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# Exclusive Semileptonic Decays of B Mesons to Orbitally and Radial Excited D

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## Abstract

In this paper we compute, within in the context of a relativistic quark model, the Isgur-Wise functions for exclusive semileptonic  $\bar{B} \rightarrow X_c$  decays, where  $X_c$  is any charmed mesons with total spin  $J = 0, 1, 2$  or one of their first excited states. The relevant matrix elements are computed by a direct numerical integration, in coordinate space, of the convolution of the wave function of the  $B$  meson at rest and the wave function of the  $X_c$  meson, boosted according with its recoil factor. Our results are compared with other predictions found in the existing literature.

## I. INTRODUCTION

There has been much progress in recent years in model-independent calculations of heavy meson decays. By using Heavy Quark Effective Theory (HQET) [1] and Lattice Gauge Theory, we can now make some definite predictions for certain processes, in a limited kinematic range. Knowledge of these decays are extremely important for particle physics, not just in their own right, but in measuring fundamental parameters such as  $V_{cb}$  and  $\sin(2\beta)$ .

Unfortunately, at present time, it is not always possible to use these techniques for all kinematic situations. One example are the Isgur-Wise (IW) functions, which relate all the different form factors for heavy-to-heavy decays to a single function, at leading order in  $\Lambda/m$ . These functions can, in principle, be determined by lattice computations but, because of limited computing resources, they have only been computed for decays into the ground state and the error is still sizable. For the moment HQET can be used to produce model independent results at zero recoil but, away from this point, the IW functions are unknown. Therefore, for the time being, we must rely on models.

We present here a study of  $\bar{B}$  decaying into excited  $D$ s and a determination of the corresponding IW functions, following the work of Ref. [2]. Our study is based on the quark model proposed in Ref. [3], where a Dirac equation was used to describe the light quark in the potential of the heavy quark and determine masses and wave functions of excited mesons. We use these wavefunctions to calculate the leading order IW functions for  $\bar{B} \rightarrow X_c$  decays, where  $X_c$  is a spin 0 – 2 charmed meson or its first radially excited state. The IW functions are computed explicitly by a three dimensional numerical integration of the relevant matrix elements expressed in terms of the wave functions derived in Ref. [3].

The paper is organized as follows. In Section II we set up the formalism and in Section III we discuss the actual calculation of the IW functions. In Section IV we discuss the results and compare with the literature. Finally in Section V we conclude.

| $j^P \equiv s_l^{\pi_l}$ | Particles  | $J^P$ | $m$ (GeV) | $m'$ (GeV) |
|--------------------------|------------|-------|-----------|------------|
| $\frac{1}{2}^-$          | $D$        | $0^-$ | 1.865     | (2.589)    |
|                          | $D^*$      | $1^-$ | 2.007     | (2.692)    |
| $\frac{1}{2}^+$          | $D_0^*$    | $0^+$ | (2.377)   | (2.949)    |
|                          | $D_1^*$    | $1^+$ | (2.490)   | (3.045)    |
| $\frac{3}{2}^+$          | $D_1$      | $1^+$ | 2.422     | (2.995)    |
|                          | $D_2^*$    | $2^+$ | 2.459     | (3.035)    |
| $\frac{3}{2}^-$          | $D_1^{**}$ | $1^-$ | (2.795)   | (3.420)    |
|                          | $D_2^{**}$ | $2^-$ | (2.833)   | (3.459)    |

TABLE I. Charmed meson spin multiplets ( $q = u, d$ ). The masses are experimental when possible, otherwise they are calculated from the model. The primed masses are for the first radial excited states. The masses in parenthesis are predictions of the in the model.

## II. THE QUARK MODEL

Due to heavy quark symmetry, we know a great deal about the spectroscopy of mesons containing a heavy quark  $Q$ . At leading order in  $\Lambda/m_Q$ , the spin and parity of the light degrees of freedom,  $s_l^{\pi_l}$ , are good quantum numbers. Thus the particles appear in multiplets labeled by both the total spin  $J$  and  $s_l^{\pi_l}$ . In this limit, the mesons come in degenerate doublets with total spin  $J_{\pm} = s_l \pm s_Q$ , where  $s_Q = 1/2$  is the heavy quark spin, a conserved quantum number at leading order in HQET. In Table I we list the low spin charmed mesons considered in this paper, their masses and corresponding quantum numbers in HQET. In those cases where the experimental mass is not known we report, in brackets, the mass predicted in Ref. [3]. For the mass of the decaying  $\bar{B}$  meson we adopt the PDG value  $m_B = 5.279$  GeV.

The model of Ref. [3] is a relativistic quark model characterized by a spin-dependent plus a spin-independent potential. The general form for these potentials has been derived by general arguments such as confinement and asymptotic freedom. The parameters that appear in the potential have been fixed by fitting the experimental spectrum of  $D$ ,  $D_s$ ,  $B$  and  $B_s$  mesons with the predicted spectrum, including  $1/m_b$  corrections [4] and mixing effects.

The most general heavy-light meson (in the  $D, D_s, B, B_s$  family),  $H$ , is a bound state of a light quark ( $q$ ) and a heavy quark ( $Q$ ). The heavy quark is treated as a static source of

chromoelectric field and the only quantum number associated with it is its spin. The light quark is treated relativistically and its state is described by the wavefunction  $\psi_{n,\ell,j,m}(r, \theta, \varphi)$ .

We introduce the following quantum numbers:

- $n$ , the number associated with the radial excitations
- $\ell$ , the orbital angular momentum
- $j$ , a short hand notation for  $s_\ell$ , the total angular momentum of the light quark
- $m$ , the component of  $j$  along the  $\hat{z}$  axis
- $J$ , the total angular momentum of the system
- $M$ , the component of  $J$  along the  $\hat{z}$  axis
- $S$ , the spin of the heavy quark along the  $\hat{z}$  axis

The parameters of the model are the masses of the light quarks ( $m_q$  for  $q = u, d$  or  $s$ ), the masses of the heavy quarks ( $m_Q$  for  $Q = c$  or  $b$ ) and the chromoelectric potential of the heavy quark ( $V(r)$ ).

The total wavefunction of the system can be decomposed as follows

$$\Psi_{n,\ell,j,J,M}(r, \theta, \varphi) = \sum_{S \in \{-\frac{1}{2}, +\frac{1}{2}\}} C_{j,m;\frac{1}{2},S}^{J,M} \psi_{n,\ell,j,m}(r, \theta, \varphi) \otimes \xi_S, \quad (1)$$

where  $C_{j,m;\frac{1}{2},S}^{J,M}$  are the usual Clebsh-Gordan coefficients and  $\xi_S$  is a two component spinor representing the heavy quark. In Eq. (1), the four spin components of the light quark wavefunction, is parametrized as follows:

$$\Psi_{n,\ell,j,J,M}(r, \theta, \varphi) = \sum_{S \in \{-\frac{1}{2}, +\frac{1}{2}\}} C_{j,m;\frac{1}{2},S}^{J,M} \begin{pmatrix} i f_{n,\ell,j}^0(r) k_{\ell,j,m}^+ Y_{m-\frac{1}{2}}^\ell(\theta, \varphi) \\ i f_{n,\ell,j}^0(r) k_{\ell,j,m}^- Y_{m+\frac{1}{2}}^\ell(\theta, \varphi) \\ f_{n,\ell,j}^1(r) k_{2j-\ell,j,m}^+ Y_{m-\frac{1}{2}}^{2j-\ell}(\theta, \varphi) \\ f_{n,\ell,j}^1(r) k_{2j-\ell,j,m}^- Y_{m+\frac{1}{2}}^{2j-\ell}(\theta, \varphi) \end{pmatrix} \otimes \xi_S. \quad (2)$$

Here  $Y_m^\ell(\theta, \varphi)$  are spherical harmonics that encode the angular dependence while  $f_{n,\ell,j}^0(r)$ ,  $f_{n,\ell,j}^1(r)$  are real functions that encode that radial dependence.  $k_{\ell,j,m}^+$  and  $k_{\ell,j,m}^-$  are fixed, up

| $\alpha_s$ | $\lambda$ | $b$   | $m_u$ | $m_s$ | $m_c$ | $M_c$ | $m_b$ | $M_b$ |
|------------|-----------|-------|-------|-------|-------|-------|-------|-------|
| 0.339      | 2.823     | 0.257 | 0.071 | 0.216 | 1.511 | 1.292 | 4.655 | 4.685 |

TABLE II. Parameters for the model. The masses and  $\lambda$  are all measured in GeV. The parameter  $b$  is measured in  $\text{GeV}^2$ .

to an overall phase, by imposing a normalization condition. Our choice of the phase is such that

$$k_{\ell,j,m}^{\pm} = \begin{cases} +\sqrt{\frac{\ell \pm m + \frac{1}{2}}{2\ell+1}} & \text{for } j = \ell + \frac{1}{2} \\ \pm\sqrt{\frac{\ell \mp m + \frac{1}{2}}{2\ell+1}} & \text{for } j = \ell - \frac{1}{2} \end{cases}. \quad (3)$$

The Hamiltonian of the most general heavy-light system, at leading order in  $1/m_b$  reads

$$\mathcal{H}^{(0)} = \gamma^0(-i\vec{\not{\partial}} + m_q) + V(r), \quad (4)$$

and the rotational-invariant potential is the sum of a constant factor ( $M_Q$ ), a scalar part ( $V_s$ ) and (the zeroth component of) a vector part ( $V_v$ )

$$V(r) = M_Q + \gamma^0 V_s(r) + V_v(r), \quad (5)$$

where

$$V_v(r) = -\frac{4}{3} \frac{\alpha_s}{r} \text{erf}(\lambda r), \quad (6)$$

$$V_s(r) = br + c. \quad (7)$$

The role of the  $\text{erf}()$  in the potential is that of regularizing the ultraviolet divergence in the  $1/m_b$  corrections to the spectrum. The parameters of the model, determined in Ref. [3], are shown in Table II.

### III. CALCULATION

The hadronic part of the semileptonic exclusive decay  $\bar{B} \rightarrow D + \ell + \bar{\nu}$  (for the most general excited D in the final state) is encoded in a matrix element of the form

$\langle D(n', \ell', j', J', M') | \Gamma | \bar{B}(n, \ell, j, J, M) \rangle$  where  $\hat{\Gamma} = \bar{u}_c(0) \Gamma u_b(0)$ . For the decays of interest  $\Gamma = \gamma^\mu$  or  $\gamma^\mu \gamma^5$  but for the purpose of this paper we consider the most general  $\Gamma$  structure. In fact, thanks to the HQET, all matrix elements that differ only for the spin structure can be related to the same IW function. The heavy-light states are normalized according with the usual non-relativistic convention.

A general formalism for the computation of matrix elements between states represented by wave functions was derived in Ref. [2]. In that paper the problem of defining equal time wave functions is discussed and the matrix elements are written as a integral in momentum space of the Fourier transformed wave functions. The general problem is greatly simplified in the specific case of heavy-light systems since it is always possible to shift a meson in time by changing the phase of the heavy quark. Hence, in this paper, we find more intuitive to express our matrix elements as integrals in coordinate space which, in the most general case of interest, look like:

$$\langle D(n', \ell', j', J', M') | \Gamma | \bar{B}(n, \ell, j, J, M) \rangle = \int \Psi_{n', \ell', j', J', M'}^{\mathbf{P}^\dagger}(\mathbf{x}) \hat{\Gamma} \Psi_{n, \ell, j, J, M}(\mathbf{x}) d^3\mathbf{x}, \quad (8)$$

where  $\Psi^{\mathbf{P}}$  is the wavefunction  $\Psi$  boosted according with the recoil momentum  $\mathbf{p}$ :

$$\Psi_{n, \ell, j, J, M}^{\mathbf{P}}(\mathbf{x}) = \sum_{S \in \{-\frac{1}{2}, +\frac{1}{2}\}} C_{j, m; \frac{1}{2}, S}^{J, M} \psi_{n, \ell, j, m}^{\mathbf{P}}(\mathbf{x}) \otimes \xi_S^{\mathbf{P}}, \quad (9)$$

and

$$\psi^{\mathbf{P}}(\mathbf{x}) = S(\Lambda_{\mathbf{p}}) \psi(\Lambda_{\mathbf{p}}^{-1} \mathbf{x}), \quad (10)$$

$$\xi_S^{\mathbf{P}} = S(\Lambda_{\mathbf{p}}) \xi_S. \quad (11)$$

where  $\Lambda_{\mathbf{p}}$  is a boost in direction  $\mathbf{p}$ .

Note for any state only that component of the spin parallel to the direction of motion  $\mathbf{p}$  is a good conserved quantum number, the helicity. Therefore in Eq. (8) we chose  $m'$ ,  $M'$  and  $S$  to be the components of the light angular momentum, the total angular momentum and the heavy quark spin respectively, parallel to the direction of the boost  $\Lambda_{\mathbf{p}}$ . We checked that, with this definition, the result for the matrix element is independent on the direction

of  $\mathbf{p}$ . In our analysis we ignore mixing in the wavefunctions and other  $O(m_b^{-1})$  corrections.<sup>1</sup> The 3D integrals are evaluated numerically using the Vegas Monte Carlo algorithm.

In order to compare matrix elements we calculate with the model to the corresponding IW functions, we need to calculate the matrix elements in HQET. This can be done by using the trace formalism [6–8]. As an example, consider the  $s_l^{\pi_l} = \frac{1}{2}^-$  doublet. The fields  $P_v$  and  $P_v^{*\mu}$  that destroy the members of this doublet with four-velocity  $v$  are grouped together in the  $4 \times 4$  matrix

$$H_v^- = \frac{1 + \not{v}}{2} [P_v^{*\mu} \gamma_\mu - P_v \gamma_5]. \quad (12)$$

The matrix  $H_v^-$  satisfies the relations  $\not{v} H_v^- = H_v^- = -H_v^- \not{v}$ . To leading order in  $\Lambda_{\text{QCD}}/m_{b,c}$  and  $\alpha_s$ , the matrix element between  $\bar{B}^{(*)}$  and  $D^{(*)}$  mesons are

$$\langle D^{(*)} | \bar{c} \Gamma b | \bar{B}^{(*)} \rangle = \xi(w) \text{Tr}[\bar{H}_{v'}^- \Gamma H_v^-]. \quad (13)$$

Here  $\xi(w \equiv v \cdot v')$  is the (dimensionless) IW function for  $\bar{B}$  decaying to the  $D^{(*)}$  multiplet. Matrix elements with any  $\Gamma$  can now easily be calculated and are related by heavy quark symmetry to  $\xi(w)$ .

For other multiplets, different  $4 \times 4$  matrices are used. The general form for arbitrary spin was developed in Ref. [8]. Below are the other matrices which are necessary to calculate relations used in this paper:

$$H_v^+ = \frac{1 + \not{v}}{2} [P_v^{*\mu} \gamma_\mu \gamma_5 - P_v], \quad (14)$$

$$F_v^{+, \mu} = \frac{1 + \not{v}}{2} \left\{ P_v^{*\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} P_v^\nu \gamma_5 \left[ g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - v^\mu) \right] \right\}, \quad (15)$$

$$F_v^{-, \mu} = \frac{1 + \not{v}}{2} \left\{ P_v^{*\mu\nu} \gamma_\nu \gamma_5 - \sqrt{\frac{3}{2}} P_v^\nu \left[ g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu + v^\mu) \right] \right\}, \quad (16)$$

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<sup>1</sup>Ref. [5] suggests that  $1/m_b$  corrections to the matrix element corresponding to  $\bar{B} \rightarrow D^{(*)}$  are smaller than expected by dimensional analysis. We do not know if this is also the case for the other matrix elements of relevance in this paper.

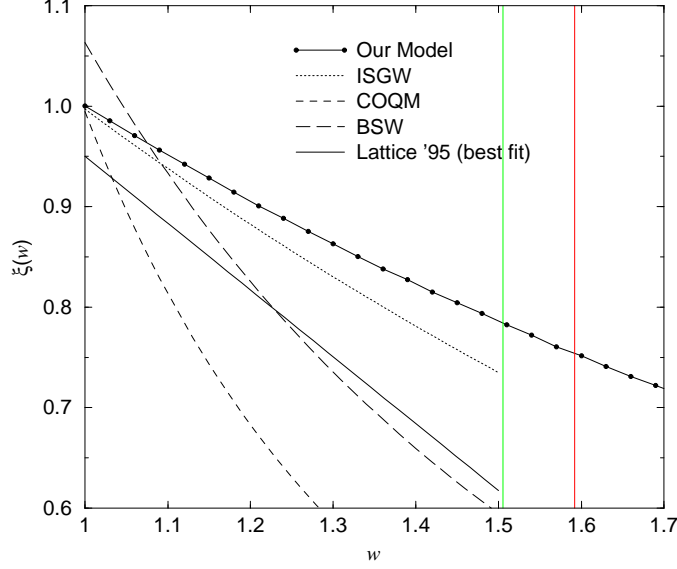


FIG. 1. IW function for the  $D - D^*$  multiplet, as predicted by different models. The vertical lines mark the end of the kinematical allowed region of the two mesons in the doublet.

where  $H_v^+$  is for the  $s_l^{\pi_l} = \frac{1}{2}^+$  doublet, and  $F_v^{\pm, \mu}$  are for the  $s_l^{\pi_l} = \frac{3}{2}^{\pm}$  doublets.

Using the trace formalism, we can relate the matrix elements calculated in the model, Eq. (8), to the IW functions. Due to heavy quark symmetry, there are many matrix elements that could be used to obtain the same IW function. By using different choices, we can check to make sure the model is giving consistent results. Also, many matrix elements are equal to zero at leading order in  $\Lambda_{\text{QCD}}/m_{b,c}$  and  $\alpha_s$ , which is another way to check the model results. In the Appendix, we show the relevant matrix elements for the different doublets and different spin structure. As an example, again consider the  $s_l^{\pi_l} = \frac{1}{2}^-$  doublet. Picking  $\Gamma = 1$  we have

$$\xi(w) = \frac{1}{1+w} \langle D | \bar{c} b | \bar{B} \rangle = \frac{1}{1+w} \int d^3 \mathbf{x} \Psi_{1,0,\frac{1}{2},0,0}^{\mathbf{P}}(\mathbf{x}) \mathbf{1} \Psi_{1,0,\frac{1}{2},0,0}(\mathbf{x}), \quad (17)$$

and similar relations can be found for other doublets.

#### IV. RESULTS



### A. The $D$ and $D^*$ multiplet

The most investigated heavy-to-heavy decays are  $\bar{B} \rightarrow D$  and  $\bar{B} \rightarrow D^*$  which corresponds to the  $s_l^{\pi_l} = \frac{1}{2}^-$  doublet and are parametrized in terms of the same IW function  $\xi(w)$ :

$$\frac{d\Gamma(\bar{B} \rightarrow D\ell\bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w-1)^{3/2} \xi^2(w), \quad (18)$$

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^*\ell\bar{\nu})}{dw} &= \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_{D^*})^2 m_{D^*}^3 \sqrt{w^2 - 1} (w+1) \\ &\times \left( 1 + \frac{4w}{w+1} \frac{m_B^2 - 2wm_B m_{D^*} + m_{D^*}^2}{(m_B + m_{D^*})^2} \right) \xi^2(w). \end{aligned} \quad (19)$$

We know from heavy quark symmetry that, ignoring perturbative corrections, the IW function for these decays,  $\xi$ , is normalized to one at zero recoil.  $|V_{cb}|$  can be determined from exclusive  $\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}$  decay channels by a direct application of Eqs. (18,19). On the experimental side it is difficult to extrapolate  $\xi(w)$  at zero recoil, since the corresponding matrix element vanishes as  $(w^2 - 1)^{1/2}$  for  $D^*$  and  $(w^2 - 1)^{3/2}$  for  $D$ . However, analyticity imposes stringent constraints [9,10].

The normalization of  $\xi$  at zero recoil is readily visible in Fig. 1, where we plot  $\xi$  as a function of  $w \equiv v \cdot v'$ . If we parameterized the shape of  $\xi$  as

$$\xi(1) \left[ 1 - \rho^2(w-1) + c(w-1)^2 + \dots \right], \quad (20)$$

we obtain for the slope parameter  $\rho^2 = 0.501$  and for the curvature  $c = 0.145$ .<sup>2</sup> In Fig. 1 the same IW function as predicted by the following alternative models:

- ISGW [12]. In this model, nonrelativistic meson wave functions are obtained using a variational approach to the Schroedinger problem and approximated with harmonic-oscillator wave functions.
- SBW [13]. In this model, the form factors are calculated assuming a pole structure.

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<sup>2</sup>Note that our result is not in agreement with Uraltsev's sum rule  $\rho^2 > 3/4$  [11].

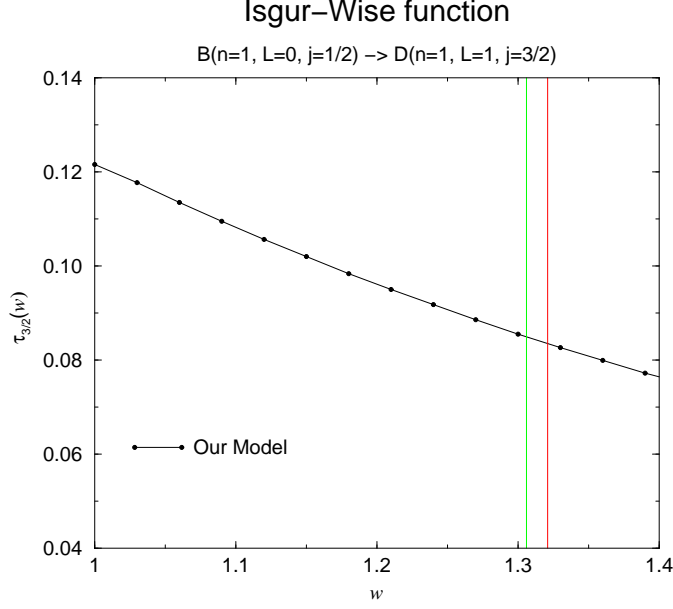


FIG. 2. IW function for the  $D_1 - D_2^*$  multiplet. The vertical lines mark the end of the kinematical allowed region of the two mesons in the doublet.

- Covariant Oscillator Quark Model or COQM [14]. This model is based on a covariant representation for nonrelativistic meson wavefunctions.

If Fig. 1 we also compare our model with the lattice prediction of Ref. [15]. The lattice result is affected by unknown systematic quenching errors and discretization errors, particularly for large momentum transferred. These errors are difficult to estimate at present and are not reported in our plot.<sup>3</sup>

We also observe that a direct lattice determinations of the slope this IW exists [16]. This computation is done with a propagating heavy quark slightly heavier than a charm meson and a light quark with a mass about the strange mass. It predicts a value of  $\rho^2 = 1.7 \pm 0.02$ , which is about a factor two larger than quark model predictions.

## B. The $D_1$ and $D_2^*$ multiplet

The narrow resonances  $D_1 - D_2^*$  with light quantum numbers  $s_l^{\pi_l} = \frac{3}{2}^+$  are important for a number of reasons. For example, it is interesting to understand the composition of the inclusive  $B$  semileptonic decay rate in terms of exclusive final states. The particles in the  $\frac{3}{2}^+$  doublet are important exclusive channels for this comparison. It is also important to know the decay spectrum for these particles as one the dominant backgrounds to  $\bar{B} \rightarrow D^{(*)}$  decays. Finally, there has been renewed effort in constraining the slope parameter  $\rho^2$  for  $\bar{B} \rightarrow D^{(*)}$  using sum rules and data on  $B$  decays to excited  $D$  states [17,11].

In Fig. 2 we plot the IW function,  $\tau_{3/2}$ , for the  $D_1 - D_2^*$  multiplet. Unlike the previous case, there is no reason why  $\tau_{3/2}$  should be normalized to one at zero recoil. If we parameterized the shape of  $\tau_{3/2}$  as  $\tau_{3/2}(w) = \tau_{3/2}(1)[1 - \rho_{3/2}^2(w - 1) + c_{3/2}(w - 1)^2 + \dots]$ , we obtain for the normalization  $\tau_{3/2}(1) = 0.122$ , for the slope parameter  $\rho_{3/2}^2 = 1.171$ , and for the curvature  $c_{3/2} = 0.601$ . In Table III we compare our result with other predictions found in the literature. Note that our value of  $\rho_{3/2}^2$  is consistent with the other models, while our  $\tau_{3/2}(1)$  is lower.

## C. The $D_0^*$ and $D_1^*$ multiplet

Only recently data have been produced on the rates to these excited mesons [26]. The  $s_l^{\pi_l} = \frac{1}{2}^+$  multiplet is very broad, as it can decay strongly to  $D^{(*)}\pi$  in an  $S$  wave [3] (the current experimental width is  $290_{-79}^{+101} \pm 26 \pm 36$  keV for the  $J = 1$  meson), while  $D_1$  and  $D_2^*$  can only decay through a  $D$  wave, thus being narrower resonances (the experimental width is  $18.9_{-3.5}^{+4.6}$  keV for the  $J = 1$  meson) [27].

In Fig. 3 we plot the IW function,  $\tau_{1/2}$ , for the  $s_l^{\pi_l} = \frac{1}{2}^+$ ,  $D_0^* - D_1^*$  multiplet. Again, there is no reason why  $\tau_{1/2}$  should be normalized to one at zero recoil. If we parameterize the shape

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<sup>3</sup>Modern computer technology allows for a noticeable improvement over these lattice results.

| Ref        | $\tau_{3/2}(1)$ | $\rho_{3/2}^2$ | $\tau_{1/2}(1)$ | $\rho_{1/2}^2$ |
|------------|-----------------|----------------|-----------------|----------------|
| Ours       | 0.12            | 1.17           | 0.094           | 0.821          |
| [19]       | 0.49            | 1.53           | 0.28            | 1.04           |
| [18]       | 0.41            | 1.5            | 0.41            | 1.0            |
| [20]       | 0.56            | 2.3            | 0.09            | 1.1            |
| [21]       | 0.66            | 1.9            | 0.41            | 1.4            |
| [22]       |                 |                | $0.35 \pm 0.08$ | $2.5 \pm 1.0$  |
| [23], [24] | 0.54            | 1.5            | 0.22            | 0.83           |
| [23], [25] | 0.52            | 1.45           | 0.06            | 0.73           |
| [12]       | 0.31            | 2.8            | 0.31            | 2.8            |

TABLE III. Comparison of IW functions  $\tau_{3/2}$  and  $\tau_{1/2}$  at zero recoil and their respective slopes  $\rho_{3/2}^2$  and  $\rho_{1/2}^2$  from different models.

of  $\tau_{1/2}$  as  $\tau_{1/2}(w) = \tau_{1/2}(1)[1 - \rho_{1/2}^2(w-1) + c_{1/2}(w-1)^2 + \dots]$ , we obtain for the normalization  $\tau_{1/2}(1) = 0.094$ , for the slope parameter  $\rho_{1/2}^2 = 0.821$ , and for the curvature  $c_{1/2} = 0.244$ . In Table III, we compare our result with other predictions found in the literature.

#### D. The $D_1^{**}$ and $D_2^{**}$ multiplet

To be complete, here we briefly mention the only other doublet containing a spin one meson, the  $s_l^{\pi_l} = \frac{3}{2}^-$ , corresponding to  $L = 2$  orbital excitations in the quark model. The states are expected to be even broader than the  $s_l^{\pi_l} = \frac{1}{2}^+$  multiplet. In Fig. 4 we show the IW function,  $\kappa$  for this doublet. If we parameterize the shape of  $\kappa(w)$  as  $\kappa(w) = \kappa(1)[1 - \rho_{3/2}^{-2}(w-1) + c_{3/2}^-(w-1)^2 + \dots]$ , we obtain for the normalization  $\kappa(1) = 0.367$ , for the slope  $\rho_{3/2}^{-2} = 0.884$ , and for the curvature  $c_{3/2}^- = 0.377$ .

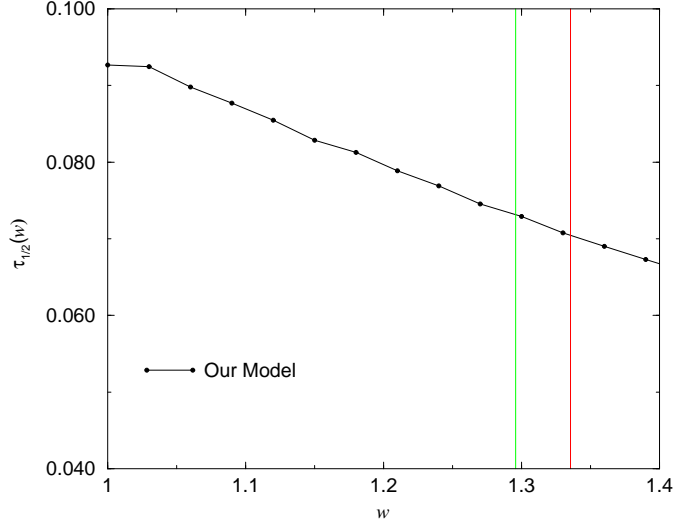


FIG. 3. IW function for the  $D_0^* - D_1^*$  multiplet. The vertical lines mark the end of the kinematical allowed region of the two mesons in the doublet.

### E. The Radial Excitations

While none have been seen, there are radial excitations of all the above mentioned doublets. It is unlikely that they will be seen in the near future, but their effects are important in reconciling the inclusive  $b \rightarrow c$  and exclusive  $\bar{B} \rightarrow X_c$  semileptonic decay rates. The radial excitations are also important as they enter into sum rule calculations. We therefore discuss the first radial excitations of the above doublets here. We denote the IW functions for the radial excitations with primed versions of the same Greek symbols as their non-radially excited counterparts.

At zero recoil, the IW function for  $\bar{B} \rightarrow D^{(*)}$  was normalized,  $\xi(1) = 1$ . For the radial excitation, there is no overlap between the  $B$  and  $D'^{(*)}$  at zero recoil, thus  $\xi'(1) = 0$ . This can be seen in Fig. 5. If we expand  $\xi'$  around  $w - 1$ , we get

$$\xi'(w) = -0.325(w - 1) + 0.213(w - 1)^2. \quad (21)$$

Our result is very different from the conclusion of Ref. [30]. They get  $\xi'(w) = 2.2(w - 1) + 2.6(w - 1)^2$ .

For the radial excitations of the other doublets  $s_l^{\pi_l} = \frac{1}{2}^+, \frac{3}{2}^\pm$  we do not have any normalization requirements at zero recoil (since the matrix elements vanish at leading order

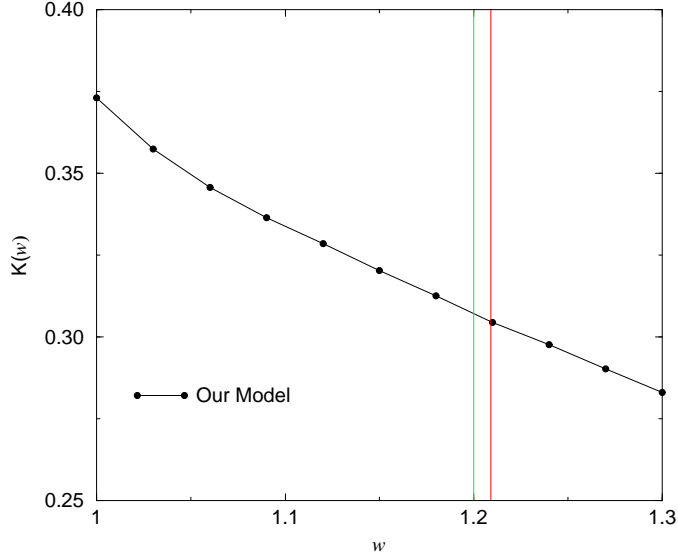


FIG. 4. IW function for the  $D_1^{**} - D_2^{**}$  multiplet. The vertical lines mark the end of the kinematical allowed region of the two mesons in the doublet.

because of the heavy quark spin symmetry). In Fig. 5 we show the IW functions for  $\xi'$ ,  $\tau'_{3/2}$ ,  $\tau'_{1/2}$  and  $\kappa'$  respectively.

## V. CONCLUSION

Information on the decays of  $B$  mesons into charmed mesons is important for a number of reasons. Using Heavy Quark Effective Theory, these decays can be written in terms of Isgur-Wise functions which parameterized the form factors. These functions cannot be calculated except, eventually, on the lattice.

In this paper we derived the Isgur-Wise functions of  $\bar{B} \rightarrow X_c$  within a quark model, where  $X_c$  can be any spin 0 – 2 charmed meson or one of their first radially excited states. Our results are compared with independent predictions found in the literature. Our model differs from the others because we derived the wavefunctions of the  $B$  meson and excited charmed mesons in a relativistic fashion by fitting the experimental spectrum with model predictions.

Our results for the Isgur-Wise function,  $\xi$ , for  $B$  to  $D^{(*)}$  decays is consistent with other model predictions but its slope is milder. Our result is also consistent with the prelimi-

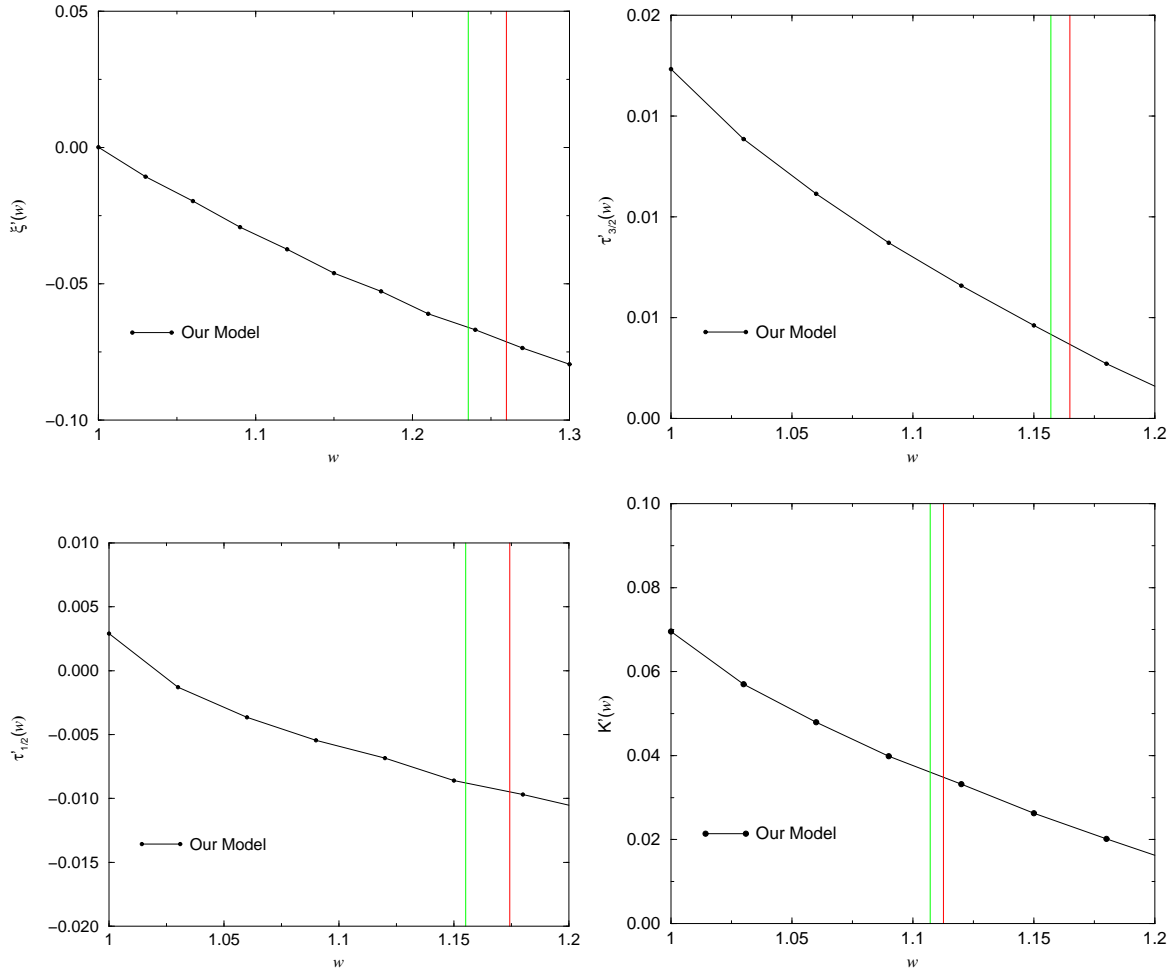


FIG. 5. IW function for the first radial excitations with spin  $J=2$ . The vertical lines mark the end of the kinematical allowed region of the two mesons in each doublet.

nary lattice results of Ref. [15]. For decays into the  $P$ -waves the situation is even more uncertain since different models predict a wide varieties of results. We believe that further model independent investigations are required in order to put more constraints important phenomenological quantities.

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## APPENDIX A: MATRIX ELEMENTS IN HQET

In this Appendix we collect the relations between the matrix elements of the form  $\langle D|\bar{c}\Gamma d|\bar{B}\rangle$  to expressions in terms of the Isgur-Wise functions in HQET at leading order in  $\Lambda_{\text{QCD}}/m_{b,c}$  and  $\alpha_s$ , in analogy with Eq. (13). The matrices containing the states for the different doublets are shown in Eqs. (12) and (14-16). The choices of Dirac structure are  $\Gamma = (1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5)$ . In this appendix  $\epsilon^\mu$  is the polarization vector for spin 1 particles and  $\epsilon^{\mu\nu}$  is the polarization tensor for spin 2 particles, while  $\epsilon^{\alpha\beta\gamma\delta}$  is the usual Minkowskian antisymmetric tensor.  $v$  is the velocity of the decaying  $B$  meson and  $v'$  is the velocity of the charmed decay product. All matrix elements not explicitly shown are zero:

$$\langle D(1/2^-) | \bar{c}b | \bar{B} \rangle = (1+w)\xi(w) \quad (\text{A1})$$

$$\langle D(1/2^-) | \bar{c}\gamma^\mu b | \bar{B} \rangle = (v+v')^\mu \xi(w) \quad (\text{A2})$$

$$\langle D^*(1/2^-) | \bar{c}\gamma^5 b | \bar{B} \rangle = -\epsilon^\mu v_\mu \xi(w) \quad (\text{A3})$$

$$\langle D^*(1/2^-) | \bar{c}\gamma^\mu b | \bar{B} \rangle = i\epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha v_\beta v'_\gamma \xi(w) \quad (\text{A4})$$

$$\langle D^*(1/2^-) | \bar{c}\gamma^\mu \gamma^5 b | \bar{B} \rangle = [\epsilon^\mu(1+w) - \epsilon^\nu v_\nu v'^\mu] \xi(w) \quad (\text{A5})$$

$$\langle D_0(1/2^+) | \bar{c}\gamma^5 b | \bar{B} \rangle = 2(w-1)\tau_{1/2}(w) \quad (\text{A6})$$

$$\langle D_0(1/2^+) | \bar{c}\gamma^\mu \gamma^5 b | \bar{B} \rangle = 2(v-v')^\mu \tau_{1/2}(w) \quad (\text{A7})$$

$$\langle D_1^*(1/2^+) | \bar{c}b | \bar{B} \rangle = -2\epsilon^\nu v_\nu \tau_{1/2}(w) \quad (\text{A8})$$

$$\langle D_1^*(1/2^+) | \bar{c}\gamma^\mu b | \bar{B} \rangle = 2[\epsilon^\mu(w-1) - \epsilon^\nu v_\nu v'^\mu] \tau_{1/2}(w) \quad (\text{A9})$$

$$\langle D_1^*(1/2^+) | \bar{c}\gamma^\mu \gamma^5 b | \bar{B} \rangle = 2i\epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha v_\beta v'_\gamma \tau_{1/2}(w) \quad (\text{A10})$$

$$\langle D_1(3/2^+) | \bar{c}b | \bar{B} \rangle = -\sqrt{2}\epsilon^\nu v_\nu(1+w)\tau_{3/2}(w) \quad (\text{A11})$$

$$\langle D_1(3/2^+) | \bar{c}\gamma^\mu b | \bar{B} \rangle = \sqrt{\frac{1}{2}}[\epsilon^\mu(1-w^2) - 3\epsilon^\nu v_\nu v'^\mu + (w-2)\epsilon^\nu v_\nu v'^\mu] \tau_{3/2}(w) \quad (\text{A12})$$



$$\langle D_1(3/2^+) | \bar{c}\gamma^\mu\gamma^5 b | \bar{B} \rangle = -\frac{i}{\sqrt{2}}(1+w)\epsilon^{\mu\alpha\beta\gamma}\epsilon_\alpha v_\beta v'_\gamma \tau_{3/2}(w) \quad (\text{A13})$$

$$\langle D_2^*(3/2^+) | \bar{c}\gamma^5 b | \bar{B} \rangle = \sqrt{3}\epsilon^{\alpha\beta}v_\alpha v_\beta \tau_{3/2}(w) \quad (\text{A14})$$

$$\langle D_2^*(3/2^+) | \bar{c}\gamma^\mu b | \bar{B} \rangle = -i\sqrt{3}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{\alpha\sigma}v^\sigma v_\beta v'_\gamma \tau_{3/2}(w) \quad (\text{A15})$$

$$\langle D_2^*(3/2^+) | \bar{c}\gamma^\mu\gamma^5 b | \bar{B} \rangle = v_\alpha[\epsilon^{\alpha\beta}v_\beta v'^\mu - \sqrt{3}(1+w)\epsilon^{\alpha\mu}]\tau_{3/2}(w) \quad (\text{A16})$$

$$\langle D_1(3/2^-) | \bar{c}\gamma^5 b | \bar{B} \rangle = \sqrt{\frac{2}{3}}(1-w)\epsilon^\nu v_\nu \kappa(w) \quad (\text{A17})$$

$$\langle D_1(3/2^-) | \bar{c}\gamma^\mu b | \bar{B} \rangle = \frac{i}{\sqrt{6}}(1-w)\epsilon^{\mu\alpha\beta\gamma}\epsilon_\alpha v_\beta v'_\gamma \kappa(w) \quad (\text{A18})$$

$$\langle D_1(3/2^-) | \bar{c}\gamma^\mu\gamma^5 b | \bar{B} \rangle = \sqrt{\frac{1}{6}}\epsilon^\nu[(2+w)v_\nu v'^\mu - 3v_\nu v^\mu + (1-w^2)g_\nu^\mu]\kappa(w) \quad (\text{A19})$$

$$\langle D_2^*(3/2^-) | \bar{c}b | \bar{B} \rangle = v_\alpha v_\beta \epsilon^{\alpha\beta} \kappa(w) \quad (\text{A20})$$

$$\langle D_2^*(3/2^-) | \bar{c}\gamma^\mu b | \bar{B} \rangle = [(1-w)\epsilon^{\alpha\mu}v_\alpha + \epsilon^{\alpha\beta}v'^\mu v_\alpha v_\beta]\kappa(w) \quad (\text{A21})$$

$$\langle D_2^*(3/2^-) | \bar{c}\gamma^\mu\gamma^5 b | \bar{B} \rangle = i\epsilon^{\mu\alpha\beta\gamma}\epsilon_{\alpha\rho}v^\rho v_\beta v'_\gamma \kappa(w) \quad (\text{A22})$$

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