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Three-dimensional Ocean Wave Simulation Based on Directional Spectrum

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Abstract. Simulating real virtual ocean environment is necessary for the research of interaction simulation of underwater gravity aided inertial navigation system. One of the key techniques of realizing virtual ocean environment is modeling and simulating three-dimensional ocean waves. Numerical simulation of three-dimensional ocean waves in the case of different wind speeds is realized using MATLAB based on directional spectrum composed of Pierson-Moscowitz frequency spectrum and directional spreading function. Experiments show that the simulation speed is fast and the simulation results are vivid if suitable simulation frequency band, interval of wave frequency and interval of direction angle are selected. The simulation can provide some technological supports for interaction simulation of gravity aided navigation system for underwater vehicles.

Introduction

Underwater vehicles (underwater surveying vessel, underwater robot, submarine, etc.) play an important role in the fields of ocean resource development and national defense military. Underwater gravity aided inertial navigation system is one of the important research fields **[1].** Due to restriction of experiment conditions of underwater surveying, currently the research of underwater aided navigation is mainly based on computer simulation, which is to set up virtual environment for underwater aided navigation system. The research will play an important role in testing feasibility and correctness of schemes and optimizing parameters of underwater aided navigation system. Research of underwater aided navigation interactive simulation firstly needs to set up vivid virtual ocean environment, which can provide virtual reality environment for operating underwater vehicles and lay the foundation for research of naval vessel simulation. Ocean waves are the most evident natural phenomenon on the sea surface, so it is one of key techniques of realizing virtual ocean environment to model and simulate three-dimensional ocean waves.

There are several methods to simulate ocean waves, such as method of computational fluid mechanics, method based on ocean wave spectrum and method based on geometric modeling, etc.[2]. Among which method based on ocean wave spectrum is the most widely used method because it needs relatively small computational load and its parameters are obtained by long-term ocean observations and simulating results have certain authenticity. The commonly used ocean wave spectrums are Pierson-Moscowitz (P-M), JONSWAP, Bretschneider, and Neumann etc. Since ocean wave frequency spectrum can not sufficiently describe the characteristics of ocean waves, directional spectrum composed of ocean wave frequency spectrum and directional spectrum function is used to simulate three-dimensional ocean waves in this paper.

Ocean Wave Directional Spectrum Functions

Sea surface fluctuation of a fixed point depends on not only wave frequency but also direction of propagation. So the frequency spectrum of ocean waves is still not enough to sufficiently describe the characteristics of ocean waves. According to linear wave theory, the field of ocean waves is characterized by the directional wave spectrum. The directional spectrum of ocean waves which considers both wave energy's frequency distribution and direction distribution can sufficiently describe the characteristics of ocean waves. According to small amplitude wave theory, the total energy provided by all the composed waves is [3]:

$$E=\int_0^\infty\int_0^\infty S(\omega,\theta)d\omega=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{2}a_{ij}^2 \tag{1}$$

Where, E is wave energy, ω is the wave frequency, θ is the directional angle, $S(\omega,\theta)$ is the directional spectrum function of random waves. The directional spectrum is expressed as the following equation:

$$S(\omega,\theta)=s(\omega)\bullet D(\omega,\theta) \tag{2}$$

Where, $s(\omega)$ is the frequency spectrum, $D(\omega,\theta)$ is called direction spreading function, $D(\omega,\theta)$ satisfies the following equation:

$$\int_{-\pi}^{\pi}D(\omega,\theta)=1 \tag{3}$$

Usually it is considered that frequency distribution of wave energy is independent of direction distribution, therefore directional spectrum can be expressed as the following equation:

$$S(\omega,\theta)=s(\omega)\bullet D(\theta) \tag{4}$$

Ocean Wave Frequency Spectrum

Pierson-Moscowitz (P-M) Spectrum

P-M spectrum is a fetch unlimited ocean wave spectrum, which is derived from measured data of north Atlantic. Its spectrum function is [3]:

$$s(\omega)=\frac{\alpha g^2}{\omega^5}\exp[-\beta(\frac{g}{U\omega})^4] \tag{5}$$

Where, $\alpha=8.10\times10^{-3}$, $\beta=0.74$, g is acceleration of gravity, ω is the wave frequency, U is the wind speed at 19.5 m above the sea surface.

Fig. 1 shows P-M spectrum energy distribution curves at different wind speeds. From Fig. 1 we can see that P-M spectrum is a narrow band spectrum, and its energy is mainly focused on some frequency band. For example, spectrum energy is mainly focused on 0.5～2.5 rad/s for a wind speed of 10 m/s, and spectrum energy is mainly focused on 0.3～1.6 rad/s for a wind speed of 16 m/s. When we simulate ocean waves, selecting limited frequency range can save simulation time and improve simulation speed.

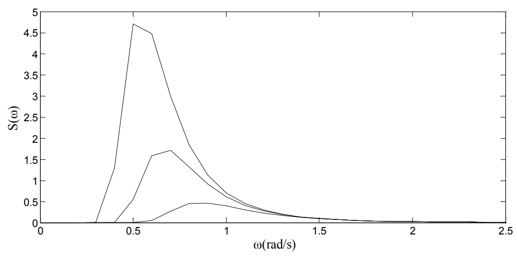


Fig.1. Energy distribution curves of P-M spectrum at wind speeds of 10m/s,13m/s and16m/s

JONSWAP spectrum

JONSWAP spectrum is a deep-water wind wave spectrum, which was developed by some institutes of England, Netherland, America and Germany after analyzing and fitting data collected during the "Joint North Sea Wave Observation Project" and is used extensively in the ocean wave research and engineering practice. JONSWAP spectrum is a fetch limited ocean wave spectrum, its spectrum function is [3]:

$$s(\omega)=\frac{ag^2}{\omega^5}\exp[-\frac{5}{4}(\frac{\omega_m}{\omega})^4]\bullet\gamma^{\exp\left(-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}\right)} \quad (6)$$

Where, $\sigma=\begin{cases}0.07, & \omega\le\omega_m\\ 0.09, & \omega>\omega_m\end{cases}$, $a=0.076\overline{X}^{-0.22}$, $\overline{X}=gx/U^2$, γ=3.3, $\omega_m=22(g/U)(\overline{X})^{-0.33}$, x is the fetch length, U is the wind speed at 10m above the sea surface, $\omega_m$is the peak wave-frequency, which is the maximum value appeared in the frequency spectrum. Fig. 2 shows JONSWAP spectrum energy distribution curves at different wind speeds.

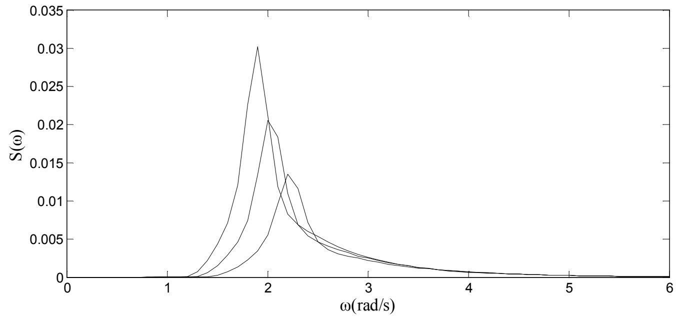


Fig.2. Energy distribution curves of JONSWAP spectrum at wind speeds of 10m/s,13m/s and16m/s

Directional Spreading Functions

The commonly used directional spreading functions are [3][4]:

Directional spreading function proposed by ITTC (International Towing Tank Conference) is:

$$D_{ITTC}(\theta)=\frac{2}{\pi}\cos^2\theta \quad (|\theta|\le\frac{\pi}{2}) \quad (7)$$

Directional spreading function proposed by ISSC (International Ship and Offshore Structures Congress) is:

$$D_{ISSC}(\theta)=\frac{8}{3\pi}\cos^4\theta \quad (|\theta|\le\frac{\pi}{2}) \quad (8)$$

Directional spreading function proposed by SWOP (Stereo Wave Observation Project) is:

$$D_{SWOP}(\theta)=\frac{1}{\pi}[1+p\cos(2\theta)+q\cos(4\theta)] \qquad (|\theta|\le\frac{\pi}{2})$$
$$p=0.5+0.82\exp[-\frac{1}{2}(\frac{\omega U}{g})^4],\ q=0.32\exp[-\frac{1}{2}(\frac{\omega U}{g})^4] \quad (9)$$

So, there are various forms of directional spectrum functions to represent ocean waves if we combine different frequency spectrum functions with direction spreading functions. For example, if P-M spectrum is chosen as frequency spectrum and $D_{ITTC}(\theta)$ and $D_{ISSC}(\theta)$ as directional spectrum function respectively, the directional spectrums of ocean waves can be expressed as:

$$S(\omega,\theta)=\frac{ag^2}{\omega^5}\exp[-0.74(\frac{g}{u\omega})^4]\frac{2}{\pi}\cos^2\theta \quad (10)$$

$$S(\omega,\theta)=\frac{ag^2}{\omega^5}\exp[-0.74(\frac{g}{u\omega})^4]\frac{8}{3\pi}\cos^4\theta \tag{11}$$

Simulation of Three-dimensional Ocean Waves

Mathematical Model of Ocean Wave Height

Sea waves are of stochastic nature in the stable sea conditions and, mathematically are represented as Gaussian stationary and ergodic processes. So sea waves can be viewed as wave superposition of infinite simple Cosine waves spreading in the direction of θ angle relatively with x axis in *(x,y)* plane, and those Cosine waves are with different amplitudes, different frequencies and different initial phases. So the sea surface elevation *H(x,y,t)* can be represented by the Double Summation Model [3,5]:

$$H(x,y,t)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{ij}\bullet\cos(k_i x\cos\theta_j+k_i y\sin\theta_j-\omega_i t-\varepsilon_{ij}) \tag{12}$$

Where, a_{ij} is the wave amplitude of frequency i and directional angle j, θ_j is spreading directional angle of a single wave, $0<\theta_j\le 2\pi$, ω_i is representative frequency in the range of frequency division. k_i is wave number, $k_i=\omega_i^2/g$, ε_{ij} is initial phase angle distributed at random($0<\varepsilon_{ij}\le 2\pi$).

According to (1):

$$a_{ij}\approx\sqrt{2S(\omega_i,\theta_i)\bullet\Delta\omega_i\bullet\Delta\theta_j} \tag{13}$$

Where, *S(ω,θ)* is directional spectrum function of ocean waves, which can be represented by (10), (11) or other forms of directional spectrum function. ω_i is representative frequency, θ_j is representative directional angle, $\Delta\omega$ and $\Delta\theta$ are increments of ω and θ respectively. The sea surface height of three-dimensional ocean waves can be represented by:

$$H(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{2S(\omega_i,\theta_j)\bullet\Delta\omega_i\bullet\Delta\theta_j}\bullet\cos(k_i x\cos\theta_j+k_i y\sin\theta_j-\omega_i t-\varepsilon_{ij}) \tag{14}$$

Where, $H(x,y,t)$ is the height of ocean wave at the point (x,y), m is division number of frequency, n is division number of directional angle.

Realization of Ocean Wave Simulation Based on MATLAB

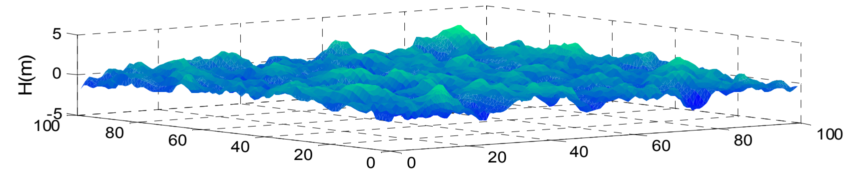
The simulation of three-dimensional ocean waves can be realized by MATLAB because MATLAB has powerful functions of calculation and provides many functions of three-dimensional graphic processing. Firstly the wind speed is determined while simulating ocean waves. Then discretization of the wave frequency and the direction angle is performed according to certain principles. Finally characteristic essential factors of each composition wave such as wave amplitude, angular frequency and phase are calculated according to (13), and furthermore the sea surface elevation at every point (x, y) is calculated according to (14) and three-dimensional ocean wave field can be plotted.

As the length of the paper is limited, only part of simulating results based on directional spectrum function (10) has been specified in this paper. According to P-M spectrum energy distribution curves and equal interval division method of frequency and direction angle dividing, the adopted frequency band, wave frequency interval, directional angle interval and division numbers of frequency and directional angle at different wind speeds are summarized in Table 1.

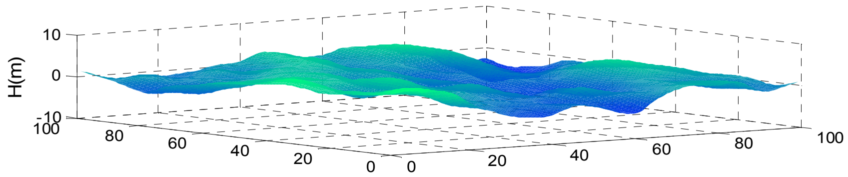
The simulation experiment is realized by dell notebook computer based on MATLAB2008 software, the computer hardware configuration is Intel Core i3, 2GB memory, ATI Mobility Radeon HD 565 chip card. The model of ocean waves is set up based on 100×100 grid. In the grid the data of every point include the information of (x, y, H), where H is wave elevation and (x, y) is horizontal coordinates of grid point. The 3D ocean wave field is plotted using 3D graphics function of surf(), and the colour of ocean waves is defined by the colour mapping function of colormap(), the function of colormap(winter) is selected when plotting ocean waves. The function of shading interp is used to color image points by interpolation form, thus the vivid wave picture can be plotted. The simulation calculation time at different conditions is listed in Table1. The simulation scenes of 3D ocean waves at different speeds are showed in Fig. 3.

Table 1 Simulation frequency band, wave frequency interval, direction angle interval, and division numbers at different wind speeds and simulation time

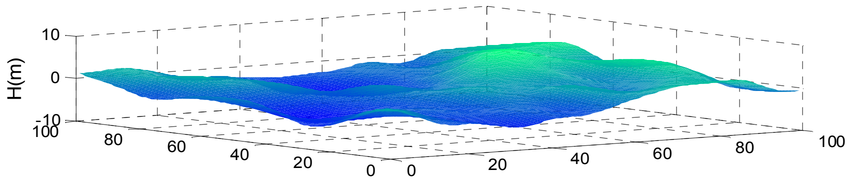
Wind speed (m/s)	Wind scale	Simulation frequency band (rad/s)	Interval of wave frequency $\triangle\omega$ (rad/s)	Division number	Interval of direction angle $\triangle\theta$ (rad)	Division number	Simulation time (s)
5	3	1.0-5.0	0.4	10	π/5	5	3
			0.2	20	π/10	10	12
			0.1	40	π/10	10	24
10	5	0.5-2.5	0.4	5	π/5	5	1.5
			0.2	10	π/10	10	6
			0.1	20	π/10	10	12
13	6	0.4-2.0	0.2	8	π/10	10	5
			0.1	16	π/10	10	10
16	7	0.3-1.6	0.1	13	π/10	10	8
			0.05	26	π/20	20	32



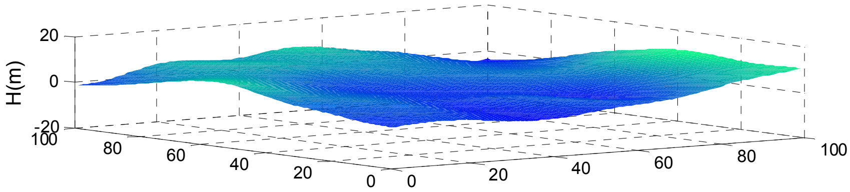
(a) u=5m/s, $\triangle\omega$=0.2, $\Delta\theta = \pi/10$



(b) u=10m/s, $\triangle\omega$=0.2, $\Delta\theta = \pi/10$



(c) u=13m/s, $\triangle\omega$=0.1, $\Delta\theta = \pi/10$



(d) u=16m/s, $\triangle\omega$=0.1, $\Delta\theta = \pi/10$

Fig. 3 Simulated 3D ocean waves at different wind speeds based on directional spectrum

Conclusions

There are various forms of directional spectrum functions to represent ocean waves if we combine different frequency spectrum functions with directional spreading functions, and thus 3D ocean waves at different conditions can be simulated, which can provide some technical supports for the research of interaction simulation of underwater gravity aided inertial navigation system.

Simulation of 3D ocean waves can be realized by MATLAB quickly and vividly based on directional spectrum if suitable simulation frequency band, interval of wave frequency and interval of direction angle are selected.

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