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Question 1) Given the differential equations, obtain the time domain step response using Laplace Transform techniques. Note that y(t) is the output and x(t)=U(t) (U(t is a unit step) is the input.

i) $5x(t) = \frac{d^3y(t)}{dt^3} + 13\frac{d^2y(t0}{dt^2} + 54\frac{dy(t)}{dt} + 72y(t)$, initial conditions zero.

ii) $0.001\frac{d^2y(t)}{dt^2} + 0.04\frac{dy(t)}{dt} + 40y(t) = x(t)$, initial conditions zero.

iii) $0.1\frac{dy(t)}{dt} + y(t) = 8x(t)$, initial condition y(t)=6.

i) $5x(t) = \frac{d^3y(t)}{dt^3} + 13\frac{d^2y(t0}{dt^2} + 54\frac{dy(t)}{dt} + 72y(t)$, initial conditions zero.

using Laplace transform

$$5X(s) = s^3Y(s) + 13Y(s)s^2 + 54sY(s) + 72Y(s)$$

$$= Y(s)\left[s^3 + 13s^2 + 54s + 72\right]$$

$$Y(s) = \frac{5X(s)}{s^3+13s^2+54s+72}$$

$$= \frac{5}{s(s^3+13s^2+54s+72)}$$

$$= \frac{5}{s(s+3)(s+4)(s+6)} = \frac{A}{s} + \frac{B}{(s+3)} + \frac{C}{(s+4)} + \frac{D}{(s+6)}$$

$$5 = A(s+3)(s+4)(s+6) + Bs(s+4)(s+6) + Cs(s+3)(s+6) + Ds(s+3)(s+4)$$

$s=0$, $s=-3$, $s=-4$, $s=-6$

$A = \frac{5}{72}$, $5 = -3B$, $5 = 8C$, $5 = -36D$

$B = -\frac{5}{9}$, $C = \frac{5}{8}$, $D = -\frac{5}{36}$

$$Y(s) = \frac{5}{72s} - \frac{5}{9(s+3)} + \frac{5}{8(s+4)} - \frac{5}{36(s+6)}$$

$$\therefore\; e^{-at}u(t) \xleftrightarrow{L} \frac{1}{s+a} \qquad u(t) \longleftrightarrow \frac{1}{s}$$

inverse Laplace

$$y(t) = \frac{5}{72}u(t) - \frac{5}{9}e^{-3t}u(t) + \frac{5}{8}e^{-4t}u(t) - \frac{5}{36}e^{-6t}u(t)$$

$$\boxed{y(t) = \left[\frac{5}{72} - \frac{5}{9}e^{-3t} + \frac{5}{8}e^{-4t} - \frac{5}{36}e^{-6t}\right]u(t)}$$

ii) $0.001\frac{d^2y(t)}{dt^2} + 0.04\frac{dy(t)}{dt} + 40y(t) = x(t)$, initial conditions zero.

using Laplace transform

$$0.001\,s^2Y(s) + 0.04\,sY(s) + 40\,Y(s) = X(s)$$

$$Y(s)\left[0.001s^2 + 0.04s + 40\right] = X(s)$$

$$Y(s) = \frac{X(s)}{0.001s^2 + 0.04s + 40} \qquad \text{where } X(s) = \frac{1}{s}$$

$$Y(s) = \frac{1000}{s(s^2+40s+40000)} = \frac{A}{s} + \frac{Bs+C}{\underbrace{s^2+40s+40000}_{(s+20)^2+(199)^2}}$$

$$1000 = A(s^2+90s+40000) + s^2B + sC$$

$$1000 = s^2(A+B) + s(40A+C) + 40000A$$

on both sides

$$40000A = 1000, \qquad B = -A, \qquad C = -1$$

$$A = \frac{1}{40} \qquad B = -\frac{1}{40}$$

$$Y(s) = \frac{1}{40s} + \frac{-\frac{1}{40}s - 1}{(s+20)^2+(199)^2} = \frac{1}{40s} - \frac{(s+20)}{40(s+20)^2+(199)^2} - \frac{1}{2[(s+20)^2+(199)^2]}$$

$$Y(s) = \frac{1}{40s} - \frac{1\,(s+20)}{40[(s+20)^2+(199)^2]} - \frac{1}{398} \cdot \frac{199}{((s+20)^2+(199)^2)}$$

inverse laplace

$$y(t) = \frac{1}{40}u(t) - \frac{1}{40}e^{-20t}\cos(199t)u(t) - \frac{1}{398}e^{-20t}\sin(199)u(t)$$

$$\boxed{y(t) = \left[\frac{1}{40} - \frac{1}{40}e^{-20t}\cos(199t) - \frac{1}{398}e^{-20t}\sin(199t)\right]u(t)}$$

$iii)\ 0.1\frac{dy(t)}{dt} + y(t) = 8x(t)$, initial condition y(t)=6.

using Laplace transform

$$0.1\left[sY(s) - y(0^-)\right] + Y(s) = 8X(s)$$

$$0.1\left[sY(s) - 6\right] + Y(s) = 8X(s)$$

$$Y(s)\left[0.1s + 1\right] = 6 \times 0.1 + 8X(s)$$

$$= 0.6 + \frac{8}{s} = \frac{0.6s + 8}{s}$$

$$Y(s) = \frac{8 + 0.6s}{s(0.1s+1)} = \frac{80 + 6s}{s(s+10)} = \frac{A}{s} + \frac{B}{(s+10)}$$

$$80 + 6s = A(s+10) + sB$$

$$80 + 6s = s(A+B) + 10A$$

on both sides

$10A = 80$, $A + B = 6$

$A = 8$ $\quad$ $8 + B = 6$

$B = -2$

$$Y(S) = \frac{8}{S} - \frac{2}{(s+10)}$$

inverse laplace

$$y(t) = 8u(t) - 2e^{-10t}u(t)$$

$$\boxed{y(t) = \left[8 - 2e^{-10t}\right]u(t)}$$

Question 2) For each of the systems in question 1 identify if the system is stable and use the Laplace Transform properties to determine the initial and final values of Y(s) and compare them with the initial and final values of y(t).

i) $y(0) = \lim_{s \to \infty} sY(s) = \lim_{s \to \infty} s \cdot \frac{5}{s(s+3)(s+6)(s+4)}$

The poles are $0, -3, -4, -6 \Rightarrow LHS + 0 \Rightarrow$ marginally stable

$$y(0) = \lim_{s \to \infty} \frac{5}{(s+3)(s+6)(s+4)}$$

$\boxed{y(0) = 0}$ initial

$$y(\infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{5}{(s+3)(s+6)(s+4)}$$

$$y(t \to \infty) = \frac{5}{3 \times 6 \times 4}$$

$y(t \to \infty) = \frac{5}{72}$ final

$$y(t \to \infty) : \lim_{s \to 0} \frac{s}{s(s+3)(s+6)(s+6)} = \infty$$

ii) $y(s) = \dfrac{1}{s(s+20+200j)(s+20-200j)}$

The poles are $0, -20+200j, -20-200j \Rightarrow$ LHS $+ 0 \Rightarrow$ marginally stable

$$y(0) = \lim_{s\to\infty} s \cdot \frac{1000}{s(s^2+40s+40000)} = \lim_{s\to\infty} \frac{1000}{s^2+40s+40000}$$

$\boxed{y(0) = 0}$ initial

$$y(\infty) = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0} \frac{1000}{s^2+40s+40000}$$

$g(t\to\infty) = \frac{1}{40}$ final

$$g(t\to\infty) : y(s) = \lim_{s\to 0} \frac{\frac{1}{40}}{s} - \frac{1}{40}\frac{s+2}{(s+20)^2+200^2} - \frac{1}{400}\frac{200}{(s+20)^2+200^2}$$

$$= \infty$$

iii) $y(s) = \dfrac{\frac{80}{s}+6}{s+10}$, pole is $-10 \Rightarrow$ LHS $\Rightarrow$ stable

$$y(0) = \lim_{s\to\infty} s\left[\frac{8}{s} - \frac{2}{(s+10)}\right]$$

$$y(0) = 8 - \lim\left[\frac{2s}{s+10}\right]$$

$$= 8 - \lim\left[\frac{2s}{1+\frac{10}{s}}\right]$$

$$= 8 - 2$$

$\boxed{y(0) = 6}$ initial

$$y(\infty) = \lim_{s\to 0} s\,Y(s) \quad \frac{s(60s+80)}{s\ (s+10)}$$

$y(t\to\infty) = 8$ final

question 3

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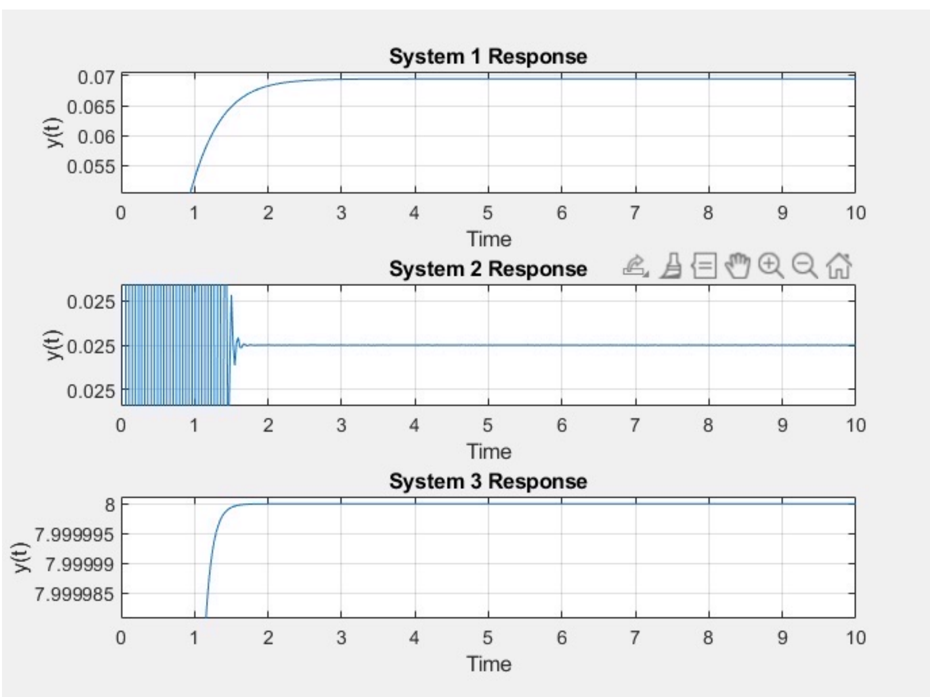
question3_MamadouKaba.m*

```
%Mamadou Diao Kaba (27070179)
% Plot 1
t = 0:0.01:10;
y1 = dsolve('D3y+13*D2y+54*Dy+72*y=5*sign(t)','y(0)=0,Dy(0)=0,D2y(0)=0');
subplot(311)
ezplot(y1, [0 10]);
grid on;
xlabel('Time');
ylabel('y(t)');
title('System 1 Response');

% Plot 2
y2 = dsolve('0.001*D2y+0.04*Dy+40*y=sign(t)','y(0)=0,Dy(0)=0');
subplot(312)
ezplot(y2, [0 10]);
grid on;
xlabel('Time');
ylabel('y(t)');
title('System 2 Response');

% Plot 3
y3 = dsolve('0.1*Dy+1*y=8*sign(t)','y(0)=6');
subplot(313)
ezplot(y3, [0 10]);
grid on;
xlabel('Time');
ylabel('y(t)');
title('System 3 Response');

```



There is no overshoot from the plots.