



F = fixed frame
M = moving frame

$$a_1 = 71 \text{ cm}$$

$$a_2 = 50 \text{ cm}$$

Since this model has 2 joints:

$$T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & a_1 \cos \theta_1 \\ \sin(\theta_1) & \cos(\theta_1) & a_1 \sin \theta_1 \\ 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & a_2 \sin \theta_2 \\ 0 & 0 & 1 \end{bmatrix}$$

ccw → femur femur → fibia

with:

$$x = a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2)$$

$$y = a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2)$$

answer: $x = 71 \cos 40 + 50 \cos(40 + 30)$
 $= 54.3$

$y = 71 \sin 40 + 50 \sin(40 + 30)$
 $= 45.6$

$$T_1 = \begin{bmatrix} \cos 40 & -\sin 40 & 54.3 \\ \sin 40 & \cos 40 & 45.6 \\ 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \cos 30 & -\sin 30 & 43.3 \\ \sin 30 & \cos 30 & 25 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 \times T_2 = \begin{bmatrix} 0.76 & -0.64 & 54.3 \\ 0.64 & 0.76 & 45.6 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.86 & -0.5 & 43.3 \\ 0.5 & 0.8 & 25 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.76 \times 0.86 & 0.64 \times -0.5 & 54.3 \times 43.3 \\ -0.64 \times 0.5 & 0.76 \times 0.8 & 45.6 \times 25 \\ 54.3 \times 0 & 45.6 \times 0 & 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.34 & -0.9 & 71.4 \\ 0.93 & 0.34 & 92.6 \\ 0 & 0 & 1 \end{bmatrix}$$

→ translation matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 71.4 \\ 92.6 \end{bmatrix}$$

rotation matrix

$$R = \begin{bmatrix} \cos 70 & -\sin 70 \\ \sin 70 & \cos 70 \end{bmatrix}$$