Grasp Metric for Gaussian Proccess Implicit Surface Representation of Shape Uncertainity

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***Abstract*— This electronic document is a live template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.**

I. Introduction

Grasp Quality Metrics: A number of metrics have been proposed to evaluate form and force closure with scalar quality measures for grasping [Bicchi and Kumar(2000)]. We use the wrench-space Ferrari-Canny force closure quality measures [Ferrari and Canny(1992)], which aims to maximize the disturbance that can be resisted given bounds on the contact forces. We also consider the general case where there is friction at the contact points.

Gaussian Process (GP) Primer: Gaussian processes (GPs) are widely used in machine learning as a nonparametric regression method for estimating continuous functions from sparse and noisy data [Rasmussen and Williams(2006)]. In a GP, a training set consists of input vectors $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, $\mathbf{x}_i \in \mathbb{R}^d$, and corresponding observations $\mathbf{y} = \{y_1, \dots, y_n\}$. The observations are assumed to be noisy measurements from the unknown target function f:

$$y_i = f(\mathbf{x}_i) + \epsilon, \tag{1}$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is Gaussian noise in the observations. A zero-mean Gaussian process is completely specified by a covariance function $k(\cdot,\cdot)$, also referred to as a kernel. Given the training data $\mathcal{D} = \{\mathcal{X}, \mathbf{y}\}$ and covariance function $k(\cdot,\cdot)$, the posterior density $p(f_*|\mathbf{x}_*, \mathcal{D})$ at a test point $\mathbf{x}_*$ is shown to be [Rasmussen and Williams(2006)]:

$$p(f_*|\mathbf{x}_*, \mathcal{D}) \sim \mathcal{N}\big(k(\mathcal{X}, \mathbf{x}_*)^\intercal (K + \sigma_\epsilon^2 I)^{-1}\mathbf{y}, \tag{2}$$

$$k(\mathbf{x}_*, \mathbf{x}_*) - k(\mathcal{X}, \mathbf{x}_*)^\intercal (K + \sigma^2 I)^{-1} k(\mathcal{X}, \mathbf{x}_*)\big),$$

where $K \in \mathbb{R}^{n\times n}$ is a matrix with entries $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$ and $k(\mathcal{X}, \mathbf{x}_*) = [k(\mathbf{x}_1, \mathbf{x}_*), \dots, k(\mathbf{x}_n, \mathbf{x}_*)]^\intercal$.

The choice of kernel is application-specific, since the function $k(\mathbf{x}_i, \mathbf{x}_j)$ is used as a measure of correlation between states $\mathbf{x}_i$ and $\mathbf{x}_j$. A common choice is the squared exponential kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \nu^2 \exp(-\frac{1}{2}(\mathbf{x}_i - \mathbf{x}_j)^\intercal \Lambda^{-1} (\mathbf{x}_i - \mathbf{x}_j)) \tag{3}$$

where $\Lambda = \mathrm{diag}(\lambda_1^2, \dots, \lambda_d^2)$ are the characteristic length scales of each dimension of $\mathbf{x}$ and ν^2 describes the variability of f. The vector of hyper-parameters $\boldsymbol{\theta} = \{\sigma, \nu, \lambda_1, \dots, \lambda_d\}$ is chosen or optimized during the training process by minimizing the log likelihood $p(\mathbf{y}|\mathcal{X}, \boldsymbol{\theta})$ [Rasmussen and Williams(2006)].

II. Related Work

III. Problem Definition

Given a grasp G on an object, we can define it by the following tuple $G = \{c_1, ..., c_m, n_1, ..., n_m, z, \tau\}$. We have a set I of m contacts on the object where $i \in I$ contact is located at c_i with surface normal n_i. The object has a center of mass z and friction coefficient μ. We demonstrate that one can efficiently compute a closed form distribution for c_i,n_i and z. We note though that our metric assumes a known μ or friction coefficient.

For the following derivations we use the following notation. $\theta(x) = \{\mu(x), \Sigma(x)\}$, hence $\theta(x)$ is a tuple consisting of the mean and covariance functions given by the trained GPIS. We further assume that the contacts on the gripper approach along a line of action: $x = \gamma(t) : R \to R^d$. The set of all elements in the range $[a, b]$ will be denoted by T. We assume a bounded rectangular workspace $\mathcal{R}$. The line segment has endpoints a, b that are defined as the start of the gripper and the intersection of the line with the end of the workspace respectively, as shown in Fig. 1. We also define the function $f(x) : \mathbb{R}^d \to \mathbb{R}$ as $f(x) = \mu(x)$ and an implicit surface $\mathcal{S} = \{x \mid f(x) = 0\}$.

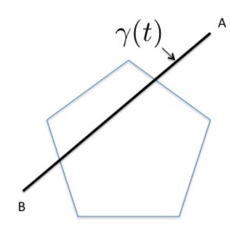

Fig. 1: Parameterized Line of Action along a Object

IV. Distribution of Grasp Parameters

A. Distribution on Contact Points

The probability along the line $\gamma(g)$ is given by the following:

$$P(f(\gamma(g))|\theta(\gamma(g))) = \mathcal{N}(\mu_T, \Sigma_T) \quad (4)$$

This gives the distributions along the entire line of action, however we want to compute the marginalization of this where at each point we simply are asking the probability $p(f(\gamma(g = k)) = 0)$. We can achieve this by marginalizing out the rest of the points along the line. With a Gaussian it has the closed form solution as follows:

$$P(c_i) = P(f(\gamma(t)) = 0|\theta(\gamma(t))) \quad (5)$$

B. Distribution on Surface Normals

The distribution of surface normals $P(n_i = k)$ can be calculate as follows. First we assume that some function exists $h(x) = \{\mu_\bigtriangledown(x), \Sigma_\bigtriangledown(x)\}$, hence given a point x it returns the parameters for a Gaussian distribution around the gradient. this function can be computed via learning the gradient [**?**] or analytical differentiation of $f(x)$. We note that both methods yield a Gaussian distribution. We now demonstrate how to marginalize out the contact distribution and compute $P(n_i = k)$

From our distribution on contact points and Bayes rule we can compute the following:

$$p(c_i = \gamma(g), n_i = k) = p(n_i = k|h(\gamma(t))) * p(c_i = \gamma(t)) \quad (6)$$

Now we can marginalize out the distribution on contacts:

$$P(n_i = k) = \int_T p(n_i = k|h(\gamma(t))) * p(c_i = \gamma(t))dg \quad (7)$$

We then discretize the curve and approximate the integral by a sum:

$$P(n_i = k) = \sum_T p(n_i = k|h(\gamma(t))) * p(c_i = \gamma(t))dg \quad (8)$$

Thus, $P(n_i = k)$ is a multi-modal distributions composed of Gaussians sum together.

C. Distribution on Center of Mass

We define the quantity $\mathcal{D}(x) = \int_{-\infty}^{0} p(f(x) = s \mid \theta(x))ds$ and note that it is equal to the probability that x is interior to the surface under the current observations. We assume that the object has uniform mass density and then $\mathcal{D}(x)$ is the expected mass density at x. Then we can find the expected center of mass as:

$$\mathcal{C} = \frac{\int_\mathcal{R} x\mathcal{D}(x)dx}{\int_\mathcal{R} \mathcal{D}(x)dx} \quad (9)$$

which can be approximated by sampling $\mathcal{R}$ uniformly in a voxel grid and approximating the spatial integral by a sum.

V. Probabilistic Bound on Grasp Metric

Following recent work on proving a Lipschitz bound on the Ferrari-Canny Metric [Pokorny and Kragic(2013)], we prove that an extension is possible to give a probabilistic bound on the change in grasp quality. We rewrite the results here:

A. Prior Work

We use the following notation $Q(g)$ refers to the exact L^1 grasp quality. It is denoted by the following

$$Q(g) = \max(0, q(g)) = -d(0, \text{Conv}(0 \cup S(g)) \quad (10)$$
$$-d(0, S) = min_{||z||=1} h_{S(z)} \quad (11)$$
$$h_{S(z)} = \sup_{s\in S} < s, z > \quad (12)$$
$$(13)$$

We denote the Ferrari-Canny version, which approximates the friction cone by a linearized set of wrenches[Ferrari and Canny(1992)], as $Q_l^-(g)$.

Theorem 1: [Pokorny and Kragic(2013)] For any grasp g, we have $0 \leq Q_l^-(g) \leq Q(g)$. Furthermore, $||Q(g) - Q_l^-(g)|| \to 0$ as $l \to \infty$ when $Q_l^-(g)$ is computed using a uniform approximation of the friction cones with l edges.

Theorem 2: For $w \in \mathbb{R}^3$, we have, for $n \in \mathbb{S}^2$ and for friction coefficient $\mu > 0$,

$$\sup_{x\in C(n)} < x, w >=< n, w > +\tau||n \times w|| \quad (14)$$

Hence, for $u = (a, b) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$, we have

$$h_{W_i(g)}(a, b) =< n_i, a + b \times (c_i - z) > +\tau||n_i \times (a + b \times (c_i - z))|| \quad (15)$$

Theorem 3: We have

$$q(g) = \min_{u\in\mathbb{R}^6,||u||=1} h_{S(g)}(u) = \min_{u\in\mathbb{R}^6,||u||=1} \max_{i=1,\dots,m} h_{W_i(g)}(u), \quad (16)$$
$$(17)$$

where $h_{S(g)}$ is convex on $\mathbb{R}^6$. Observe that q is invariant under fixed translation of the grasp center and contact positions. Furthermore, let $\mathbb{B}(r) = \{x \in \mathbb{R}^3 : ||x|| \leq r\}$. Then q is Lipschitz continuous on grasps with m contact points lying in the set $X = \{(c_1, \dots, c_m, n_1, \dots, n_m, z) : (c_i - z) \in \mathbb{B}(r), n_i \in \mathbb{S}^2\}$ with a Lipschitz constant given by $L = (1 + \mu)(1 + r)$ and where the chose distance measure

$$d(g, g') = \sum_i ||(c_i - z) - (c'_i - z')|| + \sum_i ||n_i - n'_i||.$$

We hence have

$|q(g) - q(g')| \leq Ld(g, g')$ for all $g, g' \in X$.

Since $Q(g) = max(0, q(g))$, Q is also Lipschitz continuous with the same constant L on X.

B. Our Extension

Setting $l_{i,a,b} = h_{W_i(g)}(a, b)$, we have for $||(a, b)|| \leq 1$ that is $|l_{i,a,b}(g) - l_{i,a,b}(g')|$ is bounded by $| < n_i, a + b \times (c_i - z) > - < n_i', a + b \times (c_i' - z') > | + \mu|||n_i \times (a + b \times (c_i - z)|| - ||n_i \times (a + b \times (c_i' - z'))|||$, using Theorem 2. By using the following facts $||a|| \leq 1$, $||b|| \leq 1$, $||v \times w|| \leq ||w||||v||$, $| < v, w > | \leq ||v||||w||$, we obtain:

$$\begin{aligned} |l_{i,a,b}(g) - l_{i,a,b}(g')| \leq\ & ||n_i - n_i'||(1 + ||c_i - z||) \\ & + ||c_i - z_i - c_i' - z_i'|| \\ & + \mu(||n_i - n_i'||(1 + ||c_i - z||) \\ & + ||c_i - z_i - c_i' - z'|| \end{aligned}$$

We now define the concept of a probabilistic bound by functions $\sigma(p)_{n_i}$ and $\sigma(p)_{r_i}$, given a probability p they denote the change in grasp parameter from the mean in the distribution. $\sigma(p)_{n_i}$ references the distribution defined by $p(n_i = k)$. $r = c_i - \bar{z}$ defines a moment arm, where $\bar{z}$ is the expected center of mass, then $\sigma(p)_{r_i}$ references the distribution $p(c_i = k)$. We now write the bound as follows:

$$\begin{aligned} |l_{i,a,b}(g) - l_{i,a,b}(g')| \leq\ & ||\sigma_{n_i}(p)||(1 + \bar{r}|) + ||\sigma_{r_i}(p)|| \\ & + \tau(||\sigma(p)_{n_i}||(1 + ||\bar{r}||) + ||\sigma_{r_i}(p)||) \end{aligned}$$

For convenience we rewrite the bound on a given set of graps parameters as :

$$\begin{aligned} b_i(p) =\ & ||\sigma_{n_i}(p)||(1 + \bar{r}|) + ||\sigma_{r_i}(p)|| \\ & + \tau(||\sigma(p)_{n_i}||(1 + ||\bar{r}||) + ||\sigma_{r_i}(p)||) \end{aligned}$$

To provide an upper bound overall contact parameters we introduce the following:

$$b(p) = \max_i b_i(p) \tag{18}$$

To prove if this bound is preserved on

$$q(g) = \min{(a, b) \in \mathbb{R}^6, ||u|| = 1} \max{i = 1, .., m} l_{i,a,b}(g) \tag{19}$$

we turn to the general case. $\lambda(x) = \inf_{\alpha \in A} f_\alpha(x)$ and $\lambda(x) = \sup_{\alpha \in A} f_\alpha(x)$ are bounded with $b(p)$ if $f_\alpha(x)$ for all α is bounded with $b(p)$ and $\lambda(x)$ is bounded. Since our bound is invariant to the variables (a, b) and we take the maximum set of parameters i. We can ensure that is true.

VI. References

References

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