CIS 678 - Machine Learning

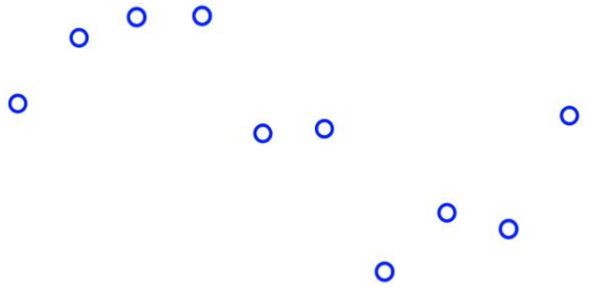
- Model Regularization (part 2)

Plan

- Regularization
  - Theory
  - Practical - Notebook presentation

Non linear data/function

- Does this data points seem familiar matching a known function?

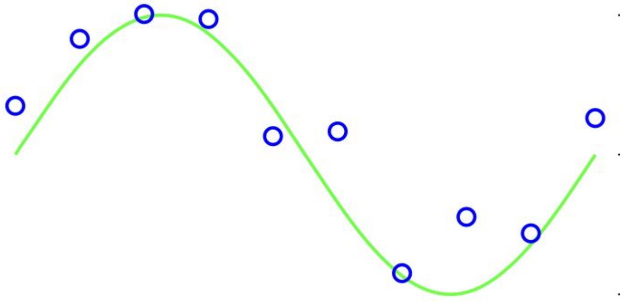


Non linear data/function

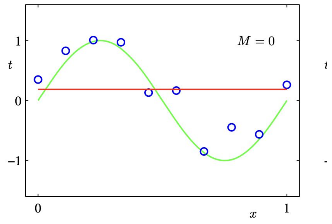
- Does this data points seem familiar matching a known function?

    - A Sinusoidal function

$$y(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi)$$

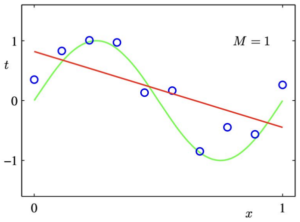


Polynomial Regression with Regularization



$$\hat{y} = \beta_0$$

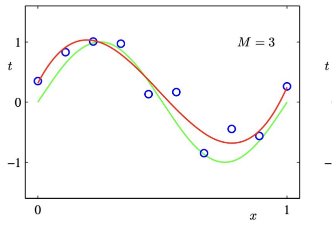
Polynomial Regression with Regularization



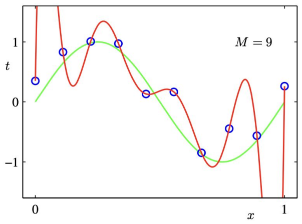
$$\hat{y} = \beta_0 + \beta_1 x$$

Polynomial Regression with Regularization

$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3$$

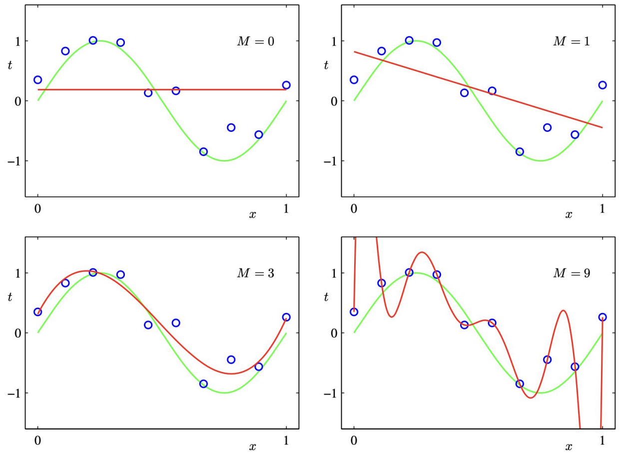


Polynomial Regression with Regularization



$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_2 x^3 + \ldots$$

Polynomial Regression with Regularization



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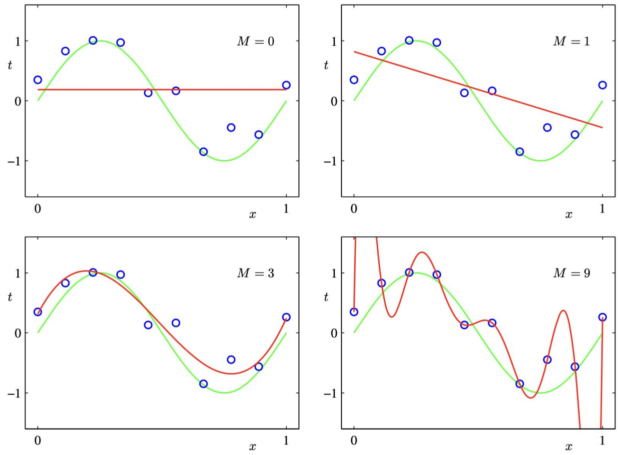
Polynomial Regression with Regularization



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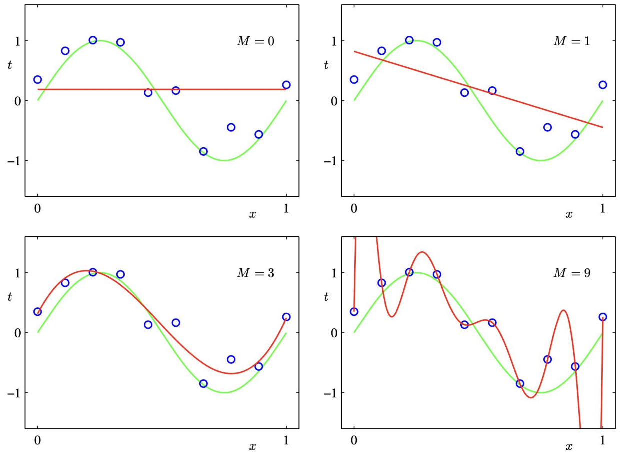
$$\boxed{\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2}$$

Polynomial Regression with Regularization



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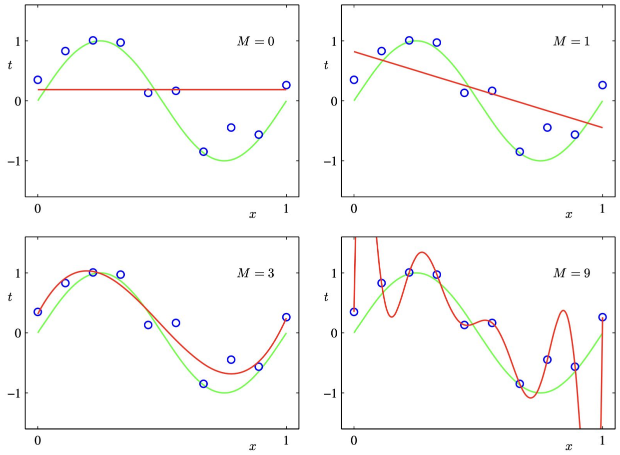
decreasing

Polynomial Regression with Regularization



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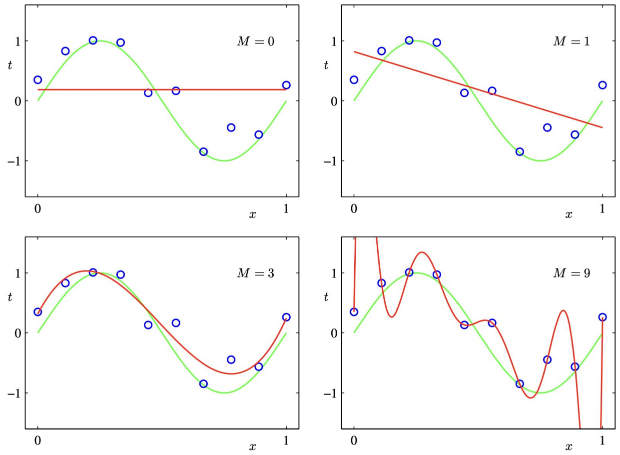
increasing

$$\boxed{\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{1}{2}\boxed{\|\mathbf{w}\|^2}}$$

decreasing

Polynomial Regression with Regularization



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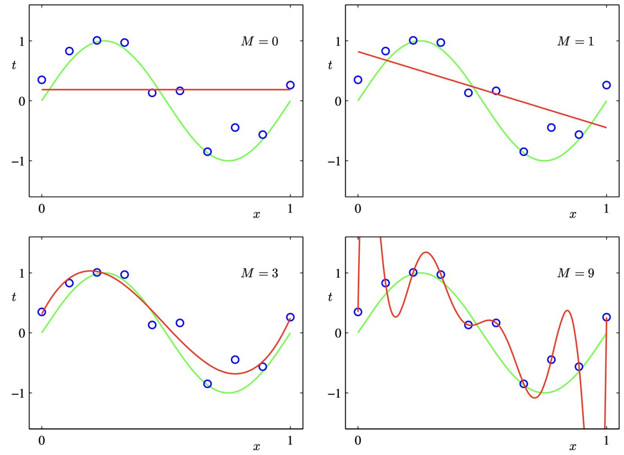
Loss function

Regularizer

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{1}{2}\boxed{\|\mathbf{w}\|^2}$$

Polynomial Regression with Regularization



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Loss function

Regularizer

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2}\boxed{\|\mathbf{w}\|^2}$$

Lamda is for balancing these two -
Hyper Parameter of this model

How to find the best HP (Lamda)?

Predictive modeling: Regression (diabetes)