Grasp Metric for Gaussian Proccess Implicit Surface Representation of Shape Uncertainity

Michael Laskey,Zoe McCarthy, Sachin Patil, Pieter Abbeel and Ken Goldberg

Abstract— **This electronic document is a live template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.**

I. INTRODUCTION

Grasp Quality Metrics: A number of metrics have been proposed to evaluate form and force closure with scalar quality measures for grasping [Bicchi and Kumar(2000)]. We use the wrench-space Ferrari-Canny force closure quality measures [Ferrari and Canny(1992)], which aims to maximize the disturbance that can be resisted given bounds on the contact forces. We also consider the general case where there is friction at the contact points. We follow the analysis of Schulman et al. [Schulman et al.(2011)Schulman, Goldberg, and Abbeel] in the discussion below.

Gaussian Process (GP) Primer: Gaussian processes (GPs) are widely used in machine learning as a nonparametric regression method for estimating continuous functions from sparse and noisy data [Rasmussen and Williams(2006)]. In a GP, a training set consists of input vectors $\mathcal{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, $\mathbf{x}_i \in \mathbb{R}^d$, and corresponding observations $\mathbf{y} = \{y_1, \ldots, y_n\}$. The observations are assumed to be noisy measurements from the unknown target function f:

$$y_i = f(\mathbf{x}_i) + \epsilon, \tag{1}$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is Gaussian noise in the observations. A zero-mean Gaussian process is completely specified by a covariance function $k(\cdot, \cdot)$, also referred to as a kernel. Given the training data $\mathcal{D} = \{\mathcal{X}, \mathbf{y}\}$ and covariance function $k(\cdot, \cdot)$, the posterior density $p(f_*|\mathbf{x}_*, \mathcal{D})$ at a test point $\mathbf{x}_*$ is shown to be [Rasmussen and Williams(2006)]:

$$p(f_*|\mathbf{x}_*, \mathcal{D}) \sim \mathcal{N}\big(k(\mathcal{X}, \mathbf{x}_*)^\mathsf{T}(K + \sigma_\epsilon^2 I)^{-1}\mathbf{y}, \tag{2}$$

$$k(\mathbf{x}_*, \mathbf{x}_*) - k(\mathcal{X}, \mathbf{x}_*)^\mathsf{T}(K + \sigma^2 I)^{-1}k(\mathcal{X}, \mathbf{x}_*)),$$

where $K \in \mathbb{R}^{n \times n}$ is a matrix with entries $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$ and $k(\mathcal{X}, \mathbf{x}_*) = [k(\mathbf{x}_1, \mathbf{x}_*), \ldots, k(\mathbf{x}_n, \mathbf{x}_*)]^\mathsf{T}$.

The choice of kernel is application-specific, since the function $k(\mathbf{x}_i, \mathbf{x}_j)$ is used as a measure of correlation between states $\mathbf{x}_i$ and $\mathbf{x}_j$. A common choice is the squared exponential kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \nu^2 \exp(-\frac{1}{2}(\mathbf{x}_i - \mathbf{x}_j)^\mathsf{T}\Lambda^{-1}(\mathbf{x}_i - \mathbf{x}_j)) \tag{3}$$

where $\Lambda = \mathrm{diag}(\lambda_1^2, \ldots, \lambda_d^2)$ are the characteristic length scales of each dimension of $\mathbf{x}$ and ν^2 describes the variability of f. The vector of hyper-parameters $\boldsymbol{\theta} = \{\sigma, \nu, \lambda_1, \ldots, \lambda_d\}$ is chosen or optimized during the training process by minimizing the log likelihood $p(\mathbf{y}|\mathcal{X}, \boldsymbol{\theta})$ [Rasmussen and Williams(2006)].

II. RELATED WORK

III. PROBLEM DEFINITION

Given a grasp G on an object, we can define it by the following tuple $G = \{c_1, ..., c_m, n_1, ..., n_m, z, \mu\}$. We have a set I of m contacts on the object where $i \in I$ contact is located at c_i with surface normal n_i. The object has a center of mass z and friction coefficient μ. We demonstrate that one can efficiently compute a closed form distribution for c_i, n_i and z. We note though that our metric assumes a given μ.

For the following derivations we use the following notation. $\theta(x) = \{\mu(x), \Sigma(x)\}$, hence $\theta(x)$ is a tuple consisting of the mean and covariance functions given by the trained GPIS. We further assume that the contacts on the gripper approach along a line of action: $x = \gamma(t) : R \to R^d$. We assume a bounded rectangular workspace. The line segment has endpoints a, b that are defined as the start of the gripper and the intersection of the line with the end of the workspace respectively, as shown in Fig. 1. We also define the function $f(x) : \mathbb{R}^d \to \mathbb{R}$ as $f(x) = \mu(x)$ and an implicit surface $\mathcal{S} = \{x \mid f(x) = 0\}$.
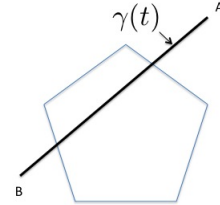

Fig. 1: Parameterized Line of Action along a Object

IV. DISTRIBUTION OF GRASP PARAMETERS

A. Distribution on Contact Points

The probability along the line $\gamma(g)$ is given by the following:

$$P(f(\gamma(g))|\theta(\gamma(g))) = \mathcal{N}(\mu_T, \Sigma_T) \qquad (4)$$

This gives the distributions along the entire line of action, however we want to compute the marginalization of this where at each point we simply are asking the probability $p(f(\gamma(g = k)) = 0)$. We can achieve this by marginalizing out the rest of the points along the line. With a Gaussian it has the closed form solution as follows:

$$P(c_{i,det} = P(f(\gamma(g)) = 0|\theta(\gamma(t))) \qquad (5)$$

B. Distribution on Surface Normals

The distribution of surface normals $P(n_i = k)$ can be calculate as follows. First we assume that some function exists $h(x) = \{\mu_\nabla(x), \Sigma_\nabla(x)\}$, hence given a point x it returns the parameters for a Gaussian distribution around the gradient. this function can be computed via learning the gradient [**?**] or analytical differentiation of $f(x)$. We note though that both methods yields a Gaussian distribution. We now demonstrate how to marginalize out the contact distribution and compute $P(n_i = k)$

From our distribution on contact points and Bayes rule we can compute the following:

$$p(c_i = \gamma(g), n_i = k) = p(n_i = k|h(\gamma(g))) * p(c_i = \gamma(g)) \qquad (6)$$

Now we can marginalize out the distribution on contacts:

$$P(n_i = k) = \int_T p(n_i = k|h(\gamma(g))) * p(c_i = \gamma(g))dg \qquad (7)$$

Which over the voxel grid turns into:

$$P(n_i = k) = \sum_T p(n_i = k|h(\gamma(g))) * p(c_i = \gamma(g))dg \qquad (8)$$

Thus, $P(n_i = k)$ is a multi-modal distributions composed of Gaussians sum together.

C. Distribution on Center of Mass

Zoe fill out or we all die :-(

D. Other Thoughts (Not Finished)

The derivation above assumes that the robot doesn't have any noise in its movements. To accommodate noise in the position of the gripper at time t, x_t given control input u_t and previous states $x_{1:t-1}$, we introduce the following probability $P(x_t = k|u_t, x_{1:t-1})$. This can be computed by methods like an Extended Kalman Filter (EKF) or a particle filter. For simplicity we assume that it is a Gaussian distribution. We know derive the following:

$$P(c_i) = \int P(c_{i,det} = \gamma(g), x_t = \gamma(g))dg \qquad (9)$$

V. PROBABILISTIC BOUND ON GRASP METRIC

Following recent work on proving a Lipschitz bound on the Ferrari-Canny Metric [Pokorny and Kragic(2013)], we prove that an extension is possible to give a probabilistic bound on the change in grasp quality. We rewrite the results here:

A. Prior Work

We use the following notation $Q(g)$ refers to the exact L^1 grasp quality. It is denoted by the following

$$Q(g) = \max(0, q(g)) = -d(0, \text{Conv}(0 \cup S(g)) \qquad (10)$$
$$-d(0, S) = min_{||z||=1} h_{S(z)} \qquad (11)$$
$$h_{S(z)} = \sup_{s \in S} <s, z> \qquad (12)$$
$$\qquad (13)$$

We denote the Ferrari-Canny version, which approximates the friction cone by a linearized set of wrenches[Ferrari and Canny(1992)], as $Q_l^-(g)$.

Theorem 1: For any grasp g, we have $0 \leq Q_l^-(g) \leq Q(g)$. Furthermore, $||Q(g) - Q_l^-(g)|| \to 0$ as $l \to \infty$ when $Q_l^-(g)$ is computed using a uniform approximation of the friction cones with l edges.

Theorem 2: We have

$$q(g) = \min_{u \in \mathbb{R}^6, ||u||=1} h_{S(g)}(u) = \min_{u \in \mathbb{R}^6, ||u||=1} \max_{i=1,\dots,m} h_{W_i(g)}(u), \qquad (14)$$
$$\qquad (15)$$

where $h_{S(g)}$ is convex on $\mathbb{R}^6$. Observe that q is invariant under fixed translation of the grasp center and contact positions. Furthermore, let $\mathbb{B}(r) = \{x \in \mathbb{R}^3 : ||x|| \leq r\}$. Then q is Lipschitz continuous on grasps with m contact points lying in the set $X = \{(c_1, \dots, c_m, n_1, \dots, n_m, z) : (c_i - z) \in \mathbb{B}(r), n_i \in \mathbb{S}^2\}$ with a Lipschitz constant given by $L = (1 + \mu)(1 + r)$ and where the chose distance measure

$$d(g, g') = \sum_i ||(c_i - z) - (c_i' - z')|| + \sum_i ||n_i - n_i'||.$$

We hence have

$$|q(g) - q(g')| \leq Ld(g, g') \text{ for all } g, g' \in X.$$

Since $Q(g) = max(0, q(g))$, Q is also Lipschitz continuous with the same constant L on X.

B. Our Extension

From our distributions $p(c_i = \gamma(t))$ and $p(n_i = x)$, we can denote the variance on the following terms as σ_{c_i} and Σ_{n_i}. Thus,

VI. REFERENCES

REFERENCES

[Bicchi and Kumar(2000)] Antonio Bicchi and Vijay Kumar. Robotic grasping and contact: A review. In *Proc. IEEE Int. Conf. Robotics and Automation (ICRA)*, pages 348–353, 2000.

[Ferrari and Canny(1992)] C. Ferrari and J. Canny. Planning optimal grasps. In *Proc. IEEE Int. Conf. Robotics and Automation (ICRA)*, pages 2290–2295, 1992.

[Pokorny and Kragic(2013)] Florian T Pokorny and Danica Kragic. Classical grasp quality evaluation: New algorithms and theory. In *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on*, pages 3493–3500. IEEE, 2013.

[Rasmussen and Williams(2006)] C Rasmussen and C Williams. *Gaussian processes for machine learning*. MIT Press, 2006.

[Schulman et al.(2011)Schulman, Goldberg, and Abbeel] John D Schulman, Ken Goldberg, and Pieter Abbeel. Grasping and fixturing as submodular coverage problems. In *Int. Symp. on Robotics Research (ISRR)*, pages 1–12, 2011.