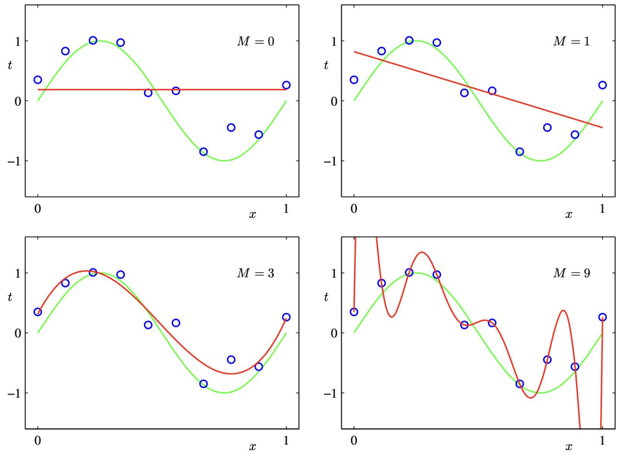
CIS 635 - Knowledge Discovery & Data Mining

Model Regularization

Plan

- Linear to Polynomial Regression
- Regularization, general concept
- Notebook presentation

Linear to Polynomial Regression



$$\hat{y} = \beta_0$$
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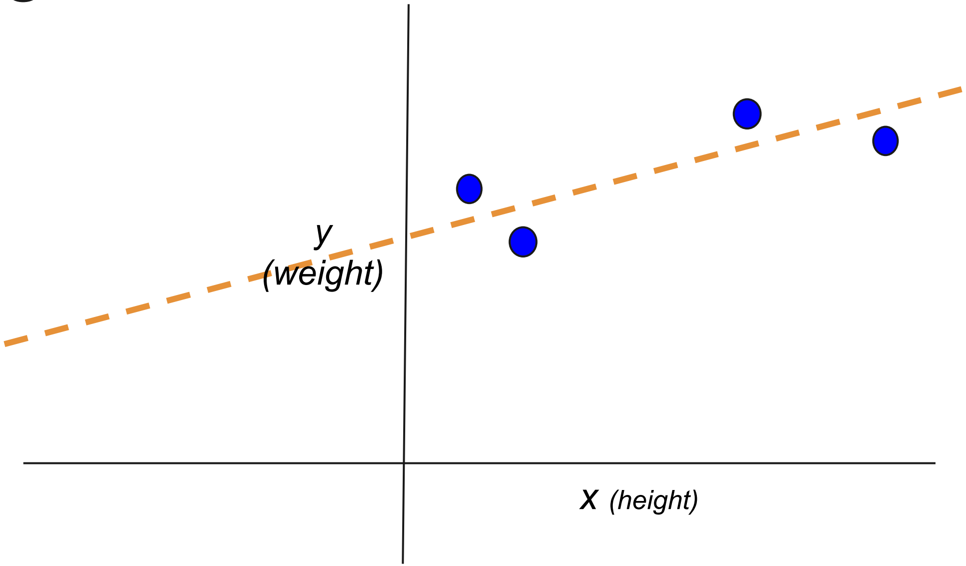
Regularization



So, essentially we are fitting a function; right?

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

Fitting Error

$$\epsilon = |\hat{y} - y|$$

Optimization function

$$E_\Theta = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$$\Theta^* = \mathrm{argmin}_\Theta\, E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

Regularization

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$$

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Essentially, the same formulation

Generally **ML** vs **Math** conventions

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Regularization

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

x : scalar
$\boldsymbol{x}$, $\mathbf{x}$: vector
$\mathbf{X}$: Matrix

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2$$

$$\epsilon = |\hat{y} - y|$$

Essentially, the same formulation

Optimization function

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Generally ML vs Math conventions

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Regularization

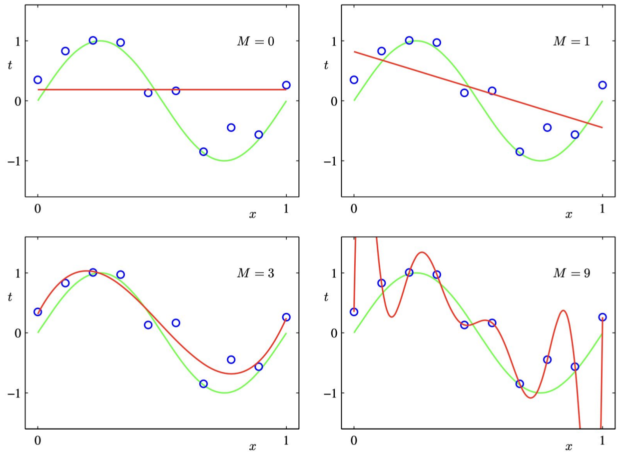


Table 1.1 Table of the coefficients $\mathbf{w}^\star$ for polynomials of various order. Observe how the typical magnitude of the coefficients increases dramatically as the order of the polynomial increases.

	$M = 0$	$M = 1$	$M = 6$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

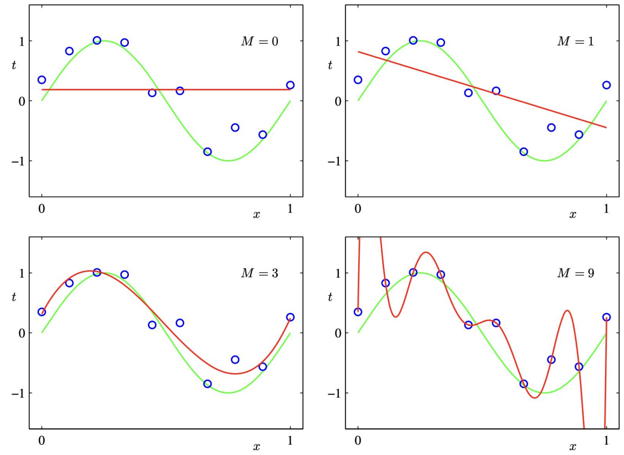
Regularization

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$$

Regularizer

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

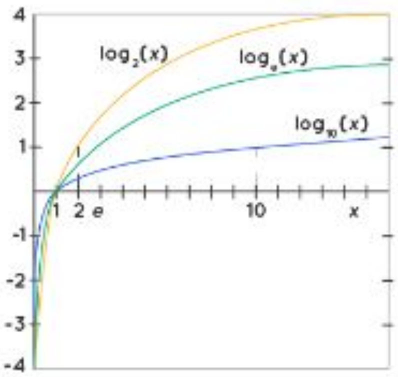
Regularization



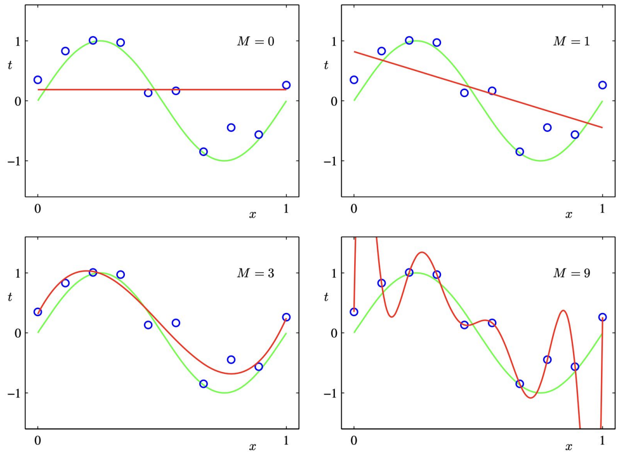
$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

Table 1.2 Table of the coefficients $\mathbf{w}^{\star}$ for $M = 9$ polynomials with various values for the regularization parameter λ. Note that $\ln\lambda = -\infty$ corresponds to a model with no regularization, i.e., to the graph at the bottom right in Figure 1.4. We see that, as the value of λ increases, the typical magnitude of the coefficients gets smaller.

	$\ln\lambda = -\infty$	$\ln\lambda = -18$	$\ln\lambda = 0$
$w_0^{\star}$	0.35	0.35	0.13
$w_1^{\star}$	232.37	4.74	-0.05
$w_2^{\star}$	-5321.83	-0.77	-0.06
$w_3^{\star}$	48568.31	-31.97	-0.05
$w_4^{\star}$	-231639.30	-3.89	-0.03
$w_5^{\star}$	640042.26	55.28	-0.02
$w_6^{\star}$	-1061800.52	41.32	-0.01
$w_7^{\star}$	1042400.18	-45.95	-0.00
$w_8^{\star}$	-557682.99	-91.53	0.00
$w_9^{\star}$	125201.43	72.68	0.01



Linear to Polynomial Regression + Regularization



Learned function is **nonlinear**

$$
\begin{aligned}
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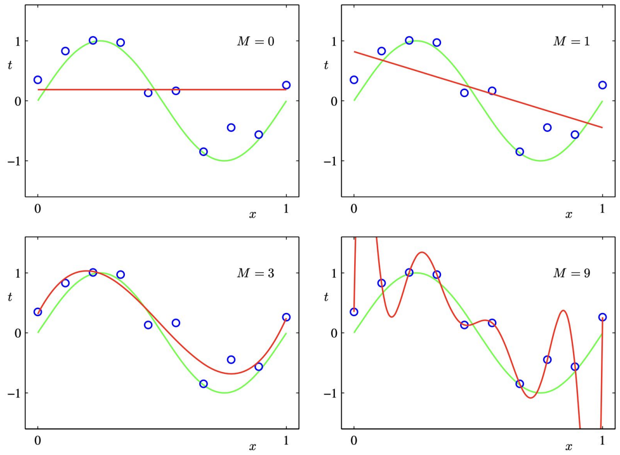
\end{aligned}
$$

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

Linear to Polynomial Regression + Regularization



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$$\widetilde{E}(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

Model (still) **linear**

Classification

- General Idea (two steps process)
  - LR (Bias Only)
  - LR (general)

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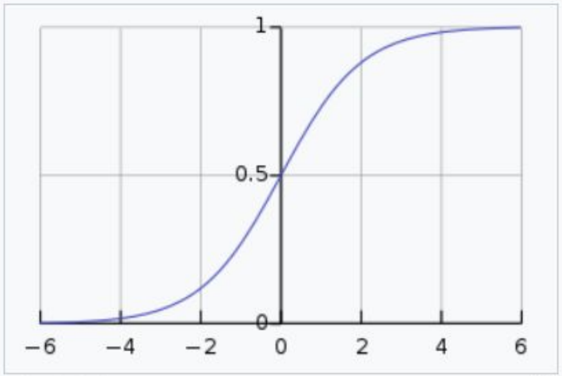
- General Idea (two steps process)
  - LR (Bias Only)
  - LR (general)
- **Logistic Regression (one single step, but with probability distribution)**
  - **The famous sigmoid function**
  - **In NN they call it activation function**



The logistic curve

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Notebook presentation

- Without regularizer
- With regularizer

Predictive modeling: [Regression (diabetes)]

Predictive modeling: [Classification]