



CIS 635 Knowledge Discovery & Data Mining

- Singular Value Decomposition (SVD)



Singular Value Decomposition (SVD)

- Another Linear Dimensionality Reduction Technique
- Different than PCA



PCA steps – Recall

1. Let assume $D_{m \times n}$ is a given data matrix
2. Apply standard scalar (normalization)
3. Estimate Covariance (matrix), $A_{n \times n}$
4. Compute Eigenvalues and Eigenvectors of the Covariance Matrix by solving the linear dynamical system at the right

If A is a $n \times n$ matrix, solving this linear dynamical system will give n **eigenvalues**, and n associated n **eigenvectors**

$$AX = \lambda X$$

$$AX - \lambda X = 0$$

or

$$(A - \lambda I) X = 0$$



SVD

- A is your Data Matrix (Transposed)
- **SVD** decomposes A into 3 matrices: U, S, V

$$A_{m \times n} = U_{m \times m} S_{m \times n} V_{n \times n}^T$$

$$\begin{array}{ccccc} & A & & U & S & V^T \\ \left(\begin{array}{ccc} x_{11} & x_{12} & x_{1n} \\ & \ddots & \\ x_{m1} & & x_{mn} \end{array} \right) & = & \left(\begin{array}{cc} u_{11} & u_{m1} \\ & \ddots \\ u_{1m} & u_{mm} \end{array} \right) & \left(\begin{array}{ccc} \sigma_1 & & 0 \\ & \ddots & \\ 0 & \sigma_r & \\ & & \ddots \\ 0 & & & 0 \end{array} \right) & \left(\begin{array}{cc} v_{11} & v_{1n} \\ & \ddots \\ v_{n1} & v_{nn} \end{array} \right) \\ & m \times n & & m \times m & m \times n & n \times n \end{array}$$

SVD

- A is your Data Matrix (Transposed)
- **SVD** decomposes A into 3 matrices: U, S, V
- Say we are given,

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

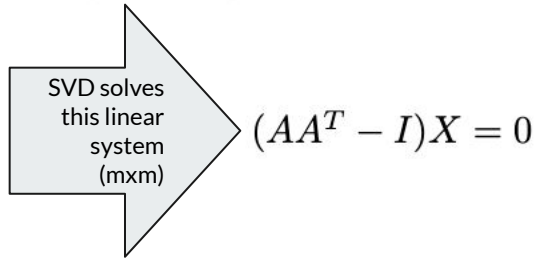
$$A_{m \times n} = U_{m \times m} S_{m \times n} V_{n \times n}^T$$

$$\begin{matrix} & A & & U & & S & & V^T \\ \begin{pmatrix} x_{11} & x_{12} & x_{1n} \\ & \ddots & \\ x_{m1} & & x_{mn} \end{pmatrix} & = & \begin{pmatrix} u_{11} & & u_{m1} \\ & \ddots & \\ u_{1m} & & u_{mm} \end{pmatrix} & \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_r & & 0 \\ & & & \ddots & \\ 0 & & & & 0 \end{pmatrix} & \begin{pmatrix} v_{11} & & v_{1n} \\ & \ddots & \\ v_{n1} & & v_{nn} \end{pmatrix} \\ & m \times n & & m \times m & & m \times n & & n \times n \end{matrix}$$

SVD

- A is your Data Matrix (Transposed)
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- Say we are given,

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$



$$AA^T = \begin{pmatrix} 17 & 8 \\ 8 & 17 \end{pmatrix}$$

eigenvalues: $\lambda_1 = 25, \lambda_2 = 9$

eigenvectors

$$u_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \quad u_2 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

SVD

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- Say we are given,

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

And

SVD solves
this linear
system (nxn)

$$(A^T A - I)X' = 0$$

$$A^T A = \begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{pmatrix}$$

eigenvalues: $\lambda_1 = 25, \lambda_2 = 9, \lambda_3 = 0$

eigenvectors

$$v_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1/\sqrt{18} \\ -1/\sqrt{18} \\ 4/\sqrt{18} \end{pmatrix} \quad v_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ -1/3 \end{pmatrix}$$

SVD

- A is your Data Matrix (Transposed)
- SVD decomposes A into 3 matrices: U, S, V
- Say we are given,

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

$$A_{m \times n} = U_{m \times m} S_{m \times n} V_{n \times n}^T$$

$$A = USV^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{18} & -1/\sqrt{18} & 4/\sqrt{18} \\ 2/3 & -2/3 & -1/3 \end{pmatrix}$$

[Full derivation](#)



SVD

[Notebook Link](#)