General Form of x in base b

$n+1$ digits — integer part
m digits — decimal part

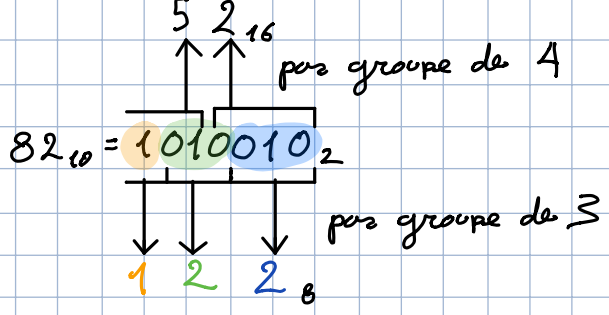
$$x = \underbrace{\sum_{i=0}^{i=n} a_i \cdot b^i}_{\text{integer part}} + \underbrace{\sum_{j=1}^{j=m} b_j \cdot b^{-j}}_{\text{decimal part}}$$

$i = n$: MSB
$i = 0$: LSB

Relevant bases : 2, 8, 10, 16

Q1 a) 82_{10}

82 : 2	0
41 : 2	1
20 : 2	0
10 : 2	0
5 : 2	1
2 : 2	0
1 : 2	1
0	



c) $100\,1110\,0001_2$ → $2^9 + 2^6 + 2^5 + 2^4 + 2^0 = 512 + 64 + 32 + 16 + 1 = 625_{10}$

$1161_8 = 271_{16}$

d) $F6D_{16}$ → $1111\,0110\,1101_2$ → X_{10} voir c)

f) $0,625_{10}$

0,625 ×2	1
1,250 ×2	0
0,500 ×2	1
1,000	

$0,101_2$

Q2 $3633_{10} + 254_{10}$

111000110001_2 $\quad$ 11111110_2

7061_8 $\quad$ 376_8

$E31_{16}$ $\quad$ FE_{16}

Base 10	Base 2	Base 8	Base 16
3633 +	111000110001 +	7061 +	E31 +
254	11111110	376	FE
3887_{10}	111100101111_2	7457_8	$F2F_{16}$

Negative numbers

Number in binary form.

Sign Bit : 0 ≡ +, 1 ≡ -

Signed Magnitude - SM

$X \longrightarrow sb \,||\, X$ ← Sur m-1 bits

↳ concatenation

Problems: - There is a positive & negative zero

- Arithmetics are complex

One's Complement - C1

if X negative $\longrightarrow X'$ ← All bits are inverted

$= (b^m - 1) - X$

(base, digits)

else $\longrightarrow X$

Problems: - Two zeros : 000000...0 / 111111...1

Good : Works for any base.

Two's Complement - C2

if X negative $\longrightarrow X' + 1$

$= (b^m - 1) - X + 1$

else $\longrightarrow X$

Arithmetics rules: Perform substractions as additions.

Overflow ($-2^m < c < 2^m - 1$) if last two report bits are not the same.

for 8 bits $-2^7 < c < 2^7 - 1$

Q3

a) $(-14)_{10}$ on 8 bits

$X_2 = 0\,0\,0\,1\,1\,1\,0$

SM = 1 0 0 0 1 1 1 0

C1 = 1 1 1 1 0 0 0 1

C2 = 1 1 1 1 0 0 1 0

Boolean Algebra

Unary complement operator : $'$

Binary operators : $+ \longrightarrow$ Inclusive OR ($\oplus \longrightarrow$ XOR)

$\cdot \longrightarrow$ AND

x	x'
0	1

x	y	x + y	x ⊕ y	x · y
0	0	0	0	0
0	1	1	1	0
1	0	1	1	0
1	1	1	0	1

Axioms :

A_1 : $x + 0 = x$; $x \cdot 1 = x$ } Identity — as std. algebra —

A_2 : $x \cdot (y + z) = xy + xz$; $x + (y \cdot z) = (x + y) \cdot (x + z)$ } Distributivity (for both ·, +)

A_3 : $x + y = y + x$; $x \cdot y = y \cdot x$ } Commutativity — as std. algebra —

A_3 : $x + \bar{x} = 1$; $x \cdot \bar{x} = 0$ } Complement

Advices :

1) Try to add another part using indentity $\begin{cases} (\dots) + 0 \\ (\dots) \cdot 1 \end{cases}$

2) Rewrite this part using complement $\begin{cases} (\dots) + (x \cdot \bar{x}) \\ (\dots) \cdot (x + \bar{x}) \end{cases}$

3) Continue with distributivity/commutativity (for instance).

RMQ : Do not skip steps !!

Ex : ~~$(\bar{x}) \cdot x + (\bar{x}) \cdot x$~~ ~~$(\bar{x}) \cdot x$~~ $(\bar{x}) \cdot x + (\bar{x}) \cdot x$ ✓

$(\bar{x}) \cdot x + 0$

Q4

T1 a : $x + x = x$

$x + x = (x + x) \cdot 1$ A2b

$= (x + x) \cdot (x + \bar{x})$ A5a

$= x + x \cdot \bar{x}$ A3b

$= x + 0$ A5b

$= x$ A2a

T1b: $x \cdot x = x$

$$
\begin{aligned}
x \cdot x &= x \cdot x + 0 && A2a \\
&= (x \cdot x) + (x \cdot \bar{x}) && A5b \\
&= x \cdot (x + \bar{x}) && A3a \\
&= x \cdot 1 && A5a \\
&= x && A2b
\end{aligned}
$$

T2a: $x + 1 = 1$

$$
\begin{aligned}
x + 1 &= (x + 0) + 1 && A2a \\
&= (x + 0) + (x + \bar{x}) && A5a \\
&= x + (x + \bar{x}) && A2a \\
&= (x + x) + \bar{x} = x + \bar{x} && T1a \\
&= 1 && A5a
\end{aligned}
$$

T2b: $x \cdot 0 = 0$

$$
\begin{aligned}
x \cdot 0 &= (x + 0) \cdot 0 && A2a \\
&= x \cdot 0 + 0 && A3a \\
&= x \cdot 0 + x \cdot \bar{x} && A5b \\
&= x \cdot (0 + \bar{x}) && A3a \\
&= x \cdot \bar{x} && A2a \\
&= 0 && A5b
\end{aligned}
$$

T3a: $x + y \cdot x = x$

$$
\begin{aligned}
x + y \cdot x &= (x \cdot 1) + (y \cdot x) && A2b \\
&= (x \cdot 1) + (x \cdot y) && A4b \\
&= x \cdot (1 + y) && A3a \\
&= x \cdot 1 && T2a \\
&= x && A2b
\end{aligned}
$$

T 3 b: $x \cdot (x+y) = x$

$$
\begin{aligned}
x \cdot (x+y) &= (x+0) \cdot (x+y) && \text{A 2a} \\
&= x + y \cdot 0 && \text{A 3b} \\
&= x + 0 && \text{T 2b} \\
&= x && \text{A 2a}
\end{aligned}
$$