Quantum Information

MA5770: Modelling Workshop

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Classical information

- Consider a physical system that stores information: let us call it X.
- Assume X can be in one of a finite number of classical states at each moment. Denote this classical state set by Σ.
- Examples:
  - If X is a bit, then its classical state set is $\Sigma = \{0, 1\}$.
  - If X is a six-sided die, then $\Sigma = \{1, 2, 3, 4, 5, 6\}$.
  - If X is a switch on a standard electric fan, then perhaps $\Sigma = \{\mathit{high}, \mathit{medium}, \mathit{low}, \mathit{off}\}$.

Classical information

- For example, if X is a bit, then perhaps it is in the classical state 0 with probability 3/4 and in the classical state 1 with probability 1/4. This is a **probabilistic state** of X.
  - $Pr(X = 0) = \frac{3}{4}$ and $Pr(X = 1) = \frac{1}{4}$
- A succinct way to represent this probabilistic state is by a column vector:
  - $$\begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} \quad \begin{array}{l} \leftarrow \text{entry corresponding to } 0 \\ \leftarrow \text{entry corresponding to } 1 \end{array}$$
- This vector is a **probability vector**:
  - All entries are nonnegative real numbers.
  - The sum of the entries is 1.

Dirac notation

- Let Σ be any classical state set, and assume the elements of Σ have been placed in correspondence with the integers $1, ..., |\Sigma|$.
- We denote by $|a\rangle$ the **column vector** having a '1' in the entry corresponding to $a \in \Sigma$, with '0' for all other entries.
- Example 1:
  - If $\Sigma = \{0, 1\}$, then $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Continue...

- Let Σ be any classical state set, and assume the elements of Σ have been placed in correspondence with the integers $1, ..., |\Sigma|$.
- We denote by $|a\rangle$ the **column vector** having a '1' in the entry corresponding to $a \in \Sigma$, with '0' for all other entries.
- Example 2:
  - If $\Sigma = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$, then we might choose to order these states like this:

$$|\clubsuit\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\diamondsuit\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |\heartsuit\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |\spadesuit\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Continue...

- Let Σ be any classical state set, and assume the elements of Σ have been placed in correspondence with the integers $1, ..., |\Sigma|$.
- We denote by $|a\rangle$ the **column vector** having a '1' in the entry corresponding to $a \in \Sigma$, with '0' for all other entries.
- Vectors of this form are called **standard basis vectors.** Every vector can be expressed uniquely as a linear combination of standard basis vectors.
- Example: $\begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix} = \frac{3}{4}|0\rangle + \frac{3}{4}|1\rangle$

Dirac notation

- Let Σ be any classical state set, and assume the elements of Σ have been placed in correspondence with the integers $1, ..., |\Sigma|$.
- We denote by $|a\rangle$ the **row vector** having a '1' in the entry corresponding to $a \in \Sigma$, with '0' for all other entries.
- Example 1:
  - If $\Sigma = \{0, 1\}$, then $\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and $\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$

Dirac notation

- $\langle a||b\rangle = \langle a|b\rangle = \begin{cases} 1 & \text{if } a = b \\ 0 & \text{if } a \neq b \end{cases}$
- $|a\rangle\langle b|$ has a 1 in the (a,b)-entry and 0 for all other entries.

Basic Terminology

Agenda
Classical information
Qubits

- **Ket:** $|*\rangle$, generally it's a column vector.
- **Bra:** $\langle *| = |*\rangle^\dagger$, generally it's a row vector.
- **Bra-Ket:** $\langle *||*\rangle$ or, $\langle *|*\rangle$
- **Ket-Bra:** $|*\rangle\langle *|$
- $|*\rangle|*\rangle = |**\rangle$
- **Vector Notations:** $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, standard Qubits.

  $\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and $\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$
- **Hadamard Qubits:** $\langle +| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ and

  $\langle -| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

Qubits

- **Definition:** Quantum analog of classical bits; exists in superposition states $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in \mathbb{C}$ $= cos(\frac{\theta}{2})|0\rangle + e^{i\varphi} sin(\frac{\theta}{2})|1\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$.

Let, $\alpha = r_1 e^{i\theta_1}$ and $\beta = r_2 e^{i\theta_2}$
Then, $r_1^2 + r_2^2 = 1$ and $\theta_1 = \theta_2 + \varphi$
Take, $r_1 = cos(\frac{\theta}{2}), r_2 = sin(\frac{\theta}{2})$

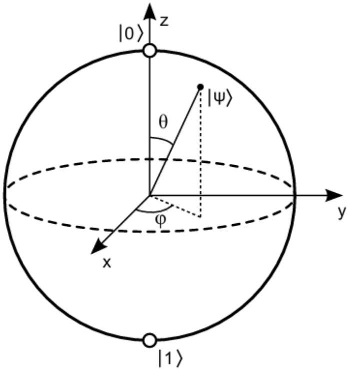

Figure: The Bloch sphere is a geometrical representation of a qubit. Qubits can take as value each point on the surface described by the two angles φ and θ. The pole points are $|0\rangle$ or $|1\rangle$.

Thank You