Semileptonic $D \to \pi/K$ and $B \to \pi/D$ decays in 2+1 flavor lattice QCD

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We present results for form factors of semileptonic decays of D and B mesons in $2+1$ flavor lattice QCD using the MILC gauge configurations. With an improved staggered action for light quarks, we successfully reduce the systematic error from the chiral extrapolation. The results for D decays are in agreement with experimental ones. The results for B decays are preliminary. Combining our results with experimental branching ratios, we then obtain the CKM matrix elements $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{ub}|$. We also check CKM unitarity, for the first time, using only lattice QCD as the theoretical input.

1. INTRODUCTION

Semileptonic decays of B and D mesons play crucial roles in CKM phenomenology. The B decays such as $B \to \pi l\nu$ and $B \to Dl\nu$ determine $|V_{ub}|$ and $|V_{cb}|$, which are essential to constrain the CKM unitarity triangle. On the other hand, the D decays such as $D \to \pi l\nu$ and $D \to Kl\nu$ provide a good test of lattice calculations because corresponding CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ are relatively well determined. In this paper, we report lattice calculations of semileptonic decays in unquenched ($n_f = 2+1$) QCD. By using a staggered-type fermion, which is fast to simulate,

$$
\left(\begin{array}{ccc}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
 & & 3.0(4)(6)\times 10^{-3} \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
0.24(3)(2) & 0.97(10)(2) & 3.8(1)(6)\times 10^{-2} \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{array}\right)
$$

Figure 1. Result for CKM matrix. The first errors are theoretical, and the second experimental.

for light quarks, we are able to reduce uncertainties from the "chiral" ($m_l \to m_{ud}$) extrapolation. We calculate form factors for the above 4 different decays, from which the 4 CKM matrix elements are determined, as summarized in Fig. 1. The results for D decays are published in Ref. [1].

2. SIMULATION DETAILS

We use $n_f = 2+1$ dynamical gauge configurations obtained with an improved staggered ("Asqtad") quark action on a lattice with $a^{-1} \approx 1.6$ GeV, generated by the MILC collaboration [2]. For the valence light quarks we use the same staggered quark action, with the valence light quark (u,d) mass $m_l^{\rm val}$ equal to the dynamical light quark mass $m_l^{\rm sea}$. The light quark masses we simulate range $\frac{m_s}{8} \leq m_l \leq \frac{3}{4} m_s$, where m_s is the strange quark mass. For the valence charm(c) and bottom(b) quarks we use a tadpole-improved clover action with the Fermilab interpretation [3]. The hopping parameter for the $c(b)$ quark is fixed from the $D_s(B_s)$ mass.

To form the heavy-light bilinears from the staggered-type light quark and the Wilson-type heavy quark, we convert the staggered-type quark to the naive-type quark, as in Refs. [4,5]. Relevant 3-point functions are then computed in the initial state meson rest frame using local sources and local sinks. We typically accumulate about 500 configurations, and results at 2-4 source times are averaged to increase the statistics.

For the matching factor of vector current $Z_{V_\mu}^{ab}$, we follow the method in Refs. [6,7], writing $Z_{V_\mu}^{ab} = \rho_{V_\mu} (Z_V^{aa} Z_V^{bb})^{1/2}$. The flavor-conserving renormalization factors $Z_V^{aa(bb)}$ are determined nonperturbatively from charge normalization conditions. For the remaining factor ρ_{V_μ} we use results in one-loop perturbation theory [8].

3. RESULTS

3.1. $D \to \pi(K)$ and $B \to \pi$

The heavy-to-light decay amplitudes are parameterized as

$$\begin{aligned} \langle P|V^\mu|H\rangle &= f_+(q^2)(p_H + p_P - \Delta)^\mu + f_0(q^2)\Delta^\mu \\ &= \sqrt{2m_H}\left[v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E)\right] \end{aligned}$$

with $q = p_H - p_P$, $\Delta^\mu = (m_H^2 - m_P^2)\, q^\mu/q^2$, $v = p_H/m_H$, $p_\perp = p_P - Ev$ and $E = E_P$. The differential decay rate $d\Gamma/dq^2$ is proportional to $|V_{CKM}|^2|f_+(q^2)|^2$. Below we briefly describe our analysis procedure; see Ref. [1] for details.

We first extract the form factors $f_\parallel$ and $f_\perp$, as in Ref. [6], and carry out the chiral extrapolation in m_l for them at fixed E. To this end, we interpolate and extrapolate the results for $f_\parallel$ and $f_\perp$ to common values of E using the parametrization of Becirevic and Kaidalov (BK) [9]. We perform the chiral extrapolation using the NLO correction in staggered chiral perturbation theory (SχPT) [10]. We try various fit forms [1], as shown in Fig. 2, and the differences between the fits are taken as associated systematic errors.

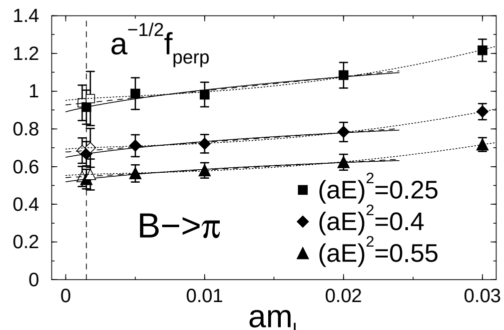

Figure 2. m_l-dependence and chiral fits for $f_\perp^{B\to\pi}$.

We then convert the results for $f_\perp$ and $f_\parallel$ at $m_l = m_{ud}$, to f_+ and f_0. To extend f_+ and f_0 to functions of q^2, we again make a fit using BK parameterization [9],

$$f_+(q^2) = \frac{f_+}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \quad f_0(q^2) = \frac{f_+}{1-\tilde{q}^2/\beta},$$

where $\tilde{q}^2 = q^2/m_{H^*}^2$. We obtain

$$f_+^{B\pi} = 0.23(2), \quad \alpha^{B\pi} = 0.63(5), \quad \beta^{B\pi} = 1.18(5),$$

for the $B \to \pi$ decay, and

$$\begin{aligned} f_+^{D\pi} &= 0.64(3), \ \ \alpha^{D\pi} = 0.44(4), \ \ \beta^{D\pi} = 1.41(6), \\ f_+^{DK} &= 0.73(3), \ \ \alpha^{DK} = 0.50(4), \ \ \beta^{DK} = 1.31(7), \end{aligned}$$

for the D decays, where the errors are statistical only. To estimate the error from BK parameterization, we also make an alternative analysis, where we perform a 2-dimensional polynomial fit in (m_l, E). A comparison between the two analyses are shown in Fig. 3.

Finally we determine the CKM matrix elements (Fig. 1) by integrating $|f_+(q^2)|^2$ over q^2 and using experimental branching ratios [11,12]. For $|V_{ub}|$ we use the branching ratio for $q^2 \geq 16$ GeV^2 in Ref. [12]. The systematic errors are summarized in Table 1. The results for D decays agree with experimental results [1].

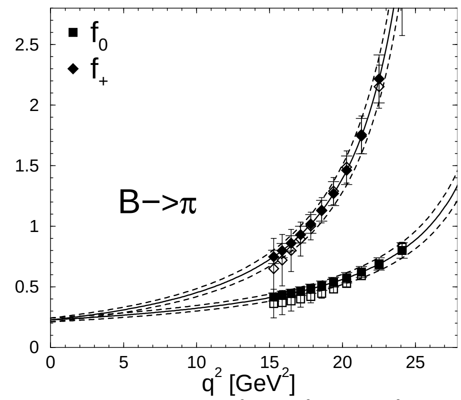

Figure 3. $B \to \pi$ form factors from BK-based (filled) and non-BK-based (open) analyses.

3.2. $B \to D$

The $B \to D$ amplitude is parameterized as

$$\langle D|V^\mu|B\rangle = \sqrt{m_B m_D} \times [h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu],$$

where $v = p_B/m_B$, $v' = p_D/m_D$ and $w = v \cdot v'$. The differential decay rate of $B \to Dl\nu$ is proportional to the square of $\mathcal{F}(w)$, which is a linear combination of $h_+(w)$ and $h_-(w)$. We calculate the form factors at $w = 1$ by employing the double ratio method [13]. The light quark mass dependence for $\mathcal{F}(1)$ is shown in Fig. 4. Extrapolating the result linearly to $m_l \to 0$, we obtain

$$\mathcal{F}^{n_f=2+1}_{B\to D}(1) = 1.074(18)(16), \tag{1}$$

where the first error is statistical, and the second is systematic summarized in Table 1. The systematic error associated with finite lattice spacing is estimated by doing quenched calculations at different lattice spacings and using different quark actions, and found to be small.

Using Eq. (1) and an experimental result for $|V_{cb}|\mathcal{F}(1)$ [14], we obtain $|V_{cb}|$ as given in Fig. 1.

Table 1
Systematic errors.

decay	$D \to \pi(K)$	$B \to \pi$	$B \to D$
3-pt function	3%	3%	1%
BK fit	2%	4%	
m_l extrap	3%(2%)	4%	1%
matching	<1%	1%	1%
a uncertainty	1%	1%	
finite a error	9%	9%	<1%
total	10%	11%	2%

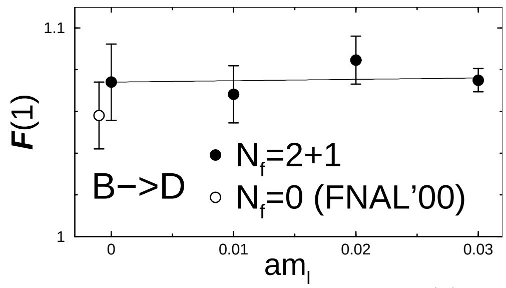

Figure 4. m_l-dependence for $\mathcal{F}_{B\to D}(1)$.

Since we have *all* 3 elements of the second row of CKM matrix, we are able to check a CKM unitarity using *only* our results as theoretical inputs; $(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(10)(2)$.

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