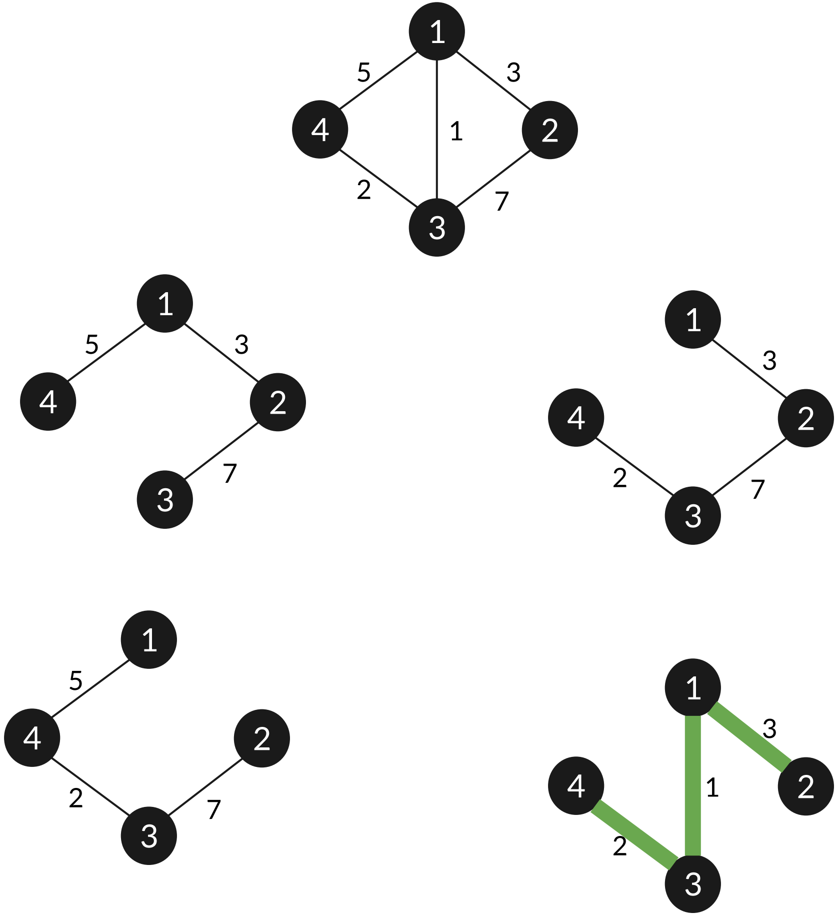
CIS 263 Introduction to Data Structures and Algorithms

Backtracking

Minimum Spanning Tree

- We have to find spanning Trees with the minimum cost
  - Search all combinations
  - Approximation Algorithms
    - **Greedy Algorithms**
      - Prim’s Algorithm
      - Kruskal’s Algorithm

- Uses heuristics
- Solution may not be optimal



Optimization Problems

Greedy Algorithms

- Mainly Heuristics based

Bruteforce

- Constraints based

Optimization Problems

Greedy Algorithms

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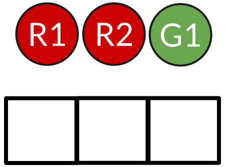
- Constraints based
- Backtracking

Backtracking

- You have to fill the boxes
- No condition
- 3! solutions



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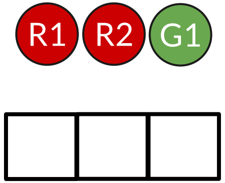
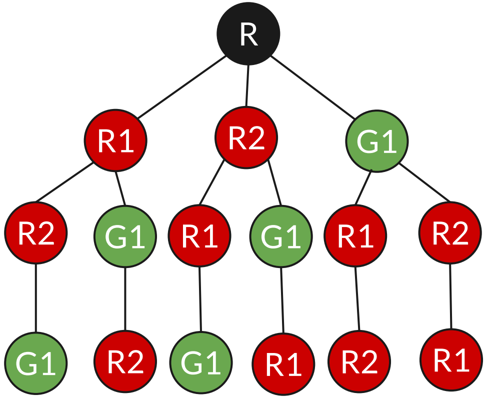
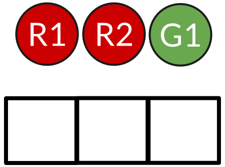


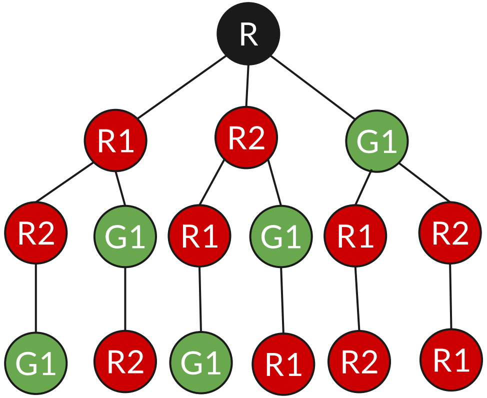


State Space Tree

Backtracking

- You have to fill the boxes
- Conditions
  - Green cannot be in the middle
- Exclude some of the 3! Solutions
- Bruteforce search
  - Apply condition as a bounding function
  - Apply Backtracking





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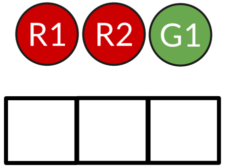
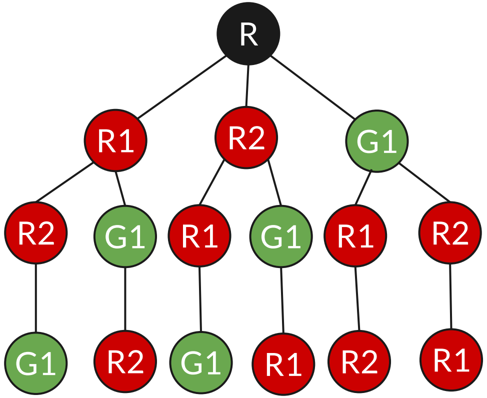
  - Apply Backtracking (Preorder Traversal)





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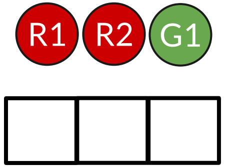
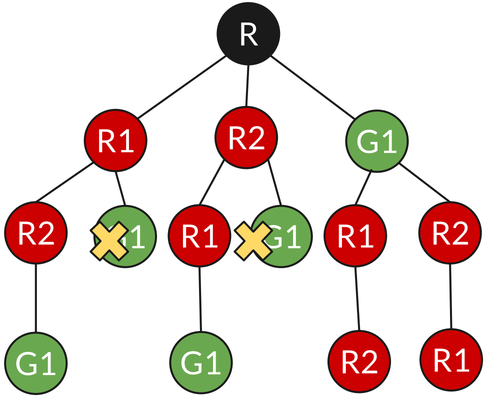
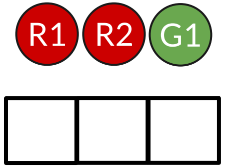


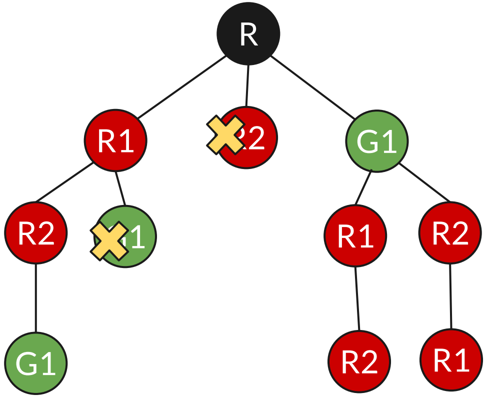


State Space Tree

Backtracking

- You have to fill the boxes
- Conditions
  - Green cannot be in the middle
  - **R2 cannot be in the first box**
- Exclude some of the 3! Solutions
- Bruteforce search
  - Apply condition as a bounding function
  - Apply Backtracking (Preorder Traversal)



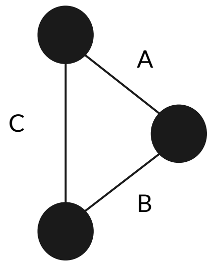


State Space Tree

Backtracking

Graph Coloring Problem

- Condition: Two adjacent nodes cannot have the same color
- Can we do it with 2 colors say {Red, Green} ?



Backtracking

?? Problem

- You are asked to make a NHL schedule, and here are the conditions:
    - You have 4 teams: {A, B, C, D}
    - A team can play only one game a day
    - Teams play following a certain format
- Q: What’s the minimum number days your schedule will require?

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- Lets first try for 3 teams

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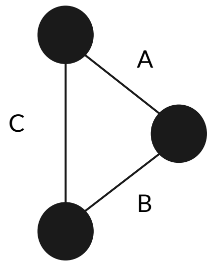

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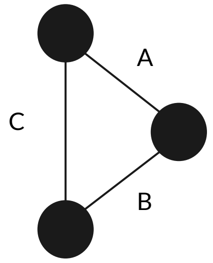
- Q: What's the minimum number days your schedule will require?
- Lets first try for 3 teams: {A, B, C}
- An equivalent question is: What's the minimum number of colors you will require so no two adjacent colors are the same?



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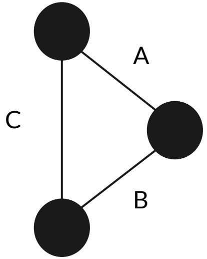

Backtracking

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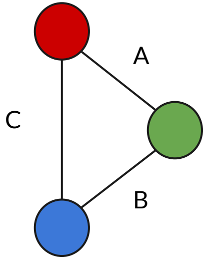
  - Graph Coloring Problem
- Answer: 3 days
- Can we do it in less than 3 days?
  - Answer: No



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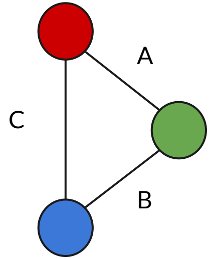
- What are your scheduling options; or How many combinations of days/dates you may have?



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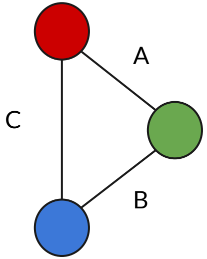
- Answer: 3 days
- Can we do it in less than 3 days?
    - Answer: No
- What are your scheduling options; or How many combinations of days/dates you may have?
    - Answer: We have to generate a SBT for all possibilities just like we did it in our toy example



Let’s go back to our original question!

Let’s go back to our exact question!

Graph Coloring Problem

- You are asked to make a NHL schedule, and here are the conditions:
  - **You have 4 teams: {A, B, C, D}**
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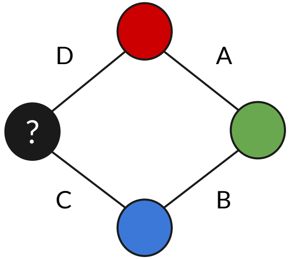
- Q: What’s the minimum number days your schedule will require? 4 or 3 or some other number of days ?



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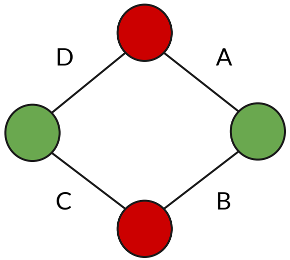
- Answer: 2 days (2 colors), for the setup on the right.



Is this the only solutions?

Let’s go back to our exact question!

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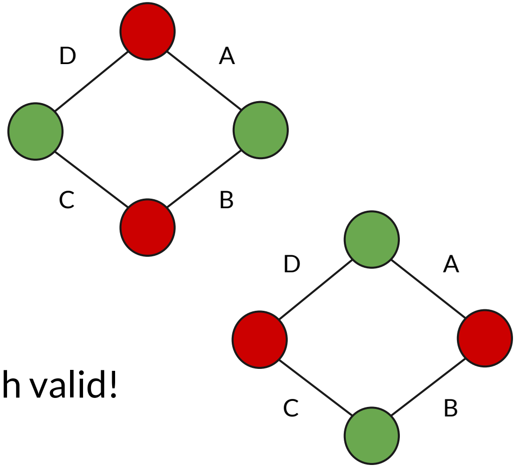

Both valid!

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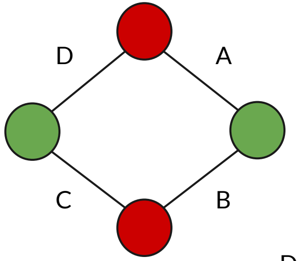
- What are your scheduling options; or How many combinations of days/dates you may have?
  - Backtracking solution



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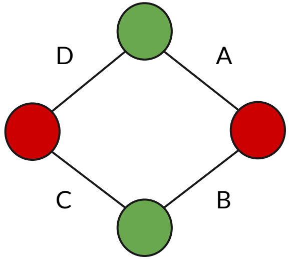
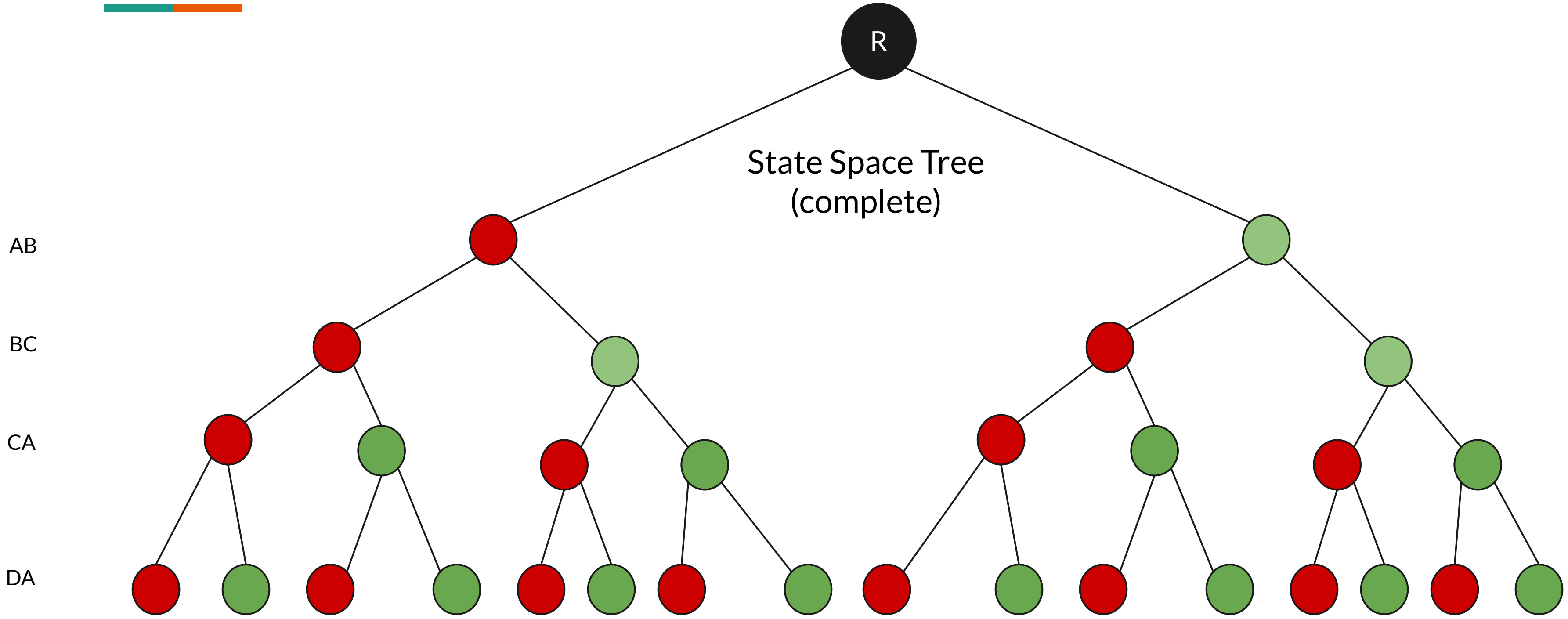
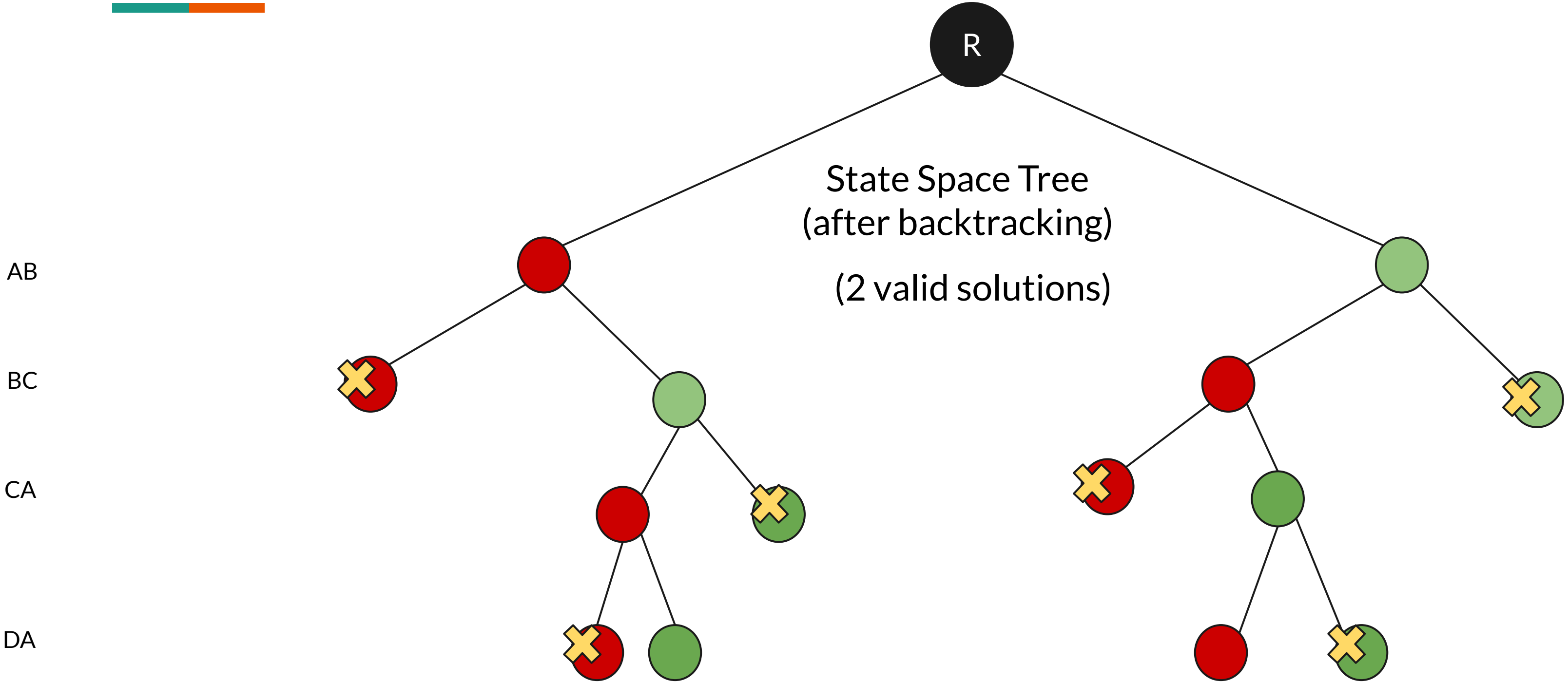



R
State Space Tree
(complete)
AB
BC
CA
DA



State Space Tree
(after backtracking)

(2 valid solutions)

Full solution

Let’s go back to our exact question!

Graph Coloring Problem

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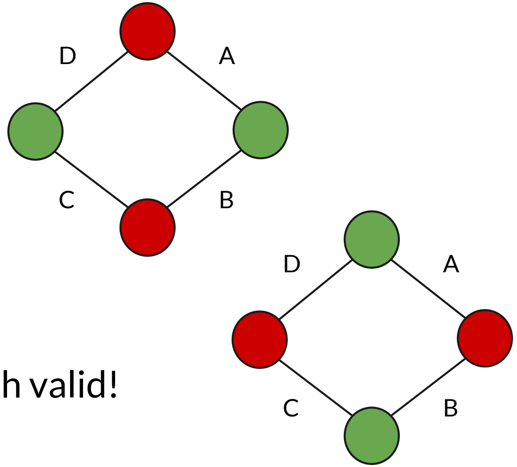

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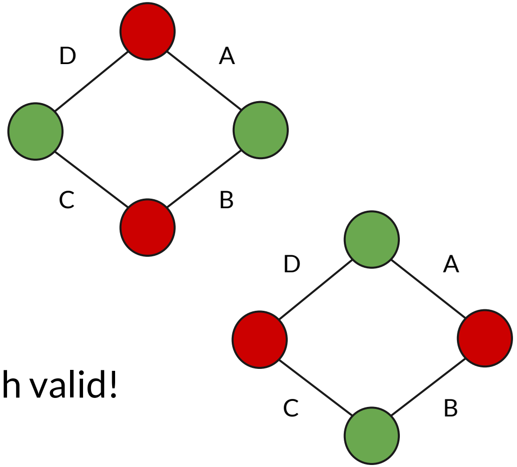
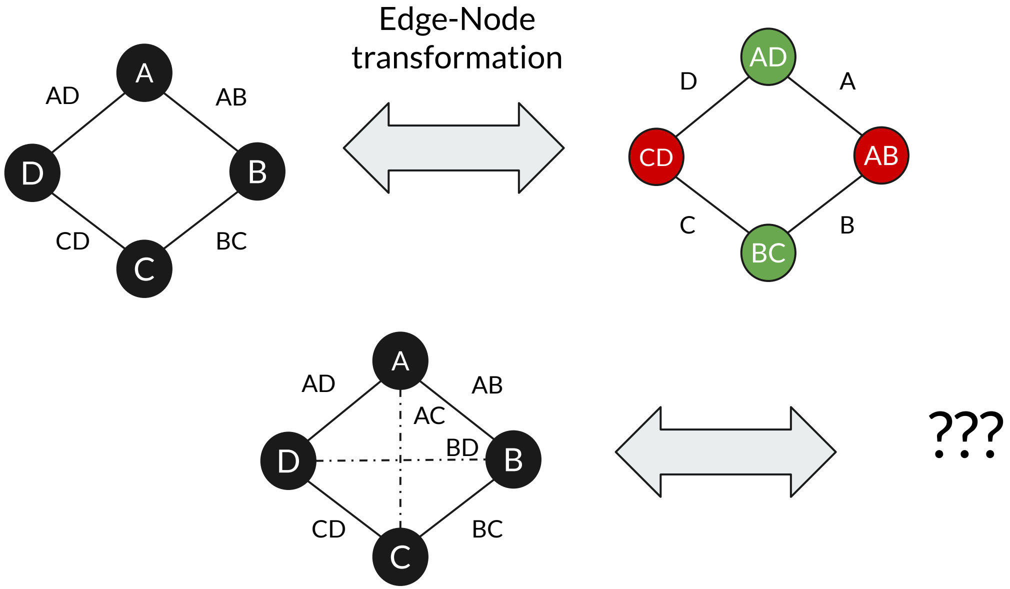


Both valid!

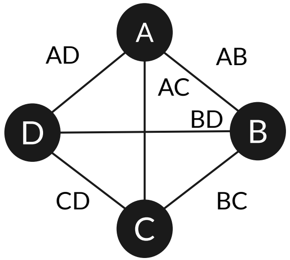
We have a partial solution

Possibilities:

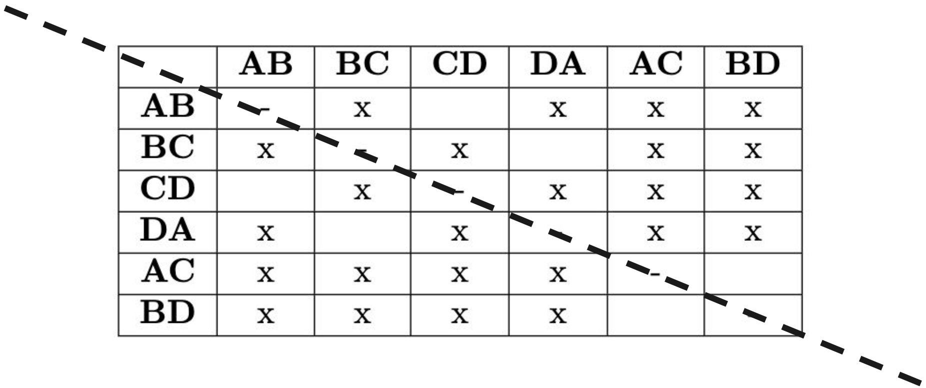
- (AB) ✅
- (BC) ✅
- (CD) ✅
- (AD) ✅
- (AC)
- (BD)



Conflict Table

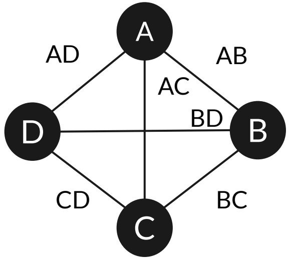


- Parallel representation
- Conflict Table
  - A team cannot participate in two games the same day
  - Will see some other examples



	AB	BC	CD	DA	AC	BD
AB	-	x		x	x	x
BC	x	-	x		x	x
CD		x	-	x	x	x
DA	x		x	-	x	x
AC	x	x	x	x	-	
BD	x	x	x	x		-

Conflict Table



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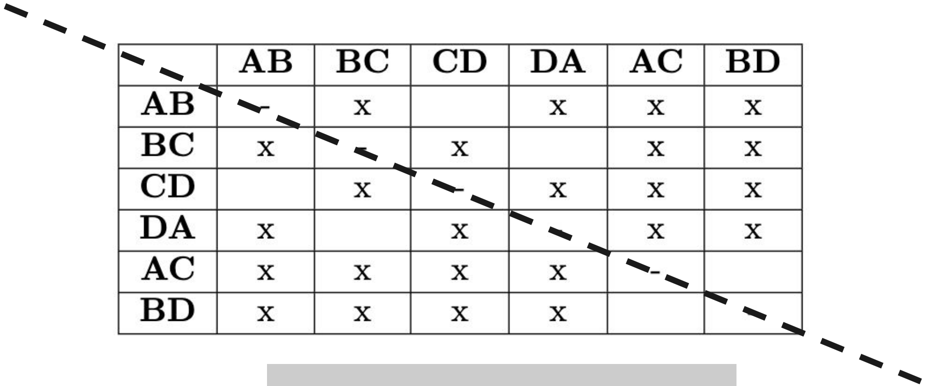
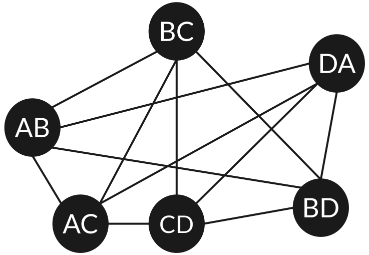


	AB	BC	CD	DA	AC	BD
AB	-	x		x	x	x
BC	x	-	x		x	x
CD		x	-	x	x	x
DA	x		x	-	x	x
AC	x	x	x	x	-	
BD	x	x	x	x		-

Matrix symmetry

Conflict Table to Solution Graph



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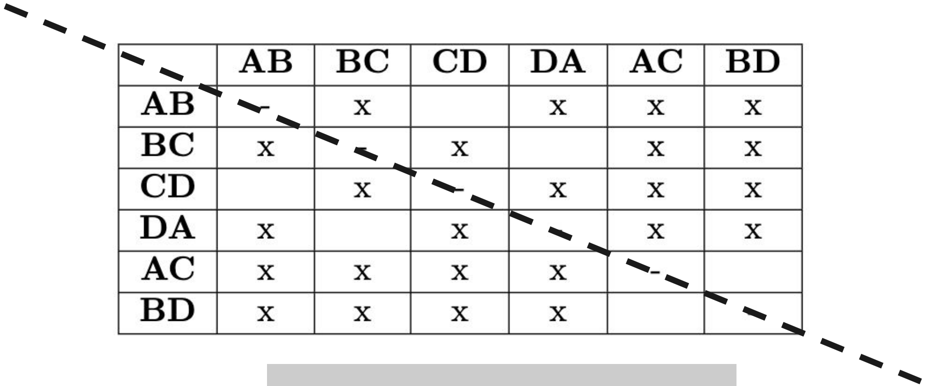


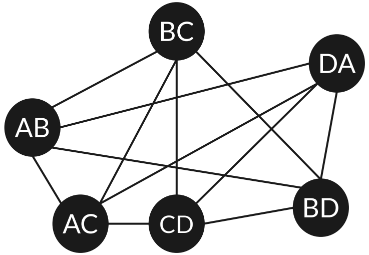
	AB	BC	CD	DA	AC	BD
AB	-	x		x	x	x
BC	x	-	x		x	x
CD		x	-	x	x	x
DA	x		x	-	x	x
AC	x	x	x	x	-	
BD	x	x	x	x		-

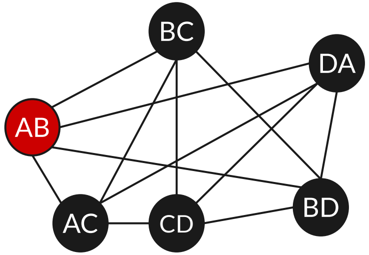
Matrix symmetry

Graph coloring

Greedy Algorithm

- Ordered list of colors
- **Starting node: color with the initial index**



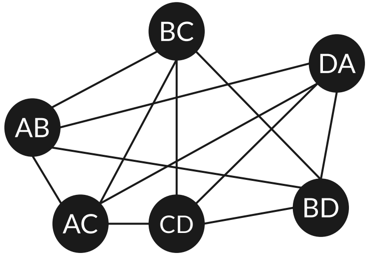


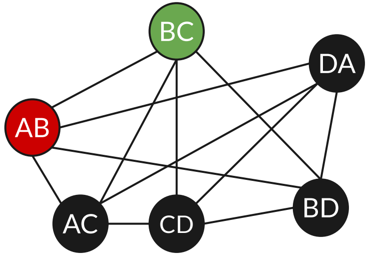
Traversal seq

AB	BC	CD	DA	AC	BD

🔴	🟢	🔵	🟣	🟠
1	2	3	4	5

Graph coloring





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Traversal seq

✖B	✖C	CD	DA	AC	BD

🔴	🟢	🔵	🟣	🟠
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Graph coloring

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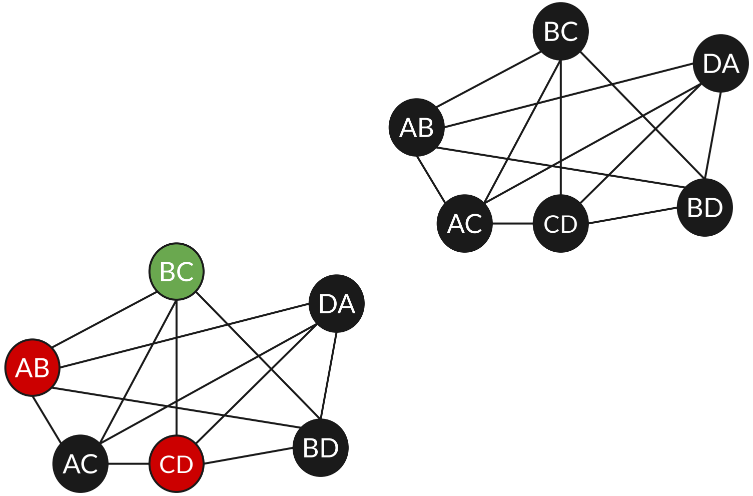

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🔴	🟢	🔵	🟣	🟠
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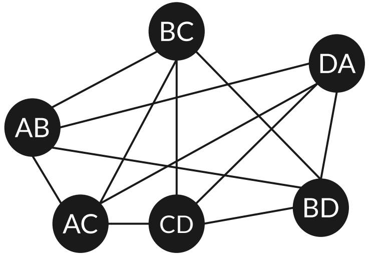

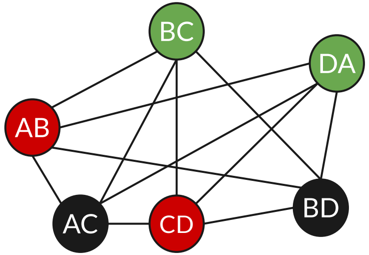

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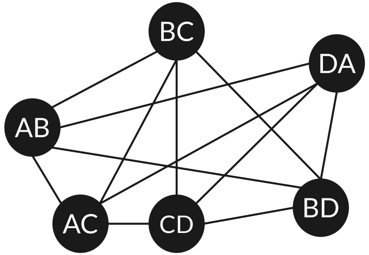

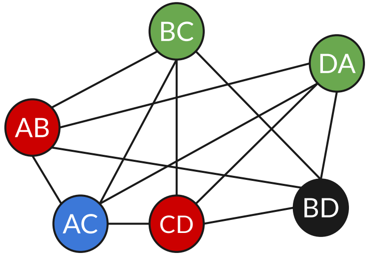

Traversal seq

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🔴	🟢	🔵	🟣	🟠
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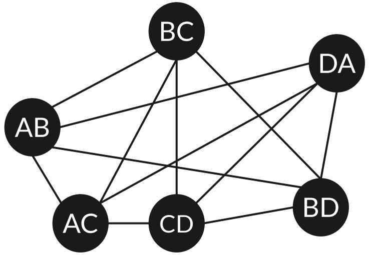
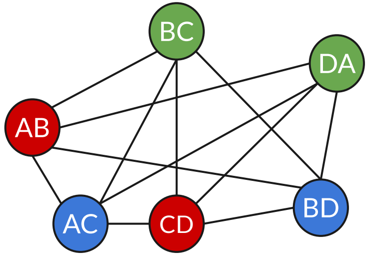
- Ordered list of colors
- **Starting node: color with the initial index**



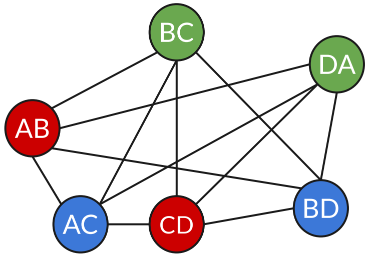


Traversal seq

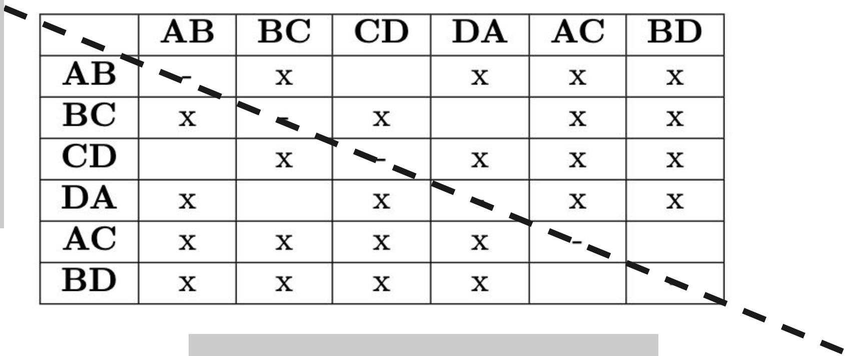
~~AB~~	~~BC~~	~~CD~~	~~DA~~	~~AC~~	~~BD~~

🔴	🟢	🔵	🟣	🟠
1	2	3	4	5

Complete solution



- 3 days (colors) as the final solution!
- Backtracking for all possible solutions
    - Bounding functions:
        - Consecutive nodes with the same color
        - A valid path should have at least 3 unique colors



	AB	BC	CD	DA	AC	BD
AB	-	x		x	x	x
BC	x	-	x		x	x
CD		x	-	x	x	x
DA	x		x	-	x	x
AC	x	x	x	x	-	
BD	x	x	x	x		-

Matrix symmetry

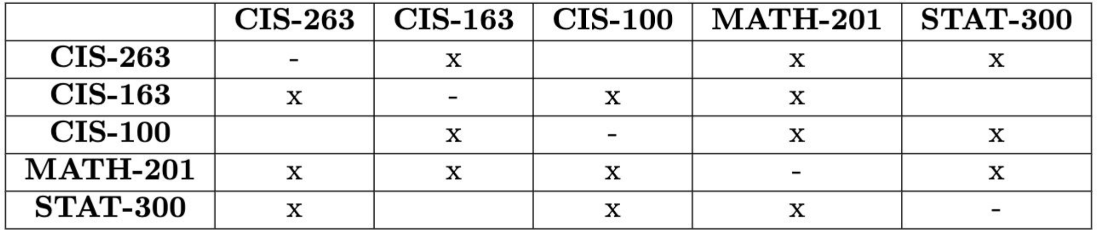
For all scheduling solutions, you have to

- Extend State Space Trees of slides 25-26 from **4 to 6** levels covering all combinations ***{AB, BC, CD, AD, BD, AC}***
- All valid paths to the leaf nodes and also with at least 3 colors will be your solutions!

Similar scheduling problems you can solve!

Similar problems

- Exam date/courses scheduling
- Courses with at least one student common



	CIS-263	CIS-163	CIS-100	MATH-201	STAT-300
CIS-263	-	x		x	x
CIS-163	x	-	x	x	
CIS-100		x	-	x	x
MATH-201	x	x	x	-	x
STAT-300	x		x	x	-

Summary of last few weeks

Graphs:

- BFS, DFS
- Minimum Spanning Trees:
    - Prim’s Algorithm,
    - Kruskal’s Algorithm,
- Backtracking
- Graph Coloring
- Solving Scheduling problems

Priority Queue, Hashing

Heap, Heap Sort

Disjoint Set Class