Grasp Metric for Shape Uncertainity with Gaussian Proccess Implicit Surface Representation (Not Finished Work)

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I. Introduction

A number of metrics have been proposed to evaluate form and force closure with scalar quality measures for grasping [1]. Many modern 3d sensors give noisy point clouds as output, so shape uncertainty is a common problem [2]. As shown in Fig. 2, the noise in our measurements of object shape can greatly change the surface normals and contact points, which are the parameters that most grasp metrics rely on. However, only recently have people started looking into a metric's robustness to uncertainty. Prior work by Zheng et al [3], looked at how to efficiently include uncertainty in friction coefficient and movement of gripper arm. However, they assumed a known surface of the object.

We use Gaussian Processes [4] to convert the point cloud measurements into an implicit surface with uncertainty in the shape. We assume friction is at the contact points. The wrench-space Ferrari-Canny force closure quality measure [5] calculates the maximum disturbance that can be resisted given bounds on the contact forces. We are working to extend this metric to incorporate shape uncertainty. For our grasping model, we make a simplifying assumption that each gripper approaches the object along a fixed line of action and thus the contact points lie on this line. We would like to analyze the induced distributions over grasp parameters and the Ferrari-Canny metric. Specifically, we would like to answer the following question: What is the probability of achieving force closure under the given grasp approach directions and implicit surface distribution? We are also interested in the grasp quality of the expected grasp. So far, we have explicit formulas for the distributions of the contact points along the line of action and surface normal vectors and the expected center of mass of the object to be grasped. We are thoroughly investigating utilizing recent results on a Lipschitz constant for the Ferrari-Canny metric [6] in order to prove probabalistic bounds on the change in grasp quality under shape uncertainty. We are also investigating efficient ways to calculate the distributions on the grasp parameters and ways to update them quickly with new observations.

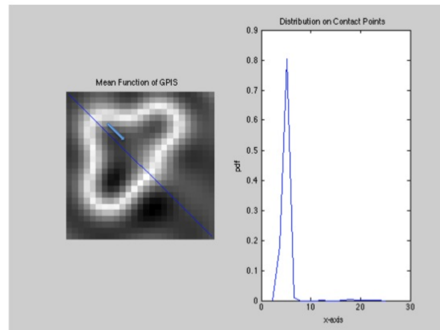

Fig. 1: Left: Grasp approach direction on an uncertain surface, represented by a Gaussian Process Implicit Surface. Right: Induced distribution on the contact point as a function of x-axis position along the approach line.

Fig. 2: An example of the noise from a Kinect-like sensor

II. Gaussian Process Primer

Gaussian processes (GPs) are widely used in machine learning as a nonparametric regression method for estimating continuous functions from sparse and noisy data [4]. In a GP, a training set consists of input vectors $\mathcal{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, $\mathbf{x}_i \in \mathbb{R}^d$, and corresponding observations $\mathbf{y} = \{y_1, \ldots, y_n\}$. The observations are assumed to be noisy measurements from the unknown target function f:

$$y_i = f(\mathbf{x}_i) + \epsilon, \tag{1}$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is Gaussian noise in the observations. A zero-mean Gaussian process is completely specified by a covariance function $k(\cdot, \cdot)$, also referred to as a kernel. Given the training data $\mathcal{D} = \{\mathcal{X}, \mathbf{y}\}$ and covariance function $k(\cdot, \cdot)$, the posterior density $p(f_*|\mathbf{x}_*, \mathcal{D})$ at a test point $\mathbf{x}_*$ is shown to be [4]:

$$p(f_*|\mathbf{x}_*, \mathcal{D}) \sim \mathcal{N}\big(k(\mathcal{X}, \mathbf{x}_*)^\intercal (K + \sigma^2 I)^{-1} \mathbf{y}, \tag{2}$$

$$k(\mathbf{x}_*, \mathbf{x}_*) - k(\mathcal{X}, \mathbf{x}_*)^\intercal (K + \sigma^2 I)^{-1} k(\mathcal{X}, \mathbf{x}_*)\big),$$

where $K \in \mathbb{R}^{n \times n}$ is a matrix with entries $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$ and $k(\mathcal{X}, \mathbf{x}_*) = [k(\mathbf{x}_1, \mathbf{x}_*), \ldots, k(\mathbf{x}_n, \mathbf{x}_*)]^\intercal$.

The choice of kernel is application-specific, since the function $k(\mathbf{x}_i, \mathbf{x}_j)$ is used as a measure of correlation between states $\mathbf{x}_i$ and $\mathbf{x}_j$. A common choice is the squared exponential kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \nu^2 \exp(-\frac{1}{2}(\mathbf{x}_i - \mathbf{x}_j)^\intercal \Lambda^{-1}(\mathbf{x}_i - \mathbf{x}_j)) \qquad (3)$$

where $\Lambda = \mathrm{diag}(\lambda_1^2, \ldots, \lambda_d^2)$ are the characteristic length scales of each dimension of $\mathbf{x}$ and ν^2 describes the variability of f. The vector of hyper-parameters $\boldsymbol{\theta} = \{\sigma, \nu, \lambda_1, \ldots, \lambda_d\}$ is chosen or optimized during the training process by minimizing the log likelihood $p(\mathbf{y}|\mathcal{X}, \boldsymbol{\theta})$ [4].

We represent surfaces as a Gaussian Process trained to predict the Truncated Signed Distance Field of a surface by noisy samples from that TSDF, as indicated in Fig. 3.

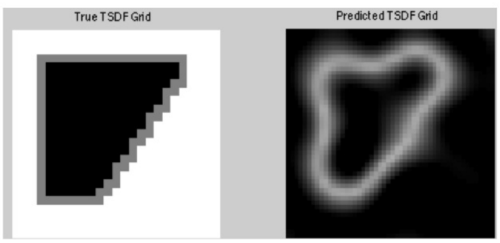

Fig. 3: Left: A surface represented as a Truncated Signed Distance Field (TSDF). Right: A GPIS reconstruction from noisy samples of the left surface's TSDF.

III. Problem Definition

The objective of our algorithm is to provide a quality of the expected grasp, $Q_l^-(\bar{g})$ [5], and the probability ζ of obtaining force closure.

We assume a bounded rectangular workspace $\mathcal{R}$. We parameterize a given grasp g on an object with the following tuple $g = \{\mathbf{c}_1, ..., \mathbf{c}_m, \mathbf{n}_1, ..., \mathbf{n}_m, \mathbf{z}, \tau\}$. We have an indexing set I of m point contacts and surface normal on the object: for $i \in I$ the contact is located at c_i with surface normal n_i. The object has a center of mass z and friction coefficient τ. The line segment $\gamma(\cdot)$ has endpoints a, b that are defined as the start of the gripper and the intersection of the line with the end of the workspace respectively, as shown in Fig. 4.

Since we have uncertainty in the shape we have a distribution on the grasp parameters, hence $p(g) = \{p(\mathbf{c}_1), ..., p(\mathbf{c}_m), p(\mathbf{n}_1), ..., p(\mathbf{n}_m), \bar{\mathbf{z}}, \tau\}$. We note that τ is considered known and $\bar{z}$ is only the expected center of mass, not a full distribution. We would like to replace this with the distribution on the center of mass, but it is unclear how to proceed in that direction. In section IV, we demonstrate how to efficiently compute these distributions on contact points and surface normals.

With distribution on $p(g)$ we then aim to use recent results on a Lipschitz constant for Ferrari-Canny Metric [6] to provide ζ or the probability the grasp will attain force closure and the quality of the expected grasp $Q_l^-(\bar{g})$, where $\bar{g} = \{\bar{\mathbf{c}}_1, ..., \bar{\mathbf{c}}_m, \bar{\mathbf{n}}_1, ..., \bar{\mathbf{n}}_m, \bar{\mathbf{z}}, \tau\}$. We can compute $Q_l^-(\bar{g})$ and have a few plans of attack to be able to use the Lipschitz bound, but it is complicated to directly apply. We should have more progress on this front by working with Florian in the next couple of days.

IV. Distribution of Grasp Parameters

For the following derivations we introduce the following, $\theta(x) = \{\mu(x), \Sigma(x)\}$, where $\theta(x)$ is a tuple consisting of the mean and covariance functions given by the trained GPIS model [4] .

To calculate $p(g)$, we assume a gripper contacts approaches along a parameterized line of action, or a 1-dimensional curve in the work space, defined by $\gamma(t)$. See Fig 4, for a detailed illustration. Each gripper contact is defined by a line of action, so we assume the following tuple is provided $\Gamma = \{\gamma_1(\cdot), ..., \gamma_m(\cdot)\}$, these approach trajectories are then used to compute a distribution on grasp parameters.

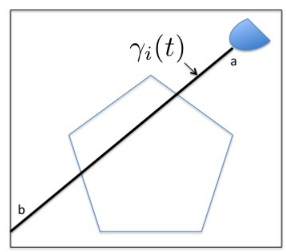

Fig. 4: Parameterized Line of Action along an object

A. Distribution on Contact Points

In our implementation we discretize along the line $\gamma(t)$ evenly but write the derivation in continuous form for generality. The probability distribution along the line $\gamma(t)$ is given by the following:

$$p\big(f(\gamma(t))|\theta(\gamma(t)) : \forall t \in [a, b]\big) = \mathcal{N}(\mu_{a:b}, \Sigma_{a:b}). \qquad (4)$$

This gives the signed distance function distributions along the entire line of action in the workspace as a multivariate gaussian. We would like to find the distribution on the first contact point, which we can define as when the signed distance function $f(\gamma(t))$ is 0 and all previous times τ we have $f(\gamma(\tau)) > 0$ for $0 \leq \tau < t$. This ensures t is at the edge of the surface (having $f(\gamma(t)) = 0$) and that for all previous τ the gripper was outside of the surface (having $f(\gamma(\tau)) > 0$). We thus compute this as the joint distribution $p\big(\mathbf{c}_i = \gamma(t)\big) = p\big(f(\gamma(t)) = 0, f(\gamma(\tau)) > 0 : \forall \tau \in [0,t)\big)$. This avoids the problem of the distribution producing multiple modes along the line: one for each intersection with the surface. We now derive this distribution

$$
\begin{aligned}
p\big(\mathbf{c}_i = \gamma(t)\big) &\propto p\big(f(\gamma(t)) = 0\big) \\
&\quad * P\big(f(\gamma(\tau)) > 0 | f(\gamma(t)) = 0 : \forall \tau \in [0,t)\big)
\end{aligned}
$$

where we only indicate proportionality and will later normalize to probability 1. Using the first product in the equation can be computed easily using the marginalization of a multivariate Gaussian distribution and the second one can be rewritten by conditioning the distribution [7].

$$
p_c\big(f(\gamma(\tau)) : \forall \tau \in [0,t)\big) = p\big(f(\gamma(\tau)) | f(\gamma(t)) = 0\big)
$$

The following can now be said:

$$
\begin{aligned}
p\big(\mathbf{c}_i = \gamma(t)\big) &= \frac{1}{\eta} p(f\gamma(t) = 0) \\
&\quad * P_c\big(f(\gamma(\tau)) > 0) : \forall \tau \in [0,t)\big)
\end{aligned}
$$

for the appropriate η normalization factor. The second product term can be evaluated by calculating the cumulative distribution of a multivariate Gaussian, which we calculate with the Matlab function $mvncdf$. We show again the theoretical distribution on $\mathbf{c}_i$ calculated for a given GPIS and approach direction in Fig. 5.

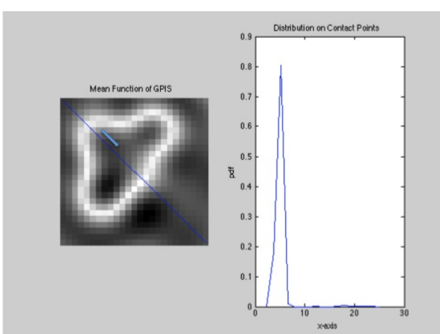

Fig. 5: Left: Grasp approach direction on an uncertain surface, represented by a Gaussian Process Implicit Surface. Right: Induced distribution on the contact point as a function of x-axis position along the approach line.

B. Distribution on Surface Normals

The distribution of surface normals $p(\mathbf{n}_i = \mathbf{k})$ can be calculate as follows. First we assume that some function exists $h(x) = \{\mu_\nabla(x), \Sigma_\nabla(x)\}$, hence given a point $\mathbf{x}$ it returns the parameters for a Gaussian distribution around the gradient. this function can be computed via learning the gradient [8] or analytical differentiation of $f(x)$. We note that both methods yield a Gaussian distribution. We now demonstrate how to marginalize out the contact distribution and compute $p(\mathbf{n}_i = \mathbf{k})$.

From our distribution on contact points and Bayes rule we can compute the following:

$$
p(\mathbf{c}_i = \gamma(t), \mathbf{n}_i = \mathbf{k}) = p\big(\mathbf{n}_i = \mathbf{k} | \mathbf{c}_i = \gamma(t)\big) * p\big(\mathbf{c}_i = \gamma(t)\big) \tag{5}
$$

Now we can marginalize out the distribution on contacts:

$$
p(\mathbf{n}_i = \mathbf{k}) = \int_a^b p\big(\mathbf{n}_i = \mathbf{k} | \mathbf{c}_i = \gamma(t)\big) * p(\mathbf{c}_i = \gamma(t)) dt \tag{6}
$$

$$
p\big(\mathbf{n}_i = \mathbf{k}\big) = \int_a^b p\big(\mathbf{n}_i = \mathbf{k} | h(\gamma(t))\big) * p\big(\mathbf{c}_i = \gamma(t)\big) dt \tag{7}
$$

We approximate this by uniformly sampling the integral along the function $\gamma(t)$ and achieve the following:

$$
p\big(\mathbf{n}_i = \mathbf{k}\big) = \sum_T p\big(\mathbf{n}_i = \mathbf{k} | h(\gamma(t))\big) * p\big(\mathbf{c}_i = \gamma(t)\big) \Delta t \tag{8}
$$

Thus, $p(\mathbf{n}_i = \mathbf{k})$ is approximated by a Gaussian Mixture Model. Note that the Ferrari-Canny metric requires $\mathbf{n}_i$ be normalized, or, equivalently, a member of $\mathcal{S}^{d-1}$. We still have to fully investigate our un-normalized normal vectors interact with the grasp metric, and possible ways to project the distribution onto the sphere. We are especially interested in the von Mises-Fisher distribution, and any role it might play here, since it is the analogue of the Gaussian distribution defined on the surface of a $d-1$ sphere. We show the theoretical distribution on $\mathbf{n}_i$ calculated for a given GPIS and approach direction in Fig. 6.

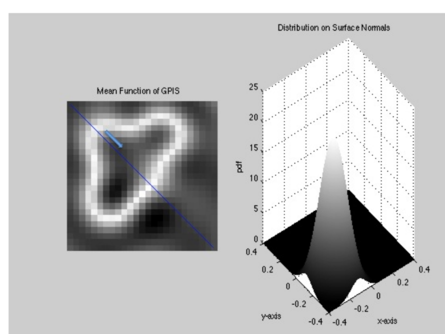

Fig. 6: Left: Grasp approach direction on an uncertain surface, represented by a Gaussian Process Implicit Surface. Right: Induced distribution on the surface normals.

C. Expected Center of Mass

We define the quantity $\mathcal{D}(x) = \int_{-\infty}^{0} p(f(x) = s \mid \theta(x))ds$ and note that it is equal to the probability that x is interior to the surface under the current observations. $\mathcal{D}(x)$ can be calculated as the CDF of $f(x)$ at 0. We assume that the object has uniform mass density and then $\mathcal{D}(x)$ is the expected mass density at x. Then we can find the expected center of mass as:

$$\bar{z} = \frac{\int_{\mathcal{R}} x\mathcal{D}(x)dx}{\int_{\mathcal{R}} \mathcal{D}(x)dx} \tag{9}$$

which can be approximated by sampling $\mathcal{R}$ uniformly in a voxel grid and approximating the spatial integral by a sum.

V. Probabilistic Bound on Grasp Metric

Following recent work on proving a Lipschitz bound on the Ferrari-Canny Metric [6], we would like to prove an extension to give a probabilistic bound on the change in grasp quality. We restate several of their results here:

A. Prior Work

We follow the notation of [6] except that their μ is our τ since we use μ to refer to means. We re-state some of their central theorems for our use here. For more detail, refer there. $Q(g)$ is the exact L^1 grasp quality. It is denoted by the following

$$Q(g) = \max(0, q(g)) = -d(0, \text{Conv}(\{0\} \cup S(g)) \tag{10}$$

$$-d(0, S) = min_{||z||=1} h_{S(z)} \tag{11}$$

$$h_{S(z)} = \sup_{s \in S} \langle s, z \rangle \tag{12}$$

$$\tag{13}$$

We denote the Ferrari-Canny version, which approximates the friction cone by a linearized set of wrenches[5], as $Q_l^-(g)$. The next theorem shows that the linearized wrench set used in Ferrari-Canny calculates a lower bound on $Q(g)$.

Theorem 1: [6] For any grasp g, we have $0 \leq Q_l^-(g) \leq Q(g)$. Furthermore, $||Q(g) - Q_l^-(g)|| \to 0$ as $l \to \infty$ when $Q_l^-(g)$ is computed using a uniform approximation of the friction cones with l edges.

The next theorems are used to show that Q is Lipschitz continuous.

Theorem 2: [6] For $w \in \mathbb{R}^3$, we have, for $n \in \mathbb{S}^2$ and for friction coefficient $\mu > 0$,

$$\sup_{x \in C(n)} \langle x, w \rangle = \langle n, w \rangle + \tau ||n \times w|| \tag{14}$$

Hence, for $u = (a, b) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$, we have

$$h_{W_i(g)}(a, b) = \langle n_i, a + b \times (c_i - z) \rangle +$$

$$+\tau ||n_i \times (a + b \times (c_i - z))|| \tag{15}$$

Theorem 3: [6] We have

$$q(g) = \min_{u \in \mathbb{R}^6, ||u||=1} h_{S(g)}(u) = \tag{16}$$

$$\min_{u \in \mathbb{R}^6, ||u||=1} \max_{i=1,\dots,m} h_{W_i(g)}(u),$$

where $h_{S(g)}$ is convex on $\mathbb{R}^6$. q is invariant under fixed translation of the grasp center and contact positions. Furthermore, let $\mathbb{B}(r) = \{x \in \mathbb{R}^3 : ||x|| \leq r\}$. Then q is Lipschitz continuous on grasps with m contact points lying in the set $X = \{(c_1, \dots, c_m, n_1, \dots, n_m, z) : (c_i - z) \in \mathbb{B}(r), n_i \in \mathbb{S}^2\}$ with a Lipschitz constant given by $L = (1 + \mu)(1 + r)$ and where we use distance measure

$$d(g, g') = \sum_i ||(c_i - z) - (c_i' - z')|| + \sum_i ||n_i - n_i'||.$$

We hence have

$$|q(g) - q(g')| \leq Ld(g, g'),\ \forall g, g' \in X.$$

Since $Q(g) = \max(0, q(g))$, Q is also Lipschitz continuous with the same constant L on X.

Setting $l_{i,a,b} = h_{W_i(g)}(a, b)$, we have for $||(a, b)|| \leq 1$ that is $|l_{i,a,b}(g) - l_{i,a,b}(g')|$ is bounded by $|\langle n_i, a + b \times (c_i - z) \rangle - \langle n_i', a + b \times (c_i' - z') \rangle| + \mu |||n_i \times (a + b \times (c_i - z))|| - ||n_i \times (a + b \times (c_i' - z'))|||$, using Theorem 2. By using the following facts $||a|| \leq 1$, $||b|| \leq 1$, $||v \times w|| \leq ||w||||v||$, $|\langle v, w \rangle| \leq ||v||||w||$, we obtain:

$$\begin{aligned} |l_{i,a,b}(g) - l_{i,a,b}(g')| \leq\ & ||n_i - n_i'||(1 + ||c_i - z||) \\ & + ||(c_i - z) - (c_i' - z')|| \\ & + \tau(||n_i - n_i'||(1 + ||c_i - z||) \\ & + ||(c_i - z) - (c_i' - z')||) \end{aligned}$$

B. Our Extension

This area is in constant flux at the moment. We are trying to obtain a lower bound on the probability that force closure is attained (ideally that exact probability, but a lower bound tells us something useful as well). The distributions on grasp parameters induce a distribution on the quality metric. We show a histogram of the quality metric for a two-finger grasp for two approach directions on a given GPIS surface in Fig. 7. The 100 different samples are over samples of the surface via the GPIS and then result in contacts and normals.

We should be able to fill in this section more after a discussion with Florian on Thursday and Friday to sort out some details with the bound calculation.

References

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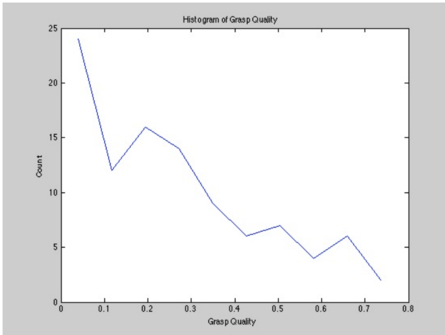

Fig. 7: Histogram of Ferrari-Canny grasp quality metric for 100 samples of a GPIS surface for the same two finger approach directions.

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