicated their opinions to them. The model assumes that the weights members accord remain constant during the course of the process. The model assumes that the process continues until further communication has no effect in altering the opinions of any member of the population.

Five restrictions are placed on the variables in the model: (1) The opinion, m_i , of a member is column vector of real numbers; (2) $0 \leq w_{ij} \leq 1$ for all ij ; (3) $\sum_{j=1}^N w_{ij} = 1$ for all i ; (4) $w_{ii} > 0$ for all i ; and (5) $w_{ij} > 0$

whenever $w_{ji} > 0$ for all ij . Restriction 1 limits the model to opinions each of which can be represented by a vector of real numbers: the vector may consist of a single number, a subjective probability distribution, or a coordinate system. Restriction 2 states that the weight a member accords to another member must have a value between zero and one, inclusive, while restriction 3 states that the total weight a person accords must sum to one. Hence, the total weight that member i may accord is assumed to be a finite resource, and the weight accorded to a particular member is assumed to be a proportion of this total weight. Restriction 4 states that all members accord a positive weight to their own opinions. Restriction 5 states that if member i accords a positive weight to member j , then j will accord a positive weight to member i .

Restriction 5 warrants more extended comment. The model excludes cases of strict asymmetry in which i accords some influence to j (i.e., $w_{ij} > 0$) while j accords no influence to i (i.e., $w_{ji} = 0$). Such occurrences are excluded on grounds of theoretical consistency. The continual communication between two members, assumed by the model, is most consistent with a situation in which there exists either a degree of mutual positive affect or pressure to reduce differences. Although strict asymmetry is a theoretical possibility, its occurrence is difficult to reconcile with the considerable literature on the attributes of interpersonal relationships in which repeated direct communications occur (see Homans 1950, Newcomb 1953, Backman 1981). Note that exclusion of strict asymmetry nevertheless permits marked differences in the magnitude of the weights a dyad's members accord to each other's opinions: if member i accords substantial influence to member j (e.g., $w_{ij} = .950$), j may accord a trivial amount of influence to i (e.g., $w_{ji} = .001$). Hence, the model asserts that some degree of mutual influence is present among dyads in continual communication but does *not* stipulate that the magnitudes of the accorded weights are similar.

Consistent with the five restrictions placed on the model, a structure of influential communications is represented as a network with labelled points (standing for the members of the population) and directed lines from member i to member j wherever i accords some positive weight to j 's opinion. A value is assigned to each line that is the accorded weight. The structure will consist exclusively of one or more strong components: at most there may be N strong components, each consisting of a single member; at least there may be one strong component, consisting of all the members of the population. (Readers should take a moment to familiarize themselves with the definition of strong

components given in the Appendix, if they have not already done so, since the concept underlies all that follows).

Research that bears on the model of opinion change has focussed on the special case of an isolated dyad. In such a case Equation 1 simplifies to

$$m_{i(t+1)} = w_{ii} m_{i(t)} + w_{ij} m_{j(t)}.$$

An alternative, more familiar, form of this equation is derived as follows: Since $w_{ii} + w_{ij} = 1$,

$$\begin{aligned} m_{i(t+1)} &= (1 - w_{ij}) m_{i(t)} + w_{ij} m_{j(t)} \\ m_{i(t+1)} &= m_{i(t)} + w_{ij} (m_{j(t)} - m_{i(t)}) \\ m_{i(t+1)} - m_{i(t)} &= w_{ij} (m_{j(t)} - m_{i(t)}). \end{aligned} \quad (2)$$

Equation 2 states that the change in member i 's opinion, from time t to $t + 1$, is a constant proportion of the discrepancy between i and j 's opinion at time t . Hunter, Danes and Cohen (1984), who have evaluated alternative models of opinion change, conclude that the evidence most strongly supports this "linear discrepancy hypothesis." However, the model has *not* been evaluated in circumstances where a person's opinion is being influenced by two or more other persons. In such circumstances the distance between the opinions of two persons in direct communication may *increase* from time t to $t + 1$ and the change is not necessarily a *constant* proportion of the distance between the two persons' opinions at time t . We shall see that these features of the general model are consistent with an eventual convergence of members' opinions.

From the process model (Equation 1) and the five restrictions on it, the following conclusions may be derived (DeGroot 1974). (1) Each member of the population will eventually settle on a final opinion, where finality is defined as the absence of further opinion change given continuation of the process described in Equation 1. (2) The final opinions of two members will be the same if at least one path of directed lines connect the two in the network, regardless of their initial opinions or the magnitudes of the weights on the lines. Hence, all members of a strong component will be in final agreement, regardless of their initial opinions and accorded weights. (3) The members of different strong components will be in agreement only under special conditions of their distributions of initial opinions and accorded weights. Henceforth, I simplify matters by assuming that members of different strong components will not share the same final opinion. (4) The final opinion of the members of a strong component with k members is a linear

combination of the initial opinions of the members of the component,

$\sum_{i=1}^k \pi_i m_{i(j)}$, in which π_i is obtained by solving the matrix equation

$\pi \mathbf{w} = \pi$ under the constraint that $\sum_{i=1}^k \pi_i = 1$, where $\pi = (\pi_1, \dots, \pi_k)$

and \mathbf{w} is the matrix of accorded weights. (5) The relative influence of the members of a strong component in determining the component's final opinion are the values $\pi_1 \dots \pi_k$.

The process model predicts that members of a strong component will be in final agreement regardless of variations in the size of the component or features of its internal structure. I have noted that this conclusion is superficially inconsistent with many empirical findings indicating that detailed variations in network structure have substantial associations with the occurrence of interpersonal agreements. The reconciliation of such evidence with the model stems from the relationship between power and influence structures.

Power Structures

One member of a population is said to have power over another member on a particular matter if the former may influence the opinion of the latter on the matter. Interpersonal power is conceptualized as *potential* influence. French deals with the distinction by considering a situation in which not all members have power over every other member and influential communication occurs wherever there is a power relation. He also considers a situation in which every member has power over every other member and influential communications do not occur wherever they might. If the total number of power relations in a population is some number R and if each power relation may either be active or inactive, then there are 2^R alternative, labelled, structures of influential communication that are consistent with a given power structure. The absence of some power relations between members of a population constrains both the number and types of influence structures that may occur. However, in a power structure with twenty-five relations, there are over thirty-three million alternative, labelled, influence structures. Many of these structures may be functionally equivalent in terms of their outcomes; nevertheless, it is evident that there are grounds for uncertainty about the outcome of a given power structure.

An analysis of the long run distributions of the outcomes of a power

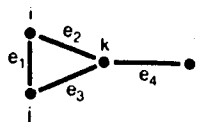
structure is called for. Recall the stipulation of mutual influence: in line with this stipulation, the power structure of a population is represented as a graph with edges between those members wherever there is a positive probability of influential communication. The occurrence of an edge indicates that mutual influential communication is possible; the absence of an edge indicates that such communication is not possible. It is assumed that members are in influential communication with themselves with probability one. It also is assumed that the probabilities of influential communication are independent; under this assumption the activation of one edge in the power structure has no effect on the activation of any other edge. Without the assumption of independence, a formal analysis of power structures is cumbersome.

With this definition of a power structure, we can determine the probability of occurrence of each of the alternative, labelled, influence structures that may arise from a given power structure. Given a particular outcome of interest, for example total consensus, the overall probability of the outcome is determined by summing the probabilities of those influence structures in which the outcome occurs.

Illustration of the Approach

Figure 1 and Table 1 provide a rudimentary illustration of the approach. We have a power structure consisting of four members (i, j, k, and l) and four edges (e_1 , e_2 , e_3 and e_4). Since each edge may be active or inactive, there are sixteen influence structures that might arise. If the probability that an edge is active is .60 for all four edges, then the probability of occurrence of each influence structure, $G_1 \dots G_{16}$ respectively, is .03, .04, .04, .06, etc. G_1 has four strong components, G_2 and G_3 have three, etc. The expected number of strong components is 1.82. From our process model, we know that i, j, k, and l must be in agreement (regardless of their initial opinions and accorded weights) in G_8 , G_{12} , G_{14} , and G_{16} . Hence, the probability of consensus is predicted to be .39. We also can find the probability that particular pairs of members will be in agreement. For example, the probability that i and j will be in agreement is .74, that is, the sum of probabilities of those influence structures in which they are connected. When intermediate subsets (or clusters) are defined *a priori*, the theory will generate a matrix of dyadic-level probabilities of agreement which, in turn, may be organized into blocks corresponding to the clusters under study. When clusters are not defined *a priori*, the theory provides an approach for an exploratory analysis of substructure. For these purposes, the

Power Structure

Possible Influence Structures
with loops on points deleted

Edges				Structures G_i ($i = 1, 16$)	
e_1	e_2	e_3	e_4		
0	0	0	0		G_1
0	0	0	1		G_2
0	0	1	0		G_3
0	0	1	1		G_4
0	1	0	0		G_5
0	1	0	1		G_6
0	1	1	0		G_7
0	1	1	1		G_8
1	0	0	0		G_9
1	0	0	1		G_{10}
1	0	1	0		G_{11}
1	0	1	1		G_{12}
1	1	0	0		G_{13}
1	1	0	1		G_{14}
1	1	1	0		G_{15}
1	1	1	1		G_{16}

FIGURE 1 Illustration of the Approach

matrix of probabilities of dyadic agreement may be used as input to Hierarchical Cluster Analysis or to algorithms that generate a spatial representation of member's relative positions. Cluster-level probabilities also may be calculated directly from an enumeration and analysis of the possible states of the power structure; for example, the probability that *i*, *j* and *k* agree is .65.

Some other outcomes pertaining to interpersonal agreement are the following. The probability that a majority (i.e., three or four) of the members in this power structure will agree is .76. The expected number of members in the largest agreeing cluster is 3.13.

Deductions need not be restricted to agreements; one also can derive expectations of the relative influence of each member in determining final opinions. However, these expectations require information on the weights members accord to other members' opinions. For purposes of

TABLE I
Analysis of the power structure in Figure 1 assuming that $P(e_i = 1) = .60$ ($i = 1, 4$) and that members change their opinions at each point in time to the mean of their own opinion and the opinion of those members who are influencing them.

Probability of Each Influence Structure		Illustrative Outcomes							
		A	B	C	D	E	F	G	H
G ₁	(.4) (.4) (.4) (.4) = .0256	4	0	0	0	0	1	0	—
G ₂	(.4) (.4) (.4) (.6) = .0384	3	0	0	0	0	2	0	—
G ₃	(.4) (.4) (.6) (.4) = .0384	3	0	0	0	0	2	0	—
G ₄	(.4) (.4) (.6) (.6) = .0576	2	0	0	0	1	3	0	—
G ₅	(.4) (.6) (.4) (.4) = .0384	3	0	0	0	0	2	0	—
G ₆	(.4) (.6) (.4) (.6) = .0576	2	0	0	0	1	3	2/7	—
G ₇	(.4) (.6) (.6) (.4) = .0576	2	0	1	1	1	3	0	—
G ₈	(.4) (.6) (.6) (.6) = .0864	1	1	1	1	1	4	1/5	2/5
G ₉	(.6) (.4) (.4) (.4) = .0384	3	0	1	0	0	2	0	—
G ₁₀	(.6) (.4) (.4) (.6) = .0576	2	0	1	0	0	2	0	—
G ₁₁	(.6) (.4) (.6) (.4) = .0576	2	0	1	1	1	3	0	—
G ₁₂	(.6) (.4) (.6) (.6) = .0864	1	1	1	1	1	4	1/5	3/10
G ₁₃	(.6) (.6) (.4) (.4) = .0576	2	0	1	1	1	3	0	—
G ₁₄	(.6) (.6) (.4) (.6) = .0864	1	1	1	1	1	4	3/10	3/10
G ₁₅	(.6) (.6) (.6) (.4) = .0864	2	0	1	1	1	3	0	—
G ₁₆	(.6) (.6) (.6) (.6) = .1296	1	1	1	1	1	4	1/4	1/3

- Notes. A is the number of strong components.
 B is consensus (*i*, *j*, *k*, and *l* agree).
 C is agreement between *i* and *j*.
 D is agreement between *i*, *j*, and *k*.
 E is majority agreement.
 F is the number of members in the largest agreeing cluster.
 G is the influence of *i* in determining *l*'s final opinion.
 H is the influence of *k* in determining the final opinion of the group, given the occurrence of consensus.

illustration, I assume that the members of each influence structure change their opinions to a value that is the *mean* of their own opinion and the opinions of those members in the influence structure who are affecting them. It follows that the expected influence of member *i* in determining the final opinion of member *l* is .11 and that, given the occurrence of consensus, the expected influence of member *i* in determining the final opinion of the group is .24. Comparable predictions may be obtained for the other members: for example, given the occurrence of consensus, the expected influences are .24, .33, and .19 of *j*, *k*, and *l* respectively.

The theory permits comparisons between different power structures as well as comparisons among subsets of members within a particular power structure. The predicted differences may or may not be intuitively obvious depending on the complexity of the structural configuration. The theory will generate conclusions when intuition cannot process the implications of a structural configuration. Whether in simple situations, the theory generates counter-intuitive conclusions is an open question.

STRUCTURAL EFFECTS

Studies of structural effects typically have been based on adjacency matrices. The orienting hypothesis is that the pattern of relationships between persons is an important determinant of various outcomes. The present theory suggests serious problems with this hypothesis that center on its incomplete specification of the process that generates a particular outcome and the contributions of each dyadic relationship to the process. The following imaginary situation may help elucidate what is at issue. Suppose I were to provide the path diagram of a recursive structural equation model (Duncan 1975), provide the scores of individuals on the exogenous variables, and not provide the values of any of the structural coefficients. It is obvious that it is impossible to predict the values of any of the endogenous variables without the structural coefficients in hand. Social network analysis, when it has treated network structure as a determinant of various outcomes, has focussed on the arrangement of interpersonal relationships while ignoring the implications of variation in the relationships' separate effects (e.g., the probability and magnitude of interpersonal influence entailed by each of them). In the absence of any information comparable to the structural coefficients in a structural equation model, network analysts

(myself included) have sought to discover purely "structural effects" (Friedkin 1984).

Under strong assumptions, different edge patterns in power structures *may* systematically differ in their outcomes regardless of the magnitudes of their probabilities of influential communication. For example, consider the six power structures shown in Figure 2. The theory predicts an invariant rank order of the structures with respect to their probabilities of consensus, under the assumption that the probabilities of influential communication are uniformly equal to some value γ other than zero or one. Their rank order is

$$CHAIN = STAR < KITE < CIRCLE < SLASH < COMCON$$

If one is unwilling to assume uniform probabilities of influential communication within and across these structures, any necessary systematic difference between them breaks down.

The problems being raised do not necessarily go away under strong assumptions. Consider two dyads, *cd* in *STAR* and *ad* in *CIRCLE* of Figure 2. Under the assumption of uniform probabilities of influential communication, is *cdSTAR* or *adCIRCLE* the more likely to agree? The answer depends on the value of γ . Assuming $\gamma = .25$, then *cdSTAR* has a higher probability of agreement (.25) than *adCIRCLE* (.12). Assuming $\gamma = .75$, then *cdSTAR* has a lower probability of agreement (.75) than *adCIRCLE* (.81). The problem is, of course, not solved by replacing the assumption of uniform probabilities with an assumption that the probability of influential communication in a dyad, $P(I_i)$, is a linear function of some observable attribute, X , of the interpersonal relationship: $P(I_i) = \beta X_i$. The reduced form equations do not necessarily imply consistent "structural effects" unless analysis shows that such effects (e.g., rank order differences) do not depend on the value of β .

Systematic differences in the outcomes of edge patterns will arise if the variation in the edge patterns overrides differences that might occur on the basis of variations in the probabilities of influential communication in them. It may be possible, for example, to assert that sufficiently different edge patterns will differ in their outcomes under most of the likely alternative patterns of probabilities of influential communication. In such cases, an edge pattern is a superordinate constraint on system outcomes. In general, however, the theory indicates that if our aim is to analyze the implications of macro structures (i.e., edge patterns), we must have knowledge about the micro events occurring in the structure — in the form of precise information on the structural

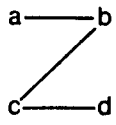
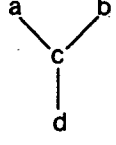
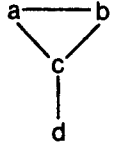
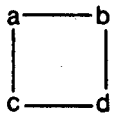
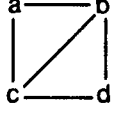
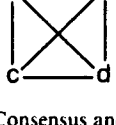
Four-Member Power Structures		Probabilities of Dyadic Agreement ($\gamma = .50$)			Probabilities of Group-Wide Consensus ($\gamma = .50$)
		b	c	d	
CHAIN		a	.50	.25	.12
		b		.50	
		c		.50	
		d			
STAR		a	.25	.50	.12
		b		.50	
		c		.50	
		d			
KITE		a	.62	.62	.25
		b		.62	
		c		.50	
		d			
CIRCLE		a	.56	.56	.31
		b		.44	
		c		.56	
		d			
SLASH		a	.66	.66	.44
		b		.72	
		c		.66	
		d			
COMCON		a	.75	.75	.59
		b		.75	
		c		.75	
		d			

FIGURE 2 Consensus and Dyadic Agreement in Four-Member Power Structures

coefficients that link attributes of interpersonal relationships to the probability of influential communication.

ASSUMING γ EQUALS .50

While formal models generally entail radical simplifications, observed phenomena sometimes behave *as if* the model were accurate. Such simplifications already have been introduced (e.g., the assumption that the probabilities of influential communication in a power structure are independent). In this section the simplification is carried a step further. I suggest, tentatively, that many power structures may behave *as if* the probabilities of influential communication in them were uniformly equal to .50.

Suppose an investigator has no information other than an adjacency matrix, indicating *opportunities* for influential communication in a population. Having no basis for asserting that the occurrence of influential communication is more or less likely than its nonoccurrence, in dyads where there is an opportunity for such communication, the investigator should assume that the chance of influential communication is .50. Though surely in error, this assumption is consistent with prevalent practice: investigators into structural effects on interpersonal agreements typically do not differentiate among the individual edges of an adjacency matrix. The assumption permits unambiguous conclusions about structural effects that are consistent with empirical findings on the distribution of interpersonal agreements in social networks. Hence, the assumption may be employed, in the absence of detailed information on probabilities of influential communication, with *some indication* that it will provide a good first approximation to an account of the effects of a power structure on interpersonal agreements.

Under the $\gamma = .50$ assumption, the probability $P(A_1 \dots N)$ that all the members of a power structure will agree is simply

$$.50^E C_{1 \dots N}$$

where E is the number of edges in the power structure and $C_{1 \dots N}$ is the frequency of those influence structures, among the alternative influence structures, in which a single strong component includes all the members. Similarly, the probability $P(A_{ij})$ that two particular members of a power structure will agree is simply

$$.50^E C_{ij}$$

where E has the same definition as before and C_{ij} is the frequency of those influence structures, among the alternatives, in which a strong component includes both i and j . In a comparison of power structures, purely structural attributes determine differences in the probability of consensus. Within a given power structure, any difference in the probability of agreement between subsets of members depends totally on the arrangement of the edges in the structure.

I am not aware of any simple formula for determining $C_1 \dots N$ (or for that matter C_{ij}) as a function of E and N . However, some deductions can be developed about structural effects on the probability of dyadic agreement in a given power structure. Three factors determine dyadic agreement: (1) path length, (2) path redundancy, and (3) path centralization. The length of a path is the number of edges involved in it (see Appendix on this application of digraph concepts to graphs). Path redundancy is the number of paths of different lengths that connect i and j in the power structure. Centralization occurs when at least one of the edges in the paths joining i and j is involved in two or more of the paths connecting them; if no edge is involved in more than one path connecting i and j , the paths connecting them are edge disjoint. While centralization is a matter of degree, I shall be concerned with a dichotomous classification of dyads: those whose connecting paths are to some extent centralized and those whose connecting paths are edge disjoint.

For the moment, let us eliminate the factor of centralization and consider a situation in which the paths connecting a dyad are edge disjoint. Equation 3 predicts the probability of dyadic agreement in this situation

$$P(A_{ij}) = 1 - (1 - \gamma)^{X_1} (1 - \gamma^2)^{X_2} \dots (1 - \gamma^D)^{X_D} \quad (3)$$

where $\gamma = .50$, X_f is the number of paths of length f that connect i and j , and D is the length of the longest path that connects i and j . It should be noted that Equation 3 holds for all values of γ , although it is being introduced as part of an approach which sets γ to .50.

Under the condition of edge disjoint paths, Equation 3 is derived from the assumptions of the theory and predicts that the probability of dyadic agreement is entirely a function of the number and length of the paths connecting two members. While it is most obviously applicable to power structures with edges that are arranged hierarchically, its conclusions also are consistent with structural cohesion models that have emphasized the effects of path length and number in nonhierarchical arrangements (Friedkin 1984). It predicts that the longer the path

connecting i and j , the smaller the path's contribution to the probability of dyadic agreement. It predicts that the greater the number of paths of a particular length connecting i and j , the more likely are i and j to be in agreement. It predicts that the probability of dyadic agreement may be high when i and j are not directly adjacent depending on the number and length of the longer paths connecting the pair. Note that, without additional assumptions concerning the distribution of path lengths and numbers among the dyads, we cannot deduce that the shorter the *distance* between two members the more likely they are to agree.

If the paths connecting i and j are *not* edge disjoint, then the probability of dyadic agreement must be less than that predicted by Equation 3: it gives the upper bound of the probability of agreement for a given combination of paths. In other words, centralization is associated with a *lower* probability of agreement, controlling for the number and length of the paths connecting i and j . The effects of centralization are illustrated in Figure 3. In each of the structures, (G_1^* , . . . G_5^*), i and j are connected by two 2-step paths and two 3-step paths; hence, Equation 3 predicts that i and j are equally likely to agree in each of these power structures. This prediction does not take into account the variation across the five structures in the degree of interdependence existing among their paths. A rough indicator of the degree of interdependence is the number of edges upon which the four paths are based: this number declines from ten to five. The probabilities of dyadic agreement, provided in the figure, illustrate that centralization has a negative effect on the occurrence of agreement.

In the absence of a control for the number and length of paths connecting i and j , the relationship between centralization and agreement may easily be confounded with the effects of path number and length. Centralized power structures, to the extent that they provide numerous short paths between two persons, are likely to entail *high* probabilities of dyadic agreement. Hence, though the zero-order association between centralization and agreement may be positive, the second-order partial association (controlling for path length and number) is predicted to be negative.

The theory suggests a different viewpoint on the concept of network centralization than that which currently prevails. In the literature on network centralization, persons are viewed as being more or less central (Freeman 1979). Edges rather than persons are the elementary units in the present approach. The two viewpoints are quite close: a person or an edge is said to be central if found in multiple paths connecting two or more members of a network. It is noteworthy that Shaw (1981) has

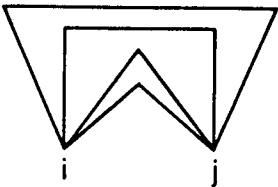
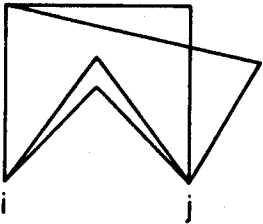
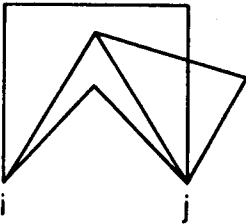
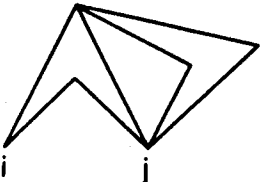
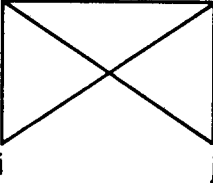
Power Structures	Number of Relationships	$P(A_{ij})$ $\gamma = .50$
G_1^*	10	.57
		
G_2^*	9	.56
		
G_3^*	9	.55
		
G_4^*	8	.52
		
G_5^*	5	.50
		

FIGURE 3 Illustration of the Effect of Centralization on Dyadic Agreement

explained the effect of network centralization on the problem-solving capability of small groups, in part, on the basis of variations in *independence*, which he defines as "the degree of freedom with which the individual may function in the group" (p. 158). From a different viewpoint, the present theory arrives at a closely related conceptualization of centralization in terms of the magnitude and pattern of nonindependence among the events determining the outcomes of a power structure.

DISCUSSION

The theory combines two general approaches to the analysis of systems that may be represented as networks. Its deductions about the effects of structures of influential communication are based on an application of mathematics involved in the Theory of Markov Chains (Kemeny and Snell 1960). Its deductions about the effects of power structures are based on an application of a system-analysis approach that has been used especially in fields of engineering; for example, in assessing the performance reliability of physical systems (Hillier and Lieberman 1980). The application of these approaches has entailed a degree of theoretical simplification that may or may not be justified. While the theory generates precise assertions, such precision should not set up an expectation of an equally precise correspondence between these assertions and the empirical world. The simplifications involved in a formal approach to a complexly determined phenomenon necessarily result in some lack of fit to reality.

Depending on the degree of inaccuracy, a formal model's lack of fit should not be a source of discomfort. Science operates under the assumption that the architect of reality is a mathematician whose works are elegant. Even if this assumption is incorrect, the process of constructing, rejecting, and reconstructing formal theories of events has an important heuristic value. For science also assumes that events are intelligible, and formal theories have a demonstrated utility in stimulating the accumulation of knowledge. They shift attention from discrete propositions to schema (entailing general viewpoints and assumptions) from which a large number of propositions are deduced. While providing plentiful matter for hypothesis testing, they transform such testing from an end to a means of evaluating approaches from which we might deduce substantial segments of reality. Although formal theories generally do not withstand disinterested scrutiny, they often make an enduring contribution to understanding as a *by-product*

of the structured speculation and empirical work that occurs when attention is focussed on them.

Empirical evaluations of the Formal Theory of Social Power might test that part of it concerned with the process of opinion change (specified in Equation 1) or that part of it concerned with the effects of power structures. The central conclusion of the process model — that consensus will form among members of strong components in a network of influential communications — is the foundation upon which the theory's deductions about power structures rest. Since the theory's deductions about the effects of power structures on interpersonal agreement depend *only* on the process model's conclusion about strong components, these deductions may be maintained under different process models so long as they generate the same conclusion about strong components.

It is, perhaps, this feature of the theory that provoked a friendly critic to comment that the process model entailed in the theory is irrelevant once the probabilities of the alternative influence structures are known: one might have started with the conceptualization of power structures, assumed that members involved in a strong component will be in agreement, and arrived at the same conclusions about interpersonal agreements. While this is obviously true, it is more satisfactory on several grounds to have *derived* the assumption about strong components from a theory about how opinions are altered. First, with the process model the connection between micro- and macro-level phenomena becomes explicit. Second, without it, the possible points at which the theory might be modified to increase its accuracy become harder to discern. For example, if the predictions about power structures are disconfirmed, do the flaws occur because its edges are not activated independently or because its assertion about strong components is inaccurate? If one had confidence in the assertion about strong components (on the basis of tests of the process model from which the assertion is derived), then attention might focus on the matter of independence. Third, although the present analysis has emphasized the effect of power structures on interpersonal agreement, the theory also generates conclusions about the relative influence of a power structure's members. It is the process model that pulls these two outcome variables together.

For the above reasons, an intensive scrutiny of the process model entailed in the theory is important. In light of Abelson's (1964) work, I suspect that the variations on the process model will differ less with respect to their conclusions about strong components than with respect

to their conclusions about the *content* of persons' settled opinions and persons' *relative influence* in determining other persons' settled opinions. At the same time, it should not be forgotten that even if the process of opinion change operates according to Equation 1, it cannot be taken for granted that the equilibrium conditions assumed by the model will be met. In fact, it is likely that these conditions are never met, but hold to varying degrees under different circumstances. These circumstances will need to be addressed in detail, should tests of the theory roughly support its predictions.

General tests of the theory may deal with its predictions about the effects of power structures. In thinking about the class of network structures to which the present conceptualization of power structures might apply, it is useful to recall French and Raven's (1959) seminal paper on the bases of interpersonal power. Coercion, French and Raven suggest, is only one of the several bases of social power. Besides coercive power, a stable potential for interpersonal influence may rest on the ability to mediate rewards, the rights to prescribe behavior, affection, or the possession of special knowledge. French and Raven refer to these noncoercive bases of social power, respectively, as reward power, legitimate power, referent power, and expert power. French and Raven's conceptualization of social power allows for stable power structures that markedly depart from a hierarchical arrangement of coercive relationships since, in their conceptualization, power relationships may be voluntary and symmetrical.

In the context of French and Raven's work and the definition of interpersonal power as potential influence, I suggest that many of the networks that have been studied in relation to the distribution of interpersonal agreements may be broadly conceived of as power structures comprised of lines that are active or inactive with probabilities that are a function of the *strength* of an interpersonal tie. The concept of tie strength, appropriately or inappropriately, incorporates the various bases of social power elaborated by French and Raven, while emphasizing the affective component (see Granovetter 1973). Along the lines of this viewpoint, the present theory considerably reinforces the theoretical significance of the concept of tie strength as being central to an understanding of structural effects upon interpersonal agreements.

In illustrating the limitations of working with adjacency matrices as a basis of predicting agreements, the theory suggests that the study of network phenomena is more complex than had been thought. At the same time, the theory offers a way to deal with these complexities to the extent that it is possible (1) to construct a reliable measure of the

strength of an interpersonal tie and (2) to estimate the parameters of the function relating tie strength and the probability of influential communication in dyads. Some progress on the first issue has occurred (Marsden and Campbell 1984). With both issues settled, the present theory will generate predictions of the effects of power structures from information about the strengths of the ties comprising the structures.

Prediction of effects of power structures is considerably simplified if they behave as if the positive probabilities of influential communication in them are all .50. This simplification appears to generate some conclusions that are in accord with available findings on the distribution of interpersonal agreements in social networks. One possible strategy for exploring the merits of the theory would be to test the fit of it on the basis of the $\gamma = .50$ assumption and then to relax this assumption (taking into account tie strength variations). An improvement in predictions would be consistent with the theory. The simultaneous assessment of the accuracy of the theory and the adequacy of the $\gamma = .50$ assumption would be useful.

APPENDIX

With three exceptions, the network terminology used in the paper conforms to Harary, Norman and Cartwright (1965). The three exceptions are: (1) the term *network* is applied to both *digraphs* and *graphs*; (2) *digraphs* are permitted to have *loops*; and (3) *graphs* are operationally treated as *symmetric digraphs*.

Network. The constituent elements of a network are a set V of points (v_1, v_2, \dots, v_n), a set L of lines (l_1, l_2, \dots, l_r), and a set P of numbers (p_1, p_2, \dots, p_r). The network is a digraph if the lines in L are directed (with a single arrowhead on each line). The network is a graph if the lines in L are undirected (with no arrowhead on each line). The term *edge* is reserved for undirected lines. It is sometimes useful to treat edges as bi-directional (as two-headed arrows); I have found it useful to do so in the paper. Each number in P is associated with its corresponding line (edge) in L . Each line (edge) is associated with a particular pair of points in V . For example, l_1 might be a line (edge) from v_1 to v_2 labelled with the number p_1 , and l_2 might be a line (edge) from v_1 to v_1 labelled with the number p_2 . A line (edge) from a point to itself is referred to as a *loop*.

Adjacency Matrix. Point i is *adjacent* to point j if there is a line (edge) from v_i to v_j . An adjacency matrix stores information on the

adjacencies in a network. A cell in the matrix is set to one if an adjacency occurs and to zero if it does not.

Path and Semipath. A path is a sequence of lines (edges) connecting two points so that, by following the direction of the arrowheads on the lines (edges), one may get from one point to the other. No line (edge) may occur more than once in the sequence. The number of lines (edges) in a path is its *length*. The *distance* between two points is the number of lines (edges) in the shortest path connecting the two points in the network. Two points are *reachable* if at least one path connects the two points. A semipath is a sequence of lines (edges) connecting two points ignoring the direction of the arrowheads on the lines (edges). As with paths, in semipaths no line (edge) may occur more than once in the sequence.

Types of Connectivity. A subset of points in a network is *strongly connected* if every two points in the subset are mutually reachable. A subset of points is *unilaterally connected* if, for any two points, at least one is reachable from the other. A subset of points is *weakly connected* if, for any two points, at least one semipath connects them. A subset of points is *disconnected* if it is not weakly connected. The subset may consist of all the points in V , in which case the network as a whole may be characterized as strong, unilateral, weak, or disconnected. The categories are not strictly mutually exclusive: strong subsets also are unilateral and weak; unilateral subsets also are weak.

Strong Component and Strongly Connected Point Basis. A strong component of a network consists of a subset of points that is strongly connected and leaves no point out the addition of which to the subset would preserve the property of strong connectivity (i.e., the component is maximally complete). A network may be comprised of a single strong component, including all the points in V , or of as many strong components as there are points in V . A strongly connected point basis is a strong component from which there are one or more paths to each point in V not included in the component.

REFERENCES

- Abelson, Robert P. 1964. Mathematical models of the distribution of attitudes under controversy. Pp. 142-160 in N. Frederiksen and H. Gulliksen (eds.), *Contributions to Mathematical Psychology*. New York, Holt, Rinehart & Winston.
- Alba, Richard D. 1981. Taking stock of network analysis: A decade's result. Pp. 39-74 in S.B. Bacharach (ed.), *Perspectives in Organizational Research*. Greenwich, CT, JAI Press.
- Backman, Carl W. 1981. Attraction in interpersonal relationships. Pp. 235-268 in

- M. Rosenberg and R.H. Turner (eds.), *Social Psychology: Sociological Perspectives*. New York, Basic Books.
- Burt, Ronald S. 1980. Models of network structure. *Annual Review of Sociology* 6, 79-141.
- DeGroot, Morris H. 1974. Reaching a consensus. *Journal of the American Statistical Association* 69, 118-21.
- Duncan, Otis Dudley. 1975. *Introduction to Structural Equation Models*. New York, Academic Press.
- Freeman, Linton C. 1979. Centrality in social networks I. Conceptual clarification. *Social Networks* 1, 215-39.
- French, John R.P. Jr. 1956. A formal theory of social power. *The Psychological Review* 63, 181-94.
- French, John R.P. Jr. and Bertram Raven. 1959. The bases of social power. Pp. 150-67 in Dorwin Cartwright (ed.), *Studies in Social Power*. Ann Arbor, MI, Institute for Social Research.
- Friedkin, Noah E. 1983. Horizons of observability and limits of informal control in organizations. *Social Forces* 62, 54-77.
- 1984. Structural cohesion and equivalence explanations of social homogeneity. *Sociological Methods & Research* 12, 235-61.
- Granovetter, Mark S. 1973. The strength of weak ties. *American Journal of Sociology* 78, 1360-80.
- Harary, Frank. 1959. A criterion for unanimity in French's theory of social power. Pp. 168-82 in Dorwin Cartwright (ed.), *Studies in Social Power*. Ann Arbor, MI, Institute for Social Research.
- Harary, Frank, Robert Z. Norman and Dorwin Cartwright. 1965. *Structural Models: An Introduction to the Theory of Directed Graphs*. New York, Wiley.
- Hillier, Frederick S. and Gerald J. Lieberman. 1980. *Introduction to Operations Research*. San Francisco, Holden-Day.
- Homans, George C. 1950. *The Human Group*. New York, Harcourt.
- Hunter, John E., Jeffery E. Danes and Stanley H. Cohen. 1984. *Mathematical Models of Attitude Change, Volume 1*. Orlando, FL, Academic.
- Kemeny, John G. and J. Laurie Snell. 1960. *Finite Markov Chains*. Princeton, NJ, D. Van Nostrand.
- Marsden, Peter V. and Karen E. Campbell. 1984. Measuring tie strength. *Social Forces* 63, 482-501.
- Marsden, Peter V. and Edward O. Laumann. 1984. Mathematical ideas in social structural analysis. *Journal of Mathematical Sociology* 10, 271-294.
- Newcomb, Theodore M. 1953. An approach to the study of communicative acts. *Psychological Review* 60, 393-404.
- Shaw, Marvin E. 1981. *Group Dynamics: The Psychology of Small Group Behavior*. Third Edition. New York, McGraw-Hill.