WHEN DO KOOPMAN EMBEDDINGS EXIST? A USER'S GUIDE

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ABSTRACT. The following list gives precise references to those results relevant to existence of continuous (or smoother) one-to-one linearizing maps from my presentation "When do Koopman embeddings exist?" at the 2025 SIAM Conference on Dynamical Systems (DS25) delivered on Sunday, May 11, 2025.

- **Slide 6**: Theorems 2.3, 2.5 and 2.6 and Remark on p. 47 of [LM13]; Propositions 2 and 3 of [KR21].
- **Slide 7**: Theorem 2 of [KS25].
- **Slide 8**: the first proposition is Proposition 1 of [KS25]. The second proposition is Theorem 10 of [Kva25]. The theorem is Theorem 1 of [Kva25].
- **Slide 10**: the first bulleted result is Corollary 6 of [KA24]. The second bulleted result is Corollary 3 of both [LOS23] and [LOS25].
- **Slide 12**: Theorem 1 of [AK23]. Footnote: [CFI83, Bel22a, Bel22b, BC23, KB24, KB25, HB25].
- **Slide 13**: Figure 1 of [AK23].
- **Slide 15**: Corollary 33 and Proposition 38 of [Mez21].
- **Slide 18**: Theorem 3 combined with Remark 3 of [KA24].
- **Slide 20**: Example 7 of [KA24].
- **Slide 21**: the theorem is a partial statement of Theorem 4 in [KA24].
- **Slide 22**: the theorem is a combination of Theorems 1 and 2 in [KA24].
- **Slide 23**: Figure 1 of [KA24].
- **Slide 24**: Corollaries 1 and 2 of [KA24].

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