

Matthew D. Kvalheim

Assistant Professor of Mathematics
University of Maryland, Baltimore County
✉ mdkvalheim.github.io
Citizenship: USA

Research Interests

Nonlinear systems in general, and in particular: dynamics, control theory, stochastic processes, robotics.

Employment

- 2023– **Assistant Professor**, *University of Maryland, Baltimore County*, Department of Mathematics and Statistics.
- 2022–2023 **Postdoctoral Assistant Professor**, *University of Michigan*, Department of Mathematics.
Mentor: Anthony M. Bloch
- 2019–2022 **Postdoctoral Researcher**, *University of Pennsylvania*, Department of Electrical and Systems Engineering.
Mentors: Yuliy Baryshnikov, Daniel E. Koditschek
- 2018–2019 **Postdoctoral Research Fellow**, *University of Michigan*, Department of Electrical Engineering and Computer Science.
Mentors: Anthony M. Bloch, Shai Revzen

Education

- 2013–2018 **Ph.D.**, *University of Michigan*, Electrical Engineering.
- 2013–2017 **M.S.**, *University of Michigan*, Mathematics.
- 2013–2017 **M.S.**, *University of Michigan*, Electrical Engineering.
- 2009–2013 **B.S.**, *Ohio University*, Electrical Engineering, *summa cum laude*, #1 EECS graduate.

Doctoral Thesis

- Title Aspects of invariant manifold theory and applications
- Adviser Shai Revzen

Published or Accepted Journal Papers

- 11 **A compositional approach to certifying the almost global asymptotic stability of cascade systems.**
Jake Welde, Matthew D. Kvalheim, and Vijay Kumar
IEEE Control Systems Letters, 7 (2023), pp. 1969–1974. ([article](#), [arXiv](#))
- 10 **Flux in tilted potential systems: negative resistance and persistence.**
Yuliy Baryshnikov and Matthew D. Kvalheim
Communications in Mathematical Physics, 400.2 (2023), pp. 853–930 ([article](#), [arXiv](#))
- 9 **Obstructions to asymptotic stabilization.**
Matthew D. Kvalheim
SIAM J. Control and Optimization, 61.2 (2023), pp. 536–542. ([article](#), [arXiv](#))
- 8 **A generalization of the Hopf degree theorem.**
Matthew D. Kvalheim
Proceedings of the American Mathematical Society, 151.1 (2023), pp. 453–454. ([article](#), [arXiv](#))

- 7 **Planning of obstacle-aided navigation for multi-legged robots using a sampling-based method over directed graphs.**
Kaustav Chakraborty, Haodi Hu, Matthew D. Kvalheim, and Feifei Qian
 IEEE Robotics and Automation Letters, 7.4 (2022), pp. 8861–8868. ([article](#))
- 6 **Necessary conditions for feedback stabilization and safety.**
Matthew D. Kvalheim and Daniel E. Koditschek
 J. Geometric Mechanics, 14.4 (2022), pp. 659–693. ([article](#), [arXiv](#))
- 5 **Families of periodic orbits: closed 1-forms and global continuability.**
Matthew D. Kvalheim and Anthony M. Bloch
 J. Differential Equations, 285 (2021), pp. 211–257. ([article](#), [arXiv](#))
- 4 **Conley’s fundamental theorem for a class of hybrid systems.**
Matthew D. Kvalheim, Paul Gustafson, and Daniel E. Koditschek
 SIAM J. Applied Dynamical Systems (SIADS), 20.2 (2021), pp. 784–825. ([article](#), [arXiv](#))
- 3 **Existence and uniqueness of global Koopman eigenfunctions for stable fixed points and periodic orbits.**
Matthew D. Kvalheim and Shai Revzen
 Physica D, 425 (2021), pp. 132959. ([article](#), [arXiv](#))
- 2 **Gait modeling and optimization for the perturbed Stokes regime.**
Matthew D. Kvalheim, Brian Bittner, and Shai Revzen
 Nonlinear Dynamics, 97.4 (2019), pp. 2249–2270. ([article](#), [arXiv](#))
- 1 **Global linearization and fiber bundle structure of invariant manifolds.**
Jaap Eldering, Matthew D. Kvalheim, and Shai Revzen
 Nonlinearity, 31.9 (2018), pp. 4202–4245. ([article](#), [arXiv](#))

Published or Accepted Peer-Reviewed Conference Papers

- 3 **The role of symmetry in constructing geometric flat outputs for free-flying robotic systems.**
Jake Welde, Matthew D. Kvalheim, and Vijay Kumar
 IEEE International Conference on Robotics and Automation, Accepted (2023). ([arXiv](#))
- 2 **Generic properties of Koopman eigenfunctions for stable fixed points and periodic orbits.**
Matthew D. Kvalheim, David A. Hong, and Shai Revzen
 IFAC-PapersOnline, 54.9 (2021), pp. 267–272. ([article](#), [arXiv](#))
- 1 **Data-driven models of legged locomotion.**
Shai Revzen and Matthew D. Kvalheim
 SPIE Defense + Security. International Society for Optics and Photonics (2015). ([article](#))

Book Chapters

- 2 **Bioinspired legged locomotion, chapter 3.2: Templates and anchors.**
Matthew D. Kvalheim and Shai Revzen
 Butterworth-Heinemann, Elsevier, Oxford (2017)
- 1 **Bioinspired legged locomotion, chapter 3.5: Locomotion as an oscillator.**
Shai Revzen and Matthew D. Kvalheim
 Butterworth-Heinemann, Elsevier, Oxford (2017)

Papers Under Review and other Preprints

- 7 **Linearizability of flows by embeddings.**
Matthew D. Kvalheim and Philip Arathoon
 (2023). ([arXiv](#))

- 6 **Koopman embedding and super-linearization counterexamples with isolated equilibria.**
Philip Arathoon and Matthew D. Kvalheim
 (2023). ([arXiv](#))
- 5 **Estimating phase from observed trajectories using the temporal 1-form.**
Simon Wilshin, Matthew D. Kvalheim, Clayton Scott, and Shai Revzen
 Physical Review E, Under Review (2022). ([arXiv](#))
- 4 **A pasting lemma for Lipschitz functions.**
Matthew D. Kvalheim, Paul Gustafson, and Samuel A. Burden
 (2021). ([arXiv](#))
- 3 **Phase response curves and the role of coordinates.**
Simon Wilshin, Matthew D. Kvalheim, and Shai Revzen
 (2021). ([arXiv](#))
- 2 **Poincaré-Hopf theorem for hybrid systems.**
Matthew D. Kvalheim
 (2021). ([arXiv](#))
- 1 **Reverse-engineering invariant manifolds with asymptotic phase.**
Matthew D. Kvalheim and Shai Revzen
 (2016). ([arXiv](#))

Teaching Experience

- 2023 **Course Instructor**, *University of Michigan*, Ann Arbor, MI.
 Instructor of record for two course sections of Differential Equations (Math 316) in Winter 2023
- 2022 **Course Instructor**, *University of Michigan*, Ann Arbor, MI.
 Instructor of record for two course sections of Calculus I (Math 115) in Fall 2022
- 2015–2022 **Private Tutor**, *WyzAnt, Inc.*
 Independent contractor; mathematics, electrical engineering;
 perfect score on student evaluations (out of 19 total); 160+hours experience
- 2019 **Guest Lecturer**, *University of Pennsylvania*, Philadelphia, PA.
 Graduate course: Dynamical Systems for Engineering and Biological Applications (ESE 512)
 Course instructor: Daniel E. Koditschek
- 2014 **Graduate Student Instructor**, *University of Michigan*, Ann Arbor, MI.
 Senior-level course: Discrete-Time Signal Processing (EECS 451)
 Course instructor: Laura Balzano
- 2011–2012 **Supplemental Instruction Leader**, *Academic Achievement Center, Ohio University*, Athens, OH.
 Sophomore-level course: Statics (CE 220)

Mentoring and Outreach

- 2023– **Co-advisor, Pi Mu Epsilon, UMBC Chapter.**
 Mathematics honor society
- 2022–2023 **Mentor of graduate student**, *University of Michigan*, Ann Arbor, MI.
 Mentored 1 graduate student in research
- 2022 **Volunteer, Females Excelling More in Math, Engineering, and Science (F.E.M.M.E.S.)**, *University of Michigan*, Ann Arbor, MI.
- 2019–2023 **Mentor of graduate students**, *University of Pennsylvania*, Philadelphia, PA.
 Mentored 2 graduate students in research

- 2019 **Volunteer, demonstrations to visiting K-12 students, Philly Robotics Expo (PRX), University of Pennsylvania, Philadelphia, PA.**
Representing GRASP Laboratory, University of Pennsylvania; robotics outreach
- 2019 **Volunteer, demonstrations to members of the National Society of Black Engineers, 45th Annual Convention of the National Society of Black Engineers (NSBE), Detroit, MI.**
Representing BIRDS Laboratory, University of Michigan; robotics outreach
- 2013–2018 **Mentor of undergraduate and graduate students, BIRDS Laboratory, University of Michigan.**
Mentored 2 undergraduate and 3 graduate students
- 2015 **Volunteer, demonstrations to visiting Chinese dignitaries, University of Michigan, Ann Arbor, MI.**
Representing University of Michigan; robotics outreach
- 2015 **Volunteer, demonstrations to alumni and industry, Engineering Graduate Symposium, Ann Arbor, MI.**
Representing BIRDS Laboratory, University of Michigan; robotics outreach
- 2015 **Volunteer, panel participant, Graduate Student Council, Ann Arbor, MI.**
Member of panel of University of Michigan graduate students; answering questions from prospective graduate students
- 2014 **Volunteer, demonstrations to elementary schoolers, Bryant Elementary School, Ann Arbor, MI.**
Representing BIRDS Laboratory, University of Michigan; robotics outreach

Talks

- 2023 **Asymptotic stabilizability, Invited talk, Banff International Research Station, Banff, AB, Canada.**
Workshop: Geometry, Topology and Control System Design ([link](#))
- 2023 **A compositional approach to certifying the almost global asymptotic stability of cascade systems, IEEE International Conference on Robotics and Automation (ICRA), London, UK.**
with Jake Welde (speaker) and Vijay Kumar
Workshop: Compositional robotics: mathematics and tools
- 2023 **Negative resistance in small-noise dynamics and obstructions to stabilization, Invited talk, George Mason University, Fairfax, Virginia, USA.**
- 2023 **Negative resistance in small-noise dynamics and obstructions to stabilization, Invited talk, University of Maryland, Baltimore County, Baltimore, Maryland, USA.**
Applied Math Colloquium
- 2023 **Negative resistance in small-noise dynamics and obstructions to stabilization, Invited talk, Loyola University Chicago, Chicago, Illinois, USA.**
- 2023 **A principal bundle perspective on differential flatness in complex robotic and biological systems, American Physical Society March Meeting, Las Vegas, Nevada, USA.**
with Jake Welde (speaker) and Vijay Kumar
- 2022 **Existence and uniqueness of Koopman eigenfunctions near stable equilibria and limit cycles, Invited talk, International Symposium on Nonlinear Theory and Its Applications (NOLTA 2022), online.**

- 2022 **Negative resistance in small-noise dynamics via persistent homology**, *Invited talk*, *Indian Institute of Technology Bombay*, online.
SysConTalks
- 2022 **Nonlinear systems**, *Invited talk*, *University of Michigan*, Ann Arbor, MI.
- 2022 **Large deviations, persistent homology, and Brownian conductors with negative resistance**, *Invited talk*, *Cornell University*, online.
Probability Seminar
- 2022 **When can hybrid systems operate safely?**, *Invited talk*, *University of Michigan*, online.
Control Seminar
- 2022 **Flux in small noise dynamics: persistence and negative resistance**, *Invited talk*, *Brown University, Division of Applied Mathematics*, online.
- 2021 **Flux in small noise dynamics: negative resistance and persistence**, *Invited talk*, *University of Illinois Urbana-Champaign*, online.
Mathematical Biology Seminar
- 2021 **When and how are hybrid dynamical systems conjugate to their classical quotients?**, *Invited talk*, *SIAM Conference on Control and its Applications*, online.
with Samuel A. Burden
Minisymposium: Physically grounded semantics for programming hybrid dynamical systems
- 2021 **Toward a physically grounded type theory for robot task composition.**, *Invited talk*, *IEEE International Conference on Robotics and Automation (ICRA)*, online.
with Daniel E. Koditschek and Paul Gustafson
Workshop: Compositional robotics: mathematics and tools
- 2021 **Toward a physically grounded type theory for robot task composition**, *Invited talk*, *IEEE International Conference on Robotics and Automation (ICRA)*, online.
with Daniel E. Koditschek and Paul Gustafson
Workshop: Compositional robotics: mathematics and tools
- 2021 **Small noise dynamics, persistence, and negative resistance**, *Invited talk*, *SIAM Conference on Applications of Dynamical Systems*, online.
with Yuliy Baryshnikov
Minisymposium: Stochastic oscillators
- 2021 **Flux in tilted potential systems: negative resistance and persistence**, *Invited talk*, *Army Research Office SLICE MURI W911NF-18-1-032 Group Meeting*, online.
- 2021 **Towards a Conley theory for hybrid dynamical systems**, *Invited talk*, *Rutgers University*, online.
TRIPODS/DATA-INSPIRE Workshop: Dynamics, Topology, and Robotic Control
- 2021 **Hierarchical composition via collapse of dimension in dynamical systems**, *Invited talk*, *ETH Zürich*, online.
Series: Autonomy Talks
- 2021 **Existence and uniqueness of global linearizing coordinates**, *Invited talk*, *Army Research Lab*, online.
Representing the MURI group funded by Army Research Office MURI 911NF-17-1-0306
- 2021 **Safety and stabilization: necessary conditions**, *Invited talk*, *Army Research Office SLICE MURI W911NF-18-1-032 Group Meeting*, online.
- 2021 **Necessary conditions for feedback stabilization and safety**, *Invited talk*, *University of Pennsylvania*, online.
Kod*Lab Research Seminar

- 2020 **Proving periodic orbits exist: global continuation and Lyapunov 1-forms**, *Invited Talk, University of Pennsylvania*, online.
Graduate Research Seminar in Applied Topology (GRST)
- 2020 **Proving periodic orbits exist: Lyapunov 1-forms and global continuation**, *Invited talk, University of Pennsylvania*, online.
Kod*Lab Research Seminar
- 2020 **Geometry and dynamics of circulant systems**, *Invited talk, AMS sectional meeting*, online.
with Anthony M. Bloch (speaker)
Special session: Geometric dynamics
- 2020 **Conley's fundamental theorem for a class of hybrid systems**, *Invited talk, Army Research Office SLICE MURI W911NF-18-1-032 Group Meeting*, online.
- 2020 **Conley's fundamental theorem for a class of hybrid systems**, *Invited talk, University of Pennsylvania*, online.
Kod*Lab Research Seminar
- 2020 **Toward a task planning theory for robot hybrid dynamics**, *Invited talk, BIRS-CMO*, online.
with Daniel E. Koditschek and Paul Gustafson
Workshop: Topological complexity and motion planning
- 2020 **Isochrons from short, noisy data**, *Invited talk, Dynamics Days Europe*, online.
with Simon Wilshin, Clayton Scott, and Shai Revzen
Minisymposium: Stochastic oscillators
- 2019 **Invariant manifolds, locomotion, and the Koopman operator**, *Invited Talk, University of Pennsylvania*, Philadelphia, PA.
Chaudhari Research Group Seminar
- 2019 **Templates, anchors, and normally hyperbolic invariant manifolds**, *Invited Talk, University of Pennsylvania*, Philadelphia, PA.
DAIR Lab Research Seminar
- 2019 **Templates, anchors, and normally hyperbolic invariant manifolds**, *Invited Talk, University of Pennsylvania*, Philadelphia, PA.
ScalAR Lab Research Seminar
- 2019 **Existence and uniqueness of linearizing semiconjugacies for stable fixed points and periodic orbits**, *Invited talk, California Institute of Technology*, Pasadena, CA.
Army Research Office MURI 911NF-17-1-0306 Group Meeting
- 2019 **Is legged locomotion almost smooth?**, *Dynamic Walking*, Calgary, AB, Canada.
with Shai Revzen (speaker) and George Council
- 2019 **Dynamics of circulant systems of ODEs**, *Invited talk, SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.
with Anthony M. Bloch (speaker)
- 2019 **Reduced-order models for locomotion in the perturbed Stokes regime**, *Invited talk, SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.
with Shai Revzen
- 2019 **Hybrid oscillators: phase and amplitude in a class of non-smooth systems**, *Invited talk, SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.
with Shai Revzen

- 2018 **Aspects of invariant manifold theory and applications**, *Invited talk*, University of California Santa Barbara, Santa Barbara, CA.
Army Research Office MURI 911NF-17-1-0306 Group Meeting
- 2018 **Oscillators, asymptotic phase, and reduction of dynamical systems**, *Invited talk*, University of Michigan, Ann Arbor, MI.
Applied and Interdisciplinary Mathematics Student Seminar
- 2018 **Testing an extended ‘Posture Principle’**, *Annual meeting of the Society for Integrative and Comparative Biology (SICB)*, San Francisco, CA.
with Shai Revzen
- 2018 **Invariant manifolds, asymptotic phase, and data-driven algorithms**, *Invited talk*, University of Pennsylvania, Philadelphia, PA.
Kod*Lab Research Seminar
- 2016 **A recipe for embedding templates in anchors**, *Invited talk*, University of Michigan, Ann Arbor, MI.
Legged Robotics Seminar
- 2015 **Introduction to homology: towards topological data analysis, pt. II/II**, *Invited talk*, University of Michigan, Ann Arbor, MI.
Signal Processing Student Seminar
- 2015 **Introduction to homology: towards topological data analysis, pt. I/II**, *Invited talk*, University of Michigan, Ann Arbor, MI.
Signal Processing Student Seminar
- 2015 **Phase-based models of rhythmic systems**, *Dynamic Walking*, Columbus, OH.
with Shai Revzen
- 2015 **Better models of rhythmic systems: predicting locomotion from phase alone**, *Annual meeting of the Society for Integrative and Comparative Biology (SICB)*, West Palm Beach, FL.
with Shai Revzen
- 2014 **Why the trot?**, *Annual meeting of the Society for Integrative and Comparative Biology (SICB)*, Austin, TX.
with Shai Revzen and Sam Burden (speakers)

Posters

- 2023 **The role of symmetry in constructing geometric flat outputs for free-flying robotic systems**, *IEEE International Conference on Robotics and Automation (ICRA)*, London, UK.
with Jake Welde (presenter) and Vijay Kumar
- 2020 **A mode map representation to predict steady states and attraction basins for legged locomotion on obstacle terrains**, *International Conference on Intelligent Robotics and Systems (IROS)*, online.
with Haodi Hu (presenter), Michelle Joyce, Simon Wilshin, Andrew Spence, and Feifei Qian
Workshop: Robotics-inspired biology
- 2019 **Principal Koopman eigenfunctions for nonlinear and nonsmooth systems**, University of California, Los Angeles, Los Angeles, CA.
with Shai Revzen
Workshop: Operator theoretic methods in dynamic data analysis and control

- 2019 **Principal Koopman eigenfunctions for nonlinear and nonsmooth systems**, *University of California, Los Angeles*, Los Angeles, CA.
with Shai Revzen
Workshop: Operator theoretic methods in dynamic data analysis and control
- 2018 **Global linearization and fiber bundle structure of invariant manifolds**, *AIMS Conference Series on Dynamical Systems and Differential Equations*, Taipei, Taiwan.
with Jaap Eldering and Shai Revzen
- 2018 **Templates and Anchors: a review of notions of model reduction**, *Dynamic Walking*, Pensicola, FL.
with Shai Revzen
- 2017 **Asymptotic phase, model reduction, and control of Templates & Anchors**, *Ohio State University, Mathematical Biosciences Institute*, Columbus, OH.
with Shai Revzen
Workshop: Sensorimotor control of animals and robots
- 2017 **A dynamical systems perspective on Templates & Anchors: some general methods for anchoring templates**, *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Vancouver, BC.
with Shai Revzen
Workshop: Robotics-inspired Biology
- 2014 **Phase-based algorithm for modeling time-rescaling distortion of rhythmic data**, *University of Michigan*, Ann Arbor, MI.
with Shai Revzen
Engineering Graduate Symposium
- 2013 **A hybrid system provides a robust alternative to a linear regulator**, *University of Michigan*, Ann Arbor, MI.
with Shai Revzen
Engineering Graduate Symposium
- 2013 **An indoor navigation device for use in GPS-denied environments**, *University of Michigan*, Ann Arbor, MI, Award: 1st place, EECS division.
with Ryan Kollar, Christopher Moore, Jessica Belzer, and Matthew Miltner
Ohio University Student Expo

Other Academic Service

Co-organizer

- 2023 **IEEE International Conference on Robotics and Automation (ICRA)**.
Workshop: "Compositional Robotics: Mathematics and Tools"
- 2022 **Festschrift in honor of Daniel E. Koditschek** ([link](#)).
- 2022 **IEEE International Conference on Robotics and Automation (ICRA)**.
Workshop: "Compositional Robotics: Mathematics and Tools"

Reviewer for

Annual Reviews in Control; ASME J. Biomechanical Engineering; Automatica; Biological Cybernetics; IEEE Conference on Decision and Control (CDC); IEEE International Conference on Robotics and Automation (ICRA); International Symposium on Nonlinear Theory and Its Applications (NOLTA); Mathematics of Control, Signals, and Systems; Robotics: Science and Systems (RSS); Scientific Reports; SIAM J. Control and Optimization (SICON); Stochastic Models, and others.

Professional Memberships

American Mathematical Society (AMS), Institute of Electrical and Electronics Engineers (IEEE), Society for Industrial and Applied Mathematics (SIAM), SIAM Activity Group on Dynamical Systems (SIAG/DS), and others.

Honors and Awards

- 2013 **Rackham Engineering Award**, *University of Michigan*.
Graduate fellowship awarded by Rackham Graduate School
- 2013 **Outstanding Senior in Electrical Engineering**, *Ohio University*.
Awarded by a faculty panel
- 2013 **First place, Undergraduate Student Expo**, *Ohio University*.
Senior design team awarded first place in the EECS division
- 2012 **Undergraduate Research Award, College of Engineering**, *Ohio University*.
My proposal was funded for a semester of undergraduate research
- 2012 **Outstanding Junior in Electrical Engineering**, *Ohio University*.
Awarded by a faculty panel
- 2009–2013 **Gateway Excellence Scholarship**, *Ohio University*.
Academic merit-based scholarship (full tuition)
- 2011 **Dean's Scholarship**, *Ohio University*.
Academic merit-based scholarship
- 2011 **Fritz/Dolores Russ Scholarship**, *Ohio University*.
Academic merit-based scholarship

Other work experience

- 2012 **Undergraduate Student Researcher**, *Ohio University*, Athens, OH.
Funded undergraduate research; sensor fusion and navigation for drones
Adviser: Maarten Uijt de Haag
- 2012 **Co-Op**, *GE Aviation Systems*, Vandalia, OH.
- 2011 **Engineering Intern**, *Lakeshore Cryotronics*, Westerville, OH.

References

Yuliy Baryshnikov

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Computer Engineering
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Frederick A. Leve

Program Officer, Dynamical Systems and
Control Theory Program; Autonomy Work-
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Beth Skubak Wolf (teaching)

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Ann Arbor, MI 48109
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