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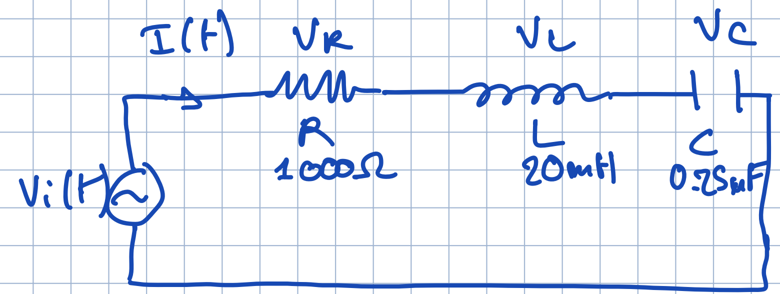
Question 1) A series RLC circuit, the resistance is 1000 ohms, the capacitor is 0.25 micro-farads and inductor is 20 millihenries, is connected to an AC voltage source that can operate between 10 Hz to 100 kHz. (8 marks total)

a) Write the differential equation for the capacitor voltage.
b) Use the Fourier transform to obtain the magnitude of frequency response of the capacitor voltage.
c) Use the Fourier transform to obtain the phase of the frequency response of the capacitor voltage.
d) Sketch the magnitude of the frequency response for the frequency range of the source.



a) $$V_i(t) = iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$C\frac{dV_c}{dt} = i(t)$$

$$\boxed{V_i(t) = RC\frac{dv(t)}{dt} + LC\frac{d^2v(t)}{dt^2} + v(t)}$$

b) Using fourier transform,

$$V_i(j\omega) = I(j\omega)R + Lj\omega \cdot I(j\omega) + \frac{1}{C}I(j\omega) \quad (i)$$

$$j\omega V_c(j\omega) = \frac{I(j\omega)}{C} \quad (ii)$$

$$V_i(j\omega) = RC \cdot j\omega V_c(j\omega) + LC\, j^2\omega^2 V_c(j\omega) + V(j\omega)$$

$$V_c(j\omega)(1 + RC\,j\omega + LC\,j^2\omega^2) = V_i(j\omega)$$

$$\frac{V_c(j\omega)}{V_i(j\omega)} = \frac{1}{1 + RC\,j\omega + LC\,j^2\omega^2}$$

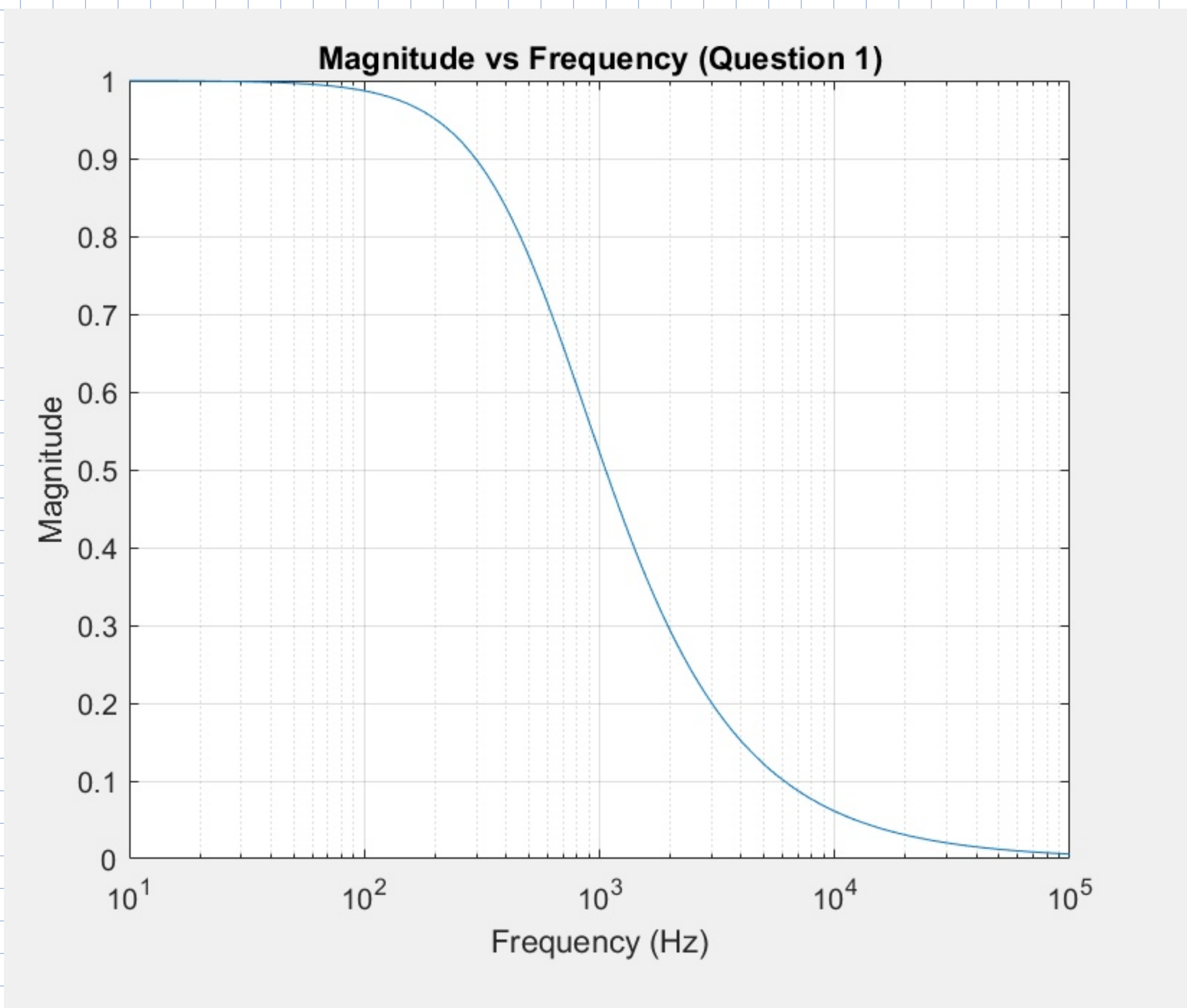
$$\left|\frac{V_c(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{(1 + LC\omega^2 + (RC\omega)^2}}$$

$$= \frac{1}{\sqrt{1 + 5\times10^{-9}\omega^2 + 250\times10^{-6}\omega}}$$

c) $$\frac{V_c(j\omega)}{V_i(j\omega)} = \frac{1}{1 + LCj^2\omega^2 + j\omega RC}$$

$$\phi = \tan^{-1}\left(\frac{-\omega RC}{1 - LC\omega^2}\right)$$

$$= \tan^{-1}\left(\frac{-250\times10^{-6}\omega}{1 - 5\times10^{9}\omega^2}\right)$$

d) 

Question 3) Fourier Transform of Signals

a) Obtain the Fourier Transform of the signal: $x(t) = e^{-a|t|}$ where "a" is a positive real number. (4 Marks)

b) Obtain the Fourier Transform of the signal: $x(t) = \delta(t) + sin(\omega_o t) + 3$. Where $\delta(t)$ is a unit impulse function. (4 Marks)

a) $$X(jw) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-jwt} dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-jwt} + \int_{0}^{\infty} e^{-at} e^{-jwt} dt$$

$$= \frac{1}{a+jw} e^{(a-jw)t} \Big|_{-\infty}^{0} + \frac{-1}{a+jw} e^{-(a+jw)t} \Big|_{0}^{\infty}$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw}$$

$$= \frac{a+jw + (a-jw)}{(a-jw)(a+jw)}$$

$$\boxed{X(jw) = \frac{2a}{a^2 + w^2}}$$

b) $$X(jw) = \int_{-\infty}^{\infty} [\delta(t) + \sin(w_o t) + 3] e^{-jwt} dt$$

$$= \int_{-\infty}^{\infty} \delta(t) e^{-jwt} dt + \int_{-\infty} \sin(w_o t) e^{-jwt} dt + \int_{-\infty}^{\infty} 3e^{-jwt} dt$$

$$\int_{-\infty}^{\infty} \delta(t) e^{-jwt} dt = \underline{1},$$

$$F.T(3) = 2\pi \delta(w) \times 3$$

$$= 6\pi \delta(w)$$

$$F.T\left(e^{j\omega_0 t}\right) = 2\pi\delta(\omega - \omega_0)$$

and $$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$F.T(\sin\omega_0 t) = \frac{1}{2j}\left[F.T\left(e^{j\omega_0 t}\right) - F.T\left(e^{-j\omega_0 t}\right)\right]$$

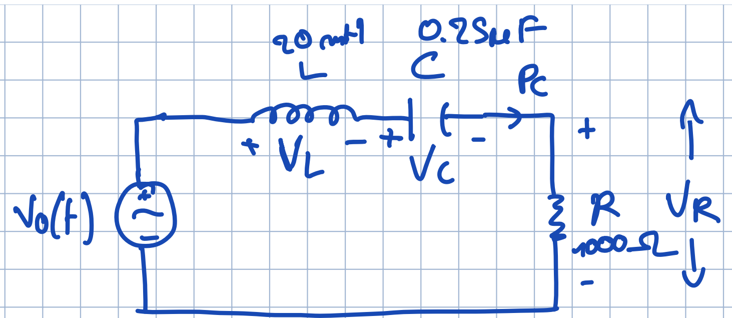
$$= \frac{1}{2j}\left[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)\right]$$

$$= \frac{\pi\delta(\omega-\omega_0) - \pi\delta(\omega+\omega_0)}{j}$$

$$\boxed{F.T(x(t)) = 1 + \frac{\pi}{j}\Big(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\Big) + 6\pi\delta(\omega)}$$

Question 2) A series parallel RLC circuit, the resistance is 1000 ohms, the capacitor is 0.25 micro-farads and inductor is 20 millihenries, is connected to an AC voltage source that can operate between 10 Hz to 100 kHz. (8 Marks total)

a) Write the differential equation for the capacitor voltage.
b) Use the Fourier transform to obtain the magnitude of frequency response of the capacitor voltage.
c) Use the Fourier transform to obtain the phase of the frequency response of the capacitor voltage.
d) Sketch the magnitude of the frequency response for the frequency range of the source.



using KVL

$$V_o(t) = V_L + V_C + V_R$$

$$V_L = L\frac{di}{dt}, \quad V_R = iR$$

$$i = i_C = C\frac{dV_C}{dt}$$

$$V_o(t) = L\frac{di}{dt} + V_c + iR$$

$$V_o(t) = LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c$$

$$\boxed{V_o(t) = 5\times10^{-9}\frac{d^2V_c}{dt^2} + 2.5\times10^{-4}\frac{dV_c}{dt} + V_c}$$

b) $$\frac{d^2x(t)}{dt^2} \Rightarrow \frac{X(\omega)}{(j\omega)^2}$$

$$V_o(\omega) = 5\times10^{-9}\frac{V_c(\omega)}{(j\omega)^2} + 2.5\times10^{-4}\frac{V_c(\omega)}{j\omega} + V_c(\omega)$$

$$V_c(\omega) = \frac{V_o(\omega)}{\frac{5\times10^{-9}}{(j\omega)^2} + \frac{2.5\times10^{-4}}{j\omega} + 1}$$

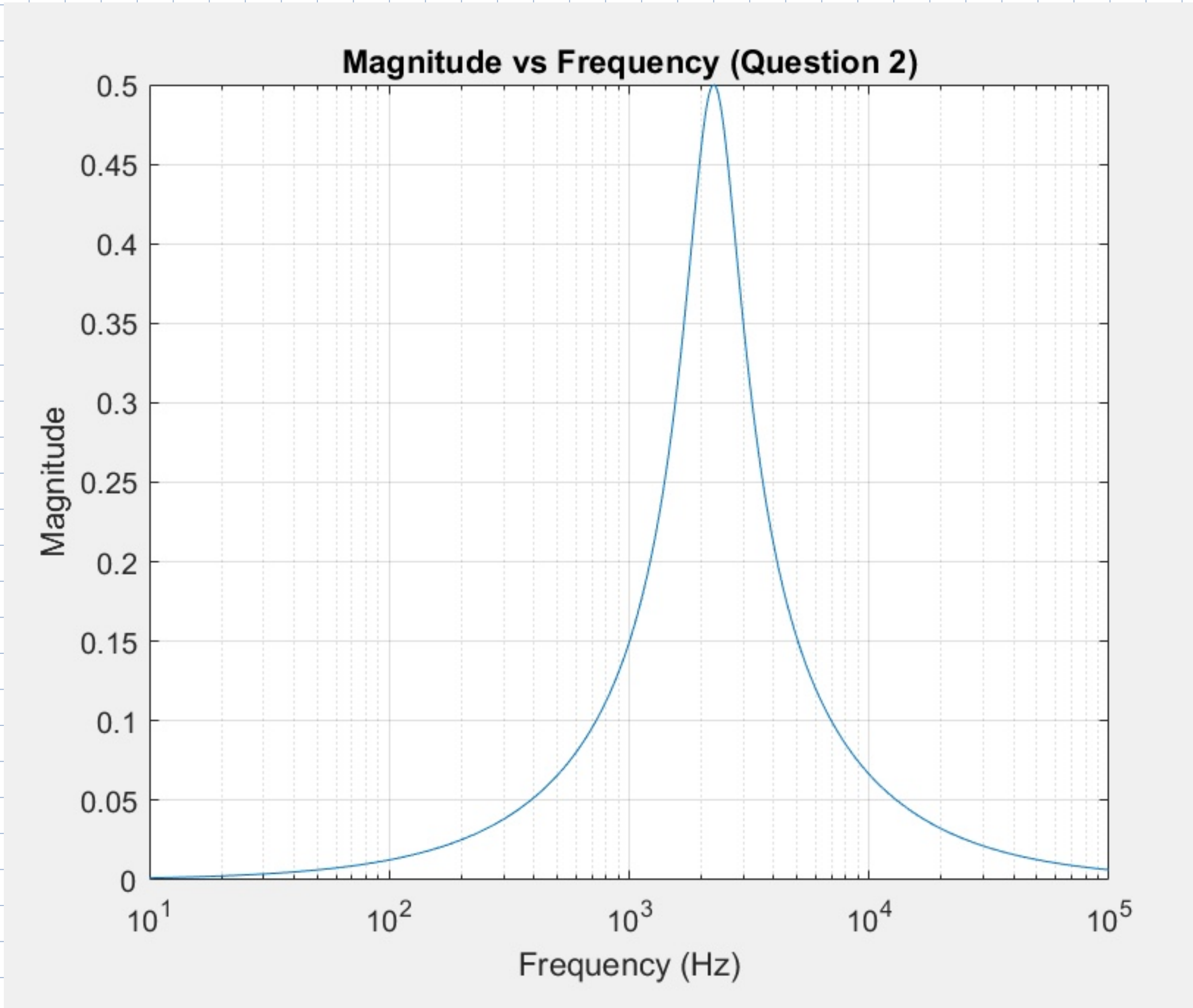
$$V_c(\omega) = \frac{V_o(\omega)\,(j\omega)^2}{5\times10^{-9} + 2.5\times10^{-4}(j\omega) + (j\omega)^2}$$

$$|V_c(\omega)| = \frac{-V_o(\omega)\,\omega^2}{\sqrt{(5\times10^{-9}-\omega^2)^2 + (2.5\times10^{-4}\omega)^2}}$$

c) $$V_c(\omega) = \frac{-\omega^2\,V_o(\omega)}{(5\times10^{-9}-\omega^2) + j(2.5\times10^{-4}\omega)}$$

$$\phi = \boxed{\tan^{-1}\left(\frac{2.5\times10^{-4}\omega}{5\times10^{-9}-\omega^2}\right)}$$

d)



Magnitude vs Frequency (Question 2)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Magnitude
10^1
10^2
10^3
10^4
10^5
Frequency (Hz)