

Homework 1

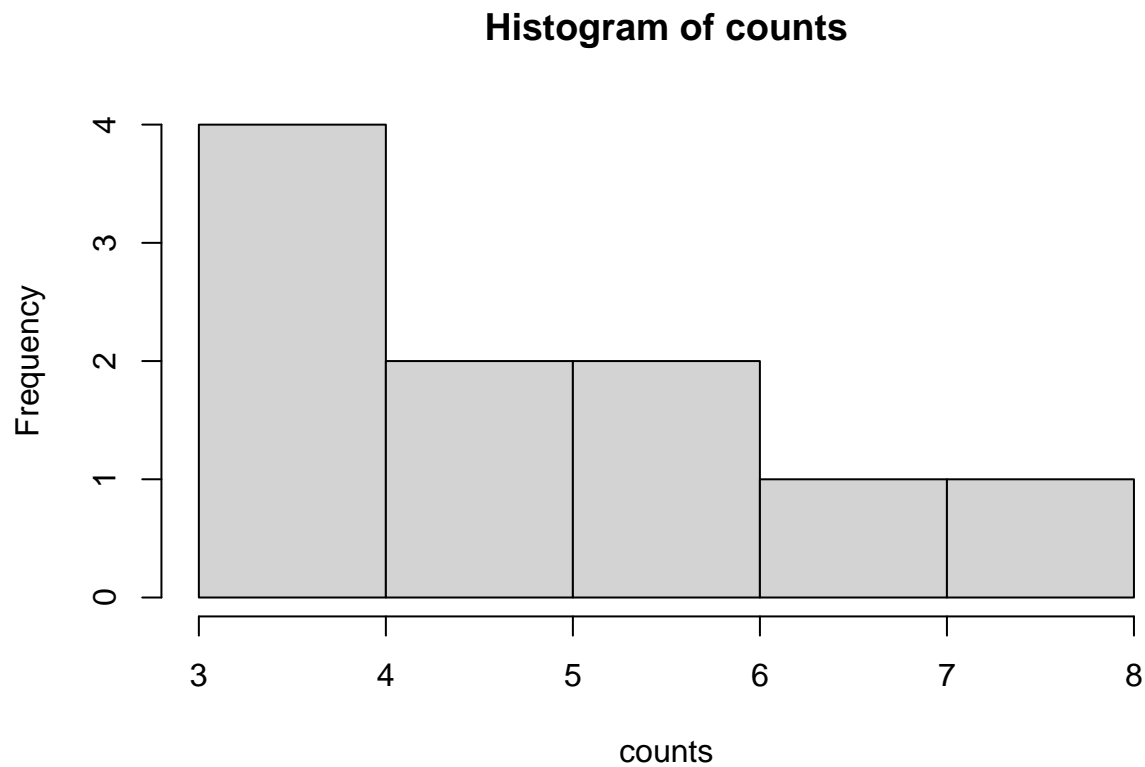
Sawyer Balint

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Contents

First, let's observe the data:

```
counts <- c(5,6,7,3,6,5,8,4,4,3)
hist(counts)
```



What is your process model?

The process model is $f(x) = \lambda$

What is your data model?

Because this is count data, it must be discrete and have a lower bound of 0. It also appears to have a higher density at lower values. Thus, it is likely a Poisson distribution:

$$x \text{ } Pois(\lambda)$$

Solve for the analytical MLE

Likelihood is equal to the Poisson distribution:

$$L = \prod Pois(x_i|\lambda)$$

which has the probability distribution function of:

$$Pois(x_i|\lambda) = \lambda^{x_i} e^{-\lambda}$$

thus likelihood is equal to:

$$L \propto \prod \lambda^{x_i} e^{-\lambda}$$

We solve for L by taking the natural log:

$$\ln L \propto \sum x_i \ln(\lambda) - \sum (\lambda)$$

Take the derivative with respect to λ :

$$\frac{d \ln L}{d \lambda} = \frac{1}{\lambda} \sum x_i - n$$

set it equal to zero:

$$0 = \frac{1}{\lambda} \sum x_i - n$$

and solve for *lambda*

$$\lambda = \frac{1}{n} \sum x_i$$

Thus, λ is equal to the sum of x over n which is the mean.

What is the estimate for this population?

```
round(mean(counts))
```

```
## [1] 5
```