T-61.3050 Exercise session 3/2013

You should attend only one problem session (Wednesday or Thursday) during a week.

The problems are divided into demonstration and home assignments. The deadline for the home assignments is noon (11:59am) the next Monday after the corresponding exercise session. Please note that late submissions will not be graded. Please, submit your answer to the course e-mail address (t613050@james.hut.fi) as pdf. Alternatively, exercises can be returned on paper to a box (labeled with the course name) in the 3rd floor B-corridor of the Computer Science Building, but we prefer electronic submission. Your answers will be graded, and returned to you at exercise sessions later on.

See `https://noppa.aalto.fi/noppa/kurssi/t-61.3050/etusivu`
for up-to-date information.

Demonstration

1. An insurance company is faced with taking one of the following 3 actions: a_1: increase sales force by 10%; a_2: maintain present sales force; a_3: decrease sales force by 10%. Depending upon whether or not the economy is good (θ_1), mediocre (θ_2) or bad (θ_3), the company would expect to lose the following amount of money in each case:

		Action Taken		
		a_1	a_2	a_3
	θ_1	-10	-5	-3
State of Economy	θ_2	-5	-5	-2
	θ_3	1	0	-1

   The company believes that θ has the probability distribution $\pi(\theta_1) = 0.2$, $\pi(\theta_2) = 0.3$, $\pi(\theta_3) = 0.5$. Find the optimal decision for the company.

2. Show that it is feasible to use the Bayes rule to update the current knowledge of model parameters θ by taking one sample at the time and using obtained posterior as a prior distribution for the next iteration. Given a sample $\boldsymbol{X}$, show that the resulting distribution $p(\theta|\boldsymbol{X})$ is same regardless of using "batch" or "iterative" update procedure. Demonstrate this with a Gaussian data set $X \sim \mathcal{N}(\theta, \sigma^2)$ and model parameter $\theta \sim \mathcal{N}(\mu, \tau^2)$, where σ is assumed to be known.

Home assignments

1. The owner of a ski shop must order skis for the upcoming season. Orders must be placed in quantities of 25 pairs of skis. The cost per pair of skis is \$50 if 25 are ordered, \$45 if 50 are ordered, and \$40 if 75 are ordered. The skis will be sold at \$75 per pair. Any skis left over at the end of the year can be sold (for sure) at \$25 a pair. If the owner runs out of skis during the season, he will suffer a loss of "goodwill"'among unsatisfied customers. He rates this loss at \$5 per unsatisfied customer. For simplicity, the owner feels that demand for the skis will be 30, 40, 50 or 60 pair of skis, with probabilities 0.2, 0.4, 0.2 and 0.2 respectively. What should be his optimum decision?

2. Estimating the sex ratio θ based on observations. Assume we have m (out of N) successful occurrences $x = 1$ of a Bernoulli process, where $x = 1$ denotes a newborn boy. This is modelled as a binomial distribution $p(m|N,\theta) = \mathrm{Bin}(m|N,\theta) = \binom{N}{m}\theta^m(1-\theta)^{N-m}$. The task is to derive the posterior distribution for θ parameter for two cases of prior distribution: 1) uniform prior $p(\theta) = \mathrm{Uni}(0,1)$ and 2) beta distribution $p(\theta) = \mathrm{Beta}(\theta|\alpha,\beta)$, where $\alpha > 0$ and $\beta > 0$. The density function of beta distribution is given by

$$\mathrm{Beta}(\theta|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}.$$

   Comment the effect of increased prior information for second case (increased α and β) on posterior distribution (assume N is fixed).

3. A large shipment of parts is received, out of which 5 are tested for defects. The number of defective parts, X, is assumed to have a $Binomial(5,\theta)$ distribution. From past shipments, it is known that θ (proportion of defective parts in large shipment) has a $Beta(1,9)$ prior distribution. Find the Bayes estimate of the proportion of defective parts in this large shipment.