3.7 Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

3.8 In some applications it is useful to model the histogram of input images as Gaussian probability density functions of the form

$$p_r(r) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-m)^2}{2\sigma^2}}$$

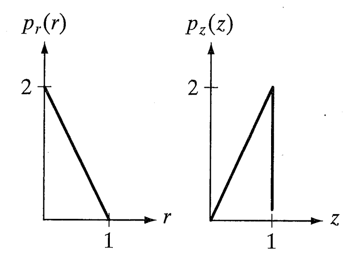
where m and σ are the mean and standard deviation of the Gaussian PDF. The approach is to let m and σ be measures of average intensity and contrast of a given image. What is the transformation function you would use for histogram equalization?

★3.9 Assuming continuous values, show by example that it is possible to have a case in which the transformation function given in Eq. (3.3-4) satisfies conditions (a) and (b) in Section 3.3.1, but its inverse may fail condition (a′).

3.10 **(a)** Show that the discrete transformation function given in Eq. (3.3-8) for histogram equalization satisfies conditions (a) and (b) in Section 3.3.1.

★(b) Show that the inverse discrete transformation in Eq. (3.3-9) satisfies conditions (a′) and (b) in Section 3.3.1 only if none of the intensity levels r_k, $k = 0, 1, \ldots, L-1$, are missing.

3.11 An image with intensities in the range [0, 1] has the PDF $p_r(r)$ shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



★3.12 Propose a method for updating the local histogram for use in the local enhancement technique discussed in Section 3.3.3.

3.13 Two images, $f(x, y)$ and $g(x, y)$, have histograms h_f and h_g. Give the conditions under which you can determine the histograms of

★(a) $f(x, y) + g(x, y)$

(b) $f(x, y) - g(x, y)$

(c) $f(x, y) \times g(x, y)$

(d) $f(x, y) \div g(x, y)$

in terms of h_f and h_g. Explain how to obtain the histogram in each case.

3.14 The images shown on the next page are quite different, but their histograms are the same. Suppose that each image is blurred with a 3×3 averaging mask.

(a) Would the histograms of the blurred images still be equal? Explain.

(b) If your answer is no, sketch the two histograms.