CSC431 Assignments for Winter 2012

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General Ramarks

- These are individual assignments. You can discuss them with your colleagues but you cannot collaborate on the solution.
- Your solutions will be cross checked using turnit-in.
- Late assignments are not accepted under any condition.
- Proofs are valid only if they include steps, and explanations.
- Handwritten formulas are not allowed. You must Latex or Word equation editors. Latex preferred.
- All programs must be indented and commented
- The total is 100 points.
- You can get extra credit by helping reporting errors, bugs and typos in class notes.
- Points will be converted to letter grades based on this scale:

A	95-100
A-	92-94
B+	88-91
B	85-87
B-	82-84
C+	78-81
C	75-77
C-	72-74
D+	68-71
D	65-67
D-	62-64
F	0-55

0.1 Using and citing electronic sources

In conducting research for this course, I encourage you to consult those standard reference tools, scholarly projects and information databases, and peer-reviewed academic journals that may be found on the Internet in addition to traditional print resources. Keep in mind, however, that those electronic sources must be acknowledged. Please see the Modern Language Academy Handbook, section 4.9, for information on the correct citation of these sources.

0.2 Cheating and Plagiarism

Academic integrity entails absolute honesty in one's intellectual efforts. The DePaul Student Handbook details the facets and ramifications of academic integrity violations, but you should be especially aware of the policies on cheating and plagiarism. Cheating is any action that violates University norms or an instructor's guidelines for the preparation and submission of assignments. Such actions may include using or providing unauthorized assistance or materials on course assignments, or possessing unauthorized materials during an examination. Plagiarism involves the representation of another's work as your own, for example: (a) submitting as one's own any material that is copied from published or unpublished sources such as the Internet, print, computer files, audio disks, video programs or musical scores without proper acknowledgement that it is someone else's; (b) paraphrasing another's views, opinions or insights without proper acknowledgement or

copying of any source in whole or in part with only minor changes in wording or syntax even with acknowledgement; (c) submitting as one's own work a report, examination, paper, computer file, lab report or other assignment which has been prepared by someone else. If you are unsure about what constitutes unauthorized help on an exam or assignment, or what information requires citation and/or attribution, please ask your instructor. Violations may result in the failure of the assignment, failure of the course, and/or additional disciplinary actions.

0.3 About Computer programs

In this class, plagiarism includes submitting as your own work a computer program that was written by someone else, or directly derived from someone else. A program is directly derived from someone else's program if it is identical to someone else's program except for minor changes such as reformatting, change of variable names, etc.

1 Assignment 1: due Jan 23, 2012

1.1 Problem 1 (7 points)

Compute the 2-norm of

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \tag{1}$$

From the definition: $||A||_2 = \max_x ||Ax||_2/||x||_2$. (You can use Wolfram Alpha to solve the cubic or quartic equation which you may obtain).

1.2 Problem 2 (4 points)

What is the condition of number of $\log(A)$ where A is a matrix? Implement the $\log(A)$ function where A is a matrix. Discuss its domain of convergence.

1.3 Problem 3 (4 points)

Check is these vectors are linearly independent: [1,2,3], [1,1,1], [2,3,4]. Find an orthonormal base for the space generated by the vectors.

2 Assignment 2: due Fab 6, 2012

2.1 Problem 1 (10 points)

Download the S&P100 data for 2008. Consider only the adjusted closing prices p_{st} where s is the stock and t is the day. For each s and each t compute the aritmetic return $r_{st} = (p_{st} - p_{s0})/p_{s0}$.

Fit all the r_{st} for each s with a quadratic curve $y_{st} = a_0 + a_1 t + a_2 t^2$ and determine the coefficients a_0, a_1, a_2. For each s you will find a new set of coeffcients a_0, a_1, a_2. Now compute the average of each one of these coefficients $\bar{a}_0$, $\bar{a}_1$, $\bar{a}_2$. What would be end of year return for a hypotetical stock with $r_t = y_t = \bar{a}_0 + \bar{a}_1 t + \bar{a}_2 t^2$? Which stocks in the S&P100 behave more similarly to the fitting curve above (minimum least squares)?

Provide code that compute the answers to all the questions.

2.2 Problem 2 (5 points)

Compute the Taylor expansion of the function of two variables $f(x, y) = \sin(xy)$ around the point $(x, y) = (0, 0)$ and write a program that implements it numerically.

3 Assignment 3: due Feb 20, 2012

3.1 Problem 1 (4 points)

Consider a contract according to which, each month, for 10 years, you will be receiving $2^{1+i/100}x$ dollars where i is the month ($i = 0$ for the first month, $i = 1$ for the second month, $i = 119$ for the last month).

If $x = \$1000$ what is the Present Value of this contract? Assume an annuarl risk free interest of 3%.

If the net present value of this contract is \$200,000, how much is x? Use the stabilized Newton algorithm to solve it and provide the code to justify your answers.

Compare the numerical result with the exact "analytical" result.

3.2 Problem 2 (6 points)

Solve numerically the following equations using both the Newton stabilized and the fixed point method:

$$\sin(x) + \cos(x^2) = 0 \quad (2)$$

$$\cos(x)(1.0 - x^2) - x = 0 \quad (3)$$

$$\sin(\cos(x^2)) - x = 0 \quad (4)$$

$$(5)$$

Find the solution closer to zero. When using the Newton stabilized justify your choice of range (a,b). When using the fixed point method justify your choice of g(x).

3.3 Problem 3 (5 points)

Consider the algorithm `option_price_call_black_scholes` descibed here:

`http://finance.bi.no/~bernt/gcc_prog/recipes/recipes/node7.html`

Write a program that given `S`, `X`, `r`, `time`, and the output `c`, solves it in `sigma`. Provide an example of usage.

4 Assignment 4: due March 10, 2012 (15 points)

4.1 Problem 1 (7 points)

Implement numerically the error function:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t \tag{6}$$

using numerical integration. Use your function to implement a solution of the Black-Scholes equation for a European Call option.

4.2 Problem 2 (8 points)

Consider a the APPL stock. Consider a European call option issued on Jan 1, 2009 for 1 share with expiration on Dec 31, 2010 and strike price of $150. Price the option using your algorithm and a standrad polynomial approximation for the error function:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{x}{2n+1} \prod_{k=1}^{n} \frac{-x^2}{k} \tag{7}$$

Use the previous year data (2008) to determine the other parameters required to price the option.

Benchmark your solution. Discuss your results.

5 Final Project: due Mar 17, 2012

Rewrite all the algorithms seen in class in C++, java, C#, D, Go, Javascript, Scala, Haskell, Erlang, or Clojure, including tests (not including functions that require data downloading, assume loading data from CSV file). The algorithms should include proper documentation strings explaining the purpose of the algorithm, the input and the output, and possible exceptions. It is not necessary to explain how the algorithm works. (30 points)

You MUST use a version control system like GIT or Mercurial and make your code publicly available. Your code MUST be released under the BSD license. I will check your commit history.

By February 6, 2012 you must have chosen the language and provide a working implementation of the Linear Algebra algorithms and a link to the code repository (I recommend GitHub). (10 points)

For the final project you can work in groups of 2 or 3 students but you have to finalize your groups by February 6 and must include a report about the roles of each member of the group.