



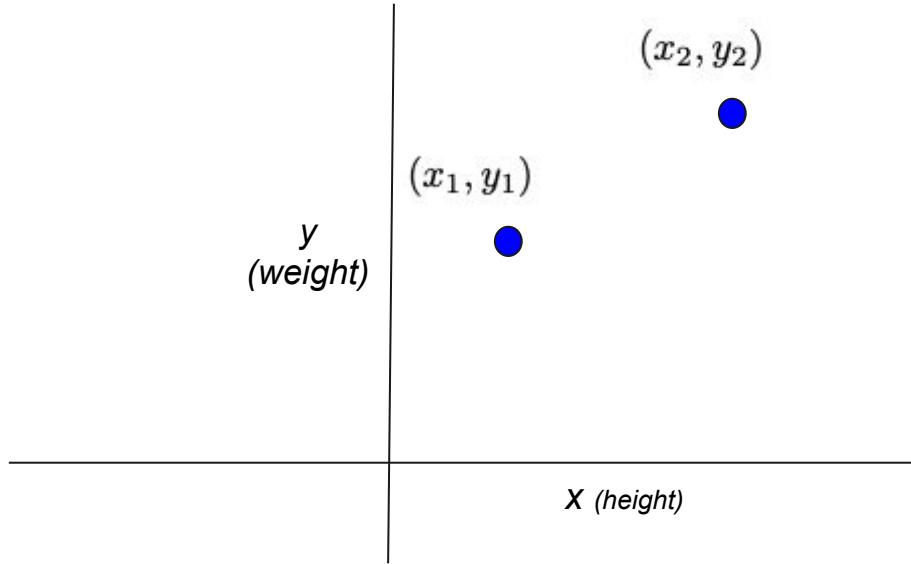
CIS 678 Machine Learning

ML Model training: Introduction to Gradient descent

What we'd like to accomplish today

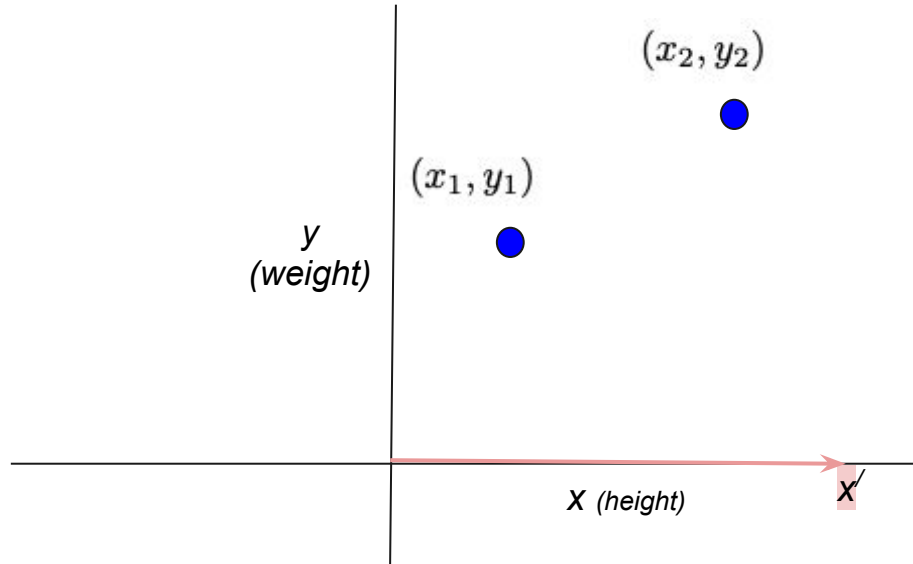
- Model training using Gradient descent
 - Refreshing some high school maths: **linear equation**
 - A simple two parameter **linear regression** model
 - The **Gradient descent algorithm**
- Hands on **Notebook implementation**
- QA

k-NN Regression



Given two known data points (x_1, y_1) , and (x_2, y_2) , and

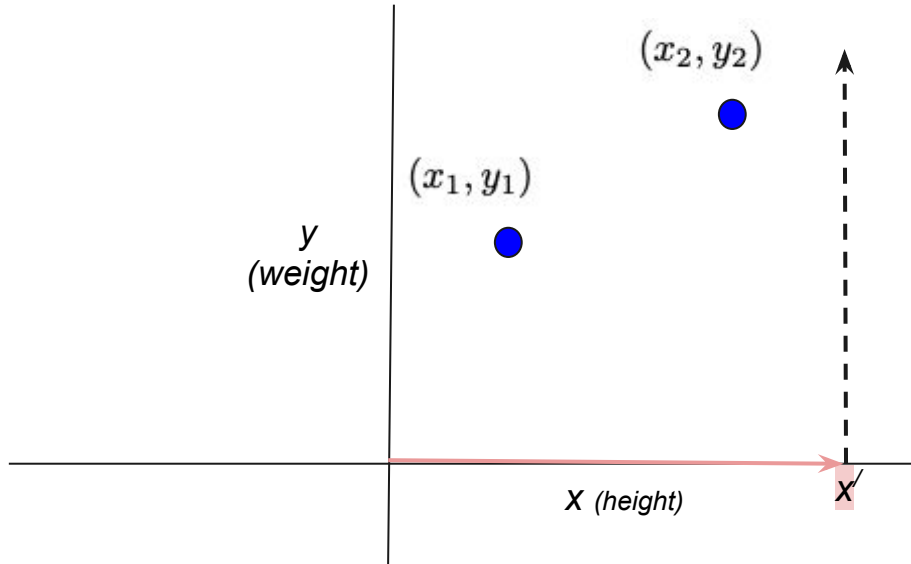
k-NN Regression



Given two known data points (x_1, y_1) , and (x_2, y_2) , and

- for test input x' , you have to predict $y(x')$.
- I.e. you have to plot $(x', ?)$

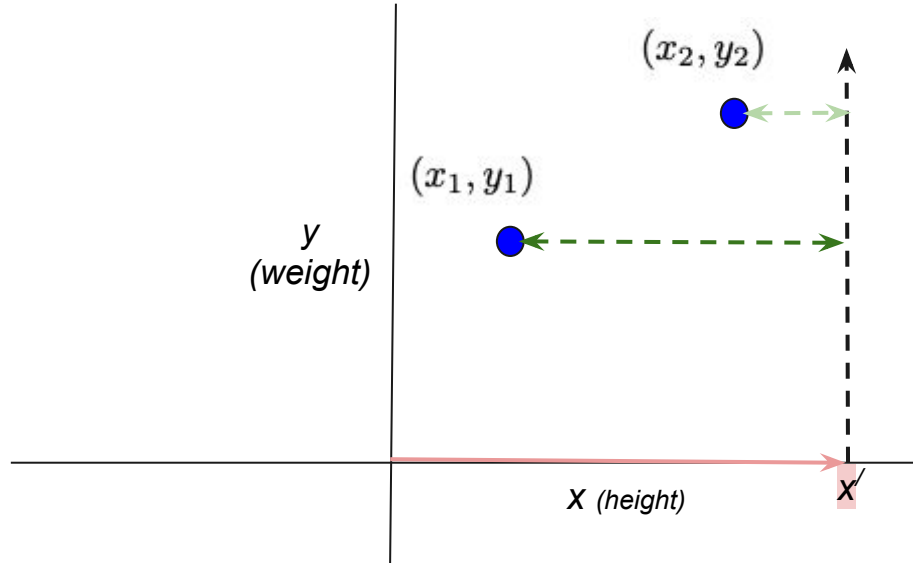
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- To estimate the distances let's draw the vertical line

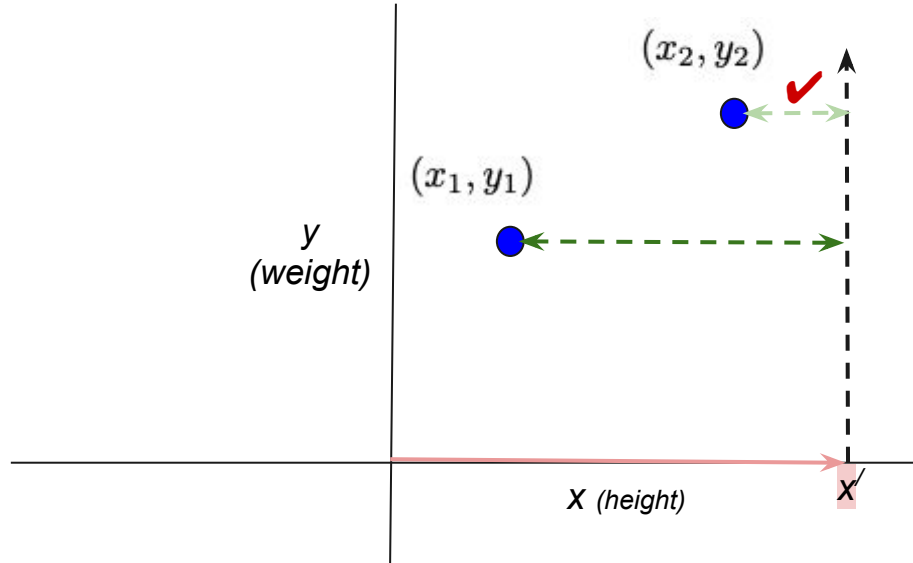
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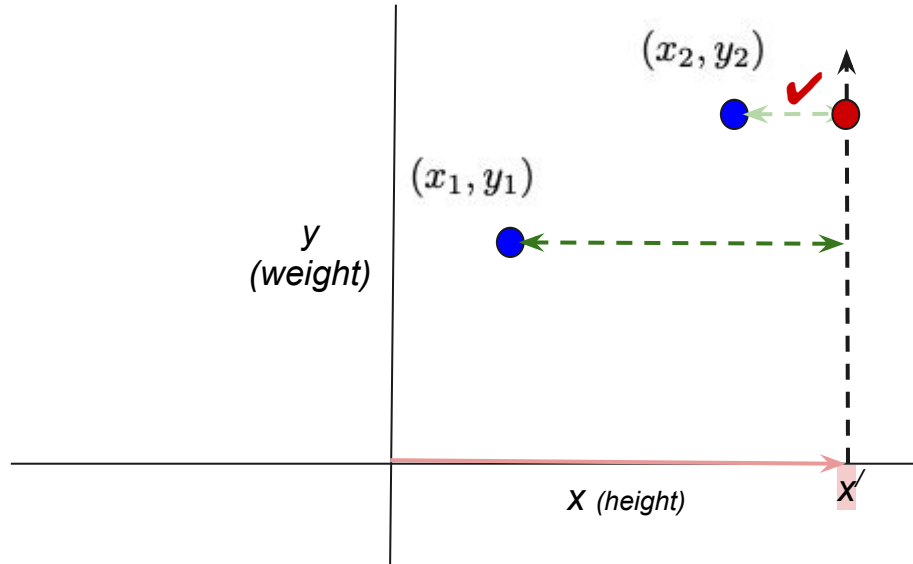
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- We find the lighter green on is the closest one [k(1)-NN]
-

k-NN Regression



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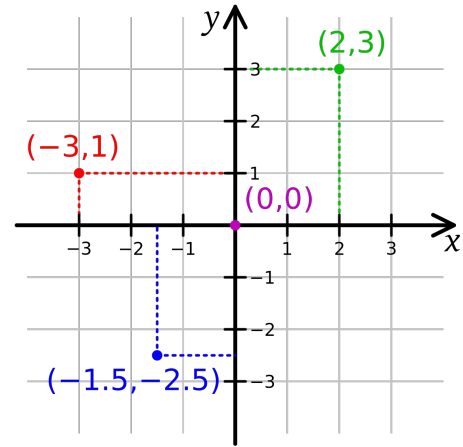
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- To estimate the distances let's draw the vertical line
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- We find the lighter green on is the closest one [k(1)-NN]
- We propagate the associated label(s), i.e.

$$y(x') = y_2$$

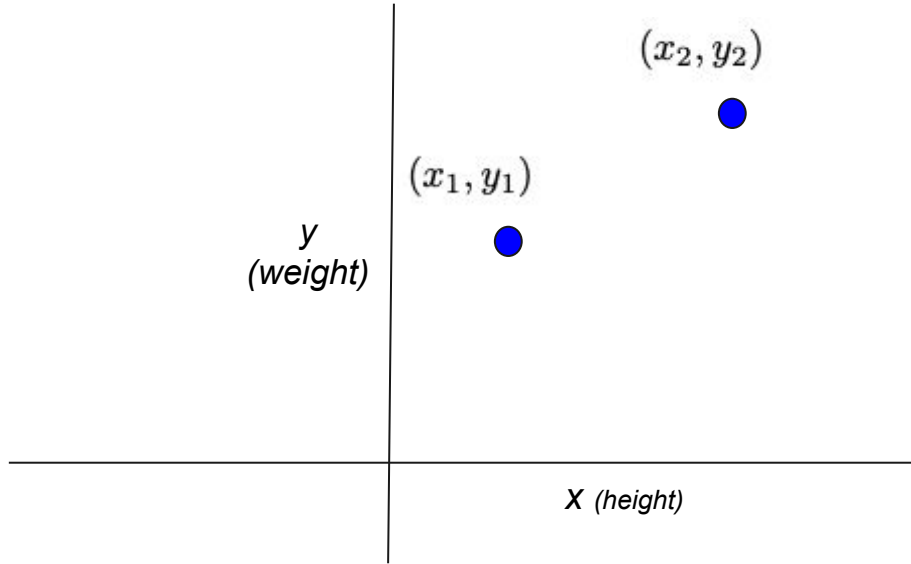
- If we have more data points we may go for a higher k , and take the average

Recall, we said k-NN is non parametric

- K-nearest neighbors (k-NN)
 - Supervised learning
 - **Non parametric**
- Based on what data (features are available) and on distance measures.

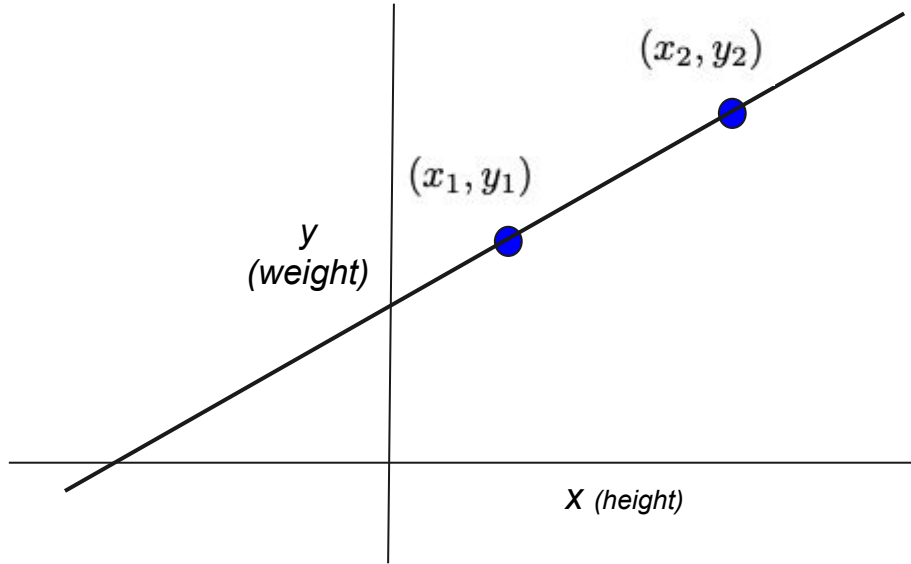


Linear equation, a quick review



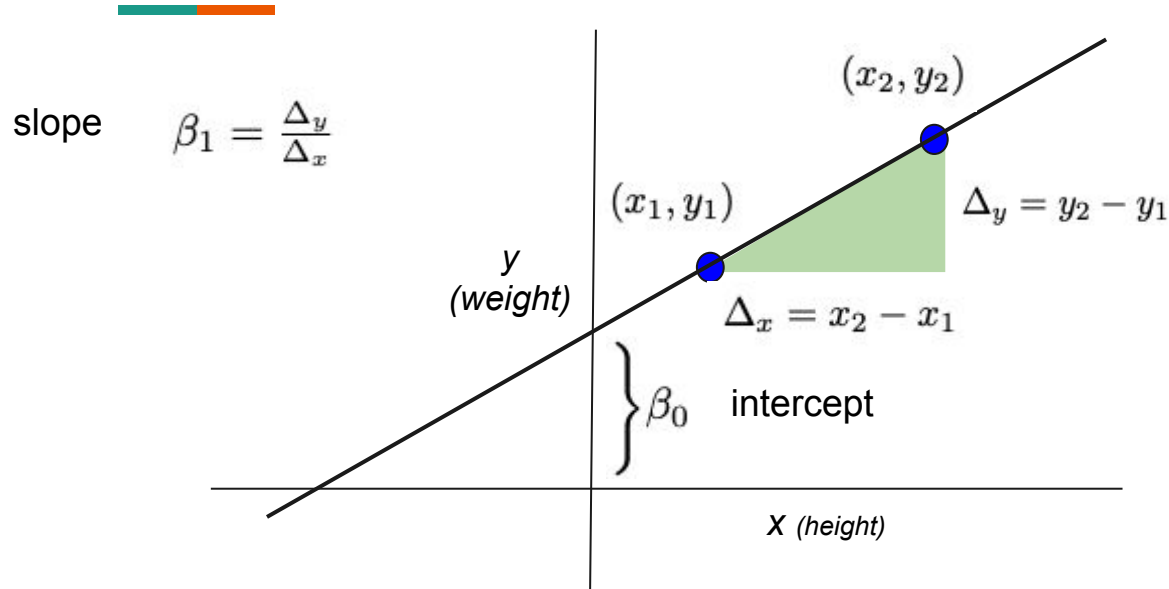
Given two data points

Linear equation, a quick review



We can fit a linear equation

Linear equation to a linear function, a quick review



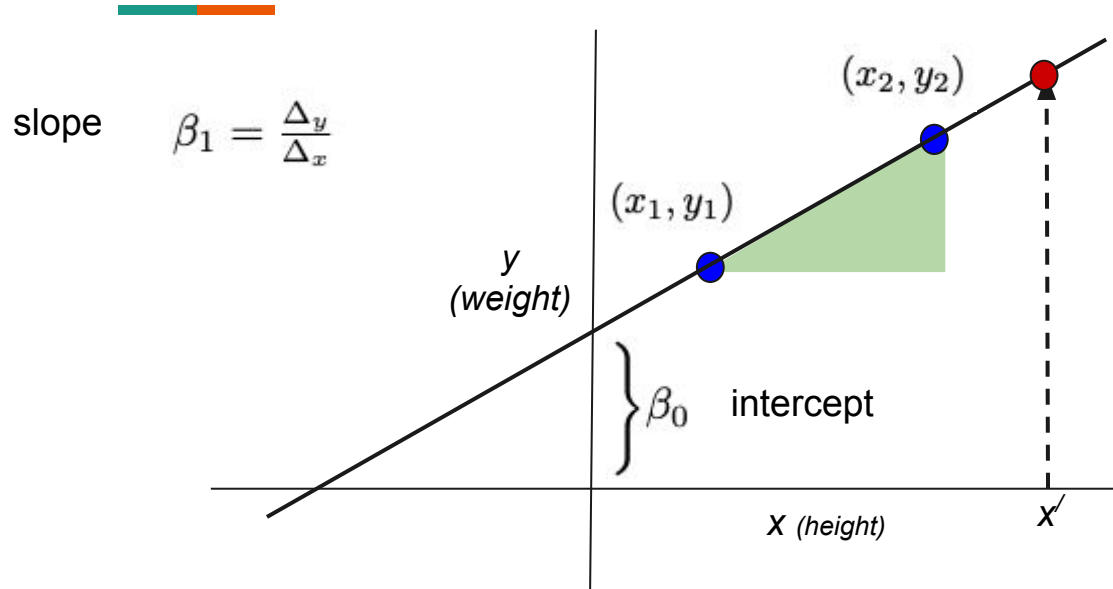
- This linear equation can be used to explain the relationship between the two axes x (independent variable) vs y (dependent variable) - as

$$y = \beta_0 + \beta_1 x$$

- A simple model with parameters: **slope**, and **intercept**

- For any given x' , this model can predict $y(x')$ using the above equation.

Linear equation to a linear function, a quick review



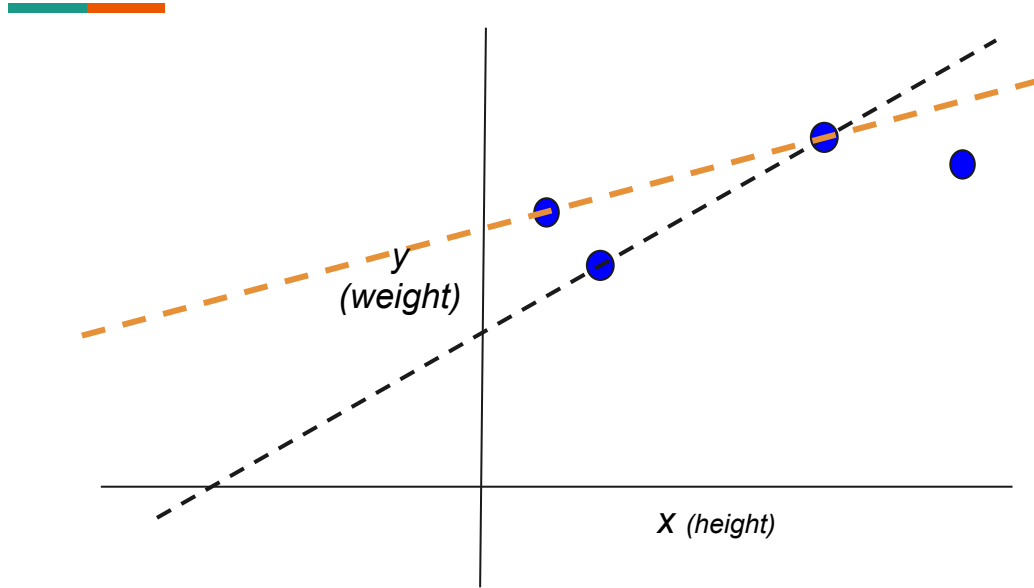
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Fitting a Linear function, a quick review



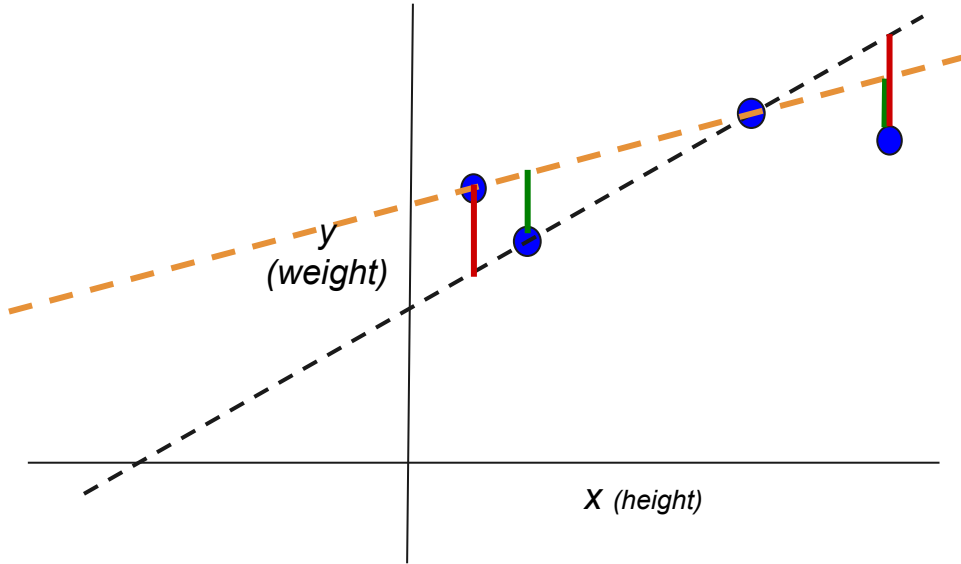
- Example of linear relationship: height vs weight of a person, marketing expense vs sales
- > 2 data points are unlikely fit perfectly on a straight line, which a straight line (2 param model) cannot fit
- We need some approximations
- Let's examine the two models for the 4 data points on the left

model β_0, β_1

model: β_0, β_1

- Both model perfectly fits 2 points each
- Orange (visual screening) model seems to be a better fit, but why?

Fitting a Linear function/function, a quick review



Based on Error is higher than Error

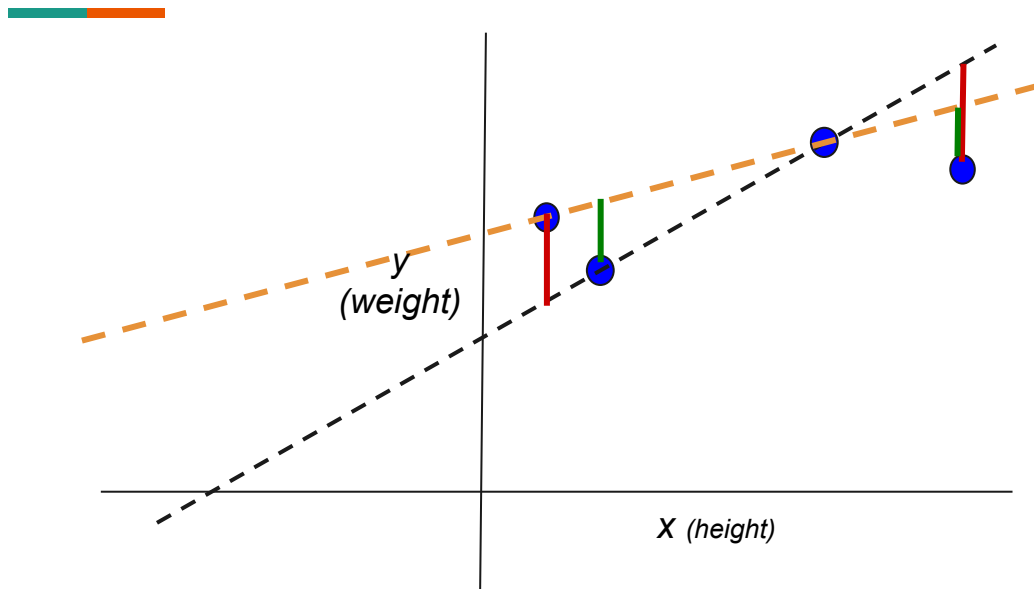
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Fitting a linear function/model



Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

Fitting Error

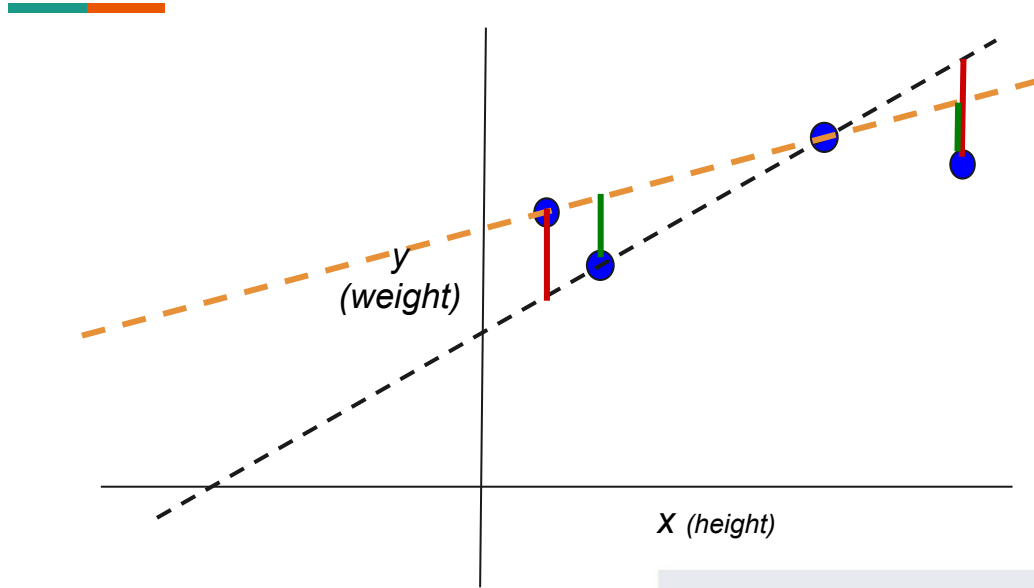
$$\epsilon = |\hat{y} - y|$$

Optimization/loss function

$$E_{\Theta} = \frac{1}{2} \sum_{i=1}^N (\hat{y}_i - y_i)^2$$

$$\Theta^* = \operatorname{argmin}_{\Theta} E\{(x_i, y_i)\}_{i=1, \dots, N}$$

Fitting a linear function/model



Ordinary Least Squares(OLS)

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y},$$

Model

$$\hat{y} = \beta_0 + \beta_1 x$$

$$\Theta = \{\beta_0, \beta_1\}$$

Fitting Error

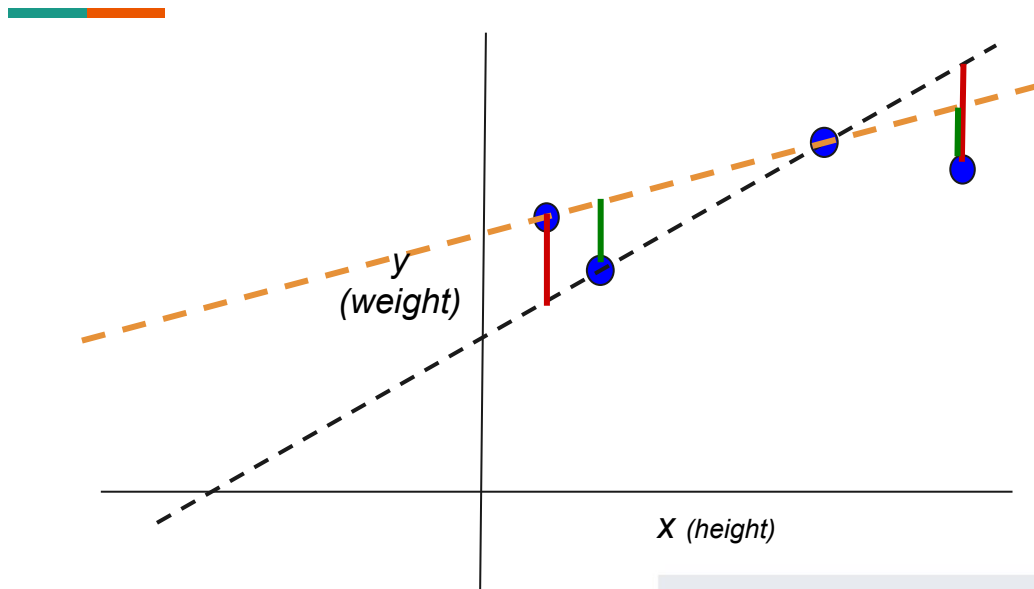
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Fitting a linear function/model



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Out of scope today

Model

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Fitting Error

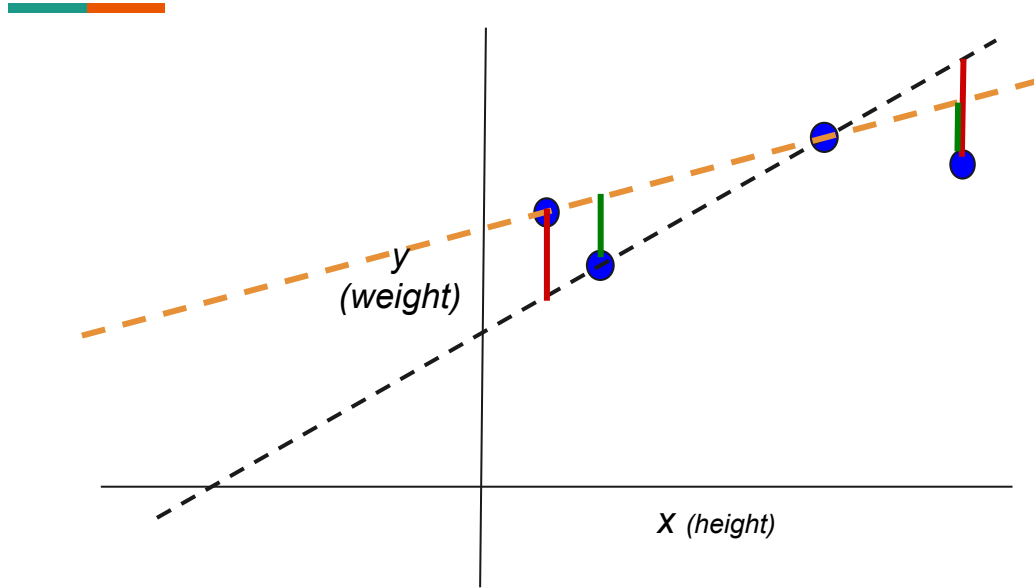
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Gradient descent (- ascent)



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Fitting Error

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Optimization/loss function

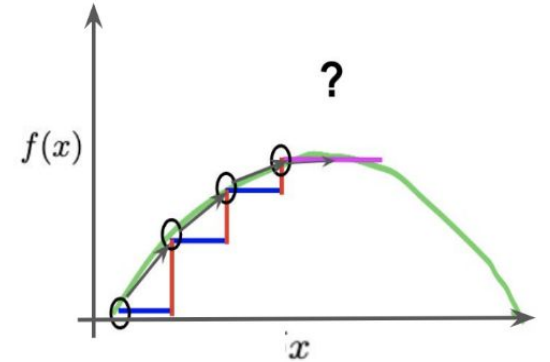
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Gradient descent (- ascent)

- We have decided to build some stairs as we need to visit the top of a mountain frequently. We want
 - To produce **equal** width stairs (**step size**)
 - To make the stair corners **touch the periphery** of the mountain (maybe someday we want recreate the shape of the mountain)

- For such a **convex** function $f(x)$, we will have varying height stairs, and at the **top of the mountain** stairs height will be **close to zero**, an indicator that we are at the top.
- Stair **height/width** is called the **gradient** of $f(x)$, the arrowhead denoting its **direction (towards an increasing value)**.
- We have to choose the **step size** sensibly: larger step size may miss the peak while smaller step size will take too much efforts to reach to the top



step size: δx
gradient: $\frac{\delta}{\delta x} f(x)$

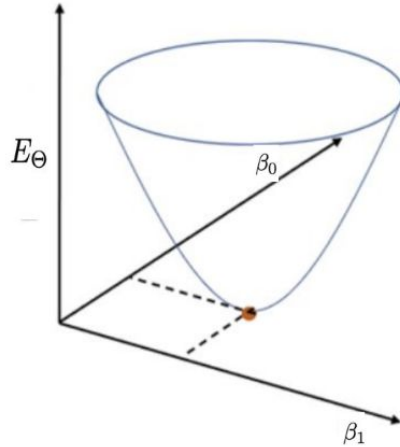
Gradient descent (- ascent)

1. Start with an initial (β_0, β_1) and a **learning rate** (L), a scalar, which controls the gradient step.
2. For N training data points, estimate the **model loss**
3. Estimate the gradient (vector of partial derivatives): $\nabla E_{\Theta} = \left[\frac{\partial}{\partial \beta_0}(E_{\Theta}), \frac{\partial}{\partial \beta_1}(E_{\Theta}) \right]$
4. Update parameters

$$\beta_0 \leftarrow \beta_0 - L * \frac{\partial}{\partial \beta_0}(E_{\Theta})$$

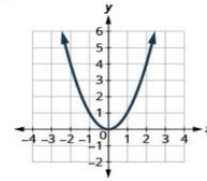
$$\beta_1 \leftarrow \beta_1 - L * \frac{\partial}{\partial \beta_1}(E_{\Theta})$$

4. Go to step 2) and iterate until the model loss reaches a predefined **threshold** or a certain **number of iterations** are executed.



Objective function

$$\Theta^* = \operatorname{argmin}_{\Theta} E\{(x_i, y_i)\}_{i=1, \dots, N}$$



Partial derivatives

$$\frac{\partial}{\partial \beta_0}(E_{\Theta}) = \sum_{i=1}^N (\beta_0 + \beta_1 x_i - y_i)$$

$$\frac{\partial}{\partial \beta_1}(E_{\Theta}) = \sum_{i=1}^N (\beta_0 + \beta_1 x_i - y_i) x_i$$

Variants of gradient descent (- ascent)

- **Batch gradient descent:** Gradient based on chunk of data
- **Stochastic gradient descent:** chunk size is 1

Notebook presentation

- Notebook github

- Gradient descent training (Linear Regression)

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- Hands on **Notebook implementation**
- **QA**



GPT

I know one of your
tricks; get you soon!!



Our model today