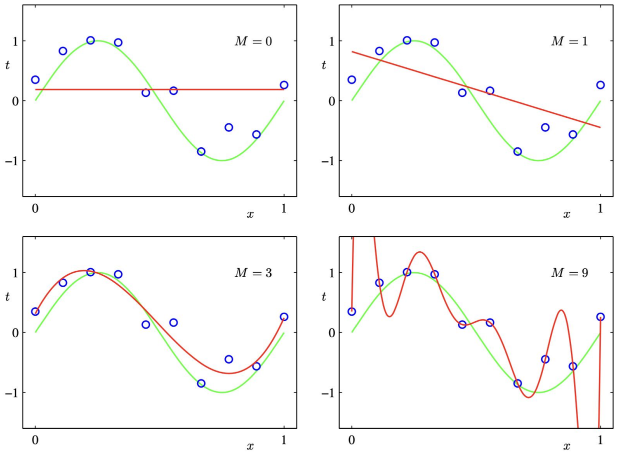
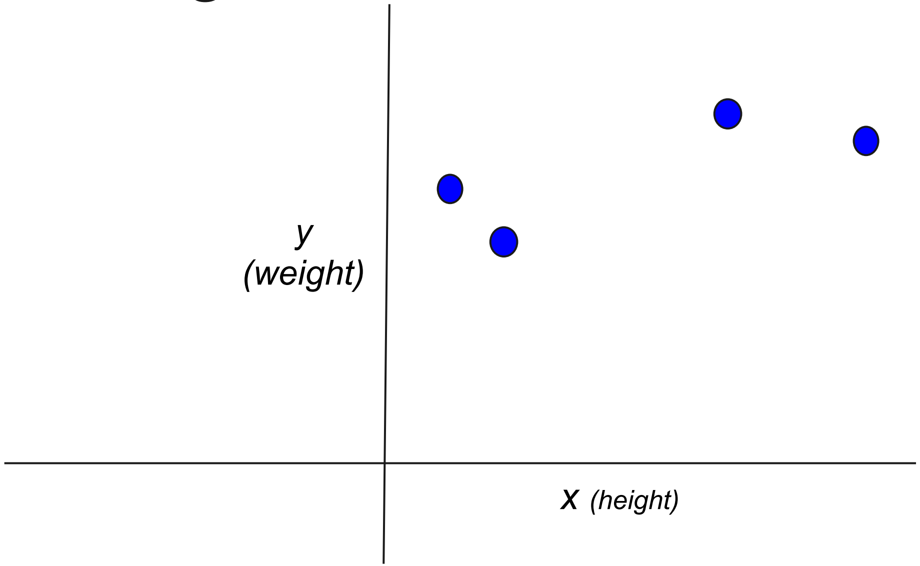
CIS 678 - Machine Learning

...

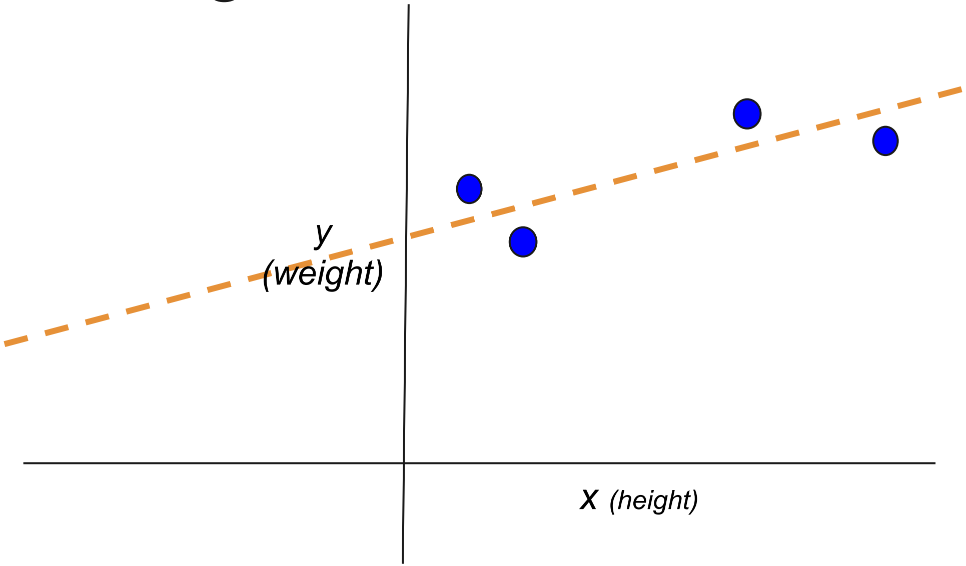
Linear to Polynomial Regression



Linear Regression



Linear Regression



Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

Fitting Error

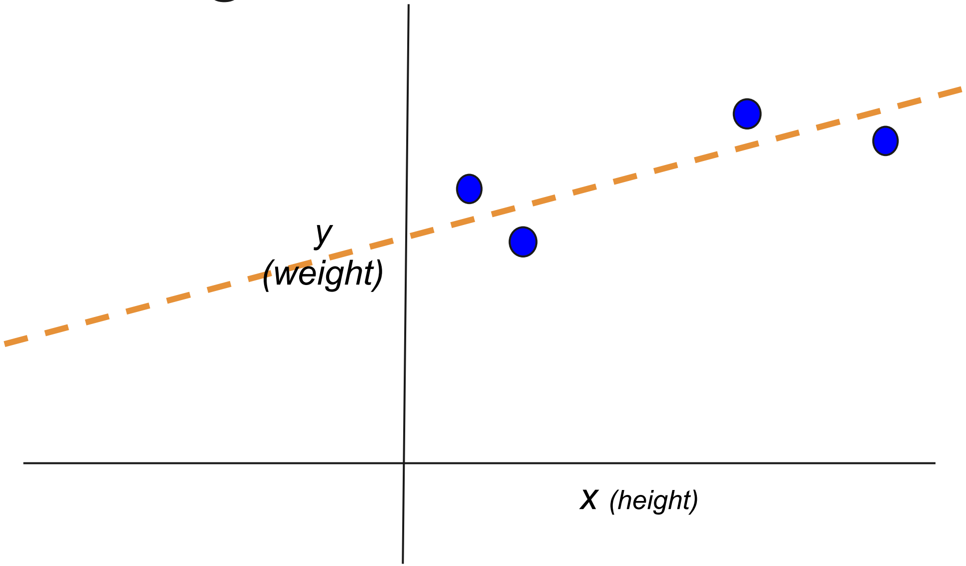
$$\epsilon = |\hat{y} - y|$$

Optimization function

$$E_\Theta = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$$\Theta^* = \mathrm{argmin}_\Theta\, E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

Linear Regression



So, essentially we are fitting a function; right?

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

Fitting Error

$$\epsilon = |\hat{y} - y|$$

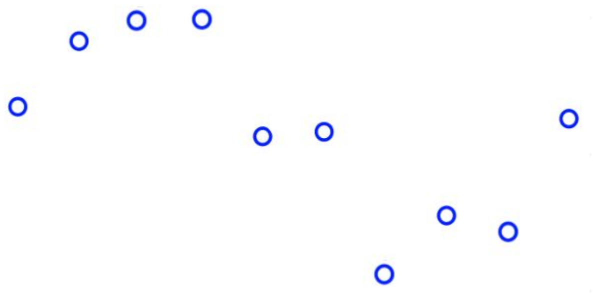
Optimization function

$$E_\Theta = \frac{1}{2} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$\Theta^* = \text{argmin}_\Theta \, E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

Non linear data/function

- Does this data points seem familiar matching a known function?

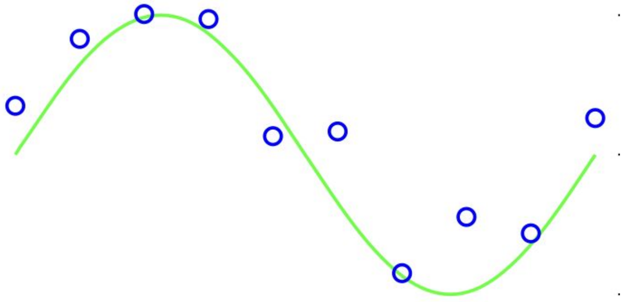


Non linear data/function

- Does this data points seem familiar matching a known function?

    - A Sinusoidal function

$$y(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi)$$

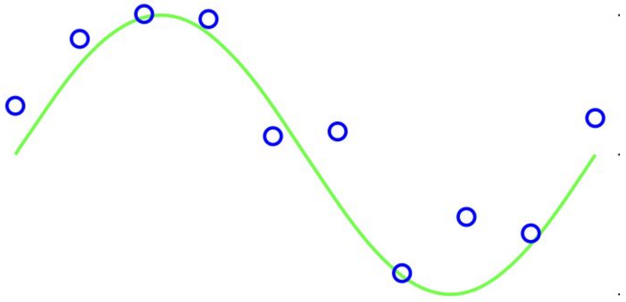


Non linear data/function

- Does this data points seem familiar matching a known function?

    - A Sinusoidal function

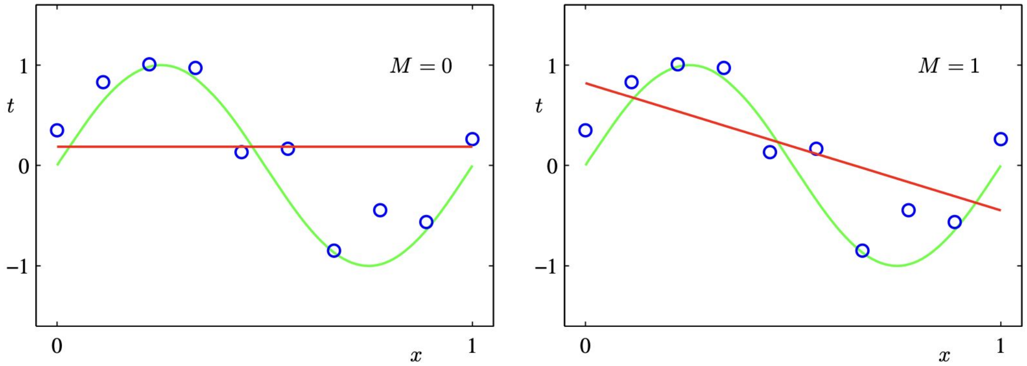
$$y(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi)$$



Clearly this is not a linear function; right?

Non linear data/function

- Does this data points seem familiar matching a known function?
- Can we approximate this function using LR ?



$$\hat{y} = \beta_0 + \beta_1 x$$

LR will not work; right?

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?

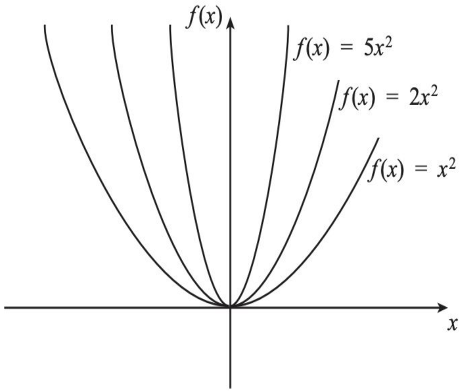
ref

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?
- **Quadratic (x^2)**

$$f(x) = x^2, \qquad f(x) = 2x^2, \qquad f(x) = 5x^2.$$

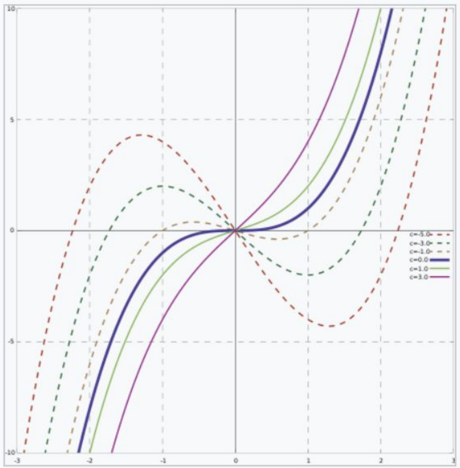
What is the impact of changing the coefficient of x^2 as we have done in these examples? One way to find out is to sketch the graphs of the functions.



[ref]

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?
- **Cubic (x^3)**



Cubic functions of the form

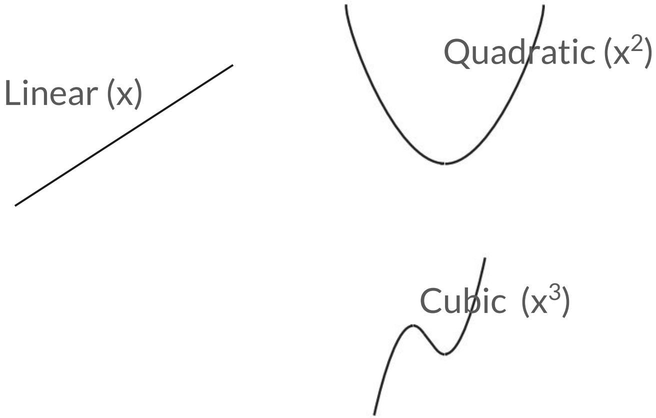
$y = x^3 + cx.$

The graph of any cubic function is similar to such a curve.

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?
- Quadratic (x^2)
- Cubic (x^3)
-

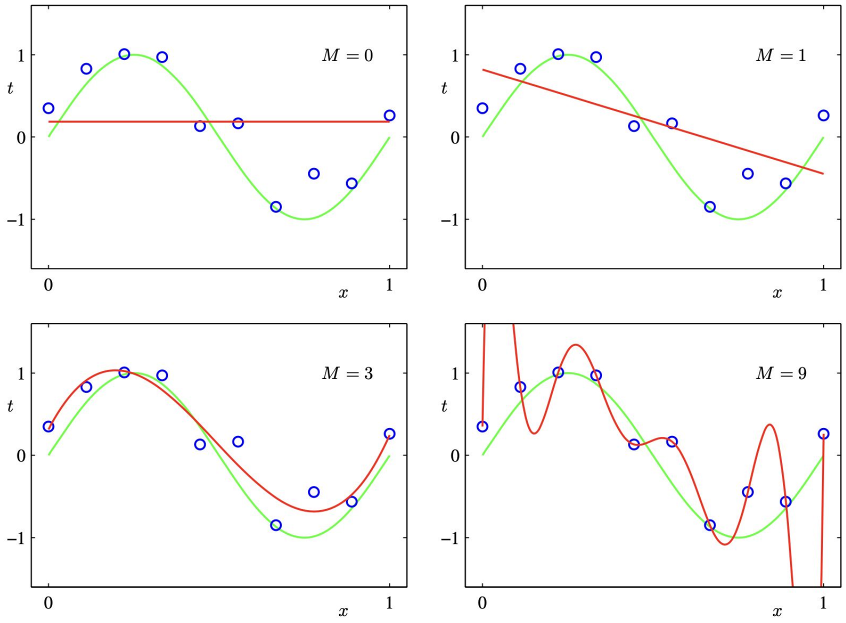
$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$



ref

LR to Polynomial Regression

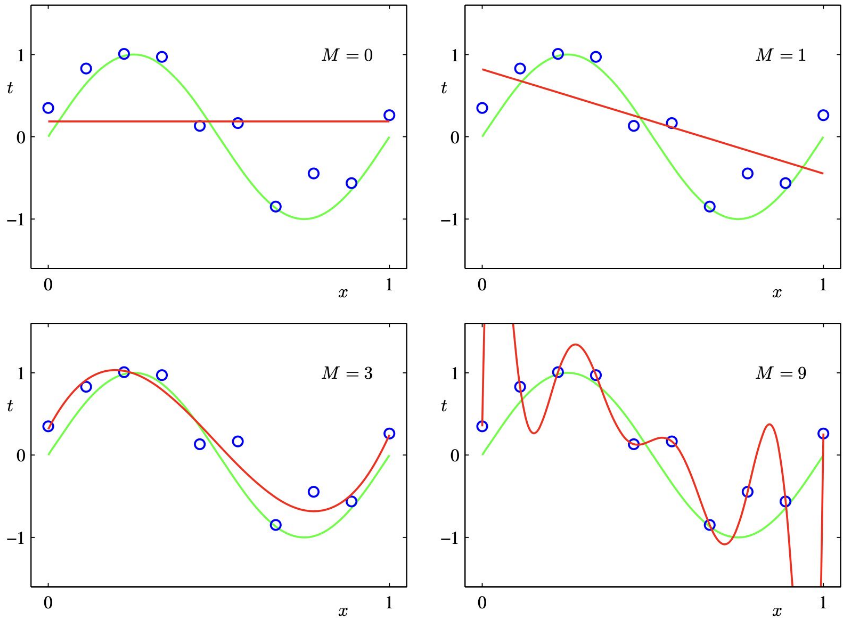
- Polynomial function
    - M is the order ..



$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

LR to Polynomial Regression

- Polynomial function
  - M is the order ..
  - **Where to stop? What is the best M?**

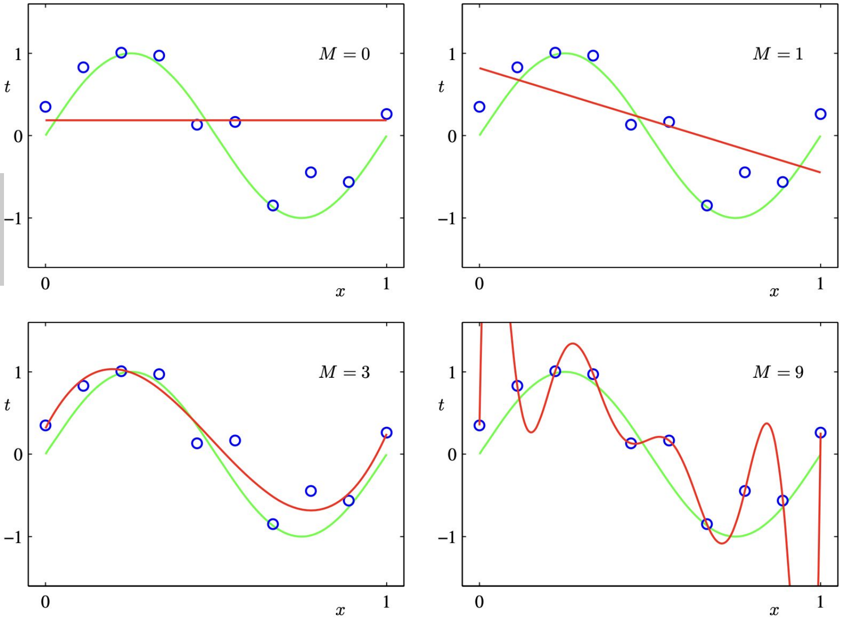


$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

LR to Polynomial Regression

- Polynomial function
  - M is the order ..
  - **Where to stop? What is the best M?**

Good news is our gradient descent (iterative learning) remains the same!



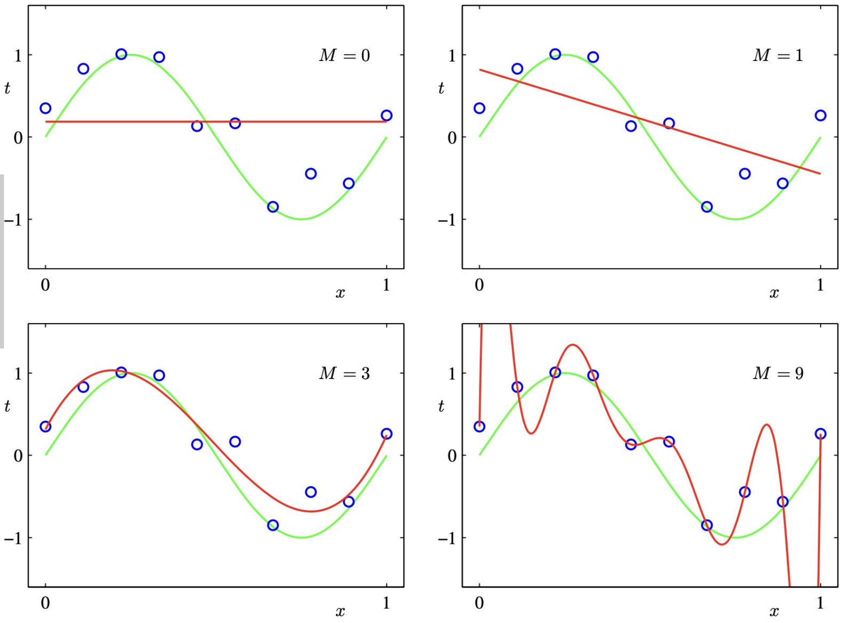
$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

LR to Polynomial Regression

- Polynomial function
    - M is the order ..
    - **Where to stop? What is the best M?**

- Good news is our gradient descent (iterative learning) remains the same!
- You only need to change your objective function (from LR to Polynomial LR)

$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$



LR to Polynomial Regression

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

$$\epsilon = |\hat{y} - y|$$

Optimization function

$$E_\Theta = \frac{1}{2} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$\Theta^* = \operatorname{argmin}_\Theta E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

Our model got a little bigger: 2 params to M param

GPT

I know one of your tricks; get you soon!!

Our model today