Glimpses of Cryptography

MAC MATH FEST 2021

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Guided by,
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- One of protection method is Cryptography.
- Cryptography is used in ATM, Email-Password, E-Payment, E-Commerce, Electronic Voting, Defense Services, Securing Data, Access Control etc.

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- It is a branch of both Mathematics and Computer science.

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- **Key:** unique info used in cipher known only sender and receiver.

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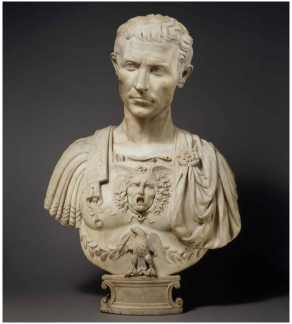

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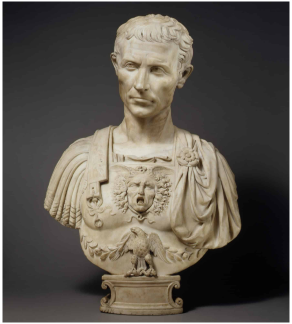

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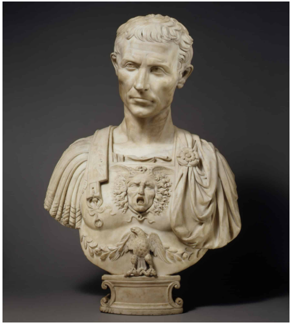
- Each character of a plaintext message is replaced by **n position down** in the alphabet.



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- Ciphertext is obtain by shifting the original letter by n position to the right.
- In this example, it is shifted by 3 to the right.
  - A becomes D
  - B becomes E
  - X becomes A
  - and so on $\cdots$

A	B	C	$\cdots$	X	Y	Z
D	E	F	$\cdots$	A	B	C

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Continue...

- Suppose the following plaintext is to be encrypted

ATTACK AT DAWN

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  ATTACK AT DAWN
- By shifting each letter by 3 to the right. The resulting ciphertext would be
  DWWDFN DW GDZQ

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- Example:
  - Ciphertext:

WJYZWS YT GFXJ

  - Key used: 5
  - Plaintext:

RETURN TO BASE

A	B	C	...	X	Y	Z
V	W	X	...	S	T	U

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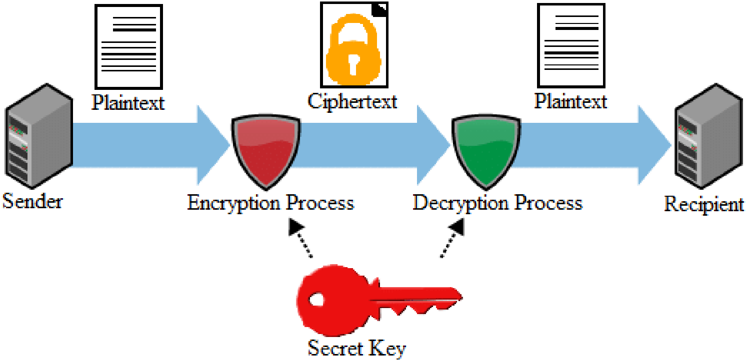
- Decryption process can be represented as:

$$x = D(y) = (y - k)(\mod 26)$$

where

- x is the plaintext
- y is the ciphertext
- k is the number of shift
- There are 26 letters in the alphabet (English alphabet).

Symmetric Ciphers



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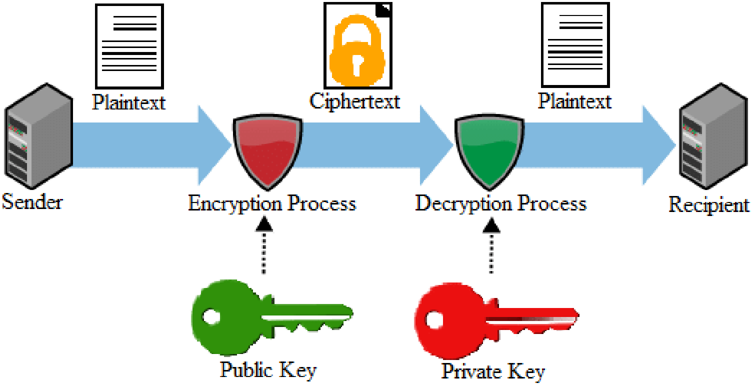

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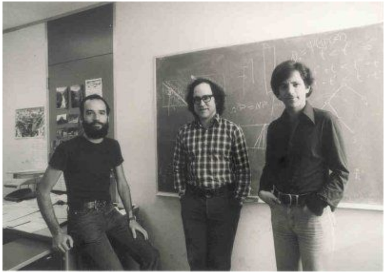

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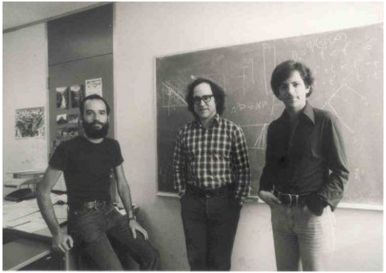

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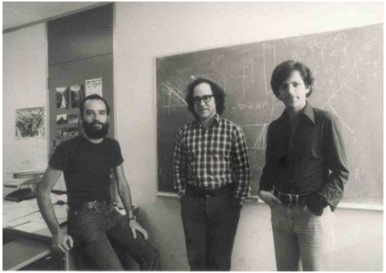

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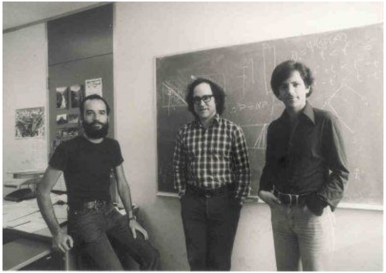

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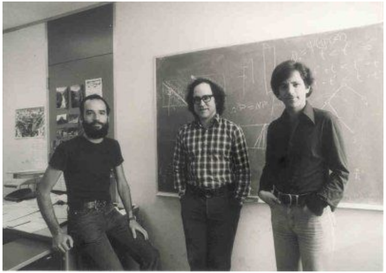

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- The modulus n must not be shared.

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- RSA-2048 has 617 decimal digits (2048 bits). It is the largest of the RSA numbers and carried the largest cash prize for its factorization, $200,000.
- As of today, no RSA number larger than 768 bits has been successfully factored, highlighting the strength of modern public-key cryptography.

References

- Yan, S. Y. Cryptanalytic Attacks on RSA. Springer, 2008.
- Delfs, H. and Knebl, H. Introduction to Cryptography: Principles and Applications. Springer, 3rd Edition, 2015.
- Stinson, D. R. Cryptography: Theory and Practice. Chapman Hall/CRC, 4th Edition, 2018.
- Lecture video (Bengali) is available at: `https://youtu.be/XRfuKKCQVBA?t=1460`
- Pictures were taken from Google Images.

Thank You