Seminar Talk
(Course: T-79.5306 Reactive Systems P)

State Space Explosion: Stubborn Set Method

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In Concurrent System:

- Effect of actions is independent of order
- All possible order of actions explodes the state space

In advanced state space methods:

- Ideally one ordering for each set of concurrent actions is considered
- At each state of the reduced state space, only a subset of semantic/structural transitions are investigated when constructing output edges from that state.

Few Advanced Space Methods

- Stubborn sets
- Persistent sets
- Ample sets

Dynamically Stubborn Set

A set $T_s \subseteq T$ of structural transition is dynamically stubborn at state s_0, iff the following hold:

D1 *If $t \in T_s, t_1, \cdots, t_n \notin T_s$ and $s_0 - t_1, t_2, \cdots, t_n \rightarrow s_n$ then there is $s_0^{'}$ such that $s_0 - t \rightarrow s_0^{'}$ and $s_0^{'} - t_1, t_2, \cdots, t_n \rightarrow s_n^{'}$*

D2 *There is at least one transition $t_k \in T_s$ such that if $t_1, \cdots, t_n \notin T_s$ and $s_0 - t_1 t_2 \cdots t_n \rightarrow s_n$, then $s_n - t_k \rightarrow$. The transition t_k is called a key transition of T_s at s_0*

- By letting n=0 in D2 we see that a key transition is enabled.
- Because a dynamically stubborn set must contain a key transition, deadlock states have no dynamically stubborn set.

Strongly Dynamically Stubborn Set

A Set of Structural Transition is Dynamically Stubborn at s if All of its enabled transitions are key transitions. That is they qualify as t_k *in D2*

Basic Stubborn Set Method

- Let $T_s \mapsto 2^T$ be a function that assigns to each non deadlock state s a dynamically stubborn set $T_s(s)$.
- The *basic stubborn set method* starts with the initial state and from each state s, constructs its successor states by firing the enabled transitions in $T_s(s)$
- And it results into a reduced state space

Deadlocks In Reduced State Space

Theorem

Let T_{do} is set of all deadlock states in the original state space and T_{dr} is the set of all the deadlock states in the reduced state space obtained by stubborn set method, Then T_{do} equals to T_{dr}

Proof.

- Let s_d be a deadlock state and s be any state in the reduced state space constructed with the basic stubborn set method.
- If $s{-}t_1 t_2 \cdots t_n \rightarrow s_d$, then D2 implies that at least one of $t_1, t_2, \cdots, t_n$ belongs to $T_s(s)$.
- Let i be as small as possible that $t_i \in T_s(s)$. D1 implies that there is an $s^{'}$ such that $s{-}t_i \rightarrow s^{'}$ and s_d is reachable from $s^{'}$ with n-1 steps, namely by firing $t_1, \cdots t_{i-1}, t_{i+1}, \cdots, t_n$
- If s is an initial state then by repeating the argument in $s^{'}$ and so on, a total of n times we see that the reduced state space contains s_d

Infinite Execution in Reduced State Space

Observation

If the transitions are deterministic then the following can be proven from D1 and D2:

D3 If T_s is dynamically stubborn at $s, t_1, t_2 \cdots \notin T_s$, t_k is a key transition of T_s at s, and s–$t_1 t_2 \cdots \rightarrow$, then there is $s^{'}$ such that s–$t_k \rightarrow s^{'}$ and $s^{'}$–$t_1 t_2 \cdots \rightarrow$.

- Now, from D1,D2 and D3 it can be proved that
A reduced state space that has been constructed with basic stubborn set method contains an infinite execution iff the full state space contains and infinite execution, provided that either all the structural transitions are deterministic or $T_s(s)$ satisfies at all state s of the reduced state space

Alternative Definitions of Stubborn Sets

- Several different definition of stubborn set of varying strength are available in literature
- We say that a definition of stubborn set is weaker than the other if every set that is stubborn according to the later definition is also stubborn according to the former but not necessarily vice versa.
- Weak definition provides better chance of few enabled transitions which results into better reduction result

Static Definition of Stubborn Sets

- An Algorithm is required to Implement the Basic Stubborn Set method
- The definition of dynamically stubbornness doesn't directly lead to an algorithm. Because it refers to states that is in the future of s and thus are not yet available
- To solve this problem, a static notion of a *stubborn set* of structural transitions has been defined:

- *If* $t \in T_s$ *and* $\neg M[t\rangle$*, then there is* $p \in \bullet t$ *such that* $M(p)$ *less than* $W(p,t)$ *and* $\bullet p \subseteq T_s$
- *If* $t \in T_s$ *and* $M[t\rangle$*, then* $(\bullet t)\bullet \subseteq T_s$ *or* $\bullet(\bullet t) \subseteq T_s$
- T_s *contains a transition* t_k *such that* $M[t_k\rangle$

Conclusion

- To use stubborn-set-type methods, it is necessary at each state to construct a good stubborn set
- It is not known what stubbor set will be the best regarding the reduction result
- If the set of enabled transition in T_{s1} is a proper subset of that of T_{s2} then T_{s1} is preferable.
- This is because the basic stubborn set method can not produce a bigger but may produce a smaller reduced state space than with T_{s2}