Michael Christensen
September 21, 2013
CS 450

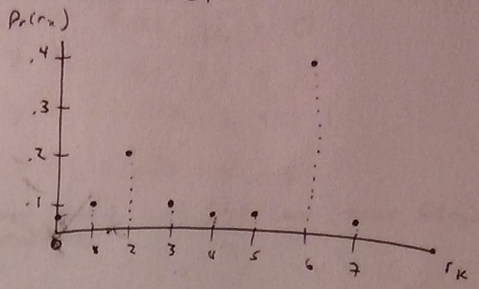
Homework #2

Written Exercises:
See the following 4 pages:

MICHAEL CHRISTENSEN SEPT-21-2013 CS450

(1) INTENSITY DISTRIBUTION:

r_k	n_k	$P_r(r_k) = n_k/MN$, $MN = 32 \times 32 = 1024$
$r_0 = 0$	62	$62/1024 = 0.06$
$r_1 = 1$	100	$100/1024 = 0.10$
$r_2 = 2$	200	$200/1024 = 0.20$
$r_3 = 3$	100	$100/1024 = 0.10$
$r_4 = 4$	50	$50/1024 = 0.05$
$r_5 = 5$	50	$50/1024 = 0.05$
$r_6 = 6$	400	$400/1024 = 0.39$
$r_7 = 7$	62	$62/1024 = 0.05$ (REALLY 0.06, BUT MADE 0.05 TO ADD UP TO 1.00)



$S_k = T(r_k) = (L-1) \sum_{j=0}^{k} P_r(r_j)$, $(L-1) = 8-1 = 7$

$S_0 = T(r_0) = 7\, p_r(r_0) = 7 * 0.06 = 0.42$

$S_1 = T(r_1) = 0.42 + 7\, p_r(r_1) = 0.42 + 7 * .1 = 1.12$

$S_2 = T(r_2) = 1.12 + 7 * (0.2) = 2.52$

$S_3 = T(r_3) = 2.52 + 7 * .1 = 3.22$

$S_4 = T(r_4) = 3.22 + 7 * .05 = 3.57$

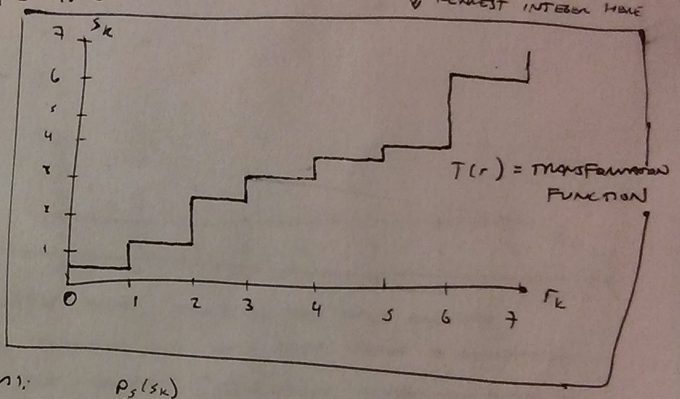
$S_5 = T(r_5) = 3.57 + 7 * .05 = 3.92$

$S_6 = T(r_6) = 3.92 + 7 * 0.39 = 6.65$

$S_7 = T(r_7) = 6.65 + 7 * .05 = 7.00$

SHOULD ROUND

I DON'T THINK THE BOOK ROUNDED TO NEAREST INTEGER HERE



(2) SCALED HISTOGRAM-EQUALIZED VALUES FROM PROBLEM 1:

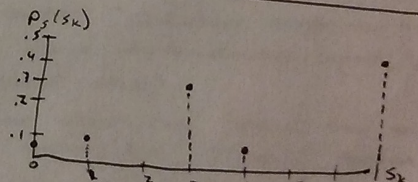
$S_0 = 0.42 \Rightarrow 0$ → $P_s(s_k) = 0.06$

$S_1 = 1.12 \Rightarrow 1 \rightarrow 0.1$

$S_2 = 2.52 \Rightarrow 3$, $S_3 = 3.22 \Rightarrow 3$ } $\frac{200+100}{1024} = 0.3$

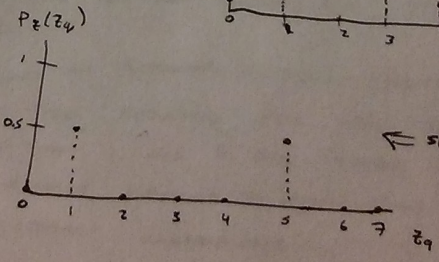
$S_4 = 3.57 \Rightarrow 4$, $S_5 = 3.92 \Rightarrow 4$ } $0.05 + 0.05 = 0.1$

$S_6 = 6.65 \Rightarrow 7$, $S_7 = 7.0 \Rightarrow 7$ } $0.39 + 0.05 = 0.44$



SPECIFIED HISTOGRAM'S INTENSITY DIST:

z_q	n_q	$P_z(z_q) = n_q/MN$
$z_0 = 0$	0	0
$z_1 = 1$	512	$512/1024 = 0.5$
$z_2 = 2$	0	0
$z_3 = 3$	0	0
$z_4 = 4$	0	0
$z_5 = 5$	512	$512/1024 = 0.5$
$z_6 = 6$	0	0
$z_7 = 7$	0	0



← SPECIFIED HISTOGRAM

VALUES OF THE TRANSFORMATION FUNCTION G:

$G(z_0) = (L-1) \sum_{j=0}^{0} p_z(z_0) = 7 * 0 = 0$

$G(z_1) = 7(0 + 0.5) = 3.5 \rightarrow 4$

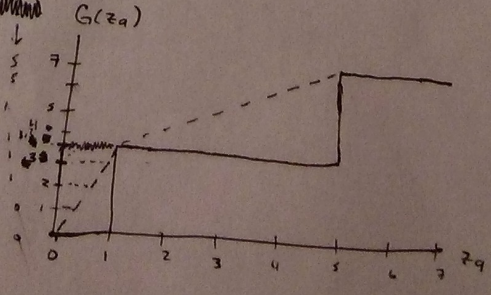
$G(z_2) = 7(0 + 0.5 + 0) = 3.5 \rightarrow 4$

$G(z_3) = 7(0 + 0.5 + 0 + 0) = 3.5 \rightarrow 4$

$G(z_4) = 7(0 + 0.5 + 0 + 0 + 0) = 3.5 \rightarrow 4$

$G(z_5) = 7(0 + 0.5 + 0 + 0 + 0 + 0.5) = 7.0$

$G(z_6) = 7.0$, $G(z_7) = 7.0$



(CONTINUED)

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	4
$z_2 = 2$	4
$z_3 = 3$	4
$z_4 = 4$	4
$z_5 = 5$	7
$z_6 = 6$	7
$z_7 = 7$	7

$G(z) = \text{round}(7 T(z))$	
0	0
1	0
2	1
3	1
4	1
5	1
6	5
7	6

$s_k \rightarrow z_q$

$s_0 = 0 = G(z_0), z_0 = 0$ $(s_0 = z_0)$

$s_1 = 1 \neq G(\ ), z_0 = 0$

$s_2 > 3 \neq G(\ ), z = 1$

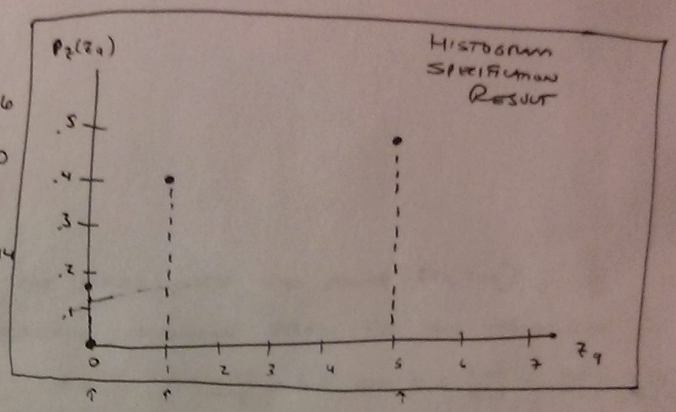
$s_3 > 4 = G(z_1), z_1 = 1$

$s_4 > 7 = G(z_4), z_4 = 5$

FIND SMALLEST VALUE OF z_q S.T. THE VALUE $G(z_q)$ IS THE CLOSEST TO s_k.

SPECIFIED & ACTUAL HISTOGRAMS

z_q	$p_z(z_q)$	$p_z(z_k)$	
$z_0 = 0$	0	$s_0 = 0$ 0.06, $s_1 = 1$ 0.10	> 0.16
$z_1 = 1$	0.5	$s_2 =$ 0.2, $s_3 =$ 0.1, $s_4 =$ 0.05, $s_5 =$ 0.05	> 0.40
$z_2 = 2$	0		
$z_3 = 3$	0		
$z_4 = 4$	0		
$z_5 = 5$	0.5	$s_6 =$ 0.39, $s_7 =$ 0.05	> 0.44
$z_6 = 6$	0		
$z_7 = 7$	0		



IF TWO INPUT VALUES MAP TO THE SAME OUTPUT VALUE, IT IMPLIES THE IMAGE CANNOT BE REPRESENTED WITH A STRICTLY MONOTONIC FUNCTION, AND YOU MUST HAVE A CONVENTION TO CHOOSE THE PARTICULAR VALUE INTO THE OUTPUT FUNCTION WHICH YOU WISH TO USE [FOR EXAMPLE, CHOOSING THE SMALLEST VALUE INTO THE SPECIFIED OUTPUT TRANSFORMATION FUNCTION WHICH GETS YOU CLOSEST TO AN INPUT VALUE].

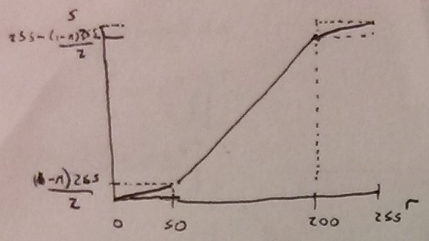
IF NO INPUT VALUE MAPS ONTO A PARTICULAR OUTPUT VALUE, THIS IMPLIES THAT YOU SHALL DON'T HAVE ANY NEED TO DO SHIFTING FOR THAT OUTPUT AND IT WON'T APPEAR IN THE NEW HISTOGRAM.

THE SPECIFIED HISTOGRAM DIFFERS FROM THE RESULTING HISTOGRAM SPECIFICATION SLIGHTLY. SINCE THIS WAS THE DISCRETE CASE, THE RESULTING HIST. SPEC. ISN'T PERFECTLY UNIFORM, BUT LOOKS SIMILAR IN THAT 1 AND 5 ARE HIGHER, BUT NOTICEABLY DIFFERENT IS HOW 0 ALSO HAS A VALUE, SEEMING TO SHOW HOW THE SCALING HISTEQUALIZED VALUE OF 0 AND 1 IN PROBLEM 1 CONTRIBUTED.

(3) THE APPROACH I WOULD USE IS A PIECE-WISE FUNCTION (SIMILAR TO WHAT IS MENTIONED IN THE SLIDES) TO STRETCH THE CONTRAST OF VALUES IN THE RANGE [50, 200], SINCE THAT IS WHERE MOST OF THE IMAGE'S VALUES ARE. THAT, PROPORTION THE OUTPUT TO GIVE MORE WEIGHT TO VALUES IN THAT RANGE.

$$s = \begin{cases} mr, & m = \left(\frac{\left(\frac{1-n}{2}\right) * 255}{2}\right) / (50-0) & \text{IF } r < 50 \\ mr, & m = \frac{n * 255}{(200-50)} & \text{IF } 50 \le r \le 200 \\ mr, & m = \left(\frac{\left(\frac{1-n}{2}\right) * 255}{2}\right) / (255-200) & \text{IF } r > 200 \end{cases}$$

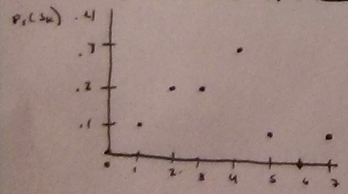
THIS DOES THE FOLLOWING, IN EFFECT:



WHERE n IS THE PERCENTAGE OF THE IMAGE INSIDE THE RANGE [50, 200], SO THAT THE SLOPE OF THE TRANSFORMATION FUNCTION BELOW 50 AND ABOVE 200 IS LESS THAN 1 (EG. IF $n = 95\%$, $1-n = 5\%$, $\frac{0.05 \times 255}{2} = 6.3$, AND $m = \frac{\text{RISE}}{\text{RUN}} = \frac{6.3}{50} = 0.13$, THE SLOPE BETWEEN [0, 50], AND THE SLOPE BETWEEN [50, 200] WOULD BE $\frac{.95 \times 255}{200-50} = 1.615$, > 1). THE CONTENT IN [50, 200] IS STRETCHED INTO MORE OUTPUT, PLACING MORE EMPHASIS ON IT. I DO THIS TO PRESERVE SOME OF THE CONTRAST IN THE LEVELS BELOW 50 AND ABOVE 200, INSTEAD OF JUST DOING A BINARY WHITE OR BLACK. I DON'T THINK HIST. EQUILIZATION WOULD NECESSARILY DO BETTER, SINCE MY APPROACH FOR WINDOWING IS BASICALLY DOING WHAT HIST. EQU. WOULD DO, RELYING ON PROPORTIONS OF THE IMAGE TO DETERMINE HOW MUCH OF THE OUTPUT TO DEVOTE TO THAT SECTION OF INPUT.

(4) GRW 3.7 - SHOW THAT A 2ND PASS OF HIST. EQUI. ON THE HIST-EQUALIZED IMAGE WILL PRODUCE EXACTLY SAME RESULT AS THE 1ST PASS.

TAKE FOR EXAMPLE AN EQUALIZED HISTOGRAM:

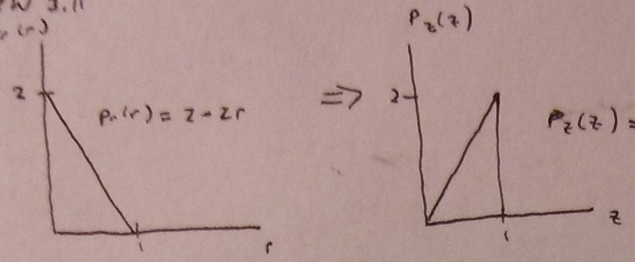


DOING HISTOGRAM EQUATION ON THIS WON'T PRODUCE ANY CHANGE, FOR EXAMPLE GIVEN AN IMAGE OF 1024 PIXELS,

s_k		
0	→	$\frac{0 * 0}{1024} = 0$, SAME AS IN EQUALIZED HISTOGRAM
1	→	$\frac{.1 * 1024}{1024} = .1$, " "
2	→	$\frac{.2 * 1024}{1024} = .2$, " "
3	→	$\frac{.2 * 1024}{1024} = .2$, " "
4	→	$\frac{.3 * 1024}{1024} = .3$, " "
5		
6		⇓
7		ETC.

BY EQUALIZING AN EQUALIZED IMAGE, YOU HAVE NO PROPORTIONS TO CHANGE OR PIXEL VALUES TO SHIFT.

⑤ CRW 3.11



ASSUME CONTINUOUS QUANTITIES, FIND TRANSFORMATION TO ACCOMPLISH THIS:

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$= (L-1)\int_0^r (2 - 2w)\,dw$$

$$= 2(L-1)\int_0^r (1-w)\,dw$$

$$= 2(L-1)\left[w - \frac{w^2}{2}\right]\Big|_0^r$$

$$s = 2(L-1)\left[r - \frac{r^2}{2}\right]$$

$$G(z) = (L-1)\int_0^z p_z(w)\,dw$$

$$= 2(L-1)\int_0^z w\,dw$$

$$= 2(L-1)\left.\frac{w^2}{2}\right|_0^z$$

$$= (L-1)\,z^2 = s$$

$$z = \left[\frac{s}{(L-1)}\right]^{1/2}$$

$$z = \left(\frac{2(L-1)}{(L-1)}\left(r - \frac{r^2}{2}\right)\right)^{1/2}$$

$$= \left(2\left(r - \frac{r^2}{2}\right)\right)^{1/2}$$

$$\boxed{z = \sqrt{2\left(r - \frac{r^2}{2}\right)} = \sqrt{2r - r^2}}$$

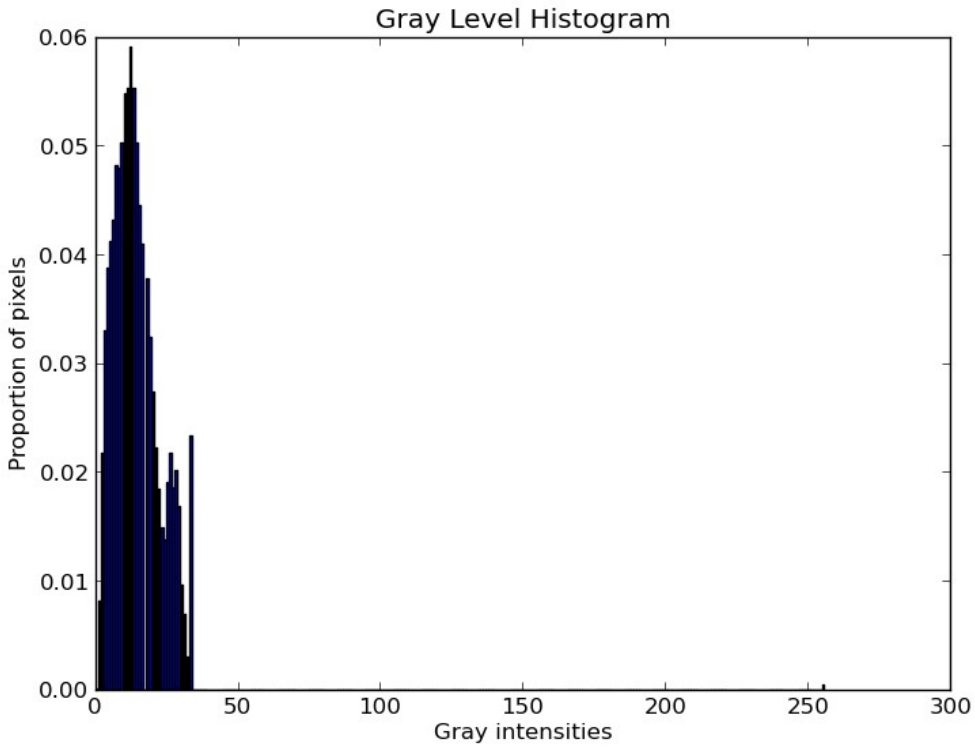
Programming Exercises:
Description of my equalization program (*source code is at end of document*):

I store the frequencies of each intensity, 0 -255, of the converted gray-scale image in an array, found by looping over the image's rows and columns and incrementing the bins of intensity values as I find them. I then create another array to store each intensity's probability density, and fill this array by iterating over the frequency array and dividing the value stored in each element by the total number of pixels in the image. I then proceed to store the transformation function in the form of an array by following equation 3.3-8 in the book, iterating the intensity probability density array one by one until I have computed every T(r_k). At the same time, I determine which integer value the current transformation value T(r_k) will map to and record how many pixels of the original image use that intensity assigned to r_k for later use in creating the equalized histogram (probability density function). Finally, I create a new image by iterating over every value in the old gray-scale image, looking up what that pixel's intensity value now maps to via the transformation lookup table, and store the associated value in the appropriate location in the new image.

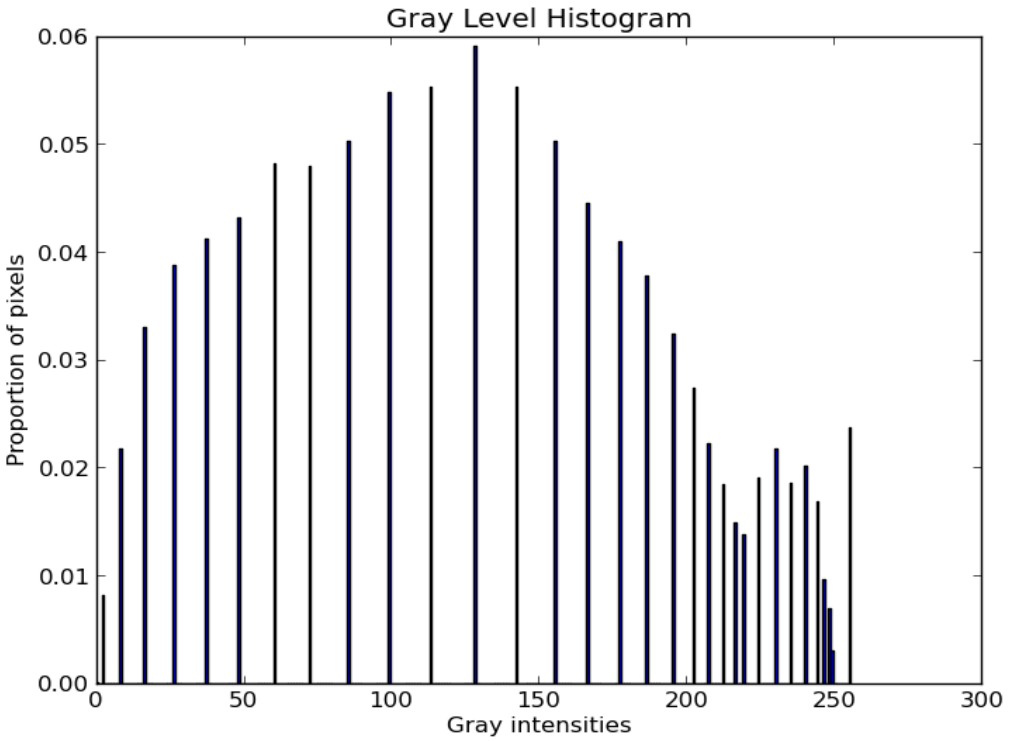
Image *mystery.png* before and after hist. Equalization:
Before:

After:

Scaled histograms of *mystery.png* before and after histogram equalization:
Before:



After:
Although theoretically it should be uniform, in reality this equalization is curved as shown below because of the binning problem. We cannot exactly assign output according the frequency of the input in this discrete case, and part of the problem is that non new intensity levels are allowed to be created. But the equalization does a good job in spreading out the parts of the input intensities where the most pixels are located, cleaning up the contrast of the image.



Lookup table used to equalize the image:

rk	sk
0	0.000000
1	2.000000
2	8.000000
3	16.000000
4	26.000000
5	37.000000
6	48.000000
7	60.000000
8	72.000000
9	85.000000
10	99.000000
11	113.000000
12	128.000000
13	142.000000
14	155.000000
15	166.000000
16	177.000000
17	177.000000
18	186.000000
19	195.000000
20	202.000000
21	207.000000
22	212.000000
23	216.000000
24	219.000000
25	224.000000
26	230.000000
27	235.000000
28	240.000000
29	244.000000
30	246.000000
31	248.000000
32	249.000000
33	255.000000
34	255.000000
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Time Spent:

Written: About 6 hours (includes reading the book beforehand)

Programming: Around 6 hours (this took so long because I encountered some trouble with Octave, and choosing not to use Matlab, I instead spent considerable time learning Numpy, PIL, and matplotlib). I think I'll be sticking with Python from now on as I feel like it's becoming less demystified to me. Also, the TA uses Python, so I feel like he's a good resource to turn to as I have questions.

Source Code (I used the TA's example code he sent out as the starting place to know how to open an image in Python and access it's data):

```
import numpy as np
import PIL.Image as pil
import matplotlib.pyplot as plt



def histogramEqualization():

    # load color image
    img = pil.open("mystery.png")
    imgData = img.getdata()
    imgPix = np.array(imgData)

    grayImgPix = np.around(np.dot(imgPix[..., :3],
        [0.299, 0.587, 0.144])).astype(int)
    # Store the frequencies for each intensity, 0 - 255
    grays = np.zeros(256, int)
    for r in range(grayImgPix.size):        # for each pixel in 1-d long array
            grayVal = grayImgPix[r]
            if (grayVal > 255):
                grayVal = 255
            grays[grayVal] = grays[grayVal] + 1
    # Store each intensity's probability density
    pdf = np.zeros(256, np.float64)
    numPixs = img.size[0] * img.size[1]
    for r in range(grays.size):
        pdf[r] = np.float64(grays[r]) / np.float64(numPixs)
    # assert summation pdf from 0 to r = 1.0--verified it does

    # Store the transformation function values, equalization histogram lookup
table
    transVals = np.zeros(256, np.float64)
    histSum = np.zeros(256)
    for r in range(pdf.size):
        prev = None
        if (r == 0.0):
            prev = 0.0
        else:
```

```
        prev = transVals[r - 1]
    transVals[r] = (255.0 * pdf[r]) + prev

    # find the rounded value of the current value we just computed
    rounded = int(round(transVals[r]))
    # be remembering what they map to to store in new histogram
    histSum[rounded] = histSum[rounded] + grays[r]

    #print (transVals[r])
    #print (rounded)
    #print (histSum[rounded])

# Store the equalized pdf
equalPdf = np.zeros(256)
a = 0.0
for r in range(equalPdf.size):
    equalPdf[r] = float(histSum[r] / float(numPixs))
    a += equalPdf[r]
# check to make sure = 1.0
#print (a)

plt.bar(list(range(transVals.size)), transVals)
plt.title("Transformation Function")
plt.xlabel("rk")
plt.ylabel("sk")
#plt.show()

plt.bar(list(range(equalPdf.size)), equalPdf)
plt.title("Gray Level Histogram")
plt.xlabel("Gray intensities")
plt.ylabel("Proportion of pixels")
plt.show()

# store it back in a new image
for r in range(grayImgPix.size):
    # print (transVals[grayImgPix[r]])
    grayVal = grayImgPix[r]
    if (grayVal > 255):  # sometimes 263 pops up...
```

```
            grayVal = 255
        grayImgPix[r] = transVals[grayVal]
    newGrayImg = pil.new("L", img.size)
    newGrayImg.putdata(grayImgPix)
    newGrayImg.show()
    newGrayImg.save("equalized-mystery.png")

    # print out transformation lookup table
    for c in range(transVals.size):
        print ("%i | %f" % (c, int(round(transVals[c]))))


def grayImgExp():
    # do stuff
    # load color image
    img = pil.open("mystery.png")
    imgData = img.getdata()
    imgPix = np.array(imgData)
    numPixs = img.size[0] * img.size[1]

    # make it grayscale, print out histogram manually
    grayImgPix = np.around(np.dot(imgPix[..., :3],
        [0.299, 0.587, 0.144])).astype(int)
    grays = np.zeros(256, int)
    for r in range(grayImgPix.size):        # for each pixel in 1-d long array
            grayVal = grayImgPix[r]
            if (grayVal > 255):
                grayVal = 255
            grays[grayVal] = grays[grayVal] + 1
    # probability distribution
    pdf = np.zeros(256, np.float64)
    for r in range(pdf.size):
        pdf[r] = np.float64(grays[r] / np.float64(numPixs))

    plt.bar(list(range(pdf.size)), pdf)
    plt.title("Gray Level Histogram")
    plt.xlabel("Gray intensities")
    plt.ylabel("Proportion of pixels")
```

```
    plt.show()

def main():
    grayImgExp()
    histogramEqualization()


if __name__ == "__main__":
    main()
```