Truth table

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Logic expressions

Standard canonical form

SoP: Sum of Products (unique)

Sum → OR

Products → Minterms: Term in which all variables appear once

Contains only
- $\overline{x}$ operator
- AND operator

Ex: if variables x, y

minterms: $\overline{x}y$, $x\overline{y}$, xy, $\overline{xy}$

X_{10}	x	y	F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0

+ where F = 1

$F = \overline{x}\overline{y} + x\overline{y}$ can be written as → $\sum(0,2)$ → Decimal form

PoM: Product of Maxterms

Product → AND

Same as SoP, but we look where F = 0 and swap operations

X_{10}	x	y	F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0

$F = (\overline{x}+y)\cdot(x+y)$

or $\prod(1,3)$

Non-standard canonical form

(better since less resources for same functionalities)

Method 1

1) Find SoP or PoM
2) Use axioms to optimize it.

K-maps

Hamming Distance

different symbols between 2 words

Ex: 1001 and 1010

There are 2 different symbols.

H.D. = 2

n-cube

has 2^n vectors as lines in TT

Q1

$\bar{a}c + \overline{abc} = \overline{ab} + \bar{a}c$

a b c	$\bar a$	$\bar a c$	+	$\overline{abc}$	$\overline{ab}$	+	$\bar a c$
0 0 0	1	0	1	1	1	1	0
0 0 1	1	1	1	0	1	1	1
0 1 0	1	0	0	0	0	0	0
0 1 1	1	1	1	0	0	1	1
1 0 0	0	0	0	0	0	0	0
1 0 1	0	0	0	0	0	0	0
1 1 0	0	0	0	0	0	0	0
1 1 1	0	0	0	0	0	0	0

Q2

a) $(a+b)\cdot(a+\bar b) = \overset{a}{aa} + a\bar b + ab + \overset{0}{b\bar b}$

$a(1+\underbrace{\bar b + b}_{1}) = a(\overset{1}{1+1}) = a\cdot 1 = a$

b) $a + \bar a b = (a+\bar a)\cdot(a+b)$

$= 1 \cdot (a+b) = a\cdot b$

c) $\overline{ab}c + \overline{abc} + \bar a b \bar c = \overline{ab} + \bar a b \bar c$

partie commune

$= \bar a(\bar b + b\bar c) = \bar a((\overset{1}{\bar b + b})\cdot(\bar b + \bar c))$

$= \bar a(1\cdot(\bar b+\bar c)) = \bar a\cdot(\bar b + \bar c)$

d) $\overline{((a+b)c\bar d + e + \bar f)}$

e) $\bar a bc + a\overline{bc} + \overline{abc} + a\bar b c + abc = bc + \overline{bc} + a\bar b c$

$= 1 + a\bar b c$

$= a\bar b c$

f) $\overline{(ab+ac)} + \overline{ab}c = (\bar a+\bar b)\cdot(\bar a+\bar c) + \overline{ab}c$

$= \bar a\cancel{\bar a} + \overline{ab} + \overline{bc} + \overline{ac} + \overline{ab}c$

$= \bar a(1 + \underbrace{\bar b + \bar c + \bar b c}_{1 \,+\, \text{something} = 1}) + \overline{bc}$

$= \bar a + \overline{bc}$

g) $\overline{(a+b)} + \overline{(\bar a + b)} = (\bar a \bar b)\cdot(a\bar b) = \underbrace{\bar a a}_{0}\,\bar b$

$0 \cdot \bar b = 0$

h) $a + \bar a b + \bar a \bar b = a + \bar a \underbrace{(b + \bar b)}_{1} = 1$

Q3 Rewrite as minterms (product term in which each term appears once)

a) $F(a,b,c,d) = a\bar b c + \overline{ab} + ab\bar c d$

Where is d & $\bar d$? Where are $cd, \bar c d, c\bar d, \overline{cd}$?

$= a\bar b c d + a\bar b c\bar d + \overline{ab}cd + \overline{ab}\bar c d + \overline{ab}c\bar d + \overline{ab}\,\overline{cd} + ab\bar c d$

b) $F(a,b,c,d) = ab + \bar b c + cd$

$= abcd + ab\bar c d + abc\bar d + ab\bar c d + ab\overline{cd} + a\bar b cd + \overline{ab}cd + a\bar b c\bar d + \overline{ab}c\bar d + \bar a bcd$

Pour ce dernier, il a fallu vérifier si les 4 expressions possibles n'avaient pas été déjà créés dans et

Q4

a) $F(a,b) = a + \bar a b + \overline{ab}$

$b \backslash a$	0	1
0		
1		

$\equiv$

	$\bar a$	a
$\bar b$	1	1
b	1	1

Interpretations : $\forall$ valeur des variables on a 1, donc $F = 1$

b) $F(a,b) = (a+b)\cdot(a+\bar b)$

$b \backslash a$	0	1
0	0	1
1	0	1

$F = a$

c) $F(a,b) = a + \bar a b$

$b \backslash a$	0	1
0	0	1
1	1	1

$F = a + b$

Q5

a) $F(a,b,c) = \bar{a}c + \overline{abc}$

$a \backslash bc$	00	01	11	10
0	1	1	1	0
1	0	0	0	0

$F = \overline{ab} + \bar{a}c$

c) $F(a,b,c) = ab\bar{c} + \overline{abc} + \bar{a}b\bar{c} + a\overline{bc}$

$a \backslash bc$	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$F = \bar{c}$

Q6

a) $F(a,b,c,d) = abd + acd + bcd + ab + \bar{a}cd + \overline{ab}cd$

$ab \backslash cd$	00	01	11	10
00			1	
01			1	
11	1	1	1	1
10			1	

ab

cd

$F = ab + cd$

c) $F(a,b,c,d) = \overline{bcd} + \bar{a}c\bar{d} + \overline{ac}d + a\bar{d} + \bar{a}b\bar{d}$

$ab \backslash cd$	00	01	11	10
00	1	1		1
01	1	1		1
11	1			1
10	1			1

$F = \bar{d} + \overline{ac}$

Q7

a) $F(a,b,c,d,e) = a\bar{e} + b\bar{e} + a\bar{b}ce + a\bar{b}cde + a\bar{b}c\bar{e} + \overline{acde} + \overline{abe} + \overline{ab}ce$

$de \backslash abc$	0 000	1 001	3 011	2 010	4 100	5 101	7 111	6 110
00	1	1	1	1	1	1	1	1
01		1				1		
11		1				1		
10	1	1	1	1	1	1	1	1

$F = \underline{\bar{e}} + \underline{\bar{b}c}$

les rectangles verts sont superposés