

# Network Science

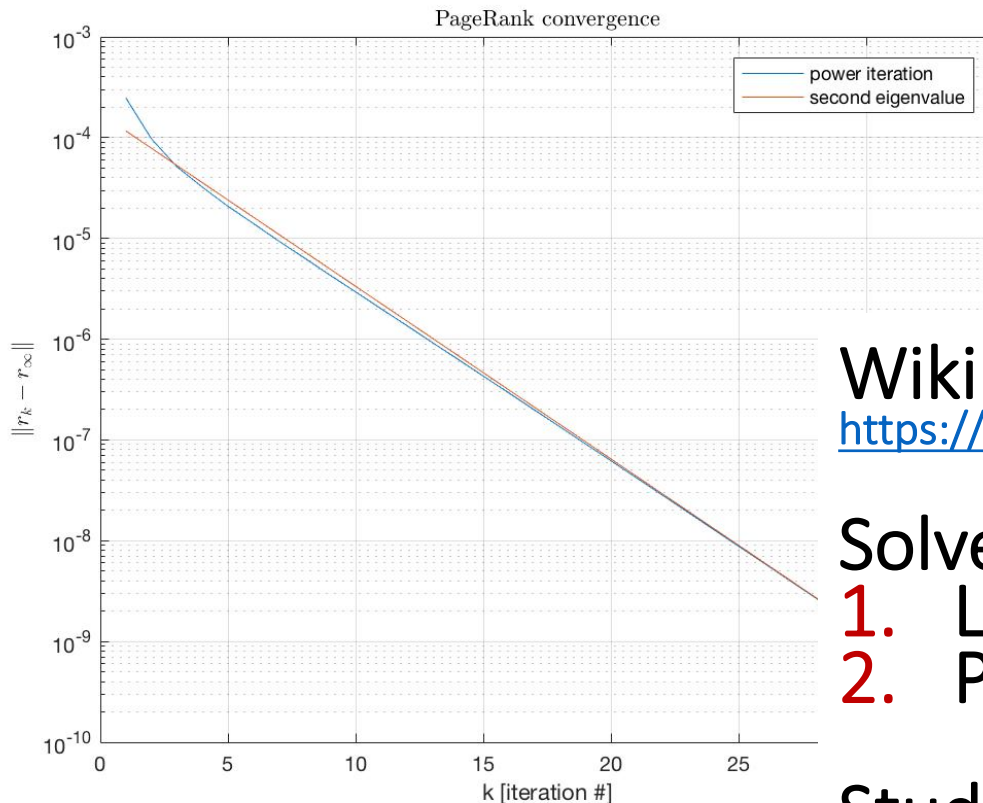
## Lab #4 Ranking

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# Timetable

- ☐ Lab 1 – Fri Oct 12  
Scale free properties
- ☐ Lab 2 – Fri Oct 19  
Albert-Baràbasi model
- ☐ Lab 3 – Fri Oct 26  
Assortativity
- ☐ Lab 4 – Fri Nov 16  
Ranking
- ☐ Lab 5 – Fri Nov 23  
Community detection – Spectral
- ☐ Lab 6 – Fri Nov 30  
Community detection – PageRank-Nibble
- ☐ Lab 7 – Fri Dec 7  
Gephi

# Lab 4 – PageRank



## ASSIGNMENT a

Wikipedia voting dataset

<https://snap.stanford.edu/data/wiki-Vote.html>

Solve the PageRank equation by:

1. Linear system solution
2. Power iteration

Study:

- **Speed** (cpu time)
- **Convergence** of the power iteration (and its dependence on the eigenvalues of  $M$ )

# Lab 4 – PageRank

PageRank equation

$$p = c M p + (1-c) q$$

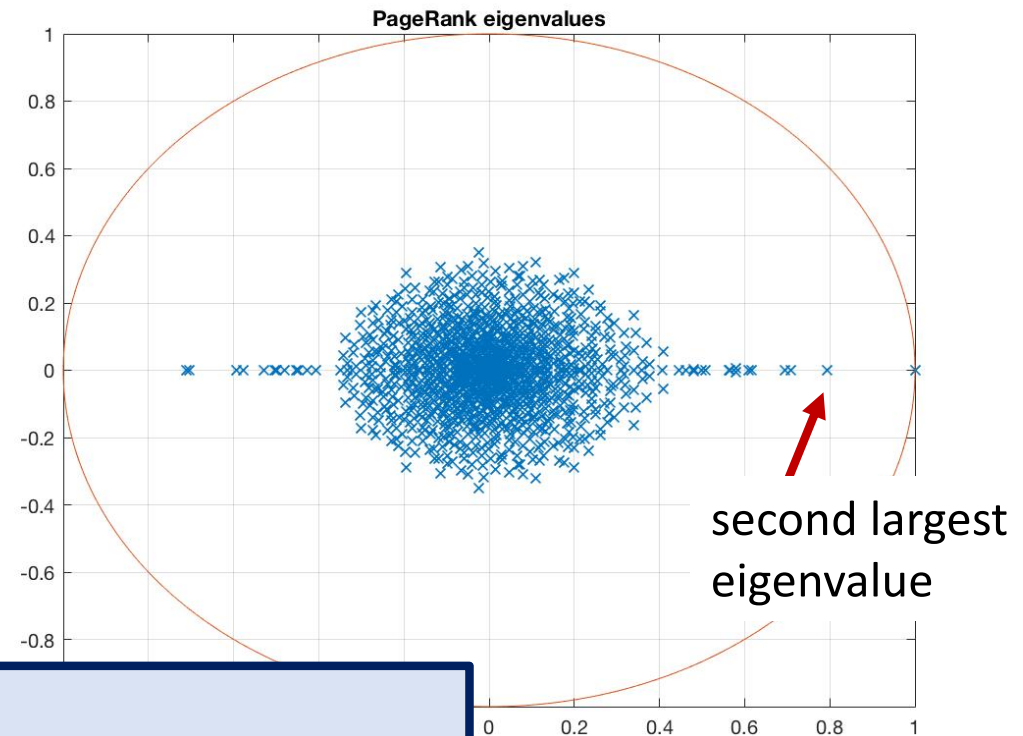
where

1.  $M = A \text{ diag}^{-1}(d)$  weighted **adjacency** matrix
2.  $d = A^T \mathbf{1}$  output **degree** vector
3.  $c = 0.85$  **damping** factor
4.  $q = 1/N$  **teleportation** vector

- Power iteration  $p_{t+1} = c M p_t + (1-c) q$

- Linear system solution  $(I - c M) p = (1-c) q$

# Lab 4 – PageRank



Convergence speed of power iteration:

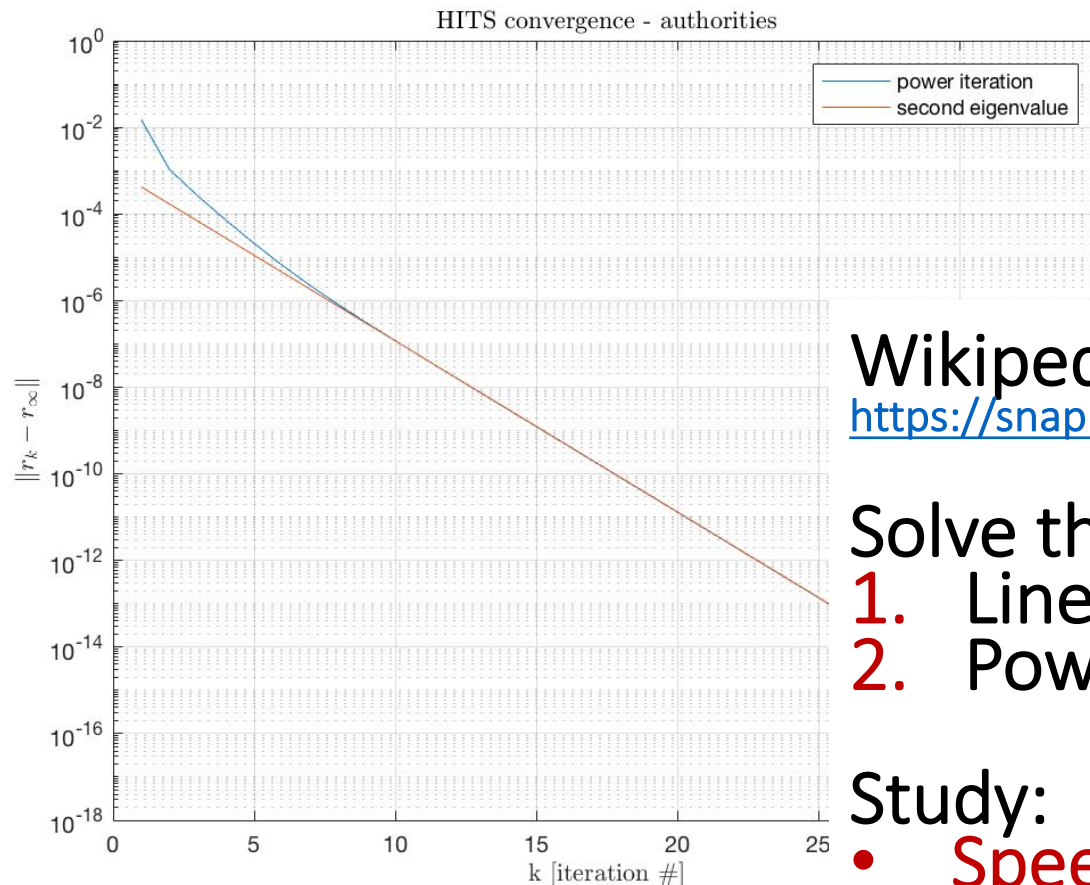
- proportional to  $[c |\lambda_2(M)|]^t$
- $\lambda_2(M)$  **second largest** eigenvalue of  $M$

# Lab 4 – MatLab hints

1. tic: starts the counter
2. toc: reads the counter
3. sparse(eye(N)) : sparse identity matrix
4. eigs: extracts (ordered) eigenvalues
5.  $B \backslash c$ : solves the linear system  $B x = c$
6. norm: evaluates the norm of a vector
7. semilogy: log plot in y axis only

# Lab 4 – HITS authorities

## ASSIGNMENT b



Wikipedia voting dataset  
<https://snap.stanford.edu/data/wiki-Vote.html>

Solve the HITS equation by:

1. Linear system solution
2. Power iteration

Study:

- **Speed** (cpu time)
- **Convergence** of the power iteration (and its dependence on the eigenvalues of  $M$ )

# Lab 4 – HITS authorities

HITS authorities equation

$$p = c M p, \quad M = A A^T$$

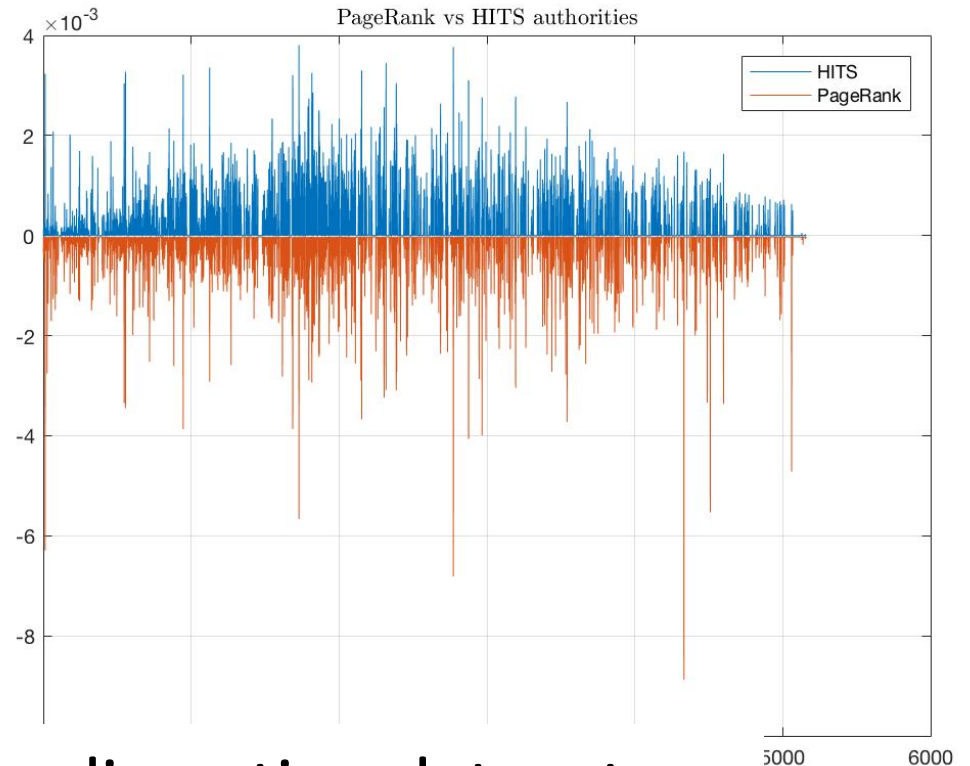
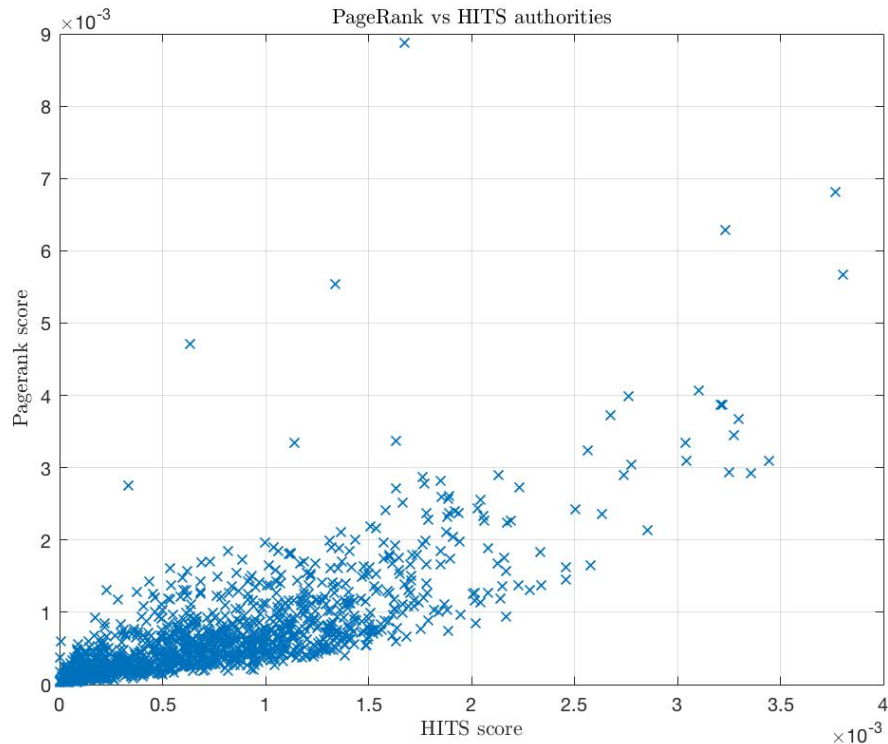
- Linear system solution `eigs(M,2)`
- Power iteration  `$p_{t+1} = c M p_t$ ;  $p_{t+1} = p_{t+1} / |p_{t+1}|$`

Convergence speed of power iteration:

- proportional to  $|\lambda_2(M)/\lambda_1(M)|^t$
- $\lambda_1(M)$  **largest** eigenvalue of  $M$
- $\lambda_2(M)$  **second largest** eigenvalue of  $M$



# Lab 4 – Comparison



Wikipedia voting dataset

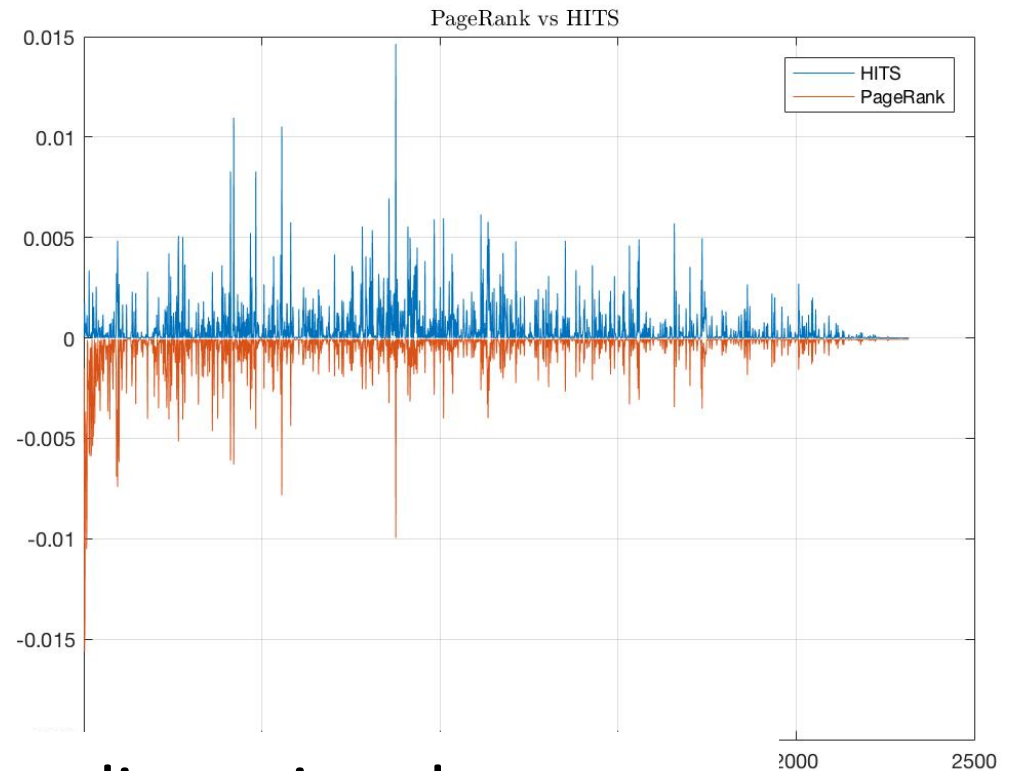
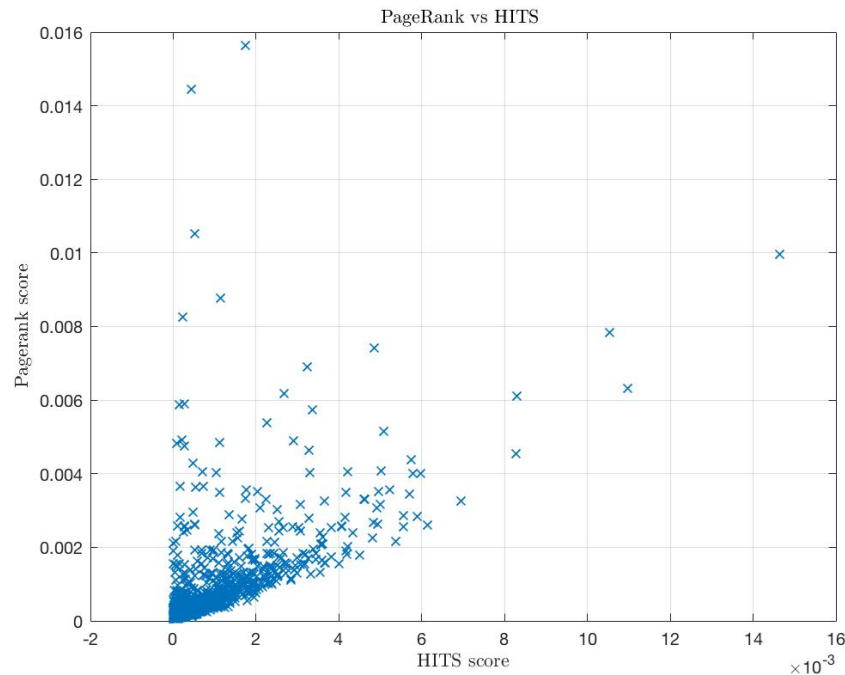
<https://snap.stanford.edu/data/wiki-Vote.html>

ASSIGNMENT c

Compare:

1. HITS authorities
2. PageRank

# Lab 4 – Comparison



Wikipedia voting dataset

<https://snap.stanford.edu/data/wiki-Vote.html>

ASSIGNMENT d

Compare:

1. HITS hubs
2. PageRank (transposed)