Michael Christensen
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CS 450

Homework 4 – Written Exercises

1. Show that the Fourier Transform is linear (use the Fourier Transform integral):

It needs to shown that *ℱ(ag + bh) = aF(g) + bF(h)*, where *F(some_function)* is the Fourier Transform for functions *g* and *h*, and *a* and *b* are constants. Therefore:

$\mathcal{F}(ag(t) + bh(t)) = \int ag(t)\, e^{-i2\pi ut}\, dt + \int bh(t)\, e^{-i2\pi ut}\, dt$ // I am replacing $\mathcal{F}(f(t))$ with $\mathcal{F}(g(t) + h(t))$

$= a \int g(t)\, e^{-i2\pi ut}\, dt + b \int bh(t)\, e^{-i2\pi ut}\, dt$ // move the constants outside of the integrals

$= a (\int g(t)\, e^{-i2\pi ut}\, dt) + b (\int bh(t)\, e^{-i2\pi ut}\, dt)$ // notice the similarity to normal Fourier transforms inside parentheses

$= aG(f(t)) + bH(f(t))$, where *G* and *H* are the Fourier Transforms

2. Compute by hand the DFT of the signal f = [1, 1, 1, 1] (show your work):

f = [1, 1, 1, 1]
M = 4

u = 0 → F(0)

x	f(x)	cos(2pi0x/M)	sin(2pi0x/M)	f(x)cos(2pi0x/M)	f(x)sin(2pi0x/M)
0	1	1	0	1	0
1	1	1	0	1	0
2	1	1	0	1	0
3	1	1	0	1	0
sum				4 (real part)	0 (imaginary part)

1/4(4 + 0j) = 1 + 0j

u = 1 → F(1)

x	f(x)	cos(2pi1x/M)	sin(2pi1x/M)	f(x)cos(2pi1x/M)	f(x)sin(2pi1x/M)
0	1	1	0	1	0
1	1	0	1	0	1
2	1	-1	0	-1	0
3	1	1	-1	0	-1
sum				0 (real part)	0 (imaginary part)

1/4(0 + 0j) = 0 + 0j

u = 2 → F(2)

x	f(x)	cos(2pi2x/M)	sin(2pi2x/M)	f(x)cos(2pi2x/M)	f(x)sin(2pi2x/M)
0	1	1	0	1	0
1	1	-1	0	-1	0
2	1	1	0	1	0
3	1	-1	0	-1	0
sum				0 (real part)	0 (imaginary part)

1/4(0 + 0j) = 0 + 0j

u = 3 → F(3)

x	f(x)	cos(2pi3x/M)	sin(2pi3x/M)	f(x)cos(2pi3x/M)	f(x)sin(2pi3x/M)
0	1	1	0	1	0
1	1	0	-1	0	-1
2	1	-1	0	-1	0
3	1	0	1	0	1
sum				0 (real part)	0 (imaginary part)

1/4(0 + 0j) = 0 + 0j

DFT of f = [1, 0, 0, 0]