CIS 678 - Machine Learning

Basics of Probability

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Head Tail

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- Range:

$$0 \leq P(\mathrm{H}) \leq 1$$

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Addition Rule (Independent Events)

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Example:

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Multiplication Rule (Independent Events)

$$P(A \cap B) = P(A) \times P(B)$$

Example: Tossing the coin twice

- $P(\text{H on 1st and H on 2nd}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Let's Try

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- Find: $P(A \cup B)$

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$$\boxed{P(A \cup B) = \frac{1}{6} + \frac{3}{6} - \frac{1}{6} = \frac{1}{2}}$$

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What is the probability of getting a 2 **and** getting an even number when rolling a fair six-sided die?

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Tossing a fair coin: *Mutually not exclusive event*

What is the probability of getting a 2 **and** getting an even number when rolling a fair six-sided die?

- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$
- Event A: Getting a 2 $A = \{2\}$
- Event B: Getting an even number $B = \{2, 4, 6\}$

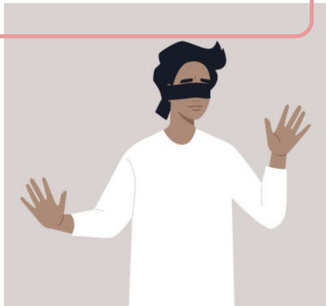
- We want $P(A \cap B)$
- $A \cap B = \{2\}$
- This means that getting a 2 automatically satisfies both conditions

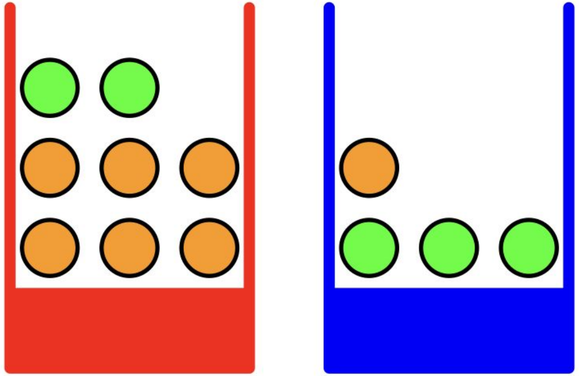
$$P(A \cap B) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{6}$$

Basics of Probability

- There are some **orange and green** balls in a ***red*** and ***blue*** box
- Someone (blinded) picked up a ball and it found to be with color orange
- ***What is is probability that the ball came from the red box?***

Let's try to solve this question…





Basics of Probability

You have to apply Bayes Rule

6.2. Bayes Rule (Theorem)

Combining equations

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; and \; P(B|A) = \frac{P(B \cap A)}{P(A)} \tag{B.17}$$

leads to the **Bayes rule (Bayes theorem)**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{B.18}$$

The Bayes rule is useful to swap events in conditional probability evaluation. The conditional probability $P(A|B)$ can be expressed by the conditional probability $P(B|A)$, $P(A)$ and $P(B)$.

The Bayes rule can be extended to a collection of events $A_1, \cdots, A_n$ conditioned on the event B

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{n}[P(A_i)P(B|A_i)]} \tag{B.19}$$

where

$$P(B) = \sum_{i=1}^{n}[P(B|A_i)P(A_i)] \tag{B.20}$$

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Solution: **Bishop:** Pattern Recognition and Machine Learning.
(Section 1.2: Probability Theory); e-copy.