$\boxed{1}$

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a) $\int_{-\infty}^{\infty}[t^3 - e^{-25t} + \sin(10\pi t)]\delta(4t-2)dt$

$$= \frac{1}{4}\int_{-\infty}^{\infty}\left[t^3 - e^{-25t} + \sin(10\pi t)\right]\delta\left(t-\frac{1}{2}\right)dt$$

$$= \frac{1}{4}\left[\left(\frac{1}{2}\right)^3 - e^{-25(\frac{1}{2})} + \underset{0}{\cancel{\sin(5\pi)}}\right]\int_{-\infty}^{\infty}\delta\left(t-\frac{1}{2}\right)dt$$

$$= \frac{1}{4}\left[\frac{1}{8} - e^{-\frac{25}{2}}\right]\times 1$$

$$\boxed{= \frac{1}{32} - \frac{e^{-\frac{25}{2}}}{4}}$$

b) $\int_{-2}^{2}[(1-t)^2 - e^{-t}][\delta(t+1) + \delta(-t+5) + \delta(4t-6)]dt$

0 out of range

$$= \int_{-2}^{2}\left[(1-t)^2 - e^{-t}\right]\delta(t+1)\,dt + \cancel{\int_{-2}^{2}\left[(1-t)^2 - e^{-t}\right]\delta(-t+5)\,dt}$$

$$+ \int_{-2}^{2}\left[(1-t)^2 - e^{-t}\right]\overset{\delta\left(4\left(t-\frac{3}{2}\right)\right)}{\delta(4t-6)}\,dt$$

$$= \left[(1-(-1))^2 - e^{+1}\right]\overset{1}{\cancel{\int_{-2}^{2}\delta(t+1)\,dt}} + \frac{1}{4}\left[\left(1-\frac{3}{2}\right)^2 - e^{-\frac{3}{2}}\right]\overset{1}{\cancel{\int_{-2}^{2}\delta\left(t-\frac{3}{2}\right)dt}}$$

$$= 4 - e + \frac{1}{4}\left[\frac{1}{4} - e^{-\frac{3}{2}}\right]$$

$$= 4 - e + \frac{1}{16} - \frac{e^{-\frac{3}{2}}}{4}$$

$$\boxed{= \frac{65}{4} - e - \frac{e^{-\frac{3}{2}}}{4}}$$

c) c) $(-t+1)u(t)[\delta(2t+1)+\delta(-2t+1)]$ (also draw the resulting function for part (c))

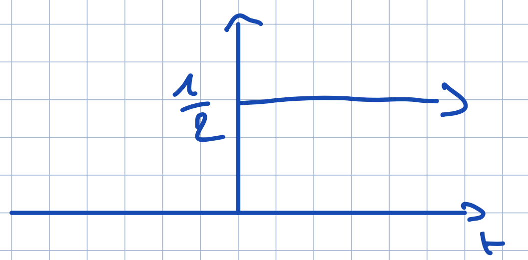
$$= (-t+1)u(t)\,\delta(2t+1) + (-t+1)u(t)\,\delta(-2t+1)$$

$$= \frac{1}{2}(-t+1)u(t)\,\delta\left(t+\frac{1}{2}\right) + \frac{1}{2}(-t+1)u(t)\,\delta\left(t-\frac{1}{2}\right)$$

$$= \frac{1}{2}\left(-\left(-\frac{1}{2}\right)+1\right)\overset{0}{\cancel{u\left(-\frac{1}{2}\right)}}\,\delta\left(t+\frac{1}{2}\right) + \frac{1}{2}\left(-\frac{1}{2}+1\right)u\left(\frac{1}{2}\right)\delta\left(t-\frac{1}{2}\right)$$

$$= 0 + \frac{1}{2}u\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}$$



$\boxed{2}$

(2) i) Express the following numbers in Cartesian form $(x + jy)$ and represent them in the complex plane.

$e^{i\theta} = \cos\theta + i\sin\theta$

(a) $e^{j\frac{\pi}{2}}$ (b) $5e^{j\frac{\pi}{3}}$ (c) $-4e^{-j\frac{2\pi}{3}}$ (d) $6e^{j\left(3\pi+\frac{\pi}{2}\right)}$ (e) $7e^{j\frac{4\pi}{3}}$

a) $e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = \boxed{j}$

b) $5e^{j\frac{\pi}{3}} = 5\left[\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right] = \boxed{\frac{5}{2} + j\frac{5\sqrt{3}}{2}}$

c) $-4e^{-j\frac{2\pi}{3}} = 4\left[\cos\frac{-2\pi}{3} + j\sin\frac{-2\pi}{3}\right] = \boxed{2 + j2\sqrt{3}}$

d) $6e^{j\left(3\pi+\frac{\pi}{2}\right)} = 6\left[\cos\left(3\pi+\frac{\pi}{2}\right) + j\sin\left(3\pi+\frac{\pi}{2}\right)\right] = \boxed{-6j}$

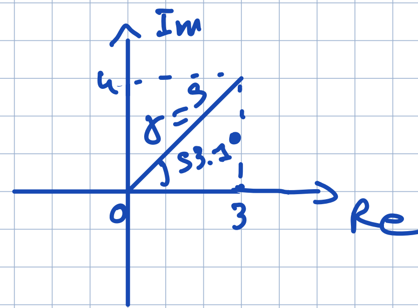
e) $7e^{j\frac{4\pi}{3}} = 7\left[\cos\left(\frac{4\pi}{3}\right) + j\sin\left(\frac{4\pi}{3}\right)\right] = \boxed{\frac{1}{2}\left[-7 + j7\sqrt{3}\right]}$

ii) Express the following numbers in polar form ($re^{j\theta}$ with $-\pi < \theta \leq \pi$) and represent them in the complex plane.

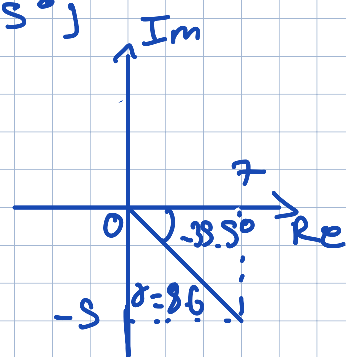
(a) $3 + j4$ (b) $7 - j5$ (c) $23 - j7$ (d) $-100 - j46$

$$x + jy = \sqrt{x^2 + y^2}\, e^{j\theta}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

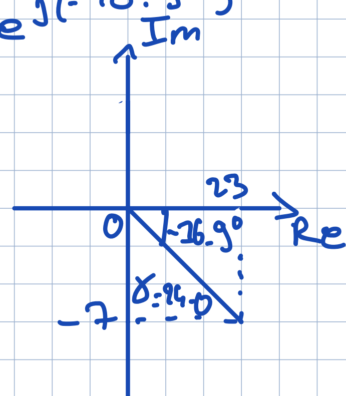
a) $3 + j4 = \sqrt{3^2 + 4^2} \angle \tan^{-1}\left(\frac{4}{3}\right) = 5 e^{j(53.1^\circ)}$



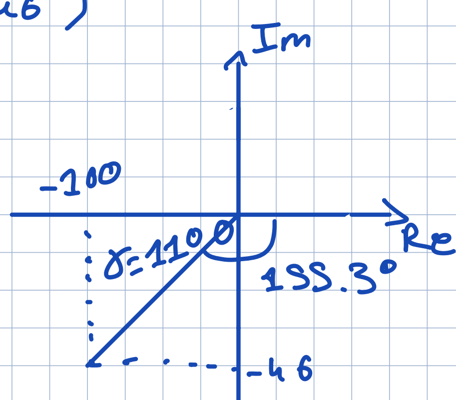
b) $7 - j5 = \sqrt{7^2 + (-5)^2} \angle \tan^{-1}\left(\frac{-5}{7}\right) = 8.6 e^{j(-35.5^\circ)}$



c) $23 - j7 = \sqrt{23^2 + (-7)^2} \angle \tan^{-1}\left(\frac{-7}{23}\right) = 24.0 e^{j(-16.9^\circ)}$



d) $-100 - j46 = \sqrt{(-100)^2 + (-46)^2} \angle \tan^{-1}\left(\frac{-100}{-46}\right) = 110 e^{j(-155.3^\circ)}$



iii) Let $z_1 = 7 - j5$ and $z_2 = -3 + j4$. Determine the following quantities in both Cartesian and polar forms:

(a) $z_1 z_2$ (b) z_1/z_2 (c) z_2/z_1

$$|z_1| = \sqrt{7^2+5^2} = 8.6$$

$$|z_2| = \sqrt{3^2+4^2} = 5$$

$$\theta_1 = \tan^{-1}\left(\frac{-5}{7}\right) = -35.5^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{4}{-3}\right) = 180 - 53.13 = 126.9^\circ$$

$$z_1 = 8.6\, e^{-j35.5^\circ}$$

$$z_2 = 5\, e^{j126.9^\circ}$$

a) $$z_1 z_2 = 8.6\, e^{-j35.5^\circ} \times 5\, e^{j126.9^\circ} = 43\, e^{j(126.9^\circ - 35.5^\circ)}$$

$$\boxed{\begin{aligned} &= 43\, e^{j91.3^\circ} \\ &= -1 + 43j \end{aligned}}$$

b) $$\frac{z_1}{z_2} = \frac{8.6 e^{-j35.5^\circ}}{5 e^{j126.9^\circ}} = 1.72\, e^{j(-35.5^\circ - 126.9^\circ)}$$

$$\boxed{\begin{aligned} &= 1.72\, e^{-j162.4^\circ} \\ &= -1.64 - 0.52j \end{aligned}}$$

c) $$\frac{z_2}{z_1} = \frac{1}{\left(\frac{z_1}{z_2}\right)} = \frac{1}{1.72\, e^{-j162.4^\circ}}$$

$$\boxed{\begin{aligned} &= 0.58\, e^{j162.4^\circ} \\ &= -0.55 + 0.172j \end{aligned}}$$

3

(3) $x(t)$ defined as follows:

$$x(t) = 2\cos(\omega t + \frac{2\pi}{3}) + 5\cos(\omega t) + 4\cos(\omega t - \frac{5\pi}{4})$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega t + \varphi)$. Use complex phasor manipulation to obtain the answer.

(b) Plot all the phasors used to solve the problem in (a) in the complex plane.

$$x(t) = \frac{2}{\sqrt{2}}\angle 120^\circ + \frac{5}{\sqrt{2}}\angle 0^\circ + \frac{4}{\sqrt{2}}\angle -225^\circ$$

$$= 3.330 \angle 75.59$$

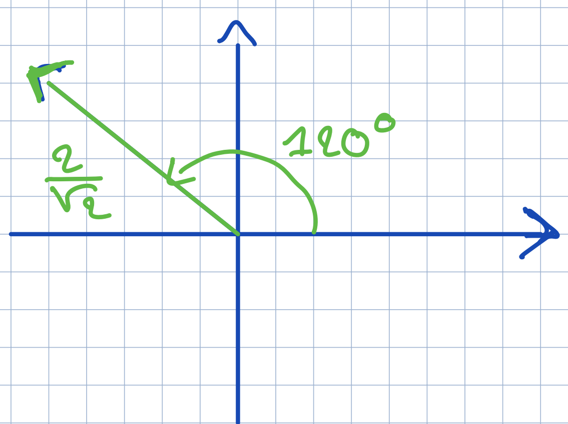
a) $x(t) = A\cos(\omega t + \phi)$

$$x(t) = 3.330\sqrt{2}\cos(\omega t + 75.59^\circ)$$

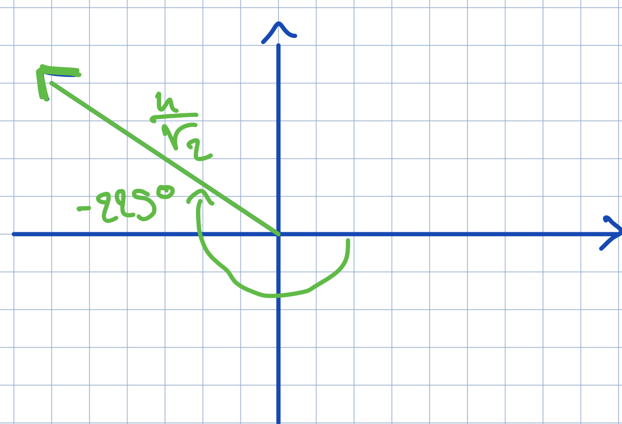
$$\boxed{= 4.710\cos(\omega t + 0.4200\pi)}$$

b) phasors: 1) $\frac{2}{\sqrt{2}}\angle 120^\circ$, 2) $\frac{5}{\sqrt{2}}\angle 0^\circ$, 3) $\frac{4}{\sqrt{2}}\angle -225^\circ$

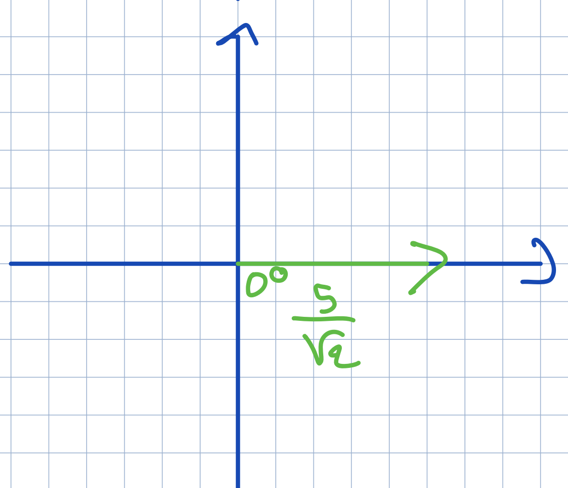
1)



3)



2)



$\boxed{4}$

(4) A series RLC circuit is powered by an ideal 10-volt source. The capacitor voltage has a natural response of $V_{cn}(t) = Ae^{-3t} + Be^{-5t}$. The capacitor is initially uncharged, and the initial current is zero. Treating the capacitor voltage as the signal, compute the signal energy from t=0 to t=1.1 seconds.

natural response   forced response

$$V(t) = V_{cn}(t) + V_{fn}(t)$$

at $t \to \infty$ $V_{fn}(t) = 10\,V$

so, $v(t) = Ae^{-3t} + Be^{-5t} + 10\,V$

at $t = 0$ $v(t) = 0$

$$A + B + 10 = 0$$

$$A + B = -10 \quad (1)$$

$$C\frac{dv}{dt} = I_c$$

at $t = 0$ $I_L(0) = I_c(0)$

then $\frac{dV}{dt} = 0$

$$\frac{d}{dt}v(t) = \frac{d}{dt}\left(Ae^{-3t} + Be^{-5t} + 10\right)$$

$$\frac{dv}{dt} = -3Ae^{-3t} - 5Be^{-5t} + 0$$

if $t = 0$ $\frac{dv}{dt} = 0 \Rightarrow$ $-3A - 5B = 0$

$$-3A = 5B$$

$$A = \frac{-5B}{3} \quad (2)$$

(2) in (1)

$$-\frac{5B}{3} + B = -10$$

$$B = 15$$

B in 1

$$A + 15 = -10$$

$$A = -25$$

so, $v(t) = -25e^{-3t} + 15e^{-5t} + 10$

$$E = \int_0^{1.1} v(t)^2 \, dt$$

$$= \int_0^{1.1} \left(15e^{-5t} - 25e^{-3t} + 10 \right)^2 dt$$

$$= \int_0^{1.1} \left(225e^{-10t} - 750e^{-8t} + 625e^{-6t} + 300e^{-5t} - 500e^{-3t} + 100 \right) dt$$

$$= 225\int_0^{1.1} e^{-10t} dt - 750\int_0^{1.1} e^{-8t} dt + 625\int_0^{1.1} e^{-6t} dt + 300\int_0^{1.1} e^{-5t} dt - 500\int_0^{1.1} e^{-3t} dt$$

$$+ \overset{110}{\int_0^{1.1} 100 \, dt}$$

$$\boxed{E = 42.02 \text{ joules}}$$