Assignment - 2

1)

Ans) The probably obtcons on.

$x = \{1, 2, 3, 4, 5, 6\}$.

1. mean.

$\mu = E(x] \quad \dfrac{1+2+3+4+5+6}{6} = \dfrac{21}{6} = 3.5.$

2. Variance

first $E[x^2] : \dfrac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \dfrac{1+4+9+16+\ldots}{6}.$

$= \dfrac{91}{6}$

Now, $var(x) = E[x^2] - (E(x])^2.$

$= \dfrac{91}{6} - (3.5)^2 = \dfrac{91}{6} - 12.25.$

$= 15.1667 - 12.25 = 2.9167.$

$\therefore$ Mean $= 3.5$. Variance $= \dfrac{35}{12} = 2.9167.$

2)

Ans) As $P[x<0] = P[x=0] = P[x>0]$.

$\therefore P[x=-1] + P(x=-2] = P[x=0] = P[x=1] + P(x=2$

$= P + P = P(x=0] = P+P.$

$= P[x=0] = 2P$.

Now, as $P[x<0] + P[x=0] + P[x>0] = 1.$

$\Rightarrow P[x=-1] + P[x=-2] + P(x=0] + P[x=1] + P[x=2]$

$\Rightarrow P+P+2P+P+P = 1.$

$\Rightarrow 6P = 1 = \boxed{P = \dfrac{1}{6}}$

$P[x=-1] = P[x=-2] = P[x=1] = P(x=2] = \dfrac{1}{6}.$

X	-2	-1	0	1	2
P(x)	1/6	1/6	2/6	1/6	1/6

3)

A) To prove :— $P(x_j) = P(X = x_j) = F(x_j) - F(x_{j-1})$

where F is df of X.

proof :— let $x_1, x_2, \dots$ be the values of r.v. X

Then $F(x_j) = P(X \le x_j) = \sum_{i=1}^{j} P(X = x_j) = \sum_{i=1}^{j} P(x_j)$

$F(x_{j-1}) = P(X \le x_{j-1}) = \sum_{i=1}^{j-1} P(X = x_j) = \sum_{i=1}^{j-1} P(x_j)$

$= F(x_j) - F(x_{j-1}) = P(x_j)$.

4)

Ans)

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find k.

if $P(x)$ is PMF.

the $\sum P(x) = 1$

$= k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$= 10k^2 + 9k = 1$

$= 10k^2 + 9k - 1 = 0$

$\boxed{k = -1}$, $\boxed{k = \frac{1}{10}}$

<u>Now</u>

X	0	1	2	3	4	5	6	7
$P(x)$	0	1/10	2/10	2/10	3/10	1/100	2/100	9/200

=

X	0	1	2	3	4	5	6	7
	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

ii) $P(X < 6) = 1 - P(X \ge 6) = 1 - [P(6) + P(7)]$

$= 1 - [0.02 + 0.17]$

$= 0.81$

$P(x \geq 6) = P(6) + P(7) = 0.02 + 0.17 = 0.19$.

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$.

$= 0.1 + 0.2 + 0.2 + 0.3$.

$= 0.8$.

iii) $P(x \leq a) > \frac{1}{2}$ min value of a.

$P(x \leq 0) = 0$.

$P(x \leq 1) = 0.1$

$P(x \leq 2) = 0.3$.

$P(x \leq 3) = 0.5$.

$P(x \leq 4) = 0.8 > \frac{1}{2}$ $\boxed{a = 4}$

iv)

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 0.1 & x \leq 1 \\ 0.3 & x \leq 2. \\ 0.5 & x \leq 3. \\ 0.8 & x \leq 4 \\ 0.81 & x \leq 5 \\ 0.83 & x \leq 6. \\ 1 & x \leq 7. \end{cases}$$

5) $f(x) = 6(1-x)$ if $0 \leq x \leq 1$.

i) to check if $f(x)$ is pdf.

to be a pdf, i) $f(x) \geq 0$ for all x.

ii) $\int_0^1 f(x)\, dx = 1$.

$\because x = [0,1]$ both $x \geq 0$ & $1 - x \geq 0$

$\therefore f(x) \geq 0$ on $[0,1]$.

also $\int_0^1 6x(1-x)\, dx = 6\int_0^1 (x - x^2)\, dx$.

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 6\left[\frac{1}{2} - \frac{1}{3}\right] = 6\left(\frac{3-2}{6}\right)$

$= 6 \times \frac{1}{6} = 1$

$\therefore f(x)$ is a pdf.

5) ii) $P(X<b) = P(X>b)$.

this mean, b is the median.

$\therefore P(X<b) = 0.5$.

$= \int_0^1 6x(1-x)\,dx = 0.5$.

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = 6\left(\frac{b^2}{2} - \frac{b^3}{3}\right) = 0.5$.

$= 6 \cdot \frac{3b^2-2b^3}{6} = 0.5$ (multiply both side by 2).

$= 6b^2 - 4b^3 = 1$

$= 4b^3 - 6b^2 + 1 = 0$.

$b = 0.5 = \frac{1}{2}$.

6)

soln:-)

$$f(x) = \begin{cases} \frac{3+2x}{18}, & 2 \le x \le 4. \\ 0, & \text{otherwise}. \end{cases}$$

$$\text{mean} = \mu_1' = \int_{-\infty}^{4} x f(x)\,dx = \int_2^4 x\left(\frac{3+2x}{18}\right)dx. = \frac{83}{27}.$$

$$= \mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_2^4 x^2\left(\frac{3+2x}{18}\right)dx = \frac{88}{9}.$$

$$\therefore \text{variance} = \mu_2' - (\mu_1')^2 = \left[\frac{88}{9} - \left(\frac{83}{27}\right)^2\right] = \frac{239}{729}.$$

$$= SD = \sqrt{\frac{239}{729}} \approx 0.57.$$

mean deviation

$$\int_{-\infty}^{\infty} |x-\bar{x}|\,f(x)\,dx = \int_2^4 \left|x - \frac{83}{27}\right|\left(\frac{3+2x}{18}\right)dx$$

$$= \int_2^{83/27} \left|\frac{83}{27} - x\right|\left(\frac{3+2x}{18}\right)dx + \int_{83/27}^{4}\left|x - \frac{83}{27}\right|\left(\frac{3+2x}{18}\right)dx$$

$= 0.4$.

7)

x	-3	-2	-1	0	1	2	3
$P(x)$	0.05	0.10	0.30	0	0.30	0.15	0.[illegible]

i) $E(x) = \sum_{i=1}^{7} x_i p_i = x_1 p_1 + x_2 p_2 + \dots x_7 p_7$

$= -3(0.05) + (-2)(0.10) + (-1)(0.30) + 1 \times (0.30) + 2 \times (0.15) + 3(0.10).$

$= 0.25.$

ii) $E(2x \pm 3) = (2 \times 0.25 \pm 3) = \cancel{0.05}$. $0.50 \pm 3 = 3.50.$

iii) $V(x) = E(x^2) - (E(x)^2]$

$E(\cancel{0.25^2})$

$E(x^2) = \cancel{(-3)^2 \times 0.05 + (-2)^2 \times 0.10 + (-1)^2 (0.30) + (1)^2 \times 0} + (2)^2 \times 0.15 + (3)^2 \times 0$

$= 2.95.$

$V(x) = 2.95 - (0.25)^2 = 2.88$

iv) $V(3x \pm 2) = 9 V(x) = 9(\cancel{2.8875})^{2.88} = 25.92.$

8)

X\Y	-1	0	1	Total
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0	0.1	0.1	0.2
Total	0.2	0.4	0.4	1.0

i) $E(x) = (-1)(0.2) + 0(0.6) + 1(0.2) = -0.2 + 0 + 0.2$
$= 0.$

$E(y) = (-1)(0.2) + 0(0.4) + 1(0.4) = 0.2 + 0.4 = 0.2$

$E(x) \neq E(y) \longrightarrow$ x & y have diff expectation

ii) ~~We~~ We calculate $Cov(x,y) = E(xy) - E(x)E(y)$.

$E(xy) = \sum_x \sum_y xy \, P(X=x, Y=y)$.

Combination $(-1)(-1) = 0 = 0$.

$(-1)(0)(0.1) = 0$.

$(-1)(1)(0.1) = -0.1$

$0(-1)(0.2) = 0$.

$0(0)(0.2) = 0$

$0(1)(0.2) = 0$.

$1(1)(0) = 0$.

$1(0)(0.1) = 0$

$1(1)(0.1) = 0.1$

$E(xy) = 0 + 0 - 0.1 + 0 + 0 + 0 + 0 + 0.1$

$= \underline{0}$.

$Cov(x,y) = E(xy) - E(x)E(y) = 0 - (0)(0.2) = 0$.

Thus, X & Y are ~~uncorre~~ uncorrelated.

iii) $Var(x)$.

$E(x^2) = (-1)^2(0.2) + 0^2(0.6) + 1^2(0.2)$.

$= 1(0.2) + 0 + 1(0.2) = 0.4$.

$Var(x) = E(x^2) - E(x)^2 = (0.4 - 0^2) = \underline{0.4}$.

$Var(y) =)$

$E(y^2) = (-1)^2(0.2) + 0^2(0.4) + 1^2(0.4)$.

$= 1(0.2) + 0 + 1(0.4) = 0.2 + 0.4 = 0.6$.

$Var(y) = E(y^2) - [E(y)]^2$

$= 0.6 - (0.2)^2 = 0.6 - 0.44 = \underline{0.56}$.

iv) $Y=0$, condition prob of X?

$P(X=x \mid Y=0)$

$P(Y=0) = 0.4$.

$P(X=-1 \mid Y=0) = P(X=-1, Y=0) / P(Y=0) = 0.1/0.4$

$= 0.25$.

$P(X=0 \mid Y=0) = 0.2/0.4 = 0.5$.

$P(X=1 \mid Y=0) = 0.1/0.4 = 0.25$.

a) Joint PDF.

$f(x,y) = \begin{cases} 2-x-y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$

a) Marginal PDF of x.

$f_x(x) = \int_0^1 f(x,y)\,dy = \int_0^1 (2-x-y)\,dy.$

$\int (2-x-y)\,dy = (2-x)y - \frac{y^2}{2}.$

$y=0$ to 1 $\quad f_x(x) = (2-x)(1) - \frac{1}{2}[0-0]$

$= (2-x) - 0.5 = 1.5x.$

Thus, $f_x(x) = 1.5 - x, \quad 0 \le x \le 1.$

Marginal PDF of y.

$f_Y(y) = \int_0^1 f(x,y)\,dx = \int_0^1 (2-x-y)\,dx$

$\int (2-x-y)\,dx = (2-y)x - \frac{x^2}{2}.$

$x=0$ to 1, $f_Y(y) = (2-y)(1) - \frac{1}{2} - [0-0]$

$= (2-y) - 0.5 = 1.5 - y.$

Thus $f_Y(y) = 1.5 - y, \quad 0 \le y \le 1.$

b) Conditional PF.

$f_{Y/X}(y|x) = \frac{f(x,y)}{f(x)}.$

$f_{Y/X}(y|x) = \frac{2-x-y}{1.5-x}, \quad 0 \le y \le 1.$

$f_{X/Y}(x/y) = \frac{f(x,y)}{f_Y(y)}$

$f_{X/Y}(x|y) = \frac{2-x-y}{1.5-y}, \quad 0 \le x \le 1$

Var (x).

$E(x) = \int_0^1 x f_x(x)\, dx = \int_0^1 x(1.5 - x)\, dx$

$= \int (1.5x - x^2)\, dx = 0.75x^2 - \frac{x^3}{3}$

limit 0 to 1

$E(x) = 0.75(1)^2 - \frac{1}{3} = 0.75 - 0.333 = 0.4167$

$E(x^2) = \int_0^1 x^2 (1.5 - x)\, dx = \int (1.5x^2 - x^3)\, dx$

$= 0.5x^2$

limit 0 to 1, $E(x^2)$

$= 0.5(1) - \frac{1}{4} = 0.5 - 0.25$

$= 0.25$

$Var(x) = E(x^2) - (E(x))^2 = 0.25 - (0.4167)^2$

$= 0.25 - 0.1736$

$= 0.0764$

Similarly, $Var(Y) = 0.0764$.

iv) Covariance between X and Y.

$E(XY) = \int_0^1 \int_0^1 xy(2 - x - y)\, dy\, dx$

Inner int wrt. y, $\int_0^1 y(2 - x - y)\, dy = \int_0^1 (2y - xy - y^2)\, dy$

$= \int_0^1 y^2 = 1 - \frac{x}{2} - \frac{1}{3} = \frac{3}{3} - \frac{x}{2} - \frac{1}{3} = \frac{2}{3} - \frac{x}{2}$.

Now, int. wrt. x.

$\int_0^1 x\left(\frac{2}{3} - \frac{x}{2}\right) dx = \int_0^1 \frac{2}{3}, \frac{x^2}{2} - \frac{1}{2}\frac{x^3}{3} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$.

$\simeq 0.1667$.

$Cov(x,y) = E(x,y) - E(x)E(y)$.

$= 0.1667 - (0.4167)(0.4167)$

$= -0.0069$

10)

So-

Moment of x are $E(x) = 0.6$ for all $r = 1, 2, \ldots$

To show,

$P(x=0) = 0.4$ & $P(x=1) = 0.6$, $P(x \geq 2) = 0$

msf of x.

$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu'_r = 1 + \sum_{r=1}^{\infty} \frac{t^r}{r!}(0.6)$

$= 0.4 + 0.6 \sum_{r=0}^{\infty} \frac{t^r}{r!} = 0.4 + 0.6$

But $M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} P(x=x)$

$= P(x=0) + e^t P(x=1) + \sum_{x=2}^{\infty} e^{tx} P(x=x)$

from ① and ②.

$P(x=0) = 0.4$, $P(x=1) = 0.6$, $P(x \geq 2) = 0$