CIS 678 - Machine Learning

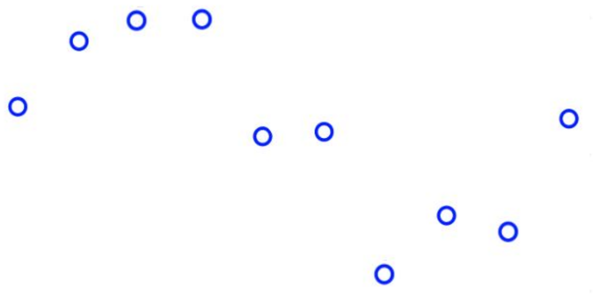
- Linear to Polynomial Regression
- Model Regularization

Plan

- LR to Polynomial Regression
- Regularization
  - Theory
  - Practical - Notebook presentation

Non linear data/function

- Does this data points seem familiar matching a known function?



Non linear data/function

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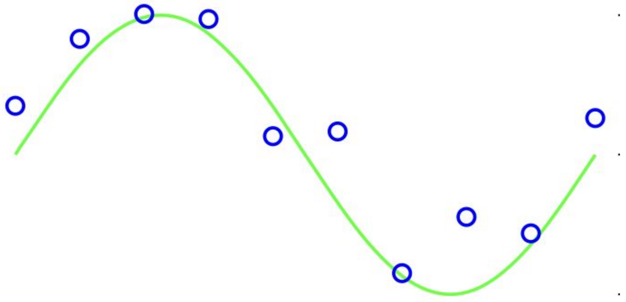
    - A Sinusoidal function

$$y(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi)$$



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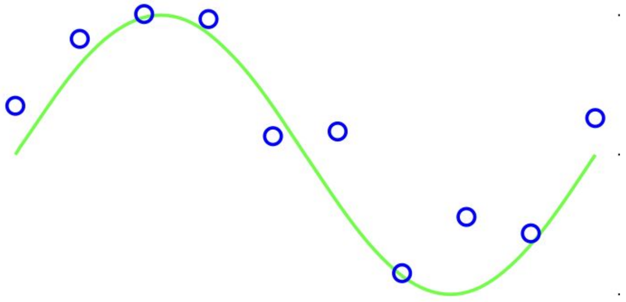
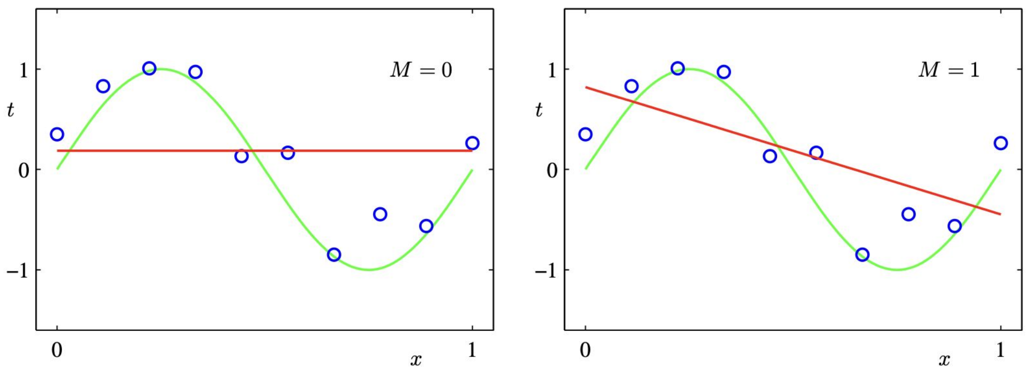


Clearly this is not a linear function; right?

Non linear data/function

- Does this data points seem familiar matching a known function?
- Can we approximate this function using LR ?

$$\hat{y} = \beta_0 + \beta_1 x$$



LR will not work; right?

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?

ref

What no-linear functions we are aware of?

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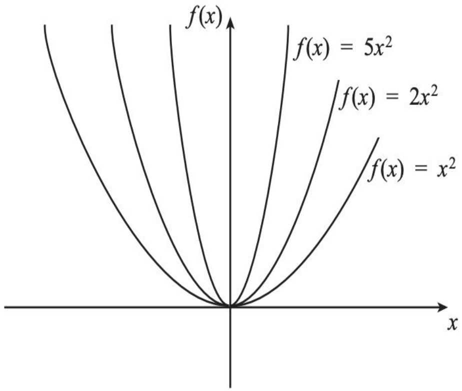
- **Quadratic (x^2)**

$$f(x) = x^2, \qquad f(x) = 2x^2, \qquad f(x) = 5x^2.$$

What is the impact of changing the coefficient of x^2 as we have done in these examples? One way to find out is to sketch the graphs of the functions.



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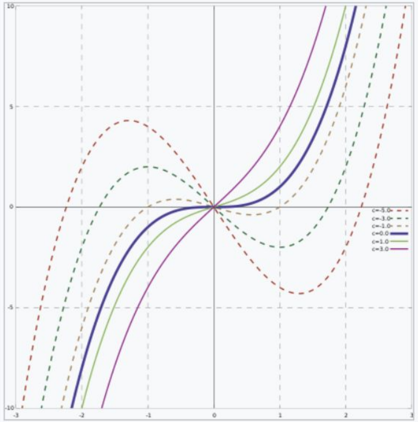
- **Cubic (x^3)**



Cubic functions of the form $y = x^3 + cx.$
The graph of any cubic function is similar to such a curve.

ref

What no-linear functions we are aware of?

- Can you recall any nonlinear function you learned at your high school/colleges?
- Quadratic (x^2)
- Cubic (x^3)
-

Linear (x)

$$\boxed{\hat{y} = \beta_0 + \beta_1 x}$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
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Cubic (x^3)

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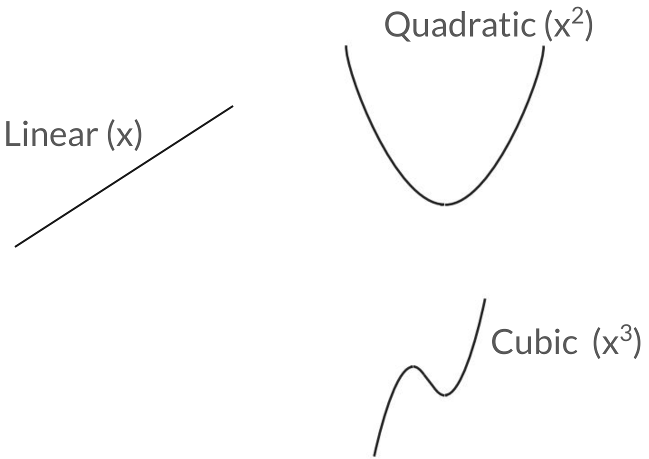

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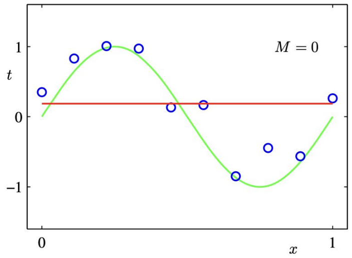
LR to Polynomial Regression

- Polynomial function
  - *M* **is the order/degree of polynomial ..**

M=0

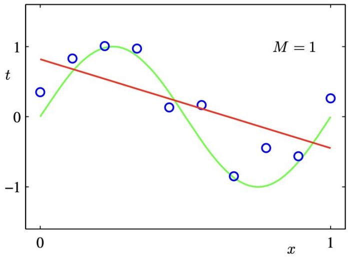


$$\hat{y} = \beta_0$$

LR to Polynomial Regression

- Polynomial function
    - *M* **is the order/degree of polynomial ..**

M=1



$$\hat{y} = \beta_0 + \beta_1 x$$

LR to Polynomial Regression

- Polynomial function
  - ***M* is the order/degree of polynomial ..**

M=3

$$\hat{y} = \beta_0 + \beta_1 x$$
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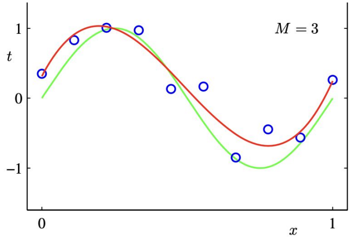

LR to Polynomial Regression

- Polynomial function
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M=9

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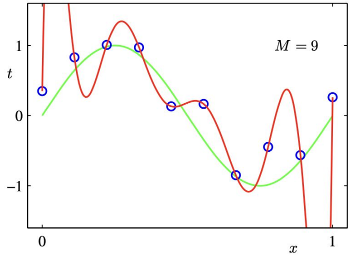
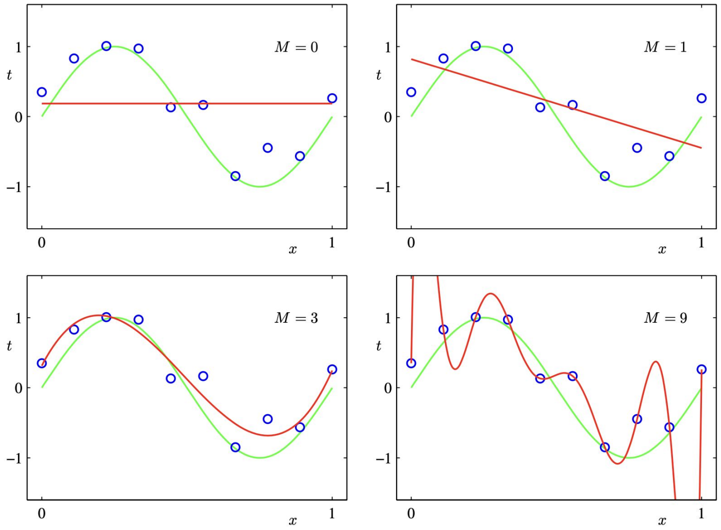


LR to Polynomial Regression

- Polynomial function
    - *M* **is the order/degree of polynomial ..**

M=?
Which one is preferable?



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Regularization

Regularization

Let's recall the definition of our optimization function

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

$$\epsilon = |\hat{y} - y|$$

Optimization function

$$E_\Theta = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$$\Theta^* = \text{argmin}_\Theta\, E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

Regularization

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$$

Same model, two different notations

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

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x : scalar
$\boldsymbol{x}$, $\mathbf{x}$: vector
$\mathbf{X}$: Matrix

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

$$\epsilon = |\hat{y} - y|$$

Essentially, the same formulation

Optimization function

$$E_\Theta = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

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Generally **ML** vs **Math** conventions

$$\boxed{W^* = \mathrm{argmin}_W\, E\{(x_i, t_i)\}_{i=1,\cdots,N}}$$

Regularization

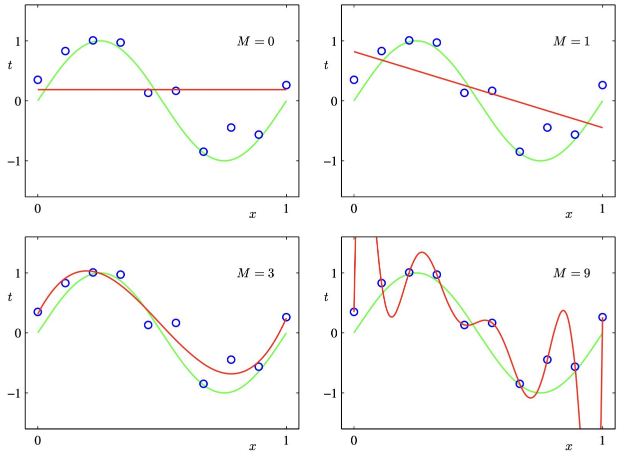

Table 1.1 Table of the coefficients $\mathbf{w}^\star$ for polynomials of various order. Observe how the typical magnitude of the coefficients increases dramatically as the order of the polynomial increases.

	$M=0$	$M=1$	$M=6$	$M=9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
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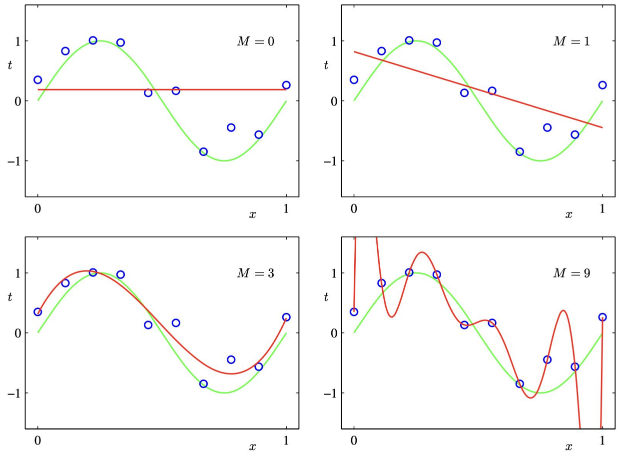
Regularization



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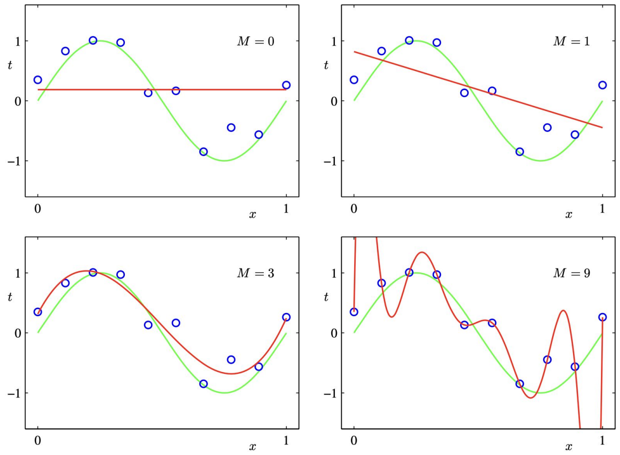
Absolute values are increasing

Regularization



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Regularizer

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

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||W||^2

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decreasing

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Absolute values are increasing

How to control this?

Regularization

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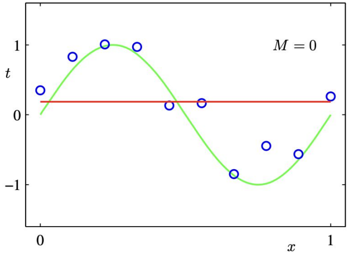
Who to control this?

Lamda is called the
Hyper Parameter of this model

LR to Polynomial Regression

- Polynomial function
  - ***M* is the order/degree of polynomial ..**

M=0



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LR to Polynomial Regression

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    - ***M* is the order/degree of polynomial ..**

M=1

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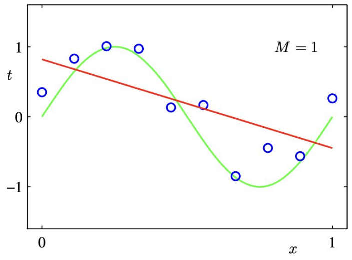
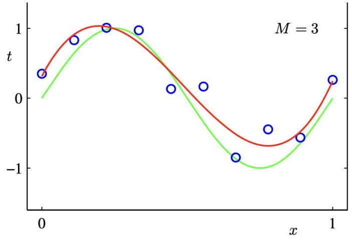


LR to Polynomial Regression

- Polynomial function
    - **M is the order/degree of polynomial ..**

M=3

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$$



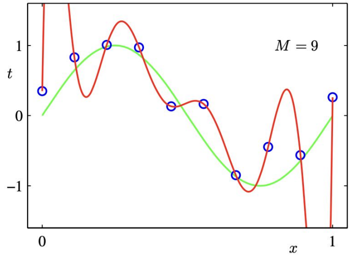
LR to Polynomial Regression

- Polynomial function
    - ***M* is the order/degree of polynomial ..**

M=9

Overfitting!!!

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$$

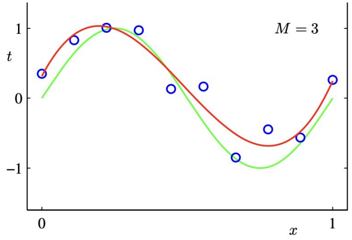


LR to Polynomial Regression

- Polynomial function
  - ***M* is the order/degree of polynomial ..**

M=3

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$



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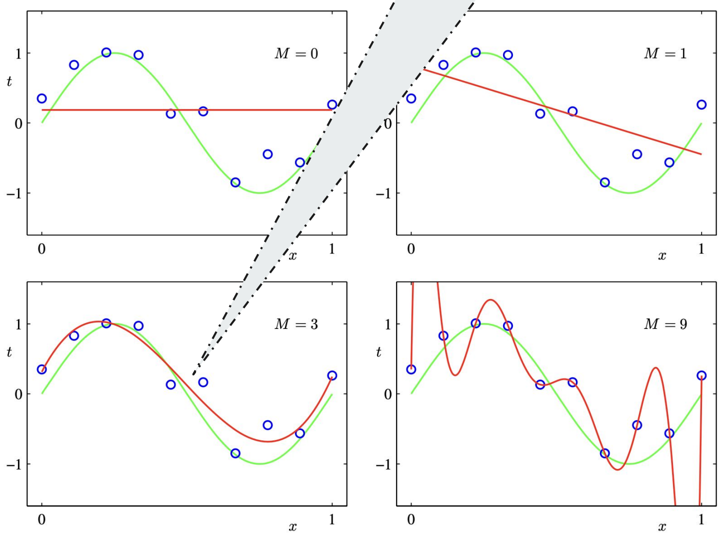
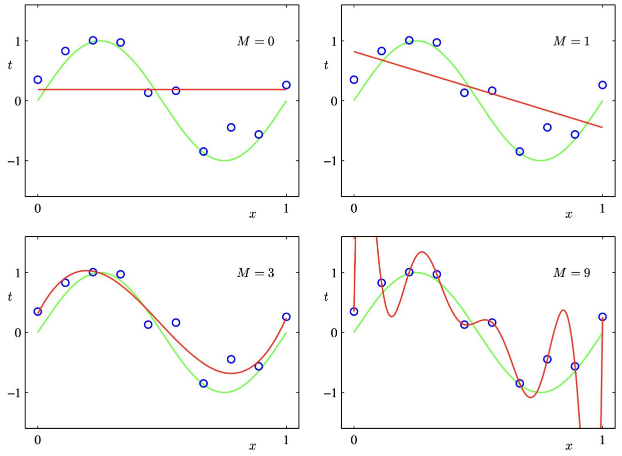
Preferred one



Polynomial Regression with Regularization



Learned function is **nonlinear**

$$\hat{y} = \beta_0$$
$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
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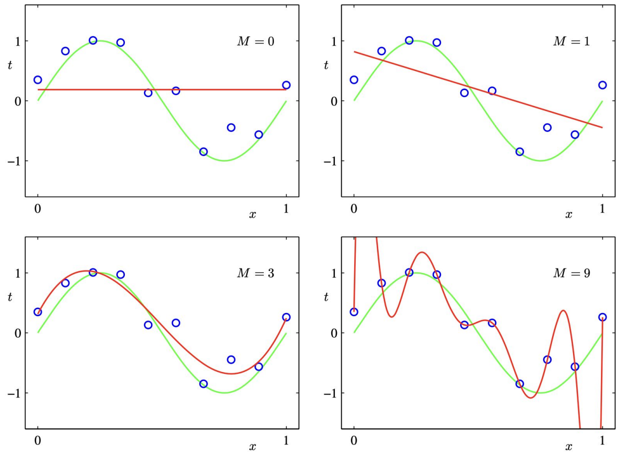
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

Polynomial Regression with Regularization



Learned function is **nonlinear**

$$
\begin{aligned}
\hat{y} &= \beta_0 \\
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\hat{y} &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots
\end{aligned}
$$

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

Model (still) **linear**

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

Objective function with the Regularizer!

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_2^2}$$

The same but with slightly different notation!

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

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L2 Regularizer

The same but with slightly different notation!

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

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The same but with slightly different notation!

L2 Regularizer

Ridge Regression

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_2^2}$$

The same but with slightly different notation!

L2 Regularizer

Ridge Regression

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_1}$$

L1 Regularizer

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_2^2}$$

The same but with slightly different notation!

L2 Regularizer | *Ridge Regression*

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_1}$$

L1 Regularizer | *Lasso Regression*

Polynomial Regression with Regularization

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

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L2 Regularizer *Ridge Regression*

$$\boxed{\tilde{E}(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|_1}$$

L1 Regularizer *Lasso Regression*

*How to choose lambda (**λ**)? Through CV*

QA

Classification

- General Idea (two steps process)
  - LR (Bias Only)
  - LR (general)

Notebook presentation

- Without regularizer
- With regularizer

Predictive modeling: Regression (diabetes)

Predictive modeling: Classification

Break!

Maximum Likelihood Learning

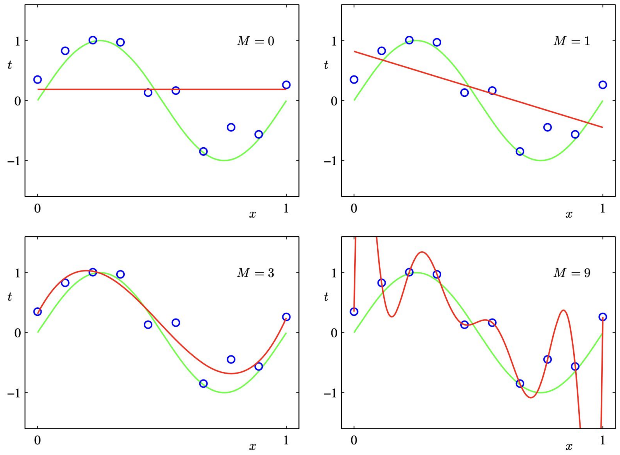
Regularization

Least Squares

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2$$

MLE EQN

Regularization



$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2$$

Regularizer

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

Our model got a little bigger: 2 params to *M* param

GPT

I know one of your
tricks; get you soon!!

Our model yesterday

Our model got a little bigger: 2 params to *M* param

GPT

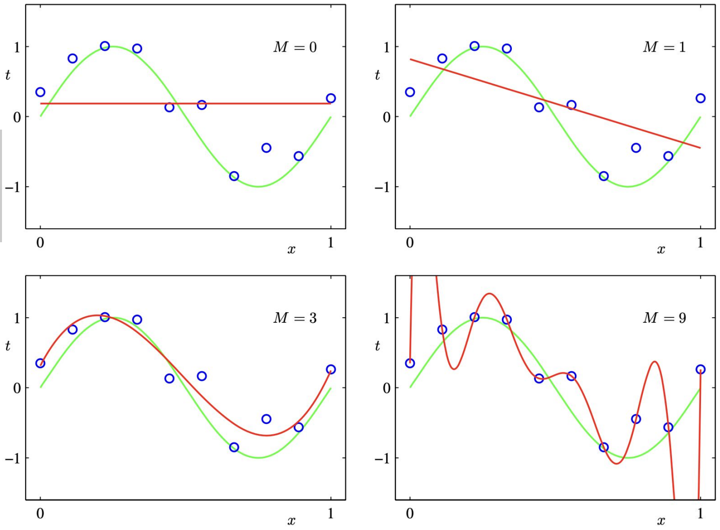
I know one of your tricks; get you soon!!

Our model today

LR to Polynomial Regression

- Polynomial function
  - *M* is the order/degree of polynomial ..
  - **Where to stop? What is the best M?**

Good news is our gradient descent (iterative learning) remains the same!



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LR to Polynomial Regression

- Polynomial function
  - M is the order ..
  - **Where to stop? What is the best M?**

- *Good news is our gradient descent (iterative learning) remains the same!*
- *You only need to change your objective function (from LR to Polynomial LR)*

$$\hat{y} = \beta_0 + \beta_1 x$$
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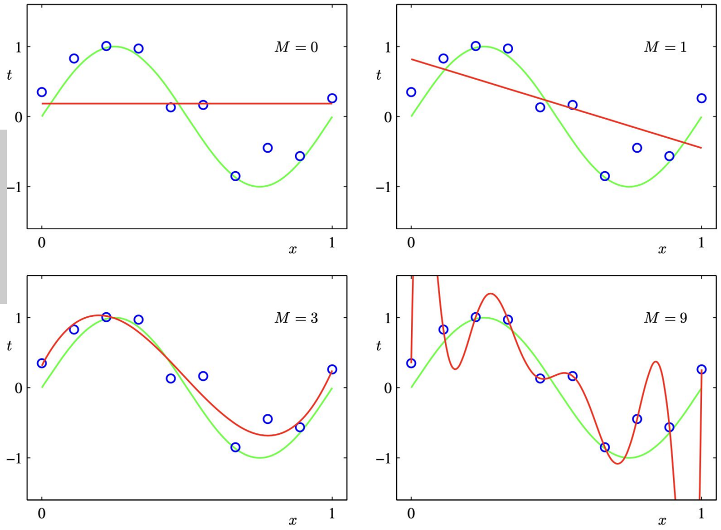

LR to Polynomial Regression

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

$$\epsilon = |\hat{y} - y|$$

Optimization function

$$E_\Theta = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$$\Theta^* = \mathrm{argmin}_\Theta\, E\{(x_i, y_i)\}_{i=1,\cdots,N}$$

$$\hat{y} = \beta_0 + \beta_1 x$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$
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