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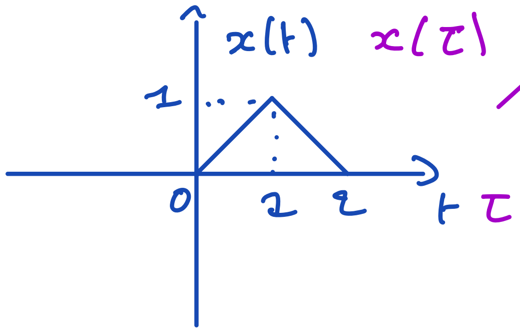
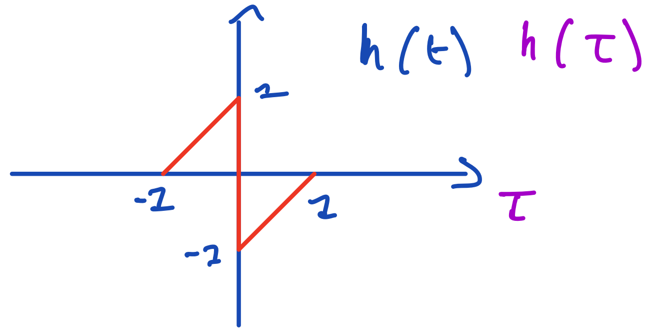
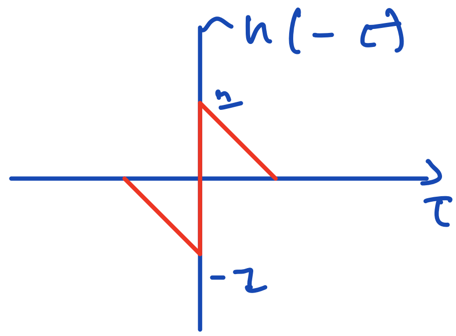
1. Using graphical method, compute the convolution integral $y(t) = x(t) * h(t)$ for the pair of signals where x(t) and h(t) are given below.

$$x(t) = \begin{Bmatrix} -1 < t < 0 & 1-t \\ 0 < t < 1 & t-1 \\ elsewhere & 0 \end{Bmatrix} \text{and } h(t) = \begin{Bmatrix} 0 < t < 1 & t \\ 1 < t < 2 & 2-t \\ elsewhere & 0 \end{Bmatrix}$$

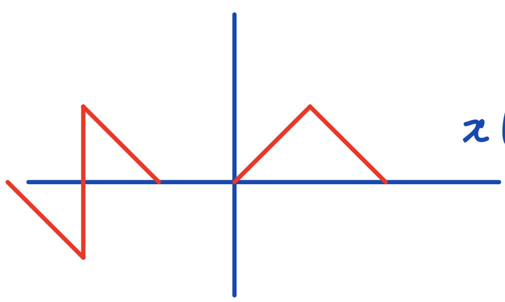
$y(t) = x(t) * h(t)$



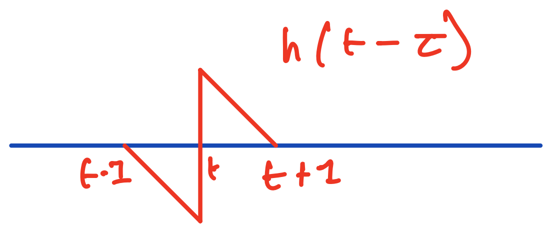




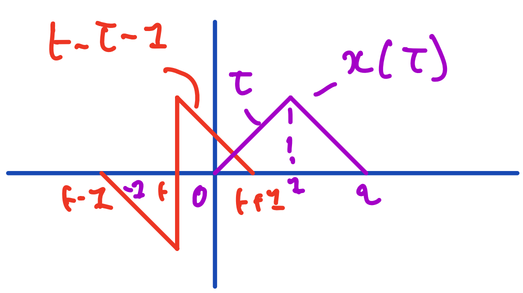
$t + 1 < 0 \Rightarrow t < -1$



$x(t) * h(t) = 0$

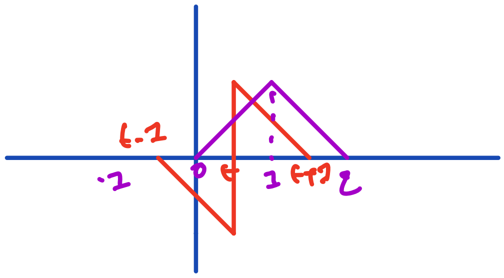


$-1 < t < 0$



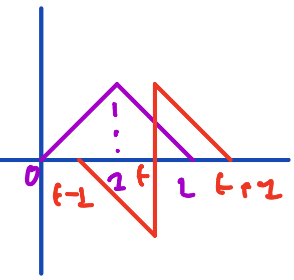
$$x(t) * h(t) = \int_0^{t+1} \tau \cdot (t - \tau + 1)\, d\tau$$

$0 \leq t < 1$



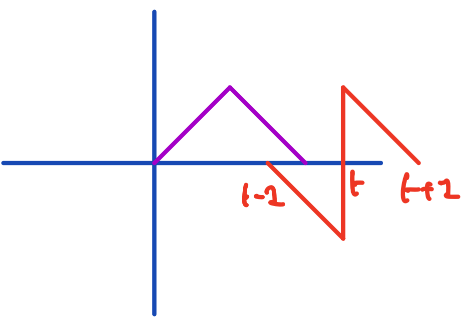
$$x(t) * h(t) = \int_0^t \tau(t-\tau-1)\,d\tau + \int_t^1 \tau(t-\tau+1)\,d\tau + \int_1^{t+1} (2-\tau)(t-\tau+1)\,d\tau$$

$1 \leq t < 2$



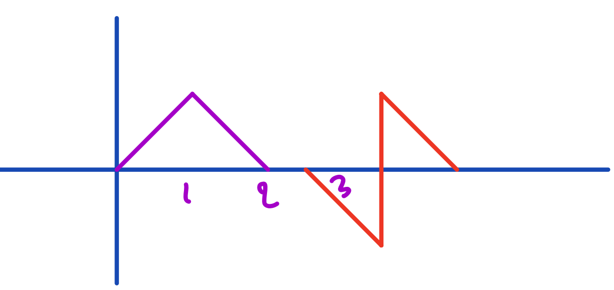
$$x(t)*h(t) = \int_{t-1}^{1} \tau\cdot(t-\tau-1)\,d\tau + \int_1^t (2-\tau)(t-\tau-1)\,dt + \int_t^2 (2-\tau)\cdot(t-\tau+1)\,d\tau$$

$2 \leq t < 3$



$$x(t)*h(t) = \int_{t-1}^{2} (2-\tau)\cdot(t-\tau-1)\,d\tau$$

$t \geq 3$



$x(t) * h(t) = 0$ (no overlap)

$\boxed{2}$

a) $x(t) = e^{j\omega_0 t} u(t)$ and $h(t) = e^{-j\omega_0 t} u(t)$

$$x(\tau) = e^{j\omega_0 \tau} u(\tau)$$

$$x(h(t-\tau) = e^{-j\omega_0 (t-\tau)} u(t-\tau)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{j\omega_0 t} u(\tau) e^{-j\omega_0 (t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t} e^{j\omega_0 \tau} e^{-j\omega_0 t} e^{j\omega_0 \tau}\, d\tau$$

$$= \int e^{-j\omega_0 t} \int_{0}^{t} e^{2j\omega_0 \tau}\, d\tau$$

$$= \frac{e^{-j\omega_0 t}}{2j\omega_0} \left[e^{2j\omega_0 \tau} \right]_0^t$$

$$= \frac{e^{-j\omega_0 t}}{2j\omega_0} \left[2e^{j\omega_0 t} - 1 \right]$$

$$y(t) = \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{\omega_0} \right] u(t)$$

$$\boxed{y(t) = \frac{\sin \omega_0 t}{\omega_0} u(t)}$$

b) $x(t) = 3u(t)$ and $h(t) = \frac{1}{3}e^{-2t}u(t-1)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left[\cancel{3}u(\tau) * \left(\frac{1}{\cancel{3}}\right) e^{-2(t-\tau)} * u(t-\tau-1)\right] d\tau$$

$$= \begin{cases} e^{-2t} \int_0^{t-1} e^{2\tau}\, d\tau , & t > 1 \\ 0 , & \text{Otherwise} \end{cases}$$

$$= [u(t-1)]\, e^{-2t} \int_0^{t-1} e^{2\tau}\, d\tau = [u(t-1)]\, e^{-2t} = \left[\frac{1}{2} e^{2\tau}\right]\Big|_0^{t-1}$$

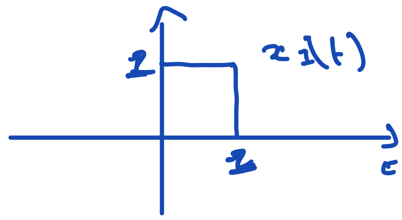
$$= \frac{1}{2}[u(t-1)]\, e^{-2t}$$

$$= e^{2t-2} - 1$$

$$= \left[-\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-2}\right] u(t-1)$$

$\boxed{3}$

$$x_1(t) = 2u(t) - 2u(t-1)$$
$$= 2(u(t) - u(t-1))$$



$$x_2(t) = x_1(t) - 2x_1(t-1) + x_1(t+1) + 2x_1(t+2)$$

from property of LTI system

$$x_1(t) \xrightarrow{S} y_1(t)$$
$$x_1(t-1) \xrightarrow{S} y_1(t-1)$$
$$-2x_1(t-1) \rightarrow -2y_1(t-1)$$
$$x_1(t+1) \rightarrow y_1(t+1)$$
$$2x_1(t+2) \rightarrow 2y_1(t+2)$$

$$x_2(t) \rightarrow \boxed{LTI} \rightarrow y_2(t)$$

$x_1(t) - 2x_1(t-1) + x_1(t+1) + 2x_1(t+2)$ $\quad$ $y_2(t) = y_1(t) + y_1(t+1) - 2y_1(t-1) + 2y_1(t+2)$

$$\boxed{y_2(t) = y_1(t) - 2y_1(t-1) + y_1(t+1) + 2y_1(t+2)}$$

$\boxed{4}$ a) $h(t) = (t+1)u(t-1)$

$h(t)$ depends on the value of t includind past and future values so, the system is not memoryless

$h(t)$ is defined only for $t>1$, so system is causal.

b) $h(t) = 2\delta(t+1)$

$\delta(t)$ represents an impulse at $t=0$

$\delta(t+1)$ depends on past value at $t=-1$, so the system is not memoryless

$h(t)$ is defined for $t \geq -1$, so system is causal

c) $h(t) = \frac{w_c}{\pi} \operatorname{sinc}(w_c t)$

$h(t)$ depends on the value of t includind past and future values so, the system is not memoryless

$\operatorname{sinc}(w_c t) = 0$ for $t < 0$, so system is causal

d) $h(t) = e^{-4t} u(t-1)$

$h(t)$ depends on the value of t includind past and future values so, the system is not memoryless

$u(t-1)$ ensures that impulse response $= 0$ for $t < 1$, so the system is causal

f) $h(t) = e^{-3|t|}$

$h(t)$ depends on the value of $|t|$ includind past and future values so, the system is not memoryless

$h(t) = e^{-3t}$ for $t > 0$

$h(t) = e^{3t}$ for $t < 0$

e) $h(t) = e^{t} u(-t-1)$

$h(t)$ depends on the value of t includind past and future values so, the system is not memoryless

$u(-t-1) = 0$ for $t < 1$, so the system is causal

g) $h(t) = 3\delta(t)$

$h(t) = 0$ for $t = 0$, so the system is not memoryless

$h(t)$ is defined for all t, so the system is non-causal.