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# CIS 678 Machine Learning

Time series data modeling

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# Outline

- How are the time series problems different than non time series problems?
- Stationary vs non-stationary signals
- AR(I)MA: Autoregressive Integrated Moving Average
  - From non no-stationary to stationary
  - Auto Regression
  - Moving Average

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# Let's look at some time-series data

[Notebook](#)



## How are the time-series problems different?

- The models (Regression and Classification), we have learned so far are of the form:

$$f(y|X)$$

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- For certain data, especially the time series, we can take advantage of the form:

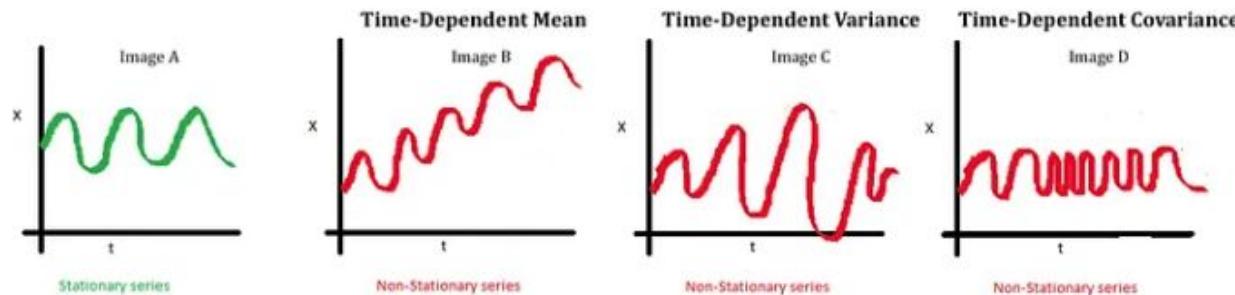
$$f(y_t|X, y_{t-1}, y_{t-2}, \dots, y_0); \text{ essentially, here the input features are } X \text{ plus the lagged instances of the target } y.$$

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# Stationarity vs non-stationary signals

- A time-series is said to be stationary if it does not display any trends or seasonality.
- One more way of defining stationarity is that it is when data does not have any time-dependent mean, variance or covariance.

## The Principles of Stationarity



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## Non stationary to stationary

- If signal is non-stationary, we can convert them into stationary signal by differencing

$$T_t = S_t - S_{t-1},$$

[ref](#)

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## Non stationary to stationary

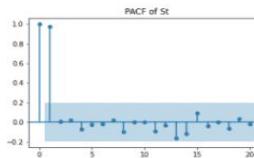
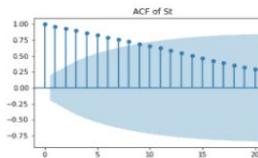
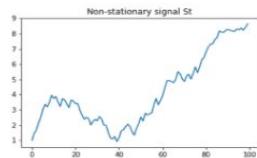
- If signal is non-stationary, we can convert them into stationary signal by differencing
- or calculating percent of change

$$T_t = S_t - S_{t-1},$$

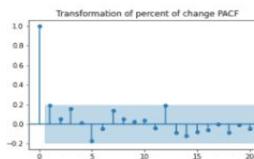
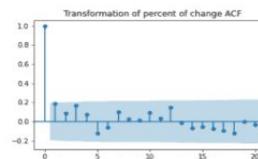
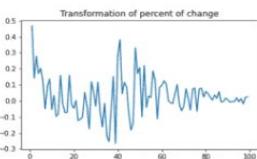
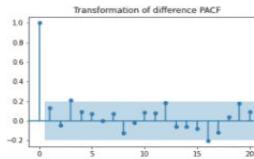
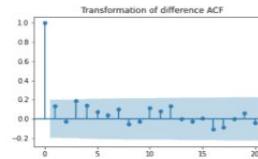
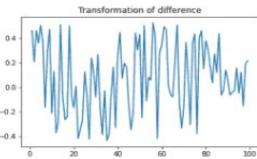
$$T_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

# Non stationary to stationary

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ref

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# Statistical time series models

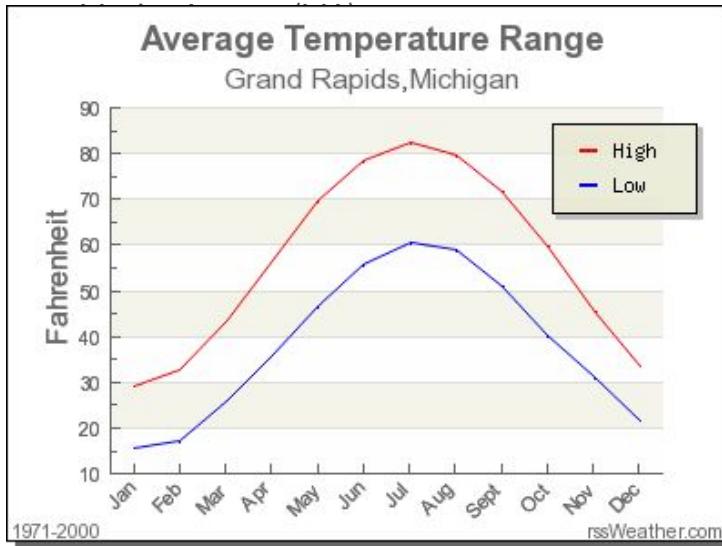
- Autoregressive Models (AR)
- Moving Average (MA)
- Autoregressive Moving Average (ARMA)
- Autoregressive Moving Integrated Average (ARIMA)

$$f(y_t | y_{t-1}, y_{t-2}, \dots, y_0)$$

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# Statistical time series models

- Autoregressive Models (AR)

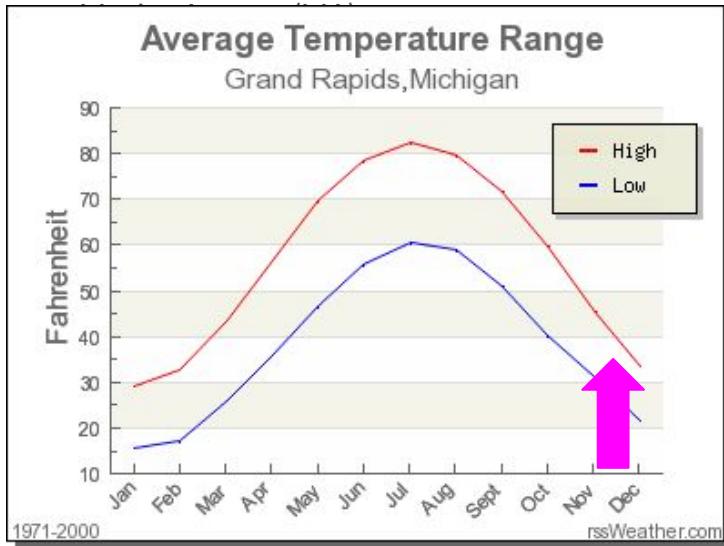


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# Statistical time series models

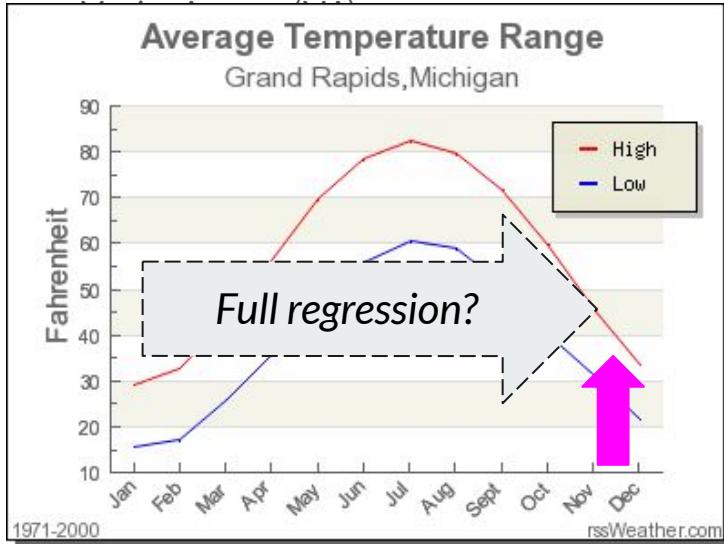
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# Statistical time series models

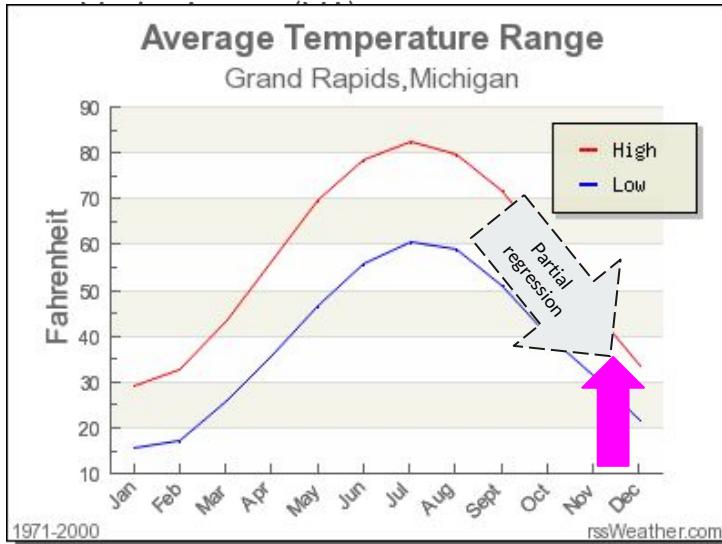
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# Statistical time series models

- Autoregressive Models (AR)



$$f(y_t | y_{t-1}, y_{t-2}, \dots, y_0)$$

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# Statistical time series models

- Autoregressive Models (AR)

The **AR( $p$ )** model is defined as,

$$X_t = \sum_{i=1}^p \varphi_i X_{t-i} + \varepsilon_t$$

*where  $p$  is the order of the model*

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# Statistical time series models

- Moving Average (MA)

*MA( $q$ ): moving average model of order  $q$ :*       $X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$

$$X_t = \mu + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t$$

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# Statistical time series models

- Autoregressive Moving Average (ARMA)

ARMA( $p, q$ ) model    
$$X_t = \varepsilon_t + \sum_{i=1}^p \varphi_i X_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i}$$

with  $p$  autoregressive terms and  $q$  moving-average terms.



# AR(I)MA

- AutoRegressive Integrated Moving Average (ARIMA) is a statistical model for forecasting time series data.

A combined model with AR, MA, but first transforming the signal to stationary.

- **AR (Autoregression):** This emphasizes the dependent relationship between an observation and its preceding or 'lagged' observations.
- **I (Integrated):** To achieve a stationary time series, one that doesn't exhibit trend or seasonality, differencing is applied. It typically involves subtracting an observation from its preceding observation.
- **MA (Moving Average):** This component zeroes in on the relationship between an observation and the residual error from a moving average model based on lagged observations.

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# AR(I)MA

The parameters of the ARIMA(p,d,q) model are defined as follows:

- **p**: The lag order, representing the number of lag observations incorporated in the model.
- **d**: Degree of differencing, denoting the number of times raw observations undergo differencing.
- **q**: Order of moving average, indicating the size of the moving average window.

A combined model with AR, MA, but first transforming the signal to stationary.

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# Notebook presentation

Notebook



# QA