Leptonic decay constants f_{D_s} and f_D in three flavor lattice QCD

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We determine the leptonic decay constants f_{D_s} and f_D in three flavor unquenched lattice QCD. We use $O(a^2)$-improved staggered light quarks and $O(a)$-improved charm quarks in the Fermilab heavy quark formalism. Our preliminary results, based upon an analysis at a single lattice spacing, are $f_{D_s} = 263^{+5}_{-9} \pm 24$ MeV and $f_D = 225^{+11}_{-13} \pm 21$ MeV. In each case, the first reported error is statistical while the second is the combined systematic uncertainty.

1. INTRODUCTION

The leptonic decay constants f_{B_s} and f_B are critical in testing the flavor sector of the Standard Model. Reliable lattice calculations are of fundamental importance since the determination of these decay constants remains beyond the reach of experiment.

Precise experimental determinations of the decay constants f_{D_s} and f_D and the semileptonic decays $D \to \pi\ell\nu$ and $D \to K\ell\nu$ will result from the high-statistics charm program of CLEO-c. Comparing these experimental results with lattice results will serve both as a critical check of lattice methods for charm and as a means of assessing the reliability of lattice calculations for the bottom quark.

This work calculates the leptonic decay constants f_{D_s} and f_D using $O(a^2)$-improved staggered light quarks and $O(a)$-improved charm quarks in the Fermilab heavy quark formalism [1]. It was done with three flavors of light sea quarks. The staggered fermion action for the light quarks makes possible calculations at lighter quarks masses than previously used [2], allowing a better controlled chiral extrapolation. We use the results of staggered, partially quenched chiral perturbation theory (SχPT) in performing the chiral extrapolations.

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2. $f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$ RATIO

We use the MILC collaboration Asqtad gauge ensembles with lattice spacing $a \approx 1/8$ fm [3]. Each ensemble has one sea quark flavor of mass, m_s, approximating the strange quark and two flavors, degenerate in mass, m_u. The five ensembles we use have light flavors in the range $0.1m_s \leq m_u \leq 0.6m_s$. For each ensemble, decay constants were computed at twelve logarithmically spaced values of the valence quark mass in the range $0.1m_s \leq m_q \leq m_s$. In all, decay constants were computed for sixty partially quenched (m_q, m_u) combinations.

The chiral expansion for the decay constants is known at leading order in both the heavy quark expansion and staggered chiral perturbation theory [4]. We take the parameterization

$$\begin{aligned} R_{q/s} \quad &= \quad 1 + a_0(\Delta f_q - \Delta f_s) + (m_q - m_s) \times \\ &\quad\quad (a_1 + a_2\tilde{m} + a_3 m_q + a_4 m_s + \ldots) \quad (1) \end{aligned}$$

for the ratio $R_{q/s} = f_{D_q}\sqrt{m_{D_q}}/f_{D_s}\sqrt{m_{D_s}}$ in our chiral extrapolation. For staggered quarks, the chiral log terms Δf_x contain discretization effects from taste violations in the pseudoscalar masses as well as explicit taste violation terms. We include quark mass terms up to $O(m^2)$ with the constraint $a_4 = a_3 - a_1^2$. The sum of sea quark masses is $\tilde{m} \equiv 2m_u + m_s$.

The parameters a_j in Eq. (1) are determined in a fully covariant χ^2 minimization using all sixty partially quenched decay constant results. Pseudoscalar masses and coefficients of the explicit taste violation terms were fixed to the values determined in an analysis of the light mesons [5]. Bayesian priors were input for each a_j. With $\xi = 1/(4\pi f)^2$, the prior for a_0 is $-0.5\xi(1 + 3g^2)(1 \pm 0.30)$ with $g^2 \approx 0.35$. Other priors are 0 ± 1 in units of $2\xi s$ where s is the slope relating the quark mass and the pion mass squared.

The fit, shown in Fig. 1, has $\chi^2/\mathrm{dof} = 0.2$. The extrapolation according to SχPT is shown along the "full QCD" direction where the valence quark mass equals the sea quark mass. The parameters in this extrapolation are determined using all sixty partially-quenched points. We obtain $R_{d/s} = 0.833^{+0.042}_{-0.036}$ in the chiral limit. The error is the combined uncertainty from statistics and

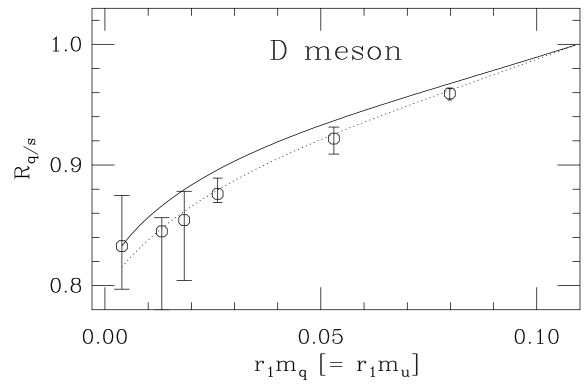

Figure 1. The chiral extrapolation of the ratio $f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$. The ratio is determined by the solid curve where staggering effects have been removed. The dashed curve includes these discretization effects.

parameter prior estimates for the extrapolation function. Systematic effects from matching the lattice theory to QCD, the lattice spacing and the tuning of charm quark mass mostly cancel in the ratio. Residual discretization effects from light quarks, not removed by the extrapolation procedure, are estimated to be 4%. Our estimate is based on the size of known taste-breaking effects shown in Fig. 1 and is similar to taste-breaking effects found in f_K and f_π [5]. The ratio was determined using the nominal strange quark mass rather than the tuned value which leads to an uncertainty of 2%.

3. $f_{D_s}\sqrt{m_{D_s}}$ DETERMINATION

We obtain $f_{D_s}\sqrt{m_{D_s}}$ by first interpolating to the tuned valence m_s value obtained from the light mesons on the same gauge ensembles. Then, a mild sea quark extrapolation (see Fig. 2) is needed to obtain f_{Ds}. We extrapolate linearly to the tuned $\hat{m} = (m_u + m_d)/2$ value obtained from the light mesons [5]. The combined statistical and extrapolation error is 3.3%. The uncertainty from the tuning of m_s and $\hat{m}$ is 1%. The tuning of the charm mass leads to a 4% error. The lattice spacing uncertainty is 2% [6]. The

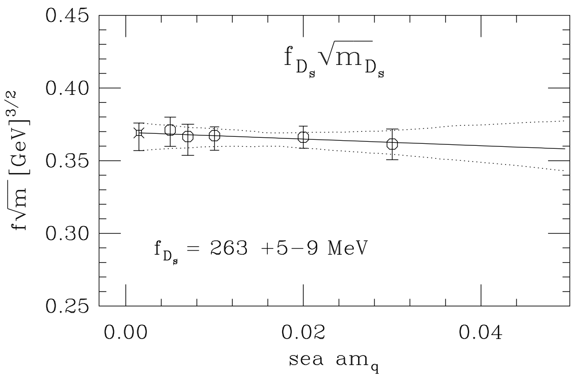

Figure 2. Extrapolation in the light sea quark mass for $f_{D_s}\sqrt{m_{D_s}}$. The curves show a linear fit (soild) and the 68% confidence level statistical error bounds (dotted).

dominant systematic uncertainty, 7%, is from the mismatch between the lattice theory and QCD, as discussed in Ref. [7]. Our final results will include an improved estimate of this uncertainty incorporating results from finer and coarser lattice spacings, which are now in progress.

4. RESULTS

Statistical and systematic uncertainties are summarized in Table 1. Our estimates of heavy quark matching effects and light quark discretization effects are based on results from a single lattice spacing. We will refine our error estimates and update our results once decay constants from additional lattice spacings are known. The heavy quark matching uncertainty can be reduced by including the higher order matchings for the action and the currents [8,9].

Combining in quadrature the systematic uncertainties shown in Table 1, we find our preliminary results:

$$\begin{aligned} \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.20 \pm 0.06 \pm 0.06 \,, \\ f_{D_s} &= 263^{+5}_{-9} \pm 24 \quad \text{MeV} \,, \\ f_D &= 225^{+11}_{-13} \pm 21 \quad \text{MeV} \,. \end{aligned}$$

Table 1
Error budget as percentage of each quantity.

source	$R_{d/s}$	$f_{D_s}\sqrt{m_{D_s}}$	$f_D\sqrt{m_D}$
stat.+extrap.	4.7	3.3	6.2
HQ matching	<1	7	7
LQ discret.	4	4	4
m_c det.	<1	4	4
val. m_s, m_d	2	1	2.2
a & sea quark	<1	2	2

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