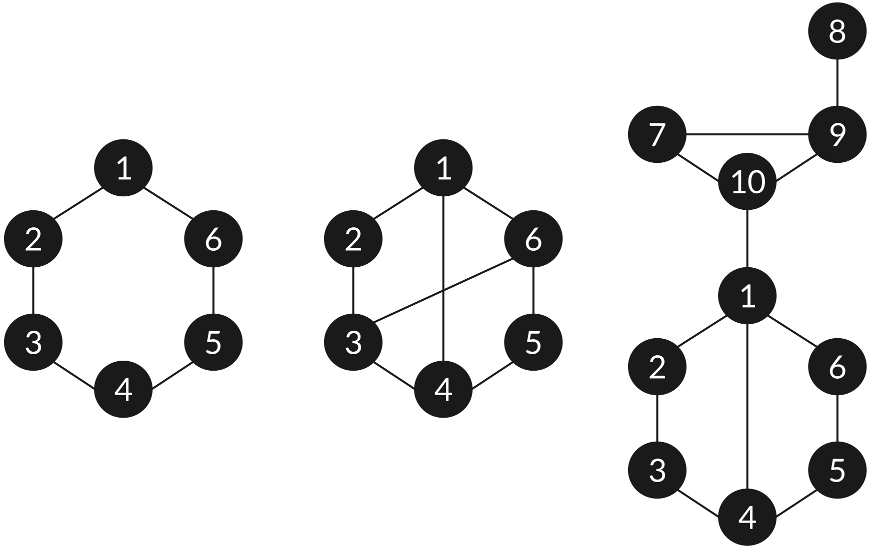
CIS 263 Introduction to Data Structures and Algorithms

Graph Algorithms (Spanning Trees)

Spanning Tree

$G = (V, E)$
$V = \{1, 2, 3, 4, 5, 6\}$
$E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{6, 1\}\}$



Graph with Cycles

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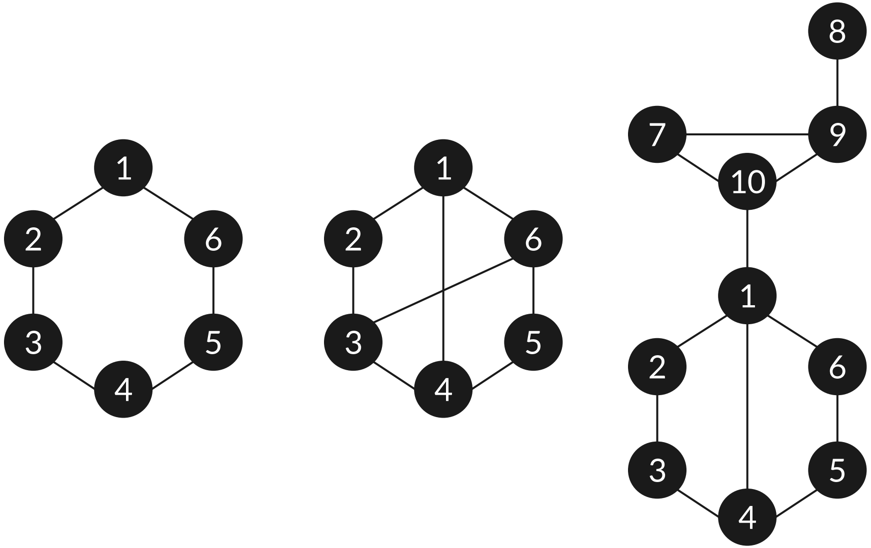
- We want to get rid of cycles in a graph
- Many applications:
  - TSP for an example
  - Networking and networking algorithms



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- Many applications:
  - TSP for an example
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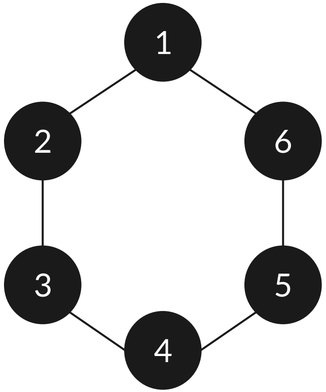
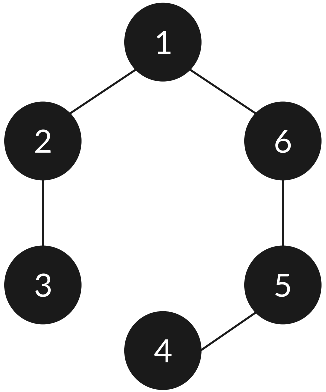
- **For this particular example, you can drop any edges**





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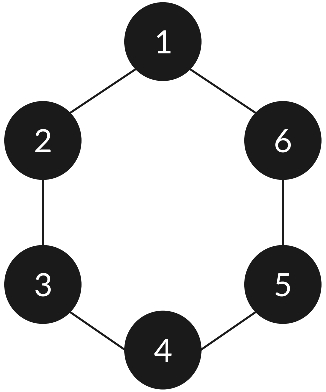
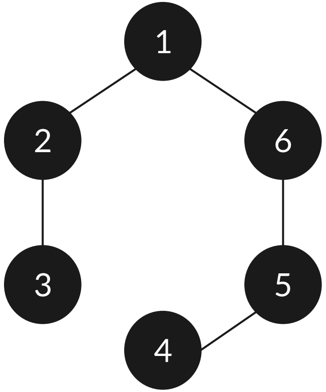
- **A Spanning Tree is a sub graph (of its parent graph)**





$S = (V', E')$
$V' = V = \{1, 2, 3, 4, 5, 6\}$
$E' = \{\{1, 2\}, \{2, 3\}, \cancel{\{3, 4\}}, \{4, 5\}, \{5, 6\}, \{6, 1\}\}$

$|V| = 6$
$|E'| = |V| - 1$

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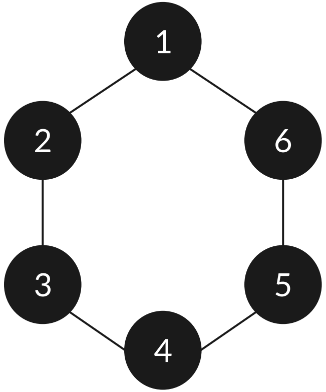
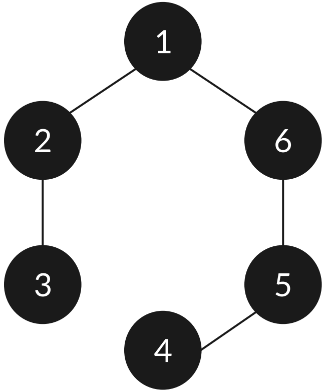
- We want to get rid of cycles in a graph
- Many applications:
  - TSP for an example
  - Networking and networking algorithms
- For this particular example, you can drop any edges
- A Spanning Tree is a sub graph (of its parent graph)
- **Nb of possibilities (for this particular example): *6***





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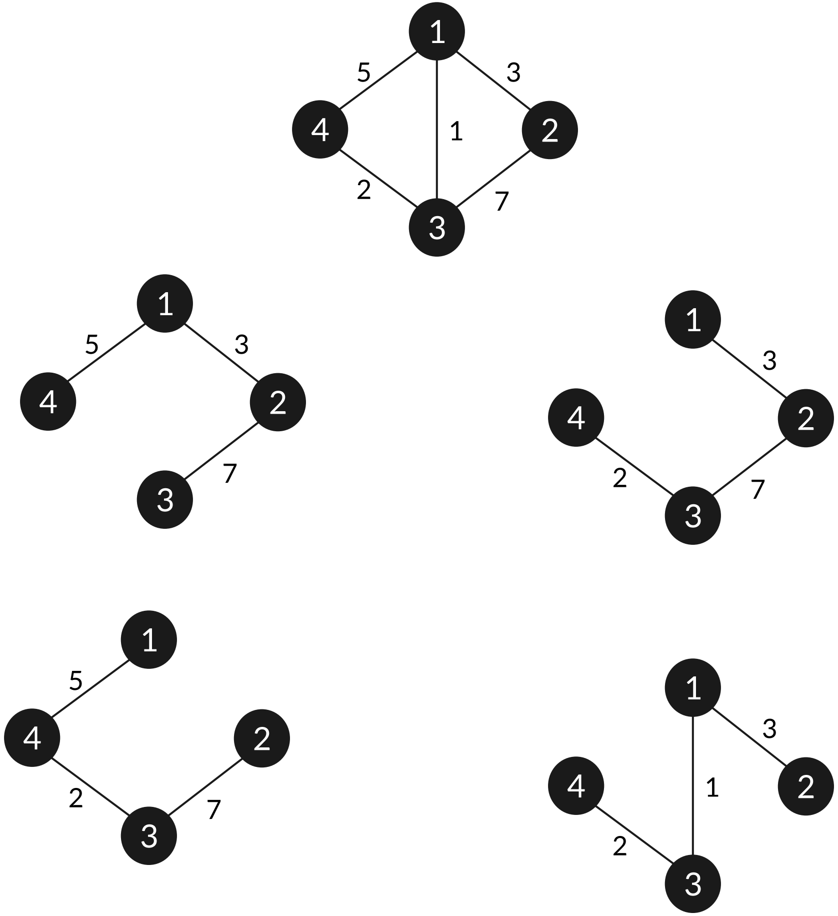
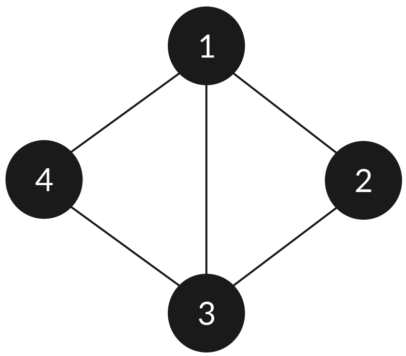
Minimum Spanning Tree

- We have to find spanning Trees with the minimum cost
  - Search all combinations
  - Approximation Algorithms
    - Greedy Algorithms
      - Prim’s Algorithm
      - Kruskal’s Algorithm



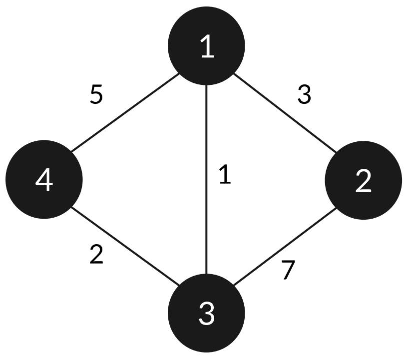
Weighted Graph

- We want to travel from (2) to (4)
- We have different paths
- **Counting the number of edges** can be assumed as an optimization metric.



Weighted Graph

- We want to travel from (2) to (4)
- We have different paths
- What if explicit weights are associated
    - (2, 1, 3, 4), 3 edges but cost is 6 (the minimum)



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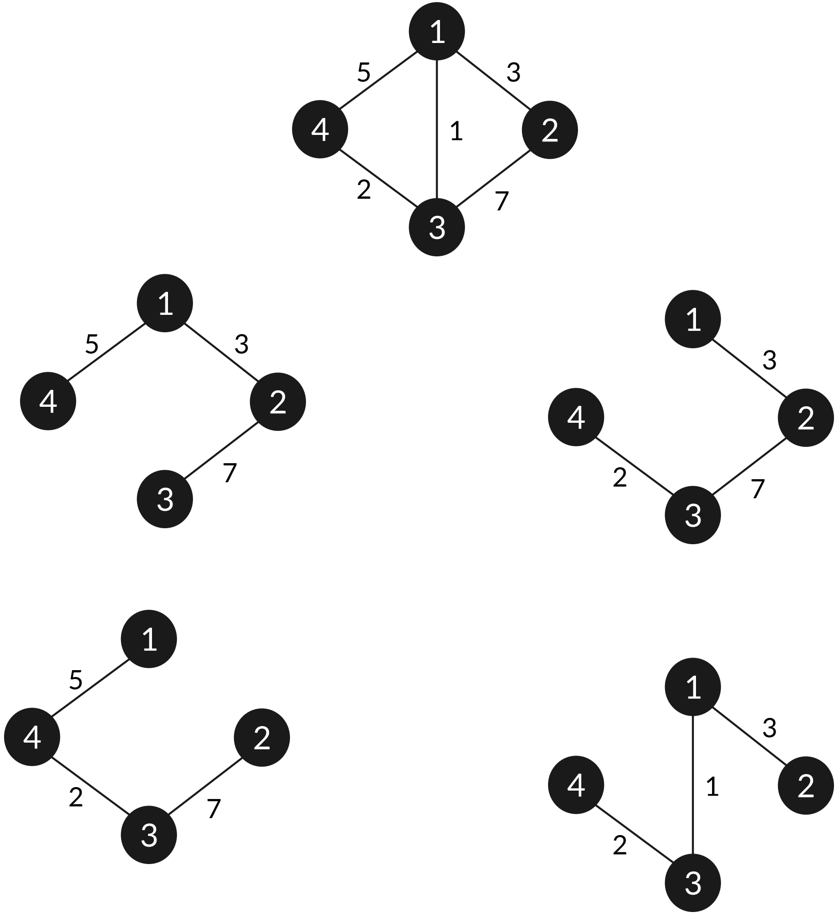

Minimum Spanning Tree

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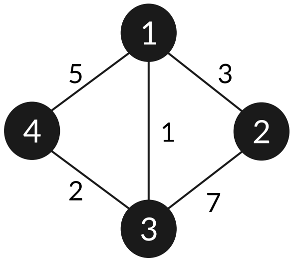

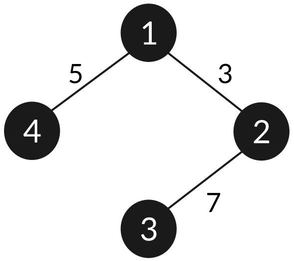

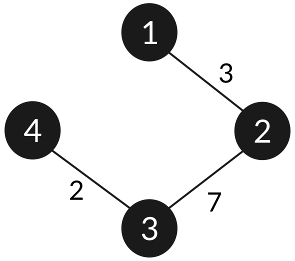

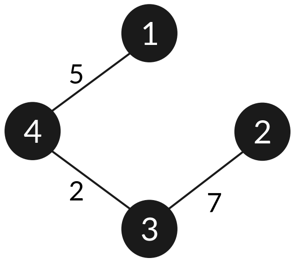

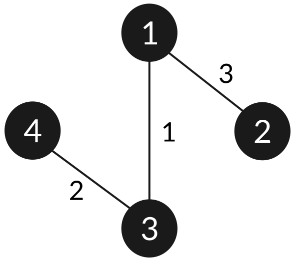

Minimum Spanning Tree

- We have to find spanning Trees with the minimum cost
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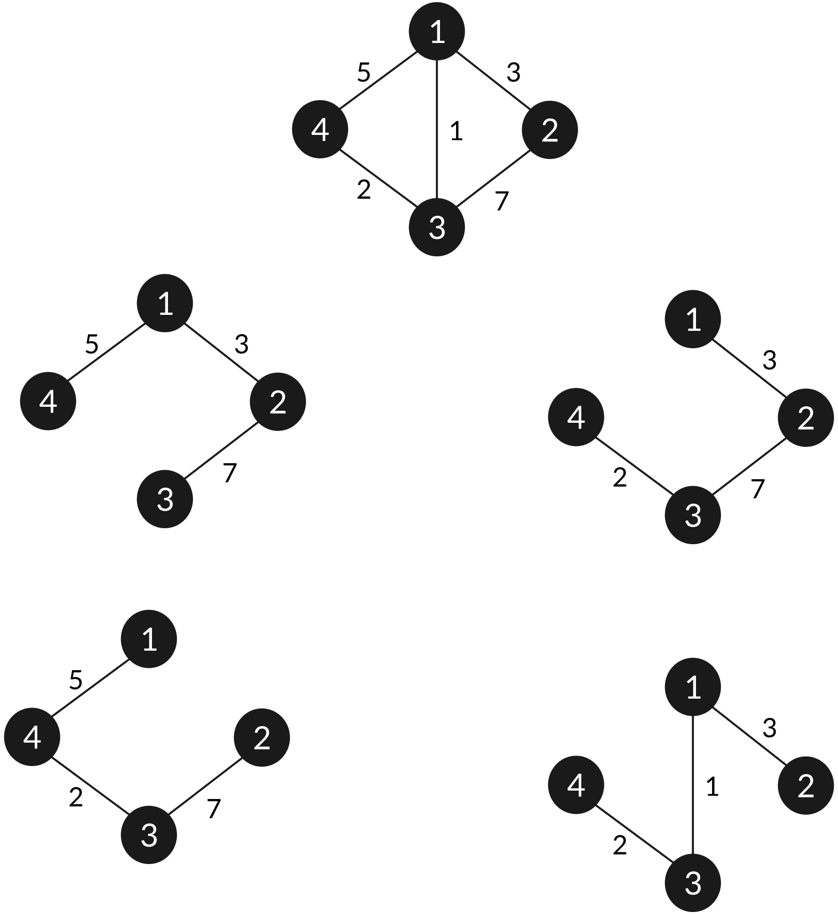
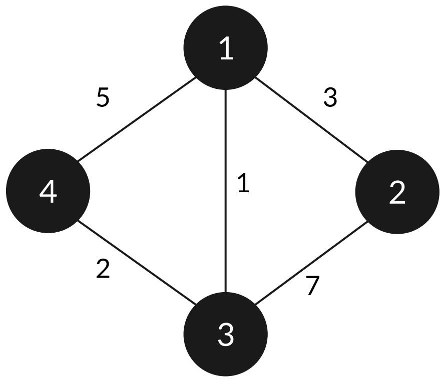


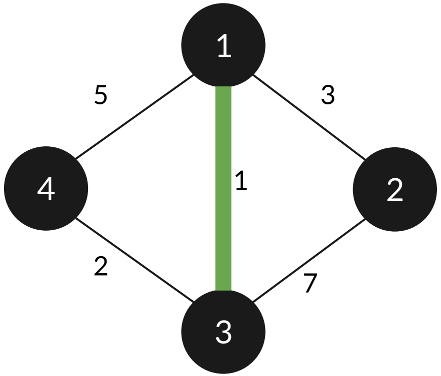
- Uses heuristics
- Solution may not be optimal

Prim’s Algorithm



Prim’s Algorithm

- **Start with the minimum cost edge**
- Pick the next minimum cost edge (but form the connected set)
  - Iterate but avoid loops



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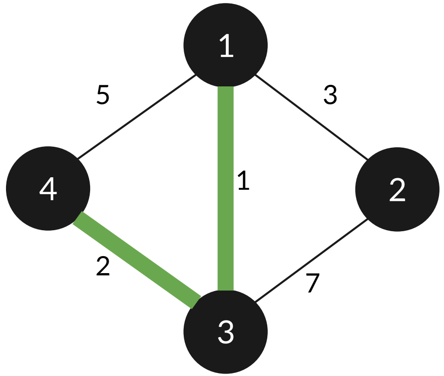

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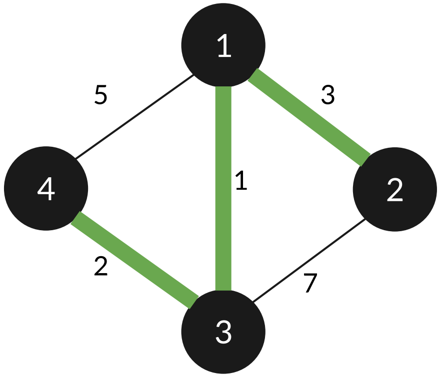

S = (*V*′, *E*′) ⊂ (*V*, *E*)
|*V*| = |*V*′| = *4*
|*E*′| = |V| - 1

Cost: 2 + 1 + 3 = 6

Kruskal's Algorithm

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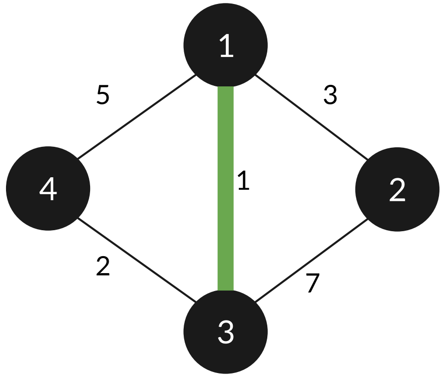

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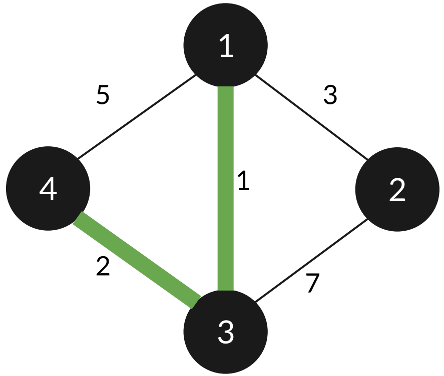

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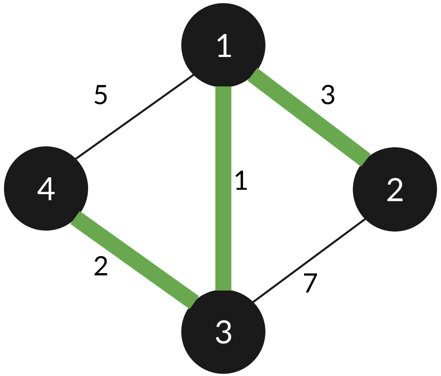

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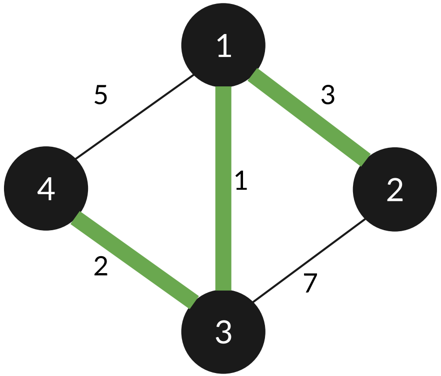

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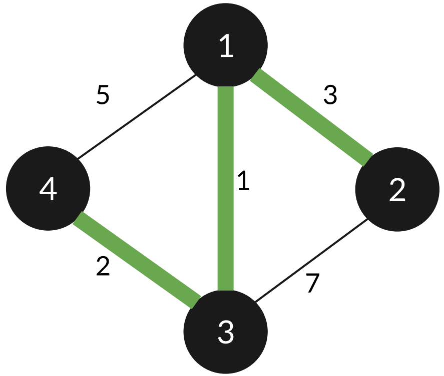

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- Exactly same solution
- Same order

Lets try a little complex one

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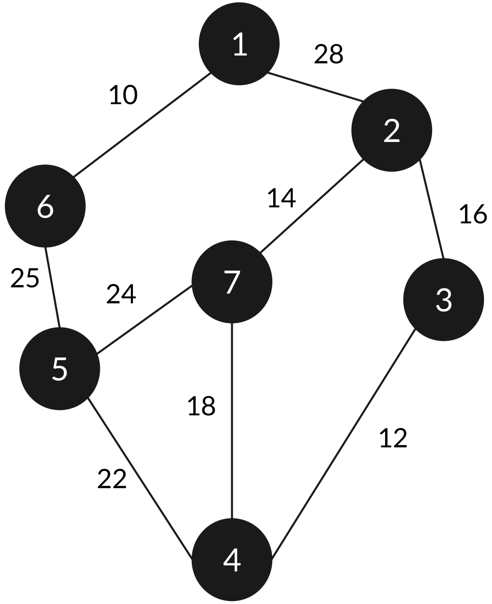

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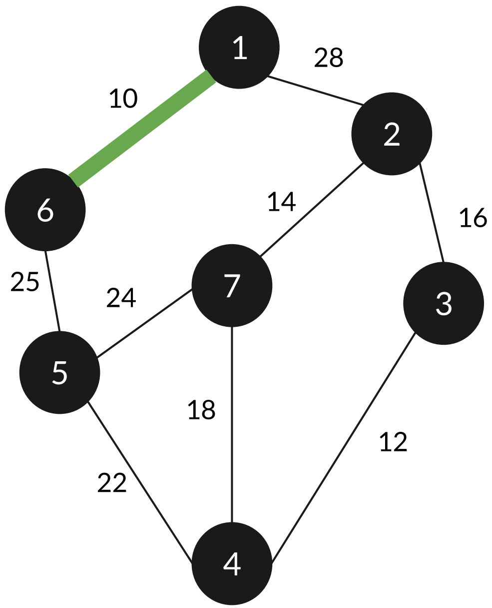

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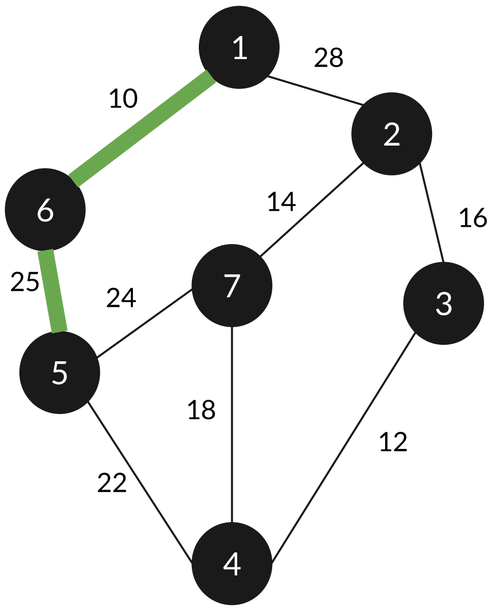

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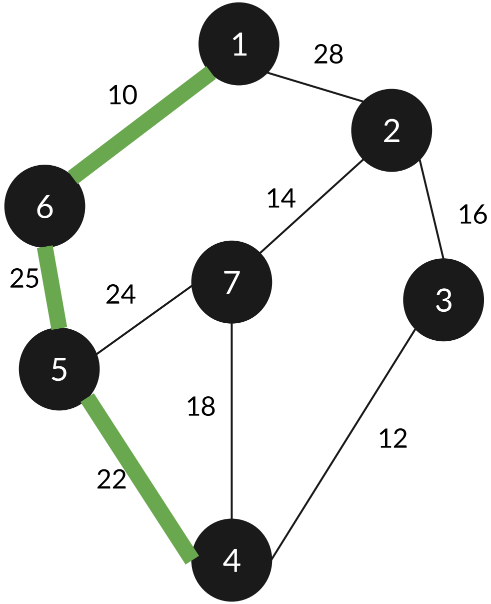

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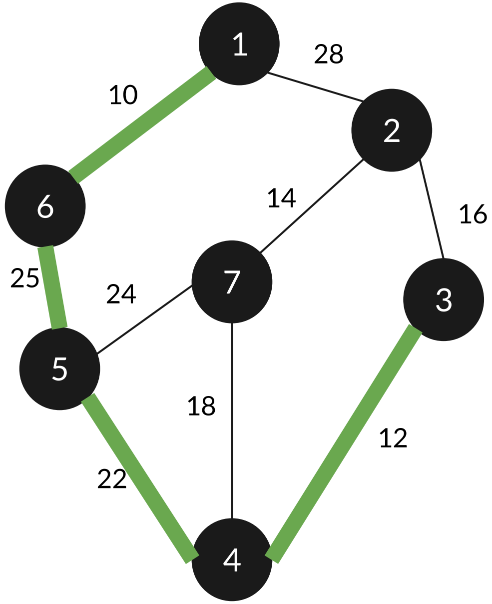

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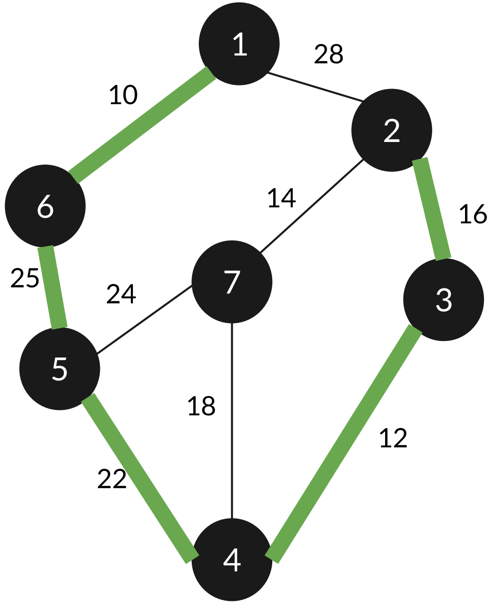

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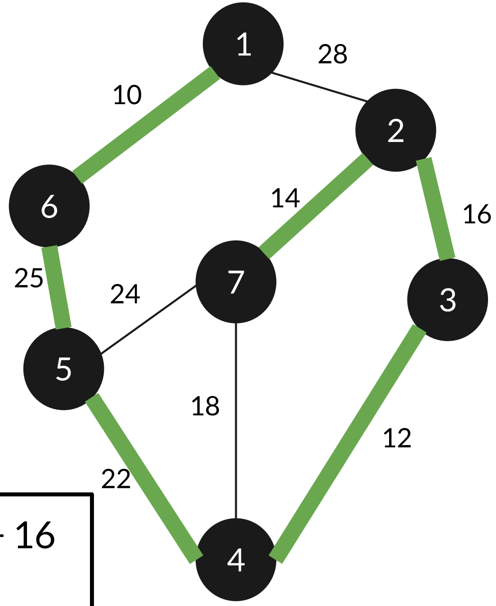

$S = (V', E') \subset (V, E)$
$|V| = |V'| = 7$
$|E'| = |V| - 1$

Cost: 10 + 25 + 22 + 12 + 16 + 14 = 99

Kruskal's Algorithm

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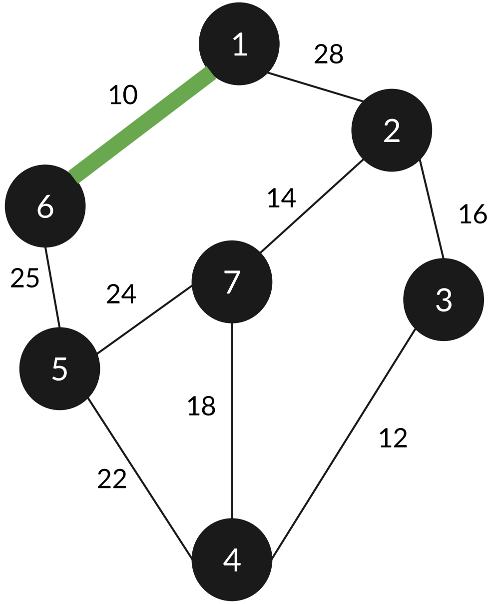

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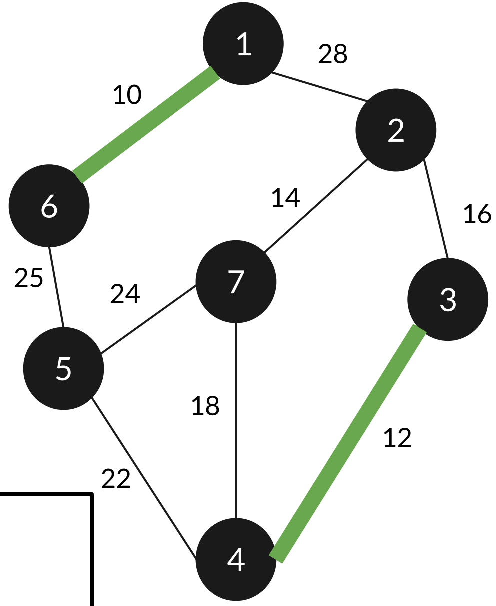

Cost: 10

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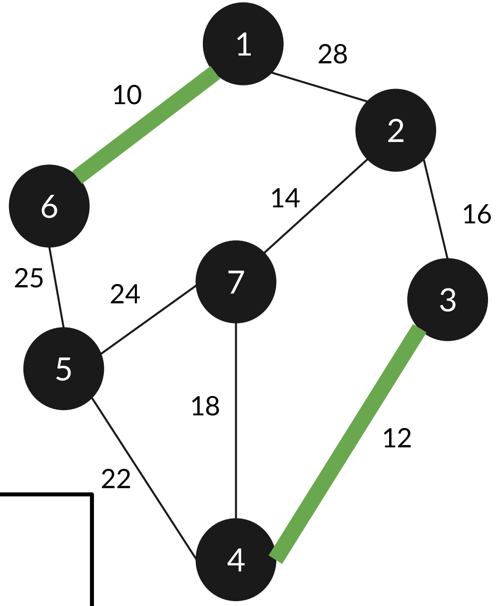

Cost: 10 + 12

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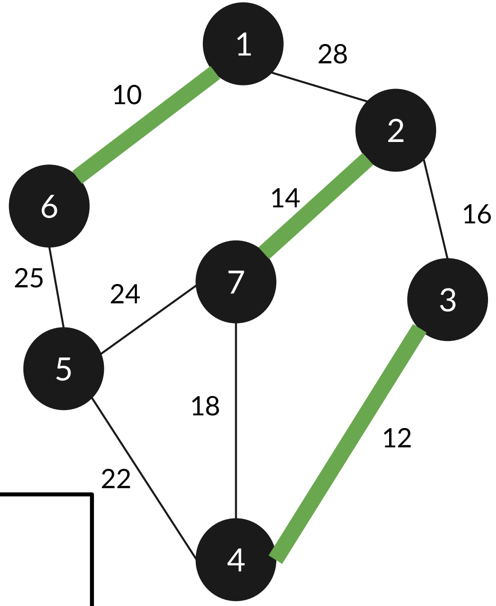

Cost: 10 + 12 + 14

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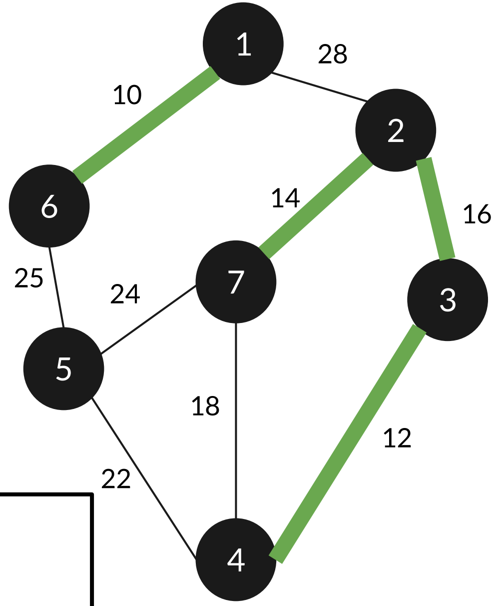

Cost: 10 + 12 + 14 + 16

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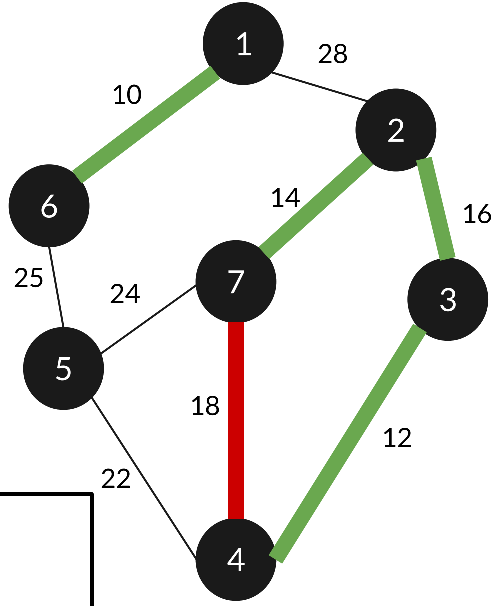

Cost: 10 + 12 + 14 + 16

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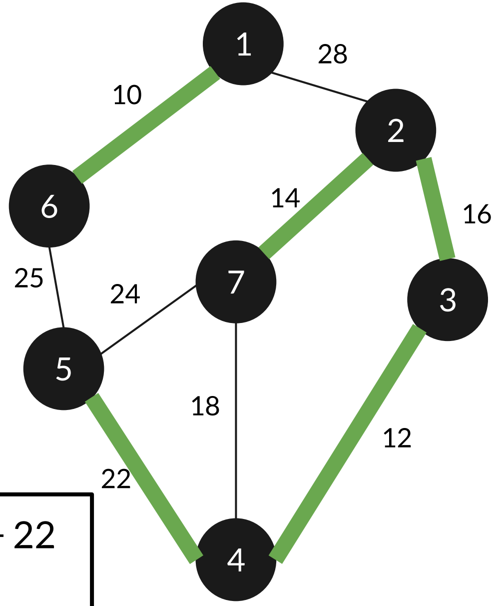

Cost: 10 + 12 + 14 + 16 + 22

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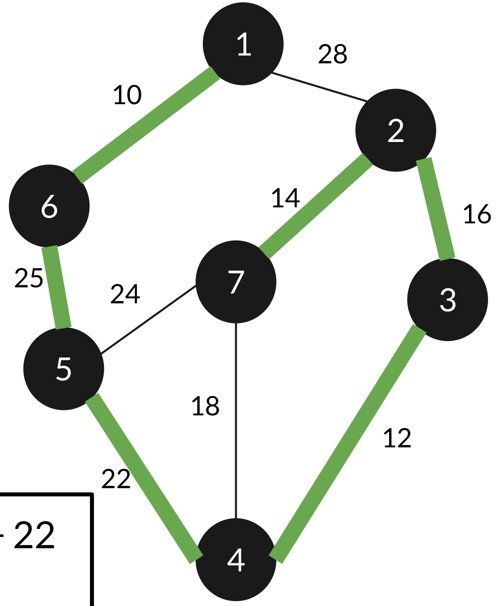

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Cost: 10 + 12 + 14 + 16 + 22 + 25 = 99

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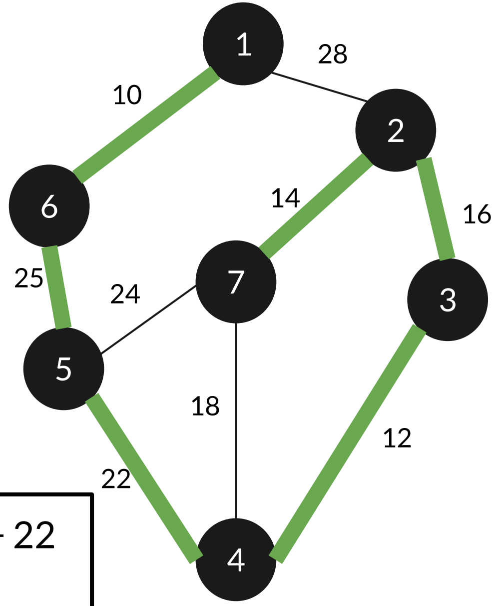

- Exactly same solution
- **Different order**

Cost: 10 + 12 + 14 + 16 + 22 + 25 = 99

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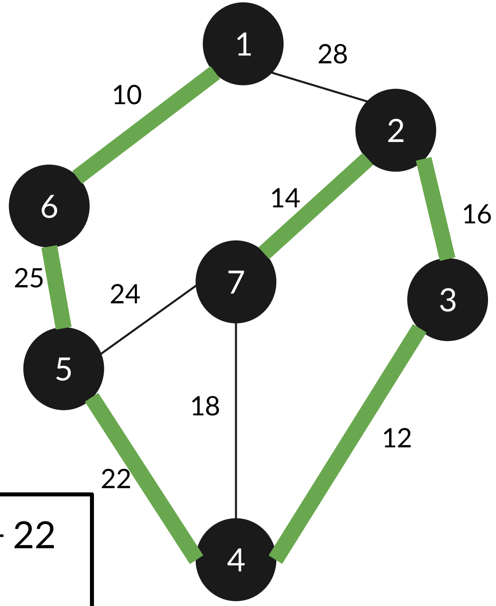

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QA