Network Power

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Abstract

This paper combines two bodies of literature to produce a measure of network power: the voting power literature and the network analysis literature. The voting power literature offers a meaningful conception of power as the potential to control social outcomes. However, it does not have a fruitful way of determining how groups that can determine such outcomes may be formed. The networks analysis literature has a meaningful way of conceptualizing the formation of groups that have the potential to control social outcomes, but it bases its conception of power strictly on the structure of ties within a network. In order to develop the concept of power in networks I first specify three basic axioms that any measure of power must satisfy. I then introduce the measure of power and prove that it satisfies these axioms. I explore some of the more interesting and counterintuitive properties of the network power index. Finally, I demonstrate the concept of power by examining two international networks over time: alliance networks and trade networks. I show how the concept of power relates to resource-based conceptions on the one hand, and to centrality-based conceptions on the other.

Network Power

1. Introduction

Power is arguably the single most important concept in the social sciences. Consequently, it has a multitude of measures. These measures differ from each other, not only across different contexts or substantive field; they also exhibit considerable variations within a given substantive context. For example, there exist many measures of national power, of political power, of economic power, and so forth.

Surprisingly, however, the concept of power is not well developed in network analysis. Network analysts implicitly assume that power is related to network centrality. However, it is not evident that this is the case. Network science is preoccupied with relationships and with social or physical interactions. Such interactions entail structures of dependence, exchange, and information flow; all of these are influenced by and reflect power relations. Consequently, the lack of an explicit measure of network power reflects a huge lacuna in this discipline.

This paper develops an axiomatic approach to the measurement of network power. This approach builds on three basic ideas. First, the power of a node in a network—as in other contexts—is based on one or more attributes of that node. These attributes define what the node can or cannot do. Second, the effect of a node's potential actions are a function of the structure of ties it has with other nodes—the number, nature, and magnitude of these ties. Third, the power of a node is based on the extent to which it affects the groups with which it is affiliated. Because there are multiple ways to define groups in networks analysis, the power of varies as a function of the precise method for deriving groups.

This paper is designed as follows. I start with a general discussion of measures of power, focusing on the conceptions that are relevant to network analysis. Second, I outline four axioms that a measure of network power must satisfy. Third, I discuss a class of measures of network power. I then prove that this class of measures satisfies these axioms. Fourth, I discuss some of the general attributes of these measures. Fifth, I apply this measure of power to domestic political and international networks. Finally, I discuss the advantages and limitations of this measure.

2. Background

It would be presumptuous to review the concept of power and its various definitions and measures in a short paper. Instead, I comment on some aspects of this concept that are relevant for the measurement of network power. Strictly speaking, there are three general conceptions of power: power as control over resources, power as control over actors, and power as control over outcomes (Hart 1976; Maoz 1989, 1990a). The difficulty of measuring power as control over actors and/or outcomes has prompted both scholars and practitioners to focus on the measurement of power as control over resources. It was assumed—implicitly or explicitly—that control over resources correlates with control over actors and control over outcomes. However, multiple paradoxes of power (Maoz 1989, 1990a: Ch. 8) suggest that not only is this assumption tenuous, but there also may be a causally inverse relationship among these conceptions. Specifically, excessive control over resources may actually *cause* loss of control over actors and/or over outcomes. Others have pointed out that power is a function of exchange relations

(Barnett and Duval 2005); one organism can influence another only if the former can provide or deny something of value to the other. If one does not have anything that the other organism desires, or does not control those things that the other values, then regardless of the access of the former to resources, its ability to influence the latter is quite limited. Conversely, an organism with few resources that is in a position to provide others what they need most can exert considerable influence on the behavior of the latter. The point is here that "Power is the production, *in and through social relations*, of effects that shape the capacities of actors to determine their circumstances and fate" (Barnett and Duvall 2005: 39, italics mine). The idea is that power can be exercised through interaction and exchange. Most measures of power that focus on the attributes of actors tend to miss this point. The problem, of course, is that such notions are seemingly impossible to measure in any *a priori* fashion.

Voting power indices represent one of the few cases of power measures based on the notion of control over outcomes. Crudely defined, voting power indices are variations on the same theme: the power of a voter is a function of her ability to affect the outcome of a decision process (e.g., passage of a bill, outcome of an election, a decision in an international organization). Indices of voting power confront head on the problem of measuring influence *ex ante* (Felsenthal and Machover 1998). The two most prominent measures of voting power are the Shapley-Shubik (1954) index and the Banzhaf (1965) index (Banzhaf 1965). Both measures conceive of the power of a voter as proportional to the number of groups in which she is pivotal. A voter is pivotal for a given coalition if her presence in the coalition makes it winning, and her defection from that coalition causes it to lose a vote.

Where these two indices differ is in their conception of the probability of a given coalition forming. Shapely and Shubik's index assumes a random roll-call process where voters choose sequentially.[1] Accordingly, assuming that a decision making body has a 51% majority rule for passing a bill, a voter is pivotal if he/she is becomes the 51st percent majority voter. Under such a circumstance, if exactly half of the voters have voted aye and another half have voted nay, and it is the focal voter's turn to cast a vote, that person determines the outcome. The probability of a voter to be pivotal in a random set of sequences is her voting power.[2]

Banzhaf's index does not assume a sequence. It counts all possible coalitions that can win a vote, regardless of how or in what order they might be formed. It then calculates, for each voter, the proportion of those winning coalitions in which the focal voter is pivotal. A number of studies compare these and other measures of power (Felsenthal and Machover 1998, Dubey and Shapley 1979). These comparisons reveal that, in general, different indices of a priori voting power are generally similar. They are also highly correlated with the attributes of voters (e.g., their weights, seat proportions).

The advantages of these indices are quite obvious. First, they tap the ability of an actor to control outcomes in settings that include electoral outcomes, coalition formation, and group decision making (in both domestic and international settings). Second, they require minimal information. The only information required for such indices includes the attributes of units, the decision rule by which outcome are obtained (e.g.,

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1 By "random" it is assumed that the order by which voters are called to vote is determined such that each voter has an equal probability of being first, second, …., last.

2 To put it in terms of social choice theory, the power of a voter is the probability of her becoming the median voter.

simple majority, special majority, quota for passing a bill), and the voting procedure (i.e., roll call, simultaneous voting). Information that is more difficult to obtain a priori (e.g., preferences) is not part of the calculation of voting power. Third, as noted, they are *ex ante* indices (Felsenthal and Machover, 1998)—they allow prediction of actors' ability to control outcomes before the process leading to outcomes was put in motion. For example, we can use these indices to predict (as many have) the ability of states in the EU Council of Ministers (where different states have different weights) the actual ability of these states to control outcome in the EU Council.

At the same time, voting power indices rely on assumptions that are quite implausible. Probably the most difficult assumption to swallow is that all coalitions are equiprobable. For example, many institutions that apply roll call voting (e.g., the U.S. Senate) carry out these votes by alphabetical order. This means that the smallest possible winning coalition in that setting is one where all senators listed alphabetically voted either aye or nay on an issue up to that Senator who is the fifty-first in the list. So, for example, in the 111th Congress, the first Senator that could cast a deciding vote under any roll call circumstances was Mike Johanns of Nebraska. Before Ted Kennedy's death, it would have been Tim Johnston of South Dakota. None of the Senators with last names preceding those of these two Senators would have been pivotal under any circumstances.

More important, voters come to elections with certain preferences. They may vote strategically, but it is implausible to assume that a voter would cast a vote in a way that was totally inconsistent with her preference, just so as to maximize her power. In reality coalitions form not by a sequence, but rather in accordance with the distribution of preferences across actors (individuals, parties). Voting power is a function of the coalitional context in which one may be pivotal. Thus, the manner in which this context emerges in of vital importance for the calculation of power. If a voter, by virtue of her preferences, becomes a viable candidate for many coalitions, then this voter has—regardless of her attributes—a higher a priori chance of casting a deciding vote than one with preferences that make her a viable partner in only a few coalitions. In contrast, a voter with preferences that diverge considerably from those of most other voters is less likely to be pivotal because it is unlikely to join or be admitted into most coalitions.

Third, voting power indices are based on single-shot cases. They assume that each voter comes to the decision as if he/she were *tabula rasa*. The participation of the voter in a given coalition is merely a function of the voter's attributes (size, weight), and the decision rule that defines the quota required for collective decisions. Given a fixed distribution of attributes over voters, a fixed quota, and a fixed voting procedure, the distribution of voting power in the decision making body is also invariant over bills, issues, and coalitions. This is so not only because the probability of a given coalition forming is the same across all possible coalitions; it is also because all coalitions are similar. No coalition is considered to be more cohesive than another. In reality, however, the likelihood of a given coalition forming is not only a function of its size; it is also due to the extent to which its members can agree on a common policy agenda. If that is the case, the preferences of coalition members define whether or not such an agreement is possible. These preferences also define the range of issues over which members can agree. The cohesion of a given coalition, however, is not a factor in the calculation of voting power.

My conception of voting power attempts to overcome these difficulties. In doing so it builds on central ideas of network science. These ideas help define the factors that affect the formation of "coalitions;" allow differentiation among coalitions in terms of cohesion and size; and exploit new aspects of the ability of individual actors to control other actors in the network. By doing so, this conception differs dramatically from the more conventional conceptions of power and influence in networks. In order to allow a more coherent comparison between the measures of network power I develop here and the more conventional conceptions of power in networks, I discuss very briefly the network analytic conceptions of influence and centrality.

Network analysis typically associates power and influence with various indices of nodal centrality. These range from simple degree centrality (defined as the ratio of the actual number of ties to the number of possible ties), to a different conceptions of centrality based on Eigenvectors (Bonacich 1987; Bonacich and Lloyd 2004; Rossman et al. 2010). The idea is that the number and type of ties of a node determines its ability to affect other nodes in certain ways. Other measures of influence (e.g., Hubbell 1965; Taylor 1969), are also based on the level of direct and indirect connectivity of nodes. The idea that power is based on connections is in line with Barnett and Duval's notion of power in the context of exchange relations.[3] Yet, focusing on connectivity alone ignores nodal attributes. It also ignores the structure and the characteristics of the groups that these connections induce—the "coalitions," that are defined by sets of ties among nodes.

This brief review suggests a general point: conceptions of power that derive from voting power logic (which itself is based on n-person games), and network conceptions of power that derive from structural patterns of ties among nodes complement each other rather nicely. Each conception addresses some of the flaws of the other. Thus, the integration of these two perspectives is the principal goal of network power.

2. Fundamental Desiderata of Network Power

2.1. Axioms

I start with a brief outline of notations. In order to provide a practical context that illustrates the logic of network power, I use an example of a multiparty political system. A network is defined by a matrix **X** that consists of a set of nodes N = [1, …., i, j, …., n] and a rule that specifies whether, in what way, and to what extent any two nodes (i, and j) are connected. The rule can specify a binary (yes/no) type of tie, a valued (0…., $t \mid t > 0$), or a signed ($-\infty \geq t \geq +\infty$) tie. For convenience, I focus only on nonnegative ties (i.e., $x_{ij} = t \geq 0$). Ties can be directional (e.g., $i \rightarrow j$ and $j \varnothing i$ for at least one $ij \in \mathbf{X}$) or symmetric ($i \leftrightarrow j \ \ \forall \ ij \in \mathbf{X}$). The attributes of the nodes are defined on attribute matrix **A** of order $n \times k$. **A**'s rows represent nodes and columns represent attributes, 1,…., k.

In our example, nodes are political parties in a multiparty political system, and the ties between them denote their ideological proximity. (For convenience, I assume that proximities range from zero—i.e., two parties are as far apart as ideologically possible—to one—the two parties are ideologically indistinguishable.) The attributes of nodes in this example are defined on an $n \times 1$ vector with entry a_i representing the share of parliamentary seats held by party i.

Any network can be partitioned into m groups. These groups may be discrete or they may partly overlap in terms of membership (such that node i can be in more than one group, and that groups g and l may have one or more nodes in common). I defer the

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3 See applications to IR by Kim (2010), Hafner-Burton and Montgomery (2009).

discussion of how these groups are extracted from the network to a subsequent section. Suffice it to note that, for any group derivation method, we obtain a group affiliation matrix **GA** of order $n \times m$, where rows stand for nodes and columns for groups. Entries in **GA** affiliation matrix are defined as $ga_{il} = 1$ if node i is affiliated with group l and zero otherwise. Denote the size of group l as $s_l = \sum_{i \in l} a_i$. In our parliamentary example, groups are non-discrete proto-coalitions (meaning that a given party can be in more than one proto-coalition). However, these measures apply equally to discrete proto-coalitions. The size of each such proto-coalition is the share of seats it would control if formed.[4]

Many—but not all—networks are emergent outcomes of nodal choices. In other words, the structure of a given network reflects the sum total of choices of individual nodes about whether and with whom to form ties. It also reflects the choices of nodes regarding the types of ties they wish to form (the magnitude and sign of the ties). Because networks are functional structures, ties serve a certain purpose for the nodes connected by them. Even in networks that are nondiscretionary in nature (Maoz 2010), that is, networks that are formed due to exogenous forces independently of nodal choices, ties may serve defined functions.

The utility assigned by node i to a tie with node j is defined by u_{ij}. For convenience, I set u_{ij} within the [0,1] interval. The utility assigned by i to not having a tie with j is denoted by $u_{i\text{-}j}$ and has the same range as u_{ij}. In our example the utility assigned by one party to another reflects their ideological similarity. Two parties that are ideologically proximate might consider cooperating across a wide range of issues on which they tend to agree; parties that are ideologically distant would find few, if any, issues on which they see eye-to-eye.

I now turn to the basic definitions and preliminary assumptions about discretionary networks:

1. $x_{ij} > 0$ *iff* (if and only if) $u_{ij} > u_{i\text{-}j}$. This means that a node will form a tie with another node if and only if the utility it derives from this tie is strictly higher than the utility it derives from not having that tie.

2. A symmetric tie can form iff both nodes prefer having a tie with each other than not having a tie. That is, $x_{ij} > 0$ *&* $x_{ji} > 0$ *iff* $u_{ij} > u_{i\text{-}j}$ *and* $u_{ji} > u_{j\text{-}i}$ $(x_{ij} \neq x_{ji})$. Two parties would not consider co-sponsoring a bill or voting together on an issue unless they agree on that bill.

3. A given group l is a subset of **X** such that $x_{ij} > 0$ *&* $x_{ji} > 0$ $\forall i, j \in l$. This means that a group can form iff all possible pairs of members prefer having ties over not having ties with each other. In our example, a proto-coalition is a subset that represents a minimal agreement between any pair of parties that are part of this coalition on a joint platform.

4. Groups can vary in terms of their cohesion. Although all dyads in group l and all dyads in group m satisfy assumption 3, the degree of common purpose among

[4] Note that the only restriction on a given proto-coalition is that it cannot be a proper subset of another proto-coalition. Thus, only proto-coalitions that differ with respect to at least two members (one in proto-coalition g but not in proto-coalition l and one in l but not in g). What these proto-coalitions are and under what circumstances they form will be discussed in the next section.

the parties making up group *l* may be different from the common purpose among the parties making up group *m*. I define common purpose in terms of *group cohesion*. The cohesion of group *l* is denoted by $C_l = \frac{\sum_{i=1}^{gl-1}\sum_{j=i+1}^{l} u_{ij}}{g_l(g_l-1)}$, where g_l is the number of nodes in group *l*. This means that the higher the average expected utility of ties between nodes in the group, the more cohesive the group.

In terms of our example, a group is a set of parties, each of which pursues a political agenda. Party *i* assigns a high utility to another party *j* if *j*'s agenda overlaps or resembles its own agenda. By extension, a high average utility within a group implies that its members pursue similar goals. Two proto-coalitions may have the same number of seats, but the range of issues on which the members of one coalition agree, or the extent of agreement on any given issue is dramatically higher than the level of agreement in another proto-coalition. So that one proto-coalition may be more cohesive ideologically than another.

5. Each group *l* confronts a potential opposition -*l*. This potential opposition is the complement of the group. It consists of all the nodes not in *l*. The opposition is also characterized by the number of members in the opposition, and their attributes, and it is also characterized by its cohesion, C_{-l}.

6. *Group Power*. The power of a given group is a function of its ability to defeat its opposition over an array of issues. This implies that—for any issue under contention—the members of a group can agree on a joint policy, and the size of the group supporting that policy should be larger than the size of the opposition to this policy. Hence, the power of a group is a combination of its size and its cohesion. Formally, $GP_l = s_l C_l$. (Note that, while $0 < GP_l \leq 1$, $\sum_{l=1}^{m} GP_l \neq 1$. This is so because groups are not necessarily discrete.) The intuition here is straightforward. Given an unknown array of issues that a group might face, the power of the group depends both on its size as well as on the extent to which its members can agree on a wide range of policies.

   Three important implications follow.

a. *The power of a fully connected binary network is 1*. A fully connected binary network reduces—by any definition of endogenously-derived groups—to a single group. Given that, the size of the group is one and its cohesion is one.
b. *No group has zero power*. For a group to have zero power, it must have either no members, or its cohesion must be zero. However, any of these two conditions violates the definition of groups. Clearly an empty group cannot exist. Moreover, a group with cohesion of zero implies that no two members have anything in common. This violates assumption 3 above.
c. *A majority group can be weaker than a minority group*. Suppose a network is divided into two groups *l* and *m,* with $s_l > s_m$ (the attributes of the members in *l* make for a large group size than the attributes of *m*'s members). However, if the cohesion of *m* is much higher than that of *l*, then it is possible that $GP_l < GP_m$. In terms of our example, this means that a coalition that has a majority of seats in parliament may be defeated by a cohesive opposition. This can happen if the opposition defines an agenda over which members of the coalition disagree, but the members of the opposition agree. A no-confidence vote on such an issue can split the coalition (some of its members may abstain, or even defect to the opposition).

I now turn to the axioms driving the concept of network power.

1. *Attribute Monotonicity*. Consider a node i with attribute a_i that participates in a set of groups $[g_1, g_2, \ldots, g_m]$, with cohesion $[c_1, c_2, \ldots, c_m]$. If the attribute of i is increased by an amount d ($a_i^* = a_i + d$), and the attributes of the other nodes decrease by the same amount $\left(\sum_{j \neq i} a_j^* = \sum_{j \neq i} a_j - d\right)$ and ties remain the same, the network power of i cannot diminish ($NP_{i|a_i} \leq NP^*_{i|a_i^*}$). In terms of our example, this means that if the ideological positions of the parties remain fixed (thus the proto-coalitions and their cohesion scores remain fixed as well), but one of the parties increases its seat share, then the power of that party cannot decline. This axiom is illustrated in Figure 1.

Figure 1 about here

2. *Affiliation Monotonicity*. Consider a node i with attribute a_i that is affiliated with group $[g_1, g_2, \ldots, g_m]$ with cohesion $[c_1, c_2, \ldots, c_m]$. If the node changes its preferences such that it continues to be affiliated with the previous groups without changing the cohesion of the groups, but now it is affiliated with at least one new group (g_{m+1}), then its network power cannot diminish ($NP_{i|g=1,m,C=c1,cm} \leq NP^*_{i|g=1,m+1,C=c1,cm,cm+1}$). In our example, assume that parties' preferences and sizes remain fixed, but one of the parties changes its ideological position such that it continues to be a member of the proto-coalitions in which it had been before, and becomes a member of at least one additional proto-coalition, then its power cannot decline. This is demonstrated in Figure 2.

Figure 2 about here

3. *Cohesion Monotonicity*. Suppose a certain distribution of cohesion scores over the groups in the network $\mathbf{C} = [c_1, \ldots, c_m]$. Suppose that the cohesion of one of the groups, g_l increases. This creates a new distribution of cohesion scores, $C^* = [c_1^* = c_1, \ldots, c_l^* > c_l, \ldots, c_m^* = c_m]$. Everything else in the system remains unchanged. In such a case, the power of the nodes in group l cannot decrease. Formally, if $c_l^* > c_l \;\&\; c_{j \neq l}^* = c_j \forall\, g_j \in G$ then $NP_i^* \geq NP_i \;\forall\, i\; \in g_l$. Figure 3 demonstrates this axiom.

Figure 3 about here

There is another axiom that serves as a desideratum in many conceptions of voting power. However, as I show below, no index of power that is based on these elements (nodes, attributes, and group cohesion) satisfies this desideratum.

4. *Population monotonicity*. This axiom has two complementary parts, one dealing with the increase in network size and the other focuses on decrease in network size.

    a. Consider a set of nodes N = $[1, \ldots, n]$ with attributes $A = [a_1, \ldots, a_n]$. This network is broken down to a set of groups $G = [g_1, \ldots, g_m]$ with corresponding cohesion scores $C = [c_1, \ldots, c_m]$. Suppose a node $n+1$ is added such that the attribute set is now $A^* = [a_1^* < a_1, \ldots, a_n^* < a_n, a_{n+1}^* > 0]$. Under such circumstances, and regardless of the effect of adding a node on the group set and their cohesion, the network power of all previously existing nodes $[x_1,\ldots,x_n]$ cannot increase.

b. Consider the same set of nodes, attributes, groups, and cohesion scores. If a node drops out of the network such that the attributes of the condensed network are $A' = [a'_1 \geq a_1,\ldots, a'_{n-1} \geq a_{n-1}]$, then regardless of the effect of a shrinking network on its group set and their cohesion, the network power of the surviving nodes cannot decrease.

In our example, consider a system with a fixed number of parties and a certain distribution of seats over these parties. Suppose a new party joins and the election results allocate seats over the parties. The new party "steals" some votes from each of the existing parties, such that no existing party changes its relative size vis-à-vis the other existing parties. Consequently, the new party gains some seats in the parliament. This axiom suggests that, under such circumstances none of the previous parties can increase its network power.

The second case represents the collapse or breakdown of a party. Consequently, its seats are distributed among the remaining parties such that each party gets at least one extra seat and the relative sizes of the parties does not change. In that case, none of the remaining parties can lose power.

I now specify the measure of network power and discuss several variations of this measure. I then proceed to prove that it satisfies the first three axioms but violates the fourth axiom. I show that any measure of power that is based on the factors discussed above violates at some point the population monotonicity axiom. In fact, this is what makes the measures of network power attractive and counterintuitive.

3. General Measures of Network Power

3.1. *Simple Network Power*

Network power concerns the ability of a node to influence outcomes. A node is powerful if it can cause desirable outcomes to happen and if can prevent the occurrence of undesirable outcomes. This conception matches most of the measures of voting power discussed above. However, the operationalization of network power is different than in the conceptions of voting power in several respects.

1. *Revealed Preferences.* Voting power indices require no information about voters' preferences. In network analysis, nodes have preferences. These preferences are revealed by the ties they have to other nodes in the network (assumption 1 above).

2. *Probability of Coalitions.* This probability is—for most voting power indices—a function of: (1) the quota—the number of votes that it takes to pass a bill, (2) the structure of voting—whether voting is unicameral or multicameral, simultaneous or sequential, and so forth, and (3) the distribution of attributes (seat shares, voting weights) over actors. Network power allows for multiple and flexible ways to define groups. The probability of a group forming is a function of both the structure of ties and of the assumptions we use to assign nodes to groups. For example, if we use the (*k*-) clique concept, then the assumption we make is that a group requires all nodes to have ties of order *k* or lower to be members. We also assume that groups are non-

discrete. On the other hand, if we use blocks or communities as our grouping concept, then we assume that groups require some minimum pattern or minimum level of ties to become members. This also implies discrete groupings.

3. *Group cohesion*. This does not play a role in voting power indices. It plays a crucial role in network power (assumption 4 above).

4. *Abstention*. Voting power indices are ambiguous on abstention (Felsenthal and Machover 1998). This causes a problem in network power conceptions as well. I deal with it by developing two versions of network power. More on this below.

Building on voting power indices, the power of a node is a function of its pivotness; that is, its effect on the ability of the groups with which it is affiliated to defeat their opposition. Accordingly, network power of node *i* is given by:

$$NP_i = \frac{\sum_{g=1}^{m} W_{ipg}}{\sum_{g=1}^{m} W_g} \qquad [1]$$

Where W_{ipg} denotes a group wherein *i* is a pivotal member, and is defined by:

$$W_{ipg} = \begin{cases} 1 & \text{if } GP_{g|i\in g} \geq GP_{\sim g} \ \textbf{\&} \ GP_{g-i} < GP_{\sim g+i} \\ 0 & \text{otherwise} \end{cases} \qquad [2]$$

And

$$W_g = \begin{cases} 1 & \text{if } GP_g \geq GP_{\sim g} \\ 0 & \text{otherwise} \end{cases} \qquad [3]$$

Simply stated, the power of a given node is the ratio of the number of groups *in which its participation is pivotal* to the number of winning groups. A node is pivotal if its shift from a given group to its (complement) opposition converts this group from winning to losing. The numerator of [1] is a count of the number of groups in which node *i* is pivotal. The denominator of [1] is the number of the groups whose power is larger than that of their respective oppositions. This is a strict definition of network power.

The underlying logic of this measure is similar to that of the measures of voting power. There are several differences, however. First, in contrast to the Shapley-Shubik index, the network power index does not consider the probability of a given coalition forming. The reason for that is simple. As we will see in the next section, groups are endogenously defined. Groups form due to the utilities nodes assign to each other. "Valid" groups are composed of nodes that have (direct or indirect) ties with each other. Accordingly, all groups that satisfy this condition are valid; no group that violates this condition can form. Moreover, the cohesion of the group provides an indicator of that probability; groups with higher cohesion are more likely to form than groups with lower cohesion. In a more strict sense, we can speculate that groups whose group power is weaker than that of their complements would not form. Likewise, all groups that defeat their complements are *ex ante* equally likely to form. Those that lose to their complements are meaningless in the calculation of network power and therefore can be said to have zero probability of forming.

3.2. *Abstention-based Network Power*

The network power measure in Equation [1] is quite demanding. A node is considered pivotal only if its presence in a group causes it to win, and its shift to the complementary group causes the former group to lose. However, in many real life situations, nodes may drop from a given group without joining the opposing group. In our example, a political party may drop from the coalition but not join the opposition. This is also a case that Felsenthal and Machover (1998) describe as *abstention-based power*. We denote abstention based network power of node *i* by NP_i^a and modify Equation [1] as

$$NP_i^a = \frac{\sum_{g=1}^{m} \widetilde{W}_{ipg}}{\sum_{g=1}^{m} W_g} \qquad [4]$$

Where,

$$W_{ipg} = \begin{cases} 1 & 1 \quad \text{if } GP_{g|i\in g} \geq GP_{\sim g} \ \textbf{\&}\ GP_{g-i} < GP_{\sim g} \\ 0 & \text{otherwise} \end{cases} \qquad [5]$$

Clearly each case in which a node is pivotal under [4] makes that node also pivotal under [1]. However, the opposite is not necessarily true. Since the complementary group in equation [4] is weaker than the complementary group in equation [1], the defection of node *i* from coalition *g* does not strengthen the opposition *~g*. Therefore, there are fewer cases that meet the condition $\sim g\,|\,i \notin \sim g \geq g–i$ (that define *i*'s pivotness under [4]) than those in which $–g\,|\,i \in \sim g \geq g–i$ (that define *i*'s pivotness under [1]).

Figure 4 about here

I illustrate this concept via Figures 4.1 and 4.2. Figure 4.1 shows a network composed of seven nodes. This network is partitioned into four cliques. A clique—equivalent to the notion of proto-coalitions discussed above—is a closed subset of a network, that is, a subset composed of nodes that are directly connected to each other. This breakdown is displayed in the clique affiliation network displayed in figure 4.2. The matrix representation of the data in Figure 4 is given in Table 1.

Table 1 about here

For the time being, I focus on the top part of this table (Table 1.1). This table reflects the network in Figure 4.1 (1.1.a). The nodal attributes for this network are given in section 1.1.b. The clique affiliation matrix induced from this network is depicted in Figure 5.1 and is represented by section 1.1.c. The bottom part of this section depicts clique sizes, clique cohesion, and group power, respectively. Each group's power is a product of its size (the sum of the sizes of its members) and its cohesion.

The complements of these cliques are given in section 1.1.d. The group power of complement cliques is the bottom row. Comparing the power of each group to the power of its complement suggests that cliques 1, 3, and 4 are stronger than their complements, hence denoted as winning coalitions (Ws in the bottom row). The other groups lose to their complements, thus denoted by L.

Consider Node #1. This node is a member of two cliques (3 and 4). Both cliques are winning with node one in them. Section 1.1.e shows these two cliques with node #1 removed from them, and section 1.1.f shows the complements of these two cliques with node #1 added to them. Comparing the group power of cliques 3 and 4 in section 1.1.e to the clique power of the complement groups in section 1.1.f, we note that without node #1 both cliques lose to their complement. Thus, node #1 is pivotal in both cliques of which it is a member and in two out of the three winning coalitions. The result is that node's 1 network power is 2/3 = 0.67.

With respect to abstention-based network power, we compare the clique affiliation matrix in which node 1 is removed (section 1.1.e) to the complement groups without node 1 in them. (This is not shown in the table, but it can be easily visualized by dropping node 1 from the two complement cliques in section 1.1.f.) The result is that clique 3 without node #1 is still larger than its complement (without node 1), but clique 4 is weaker than its complement. Hence, node 1's abstention-based power is $NP_1^a = {}^1\!/_3$.

The lower part of Table 1 shows the same analysis for node #7. Table 2 shows the network power of all nodes in this example.

Table 2 about here

4. Group Derivation

The structure of groups has an important impact on the measure of power. As I mentioned above, one of the key contributions of the networks framework to the concept of power is that it allows endogenous derivation of groups. We do not have to determine which groups would form in a specific social context. There is also no need to exogenously define the probability of certain groups forming or their probability of sustaining their membership across the array of circumstances that require their collective action. The catch of a networks framework, however, is that there is no single best way to derive groups. In fact, there are quite a few conceptions of what constitute meaningful groups in a network context.[5] I discuss here very briefly a few of them, focusing on their implications for the measurement of network power.

The first type of group—and the one I think is probably the most appropriate for many political contexts—is that of cliques (or *k*-cliques). As mentioned above, a clique is a closed subset of connected nodes at a level of *w* or above (for valued networks), at distance $k = 1, \ldots, n\text{-}1$.[6] A given node can be a member of multiple cliques, and any pair of cliques can share one or several members in common. The only limitation is that no clique can be a subset of another clique. This implies that any two cliques *r* and q must differ with respect to at least two nodes, one node that is in *r* but not in q and one node that is in q but not in *r*.

What makes cliques attractive with respect to the concept of network power is that a node can be a member of multiple cliques. The more central a node, the more likely it is to be a member of multiple cliques. This corresponds to the conception that ties power to relational centrality. At the same time multiple clique membership—just like centrality—does not define the probability of a node actually being pivotal in any of the cliques of which it is a member.

5 A more extensive discussion of different groups and various methods of partitioning a network into such groups is given in Wasserman and Faust (1997) and Jackson (2008).

6 Unless otherwise specified, I use the concept of clique as equivalent to a *1-clique*—a subset of nodes all of which are *directly* connected to each other.

The downside of using cliques as the grouping principle has to do with the alteration in the nature of network data required to extract such groups. First, valued networks must be binarized at some user-defined cutoff *w* (such that only ties of value *w* or higher can be considered members of cliques). Second, directional networks must be symmetrized via some user-defined rule.[7] This leads both to the loss of information in valued networks and to a redefinition of the rule that defines network ties. However, once this is done, the information that is lost in valued or directed networks is recaptured when we calculate clique cohesion because structural equivalence algorithms that are used to do so take into account both valued and directional ties.

Another set of groups consists of blocks, clusters, or communities (Karrer *et al.* 2008; Leicht and Newman 2008). These groups are derived in different ways, but they all share two important things in common. First, they are disjoint, or mutually exclusive. Any given node can be a member of one and only one such group, and no two groups can share any node in common. Second, they are all based on the concept of maximal similarity or minimal dissimilarity between nodes (or on the maximal difference between the structure of ties in a given network and randomness in communities). Some of the group extraction algorithms require some stopping principle that define the number of groups that are induced by the specific algorithm. This implies that such groups are not "natural" in the sense of cliques. Rather, their number and composition depend to a large extent on the method by which they are derived: change the method, you will get a different set of groups. In many cases, their disjoint nature becomes a problem. For example, the density of blocks (the proportion of actual ties between nodes making up a given block) may be much lower than inter-block density (the proportion of ties between nodes making up a block and the nodes making up another block). Nevertheless, the concept of network power applies to all kind of network partitions, as I show below.

5. Properties of Network Power

Clearly, the measures of network power are far more complicated than the voting power indices, and—to a large extent—more complex than the centrality indices in network analysis. More important, as we will see, network power measures do not correlate very highly with the latter and only moderately with nodal attributes. Let us now examine whether the network power indices satisfy the axioms set forth in the previous section.

Proposition 1: Network power measures satisfy Axioms 1-3.

Proof:

(1) *Attribute Monotonicity*. Assume a network of *n* nodes with attributes $A = [a_1, a_n]$. Denote the set of Groups that are induced from this network by G and the winning Groups by $W \subseteq G = [w_1, w_m]$. Let the winning groups in which node *i* is a member be denoted by W_i ($W_i \subseteq W$). Denote the winning groups in which node *i* is pivotal by W_{ip} ($W_{ip} \subseteq W_i$). According to Axiom 1, we increase *i*'s attribute by $d > 0$ ($0 < a_i + d < 1$). The set of groups in the new setup does not change because the ties are the same. The set of cohesion scores $C = [c_1, \ldots, c_k]$ also remains identical to that of the baseline condition. The set of winning groups under the new condition can be larger, equal, or smaller, than the set of winning groups under the baseline condi-

[7] E.g., *the strong tie rule:* two nodes are considered to have a tie if and only iff $i \rightarrow j$ and $j \rightarrow i$, or *the weak tie rule*: two nodes are considered to have a tie if either $i \rightarrow j$ or $j \rightarrow i$.

tion. This is so because, the increase in the size of node i under the new condition is accompanied by a proportional decline in the size of other nodes. Hence, $GP_{k|i\notin k} \leq GP^*_{k|i\notin k}$ and $GP^*_{k|i\in k} > GP_{k|i\in k}$. The change in the size of node i can yield three outcomes regarding the relationship between W^*_{ip} and W_{ip}, and between W* and W.

a. $W^*_{ip} = W_{ip};\ W^* \leq W$
b. $W^*_{ip} \geq W_{ip};\ W^* = W$
c. $W^*_{ip} \geq W_{ip};\ W^* > W;\ W^*_{ip} - W_{ip} \geq W^* - W$

All three scenarios imply that $NP^*_i \geq NP_i$. (Note that condition c holds only for cases where the numerator is smaller than or equal to the denominator. This is, by definition, the case with NP. So that the proof is valid for this range.)

There are cases, however, where the network power of node i under the new condition will be strictly larger than its network power in the base condition. The numerator of [1] and/or [4] may increase while the denominator does not, or the numerator's increase under the new condition may be larger than the increase in the denominator. This may happen if $W^*_{ip} > W_{ip}$, but the increase in W^*_{ip} is larger than the increase of W* (or W* = W). Hence in such cases $NP^*_i > NP_i$.

(2) *Affiliation Monotonicity*. This proof resembles the previous one. Assume that the base condition is defined by a number of winning groups W, a subset of winning groups in which node i is a member W_i, and a subset of W_i wherein node i is pivotal (W_{ip}). The alternative condition—when i's group membership increases—is denoted by an asterisk that accompanies all notations. Here again, it is impossible for the denominator of equations [1] and/or [2] to increase without at least the same increase in the value of the numerator. This is so because any increase in the number of winning groups is due to the fact that i is a member of these groups $(\Delta W^* = \Delta W^*_i)$. However, it is possible that W* = W, while $W^*_{ip} > W_{ip}$. Hence, here too, $NP^*_i \geq NP_i$.

(3) *Cohesion Monotonicity*. The proof here is trivial. We have the same notations here as in the previous proofs. The new condition is defined by the fact that the set of group cohesion scores under the new condition $(C^* = [c^*_1, \ldots, c^*_l, \ldots, c^*_m])$ is related to the group cohesion scores in the baseline condition such that $[c^*_1 = c_1, \ldots, c^*_l >$ *cl, …, cm*=cm.* Given the definition of a winning group (W) in equation [3], and the definition of group power in assumption 6, an increase in the cohesion of group g_l increases the power of this group. Consequently, one of two things might happen given this change (1) $GP^*_l \leq GP^*_{-l}$, that is, the increased cohesion of group l still leaves it with less power than its complement. In this case, W* = W, and thus $NP^*_{i\in l} = NP_{i\in l}$, or (2) $GP^*_l > GP^*_{-l}$, whereas $GP_l < GP_{-l}$, in which case W* > W. In this case, ΔW^* is due to the fact that, whereas $g_l \notin W$, the increase in c^*_l induces $g^*_l \in W^*$. Thus, $NP^*_{i\in l} \geq NP_{i\in l}$. Each node that is pivotal in group g^*_l, gains power. Those nodes that are not pivotal, do not lose power. In both cases, we have the same process as in the proof of the previous two axioms. Thus $NP^*_{i\in l} \geq NP_{i\in l}$.
Q.E.D.

Before we proceed, a general comment about other group types is in order. Since blocks, clusters, or communities are disjoint, the proofs above apply somewhat differently to these groups. Specifically, for disjoint groups, the following versions of this proposition apply.

1. *Attribute Monotonicity*. A change in the attribute of one of the nodes—at the expense of the attributes of the other nodes, without a change in the structure of ties in the network, induces $NP_i^* > NP_i$. The proof here is simple. In the case of disjoint groups, the strict version of condition *b* above applies.
2. *Affiliation Monotonicity*. This axiom does not apply to disjoint groups. In the case of disjoint groups, a change in the affiliation of a node can mean only that a node that was in one group moves to another group. This exceeds the parameters of this axiom.
3. *Cohesion Monotonicity*. This axiom applies to disjoint groups in the same way that it applies to overlapping groups. The proof here is the same as the proof for this axiom above.

Proposition 2: Measures of network power violate the population monotonicity axiom.

The proof here is through a simple example. Consider a network of 6 nodes (shown in the top part of Table 3). The attributes of the nodes are given in section 3.1.2. The network induces three cliques (section 3.1.3) with their size, cohesion, and GP at the bottom of each section. We are now examining the power of node #5. As can be seen, node #5 is a member of clique 1 only. However, clique 1—with node #5 in it—loses against its complement group (section 1.4). Hence, node #5 is the second-largest node in the network, but it has no network power ($NP_5 = 0$).

Table 3 about here

Now, consider the bottom section of the Table. Here node #7 is added to the network, and the attributes of some of the other nodes decline. Node #5 in this new setup one of the victims of the addition of node #7; its attribute now is 0.18, smaller than its previous attribute (0.24). The new network induces six cliques, three of which are winning. (Not shown in the table because they are not relevant for the assessment of NP_5.) Node #5 is in two of these six cliques (section 3.2.3). Clique #1 with node #5 is now winning against its complement group. Yet, clique #2 is losing to its complement (bottom of section 3.2.3). Now, once we remove node #5 from both cliques in which it is a member and add it to the complement group, clique #1 without node #5 is losing. This means that node #5 is pivotal for this clique. This means that node #5 is pivotal in one of the three winning cliques, and its network power is $NP_5 = {}^1\!/_3 = 0.333$ (its normalized power is 0.2). Hence, due to population increas, the attribute of node #5 shrunk, but its power increased substantially.

This example proves both parts of proposition 2. If we flip the demonstration above to have the bottom part constitute the baseline condition and we drop node #7, we get the top case to prove violation of the second part of the population monotonicity axiom. Node #5 increased its attribute but lost all of its network power (and also some of its spoiling power). This concludes the proof.

General properties of network power. The following propositions outline some of the more interesting and central properties of network power. Here I discuss the general implications of these propositions and their general import. The proofs are given in the appendix.

Proposition 3: *A node with an attribute that is higher than the sum of attributes of all other nodes has absolute power regardless of the structure of ties in the network. Also, all other nodes have zero network power. Formally,* $a_i > \Sigma_{j\neq i}a_j, \Rightarrow NP_i = 1, NP_j = 0 \;\; \forall j \neq i \in N.$

This proposition defines the limits of network structure effect on nodal power. When a node's attribute offsets the combined attributes of all other nodes—regardless of the number and nature of network ties—it will have absolute power. Consequently, none of the nodes has any power. In other words, in extreme cases of the distribution of an attribute (as specified in the proposition), network structure will have no impact on the distribution of power.

The remaining propositions outline the impact of network structure on the power of nodes, as well as the limitations of attribute-based power.

Proposition 4: *An isolate with an attribute that is lower than the sum of attributes of the other nodes may have absolute power. Formally, if* $i \in g_1$ *&* $I \notin g_{2,..}g_k \in G$ *&* $a_i < \Sigma_{j\neq i}a_j$, there exists a case where $NP_i = 1, NP_j = 0 \;\; \forall j \neq i \in N.$

The extreme case here is one where the focal node with the high attribute is an isolate. In such a case, the attribute of the focal node may be offset by a countervailing power by a coalition of the remaining nodes. However, such a countervailing coalition is "weak" due to the structure of network ties between the remaining nodes. Specifically, the low cohesion of the countervailing coalition reduces its group power to a point that makes it weaker from the "cohesive" coalition of the strong isolate. This enables an "attribute-strong" node to attain absolute power, and renders all other nodes powerless due to the structure of network ties.

Proposition 5: *A node with the highest value of an attribute that is less than sum of the attributes of all other nodes can have zero power. Formally: if* $a_i > a_j$ *&* $a_i < \Sigma_{j\neq i}a_j \;\; \forall j\neq i \in N$, *there exists a case where* $NP_i = 0$.

This case reflects situations where an "attribute-strong" node is rendered powerless by virtue of its network ties—or lack thereof.

Proposition 6: *A spoke-hub network produces an empty vector of network power if no node is stronger than the sum of all other nodes. Formally: if* $x_{ij} > 0$ *for i and* $\forall j \in N$, *&* $x_{jk} = 0 \;\; \forall j, k \in N$ *&* $a_i < \Sigma_{j\neq i}a_j$, *then* $\mathbf{NP} = [np_1 = np_2 \ldots = np_n = 0]$.

This is an extreme case of the impact of network structure on the distribution of power. Here, the structure of ties prevents any of the nodes—be it the most central node or any of the peripheral nodes—to be pivotal. This too is due to the low (or zero) cohesiveness of the groups induced by such a network. Such low cohesiveness renders each losing against its complement group. Alternatively, it prevents each node from being pivotal in the groups induced by some of the grouping methods.

Proposition 7: *A node that is the weakest in the network, but—at the same time—it is highly central, may have more power than strong but low-centrality nodes. Formally: if* $a_i < a_j \;\; \forall j \in N$, *and* $\Sigma x_{ik} > \Sigma x_{jk}$ *for some j, k* $\in N$, *there exist a case where* $NP_i > NP_j$ *for some j* $\in N$.

The last proposition centers on a key aspect of the relationship between attribute-based strength and network power. It shows that the structure of network ties can impose an almost complete rank-inversion between attribute and network power.

Taken together, these propositions establish the uniqueness of the concept of network power. This concept incorporates attribute-based notions of power as control over resources with network-based notions of power as control over actors and outcome. With the exception of extreme distributions of attributes or extreme types of network structure, neither of the distinct elements of this index can uniquely define network power. Network power is correlated with attribute based power, but the limits of attribute-related effects on network power are so significant (e.g., propositions 2, 5, and 7) as to suggest that this correlation is not sufficient to capture network power by attributes alone. The same applies to concepts of centrality as power in networks. Here too, there is a relationship between centrality and power, but this relationship does not capture the nuances of network power.

5. Network Power in International Relations

In order to illustrate the value of network power, I employ two international networks: alliances and trade. These are discussed more extensively in Maoz (2010). Here I discuss briefly how network power is obtained in these networks. Before doing that, however, I offer a brief theoretical discussion of two possible consequences of network power. First, conventional wisdom has it that material capabilities are important determinants of international outcomes. Network ideas suggest that the structure of ties in networks determines the ability of a node to determine collective outcomes as well. If that is the case, then we need to see how the concept of network power performs compared to the material capabilities and network centrality concepts in such settings. We thus set out to test two hypotheses.

H1. *The more network power a given state possesses, the more likely it is to win in militarized disputes it participates in.*

H2. *The more network power a given state possesses, the more likely it is to carry votes in international organizations.*

I now turn to the discussion of the empirical domain and the measurement of the variables in these applications.

Dependent Variables. I use two dependent variables: Dispute Outcomes and UN General Assembly Voting Outcomes.

Dispute outcomes are derived from the dyadic MID dataset (Maoz 2005), and are coded as in Maoz 1982). A victory for a given state is defined either as the accomplishment of the aims of a set of military operations in a dispute or as a settlement favoring the demands of the focal state. A defeat is defined as the opposite of these conditions. A draw is defined either as a stalemated outcome or as a symmetrical settlement that favors neither side in the dispute. I focus on the outcome for State A in the MID regardless of whether it is the initiator or the target in the dispute.

UN General Assembly (UNGA) Roll Call dataset (Voeten 2004) A detailed discussion of the measurement of voting outcomes is provided in Maoz (2010: 243). Simply put, voting record is measured as the proportion of the UNGA resolutions that went the way the focal state had voted.

Independent Variables.

Alliance Network Power. Data for alliances is derived from the Alliance Treaty Obligations and Provisions Project (Leeds 2005). The data cover the 1816-2001 period. I convert the ordinal rankings of alliances into an alliance commitment measure following Maoz (2009, 2010), such that higher commitments receive a greater reliability score and multiple commitments are aggregated. This commitment index varies between zero (no commitment) and one (all possible levels of commitment). Self-ties are assigned a score of 1, by definition. An alliance network at year t is defined by a $n_t \times n_t$ matrix $\mathbf{A}_t$, where n_t is the number of independent states at year t. Entry a_{ijt} is the level of alliance commitment between states i and j. Matrix $\mathbf{A}$ is symmetrical ($a_{ijt} = a_{jit}\ \forall\, i, j \in \mathbf{A}$). For each year a clique affiliation network is derived on the basis of 2-clique definition.[8] The cohesion of the clique is measured by the average Euclidean Distance Structural Equivalence of the dyads composing it (Maoz 2010: 58). A highly cohesive clique is one composed of states that, on average, have very similar alliance profiles with all other states in the system. The attributes of the nodes are their Composite Index of Military Capabilities (CINC, COW 2003). This is an index that measures the average share of the system's economic, military, and demographic resources accounted for by a given state. A clique's power is the product of the sum of the CINC scores of its members and the clique's cohesion. Once these variables are entered, network power and abstention-based power are calculated according to the algorithms discussed above.

Trade Network Power. Trade data are based on Oneal and Russett (2005) who employ and extend the Gleditsch (2002) dataset. These data are updated by incorporating the Barbieri, Keshk, and Pollins (2009) trade dataset. These data cover the 1870-2008 period. Each year is reflected by a trade network matrix $\mathbf{T}_t$ of order $n_t \times n_t$. Entry t_{ijt} denotes the relative share of i's exports going to state j. Here too trade networks are partitioned into trade cliques of order $k = 1$. (If we go beyond direct-trade cliques, the network collapses into a single clique for much of the period.) The cohesion of trade cliques is measured in the same way as is the cohesion of alliance cliques. The size of the clique is the share of the world's GDP accounted for by clique members. Here too, once cliques are extracted, trade network power is calculated according to the algorithms discussed above.

Control Variables.
Control variables vary depending on the type of dependent variable being estimated. With MID outcomes, the following controls are used:
Capability Ratio State A to State B. The Combined Index of National Capabilities (COW 2008) is used to measure the capability ratio of the dispute dyad.
GDP Share of States in MID. Using the Maddison World Economy dataset (Maddison 2008) I calculated the GDP share of each state in the system.
Degree Centrality. I use the alliance and trade indegree centrality of states, as defined in Maoz (2010: 53-54).
Democracy. Using the Polity IV data (Marshall 2004), I apply the coding by Maoz (1998) to assign a state a binary democracy score (1 = democracy, 0 = otherwise).

[8] A 2-cliques consists of nodes that are either directly linked or that are linked through one other node at most (the ally of my ally).

Reputational Status: Coded 1 if state is a major power and zero otherwise (Maoz 2010: 243 and website—http://psfaculty.ucdavis.edu/zmaoz/networksbook/suppmaterials.htm).

The expectation is that the probability of a state obtaining a satisfactory outcome in a dispute increases with the capability ratio with its GDP (which is another way to measure national capabilities, e.g., Organski and Kugler 1980), with its degree centrality, with its democracy score (Lake 1992), and with its reputational status.

Autocracy, past voting success, alliance and trade indegree centrality, GDP share, and reputational status are expected to have a positive effect on UNGA voting performance.

6. Results

Table 4 shows the correlations between the network power indices of states across these two networks. It also displays the correlations between the measures of network power and nodal attributes (capabilities for alliance NP and per capita GDP for trade NP). Finally, the table displays correlations between measures of network power and centrality scores. The results suggest several things. First, slightly different versions of network power produce relatively similar results across networks. This is indicated by the high correlations among the various versions of alliance network power and trade network power. It is also confirmed by a simulation based on random networks (results are not displayed herein). Second, the correlations between alliance network power and trade network power are moderate but statistically significant. This corroborates a stream of findings that show cross network spillover effects (Maoz 2010: Ch. 7).

Table 4 about here

Third, the correlations between alliance NP and the attribute-based measure of national capabilities are moderate. In contrast, the correlations between trade NP and the share of system's GDP are quite high. This suggests that trade cliques are typically made up of a few dominant states with high GDP and many medium or low-developed states. This point offers some insights into the dependent structure of trade in the international system. On the other hand, the correlations between network power indices and centrality scores are low to moderate for both alliance and trade networks. This again is confirmed in simulations based on random networks. It seems, therefore, that network power indices offer in many ways an aspect of nodal characteristics that are different from either their resource-related attributes or their network-based prestige.

I now turn to the analyses of the impact of network power on outcomes of disputes and UNGA voting. Table 5 provides the results of the MID outcomes.

Table 5 about here

A number of interesting things emerge from the results reported in Table 5. First, let us consider the baseline models. These models contain a number of surprises. The first (actually confirming the results of a very old analysis of dispute outcomes—Maoz 1983), is that the capability ratio of the focal state to its opponent does not affect dispute outcomes. This is true both for disputes short of war and wars. Second, does not have a robust effect on the outcomes of MIDs. The reputational status of the focal state affects its probability of winning (vs. losing) in short of war disputes, but it has a negative or in-significant effect on the probability of winning in wars. The paradox of power—defined as control over resources—and dispute outcomes (Maoz 1989) seems to be quite real.

Related to that, reputational status does not seem to have a consistent, robust, or even significant effect on outcomes across all categories and types of conflicts. The major power status of the focal state significantly increases the probability of winning in non-war MIDs but has no effect on the probability of winning in wars. This too accentuates the paradox of power as control over resources.

Centrality scores do seem to have an effect of dispute outcomes, but not always in the expected direction. For example, but the alliance indegree centrality of the focal state and the degree centrality of the opponent affect the probability of a shift from losing to drawing and from winning to losing in short of war MIDs. However, in wars, the probability of a draw or a win compared to lose increases only with the alliance indegree centrality of the focal state and declines with the indegree centrality of its rival. The effects of trade centrality on outcomes is inconsistent and not robust.

Finally, and also consistent with previous findings, the regime of states has a consistent and statistically significant effect on outcomes. Democracies tend to do consistently better than autocracies, and this applies both to the probability of winning in short of war MIDs and in wars.

When we substitute the capability and centrality scores with the network power indices, the results suggest that alliance network power has a consistent and statistically significant effect on both the shift from losing to drawing and from losing to winning. This applies both to MIDs short of war and to wars. As the network power of the focal state goes up, its probability of drawing or winning increases. As the network power of its enemy goes up, its probability of losing increases. The trade-related network power of the focal state does not seem to have a consistent effect on the outcomes it obtains in MIDs.

Finally, we examine now the effect of network power on the ability of states to exert peaceful influence in international organizations. The results of this analysis are displayed in Table 6.

Table 6 about here

The results of the baseline model largely replicate Maoz's results (2010: 234-236). Capabilities (here displayed as the system's GDP share possessed by the focal state) and trade degree centrality have a statistically significant effect on the UNGA voting record. Substituting these variables by network power suggests that only alliance network power has a statistically significant effect on a state's voting success. Trade power does not seem to have a significant effect on this record.

7. Conclusion

This study offered a new conception of network power. Since politics are about networks, the idea of power in networks is based on the ability of nodes to determine the outcome of group action via the ties that nodes—that is, people, groups, or states—forge with each other. Network power is also based on the ability of a given unit to affect the power of other units. The ideas explored in this paper are quite preliminary. They do offer, however, a new strategy for thinking about power as control over outcomes and over actors via network ties. The empirical analyses illustrate the fact that network power is based on but not equivalent to the attributes of states or to the pattern of ties they have in various networks. As such these conceptions of network power offer new avenues of research in networks in general, and on international networks, in particular. More important, some aspects—particularly alliance network— of network power seem to have a statistically significant effect on the probability of obtaining favorable outcomes in both peaceful and hostile international interactions. This suggests that the concept has not only important theoretical implications, it may provide important insights into otherwise puzzling international phenomena.

Appendix

This appendix provides proofs of the propositions regarding the properties of network power.

Proof of Proposition 3: *If* $a_i > \Sigma_{j\neq i}a_j, \Rightarrow NP_i = 1, NP_j = 0\ \forall j \neq i \in N$.
The simplest way to prove this property is to assume that node i is an isolate. If this proposition applies to an isolate, it must also apply to a node with one or more ties to other nodes. With cliques as the grouping principle, an isolate is in a clique with itself (for convenience, define $i \in g_1$ and $i \notin g_{r|r\neq 1}$). It thus faces the remaining nodes as the complementary group. The complementary group is defined as $j \in g_{\sim 1}\ \forall\ j \neq i \in N$. Since—by definition—the cohesion of group 1 is $c_1 = 1$, the power of this group is $GP_1 = a_i > q$. This implies that $W_g = g_1$ and $W_{g|i\in g} = g_1$. It also implies that each $g_{r|r\neq 1} \notin W_g$. Thus, $NP_i = 1/1$, and $NP_{j\neq 1} = 0/1$.

Proof of Proposition 4: *If* $i \in g_1$ *&* $i \notin g_{2,..}g_k \subseteq G$ *&* $a_i < \Sigma_{j\neq i}a_j$, there exists a case where $NP_i = 1, NP_j = 0\ \forall j \neq i \in N$.

As in the previous proposition, $c_1 = 1$, thus $GP_1 = a_i$. On the other hand, $GP_{\sim 1} = (\Sigma_{j\neq i}a_j)c_{\sim 1}$. The first element in the product is $\Sigma_{j\neq i}a_j > a_i$. However, $c_{\sim 1}$ can be, and typically is significantly smaller than one, depending on the pattern of ties between the other nodes in the network. Consequently, it is possible that $GP_1 > GP_{\sim 1}$. Since node i is only a member of group 1, we have the same set of winning groups as in proposition 3 ($W_g = g_1$ and $W_{g|i\in g} = g_1$). Consequently, $NP_i = 1/1$, and $NP_{j\neq 1} = 0/1$.

Proof of Proposition 5: *If* $a_i > a_j\ \forall j\neq i \in N$, *there exists a case where* $NP_i = 0$.

The proof of this proposition covers both isolates that are characterized by the conditions stated in the proof of the previous proposition (i.e., $i \in g_1$ *&* $i \notin g_{2,..}g_k \subseteq G$ *&* $a_i < \Sigma_{j\neq i}a_j$) as well as connected nodes (i.e., $i \in g_1, g_r\ldots \subseteq G$). The simplest way of proving this proposition concerns isolates meeting the condition stated in the proposition. Again, an isolate is in a group with itself, and this group has a cohesion score of 1. Thus, $GP_1 = a_i$ and $GP_{\sim 1} = (\Sigma_{j\neq i}a_j)c_{\sim 1}$. While $a_i > a_j\ \forall j\neq i \in N$, it is possible that $a_i < \Sigma a_{j\neq i}$. Given a sufficiently high $c_{\sim 1}$, we can have $GP_{\sim=1} > GP_1$. Thus, $NP_i = 0$. A simple example would be a network of 5 nodes with the attribute vector defined as **A** = [0.4, 0.2, 0.2, 0.1, 0.1]. Let node 1 be an isolate and all other nodes fully connected. This induces two groups $g_1 = [1]$, $g_2 = [2,3,4,5]$. It is also the case that $g_2 = g_{\sim 1}$. Given that g_2 is composed of a set of nodes all of which are connected to each other and none is connected to node 1, we have **C** = [1, 1]. It follows that $GP_1 = .4$, $GP_2 = GP_{\sim 1} = 0.6$. Thus, $NP_1 = 0$ (and $NP_{j\neq 1} = 1$).

Proof of Proposition 6: *If* $x_{ij} > 0$ *for* i *and* $\forall j \in N$, *&* $x_{jk} = 0\ \forall j, k \in N$ *&* $a_i < \Sigma_{j\neq i}a_j$, *then* $\boldsymbol{NP} = [np_1 = np_2 \ldots = np_n = 0]$.

The proof covers two possible cases depending on how such a spoke-hub network is partitioned into groups. Using cliques as our definition of a group, a network with n nodes is partitioned into $n(n-1)/2$ dyadic cliques with the hub a member of each. A block, or cluster partitioning induces two blocks one with the hub as the sole member, and the other with the remaining $n-1$ nodes.

6.1 With cliques the proof is simple. A spoke-hub network induces an empty cohesion vector (**C** = [$c_1=c_2=$ .., $c_k = 0$]). This means that each group's power is also zero ($GP_r = (a_1+a_j)c_r = 0\ \forall\, r \subseteq G$). This means that **W** = $\varnothing$, and consequently $NP_i = 0\ \forall\, i \in N$.

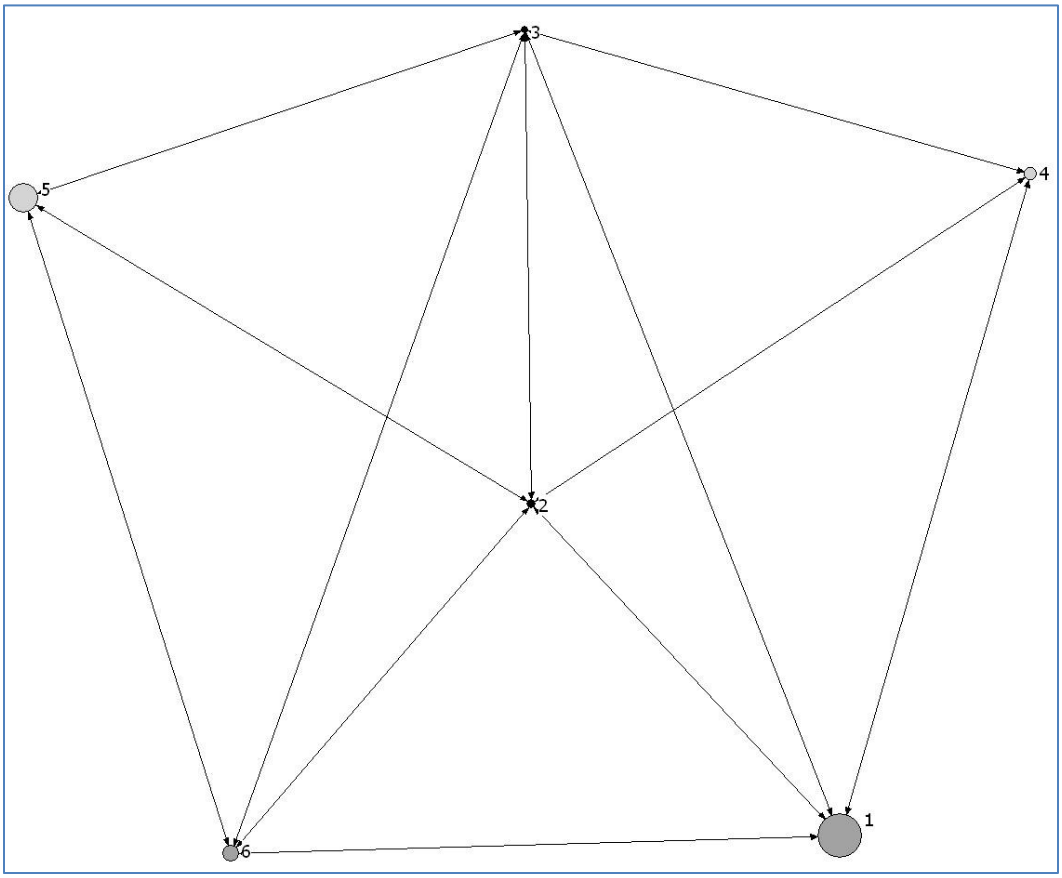
6.2 With blocks or clusters, the cohesion vector is **C** = [1, 1], thus $GP_1 = a_1c_1 = a_1$, and $GP_2 = (\Sigma_{j\neq 1}a_j = 1-a_1)c_2 = 1-a_1$. If $a_1 < \Sigma_{j\neq 1}a_j$, then $GP_1 < GP_{\sim 1}$. While $GP_2 > GP_{\sim 2}$, none of the nodes in G_2 is pivotal. This is so because the defection of any node from g_{2-j} to $g_{\sim 2+j}$ induces $c_{\sim 2+j} = 0$. Thus $GP_{\sim 2+j} = 0$ regardless of a_j.

Note: All networks with $a_i > \Sigma_{j\neq i}a_j$, including a spoke-wheel network, are covered by proposition 3 above.

Proof of Proposition 7: *If $a_i < a_j\ \forall\, j \in N$, and $\Sigma x_{ik} > \Sigma x_{jk}$ for some $j, k \in N$, there exist a case where $NP_i > NP_j$ for some $j \in N$.*

I prove this proposition by example. Consider the following network:

Figure A1: A Hypothetical Six-Member Network



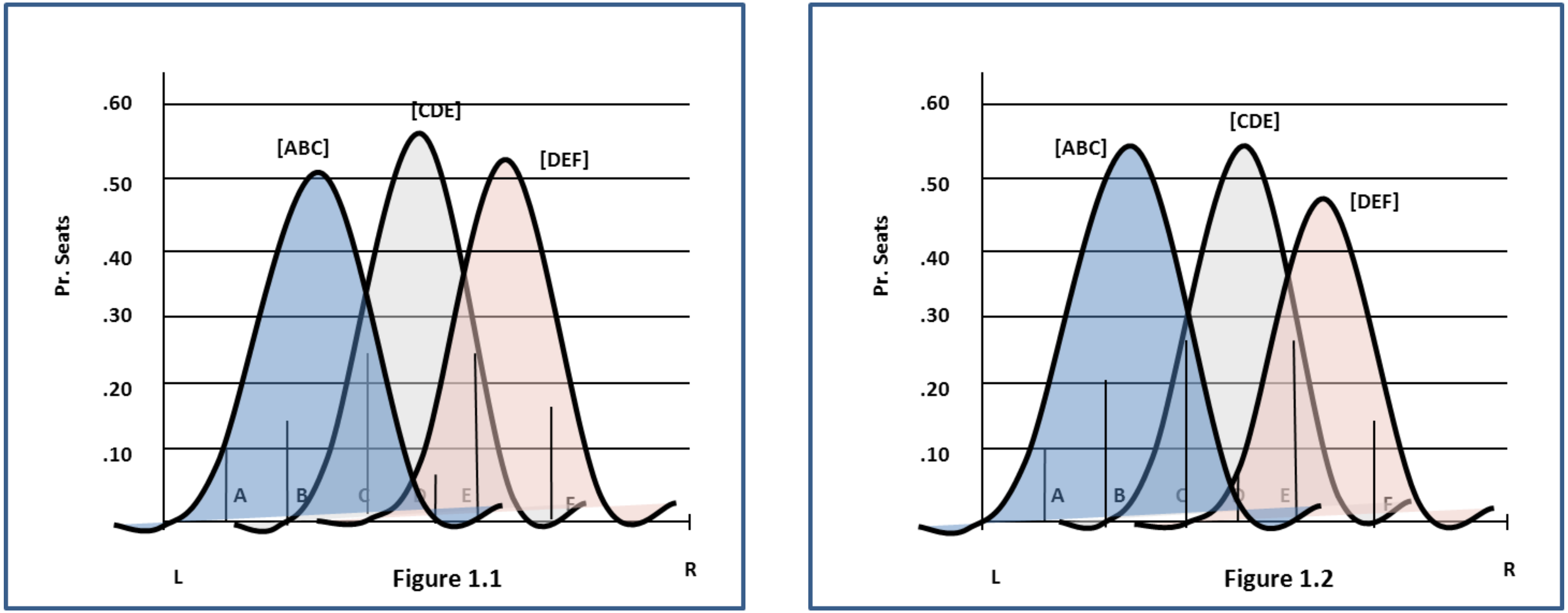
Nodes' sizes reflect their attributes, and their color reflects their centrality (with more central nodes getting a darker color). The clique breakdown of these nodes is given in Table A1 below. This table shows very clearly the discrepancy between attributes and power in networks. Nodes 2 and 3 are the most central nodes in the network, but also the weakest in terms of their attributes. However, node 2 is pivotal in all three groups in which it is a member and node 3 is pivotal in two of the three groups in which it is a member. Thus, node #2, the second weakest node in terms of its attributes becomes the most powerful node in the network, and node #3, the weakest node becomes the second most powerful node in the network. (Incidentally, the same relationship exists between attribute size and spoiling power.)

Table A1: Calculation of Network Power for Node #3 in the network of Figure A1

	Group Power				**Complementary Group**				g_{i-3}				$g_{\sim i+3}$		
	I	II	III		I	II	III		I	II	III		I	II	III
1	0	0.35	0.35	**1**	0.35	0	0	**1**	0	0.35	0.35	**1**	0.35	0	0
2	0.09	0.09	0.09	**2**	0	0	0	**2**	0.09	0.09	0.09	**2**	0	0	0
3	0.07	0.07	0.07	**3**	0	0	0	**3**	0	0	0	**3**	0.07	0.07	0.07
4	0	0	0.11	**4**	0.11	0.11	0	**4**	0	0	0.11	**4**	0.11	0.11	0
5	0.24	0	0	**5**	0	0.24	0.24	**5**	0.24	0	0	**5**	0	0.24	0.24
6	0.14	0.14	0	**6**	0	0	0.14	**6**	0.14	0.14	0	**6**	0	0	0.14
GA	0.54	0.65	0.62	**GA**	0.46	0.35	0.38	**GA**	0.47	0.58	0.55	**GA**	0.53	0.42	0.45
Coh	0.472	0.500	0.472	**Coh**	0.5	0.667	0.5	**Coh**	0.444	0.444	0.333	**Coh**	0.444	0.444	0.444
GP	0.255	0.325	0.293	**GP**	0.23	0.233	0.19	**GP**	0.209	0.258	0.182	**GP**	0.236	0.187	0.2
Status of Coalition	**W**	**W**	**W**						**L**	**W**	**L**				

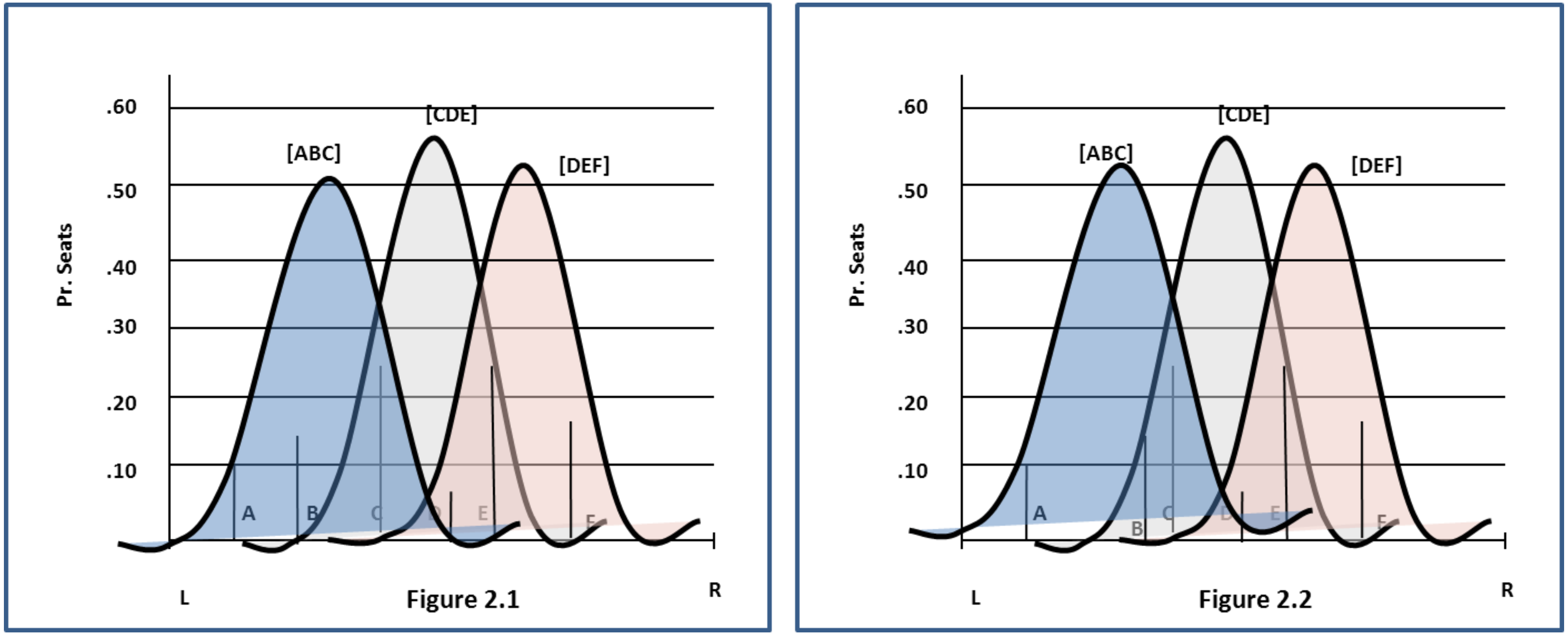
Node	**Attribute**	**NP**	**Norm. NP**	**SP**	**Norm. SP**	**Attribute Rank**	**NP Rank**	**SP Rank**
1	0.35	0.667	0.182	0.292	0.219	1	3	3
2	0.09	1.000	0.273	0.433	0.326	5	1	1
3	0.07	0.667	0.182	0.300	0.225	6	3	2
4	0.11	0.333	0.091	0.00	0.00	4	5.5	6
5	0.24	0.333	0.091	0.056	0.042	2	5.5	5
6	0.14	0.667	0.182	0.250	0.188	3	3	4

Figure 1: Axiom 1—Attribute Monotonicity



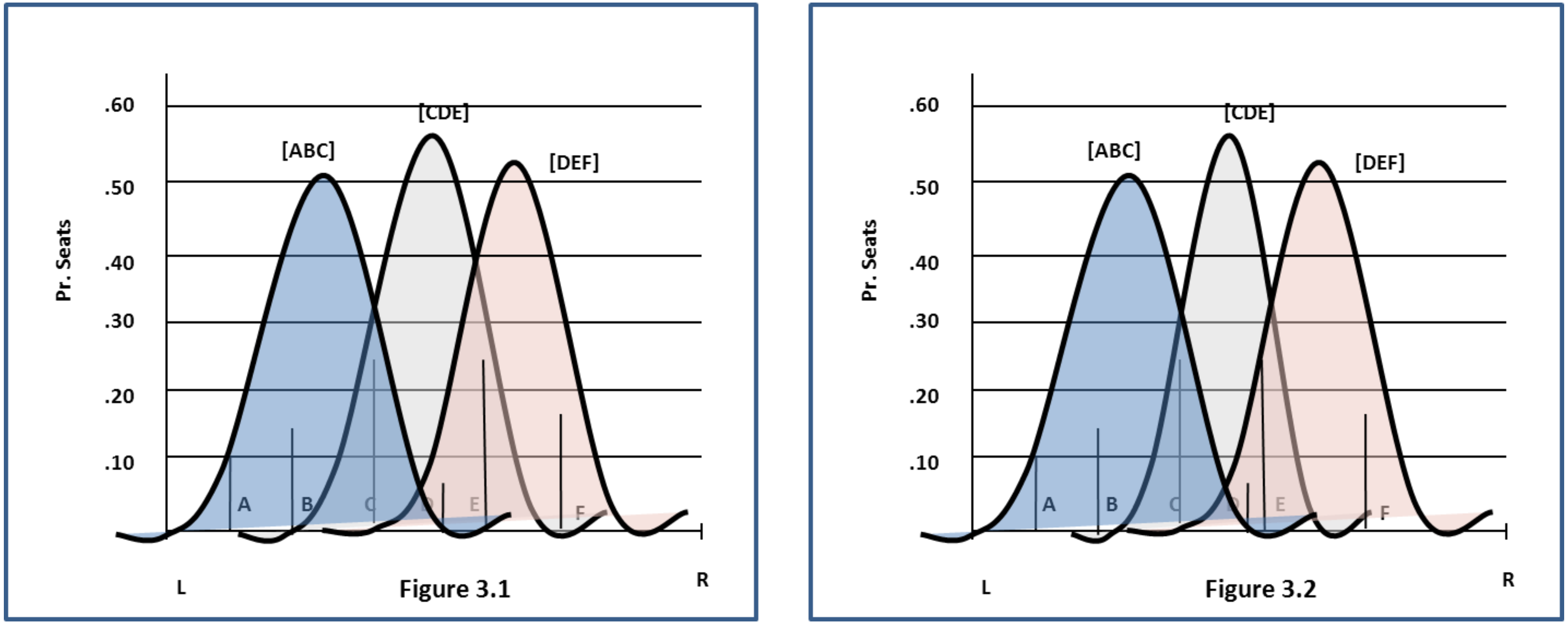
Explanation: This is a political system of six parties (**A-F**) located along a left-right continuum. Groups must be winning. There exist three proto-groups of 50% or larger ([**ABC**], **[CDE**], and [**DEF**]. In Figure 1.2, Party **B** increases its seat share from 0.15 to 0.2 while other parties (**E** and **F**) declined proportionately. The groups did not change in composition or cohesion. The axiom says that under these conditions, **B'**s network power cannot decline.

Figure 2: Affiliation monotonicity



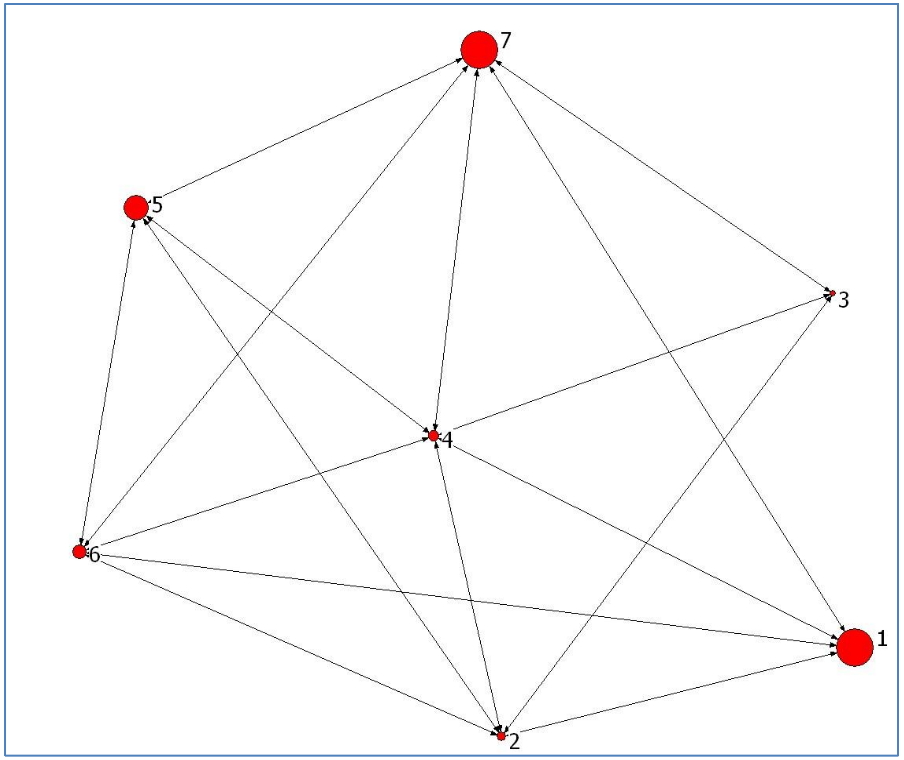
Explanation: We have the same group structure as in Figure 1.1. In Figure 2.2, party **B** shifts to the right. This makes it a member of both groups [**ABC**] and [**BCDE**]. The sizes of the parties and the coalitional structure remain the same. The axiom claims that **B'**s power cannot decline by this shift.

Figure 3: Cohesion Monotonicity

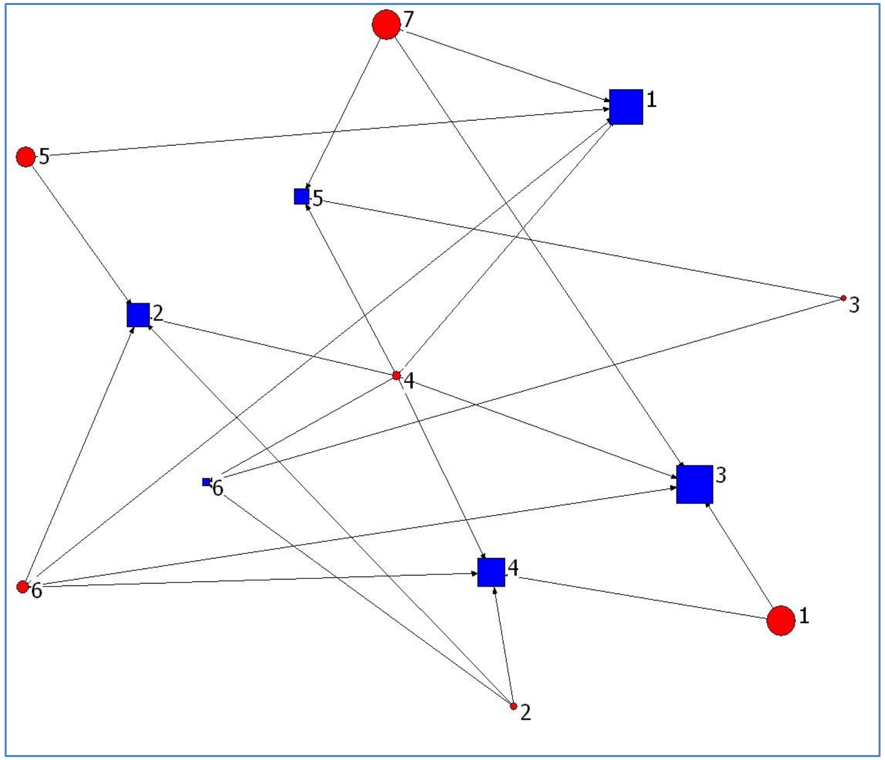


Explanation: In Figure 3.2, party **A** moves to the right, while all other parties remain in the same place. This makes the [**ABC**] group in Figure 3.2 more cohesive than it had been in 3.1. The axiom requires that none of the parties in group **[ABC]** lose power.

Figure 4: A Simple Network



4.1 Original Network



4.2 Clique Affiliations

Note: In both figures: Circles are nodes; circle size reflects nodal attribute. Larger circles represent nodes with sizable attributes.
In Figure 4.2 squares represent cliques with sizes reflecting clique power.

Table 1: Network Power Index for Nodes #1 and #7 (Figure 4)

1.1.a Basic Matrix	1	2	3	4	5	6	7
1	0	1	0	1	0	1	1
2	1	0	1	1	1	1	0
3	0	1	0	1	0	0	1
4	1	1	1	0	1	1	1
5	0	1	0	1	0	1	1
6	1	1	0	1	1	0	1
7	1	0	1	1	1	1	0

1.1.b Attribute	
1	0.26
2	0.07
3	0.05
4	0.08
5	0.18
6	0.1
7	0.26

1.1.c Clique Affiliation	1	2	3	4	5	6
1	0	0	1	1	0	0
2	0	1	0	1	0	1
3	0	0	0	0	1	1
4	1	1	1	1	1	1
5	1	1	0	0	0	0
6	1	1	1	1	0	0
7	1	0	1	0	1	0
Size	0.62	0.43	0.7	0.51	0.39	0.2
COH	0.48	0.48	0.48	0.48	0.33	0.33
GP	0.3	0.2	0.33	0.24	0.13	0.07
WS	W	L	W	W	L	L

1.1.d Complements	1	2	3	4	5	6
1	0.26	0.26	0	0	0.26	0.26
2	0.07	0	0.07	0	0.07	0
3	0.05	0.05	0.05	0.05	0	0
4	0	0	0	0	0	0
5	0	0	0.18	0.18	0.18	0.18
6	0	0	0	0	0.1	0.1
7	0	0.26	0	0.26	0	0.26
Size	0.38	0.57	0.3	0.49	0.61	0.8
COH	0.43	0.43	0.43	0.43	0.5	0.52
GP	0.16	0.24	0.13	0.21	0.31	0.42

1.1.e CA/wo #1	3	4
1	0	0
2	0	0.07
3	0	0
4	0.08	0.08
5	0	0
6	0.1	0.1
7	0.26	0
Size	0.44	0.25
COH	0.52	0.52
GP	0.23	0.13
WS	L	L

1.1.f Comp. w #1	3	4
1	0.26	0.26
2	0.07	0
3	0.05	0.05
4	0	0
5	0.18	0.18
6	0	0
7	0	0.26
Size	0.56	0.75
COH	0.57	0.9
GP	0.32	0.68

1.2.c Cliques	1	2	3	4	5	6
1	0	0	0.2	0.26	0	0
2	0	0.0	0	0.07	0	0.0
3	0	0	0	0	0.0	0.0
4	0.08	0.0	0.0	0.08	0.0	0.0
5	0.18	0.1	0	0	0	0
6	0.1	0.1	0.1	0.1	0	0
7	0.26	0	0.2	0	0.2	0
Size	**0.62**	**0.4**	**0.7**	**0.51**	**0.3**	**0.2**
COH	**0.48**	**0.4**	**0.4**	**0.48**	**0.3**	**0.3**
GP	**0.30**	**0.21**	**0.3**	**0.24**	**0.13**	**0.0**
	W	**L**	**W**	**W**	**L**	**L**

1.2.d Complementary Cliques	1	2	3	4	5	6
1	0.2	0.26	0	0	0.26	0.2
2	0.0	0	0.07	0	0.07	0
3	0.0	0.05	0.05	0.05	0	0
4	0	0	0	0	0	0
5	0	0	0.18	0.18	0.18	0.1
6	0	0	0	0	0.1	0.1
7	0	0.26	0	0.26	0	0.2
Size	**0.3**	**0.57**	**0.3**	**0.49**	**0.61**	**0.8**
COH	**0.4**	**0.43**	**0.43**	**0.43**	**0.52**	**0.5**
GP	**0.16**	**0.24**	**0.13**	**0.21**	**0.32**	**0.4**

1.2.e CA Without #7	1	3
1	0	0
2	0.07	0.07
3	0	0
4	0.08	0.08
5	0	0
6	0.1	0
7	0	0
Size	**0.25**	**0.15**
CO	**0.71**	**0.57**
GP	**0.18**	**0.09**
	L	**L**

Comp. Cliques + #7	1	3
1	0.26	0.26
2	0	0
3	0.05	0.05
4	0	0
5	0.18	0.18
6	0	0.1
7	0.26	0.26
Size	**0.75**	**0.85**
COH	**0.57**	**0.59**
GP	**0.43**	**0.5**

Table 2: Power Indices for Baseline Network (Figure 4.1)

Node	Attribute	Network Power (NP)	Normalized Network Power (NNP)	Abstention-based Power NP(A)	Abstention-based normalized Power NNP(A)
1	0.26	0.67	0.20	0.50	0.27
2	0.07	0.33	0.10	0.00	0.00
3	0.05	0.00	0.00	0.00	0.00
4	0.08	0.67	0.20	0.50	0.27
5	0.18	0.33	0.10	0.00	0.00
6	0.10	0.67	0.20	0.33	0.18
7	0.26	0.67	0.20	0.50	0.27

Table 3: Proof of Proposition #2

	3.1.1 Network Matrix 1	2	3	4	5	6	3.1.2 Attribute
1	0	1	0	1	0	1	0.35
2	1	0	1	1	1	1	0.09
3	0	1	0	1	0	0	0.07
4	1	1	1	0	1	1	0.11
5	0	1	0	1	0	1	0.24
6	1	1	0	1	1	0	0.14

	3.1.3 Cliques 1	2	3	3.1.4 Comp. Groups 1	2	3	3.1.5 Cliques w/o Node #5 1	2	3	3.1.6 Comp Groups w. Node #5 1	2	3
1	0	1	0	0.4	0	0.4	0	0.4	0	0	0.4	0
2	1	1	1	0	0	0	0.1	0.1	0.1	0.1	0.1	0.1
3	0	0	1	0.1	0.1	0	0	0	0.1	0	0	0.1
4	1	1	1	0	0	0	0.1	0.1	0.1	0.1	0.1	0.1
5	1	0	0	0	0.2	0.2	0	0	0	0.2	0.2	0.2
6	1	1	0	0	0	0.1	0.1	0.1	0	0.1	0.1	0
Size	0.6	0.7	0.3	0.4	0.3	0.7	0.3	0.7	0.3	0.6	0.9	0.5
COH	0.5	0.5	0.3	0.8	0.7	0.7	0.8	0.5	0.3	0.5	0.5	0.4
GP	0.3	0.3	0.1	0.4	0.2	0.5	0.3	0.3	0.1	0.3	0.5	0.2
Win?	**L**	**W**	**L**				**L**	**L**	**L**			

Node #7 added: Attributes of other nodes modified

	3.2.1 Network Matrix 1	2	3	4	5	6	7	3.2.2 Attribute
1	0	1	0	1	0	1	1	0.26
2	1	0	1	1	1	1	0	0.07
3	0	1	0	1	0	0	1	0.05
4	1	1	1	0	1	1	1	0.08
5	0	1	0	1	0	1	1	0.18
6	1	1	0	1	1	0	1	0.12
7	1	0	1	1	1	1	0	0.24

	3.2.3 Cliques 1	2	3	4	5	6	3.2.4 Comp. Groups 1	2	3.2.5 Cliques w/o Node #5 1	2	3.2.6. Comp Groups w. Node #5 1	2
1	0	0	1	1	0	0	0.26	0.26	0	0	0.26	0.26
2	0	1	0	1	0	1	0.07	0	0	0.07	0.07	0
3	0	0	0	0	1	1	0.05	0.05	0	0	0.05	0.05
4	1	1	1	1	1	1	0	0	0.08	0.08	0	0
5	1	1	0	0	0	0	0	0	0	0	0.18	0.18
6	1	1	1	1	0	0	0	0	0.12	0.12	0	0
7	1	0	1	0	1	0	0	0.24	0.24	0	0	0.24
Size	0.62	0.45	0.7	0.53	0.37	0.2	0.38	0.55	0.44	0.27	0.56	0.73
COH	0.48	0.48	0.48	0.48	0.33	0.33	0.43	0.43	0.57	0.71	0.57	0.57
GP	0.3	0.21	0.33	0.25	0.12	0.07	0.16	0.24	0.30	0.21	0.32	0.42
Win?	**W**	**L**	**W**	**W**	**L**	**L**			**L**	**L**		

Table 4: Correlations between alliance and trade network power indices

No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Alliance															
1	Network Power 1	1.000														
2	Normalized NP1	0.801	1.000													
3	Abstention-Based NP	0.830	0.753	1.000												
4	Normalized ABNP	0.801	1.000	0.753	1.000											
5	Defense/Offense InDegree	0.140	0.113	0.104	0.113	1.000										
6	Defense/Offense Betweenness	0.155	0.140	0.156	0.140	0.257	1.000									
7	Defense/Offense Eigenvector	0.041	0.074	0.083	0.074	0.419	0.377	1.000								
8	National Capability	**0.433**	**0.490**	**0.407**	**0.490**	0.061	0.301	0.164	1.000							
	Trade															
9	Network Power 1	**0.331**	**0.384**	**0.236**	**0.384**	0.165	0.332	0.098	0.633	1.000						
10	Normalized NP1	**0.393**	**0.430**	**0.302**	**0.430**	0.150	0.327	0.094	0.658	0.866	1.000					
11	Abstention-Based NP	**0.227**	**0.285**	**0.190**	**0.285**	0.081	0.283	0.130	0.621	0.827	0.761	1.000				
12	Normalized ABNP	**0.393**	**0.430**	**0.302**	**0.430**	0.150	0.327	0.094	0.658	0.866	1.000	0.761	1.000			
13	Trade Degree Centrality	0.337	0.262	0.319	0.262	0.065	0.281	0.124	0.422	0.212	0.276	0.178	0.276	1.000		
14	Trade Betweenness Centrality	0.109	0.154	0.180	0.154	0.008	0.350	0.256	0.426	0.170	0.181	0.311	0.181	0.428	1.000	
15	Trade Eigenvector Centrality	0.078	0.120	0.020	0.120	0.020	0.092	0.133	0.243	0.272	0.213	0.280	0.213	0.107	0.155	1.000
16	Share of system GDP	0.413	0.444	0.340	0.444	0.100	0.364	0.169	**0.863**	**0.767**	**0.822**	**0.738**	**0.822**	0.411	0.282	0.282

Notes: [1] Boldface entries are not statistically significant at $p < .05$.
Ns vary between 9,317 and 13,011.

Table 5: The Effect of Network Power on Dispute and War Outcomes, 1816-2001: Multinomial Logit Analysis

	Dispute Outcomes			
	No War		**War**	
	Baseline	**NP**	**Baseline**	**NP**
	Draw vs. Lose			
Alliance Normalized NP State A		**3.786****		**18.351****
		(1.171)		**(4.187)**
Alliance Normalized NP State B		**1.184**		**8.368****
		(0.748)		**(2.027)**
Trade Normalized NP State A		**0.827+**		**-2.094**
		(0.464)		**(2.32)**
Trade Normalized NP State B		**0.384**		**-3.318***
		(0.259)		**(1.355)**
Capability Ratio A/B	0.001		-0.005	
	(0)		(0.005)	
Democracy State A	0.261	0.495**	1.264**	1.585**
	(0.144)	(0.145)	(0.438)	(0.476)
Democracy State B	-0.37**	-0.21+	-0.51	-0.128
	(0.122)	(0.114)	(0.338)	(0.331)
Alliance Indegree Cent. State A	5.181**		3.084*	
	(0.827)		(1.476)	
Alliance Indegree Cent. State B	2.197**		-1.723+	
	(0.569)		(0.906)	
Trade Indegree Cent. State A	1.109**		1.317	
	(0.291)		(0.975)	
Trade Indegree Cent. State B	0.936**		0.989	
	(0.272)		(0.918)	
Reputational Status State A	-0.404**	-0.2**	-0.527*	-0.7**
	(0.085)	(0.075)	(0.247)	(0.219)
Reputational Status State B	-0.7**	-0.548**	-0.557**	-0.614**
	(0.079)	(0.067)	(0.215)	(0.206)
No. Initiators	-0.24**	-0.237**	-0.208**	-0.277**
	(0.016)	(0.015)	(0.062)	(0.069)
No. Targets	-0.144**	-0.127**	0.057*	0.074**
	(0.008)	(0.007)	(0.027)	(0.025)
Constant	2.836**	3.492**	0.07	0.958**
	(0.097)	(0.106)	(0.289)	(0.308)

	Dispute Outcomes			
	No War		War	
	Baseline	NP	Baseline	NP
	Win vs. Lose			
Alliance Normalized NP State A		**2.589***		**10.476****
		(1.314)		**(3.925)**
Alliance Normalized NP State B		**-2.482+**		**-8.5****
		(1.279)		**(3.287)**
Trade Normalized NP State A		**0.506**		**1.428**
		(0.514)		**(2.37)**
Trade Normalized NP State B		**-0.37**		**-0.372**
		(0.504)		**(1.78)**
Capability Ratio A/B	0.001		-0.004	
	(0)		(0.004)	
Democracy State A	0.552**	0.661**	1.664**	1.739**
	(0.169)	(0.162)	(0.383)	(0.424)
Democracy State B	-0.773**	-0.794**	-1.754**	-1.946**
	(0.17)	(0.165)	(0.382)	(0.435)
Alliance Indegree Cent. State A	4.567**		4.172**	
	(0.973)		(1.131)	
Alliance Indegree Cent. State B	2.914**		-3.665**	
	(0.85)		(1.029)	
Trade Indegree Cent. State A	0.202		0.459	
	(0.349)		(0.877)	
Trade Indegree Cent. State B	-0.063		-0.191	
	(0.349)		(0.881)	
Reputational Status State A	0.3**	0.346**	-0.048	-0.09
	(0.106)	(0.091)	(0.209)	(0.19)
Reputational Status State B	-0.297**	-0.331**	0.002	0.032
	(0.106)	(0.09)	(0.209)	(0.187)
No. Initiators	0.002	-0.008	0.001	-0.011
	(0.011)	(0.01)	(0.017)	(0.018)
No. Targets	0.002	0.003	0	-0.014
	(0.006)	(0.006)	(0.027)	(0.029)
Constant	-0.005	0.277*	0.016	-1.526**
	(0.126)	(0.134)	(0.268)	(0.319)
Model Statistics				
N	5,686	5,666	455	453
Chi-Square	830.45	887.24	116.81	139.32
Pseudo R-Squared	0.189	0.171	0.211	0.260

Notes: Numbers in parentheses are robust standard errors.

$^{+}$ *p < .10*   * *p < .05;*   ** *p < .01;*

Table 6: The Effects of Network Power on Peaceful Influence: UNGA Voting Success, 1945-2002

	Baseline	**Network Power**
Alliance Normalized NP		0.152*
		(0.065)
Trade Normalized NP		0.055
		(0.08)
Share of System's GDP	0.547*	
	(0.232)	
Democracy	0.005	0.005
	(0.007)	(0.007)
Alliance Indegree Centrality	-0.022	
	(0.053)	
Trade Outdegree Centrality	0.149**	
	(0.037)	
Regional Power?	-0.019	-0.020
	(0.026)	(0.026)
Major Power	-0.058	-0.044
	(0.054)	(0.053)
Lagged Pr. Resolutions Won	0.721**	0.725**
	(0.008)	(0.008)
Constant	0.170**	0.192**
	(0.009)	(0.006)
Model Statistics		
N	6,519	6,519
No. of States	165	165
F Statistic	1,146.61	1,331.55
Adjusted R-Squared	0.719	0.726

Notes: Numbers in parentheses are robust standard errors.

$^{+}p < .10$ $\quad$ $*\,p < .05$; $\quad$ $**\,p < .01$;

Works Cited

Banzhaf, John F. 1965. Weighted Voting Doesn't Work: A Mathematical Analysis. *Rutgers Law Review* 19 (2):317-43.

Barbieri, Katherine, Omar M.G. Keshk, and Brian Pollins. 2009. Trading Data: Evaluating our Assumptions and Coding Rules. *Conflict Management and Peace Science* 26 (5):471-91.

Barnett, Michael , and Raymond Duvall. 2005. Power in International Politics. *International Organization* 59 (1):39-75.

Bonacich, Phillip B. 1987. Power and Centrality: A Family of Measures. *American Journal of Soicology* 92 (4):1170-82.

Bonacich, Phillip B., and Paulette Lloyd. 2001. Eigenvector-Like Measures of Centrality for Asymmetric Relations. *Social Networks* 23 (2):191-201.

Correlates of War. *The New COW War Data, 1816-2007* 2008 [cited September 30, 2010. Available from http://www.correlatesofwar.org/.

Dubey, Pardeep , and Lloyd S. Shapley. 1979. Mathematical Properties of the Banzhaf Power Index. *Mathematics of Operations Research* 4 (2):99-131.

Felsenthal, Dan S., and Moshe Machover. 1998. *The Measurement of Voting Power.* Cheltenham UK: Edward Elgar.

Gleditsch, Kristian S. 2002. Expanded Trade and GDP Data. *Journal of Conflict Resolution* 46 (5):712-24.

Hafner-Burton, Emilie, and Alexander Montgomery. 2011. War, Trade, and Envy: Why Trade Agreements Don't Always Keep the Peace. *Conflict Management and Peace Science*.

Hart, Jeffrey. 1976. Three Approaches to the Measurement of Power in International Relations. *International Organization* 30 (2):299-305.

Hubbell, Charles H. 1965. An Input-Output Approach to Clique Identification. *Sociometry* 28 (4):377-99.

Jackson, Matthew O. 2008. *Social and Economic Networks.* Princeton: Princeton University Press.

Karrer, Brian, Elizaveta Levina, and Mark J. Newman. 2008. Robustness of Community Structure in Networks. *Physical Review E* 77 (046119):1-9.

Kim, Hyung Min. 2010. Comparing Measures of National Power. *International Political Science Review* 31 (4):405-27.

Lake, David A. 1992. Powerful Pacifists: Democratic States and War. *American Political Science Review* 86 (1):24-37.

Leeds, Brett Ashley *Alliance Treaty Obligations and Provisions (ATOP) Codebook*. Rice University 2005 [cited. Available from http://atop.rice.edu/home.

Leicht, Elizabeth A., and Mark E. J. Newman. 2008. Community Structure in Directed Networks. *Physical Review Letters* 100 (11):118703, 1-5.

Maddison, Angus 2008. *Statistics on World Population, GDP, and Per Capita GDP, 1-2006 AD* [cited November 14, 2010. Available from http://www.ggdc.net/maddison/.

Maoz, Zeev. 1982. *Paths to Conflict: Interstate Dispute Initiation, 1816-1976*. Boulder, CO: Westview Press.

———. 1983. Resolve, Capabilities, and the Outcomes of Interstate Disputes, 1816-1976. *Journal of Conflict Resolution* 27 (2):195-229.

———. 1989. Power, Capabilities, and Paradoxical Conflict Outcomes. *World Politics* 41 (2):239-66.

———. 1990a. *National Choices and International Processes.* Cambridge: Cambridge University Press.

———. 1990b. *Paradoxes of War: On the Art of National Self-Entrapment.* Boston: Unwin Hymann.

———. 2005. *The Dyadic Militarized Interstate Dispute Dataset, Version 2.0* [Available from http://psfaculty.ucdavis.edu/zmaoz/dyadmid.html.]

———. 2009. The Effects of Strategic and Economic Interdependence on International Conflict Across Levels of Analysis. *American Jounral of Political Science* 53 (1):223-40.

———. 2010. *Networks of Nations: The Evolution, Structure, and Impact of International Networks, 1816-2001*. New York: Cambridge University Press.

Marshall, Monty G., and Keith Jaggers. *Polity IV Project.* Center for International Development and Conflict Management, University of Maryland 2004 [cited. Available from http://www.cidcm.umd.edu/inscr/polity/.

Oneal, John, and Bruce Russett. 2005. Rule of Three, Let it Be: When More Really Is Better. *Conflict Management and Peace Science* 22 (4):293-310.

Organski, A.F.K, and Jacek Kugler. 1980. *The War Ledger.* Chicago: University of Chicago Press.

Rossman, Gabriel, Nicole Esparza, and Phillip Bonacich. 2010. I'd Like to Thank the Academy: Team Spillovers, and Network Centrality. *American Sociological Review* 75 (1):31-51.

Shapley, Lloyd S., and Martin Shubik. 1954. A Method for Evaluating the Distribution of Power in a Committee System. *American Political Science Review* 48 (3):787-92.

Taylor, Michael. 1969. Influence Structures. *Sociometry* 32 (4):490-502.

Voeten, Erik. *UN General Assembly Roll Call Dataset* 2004 [cited August 22, 2008. Available from http://www9.georgetown.edu/faculty/ev42/UNVoting.htm.

Wasserman, Stanley and Katherine Faust. 1997. *Social Networks Analysis: Methods and Applications* Second ed. New York: Cambridge University Press.