Q1 Find all prime implicants

a) $f(a,b,c,d) = \sum_m (0,4,5,8,12,13)$

1) Build the corresponding truth table:

$n=4$

	a	b	c	d	f
0	0	0	0	0	1
4	0	1	0	0	1
5	0	1	0	1	1
8	1	0	0	0	1
12	1	1	0	0	1
13	1	1	0	1	1

2) Group minterms according to # "1" in each. There will be at least n+1 G.

G_i groups

	a	b	c	d	
G_0	0	0	0	0	(0)
G_1	0	1	0	0	(4)
	1	0	0	0	(8)
G_2	0	1	0	1	(5)
	1	1	0	0	(12)
G_3	1	1	0	1	(13)

3) We compare the Hamming distance of every minterm of a group G_i with minterms of G_{i+1}.
If Hamming distance is 1 $\rightarrow$ We group in G'

	a	b	c	d		
G_0'	0	-	0	0	(0,4)	X
	-	0	0	0	(0,8)	X
G_1'	0	1	0	-	(4,5)	X
	-	1	0	0	(4,12)	X
	1	-	0	0	(8,12)	X
G_2'	-	1	0	1	(5,13)	X
	1	1	0	-	(12,13)	X

4) Repeat for G''

	a	b	c	d		
G_0''	-	-	0	0	(0,4 | 8,12)	Same
	-	-	0	0	(0,8 | 4,12)	
G_1''	-	1	0	-	(4,5 | 12,13)	Same
	-	1	0	-	(4,12 | 5,13)	

Since the same, we only choose one in each group.

IP1 : $\bar{c}\bar{d}$

IP2 : $b\bar{c}$

b) $f(a,b,c,d) = \sum_m (2,3,4,10,12,13) + \sum_d (11,14,15)$

In this case we make an hypothesis that all don't cares are "1".

	a	b	c	d
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

	a	b	c	d		
G_1	0	0	1	0	(2)	x
	0	1	0	0	(4)	x
G_2	0	0	1	1	(3)	x
	1	0	1	0	(10)	x
	1	1	0	0	(12)	x
G_3	1	0	1	1	(11)	x
	1	1	0	1	(13)	x
	1	1	1	0	(14)	x
G_4	1	1	1	1	(15)	x

	a	b	c	d		
G_1'	0	0	1	-	(2,3)	x
	-	0	1	0	(2,10)	x
	-	1	0	0	(4,12)	Never Used so: IP
G_2'	-	0	1	1	(3,11)	x
	1	0	1	-	(10,11)	x
	1	-	1	0	(10,14)	
	1	1	0	-	(12,13)	
	1	1	-	0	(12,14)	
G_3'	1	-	1	1	(11,15)	x
	1	1	-	1	(13,15)	x
	1	1	1	-	(14,15)	x

	a	b	c	d		
G_1''	-	0	1	-	(2,3 | 10,11)	IP1 : $\bar{b}c$
	-	0	1	-	(2,10 | 3,11)	
G_2''	1	-	1	-	(10,14 | 11,15)	IP2 : ac
	1	-	1	-	(11,10 | 14,15)	
	1	1	-	-	(12,14 | 13,15)	IP3 : ab
	1	1	-	-	(12,13 | 14,15)	

IP4 : -100 $(12,14)$: $b\overline{cd}$

$$f(a,b,c,d) = \bar{b}c + ac + ab + b\overline{cd}$$

Q 2 Find the simplified function.

b) $f(a,b,c,d) = \sum_m (2,3,4,10,12,13) + \sum_d (11,14,15)$

1) Build a Table that shows relationships between minterms and prime implicants: for each IP we indicate all covered minterms.

PI1 = $\bar{b}c$
PI2 = ac
PI3 = ab
PI4 = $b\overline{cd}$

a b c d	X_{10}	PI1	PI2	PI3	PI4
0 0 1 0	2	X			
0 0 1 1	3	X			
0 1 0 0	4				X
1 0 1 0	10	X	x		
1 0 1 1	11	X	x		
1 1 0 0	12			X	x
1 1 0 1	13			X	
1 1 1 0	14		x	X	
1 1 1 1	15		x	X	

2) Look at essential PI: the ones that are the only to cover a minterm.

3) We have to cover all PI.

Nous pourrons selectionner directement ceux qui couvrent le plus, soit passer au step 4)

4) Formalise the choice of PIs using minimal cover equation.

if i_1, i_2 are in different lines : $i_1 \cdot i_2$

if i_2, i_2 are in the same line : $i_1 + i_2$

Different line   Same

$$1 = i_1 \cdot i_4 \cdot (i_1 + i_2) \cdot (i_3 + i_4) \cdot i_3 \cdot (i_2 + i_3)$$

5) Simplification

$$1 = i_1 \cdot i_4 \cdot \cancel{(i_1 + i_2)} \cdot \cancel{(i_3 + i_4)} \cdot i_3 \cdot \cancel{(i_2 + i_3)}$$

$$1 = \underbrace{i_1 \cdot i_4 \cdot i_3}_{} + \underbrace{\dots}_{} + \underbrace{\dots}_{}$$

Each product of term is a possible solution

Here we have only one : $i_1 i_4 i_3$

6) Finding expressions : We just have to rewrite the product of i_i as a OR of i_i

So : $i_1 i_4 i_3 = PI_1 + PI_4 + PI_3 = \bar{b}c + b\overline{cd} + ab$

$$f(a,b,c,d) = \bar{b}c + b\overline{cd} + ab$$

Q3) K-maps, optimize functions, avoid glitches

a) $f(a,b,c,d) = \sum_m (0, 1, 2, 6, 8, 9, 10, 14)$

	a	b	c	d
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
6	0	1	1	0
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
14	1	1	1	0

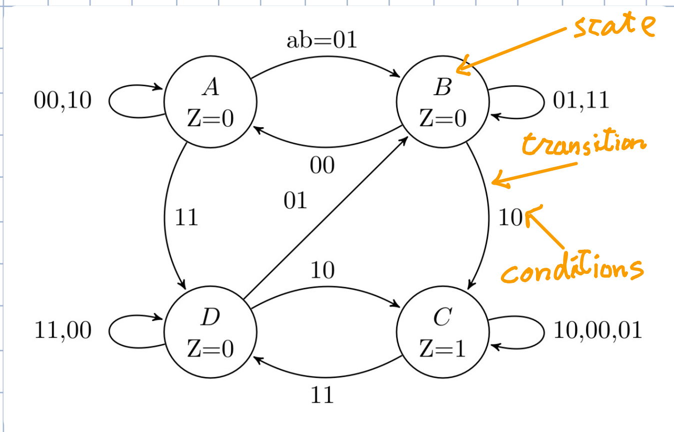
ab \ cd	00	01	11	10
00	1	1		1
01				1
11				1
10	1	1		1

$$F = \bar{b}\bar{c} + \bar{b}\bar{d} + c\bar{d}$$

Redundant term that avoid glitches because "a part is included both in ● and ●"

Rule : Add a redundant term to maintain the output to 1 during the transition.

Q 4) Huffman table from state graph



All 2^n possibilities of n. inputs var: a,b

→ output

States

	00	01	11	10	Z
A	A	B	D	A	0
B	A	B	B	C	0
C	C	C	D	C	1
D	D	B	D	C	0

States like that are STABLES STATES (system remains stable as long as input remains unchanged)

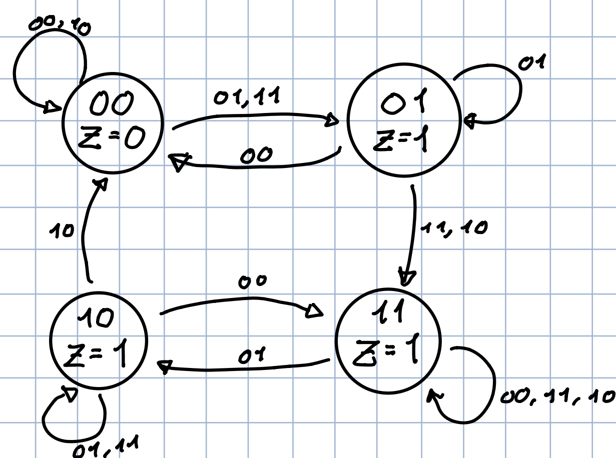
Q 5 Huffman table to state graph & equations

Y_1 Y_2	00	01	11	10	ab	Z
00	**00**	01	01	**00**		0
01	00	**01**	11	11		1
11	**11**	10	**11**	**11**		1
10	11	**10**	**10**	00		1

y_1 y_2

State x "goes to state 11" with the condition 00.

Interpretation



Conversion To K-map: Make sure to have a Huffman Table

* Build the K-table from Huffmann Table hiding all $K_{\neq i}$

DO IT: for each input Y_i and output Z_i

Y_1 (* second bit of Y_2 is hidden)

y_1y_2 \ ab	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	1	0

Y_2

y_1y_2 \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	1	0	1	1
10	1	0	0	0

Z

Value of Z in stable states

y_1y_2 \ ab	00	01	11	10
00	0	-	-	0
01	-	1	1	1
11	1	1	1	1
10	1	1	1	-

→ States That can be stable

$$Y_1 = y_2a + y_1b + y_1\bar{a}$$

$$Y_2 = \bar{y}_1b + y_2a + y_1\overline{ab}$$

$$Z = y_1 + y_2$$