Heaven’s light is our guide

Rajshahi University of Engineering & Technology

Course Code: EEE 4184

Course Title: Digital Communication Sessional

Experiment No: 01

Submitted To:

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1. **Problem Statement:** The probability density function of the signal-to-noise ratio of Rayleigh Fading SISO is given by

f_γ (γ)=1/γ¯ e^(-γ/γ¯), γ≥0

(a) Derive the expressions of
- (i) The cumulative distribution function of γ.
- (ii) The moment generation function of γ.
- (iii) The amount of fading in Rayleigh fading SISO channel.

(b) write the programs of
- (i) The probability density function of γ.
- (ii) The cumulative distribution function of γ.
- (iii) The moment generation function of γ.

(c) Explain the numerical results of
- (i) The probability density function of γ.
- (ii) The cumulative distribution function of γ.
- (iii) The moment generation function of γ.

2. Derivation of Cumulative Distribution Function (CDF) of $\boldsymbol{\gamma}$

The formula for finding CDF is given by-

$$\boldsymbol{F_\gamma(\gamma) = \int_0^\gamma f_\gamma(\gamma)\, d\gamma \dots\dots\dots (1)}$$

By using the above formula we can find the expression of CDF as follows-

$$\Rightarrow F_\gamma(\gamma) = \int_0^\gamma \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\gamma e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma$$

$$= \frac{1}{\bar{\gamma}} \left[\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{-\frac{1}{\bar{\gamma}}} \right]_0^\gamma$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}}\right]_0^\gamma$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}} - e^0\right]$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}} - 1\right]$$

$$\Rightarrow \boldsymbol{F_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}}} \dots\dots\dots\dots (2)}$$

Equation-2 is the Cumulative Distribution Function (CDF) of Rayleigh Fading SISO channel.

3. Derivation of Moment Generating Function (MGF) of $\boldsymbol{\gamma}$

The formula for finding MGF is given by-

$$M_\gamma(s) = \int_0^\infty f_\gamma(\gamma) e^{s\gamma}\, d\gamma$$

By using the above formula we can find the expression of MGF as follows-

$$M_\gamma(s) = \int_0^\infty \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} e^{s\gamma}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-\frac{\gamma}{\bar{\gamma}}} e^{s\gamma}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{(s\gamma - \frac{\gamma}{\bar{\gamma}})}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-(\frac{1}{\bar{\gamma}} - s)\gamma}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-(\frac{1-s\bar{\gamma}}{\bar{\gamma}})\gamma}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \cdot \frac{1}{-(\frac{1-s\bar{\gamma}}{\bar{\gamma}})} \left[e^{-(\frac{1-s\bar{\gamma}}{\bar{\gamma}})\gamma} \right]_0^\infty$$

$$= -\frac{1}{(1-s\bar{\gamma})} [e^{-\infty} - e^0]$$

$$= -\frac{1}{(1-s\bar{\gamma})} [0 - 1]$$

$$= \frac{1}{(1-s\bar{\gamma})}$$

$$\Rightarrow \boldsymbol{M_\gamma(s) = (1 - s\bar{\gamma})^{-1}} \ \ldots\ldots\ldots\ldots\ldots \mathbf{(3)}$$

Equation-3 is the Moment Generating Function (MGF) of Rayleigh Fading SISO channel.

4. Derivation of Amount of Fading (AF) of γ

The formula for finding AF is given by-

$$AF = \frac{\left.\frac{d^2 M_\gamma(s)}{ds^2}\right|_{s=0} - \left(\left.\frac{dM_\gamma(s)}{ds}\right|_{s=0}\right)^2}{\left(\left.\frac{dM_\gamma(s)}{ds}\right|_{s=0}\right)^2} \ldots\ldots\ldots (4)$$

Now, $\frac{d(M_\gamma(s))}{ds} = \frac{d}{ds}(1 - s\bar{\gamma})^{-1} = \frac{\bar{\gamma}}{(1-s\bar{\gamma})^2}$

And, $\frac{d^2(M_\gamma(s))}{ds^2} = \frac{d}{ds}\left\{\frac{\bar{\gamma}}{(1-s\bar{\gamma})^2}\right\} = \frac{2\bar{\gamma}^2}{(1-s\bar{\gamma})^3}$

$$\left.\frac{d(M_\gamma(s))}{ds}\right|_{s=0} = \frac{\bar{\gamma}}{(1-0.\bar{\gamma})^2} = \bar{\gamma}$$

And, $\left.\frac{d^2(M_\gamma(s))}{ds^2}\right|_{s=0} = \frac{2\bar{\gamma}^2}{(1-0.\bar{\gamma})^3} = 2\bar{\gamma}^2$

Now, putting these values in equation 4, we get

$$AF = \frac{2\bar{\gamma}^2 - \bar{\gamma}^2}{\bar{\gamma}^2} = 1$$

$$\boldsymbol{AF = 1} \dots \dots \dots (\mathbf{5})$$

So, the Amount of Fading of Rayleigh Fading SISO channel is 1.

5. Program for probability density function of γ

```
SNRADB = 5;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\PDF.txt"];
For[SNRdB = 0, SNRdB < 35.1, SNRdB++,
  snr = 10^(SNRdB/10) // N;
  PDF1 = 1/snra × Exp[-snr/snra];
  Print["SNR=", SNRdB, "dB\t", PDF1];
  Write[strm, PDF1];
 ];
Close[strm];
```

6. Program for cumulative distribution function of γ

```
SNRADB = -5;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\CDF.txt"];
For[SNRdB = 0, SNRdB < 23.1, SNRdB++,
  snr = 10^(SNRdB/10) // N;
  CDF1 = 1 - Exp[-snr/snra];
  Print["SNR=", SNRdB, "dB\t", CDF1];
  Write[strm, CDF1];
 ];
Close[strm];
```

7. Program for moment generation function of γ

```
SNRADB = -1;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\MGF.txt"];
For[s = -15, s < 3.1,  s = s + 0.5,
  SN = 1 - s × snra // N;
  MGF = 1/SN;
  Print["S=", s, "\t", MGF];
  Write[strm, MGF];
 ];
Close[strm];
```

8. Numerical results of probability density function of γ

a) Numerical data

SNR	PDF value at SNRADB=5	PDF value at SNRADB=10	PDF value at SNRADB=15	SNR	PDF value at SNRADB=5	PDF value at SNRADB=10	PDF value at SNRADB=15
0	0.230496336	0.090483742	0.030638423	19	3.90E-12	3.55039E-05	0.002565084
1	0.212375421	0.088170959	0.030388581	20	5.84E-15	4.54E-06	0.001338567
2	0.191574257	0.085343208	0.030076945	21	1.62E-18	3.41E-07	0.000590259
3	0.168259156	0.081911873	0.029689157	22	5.42E-23	1.31E-08	0.000210558
4	0.142898487	0.077787562	0.029208063	23	1.25E-28	2.16E-10	5.75158E-05
5	0.116333694	0.072889341	0.028613472	24	1.01E-35	1.23E-12	1.12273E-05
6	0.089795721	0.067159005	0.027882105	25	1.18E-44	1.85E-15	1.44E-06
7	0.064817105	0.060581099	0.026987892	26	6.69E-56	5.13E-19	1.08E-07
8	0.043000013	0.053208217	0.025902809	27	4.67E-70	1.71E-23	4.14E-09
9	0.02565084	0.045188469	0.024598587	28	7.03E-88	3.96E-29	6.83E-11
10	0.013385675	0.036787944	0.023049634	29	2.57E-110	3.18E-36	3.90E-13
11	0.005902589	0.0283959	0.021237542	30	1.46E-138	3.72E-45	5.84E-16
12	0.002105579	0.020496968	0.019157426	31	4.02E-174	2.12E-56	1.62E-19
13	0.000575158	0.013597798	0.016825916	32	6.87E-219	1.48E-70	5.42E-24
14	0.000112273	0.008111508	0.014289849	33	3.01E-275	2.22E-88	1.25E-29
15	1.43567E-05	0.004232922	0.011633369	34	3.37E-246	8.13E-111	1.01E-36
16	1.08E-06	0.001866562	0.008979572	35	1.61E-435	4.61E-139	1.18E-45
17	4.14E-08	0.000665842	0.006481711				

Graphical representation

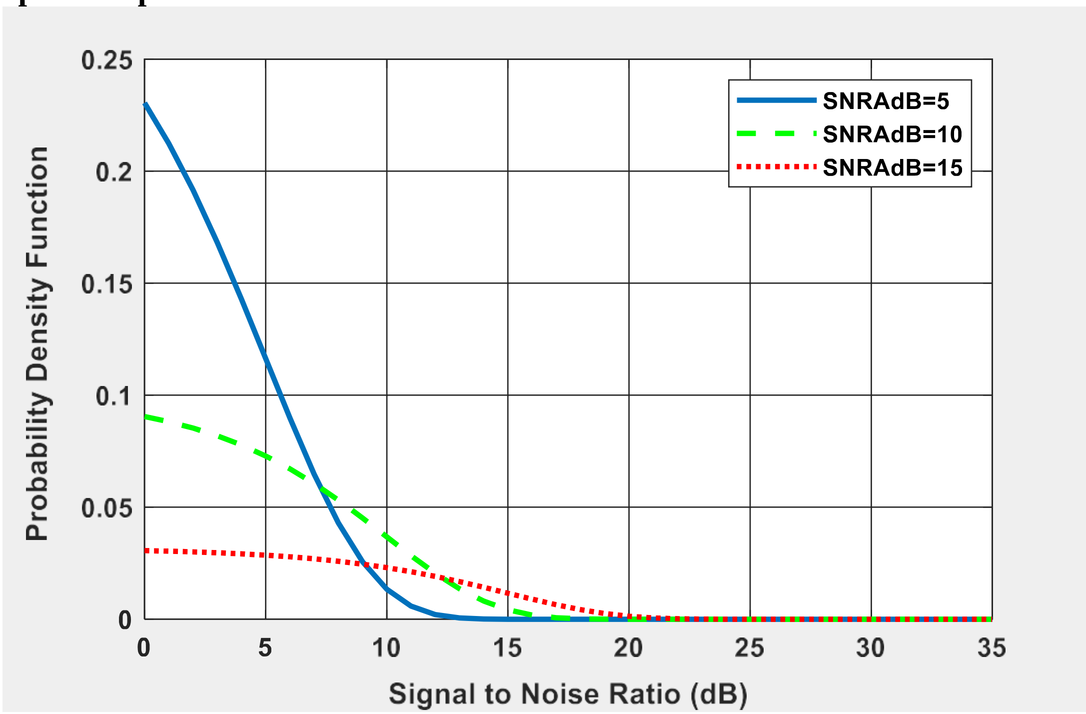

Figure 1.1. Probability Density Function vs Signal to Noise Ratio plot for Rayleigh fading SISO channel.

b) Description of Figure 1.1: This is a plot of Probability Density Function (PDF) as a function of Signal to Noise Ratio (SNR or γ) for selected values of average value of SNR (SNRADB). This figure describes the effects of SNR on PDF. In the context of wireless communication systems and Rayleigh fading channels, the PDF vs. SNR graph provides information about the probability distribution of the fading envelope at varying SNR levels. The graph shown above shows that PDF value decreases when SNR value increases. The rate of decreasing of PDF value depends on average SNR value. When SNRADB (e.g. 5) value is small, this time PDF value decreases rapidly than the other larger values of SNRADB (e.g. 10 or 15).

The peak value of PDF at 0dB depends on the average SNR value. As the SNR increases, the impact of noise diminishes, resulting in a more reliable communication link. When the SNR is low, the received signal is dominated by noise, and the fading envelope experiences deep fades. As the SNR increases, the fading envelope becomes less severe, leading to a shift in the PDF towards higher envelope values.

9. Numerical results of cumulative distribution function of γ

a) Numerical data

SNR	CDF value at SNRADB= -5	CDF value at SNRADB= -3	CDF value at SNRADB=-1	SNR	CDF value at SNRADB=-5	CDF value at SNRADB=-3	CDF value at SNRADB=-1

0	0.95767078	0.864022	0.716041	7	0.999999869	0.999955	0.998181
1	0.981334375	0.918885	0.79503	8	0.999999998	0.999997	0.999645
2	0.993341575	0.957671	0.864022	9	1	1	0.999955
3	0.998181191	0.981334	0.918885	10	1	1	0.999997
4	0.999644961	0.993342	0.957671	11	1	1	1
5	0.9999546	0.998181	0.981334	12	1	1	1
6	0.999996592	0.999645	0.993342	13	1	1	1

b) Graphical representation

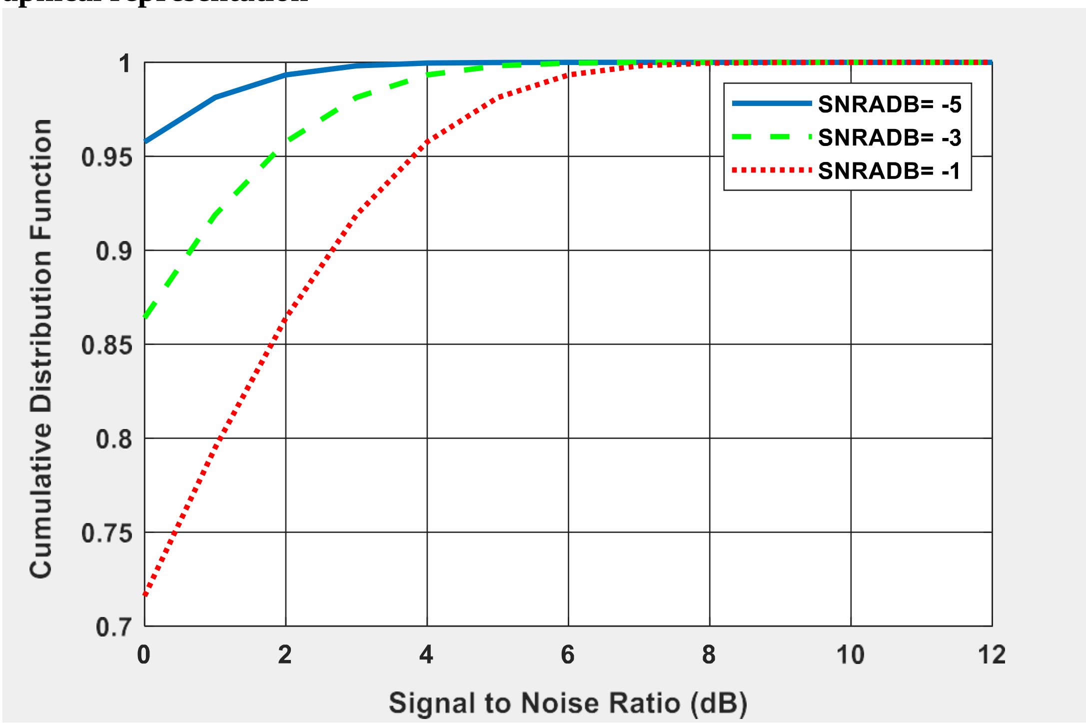

Figure 1.2. Cumulative Distribution Function vs Signal to Noise Ratio plot for Rayleigh fading SISO channel.

c) Description of Figure 1.2: This is a plot of Cumulative Distribution Function (CDF) as a function of Signal to Noise Ratio (SNR or γ) for selected value of average value of SNR (SNRADB). This figure describes the effects of SNR on CDF.

The CDF vs SNR graph provides information about the cumulative probability of encountering different fading envelope values at varying Signal to Noise Ratio (SNR) levels. E.g., If the CDF value at SNR = 10 dB is 0.8, it indicates an 80% probability that the fading envelope R is less than or equal to r, at SNR = 10 dB where we denote the fading envelope as R and its cumulative distribution function (CDF) as $F_R(r)$. From the CDF vs. SNR graph, it was observed the following trends:

1. At low SNR levels, the CDF rises slowly and steadily, indicating a higher probability of encountering small fading envelope values.
2. As SNR increases, the CDF exhibits a steeper slope, suggesting a reduced probability of encountering small envelope values.

3. At high SNR levels, the CDF tends to plateau, indicating that the probability of encountering extremely low fading envelope values becomes negligible. The channel experiences only marginal fading effects, and the signal quality is significantly enhanced.

It was also noticed from the graph that as lower the value of SNRADB (e.g. -5) the rate of increasing of CDF is higher than the other higher values of SNRADB (e.g. -3 or -1). Finally, all the CDF values saturated at 1 which justify that the plot was right according to the theory of CDF. Initial values of CDF at SNR=0dB depends on SNRADB. At SNR=0dB the signal may goes to outage.

10. Numerical results of moment generation function of γ

a) Numerical data

s	MGF value at SNRADB= -5	MGF value at SNRADB= -3	MGF value at SNRADB= -1	s	MGF value at SNRADB= -5	MGF value at SNRADB= -3	MGF value at SNRADB= -1
-15	0.174112395	0.11740109	0.077429804	-5.5	0.365063068	0.266203134	0.186261178
-15	0.174112395	0.11740109	0.077429804	-5	0.387425887	0.285230521	0.201140824
-14.5	0.179041329	0.12095972	0.079886501	-4.5	0.412707265	0.307187334	0.218604222
-14	0.184257458	0.124740831	0.082504199	-4	0.44151844	0.332806508	0.239388338
-13.5	0.189786638	0.128765959	0.08529926	-3.5	0.474654138	0.363087729	0.264539849
-13	0.195657922	0.133059514	0.088290343	-3	0.513167019	0.399430939	0.295596962
-12.5	0.201904074	0.137649273	0.091498818	-2.5	0.55848156	0.443858929	0.334916306
-12	0.208562179	0.142566982	0.094949279	-2	0.612574113	0.499407087	0.386300775
-11.5	0.215674381	0.147849095	0.098670176	-1.5	0.67826884	0.57084766	0.456310057
-11	0.223288777	0.153537671	0.102694598	-1	0.759746927	0.666139425	0.557311634
-10.5	0.231460503	0.159681507	0.107061263	-0.5	0.863472941	0.799620266	0.71573553
-10	0.240253073	0.166337531	0.11181577	0	1	1	1
-9.5	0.249740035	0.173572578	0.117012189	0.5	1.187809111	1.334389488	1.658826272
-9	0.260007027	0.18146564	0.122715135	1	1.462475296	2.004760238	4.862116094
-8.5	0.271154379	0.190110762	0.129002463	1.5	1.902376321	4.028698035	-5.222140671
-8	0.283300394	0.199620806	0.135968847	2	2.72075922	-421.1471007	-1.698783674
-7.5	0.296585567	0.210132406	0.143730578	2.5	4.774851773	-3.953067849	-1.014383361
-7	0.311178042	0.221812577	0.152432108	3	19.48683298	-1.985853964	-0.723073796
-6.5	0.327280769	0.234867652	0.162255125				

b) Graphical representation

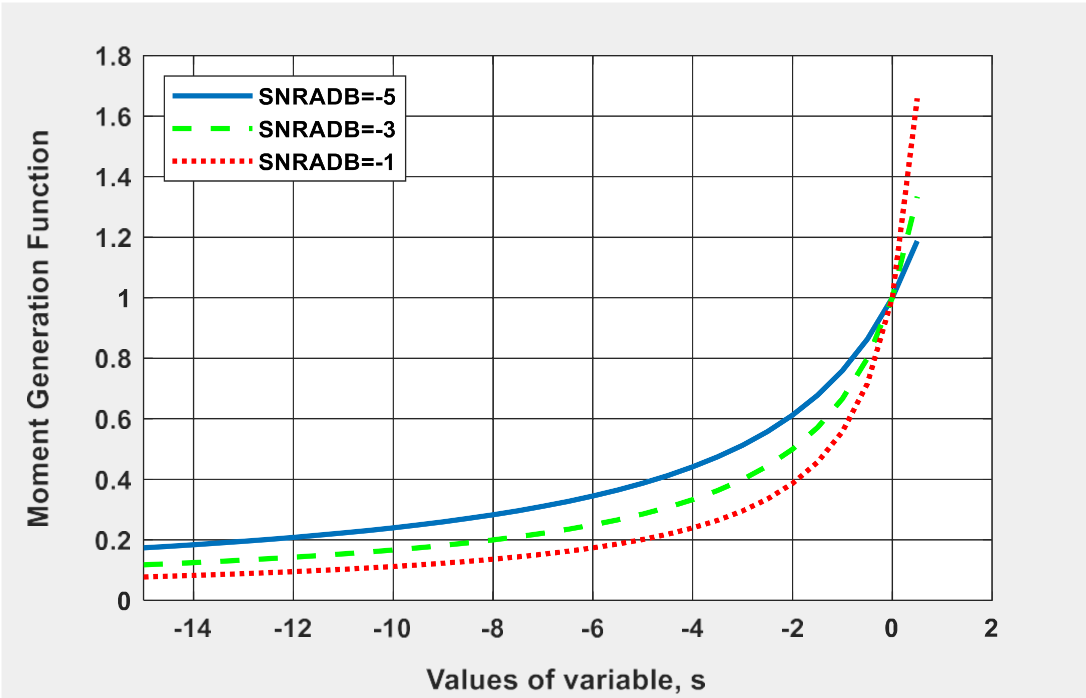

Figure 1.3. Moment Generation Function vs Variable (s) plot for Rayleigh fading SISO channel.

c) Description of Figure 1.3: This is a plot of Moment Generating Function (MGF) as a function of variable s for selected value of average value of SNR (SNRADB). This figure describes the effects of SNR on MGF. The graph shown above shows that MGF value increases when "s" value increases. At the negative portion of 's', the rate of increasing of MGF value depends on average SNR value. When SNRADB (e.g. -5) value is small, this time MGF value increases slower than the other larger values of SNRADB (e.g. -3 or -1), but in the positive portion of 's' the situation totally reversed. That mean lower valued SNRADB MGF function reaches saturation faster than other values of SNRADB.

The peak value of MGF at 0dB depends on the average SNR value. At lower SNRADB, initial value of MGF (at 0dB) was higher than other higher values of SNRADB.

11. Discussion and Conclusion

a) Discussion: In wireless communication, signals experience random fluctuations in amplitude due to the varying phase and magnitude of multipath components. The Rayleigh fading model is widely used to describe this phenomenon in line-of-sight (LOS)-free environments. In this report, we will focus on the SISO channel, where there is a single transmit antenna and a single receive antenna, and investigate the impact of SNR on the probability density function, cumulative distribution function, moment generating function and amount of fading of the fading envelope.

Figure 1.1 shows the effect of SNR on PDF. As SNR increases, the PDF value decreases. The similar characteristics is shown for different average values of SNR. The shape, or the peak point or the sharpness depends on the average SNR value of the PDF function.

Figure 1.2 shows the effect of SNR on CDF. As SNR increases, the CDF value may increase or decrease (in figure only increasing value is shown) according to instantaneous SNR magnitude. The similar characteristics is shown for different average values of SNR. The shape or the initial CDF value or the rate of CDF value changes depend on the average SNR value of the PDF function.

Figure 1.3 shows the effect of SNR on MGF. As SNR increases, the MGF value increases. The similar characteristics is shown for different average values of SNR. The shape, or the peak point or the sharpness depends on the average SNR value of the PDF function.

For the Rayleigh Fading SISO channel amount of fading is unity and independent of SNR.

b) Conclusion: In conclusion, our analysis of various channel parameters, including Probability Density Function (PDF), Cumulative Density Function (CDF), Moment Generating Function (MGF), and Amount of Fading (AF) in a Rayleigh Fading Single Input Single Output (SISO) channel, has provided valuable insights into the behavior of the channel under different Signal to Noise Ratio (SNR) conditions.
From our investigation, we can draw the following conclusions:

1. **Channel Parameters and Instantaneous SNR:** We observed that the channel parameters, as represented by the PDF, CDF, and MGF, are functions of the instantaneous SNR of the channel. The instantaneous SNR, which is the ratio of signal power to noise power at a particular time instance, plays a crucial role in determining the statistical behavior of the fading envelope and the reliability of the communication link.

2. **Importance of High SNR:** Our analysis demonstrated that for a good communication channel, a higher SNR is desirable. When the SNR is high, the signal power dominates over the noise power, leading to improved communication performance. This results in a higher probability of encountering stronger envelope values, reduced fading effects, and a more reliable wireless communication link.

3. **Unity Amount of Fading for Rayleigh Fading SISO Channel:** We found that for the Rayleigh fading SISO channel, the amount of fading is independent of the SNR. The fading envelope experiences unity amount of fading, meaning that its magnitude fluctuates around its mean value without being affected by changes in the SNR.

4. **Dependence on Average SNR:** We observed that the PDF, CDF, and MGF are dependent on the average SNR of the channel. The shape of these curves varies based on the constant parameter of the average SNR. The average SNR provides insights into the overall behavior of the channel over an extended period, influencing the statistical distribution of the fading envelope.

In conclusion, understanding the statistical properties of the Rayleigh fading SISO channel, as revealed through the PDF, CDF, and MGF analysis, is crucial for designing robust and efficient digital communication systems.