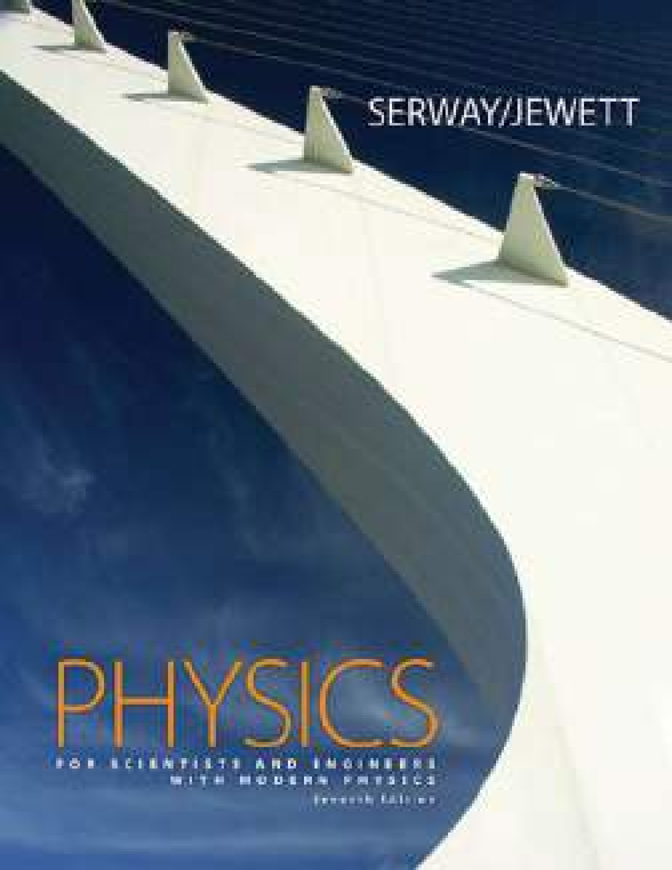

SERWAY/JEWETT

PHYSICS

FOR SCIENTISTS AND ENGINEERS
WITH MODERN PHYSICS

Seventh Edition

Pedagogical Color Chart

Mechanics

- Displacement and position vectors
- Linear ($\vec{\mathbf{v}}$) and angular ($\vec{\boldsymbol{\omega}}$) velocity vectors
- Velocity component vectors
- Force vectors ($\vec{\mathbf{F}}$)
- Force component vectors
- Acceleration vectors ($\vec{\mathbf{a}}$)
- Acceleration component vectors
- Linear ($\vec{\mathbf{p}}$) and angular ($\vec{\mathbf{L}}$) momentum vectors
- Torque vectors ($\vec{\boldsymbol{\tau}}$)
- Linear or rotational motion directions
- Springs
- Pulleys

Electricity and Magnetism

- Electric fields
- Magnetic fields
- Positive charges (+)
- Negative charges (–)
- Resistors
- Batteries and other DC power supplies (– +)
- Switches
- Capacitors
- Inductors (coils)
- Voltmeters (V)
- Ammeters (A)
- AC Sources
- Ground symbol
- Current

Light and Optics

- Light rays
- Lenses and prisms
- Mirrors
- Objects
- Images

Some Physical Constants

Quantity	Symbol	Value[a]
Atomic mass unit	u	$1.660\ 538\ 86\ (28) \times 10^{-27}$ kg $931.494\ 043\ (80)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 141\ 5\ (10) \times 10^{23}$ particles/mol
Bohr magneton	$\mu_B = \dfrac{e\hbar}{2m_e}$	$9.274\ 009\ 49\ (80) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \dfrac{\hbar^2}{m_e e^2 k_e}$	$5.291\ 772\ 108\ (18) \times 10^{-11}$ m
Boltzmann's constant	$k_B = \dfrac{R}{N_A}$	$1.380\ 650\ 5\ (24) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \dfrac{h}{m_e c}$	$2.426\ 310\ 238\ (16) \times 10^{-12}$ m
Coulomb constant	$k_e = \dfrac{1}{4\pi\epsilon_0}$	$8.987\ 551\ 788 \ldots \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$ (exact)
Deuteron mass	m_d	$3.343\ 583\ 35\ (57) \times 10^{-27}$ kg $2.013\ 553\ 212\ 70\ (35)$ u
Electron mass	m_e	$9.109\ 382\ 6\ (16) \times 10^{-31}$ kg $5.485\ 799\ 094\ 5\ (24) \times 10^{-4}$ u $0.510\ 998\ 918\ (44)$ MeV/c^2
Electron volt	eV	$1.602\ 176\ 53\ (14) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 176\ 53\ (14) \times 10^{-19}$ C
Gas constant	R	$8.314\ 472\ (15)$ J/mol·K
Gravitational constant	G	$6.674\ 2\ (10) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Josephson frequency–voltage ratio	$\dfrac{2e}{h}$	$4.835\ 978\ 79\ (41) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \dfrac{h}{2e}$	$2.067\ 833\ 72\ (18) \times 10^{-15}\ \mathrm{T \cdot m^2}$
Neutron mass	m_n	$1.674\ 927\ 28\ (29) \times 10^{-27}$ kg $1.008\ 664\ 915\ 60\ (55)$ u $939.565\ 360\ (81)$ MeV/c^2
Nuclear magneton	$\mu_n = \dfrac{e\hbar}{2m_p}$	$5.050\ 783\ 43\ (43) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A (exact)
Permittivity of free space	$\epsilon_0 = \dfrac{1}{\mu_0 c^2}$	$8.854\ 187\ 817 \ldots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ (exact)
Planck's constant	h $\hbar = \dfrac{h}{2\pi}$	$6.626\ 069\ 3\ (11) \times 10^{-34}$ J·s $1.054\ 571\ 68\ (18) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 621\ 71\ (29) \times 10^{-27}$ kg $1.007\ 276\ 466\ 88\ (13)$ u $938.272\ 029\ (80)$ MeV/c^2
Rydberg constant	R_H	$1.097\ 373\ 156\ 852\ 5\ (73) \times 10^{7}\ \mathrm{m^{-1}}$
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^{8}$ m/s (exact)

Note: These constants are the values recommended in 2002 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see P. J. Mohr and B. N. Taylor, "CODATA Recommended Values of the Fundamental Physical Constants: 2002." *Rev. Mod. Phys.* **77:**1, 2005.

[a] The numbers in parentheses for the values represent the uncertainties of the last two digits.

Solar System Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Distance from the Sun (m)
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}
Earth	5.98×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}
Pluto[a]	$\approx 1.4 \times 10^{22}$	$\approx 1.5 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}
Moon	7.36×10^{22}	1.74×10^{6}	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—

[a] In August 2006, the International Astronomical Union adopted a definition of a planet that separates Pluto from the other eight planets. Pluto is now defined as a "dwarf planet" (like the asteroid Ceres).

Physical Data Often Used

Average Earth–Moon distance	3.84×10^{8} m
Average Earth–Sun distance	1.496×10^{11} m
Average radius of the Earth	6.37×10^{6} m
Density of air (20°C and 1 atm)	1.20 kg/m^3
Density of water (20°C and 1 atm)	1.00×10^{3} kg/m^3
Free-fall acceleration	9.80 m/s^2
Mass of the Earth	5.98×10^{24} kg
Mass of the Moon	7.36×10^{22} kg
Mass of the Sun	1.99×10^{30} kg
Standard atmospheric pressure	1.013×10^{5} Pa

Note: These values are the ones used in the text.

Some Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{1}	deka	da
10^{-21}	zepto	z	10^{2}	hecto	h
10^{-18}	atto	a	10^{3}	kilo	k
10^{-15}	femto	f	10^{6}	mega	M
10^{-12}	pico	p	10^{9}	giga	G
10^{-9}	nano	n	10^{12}	tera	T
10^{-6}	micro	μ	10^{15}	peta	P
10^{-3}	milli	m	10^{18}	exa	E
10^{-2}	centi	c	10^{21}	zetta	Z
10^{-1}	deci	d	10^{24}	yotta	Y

PHYSICS

for Scientists and Engineers with Modern Physics

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for Scientists and Engineers with Modern Physics

Seventh Edition

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California State Polytechnic University, Pomona

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Raymond A. Serway and John W. Jewett, Jr.

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We dedicate this book to our wives Elizabeth and Lisa and all our children and grandchildren for their loving understanding when we spent time on writing instead of being with them.

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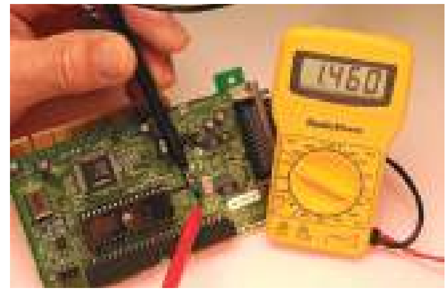
Brief Contents

John W. Jewett, Jr.

Courtesy of NASA



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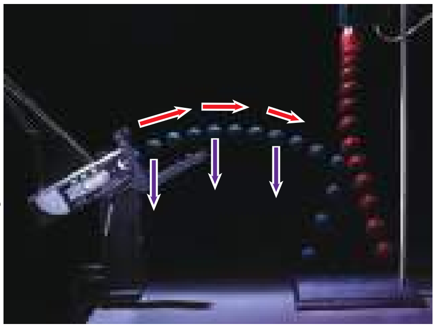
Contents



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PART 6 MODERN PHYSICS 1111


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Raymond A. Serway received his doctorate at Illinois Institute of Technology and is Professor Emeritus at James Madison University. In 1990, he received the Madison Scholar Award at James Madison University, where he taught for 17 years. Dr. Serway began his teaching career at Clarkson University, where he conducted research and taught from 1967 to 1980. He was the recipient of the Distinguished Teaching Award at Clarkson University in 1977 and of the Alumni Achievement Award from Utica College in 1985. As Guest Scientist at the IBM Research Laboratory in Zurich, Switzerland, he worked with K. Alex Müller, 1987 Nobel Prize recipient. Dr. Serway also was a visiting scientist at Argonne National Laboratory, where he collaborated with his mentor and friend, Sam Marshall. In addition to earlier editions of this textbook, Dr. Serway is the coauthor of *Principles of Physics,* fourth edition; *College Physics,* seventh edition; *Essentials of College Physics;* and *Modern Physics,* third edition. He also is the coauthor of the high school textbook *Physics,* published by Holt, Rinehart, & Winston. In addition, Dr. Serway has published more than 40 research papers in the field of condensed matter physics and has given more than 70 presentations at professional meetings. Dr. Serway and his wife, Elizabeth, enjoy traveling, golf, singing in a church choir, and spending quality time with their four children and eight grandchildren.

John W. Jewett, Jr., earned his doctorate at Ohio State University, specializing in optical and magnetic properties of condensed matter. Dr. Jewett began his academic career at Richard Stockton College of New Jersey, where he taught from 1974 to 1984. He is currently Professor of Physics at California State Polytechnic University, Pomona. Throughout his teaching career, Dr. Jewett has been active in promoting science education. In addition to receiving four National Science Foundation grants, he helped found and direct the Southern California Area Modern Physics Institute. He also directed Science IMPACT (Institute for Modern Pedagogy and Creative Teaching), which works with teachers and schools to develop effective science curricula. Dr. Jewett's honors include the Stockton Merit Award at Richard Stockton College in 1980, the Outstanding Professor Award at California State Polytechnic University for 1991–1992, and the Excellence in Undergraduate Physics Teaching Award from the American Association of Physics Teachers in 1998. He has given more than 80 presentations at professional meetings, including presentations at international conferences in China and Japan. In addition to his work on this textbook, he is coauthor of *Principles of Physics,* fourth edition, with Dr. Serway and author of *The World of Physics . . . Mysteries, Magic, and Myth.* Dr. Jewett enjoys playing keyboard with his all-physicist band, traveling, and collecting antiques that can be used as demonstration apparatus in physics lectures. Most importantly, he relishes spending time with his wife, Lisa, and their children and grandchildren.

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In writing this seventh edition of *Physics for Scientists and Engineers,* we continue our ongoing efforts to improve the clarity of presentation and include new pedagogical features that help support the learning and teaching processes. Drawing on positive feedback from users of the sixth edition and reviewers' suggestions, we have refined the text to better meet the needs of students and teachers.

This textbook is intended for a course in introductory physics for students majoring in science or engineering. The entire contents of the book in its extended version could be covered in a three-semester course, but it is possible to use the material in shorter sequences with the omission of selected chapters and sections. The mathematical background of the student taking this course should ideally include one semester of calculus. If that is not possible, the student should be enrolled in a concurrent course in introductory calculus.

Objectives

This introductory physics textbook has two main objectives: to provide the student with a clear and logical presentation of the basic concepts and principles of physics and to strengthen an understanding of the concepts and principles through a broad range of interesting applications to the real world. To meet these objectives, we have placed emphasis on sound physical arguments and problem-solving methodology. At the same time, we have attempted to motivate the student through practical examples that demonstrate the role of physics in other disciplines, including engineering, chemistry, and medicine.

Changes in the Seventh Edition

A large number of changes and improvements have been made in preparing the seventh edition of this text. Some of the new features are based on our experiences and on current trends in science education. Other changes have been incorporated in response to comments and suggestions offered by users of the sixth edition and by reviewers of the manuscript. The features listed here represent the major changes in the seventh edition.

QUESTIONS AND PROBLEMS A substantial revision to the end-of-chapter questions and problems was made in an effort to improve their variety, interest, and pedagogical value, while maintaining their clarity and quality. Approximately 23% of the questions and problems are new or substantially changed. Several of the questions for each chapter are in objective format. Several problems in each chapter explicitly ask for qualitative reasoning in some parts as well as for quantitative answers in other parts:

> **19.** ● Assume a parcel of air in a straight tube moves with a constant acceleration of $-4.00\ \text{m/s}^2$ and has a velocity of 13.0 m/s at 10:05:00 a.m. on a certain date. (a) What is its velocity at 10:05:01 a.m.? (b) At 10:05:02 a.m.? (c) At 10:05:02.5 a.m.? (d) At 10:05:04 a.m.? (e) At 10:04:59 a.m.? (f) Describe the shape of a graph of velocity versus time for this parcel of air. (g) Argue for or against the statement, "Knowing the single value of an object's constant acceleration is like knowing a whole list of values for its velocity."

WORKED EXAMPLES All in-text worked examples have been recast and are now presented in a two-column format to better reinforce physical concepts. The left column shows textual information that describes the steps for solving the problem. The right column shows the mathematical manipulations and results of taking these steps. This layout facilitates matching the concept with its mathematical execution and helps students organize their work. These reconstituted examples closely follow a General Problem-Solving Strategy introduced in Chapter 2 to reinforce effective problem-solving habits. A sample of a worked example can be found on the next page.

Each solution has been reconstituted to more closely follow the General Problem-Solving Strategy as outlined in Chapter 2, to reinforce good problem-solving habits.

EXAMPLE 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in Figure 3.11a. Find the magnitude and direction of the car's resultant displacement.

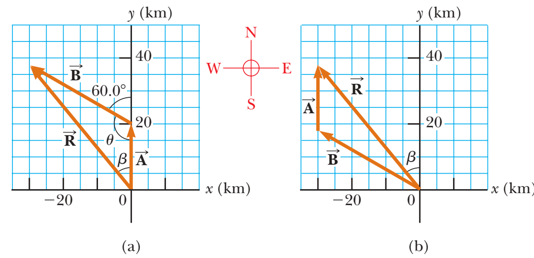

Figure 3.11 (Example 3.2) (a) Graphical method for finding the resultant displacement vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (b) Adding the vectors in reverse order $(\vec{\mathbf{B}} + \vec{\mathbf{A}})$ gives the same result for $\vec{\mathbf{R}}$.

SOLUTION

Conceptualize The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ drawn in Figure 3.11a help us conceptualize the problem.

Categorize We can categorize this example as a simple analysis problem in vector addition. The displacement $\vec{\mathbf{R}}$ is the resultant when the two individual displacements $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added. We can further categorize it as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

Each step of the solution is detailed in a two-column format. The left column provides an explanation for each mathematical step in the right column, to better reinforce the physical concepts.

Analyze In this example, we show two ways to analyze the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of $\vec{\mathbf{R}}$ and its direction in Figure 3.11a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of $\vec{\mathbf{R}}$ can be obtained from the law of cosines as applied to the triangle (see Appendix B.4).

Use $R^2 = A^2 + B^2 - 2AB\cos\theta$ from the law of cosines to find R:

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Substitute numerical values, noting that $\theta = 180° - 60° = 120°$:

$$R = \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$

$$= \boxed{48.2\text{ km}}$$

Use the law of sines (Appendix B.4) to find the direction of $\vec{\mathbf{R}}$ measured from the northerly direction:

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = \boxed{38.9°}$$

The resultant displacement of the car is 48.2 km in a direction 38.9° west of north.

Finalize Does the angle β that we calculated agree with an estimate made by looking at Figure 3.11a or with an actual angle measured from the diagram using the graphical method? Is it reasonable that the magnitude of $\vec{\mathbf{R}}$ is larger than that of both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$? Are the units of $\vec{\mathbf{R}}$ correct?

Although the graphical method of adding vectors works well, it suffers from two disadvantages. First, some people find using the laws of cosines and sines to be awkward. Second, a triangle only results if you are adding two vectors. If you are adding three or more vectors, the resulting geometric shape is usually not a triangle. In Section 3.4, we explore a new method of adding vectors that will address both of these disadvantages.

What If? Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

Answer They would not change. The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant. Graphically, Figure 3.11b shows that the vectors added in the reverse order give us the same resultant vector.

What If? statements appear in about 1/3 of the worked examples and offer a variation on the situation posed in the text of the example. For instance, this feature might explore the effects of changing the conditions of the situation, determine what happens when a quantity is taken to a particular limiting value, or question whether additional information can be determined about the problem situation. This feature encourages students to think about the results of the example and assists in conceptual understanding of the principles.

All worked examples are also available to be assigned as interactive examples in the Enhanced WebAssign homework management system (visit **www.pse7.com** for more details).

ONLINE HOMEWORK It is now easier to assign online homework with Serway and Jewett and Enhanced WebAssign. All worked examples, end-of-chapter problems, active figures, quick quizzes, and most questions are available in WebAssign. Most problems include hints and feedback to provide instantaneous reinforcement or direction for that problem. In addition to the text content, we have also added math remediation tools to help students get up to speed in algebra, trigonometry, and calculus.

SUMMARIES Each chapter contains a summary that reviews the important concepts and equations discussed in that chapter. A marginal note next to each chapter summary directs students to additional quizzes, animations, and interactive exercises for that chapter on the book's companion Web site. The format of the end-of-chapter summary has been completely revised for this edition. The summary is divided into three sections: Definitions, Concepts and Principles, and Analysis Models for Problem-Solving. In each section, flashcard-type boxes focus on each separate definition, concept, principle, or analysis model.

MATH APPENDIX The math appendix, a valuable tool for students, has been updated to show the math tools in a physics context. This resource is ideal for students who need a quick review on topics such as algebra, trigonometry, and calculus.

CONTENT CHANGES The content and organization of the textbook are essentially the same as in the sixth edition. Many sections in various chapters have been streamlined, deleted, or combined with other sections to allow for a more balanced presentation. Vectors are now denoted in boldface with an arrow over them (for example, $\vec{\mathbf{v}}$), making them easier to recognize. Chapters 7 and 8 have been completely reorganized to prepare students for a unified approach to energy that is used throughout the text. A new section in Chapter 9 teaches students how to analyze deformable systems with the conservation of energy equation and the impulse-momentum theorem. Chapter 34 is longer than in the sixth edition because of the movement into that chapter of the material on displacement current from Chapter 30 and Maxwell's equations from Chapter 31. A more detailed list of content changes can be found on the instructor's companion Web site.

Content

The material in this book covers fundamental topics in classical physics and provides an introduction to modern physics. The book is divided into six parts. Part 1 (Chapters 1 to 14) deals with the fundamentals of Newtonian mechanics and the physics of fluids; Part 2 (Chapters 15 to 18) covers oscillations, mechanical waves, and sound; Part 3 (Chapters 19 to 22) addresses heat and thermodynamics; Part 4 (Chapters 23 to 34) treats electricity and magnetism; Part 5 (Chapters 35 to 38) covers light and optics; and Part 6 (Chapters 39 to 46) deals with relativity and modern physics.

Text Features

Most instructors believe that the textbook selected for a course should be the student's primary guide for understanding and learning the subject matter. Furthermore, the textbook should be easily accessible and should be styled and written to facilitate instruction and learning. With these points in mind, we have included many pedagogical features, listed below, that are intended to enhance its usefulness to both students and instructors.

Problem Solving and Conceptual Understanding

GENERAL PROBLEM-SOLVING STRATEGY A general strategy outlined at the end of Chapter 2 provides students with a structured process for solving problems. In all remaining chapters, the strategy is employed explicitly in every example so that students learn how it is applied. Students are encouraged to follow this strategy when working end-of-chapter problems.

MODELING Although students are faced with hundreds of problems during their physics courses, instructors realize that a relatively small number of physical situations form the basis of these problems. When faced with a new problem, a physicist forms a *model* of the problem that can be solved in a simple way by identifying the common physical situation that occurs in the problem. For example, many problems involve particles under constant acceleration, isolated systems, or waves under refraction. Because the physicist has studied these situations extensively and understands the associated behavior, he or she can apply this knowledge as a model for solving a new problem. In certain chapters, this edition identifies Analysis Models, which are physical situations (such as the particle under constant acceleration, the isolated system, or the wave under refraction) that occur so often that they can be used as a model for solving an unfamiliar problem. These models are discussed in the chapter text, and the student is reminded of them in the end-of-chapter summary under the heading "Analysis Models for Problem-Solving."

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PROBLEMS An extensive set of problems is included at the end of each chapter; in all, the text contains approximately three thousand problems. Answers to odd-numbered problems are provided at the end of the book. For the convenience of both the student and the instructor, about two-thirds of the problems are keyed to specific sections of the chapter. The remaining problems, labeled "Additional Problems," are not keyed to specific sections. The problem numbers for straightforward problems are printed in black, intermediate-level problems are in blue, and challenging problems are in magenta.

- **"Not-just-a-number" problems** Each chapter includes several marked problems that require students to think qualitatively in some parts and quantitatively in others. Instructors can assign such problems to guide students to display deeper understanding, practice good problem-solving techniques, and prepare for exams.
- **Problems for developing symbolic reasoning** Each chapter contains problems that ask for solutions in symbolic form as well as many problems asking for numerical answers. To help students develop skill in symbolic reasoning, each chapter contains a pair of otherwise identical problems, one asking for a numerical solution and one asking for a symbolic derivation. In this edition, each chapter also contains a problem giving a numerical value for every datum but one so that the answer displays how the unknown depends on the datum represented symbolically. The answer to such a problem has the form of a function of one variable. Reasoning about the behavior of this function puts emphasis on the *Finalize* step of the General Problem-Solving Strategy. All problems developing symbolic reasoning are identified by a tan background screen:

> **53.** ● A light spring has an unstressed length of 15.5 cm. It is described by Hooke's law with spring constant 4.30 N/m. One end of the horizontal spring is held on a fixed vertical axle, and the other end is attached to a puck of mass m that can move without friction over a horizontal surface. The puck is set into motion in a circle with a period of 1.30 s. (a) Find the extension of the spring x as it depends on m. Evaluate x for (b) $m = 0.070\,0$ kg, (c) $m = 0.140$ kg, (d) $m = 0.180$ kg, and (e) $m = 0.190$ kg. (f) Describe the pattern of variation of x as it depends on m.

- **Review problems** Many chapters include review problems requiring the student to combine concepts covered in the chapter with those discussed in previous chapters. These problems reflect the cohesive nature of the principles in the text and verify that physics is not a scattered set of ideas. When facing a real-world issue such as global warming or nuclear weapons, it may be necessary to call on ideas in physics from several parts of a textbook such as this one.
- **"Fermi problems"** As in previous editions, at least one problem in each chapter asks the student to reason in order-of-magnitude terms.

- **Design problems** Several chapters contain problems that ask the student to determine design parameters for a practical device so that it can function as required.
- **"*Jeopardy!*" problems** Some chapters give students practice in changing between different representations by stating equations and asking for a description of a situation to which they apply as well as for a numerical answer.
- **Calculus-based problems** Every chapter contains at least one problem applying ideas and methods from differential calculus and one problem using integral calculus.

The instructor's Web site, **www.thomsonedu.com/physics/serway,** provides lists of problems using calculus, problems encouraging or requiring computer use, problems with "**What If?**" parts, problems referred to in the chapter text, problems based on experimental data, order-of-magnitude problems, problems about biological applications, design problems, *Jeopardy!* problems, review problems, problems reflecting historical reasoning about confusing ideas, problems developing symbolic reasoning skill, problems with qualitative parts, ranking questions, and other objective questions.

QUESTIONS The questions section at the end of each chapter has been significantly revised. Multiple-choice, ranking, and true–false questions have been added. The instructor may select items to assign as homework or use in the classroom, possibly with "peer instruction" methods and possibly with "clicker" systems. More than eight hundred questions are included in this edition. Answers to selected questions are included in the *Student Solutions Manual/Study Guide,* and answers to all questions are found in the *Instructor's Solutions Manual.*

19. O (i) Rank the gravitational accelerations you would measure for (a) a 2-kg object 5 cm above the floor, (b) a 2-kg object 120 cm above the floor, (c) a 3-kg object 120 cm above the floor, and (d) a 3-kg object 80 cm above the floor. List the one with the largest-magnitude acceleration first. If two are equal, show their equality in your list. **(ii)** Rank the gravitational forces on the same four objects, largest magnitude first. **(iii)** Rank the gravitational potential energies (of the object–Earth system) for the same four objects, largest first, taking $y = 0$ at the floor.

23. O An ice cube has been given a push and slides without friction on a level table. Which is correct? (a) It is in stable equilibrium. (b) It is in unstable equilibrium. (c) It is in neutral equilibrium (d) It is not in equilibrium.

WORKED EXAMPLES Two types of worked examples are presented to aid student comprehension. All worked examples in the text may be assigned for homework in WebAssign.

The first example type presents a problem and numerical answer. As discussed earlier, solutions to these examples have been altered in this edition to feature a two-column layout to explain the physical concepts and the mathematical steps side by side. Every example follows the explicit steps of the General Problem-Solving Strategy outlined in Chapter 2.

The second type of example is conceptual in nature. To accommodate increased emphasis on understanding physical concepts, the many conceptual examples are labeled as such, set off in boxes, and designed to focus students on the physical situation in the problem.

WHAT IF? Approximately one-third of the worked examples in the text contain a **What If?** feature. At the completion of the example solution, a **What If?** question offers a variation on the situation posed in the text of the example. For instance, this feature might explore the effects of changing the conditions of the situation, determine what happens when a quantity is taken to a particular limiting value, or question whether additional information can be determined about the situation. This feature encourages students to think about the results of the example, and it also assists in conceptual understanding of the principles. **What If?** questions also prepare students to encounter novel problems that may be included on exams. Some of the end-of-chapter problems also include this feature.

QUICK QUIZZES Quick Quizzes provide students an opportunity to test their understanding of the physical concepts presented. The questions require students to make decisions on the basis of sound reasoning, and some of the questions have been written to help students overcome common misconceptions. Quick Quizzes have been cast in an objective format, including multiple-choice, true–false, and ranking. Answers to all Quick Quiz questions are found at the end of each chapter. Additional Quick Quizzes that can be used in classroom teaching are available on the instructor's companion Web site. Many instructors choose to use such questions in a "peer instruction" teaching style or with the use of personal response system "clickers," but they can be used in standard quiz format as well. Quick Quizzes are set off from the text by horizontal lines:

Quick Quiz 7.5 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance x. For the next loading, the spring is compressed a distance $2x$. How much faster does the second dart leave the gun compared with the first? (a) four times as fast (b) two times as fast (c) the same (d) half as fast (e) one-fourth as fast

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PITFALL PREVENTION 16.2

Two Kinds of Speed/Velocity

Do not confuse v, the speed of the wave as it propagates along the string, with v_y, the transverse velocity of a point on the string. The speed v is constant for a uniform medium, whereas v_y varies sinusoidally.

PITFALL PREVENTIONS More than two hundred Pitfall Preventions (such as the one to the left) are provided to help students avoid common mistakes and misunderstandings. These features, which are placed in the margins of the text, address both common student misconceptions and situations in which students often follow unproductive paths.

Helpful Features

STYLE To facilitate rapid comprehension, we have written the book in a clear, logical, and engaging style. We have chosen a writing style that is somewhat informal and relaxed so that students will find the text appealing and enjoyable to read. New terms are carefully defined, and we have avoided the use of jargon.

IMPORTANT STATEMENTS AND EQUATIONS Most important statements and definitions are set in **boldface** or are highlighted with a background screen for added emphasis and ease of review. Similarly, important equations are highlighted with a background screen to facilitate location.

MARGINAL NOTES Comments and notes appearing in the margin with a ▶ icon can be used to locate important statements, equations, and concepts in the text.

PEDAGOGICAL USE OF COLOR Readers should consult the **pedagogical color chart** (inside the front cover) for a listing of the color-coded symbols used in the text diagrams. This system is followed consistently throughout the text.

MATHEMATICAL LEVEL We have introduced calculus gradually, keeping in mind that students often take introductory courses in calculus and physics concurrently. Most steps are shown when basic equations are developed, and reference is often made to mathematical appendices near the end of the textbook. Vector products are introduced later in the text, where they are needed in physical applications. The dot product is introduced in Chapter 7, which addresses energy of a system; the cross product is introduced in Chapter 11, which deals with angular momentum.

SIGNIFICANT FIGURES Significant figures in both worked examples and end-of-chapter problems have been handled with care. Most numerical examples are worked to either two or three significant figures, depending on the precision of the data provided. End-of-chapter problems regularly state data and answers to three-digit precision.

UNITS The international system of units (SI) is used throughout the text. The U.S. customary system of units is used only to a limited extent in the chapters on mechanics and thermodynamics.

APPENDICES AND ENDPAPERS Several appendices are provided near the end of the textbook. Most of the appendix material represents a review of mathematical concepts and techniques used in the text, including scientific notation, algebra, geometry, trigonometry, differential calculus, and integral calculus. Reference to these appendices is made throughout the text. Most mathematical review sections in the appendices include worked examples and exercises with answers. In addition to the mathematical reviews, the appendices contain tables of physical data, conversion factors, and the SI units of physical quantities as well as a periodic table of the elements. Other useful information—fundamental constants and physical data, planetary data, a list of standard prefixes, mathematical symbols, the Greek alphabet, and standard abbreviations of units of measure—appears on the endpapers.

Course Solutions That Fit Your Teaching Goals and Your Students' Learning Needs

Recent advances in educational technology have made homework management systems and audience response systems powerful and affordable tools to enhance the way you teach your course. Whether you offer a more traditional text-based course, are interested in using or are currently using an online homework management system such as WebAssign, or are ready to turn your lecture into an interactive learning environment with JoinIn on TurningPoint, you can be confident that the text's proven content provides the foundation for each and every component of our technology and ancillary package.

Homework Management Systems

Enhanced WebAssign Whether you're an experienced veteran or a beginner, Enhanced WebAssign is the perfect solution to fit your homework management needs. Designed by physicists for physicists, this system is a reliable and user-friendly teaching companion. Enhanced WebAssign is available for *Physics for Scientists and Engineers*, giving you the freedom to assign

- every end-of-chapter Problem and Question, enhanced with hints and feedback
- every worked example, enhanced with hints and feedback, to help strengthen students' problem-solving skills
- every Quick Quiz, giving your students ample opportunity to test their conceptual understanding.

- animated Active Figures, enhanced with hints and feedback, to help students develop their visualization skills
- a math review to help students brush up on key quantitative concepts

Please visit **www.thomsonedu.com/physics/serway** to view a live demonstration of Enhanced WebAssign.

The text also supports the following Homework Management Systems:

LON-CAPA: A Computer-Assisted Personalized Approach
http://www.lon-capa.org/

The University of Texas Homework Service
contact **moore@physics.utexas.edu**

Personal Response Systems

JoinIn on TurningPoint Pose book-specific questions and display students' answers seamlessly within the Microsoft® PowerPoint slides of your own lecture in conjunction with the "clicker" hardware of your choice. JoinIn on TurningPoint works with most infrared or radio frequency keypad systems, including Responsecard, EduCue, H-ITT, and even laptops. Contact your local sales representative to learn more about our personal response software and hardware.

Personal Response System Content Regardless of the response system you are using, we provide the tested content to support it. Our ready-to-go content includes all the questions from the Quick Quizzes, test questions, and a selection of end-of-chapter questions to provide helpful conceptual checkpoints to drop into your lecture. Our series of Active Figure animations have also been enhanced with multiple-choice questions to help test students' observational skills.

We also feature the Assessing to Learn in the Classroom content from the University of Massachusetts at Amherst. This collection of 250 advanced conceptual questions has been tested in the classroom for more than ten years and takes peer learning to a new level.

Visit **www.thomsonedu.com/physics/serway** to download samples of our personal response system content.

Lecture Presentation Resources

The following resources provide support for your presentations in lecture.

MULTIMEDIA MANAGER INSTRUCTOR'S RESOURCE CD An easy-to-use multimedia lecture tool, the Multimedia Manager Instructor's Resource CD allows you to quickly assemble art, animations, digital video, and database files with notes to create fluid lectures. The two-volume set (Volume 1: Chapters 1–22; Volume 2: Chapters 23–46) includes prebuilt PowerPoint lectures, a database of animations, video clips, and digital art from the text as well as editable electronic files of the *Instructor's Solutions Manual* and *Test Bank*.

TRANSPARENCY ACETATES Each volume contains approximately one hundred transparency acetates featuring art from the text. Volume 1 contains Chapters 1 through 22, and Volume 2 contains Chapters 23 through 46.

Assessment and Course Preparation Resources

A number of resources listed below will assist with your assessment and preparation processes.

INSTRUCTOR'S SOLUTIONS MANUAL by Ralph McGrew. This two-volume manual contains complete worked solutions to all end-of-chapter problems in the textbook as well as answers to the even-numbered problems and all the questions. The solutions to problems new to the seventh edition are marked for easy identification. Volume 1 contains

Chapters 1 through 22, and Volume 2 contains Chapters 23 through 46. Electronic files of the Instructor's Solutions are available on the Multimedia Manager CD as well.

PRINTED TEST BANK by Edward Adelson. This two-volume test bank contains approximately 2 200 multiple-choice questions. These questions are also available in electronic format with complete answers and solutions in the ExamView test software and as editable Word® files on the Multimedia Manager CD. Volume 1 contains Chapters 1 through 22, and Volume 2 contains Chapters 23 through 46.

EXAMVIEW This easy-to-use test generator CD features all of the questions from the printed test bank in an editable format.

WEBCT AND BLACKBOARD CONTENT For users of either course management system, we provide our test bank questions in the proper format for easy upload into your online course. In addition, you can integrate the ThomsonNOW for Physics student tutorial content into your WebCT or Blackboard course, providing your students a single sign-on to all their Web-based learning resources. Contact your local sales representative to learn more about our WebCT and Blackboard resources.

INSTRUCTOR'S COMPANION WEB SITE Consult the instructor's site by pointing your browser to **www.thomsonedu.com/physics/serway** for additional Quick Quiz questions, a detailed list of content changes since the sixth edition, a problem correlation guide, images from the text, and sample PowerPoint lectures. Instructors adopting the seventh edition of *Physics for Scientists and Engineers* may download these materials after securing the appropriate password from their local Thomson•Brooks/Cole sales representative.

Student Resources

STUDENT SOLUTIONS MANUAL/STUDY GUIDE by John R. Gordon, Ralph McGrew, Raymond Serway, and John W. Jewett, Jr. This two-volume manual features detailed solutions to 20% of the end-of-chapter problems from the text. The manual also features a list of important equations, concepts, and notes from key sections of the text in addition to answers to selected end-of-chapter questions. Volume 1 contains Chapters 1 through 22, and Volume 2 contains Chapters 23 through 46.

THOMSONNOW PERSONAL STUDY This assessment-based student tutorial system provides students with a personalized learning plan based on their performance on a series of diagnostic pre-tests. Rich interactive content, including Active Figures, Coached Problems, and Interactive Examples, helps students prepare for tests and exams.

Teaching Options

The topics in this textbook are presented in the following sequence: classical mechanics, oscillations and mechanical waves, and heat and thermodynamics followed by electricity and magnetism, electromagnetic waves, optics, relativity, and modern physics. This presentation represents a traditional sequence, with the subject of mechanical waves being presented before electricity and magnetism. Some instructors may prefer to discuss both mechanical and electromagnetic waves together after completing electricity and magnetism. In this case, Chapters 16 through 18 could be covered along with Chapter 34. The chapter on relativity is placed near the end of the text because this topic often is treated as an introduction to the era of "modern physics." If time permits, instructors may choose to cover Chapter 39 after completing Chapter 13 as a conclusion to the material on Newtonian mechanics.

For those instructors teaching a two-semester sequence, some sections and chapters could be deleted without any loss of continuity. The following sections can be considered optional for this purpose:

2.8	Kinematic Equations Derived from Calculus
4.6	Relative Velocity and Relative Acceleration
6.3	Motion in Accelerated Frames
6.4	Motion in the Presence of Resistive Forces
7.9	Energy Diagrams and Equilibrium of a System
9.8	Rocket Propulsion
11.5	The Motion of Gyroscopes and Tops
14.7	Other Applications of Fluid Dynamics
15.6	Damped Oscillations
15.7	Forced Oscillations
17.5	Digital Sound Recording
17.6	Motion Picture Sound
18.6	Standing Waves in Rods and Membranes
18.8	Nonsinusoidal Wave Patterns
22.8	Entropy on a Microscopic Scale
25.7	The Millikan Oil-Drop Experiment
25.8	Applications of Electrostatics
26.7	An Atomic Description of Dielectrics
27.5	Superconductors
28.5	Electrical Meters
28.6	Household Wiring and Electrical Safety
29.3	Applications Involving Charged Particles Moving in a Magnetic Field
29.6	The Hall Effect
30.6	Magnetism in Matter
30.7	The Magnetic Field of the Earth
31.6	Eddy Currents
33.9	Rectifiers and Filters
34.6	Production of Electromagnetic Waves by an Antenna
36.5	Lens Aberrations
36.6	The Camera
36.7	The Eye
36.8	The Simple Magnifier
36.9	The Compound Microscope
36.10	The Telescope
38.5	Diffraction of X-Rays by Crystals
39.10	The General Theory of Relativity
41.6	Applications of Tunneling
42.9	Spontaneous and Stimulated Transitions
42.10	Lasers
43.7	Semiconductor Devices
43.8	Superconductivity
44.8	Nuclear Magnetic Resonance and Magnetic Resonance Imaging
45.5	Radiation Damage
45.6	Radiation Detectors
45.7	Uses of Radiation

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Raymond A. Serway
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Pomona, California

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It is appropriate to offer some words of advice that should be of benefit to you, the student. Before doing so, we assume you have read the Preface, which describes the various features of the text and support materials that will help you through the course.

How to Study

Instructors are often asked, "How should I study physics and prepare for examinations?" There is no simple answer to this question, but we can offer some suggestions based on our own experiences in learning and teaching over the years.

First and foremost, maintain a positive attitude toward the subject matter, keeping in mind that physics is the most fundamental of all natural sciences. Other science courses that follow will use the same physical principles, so it is important that you understand and are able to apply the various concepts and theories discussed in the text.

Concepts and Principles

It is essential that you understand the basic concepts and principles before attempting to solve assigned problems. You can best accomplish this goal by carefully reading the textbook before you attend your lecture on the covered material. When reading the text, you should jot down those points that are not clear to you. Also be sure to make a diligent attempt at answering the questions in the Quick Quizzes as you come to them in your reading. We have worked hard to prepare questions that help you judge for yourself how well you understand the material. Study the **What If?** features that appear in many of the worked examples carefully. They will help you extend your understanding beyond the simple act of arriving at a numerical result. The Pitfall Preventions will also help guide you away from common misunderstandings about physics. During class, take careful notes and ask questions about those ideas that are unclear to you. Keep in mind that few people are able to absorb the full meaning of scientific material after only one reading; several readings of the text and your notes may be necessary. Your lectures and laboratory work supplement the textbook and should clarify some of the more difficult material. You should minimize your memorization of material. Successful memorization of passages from the text, equations, and derivations does not necessarily indicate that you understand the material. Your understanding of the material will be enhanced through a combination of efficient study habits, discussions with other students and with instructors, and your ability to solve the problems presented in the textbook. Ask questions whenever you believe that clarification of a concept is necessary.

Study Schedule

It is important that you set up a regular study schedule, preferably a daily one. Make sure that you read the syllabus for the course and adhere to the schedule set by your instructor. The lectures will make much more sense if you read the corresponding text material *before* attending them. As a general rule, you should devote about two hours of study time for each hour you are in class. If you are having trouble with the course, seek the advice of the instructor or other students who have taken the course. You may find it necessary to seek further instruction from experienced students. Very often, instructors offer review sessions in addition to regular class periods. Avoid the practice of delaying study until a day or two before an exam. More often than not, this approach has disastrous results. Rather than undertake an all-night study session before a test, briefly review the basic concepts and equations, and then get a good night's rest. If you believe that you need additional help in understanding the concepts, in preparing for exams, or in problem solving, we suggest that you acquire a copy of the *Student Solutions Manual/Study Guide* that accompanies this textbook; this manual should be available at your college bookstore or through the publisher.

Use the Features

You should make full use of the various features of the text discussed in the Preface. For example, marginal notes are useful for locating and describing important equations and concepts, and **boldface** indicates important statements and definitions. Many useful tables are contained in the appendices, but most are incorporated in the text where they are most often referenced. Appendix B is a convenient review of mathematical tools used in the text.

Answers to odd-numbered problems are given at the end of the textbook, answers to Quick Quizzes are located at the end of each chapter, and solutions to selected end-of-chapter questions and problems are provided in the *Student Solutions Manual/Study Guide*. The table of contents provides an overview of the entire text, and the index enables you to locate specific material quickly. Footnotes are sometimes used to supplement the text or to cite other references on the subject discussed.

After reading a chapter, you should be able to define any new quantities introduced in that chapter and discuss the principles and assumptions that were used to arrive at certain key relations. The chapter summaries and the review sections of the *Student Solutions Manual/Study Guide* should help you in this regard. In some cases, you may find it necessary to refer to the textbook's index to locate certain topics. You should be able to associate with each physical quantity the correct symbol used to represent that quantity and the unit in which the quantity is specified. Furthermore, you should be able to express each important equation in concise and accurate prose.

Problem Solving

R. P. Feynman, Nobel laureate in physics, once said, "You do not know anything until you have practiced." In keeping with this statement, we strongly advise you to develop the skills necessary to solve a wide range of problems. Your ability to solve problems will be one of the main tests of your knowledge of physics; therefore, you should try to solve as many problems as possible. It is essential that you understand basic concepts and principles before attempting to solve problems. It is good practice to try to find alternate solutions to the same problem. For example, you can solve problems in mechanics using Newton's laws, but very often an alternative method that draws on energy considerations is more direct. You should not deceive yourself into thinking that you understand a problem merely because you have seen it solved in class. You must be able to solve the problem and similar problems on your own.

The approach to solving problems should be carefully planned. A systematic plan is especially important when a problem involves several concepts. First, read the problem several times until you are confident you understand what is being asked. Look for any key words that will help you interpret the problem and perhaps allow you to make certain assumptions. Your ability to interpret a question properly is an integral part of problem solving. Second, you should acquire the habit of writing down the information given in a problem and those quantities that need to be found; for example, you might construct a table listing both the quantities given and the quantities to be found. This procedure is sometimes used in the worked examples of the textbook. Finally, after you have decided on the method you believe is appropriate for a given problem, proceed with your solution. The General Problem-Solving Strategy will guide you through complex problems. If you follow the steps of this procedure *(Conceptualize, Categorize, Analyze, Finalize)*, you will find it easier to come up with a solution and gain more from your efforts. This Strategy, located at the end of Chapter 2, is used in all worked examples in the remaining chapters so that you can learn how to apply it. Specific problem-solving strategies for certain types of situations are included in the text and appear with a blue heading. These specific strategies follow the outline of the General Problem-Solving Strategy.

Often, students fail to recognize the limitations of certain equations or physical laws in a particular situation. It is very important that you understand and remember the assumptions that underlie a particular theory or formalism. For example, certain equations in kinematics apply only to a particle moving with constant acceleration. These equations are not valid for describing motion whose acceleration is not constant such as the motion of an object connected to a spring or the motion of an object through a fluid. Study the Analysis Models for Problem-Solving in the chapter summaries carefully so that you know how each model can be applied to a specific situation.

Experiments

Physics is a science based on experimental observations. Therefore, we recommend that you try to supplement the text by performing various types of "hands-on" experiments either at home or in the laboratory. These experiments can be used to test ideas and models discussed in class or in the textbook. For example, the common Slinky toy is excellent for studying traveling waves, a ball swinging on the end of a long string can be used to investigate pendulum motion, various masses attached to the end of a vertical spring or rubber band can be used to determine their elastic nature, an old pair of Polaroid sunglasses and some discarded lenses and a magnifying glass are the components of various experiments in optics, and an approximate measure of the free-fall acceleration can be determined simply by measuring with a stopwatch the time it takes for a ball to drop from a known height. The list of such experiments is endless. When physical models are not available, be imaginative and try to develop models of your own.

New Media

We strongly encourage you to use the **ThomsonNOW** Web-based learning system that accompanies this textbook. It is far easier to understand physics if you see it in action, and these new materials will enable you to become a part of that action. **ThomsonNOW** media described in the Preface and accessed at **www.thomsonedu.com/physics/serway** feature a three-step learning process consisting of a pre-test, a personalized learning plan, and a post-test.

It is our sincere hope that you will find physics an exciting and enjoyable experience and that you will benefit from this experience, regardless of your chosen profession. Welcome to the exciting world of physics!

> *The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.*
>
> **—Henri Poincaré**

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Mechanics

Physics, the most fundamental physical science, is concerned with the fundamental principles of the Universe. It is the foundation upon which the other sciences—astronomy, biology, chemistry, and geology—are based. The beauty of physics lies in the simplicity of its fundamental principles and in the manner in which just a small number of concepts and models can alter and expand our view of the world around us.

The study of physics can be divided into six main areas:

1. *classical mechanics,* concerning the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light;
2. *relativity,* a theory describing objects moving at any speed, even speeds approaching the speed of light;
3. *thermodynamics,* dealing with heat, work, temperature, and the statistical behavior of systems with large numbers of particles;
4. *electromagnetism,* concerned with electricity, magnetism, and electromagnetic fields;
5. *optics,* the study of the behavior of light and its interaction with materials;
6. *quantum mechanics,* a collection of theories connecting the behavior of matter at the submicroscopic level to macroscopic observations.

The disciplines of mechanics and electromagnetism are basic to all other branches of classical physics (developed before 1900) and modern physics (c. 1900–present). The first part of this textbook deals with classical mechanics, sometimes referred to as *Newtonian mechanics* or simply *mechanics*. Many principles and models used to understand mechanical systems retain their importance in the theories of other areas of physics and can later be used to describe many natural phenomena. Therefore, classical mechanics is of vital importance to students from all disciplines.

Image not available due to copyright restrictions

A close-up of the gears inside a mechanical clock. Complicated timepieces have been built for centuries in an effort to measure time accurately. Time is one of the basic quantities that we use in studying the motion of objects. (© Photographer's Choice/Getty Images)

1 Physics and Measurement

ThomsonNOW™ Throughout this chapter and others, there are opportunities for online self-study, linking you to interactive tutorials based on your level of understanding. Sign in at **www.thomsonedu.com** to view tutorials and simulations, develop problem-solving skills, and test your knowledge with these interactive resources.

WebAssign Interactive content from this chapter and others may be assigned online in WebAssign.

Like all other sciences, physics is based on experimental observations and quanti-tative measurements. The main objectives of physics are to identify a limited number of fundamental laws that govern natural phenomena and use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When there is a discrepancy between the prediction of a theory and experimental results, new or modified theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed later by Albert Einstein (1879–1955) gives the same results as Newton's laws at low speeds but also correctly describes the motion of objects at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion than that formed from Newton's laws.

Classical physics includes the principles of classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics were provided by Newton, who was also one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electromagnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments in these disciplines was either too crude or unavailable.

A major revolution in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's special theory of relativity not only correctly describes the motion of objects moving at speeds comparable to the speed of light; it also completely modifies the traditional concepts of space, time, and energy. The theory also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level. Many practical devices have been developed using the principles of quantum mechanics.

Scientists continually work at improving our understanding of fundamental laws. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians, such as unmanned planetary explorations and manned moon landings, microcircuitry and high-speed computers, sophisticated imaging techniques used in scientific research and medicine, and several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1.1 Standards of Length, Mass, and Time

To describe natural phenomena, we must make measurements of various aspects of nature. Each measurement is associated with a physical quantity, such as the length of an object.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Whatever is chosen as a standard must be readily accessible and must possess some property that can be measured reliably. Measurement standards used by different people in different places—throughout the Universe—must yield the same result. In addition, standards used for measurements must not change with time.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its fundamental units of length, mass, and time are the *meter*, *kilogram*, and *second*, respectively. Other standards for SI fundamental units established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. In mechanics, the three fundamental quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

Length

We can identify **length** as the distance between two points in space. In 1120, the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. Neither of these standards is constant in time; when a new king took the throne, length measurements changed! The French standard prevailed until 1799, when the legal standard of length in France became the **meter** (m), defined as one ten-millionth of the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Notice that this value is an Earth-based standard that does not satisfy the requirement that it can be used throughout the universe.

As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. Current requirements of science and technology, however, necessitate more accuracy than that with which the separation between the lines on the bar can be determined. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths[1] of orange-red light emitted from a krypton-86 lamp. In October 1983, however, the meter was redefined as **the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. This definition of the meter is valid throughout the Universe based on our assumption that light is the same everywhere.

Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by, for example, a length of 20 centimeters, a mass of 100 kilograms, or a time interval of 3.2×10^7 seconds.

PITFALL PREVENTION 1.1
Reasonable Values

Generating intuition about typical values of quantities when solving problems is important because you must think about your end result and determine if it seems reasonable. If you are calculating the mass of a housefly and arrive at a value of 100 kg, this answer is *unreasonable* and there is an error somewhere.

TABLE 1.1
Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (Andromeda)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One light-year	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

[1] We will use the standard international notation for numbers with more than three digits, in which groups of three digits are separated by spaces rather than commas. Therefore, 10 000 is the same as the common American notation of 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

(a)

(b)

Figure 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The primary time standard in the United States is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. The clock will neither gain nor lose a second in 20 million years.

Mass

The SI fundamental unit of **mass,** the **kilogram** (kg), is defined as **the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France**. This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a). Table 1.2 lists approximate values of the masses of various objects.

Time

Before 1960, the standard of **time** was defined in terms of the *mean solar day* for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The fundamental unit of a **second** (s) was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time. Therefore, this motion does not provide a time standard that is constant.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an *atomic clock* (Fig. 1.1b), which measures vibrations of cesium atoms. One second is now defined as **9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.**[2] Approximate values of time intervals are presented in Table 1.3.

TABLE 1.2

Approximate Masses of Various Objects

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

TABLE 1.3

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

[2] *Period* is defined as the time interval needed for one complete vibration.

TABLE 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

In addition to SI, another system of units, the *U.S. customary system,* is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this book, we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the fundamental SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^{3} m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^{3} grams (g), and 1 megavolt (MV) is 10^{6} volts (V).

The variables length, time, and mass are examples of *fundamental quantities.* Most other variables are *derived quantities,* those that can be expressed as a mathematical combination of fundamental quantities. Common examples are *area* (a product of two lengths) and *speed* (a ratio of a length to a time interval).

A table of the letters in the Greek alphabet is provided on the back endpaper of this book. ▶

Another example of a derived quantity is **density.** The density ρ (Greek letter rho) of any substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

In terms of fundamental quantities, density is a ratio of a mass to a product of three lengths. Aluminum, for example, has a density of 2.70×10^{3} kg/m^3, and iron has a density of 7.86×10^{3} kg/m^3. An extreme difference in density can be imagined by thinking about holding a 10-centimeter (cm) cube of Styrofoam in one hand and a 10-cm cube of lead in the other. See Table 14.1 in Chapter 14 for densities of several materials.

Quick Quiz 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) The aluminum cam is larger. (b) The iron cam is larger. (c) Both cams are the same size.

1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a **model** for a physical system that is related to the phenomenon. For example, we cannot interact directly with atoms because they are too small. Therefore, we build a mental model of an atom based on a system of a nucleus and one or more electrons outside the nucleus. Once we have identified the physical components of the model, we make predictions about its behavior based on the interactions among the components of the system or the interaction between the system and the environment outside the system.

As an example, consider the behavior of *matter*. A 1-kg cube of solid gold, such as that at the top of Figure 1.2, has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. What if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Such questions can be traced to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They developed a model for matter by speculating that the process ultimately must end when it produces a particle that can no longer be cut. In Greek, *atomos* means "not sliceable." From this Greek term comes our English word *atom.*

The Greek model of the structure of matter was that all ordinary matter consists of atoms, as suggested in the middle of Figure 1.2. Beyond that, no additional structure was specified in the model; atoms acted as small particles that interacted with one another, but internal structure of the atom was not a part of the model.

In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom. This led to the first atomic model that contained internal structure. We shall discuss this model in Chapter 42.

Following the discovery of the nucleus in 1911, an atomic model was developed in which each atom is made up of electrons surrounding a central nucleus. A nucleus of gold is shown in Figure 1.2. This model leads, however, to a new question: Does the nucleus have structure? That is, is the nucleus a single particle or a collection of particles? By the early 1930s, a model evolved that described two basic entities in the nucleus: protons and neutrons. The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element. For instance, the nucleus of a hydrogen atom contains one proton (so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, a second number—**mass number,** defined as the number of protons plus neutrons in a nucleus—characterizes atoms. The atomic number of a specific element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies).

Is that, however, where the process of breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down, strange, charmed, bottom,* and *top*. The up, charmed, and top quarks have electric charges of $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $-\frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark, as shown at the bottom of Figure 1.2 and labeled u and d. This structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

You should develop a process of building models as you study physics. In this study, you will be challenged with many mathematical problems to solve. One of the most important problem-solving techniques is to build a model for the problem: identify a system of physical components for the problem and make predictions of the behavior of the system based on the interactions among its components or the interaction between the system and its surrounding environment.

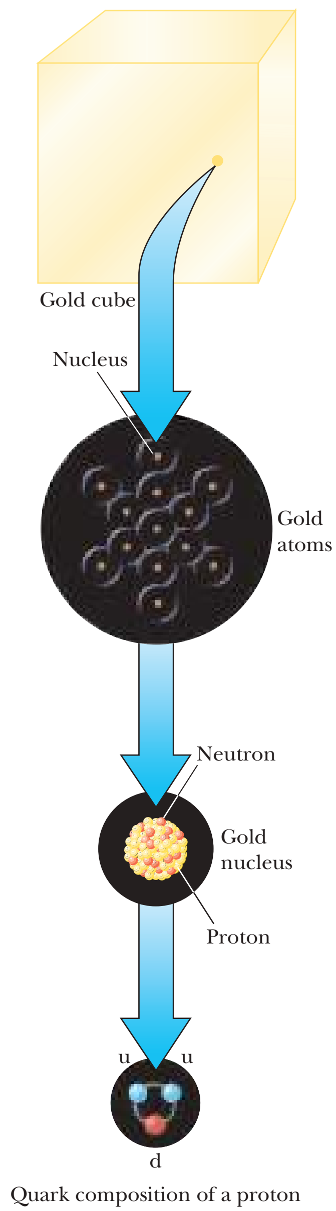

Figure 1.2 Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.

1.3 Dimensional Analysis

The word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is *length.*

TABLE 1.5

Dimensions and Units of Four Derived Quantities

Quantity	Area	Volume	Speed	Acceleration
Dimensions	L^2	L^3	L/T	L/T^2
SI units	m^2	m^3	m/s	m/s^2
U.S. customary units	ft^2	ft^3	ft/s	ft/s^2

PITFALL PREVENTION 1.2

Symbols for Quantities

Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always *t*. Others quantities might have various symbols depending on the usage. Length may be described with symbols such as *x*, *y*, and *z* (for position); *r* (for radius); *a*, *b*, and *c* (for the legs of a right triangle); ℓ (for the length of an object); *d* (for a distance); *h* (for a height); and so forth.

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.[3] We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation, the dimensions of speed are written $[v] = L/T$. As another example, the dimensions of area A are $[A] = L^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.5. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to check a specific equation to see if it matches your expectations. A useful and powerful procedure called **dimensional analysis** can assist in this check because **dimensions can be treated as algebraic quantities.** For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to determine whether an expression has the correct form. Any relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you are interested in an equation for the position x of a car at a time t if the car starts from rest at $x = 0$ and moves with constant acceleration a. The correct expression for this situation is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.5), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side to match that on the left.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = L^1T^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$(L/T^2)^n T^m = L^1T^0 \quad \rightarrow \quad (L^n T^{m-2n}) = L^1T^0$$

[3] The *dimensions* of a quantity will be symbolized by a capitalized, nonitalic letter, such as L or T. The *algebraic symbol* for the quantity itself will be italicized, such as L for the length of an object or t for time.

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$.

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

EXAMPLE 1.1 Analysis of an Equation

Show that the expression $v = at$, where v represents speed, a acceleration, and t an instant of time, is dimensionally correct.

SOLUTION

Identify the dimensions of v from Table 1.5:

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

Identify the dimensions of a from Table 1.5 and multiply by the dimensions of t:

$$[at] = \frac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\cancel{\mathrm{T}} = \frac{\mathrm{L}}{\mathrm{T}}$$

Therefore, $v = at$ is dimensionally correct because we have the same dimensions on both sides. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect*. Try it and see!)

EXAMPLE 1.2 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

SOLUTION

Write an expression for a with a dimensionless constant of proportionality k:

$$a = kr^n v^m$$

Substitute the dimensions of a, r, and v:

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

Equate the exponents of L and T so that the dimensional equation is balanced:

$$n + m = 1 \quad \text{and} \quad m = \boxed{2}$$

Solve the two equations for n:

$$n = \boxed{-1}$$

Write the acceleration expression:

$$a = kr^{-1}v^2 = \boxed{k\frac{v^2}{r}}$$

In Section 4.4 on uniform circular motion, we show that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2.

PITFALL PREVENTION 1.3

Always Include Units

When performing calculations, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

1.4 Conversion of Units

Sometimes you must convert units from one measurement system to another or convert within a system (for example, from kilometers to meters). Equalities between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Like dimensions, units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \cancel{\text{in.}})\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. We must place the unit "inch" in the denominator so that it cancels with the unit in the original quantity. The remaining unit is the centimeter, our desired result.

Quick Quiz 1.3 The distance between two cities is 100 mi. What is the number of kilometers between the two cities? (a) smaller than 100 (b) larger than 100 (c) equal to 100

EXAMPLE 1.3 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit of 75.0 mi/h?

SOLUTION

Convert meters in the speed to miles:

$$(38.0 \cancel{\text{m}}/\text{s})\left(\frac{1 \text{ mi}}{1\,609 \cancel{\text{m}}}\right) = 2.36 \times 10^{-2} \text{ mi/s}$$

Convert seconds to hours:

$$(2.36 \times 10^{-2} \text{ mi}/\cancel{\text{s}})\left(\frac{60 \cancel{\text{s}}}{1 \cancel{\text{min}}}\right)\left(\frac{60 \cancel{\text{min}}}{1 \text{ h}}\right) = 85.0 \text{ mi/h}$$

The driver is indeed exceeding the speed limit and should slow down.

What If? What if the driver were from outside the United States and is familiar with speeds measured in km/h? What is the speed of the car in km/h?

Answer We can convert our final answer to the appropriate units:

$$(85.0 \cancel{\text{mi}}/\text{h})\left(\frac{1.609 \text{ km}}{1 \cancel{\text{mi}}}\right) = 137 \text{ km/h}$$

Figure 1.3 shows an automobile speedometer displaying speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?

Phil Boorman/Getty Images

Figure 1.3 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

1.5 Estimates and Order-of-Magnitude Calculations

Suppose someone asks you the number of bits of data on a typical musical compact disc. In response, it is not generally expected that you would provide the exact number but rather an estimate, which may be expressed in scientific notation. An *order of magnitude* of a number is determined as follows:

1. Express the number in scientific notation, with the multiplier of the power of ten between 1 and 10 and a unit.
2. If the multiplier is less than 3.162 (the square root of ten), the order of magnitude of the number is the power of ten in the scientific notation. If the multiplier is greater than 3.162, the order of magnitude is one larger than the power of ten in the scientific notation.

We use the symbol ~ for "is on the order of." Use the procedure above to verify the orders of magnitude for the following lengths:

$$0.008\,6 \text{ m} \sim 10^{-2} \text{ m} \qquad 0.002\,1 \text{ m} \sim 10^{-3} \text{ m} \qquad 720 \text{ m} \sim 10^{3} \text{ m}$$

Usually, when an order-of-magnitude estimate is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, its value increases by a factor of about $10^3 = 1\,000$.

Inaccuracies caused by guessing too low for one number are often canceled by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work because you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a *small* scrap of paper and are often called "back-of-the-envelope calculations."

EXAMPLE 1.4 Breaths in a Lifetime

Estimate the number of breaths taken during an average human life span.

SOLUTION

We start by guessing that the typical human life span is about 70 years. Think about the average number of breaths that a person takes in 1 min. This number varies depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate. (This estimate is certainly closer to the true average value than 1 breath per minute or 100 breaths per minute.)

Find the approximate number of minutes in a year:

$$1\ \cancel{\text{yr}}\left(\frac{400\ \cancel{\text{days}}}{1\ \cancel{\text{yr}}}\right)\left(\frac{25\ \cancel{\text{h}}}{1\ \cancel{\text{day}}}\right)\left(\frac{60 \text{ min}}{1\ \cancel{\text{h}}}\right) = 6 \times 10^5 \text{ min}$$

Find the approximate number of minutes in a 70-year lifetime:

$$\begin{aligned}\text{number of minutes} &= (70 \text{ yr})(6 \times 10^5 \text{ min/yr})\\ &= 4 \times 10^7 \text{ min}\end{aligned}$$

Find the approximate number of breaths in a lifetime:

$$\begin{aligned}\text{number of breaths} &= (10 \text{ breaths/min})(4 \times 10^7 \text{ min})\\ &= 4 \times 10^8 \text{ breaths}\end{aligned}$$

Therefore, a person takes on the order of 10^9 breaths in a lifetime. Notice how much simpler it is in the first calculation above to multiply 400×25 than it is to work with the more accurate 365×24.

What If? What if the average life span were estimated as 80 years instead of 70? Would that change our final estimate?

Answer We could claim that $(80 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 5 \times 10^7$ min, so our final estimate should be 5×10^8 breaths. This answer is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged.

1.6 Significant Figures

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed. The number of **significant figures** in a measurement can be used to express something about the uncertainty.

As an example of significant figures, suppose we are asked to measure the area of a compact disc using a meter stick as a measuring instrument. Let us assume the accuracy to which we can measure the radius of the disc is ± 0.1 cm. Because of the uncertainty of ± 0.1 cm, if the radius is measured to be 6.0 cm, we can claim only that its radius lies somewhere between 5.9 cm and 6.1 cm. In this case, we say that the measured value of 6.0 cm has two significant figures. Note that **the significant figures include the first estimated digit.** Therefore, we could write the radius as (6.0 ± 0.1) cm.

Now we find the area of the disc by using the equation for the area of a circle. If we were to claim the area is $A = \pi r^2 = \pi(6.0 \text{ cm})^2 = 113 \text{ cm}^2$, our answer would be unjustifiable because it contains three significant figures, which is greater than the number of significant figures in the radius. A good rule of thumb to use in determining the number of significant figures that can be claimed in a multiplication or a division is as follows:

> When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to division.

Applying this rule to the area of the compact disc, we see that the answer for the area can have only two significant figures because our measured radius has only two significant figures. Therefore, all we can claim is that the area is $1.1 \times 10^2 \text{ cm}^2$.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Therefore, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. The same rule holds for numbers less than 1, so 2.3×10^{-4} has two significant figures (and therefore could be written 0.000 23) and 2.30×10^{-4} has three significant figures (also written 0.000 230).

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report:

> When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

For example, if we wish to compute 123 + 5.35, the answer is 128 and not 128.35. If we compute the sum 1.000 1 + 0.000 3 = 1.000 4, the result has five significant figures even though one of the terms in the sum, 0.000 3, has only one significant figure. Likewise, if we perform the subtraction 1.002 − 0.998 = 0.004, the result

PITFALL PREVENTION 1.4

Read Carefully

Notice that the rule for addition and subtraction is different from that for multiplication and division. For addition and subtraction, the important consideration is the number of *decimal places,* not the number of *significant figures.*

has only one significant figure even though one term has four significant figures and the other has three.

In this book, most of the numerical examples and end-of-chapter problems will yield answers having three significant figures. When carrying out order-of-magnitude calculations, we shall typically work with a single significant figure.

If the number of significant figures in the result of an addition or subtraction must be reduced, there is a general rule for rounding numbers: the last digit retained is increased by 1 if the last digit dropped is greater than 5. If the last digit dropped is less than 5, the last digit retained remains as it is. If the last digit dropped is equal to 5, the remaining digit should be rounded to the nearest even number. (This rule helps avoid accumulation of errors in long arithmetic processes.)

A technique for avoiding error accumulation is to delay rounding of numbers in a long calculation until you have the final result. Wait until you are ready to copy the final answer from your calculator before rounding to the correct number of significant figures.

EXAMPLE 1.5 Installing a Carpet

A carpet is to be installed in a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

SOLUTION

If you multiply 12.71 m by 3.46 m on your calculator, you will see an answer of 43.976 6 m^2. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in your answer as are present in the measured quantity having the lowest number of significant figures. In this example, the lowest number of significant figures is three in 3.46 m, so we should express our final answer as 44.0 m^2.

Summary

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DEFINITIONS

The three fundamental physical quantities of mechanics are **length, mass,** and **time,** which in the SI system have the units **meter** (m), **kilogram** (kg), and **second** (s), respectively. These fundamental quantities cannot be defined in terms of more basic quantities.

The **density** of a substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \qquad \textbf{(1.1)}$$

CONCEPTS AND PRINCIPLES

The method of **dimensional analysis** is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and performing order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to specify an exact solution completely.

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of **significant figures.** When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to division. When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

Questions

☐ denotes answer available in *Student Solutions Manual/Study Guide;* **O** denotes objective question

1. Suppose the three fundamental standards of the metric system were length, *density,* and time rather than length, *mass,* and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure the standard of density is as accurate as possible?

2. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m (b) 5×10^{-5} s (c) 72×10^{2} g

3. **O** Rank the following five quantities in order from the largest to the smallest: (a) 0.032 kg (b) 15 g (c) 2.7×10^{5} mg (d) 4.1×10^{-8} Gg (e) 2.7×10^{8} μg. If two of the masses are equal, give them equal rank in your list.

4. **O** If an equation is dimensionally correct, does that mean that the equation must be true? If an equation is not dimensionally correct, does that mean that the equation cannot be true?

5. **O** Answer each question yes or no. Must two quantities have the same dimensions (a) if you are adding them? (b) If you are multiplying them? (c) If you are subtracting them? (d) If you are dividing them? (e) If you are using one quantity as an exponent in raising the other to a power? (f) If you are equating them?

6. **O** The price of gasoline at a particular station is 1.3 euros per liter. An American student can use 41 euros to buy gasoline. Knowing that 4 quarts make a gallon and that 1 liter is close to 1 quart, she quickly reasons that she can buy (choose one) (a) less than 1 gallon of gasoline, (b) about 5 gallons of gasoline, (c) about 8 gallons of gasoline, (d) more than 10 gallons of gasoline.

7. **O** One student uses a meterstick to measure the thickness of a textbook and finds it to be 4.3 cm ± 0.1 cm. Other students measure the thickness with vernier calipers and obtain (a) 4.32 cm ± 0.01 cm, (b) 4.31 cm ± 0.01 cm, (c) 4.24 cm ± 0.01 cm, and (d) 4.43 cm ± 0.01 cm. Which of these four measurements, if any, agree with that obtained by the first student?

8. **O** A calculator displays a result as $1.365\,248\,0 \times 10^{7}$ kg. The estimated uncertainty in the result is ±2%. How many digits should be included as significant when the result is written down? Choose one: (a) zero (b) one (c) two (d) three (e) four (f) five (g) the number cannot be determined

Problems

WebAssign The Problems from this chapter may be assigned online in WebAssign.

ThomsonNOW™ Sign in at **www.thomsonedu.com** and go to ThomsonNOW to assess your understanding of this chapter's topics with additional quizzing and conceptual questions.

1, 2, 3 denotes straightforward, intermediate, challenging; ☐ denotes full solution available in *Student Solutions Manual/Study Guide;* ▲ denotes coached solution with hints available at **www.thomsonedu.com;** ▭ denotes developing symbolic reasoning; ● denotes asking for qualitative reasoning; 🖥 denotes computer useful in solving problem

Section 1.1 Standards of Length, Mass, and Time

Note: Consult the endpapers, appendices, and tables in the text whenever necessary in solving problems. For this chapter, Table 14.1 and Appendix B.3 may be particularly useful. Answers to odd-numbered problems appear in the back of the book.

1. ● Use information on the endpapers of this book to calculate the average density of the Earth. Where does the value fit among those listed in Table 14.1? Look up the density of a typical surface rock, such as granite, in another source and compare the density of the Earth to it.

2. The standard kilogram is a platinum-iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?

3. A major motor company displays a die-cast model of its first automobile, made from 9.35 kg of iron. To celebrate its one-hundredth year in business, a worker will recast the model in gold from the original dies. What mass of gold is needed to make the new model?

4. ● A proton, which is the nucleus of a hydrogen atom, can be modeled as a sphere with a diameter of 2.4 fm and a mass of 1.67×10^{-27} kg. Determine the density of the proton and state how it compares with the density of lead, which is given in Table 14.1.

5. Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the second sphere is five times greater. Find the radius of the second sphere.

Section 1.2 Matter and Model Building

6. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.6a. The atoms reside at the corners of cubes of side L = 0.200 nm. One piece of evidence for the regular

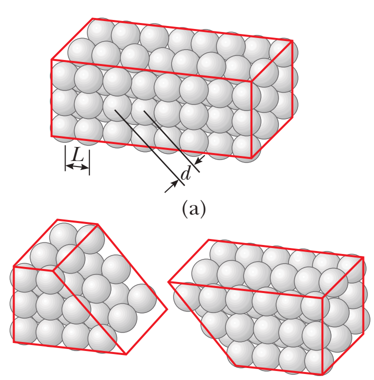


Figure P1.6

arrangement of atoms comes from the flat surfaces along which a crystal separates, or cleaves, when it is broken. Suppose this crystal cleaves along a face diagonal as shown in Figure P1.6b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

Section 1.3 Dimensional Analysis

7. Which of the following equations are dimensionally correct? (a) $v_f = v_i + ax$ (b) $y = (2\text{ m}) \cos (kx)$, where $k = 2\text{ m}^{-1}$

8. Figure P1.8 shows a *frustum of a cone.* Of the following mensuration (geometrical) expressions, which describes **(i)** the total circumference of the flat circular faces, **(ii)** the volume, and **(iii)** the area of the curved surface? (a) $\pi(r_1 + r_2)[h^2 + (r_2 - r_1)^2]^{1/2}$, (b) $2\pi(r_1 + r_2)$ (c) $\pi h(r_1^2 + r_1 r_2 + r_2^2)/3$

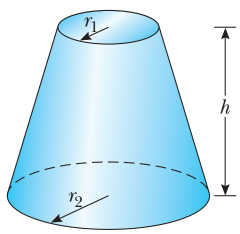


Figure P1.8

9. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the magnitude of the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units $\text{kg} \cdot \text{m/s}^2$. What are the SI units of the proportionality constant G?

Section 1.4 Conversion of Units

10. Suppose your hair grows at the rate $^1/_{32}$ in. per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

11. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in square meters.

12. An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m³. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

13. ● A room measures 3.8 m by 3.6 m, and its ceiling is 2.5 m high. Is it possible to completely wallpaper the walls of this room with the pages of this book? Explain your answer.

14. Assume it takes 7.00 min to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time interval, in hours, required to fill a 1.00-m³ volume at the same rate. (1 U.S. gal = 231 in.³)

15. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm³. From these data, calculate the density of lead in SI units (kg/m³).

16. An ore loader moves 1 200 tons/h from a mine to the surface. Convert this rate to pounds per second, using 1 ton = 2 000 lb.

17. At the time of this book's printing, the U.S. national debt is about \$8 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If eight trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the planet? Take the radius of the Earth at the equator to be 6 378 km. *Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.

18. A pyramid has a height of 481 ft, and its base covers an area of 13.0 acres (Fig. P1.18). The volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. Find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft²)

Figure P1.18 Problems 18 and 19.

19. The pyramid described in Problem 18 contains approximately 2 million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

20. A hydrogen atom has a diameter of 1.06×10^{-10} m as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.40×10^{-15} m. (a) For a scale model, represent the diameter of the hydrogen atom by the playing length of an American football field (100 yards = 300 ft) and determine the diameter of the nucleus in millimeters. (b) The atom is how many times larger in volume than its nucleus?

21. ▲ One gallon of paint (volume = 3.78×10^{-3} m³) covers an area of 25.0 m². What is the thickness of the fresh paint on the wall?

22. The mean radius of the Earth is 6.37×10^6 m and that of the Moon is 1.74×10^8 cm. From these data calculate (a) the ratio of the Earth's surface area to that of the Moon and (b) the ratio of the Earth's volume to that of the Moon. Recall that the surface area of a sphere is $4\pi r^2$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

23. ▲ One cubic meter (1.00 m³) of aluminum has a mass of 2.70×10^3 kg, and the same volume of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.

24. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

Section 1.5 Estimates and Order-of-Magnitude Calculations

25. ▲ Find the order of magnitude of the number of table-tennis balls that would fit into a typical-size room (without being crushed). In your solution, state the quantities you measure or estimate and the values you take for them.

26. An automobile tire is rated to last for 50 000 miles. To an order of magnitude, through how many revolutions will it turn? In your solution, state the quantities you measure or estimate and the values you take for them.

27. Compute the order of magnitude of the mass of a bathtub half full of water. Compute the order of magnitude of the mass of a bathtub half full of pennies. In your solution, list the quantities you take as data and the value you measure or estimate for each.

28. ● Suppose Bill Gates offers to give you $1 billion if you can finish counting it out using only one-dollar bills. Should you accept his offer? Explain your answer. Assume you can count one bill every second, and note that you need at least 8 hours a day for sleeping and eating.

29. To an order of magnitude, how many piano tuners are in New York City? Physicist Enrico Fermi was famous for asking questions like this one on oral doctorate qualifying examinations. His own facility in making order-of-magnitude calculations is exemplified in Problem 48 of Chapter 45.

Section 1.6 Significant Figures

Note: Appendix B.8 on propagation of uncertainty may be useful in solving some problems in this section.

30. A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.8 ± 0.1) cm. Calculate the area of the plate, including its uncertainty.

31. How many significant figures are in the following numbers: (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.46×10^{-6} (d) 0.005 3?

32. The radius of a uniform solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

33. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2 (b) the product $0.003\,2 \times 356.3$ (c) the product $5.620 \times \pi$

34. The *tropical year,* the time interval from one vernal equinox to the next vernal equinox, is the basis for our calendar. It contains 365.242 199 days. Find the number of seconds in a tropical year.

Note: The next 11 problems call on mathematical skills that will be useful throughout the course.

35. **Review problem.** A child is surprised that she must pay \$1.36 for a toy marked \$1.25 because of sales tax. What is the effective tax rate on this purchase, expressed as a percentage?

36. ● **Review problem.** A student is supplied with a stack of copy paper, ruler, compass, scissors, and a sensitive balance. He cuts out various shapes in various sizes, calculates their areas, measures their masses, and prepares the graph of Figure P1.36. Consider the fourth experimental point from the top. How far is it from the best-fit straight line? (a) Express your answer as a difference in vertical-axis coordinate. (b) Express your answer as a difference in horizontal-axis coordinate. (c) Express both of the answers to parts (a) and (b) as a percentage. (d) Calculate the slope of the line. (e) State what the graph demonstrates, referring to the shape of the graph and the results of parts (c) and (d). (f) Describe whether this result should be expected theoretically. Describe the physical meaning of the slope.

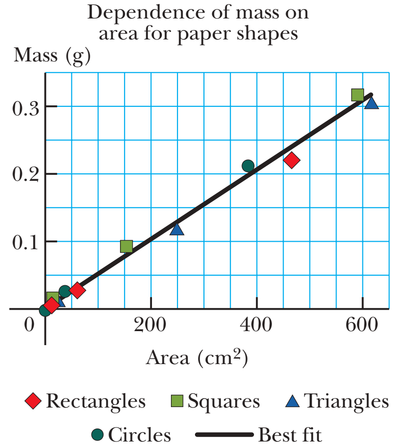


Figure P1.36

37. **Review problem.** A young immigrant works overtime, earning money to buy portable MP3 players to send home as gifts for family members. For each extra shift he works, he has figured out that he can buy one player and two-thirds of another one. An e-mail from his mother informs him that the players are so popular that each of 15 young neighborhood friends wants one. How many more shifts will he have to work?

38. **Review problem.** In a college parking lot, the number of ordinary cars is larger than the number of sport utility vehicles by 94.7%. The difference between the number of cars and the number of SUVs is 18. Find the number of SUVs in the lot.

39. **Review problem.** The ratio of the number of sparrows visiting a bird feeder to the number of more interesting birds is 2.25. On a morning when altogether 91 birds visit the feeder, what is the number of sparrows?

40. **Review problem.** Prove that one solution of the equation

$$2.00x^4 - 3.00x^3 + 5.00x = 70.0$$

is $x = -2.22$.

41. **Review problem.** Find every angle θ between 0 and 360° for which the ratio of $\sin\theta$ to $\cos\theta$ is -3.00.

42. **Review problem.** A highway curve forms a section of a circle. A car goes around the curve. Its dashboard compass shows that the car is initially heading due east. After it travels 840 m, it is heading 35.0° south of east. Find the radius of curvature of its path. *Suggestion:* You may find it useful to learn a geometric theorem stated in Appendix B.3.

43. **Review problem.** For a period of time as an alligator grows, its mass is proportional to the cube of its length. When the alligator's length changes by 15.8%, its mass increases by 17.3 kg. Find its mass at the end of this process.

44. Review problem. From the set of equations

$$p = 3q$$
$$pr = qs$$
$$\tfrac{1}{2}pr^2 + \tfrac{1}{2}qs^2 = \tfrac{1}{2}qt^2$$

involving the unknowns p, q, r, s, and t, find the value of the ratio of t to r.

45. ● **Review problem.** In a particular set of experimental trials, students examine a system described by the equation

$$\frac{Q}{\Delta t} = \frac{k\pi\, d^2\,(T_h - T_c)}{4L}$$

We will see this equation and the various quantities in it in Chapter 20. For experimental control, in these trials all quantities except d and Δt are constant. (a) If d is made three times larger, does the equation predict that Δt will get larger or smaller? By what factor? (b) What pattern of proportionality of Δt to d does the equation predict? (c) To display this proportionality as a straight line on a graph, what quantities should you plot on the horizontal and vertical axes? (d) What expression represents the theoretical slope of this graph?

Additional Problems

46. In a situation in which data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," because we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which factors of change rather than increments are important. We write 500 m ~ 10^3 m because 500 differs from 100 by a factor of 5, whereas it differs from 1 000 by only a factor of 2. We write 437 m ~ 10^3 m and 305 m ~ 10^2 m. What distance differs from 100 m and from 1 000 m by equal factors so that we could equally well choose to represent its order of magnitude either as ~10^2 m or as ~10^3 m?

47. ● A spherical shell has an outside radius of 2.60 cm and an inside radius of a. The shell wall has uniform thickness and is made of a material with density 4.70 g/cm^3. The space inside the shell is filled with a liquid having a density of 1.23 g/cm^3. (a) Find the mass m of the sphere, including its contents, as a function of a. (b) In the answer to part (a), if a is regarded as a variable, for what value of a does m have its maximum possible value? (c) What is this maximum mass? (d) Does the value from part (b) agree with the result of a direct calculation of the mass of a sphere of uniform density? (e) For what value of a does the answer to part (a) have its minimum possible value? (f) What is this minimum mass? (g) Does the value from part (f) agree with the result of a direct calculation of the mass of a uniform sphere? (h) What value of m is halfway between the maximum and minimum possible values? (i) Does this mass agree with the result of part (a) evaluated for a = 2.60 cm/2 = 1.30 cm? (j) Explain whether you should expect agreement in each of parts (d), (g), and (i). (k) **What If?** In part (a), would the answer change if the inner wall of the shell were not concentric with the outer wall?

48. A rod extending between $x = 0$ and $x = 14.0$ cm has uniform cross-sectional area $A = 9.00$ cm^2. It is made from a continuously changing alloy of metals so that along its length its density changes steadily from 2.70 g/cm^3 to 19.3 g/cm^3. (a) Identify the constants B and C required in the expression $\rho = B + Cx$ to describe the variable density. (b) The mass of the rod is given by

$$m = \int_{\text{all material}} \rho\, dV = \int_{\text{all } x} \rho A\, dx = \int_0^{14\text{ cm}} (B + Cx)\,(9.00\text{ cm}^2)\, dx$$

Carry out the integration to find the mass of the rod.

49. ☐ The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 light-years (ly). The distance to Andromeda, which is the spiral galaxy nearest to the Milky Way, is about 2.0 million ly. If a scale model represents the Milky Way and Andromeda galaxies as dinner plates 25 cm in diameter, determine the distance between the centers of the two plates.

50. ● Air is blown into a spherical balloon so that, when its radius is 6.50 cm, its radius is increasing at the rate 0.900 cm/s. (a) Find the rate at which the volume of the balloon is increasing. (b) If this volume flow rate of air entering the balloon is constant, at what rate will the radius be increasing when the radius is 13.0 cm? (c) Explain physically why the answer to part (b) is larger or smaller than 0.9 cm/s, if it is different.

51. ☐ ▲ The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume in millions of cubic feet and t is the time in months. Express this equation in units of cubic feet and seconds. Assign proper units to the coefficients. Assume a month is 30.0 days.

52. In physics it is important to use mathematical approximations. Demonstrate that for small angles (< 20°),

$$\tan\alpha \approx \sin\alpha \approx \alpha = \frac{\pi\alpha'}{180°}$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan\alpha$ may be approximated by α with an error less than 10.0%.

53. ☐ A high fountain of water is located at the center of a circular pool as shown in Figure P1.53. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

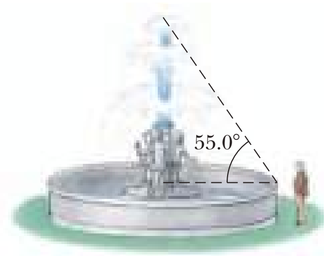


Figure P1.53

54. ● Collectible coins are sometimes plated with gold to enhance their beauty and value. Consider a commemorative quarter-dollar advertised for sale at \$4.98. It has a diameter of 24.1 mm and a thickness of 1.78 mm, and it is completely covered with a layer of pure gold 0.180 μm thick. The volume of the plating is equal to the thickness of the layer times the area to which it is applied. The patterns on the faces of the coin and the grooves on its edge have a negligible effect on its area. Assume the price of gold is \$10.0 per gram. Find the cost of the gold added to the coin. Does the cost of the gold significantly enhance the value of the coin? Explain your answer.

55. One year is nearly $\pi \times 10^7$ s. Find the percentage error in this approximation, where "percentage error" is defined as

$$\text{Percentage error} = \frac{|\text{assumed value} - \text{true value}|}{\text{true value}} \times 100\%$$

56. ● A creature moves at a speed of 5.00 furlongs per fortnight (not a very common unit of speed). Given that 1 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in meters per second. Explain what kind of creature you think it might be.

57. A child loves to watch as you fill a transparent plastic bottle with shampoo. Horizontal cross sections of the bottle are circles with varying diameters because the bottle is much wider in some places than others. You pour in bright green shampoo with constant volume flow rate 16.5 cm^3/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

58. ● The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of these substances. State how your results for aluminum, copper, and iron compare with those given in Table 14.1.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

59. Assume there are 100 million passenger cars in the United States and the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

60. The distance from the Sun to the nearest star is about 4×10^{16} m. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

Answers to Quick Quizzes

1.1 (a). Because the density of aluminum is smaller than that of iron, a larger volume of aluminum than iron is required for a given mass.

1.2 False. Dimensional analysis gives the units of the proportionality constant but provides no information about its numerical value. To determine its numerical value requires either experimental data or geometrical reasoning. For example, in the generation of the equation $x = \frac{1}{2}at^2$, because the factor $\frac{1}{2}$ is dimensionless there is no way to determine it using dimensional analysis.

1.3 (b). Because there are 1.609 km in 1 mi, a larger number of kilometers than miles is required for a given distance.

In drag racing, a driver wants as large an acceleration as possible. In a distance of one-quarter mile, a vehicle reaches speeds of more than 320 mi/h, covering the entire distance in under 5 s. (George Lepp/Stone/Getty)

2 Motion in One Dimension

As a first step in studying classical mechanics, we describe the motion of an object while ignoring the interactions with external agents that might be causing or modifying that motion. This portion of classical mechanics is called *kinematics.* (The word *kinematics* has the same root as *cinema.* Can you see why?) In this chapter, we consider only motion in one dimension, that is, motion of an object along a straight line.

From everyday experience we recognize that motion of an object represents a continuous change in the object's position. In physics, we can categorize motion into three types: translational, rotational, and vibrational. A car traveling on a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion. In this and the next few chapters, we are concerned only with translational motion. (Later in the book we shall discuss rotational and vibrational motions.)

In our study of translational motion, we use what is called the **particle model** and describe the moving object as a *particle* regardless of its size. In general, **a particle is a point-like object, that is, an object that has mass but is of infinitesimal size.** For example, if we wish to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules.

2.1 Position, Velocity, and Speed

Position ▶

The motion of a particle is completely known if the particle's position in space is known at all times. A particle's **position** is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

Consider a car moving back and forth along the x axis as in Active Figure 2.1a. When we begin collecting position data, the car is 30 m to the right of a road sign, which we will use to identify the reference position $x = 0$. We will use the particle model by identifying some point on the car, perhaps the front door handle, as a particle representing the entire car.

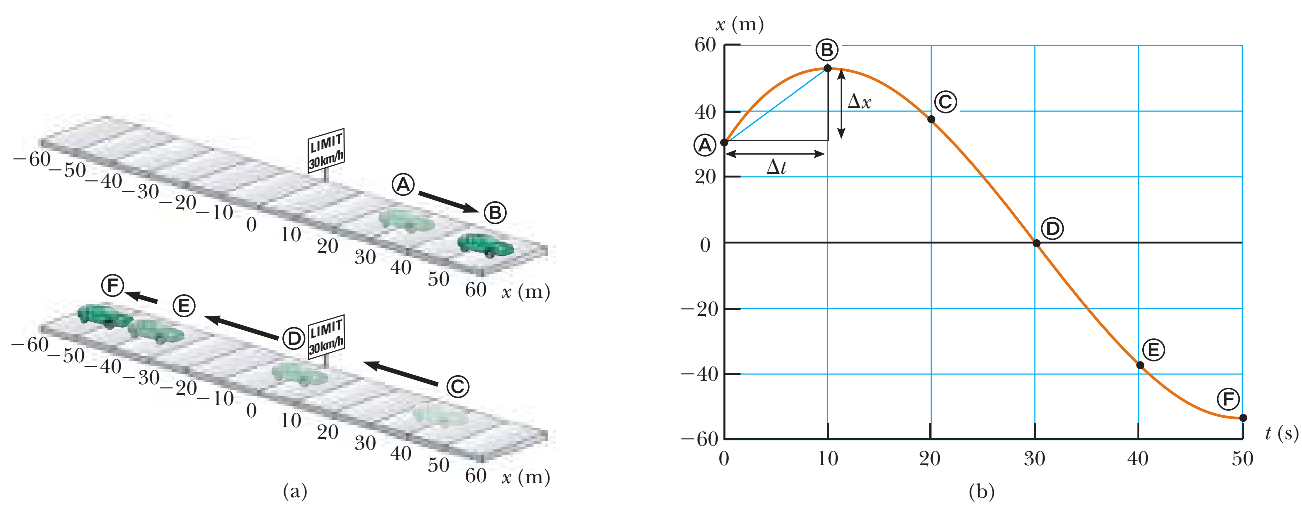
We start our clock, and once every 10 s we note the car's position relative to the sign at $x = 0$. As you can see from Table 2.1, the car moves to the right (which we have defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. After Ⓑ, the position values begin to decrease, suggesting the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is alongside the road sign that we are using to mark our origin of coordinates (see Active Figure 2.1a). It continues moving to the left and is more than 50 m to the left of the sign when we stop recording information after our sixth data point. A graphical representation of this information is presented in Active Figure 2.1b. Such a plot is called a *position–time graph.*

Notice the *alternative representations* of information that we have used for the motion of the car. Active Figure 2.1a is a *pictorial representation,* whereas Active Figure 2.1b is a *graphical representation.* Table 2.1 is a *tabular representation* of the same information. Using an alternative representation is often an excellent strategy for understanding the situation in a given problem. The ultimate goal in many problems is a *mathematical representation,* which can be analyzed to solve for some requested piece of information.

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



ACTIVE FIGURE 2.1

A car moves back and forth along a straight line. Because we are interested only in the car's translational motion, we can model it as a particle. Several representations of the information about the motion of the car can be used. Table 2.1 is a tabular representation of the information. (a) A pictorial representation of the motion of the car. (b) A graphical representation (position–time graph) of the motion of the car.

Sign in at www.thomsonedu.com and go to ThomsonNOW to move each of the six points Ⓐ through Ⓕ and observe the motion of the car in both a pictorial and a graphical representation as it follows a smooth path through the six points.

Given the data in Table 2.1, we can easily determine the change in position of the car for various time intervals. The **displacement** of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f, its displacement is given by

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

◀ Displacement

We use the capital Greek letter delta (Δ) to denote the *change* in a quantity. From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

It is very important to recognize the difference between displacement and distance traveled. **Distance** is the length of a path followed by a particle. Consider, for example, the basketball players in Figure 2.2. If a player runs from his own team's basket down the court to the other team's basket and then returns to his own basket, the *displacement* of the player during this time interval is zero because he ended up at the same point as he started: $x_f = x_i$, so $\Delta x = 0$. During this time interval, however, he moved through a *distance* of twice the length of the basketball court. Distance is always represented as a positive number, whereas displacement can be either positive or negative.

Image not available due to copyright restrictions

Displacement is an example of a vector quantity. Many other physical quantities, including position, velocity, and acceleration, also are vectors. In general, **a vector quantity requires the specification of both direction and magnitude**. By contrast, **a scalar quantity has a numerical value and no direction**. In this chapter, we use positive (+) and negative (−) signs to indicate vector direction. For example, for horizontal motion let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x > 0$, and any object moving to the left undergoes a negative displacement so that $\Delta x < 0$. We shall treat vector quantities in greater detail in Chapter 3.

One very important point has not yet been mentioned. Notice that the data in Table 2.1 result only in the six data points in the graph in Active Figure 2.1b. The smooth curve drawn through the six points in the graph is only a *possibility* of the actual motion of the car. We only have information about six instants of time; we have no idea what happened in between the data points. The smooth curve is a *guess* as to what happened, but keep in mind that it is *only* a guess.

If the smooth curve does represent the actual motion of the car, the graph contains information about the entire 50-s interval during which we watch the car move. It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car covers more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car travels almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it moves less than half that far. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the value of that particular time interval Δt. The result turns out to be a very useful ratio, one that we shall use many times. This ratio has been given a special name: the *average velocity*. **The average velocity $v_{x,\,\text{avg}}$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:**

$$v_{x,\,\text{avg}} \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

◀ Average velocity

where the subscript x indicates motion along the x axis. From this definition we see that average velocity has dimensions of length divided by time (L/T), or meters per second in SI units.

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), Δx is positive and $v_{x,\,\text{avg}} = \Delta x/\Delta t$ is positive. This case corresponds to a particle moving in the positive x direction, that is, toward larger values of x. If the coordinate decreases in time (that is, if $x_f < x_i$), Δx is negative and hence $v_{x,\,\text{avg}}$ is negative. This case corresponds to a particle moving in the negative x direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Active Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$, which is what we have defined as average velocity in Equation 2.2. For example, the line between positions Ⓐ and Ⓑ in Active Figure 2.1b has a slope equal to the average velocity of the car between those two times, $(52 \text{ m} - 30 \text{ m})/(10 \text{ s} - 0) = 2.2 \text{ m/s}$.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs a distance d of more than 40 km and yet ends up at her starting point. Her total displacement is zero, so her average velocity is zero! Nonetheless, we need to be able to quantify how fast she was running. A slightly different ratio accomplishes that for us. The **average speed** v_{avg} of a particle, a scalar quantity, is defined as **the total distance traveled divided by the total time interval required to travel that distance:**

Average speed ►

$$v_{\text{avg}} \equiv \frac{d}{\Delta t} \tag{2.3}$$

PITFALL PREVENTION 2.1

Average Speed and Average Velocity

The magnitude of the average velocity is *not* the average speed. For example, consider the marathon runner discussed before Equation 2.3. The magnitude of her average velocity is zero, but her average speed is clearly not zero.

The SI unit of average speed is the same as the unit of average velocity: meters per second. Unlike average velocity, however, average speed has no direction and is always expressed as a positive number. Notice the clear distinction between the definitions of average velocity and average speed: average velocity (Eq. 2.2) is the *displacement* divided by the time interval, whereas average speed (Eq. 2.3) is the *distance* divided by the time interval.

Knowledge of the average velocity or average speed of a particle does not provide information about the details of the trip. For example, suppose it takes you 45.0 s to travel 100 m down a long, straight hallway toward your departure gate at an airport. At the 100-m mark, you realize you missed the restroom, and you return back 25.0 m along the same hallway, taking 10.0 s to make the return trip. The magnitude of your average *velocity* is $+75.0 \text{ m}/55.0 \text{ s} = +1.36 \text{ m/s}$. The average *speed* for your trip is $125 \text{ m}/55.0 \text{ s} = 2.27 \text{ m/s}$. You may have traveled at various speeds during the walk. Neither average velocity nor average speed provides information about these details.

Quick Quiz 2.1 Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval? (a) a particle moves in the $+x$ direction without reversing (b) a particle moves in the $-x$ direction without reversing (c) a particle moves in the $+x$ direction and then reverses the direction of its motion (d) there are no conditions for which this is true

EXAMPLE 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Active Figure 2.1a between positions Ⓐ and Ⓕ.

SOLUTION

Consult Active Figure 2.1 to form a mental image of the car and its motion. We model the car as a particle. From the position–time graph given in Active Figure 2.1b, notice that $x_Ⓐ = 30$ m at $t_Ⓐ = 0$ s and that $x_Ⓕ = -53$ m at $t_Ⓕ = 50$ s.

Use Equation 2.1 to find the displacement of the car:

$$\Delta x = x_Ⓕ - x_Ⓐ = -53 \text{ m} - 30 \text{ m} = \boxed{-83 \text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Active Figure 2.1a indicates that it is the correct answer.

Use Equation 2.2 to find the average velocity:

$$v_{x,\,\text{avg}} = \frac{x_{Ⓕ} - x_{Ⓐ}}{t_{Ⓕ} - t_{Ⓐ}}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}} = \boxed{-1.7\text{ m/s}}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1 because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Active Figure 2.1b, the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ), for a total of 127 m.

Use Equation 2.3 to find the car's average speed:

$$v_{\text{avg}} = \frac{127\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$

Notice that the average speed is positive, as it must be. Suppose the brown curve in Active Figure 2.1b were different so that between 0 s and 10 s it went from Ⓐ up to 100 m and then came back down to Ⓑ. The average speed of the car would change because the distance is different, but the average velocity would not change.

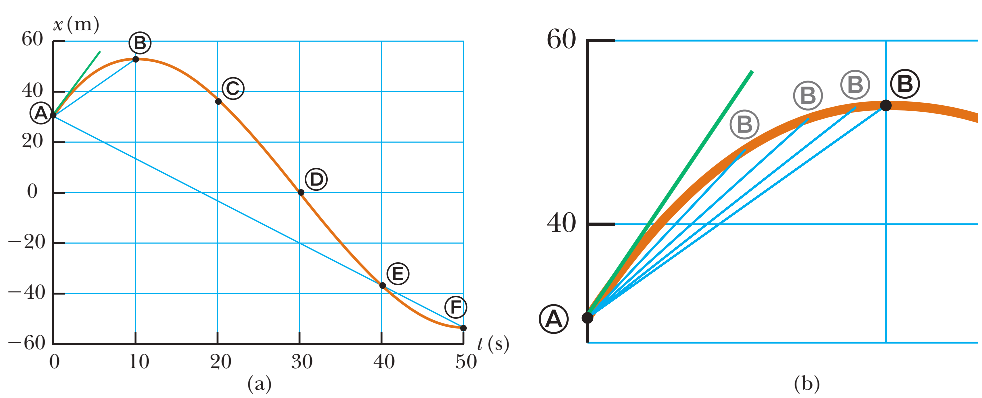
2.2 Instantaneous Velocity and Speed

Often we need to know the velocity of a particle at a particular instant in time rather than the average velocity over a finite time interval. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading—that is, at some specific instant. What does it mean to talk about how quickly something is moving if we "freeze time" and talk only about an individual instant? In the late 1600s, with the invention of calculus, scientists began to understand how to describe an object's motion at any moment in time.

To see how that is done, consider Active Figure 2.3a, which is a reproduction of the graph in Active Figure 2.1b. We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the longer blue line and calculated in Example 2.1). The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the interval from Ⓐ to Ⓑ is more representative of the initial velocity than is the value

PITFALL PREVENTION 2.2
Slopes of Graphs

In any graph of physical data, the *slope* represents the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis. Remember that *a slope has units* (unless both axes have the same units). The units of slope in Active Figure 2.1b and Active Figure 2.3 are meters per second, the units of velocity.



ACTIVE FIGURE 2.3

(a) Graph representing the motion of the car in Active Figure 2.1. (b) An enlargement of the upper-left-hand corner of the graph shows how the blue line between positions Ⓐ and Ⓑ approaches the green tangent line as point Ⓑ is moved closer to point Ⓐ.

Sign in at www.thomsonedu.com and go to ThomsonNOW to move point Ⓑ as suggested in part (b) and observe the blue line approaching the green tangent line.

of the average velocity during the interval from Ⓐ to Ⓕ, which we determined to be negative in Example 2.1. Now let us focus on the short blue line and slide point Ⓑ to the left along the curve, toward point Ⓐ, as in Active Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve, indicated by the green line in Active Figure 2.3b. The slope of this tangent line represents the velocity of the car at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, **the instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:**[1]

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{2.4}$$

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

Instantaneous velocity ▶

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

PITFALL PREVENTION 2.3

Instantaneous Speed and Instantaneous Velocity

In Pitfall Prevention 2.1, we argued that the magnitude of the average velocity is not the average speed. The magnitude of the instantaneous velocity, however, *is* the instantaneous speed. In an infinitesimal time interval, the magnitude of the displacement is equal to the distance traveled by the particle.

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Active Figure 2.3, v_x is positive and the car is moving toward larger values of x. After point Ⓑ, v_x is negative because the slope is negative and the car is moving toward smaller values of x. At point Ⓑ, the slope and the instantaneous velocity are zero and the car is momentarily at rest.

From here on, we use the word *velocity* to designate instantaneous velocity. When we are interested in *average velocity*, we shall always use the adjective *average*.

The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity. As with average speed, instantaneous speed has no direction associated with it. For example, if one particle has an instantaneous velocity of $+25$ m/s along a given line and another particle has an instantaneous velocity of -25 m/s along the same line, both have a speed[2] of 25 m/s.

Quick Quiz 2.2 Are members of the highway patrol more interested in (a) your average speed or (b) your instantaneous speed as you drive?

CONCEPTUAL EXAMPLE 2.2 The Velocity of Different Objects

Consider the following one-dimensional motions: **(A)** a ball thrown directly upward rises to a highest point and falls back into the thrower's hand; **(B)** a race car starts from rest and speeds up to 100 m/s; and **(C)** a spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

SOLUTION

(A) The average velocity for the thrown ball is zero because the ball returns to the starting point; therefore, its displacement is zero. There is one point at which the instantaneous velocity is zero: at the top of the motion.

(B) The car's average velocity cannot be evaluated unambiguously with the information given, but it must have some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity over the entire motion.

(C) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

[1] Notice that the displacement Δx also approaches zero as Δt approaches zero, so the ratio looks like 0/0. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

[2] As with velocity, we drop the adjective for instantaneous speed. "Speed" means instantaneous speed.

EXAMPLE 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

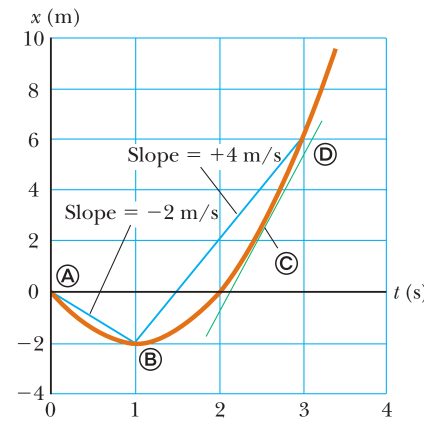


Figure 2.4 (Example 2.3) Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

SOLUTION

From the graph in Figure 2.4, form a mental representation of the motion of the particle. Keep in mind that the particle does not move in a curved path in space such as that shown by the brown curve in the graphical representation. The particle moves only along the x axis in one dimension. At $t = 0$, is it moving to the right or to the left?

During the first time interval, the slope is negative and hence the average velocity is negative. Therefore, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, set $t_i = t_{Ⓐ} = 0$ and $t_f = t_{Ⓑ} = 1$ s and use Equation 2.1 to find the displacement:

$$\Delta x_{Ⓐ\to Ⓑ} = x_f - x_i = x_{Ⓑ} - x_{Ⓐ}$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = \boxed{-2 \text{ m}}$$

For the second time interval ($t = 1$ s to $t = 3$ s), set $t_i = t_{Ⓑ} = 1$ s and $t_f = t_{Ⓓ} = 3$ s:

$$\Delta x_{Ⓑ\to Ⓓ} = x_f - x_i = x_{Ⓓ} - x_{Ⓑ}$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = \boxed{+8 \text{ m}}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

SOLUTION

In the first time interval, use Equation 2.2 with $\Delta t = t_f - t_i = t_{Ⓑ} - t_{Ⓐ} = 1$ s:

$$v_{x,\text{ avg }(Ⓐ\to Ⓑ)} = \frac{\Delta x_{Ⓐ\to Ⓑ}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = \boxed{-2 \text{ m/s}}$$

In the second time interval, $\Delta t = 2$ s:

$$v_{x,\text{ avg }(Ⓑ\to Ⓓ)} = \frac{\Delta x_{Ⓑ\to Ⓓ}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = \boxed{+4 \text{ m/s}}$$

These values are the same as the slopes of the lines joining these points in Figure 2.4.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

SOLUTION

Measure the slope of the green line at $t = 2.5$ s (point Ⓒ) in Figure 2.4:

$$v_x = \boxed{+6 \text{ m/s}}$$

Notice that this instantaneous velocity is on the same order of magnitude as our previous results, that is, a few meters per second. Is that what you would have expected?

[3] Simply to make it easier to read, we write the expression as $x = -4t + 2t^2$ rather than as $x = (-4.00 \text{ m/s})t + (2.00 \text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

2.3 Analysis Models: The Particle Under Constant Velocity

An important technique in the solution to physics problems is the use of *analysis models*. Such models help us analyze common situations in physics problems and guide us toward a solution. An **analysis model** is a problem we have solved before. It is a description of either (1) the behavior of some physical entity or (2) the interaction between that entity and the environment. When you encounter a new problem, you should identify the fundamental details of the problem and attempt to recognize which of the types of problems you have already solved might be used as a model for the new problem. For example, suppose an automobile is moving along a straight freeway at a constant speed. Is it important that it is an automobile? Is it important that it is a freeway? If the answers to both questions are no, we model the automobile as a *particle under constant velocity*, which we will discuss in this section.

This method is somewhat similar to the common practice in the legal profession of finding "legal precedents." If a previously resolved case can be found that is very similar legally to the current one, it is offered as a model and an argument is made in court to link them logically. The finding in the previous case can then be used to sway the finding in the current case. We will do something similar in physics. For a given problem, we search for a "physics precedent," a model with which we are already familiar and that can be applied to the current problem.

We shall generate analysis models based on four fundamental simplification models. The first is the particle model discussed in the introduction to this chapter. We will look at a particle under various behaviors and environmental interactions. Further analysis models are introduced in later chapters based on simplification models of a *system*, a *rigid object*, and a *wave*. Once we have introduced these analysis models, we shall see that they appear again and again in different problem situations.

Let us use Equation 2.2 to build our first analysis model for solving problems. We imagine a particle moving with a constant velocity. The **particle under constant velocity** model can be applied in *any* situation in which an entity that can be modeled as a particle is moving with constant velocity. This situation occurs frequently, so this model is important.

If the velocity of a particle is constant, its instantaneous velocity at any instant during a time interval is the same as the average velocity over the interval. That is, $v_x = v_{x,\,\text{avg}}$. Therefore, Equation 2.2 gives us an equation to be used in the mathematical representation of this situation:

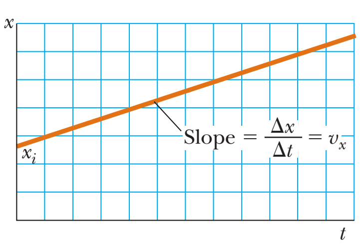


Figure 2.5 Position–time graph for a particle under constant velocity. The value of the constant velocity is the slope of the line.

$$v_x = \frac{\Delta x}{\Delta t} \tag{2.6}$$

Remembering that $\Delta x = x_f - x_i$, we see that $v_x = (x_f - x_i)/\Delta t$, or

$$x_f = x_i + v_x\,\Delta t$$

This equation tells us that the position of the particle is given by the sum of its original position x_i at time $t = 0$ plus the displacement $v_x\,\Delta t$ that occurs during the time interval Δt. In practice, we usually choose the time at the beginning of the interval to be $t_i = 0$ and the time at the end of the interval to be $t_f = t$, so our equation becomes

Position as a function of time ▶

$$x_f = x_i + v_x t \quad (\text{for constant } v_x) \tag{2.7}$$

Equations 2.6 and 2.7 are the primary equations used in the model of a particle under constant velocity. They can be applied to particles or objects that can be modeled as particles.

Figure 2.5 is a graphical representation of the particle under constant velocity. On this position–time graph, the slope of the line representing the motion is constant and equal to the magnitude of the velocity. Equation 2.7, which is the equation of a straight line, is the mathematical representation of the particle under constant velocity model. The slope of the straight line is v_x and the y intercept is x_i in both representations.

EXAMPLE 2.4 **Modeling a Runner as a Particle**

A scientist is studying the biomechanics of the human body. She determines the velocity of an experimental subject while he runs along a straight line at a constant rate. The scientist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

(A) What is the runner's velocity?

SOLUTION

Think about the moving runner. We model the runner as a particle because the size of the runner and the movement of arms and legs are unnecessary details. Because the problem states that the subject runs at a constant rate, we can model him as a particle under constant velocity.

Use Equation 2.6 to find the constant velocity of the runner:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{20 \text{ m} - 0}{4.0 \text{ s}} = \boxed{5.0 \text{ m/s}}$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s has passed?

SOLUTION

Use Equation 2.7 and the velocity found in part (A) to find the position of the particle at time $t = 10$ s:

$$x_f = x_i + v_x t = 0 + (5.0 \text{ m/s})(10 \text{ s}) = \boxed{50 \text{ m}}$$

Notice that this value is more than twice that of the 20-m position at which the stopwatch was stopped. Is this value consistent with the time of 10 s being more than twice the time of 4.0 s?

The mathematical manipulations for the particle under constant velocity stem from Equation 2.6 and its descendent, Equation 2.7. These equations can be used to solve for any variable in the equations that happens to be unknown if the other variables are known. For example, in part (B) of Example 2.4, we find the position when the velocity and the time are known. Similarly, if we know the velocity and the final position, we could use Equation 2.7 to find the time at which the runner is at this position.

A particle under constant velocity moves with a constant speed along a straight line. Now consider a particle moving with a constant speed along a curved path. This situation can be represented with the **particle under constant speed** model. The primary equation for this model is Equation 2.3, with the average speed v_{avg} replaced by the constant speed v:

$$v = \frac{d}{\Delta t} \qquad \textbf{(2.8)}$$

As an example, imagine a particle moving at a constant speed in a circular path. If the speed is 5.00 m/s and the radius of the path is 10.0 m, we can calculate the time interval required to complete one trip around the circle:

$$v = \frac{d}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi \, (10.0 \text{ m})}{5.00 \text{ m/s}} = 12.6 \text{ s}$$

2.4 Acceleration

In Example 2.3, we worked with a common situation in which the velocity of a particle changes while the particle is moving. When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the magnitude of the velocity of a car increases when you step on the gas and decreases when you apply the brakes. Let us see how to quantify acceleration.

Suppose an object that can be modeled as a particle moving along the x axis has an initial velocity v_{xi} at time t_i and a final velocity v_{xf} at time t_f, as in Figure 2.6a. The **average acceleration** $a_{x,\,\text{avg}}$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

Average acceleration ▶

$$a_{x,\,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.9}$$

As with velocity, when the motion being analyzed is one dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of $+2\ m/s^2$. You should form a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every interval of 1 s. If the object starts from rest, you should be able to picture it moving at a velocity of +2 m/s after 1 s, at +4 m/s after 2 s, and so on.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the **instantaneous acceleration** as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in Section 2.2. If we imagine that point Ⓐ is brought closer and closer to point Ⓑ in Figure 2.6a and we take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration at point Ⓑ:

Instantaneous acceleration ▶

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.10}$$

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time**, which by definition is the slope of the velocity–time graph. The slope of the green line in Figure 2.6b is equal to the instantaneous acceleration at point Ⓑ. Therefore, we see that just as the velocity of a moving particle is the slope at a point on the particle's x–t graph, the acceleration of a particle is the slope at a point on the particle's v_x–t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, the acceleration is in the positive x direction; if a_x is negative, the acceleration is in the negative x direction.

For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows. **When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.**

PITFALL PREVENTION 2.4
Negative Acceleration

Keep in mind that *negative acceleration does not necessarily mean that an object is slowing down.* If the acceleration is negative and the velocity is negative, the object is speeding up!

PITFALL PREVENTION 2.5
Deceleration

The word *deceleration* has the common popular connotation of *slowing down.* We will not use this word in this book because it confuses the definition we have given for negative acceleration.

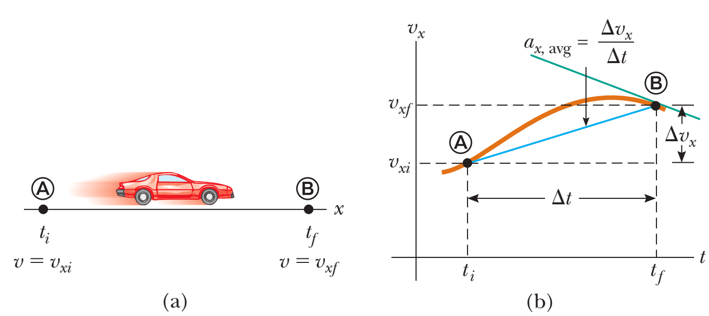


Figure 2.6 (a) A car, modeled as a particle, moving along the x axis from Ⓐ to Ⓑ, has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph (brown) for the particle moving in a straight line. The slope of the blue straight line connecting Ⓐ and Ⓑ is the average acceleration of the car during the time interval $\Delta t = t_f - t_i$. The slope of the green line is the instantaneous acceleration of the car at point Ⓑ.

To help with this discussion of the signs of velocity and acceleration, we can relate the acceleration of an object to the total *force* exerted on the object. In Chapter 5, we formally establish that **force is proportional to acceleration:**

$$F_x \propto a_x \tag{2.11}$$

This proportionality indicates that acceleration is caused by force. Furthermore, force and acceleration are both vectors and the vectors act in the same direction. Therefore, let us think about the signs of velocity and acceleration by imagining a force applied to an object and causing it to accelerate. Let us assume the velocity and acceleration are in the same direction. This situation corresponds to an object that experiences a force acting in the same direction as its velocity. In this case, the object speeds up! Now suppose the velocity and acceleration are in opposite directions. In this situation, the object moves in some direction and experiences a force acting in the opposite direction. Therefore, the object slows down! It is very useful to equate the direction of the acceleration to the direction of a force, because it is easier from our everyday experience to think about what effect a force will have on an object than to think only in terms of the direction of the acceleration.

Quick Quiz 2.3 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? (a) eastward (b) westward (c) neither eastward nor westward

From now on we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average*. Because $v_x = dx/dt$, the acceleration can also be written as

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.12}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of x with respect to time.

Figure 2.7 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.7a where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time $t_{Ⓐ}$, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time $t_{Ⓑ}$, when the velocity is a maximum (that is, when the slope of the v_x–t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time $t_{Ⓒ}$.

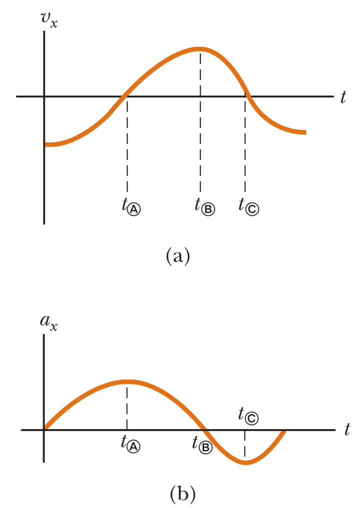


Figure 2.7 The instantaneous acceleration can be obtained from the velocity–time graph (a). At each instant, the acceleration in the graph of a_x versus t (b) equals the slope of the line tangent to the curve of v_x versus t (a).

Quick Quiz 2.4 Make a velocity–time graph for the car in Active Figure 2.1a. The speed limit posted on the road sign is 30 km/h. True or False? The car exceeds the speed limit at some time within the time interval 0 − 50 s.

CONCEPTUAL EXAMPLE 2.5 Graphical Relationships Between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.8a (page 30). Graph the velocity versus time and the acceleration versus time for the object.

SOLUTION

The velocity at any instant is the slope of the tangent to the x–t graph at that instant. Between $t = 0$ and $t = t_{Ⓐ}$, the slope of the x–t graph increases uniformly, so the velocity increases linearly as shown in Figure 2.8b. Between $t_{Ⓐ}$ and $t_{Ⓑ}$, the slope of the x–t graph is constant, so the velocity remains constant. Between $t_{Ⓑ}$ and $t_{Ⓓ}$, the slope of the x–t graph decreases, so the value of the velocity in the v_x–t graph decreases. At $t_{Ⓓ}$, the slope of the x–t graph is zero, so the velocity is zero at that instant. Between $t_{Ⓓ}$ and $t_{Ⓔ}$, the slope of the x–t graph and therefore the velocity are negative and decrease uniformly in this interval. In the interval $t_{Ⓔ}$ to $t_{Ⓕ}$, the slope of the x–t graph is still negative, and at $t_{Ⓕ}$ it goes to zero. Finally, after $t_{Ⓕ}$, the slope of the x–t graph is zero, meaning that the object is at rest for $t > t_{Ⓕ}$.

The acceleration at any instant is the slope of the tangent to the v_x–t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.8c. The acceleration is constant and positive between 0 and $t_{Ⓐ}$, where the slope of the v_x–t graph is positive. It is zero between $t_{Ⓐ}$ and $t_{Ⓑ}$ and for $t > t_{Ⓕ}$ because the slope of the v_x–t graph is zero at these times. It is negative between $t_{Ⓑ}$ and $t_{Ⓔ}$ because the slope of the v_x–t graph is negative during this interval. Between $t_{Ⓔ}$ and $t_{Ⓕ}$, the acceleration is positive like it is between 0 and $t_{Ⓐ}$, but higher in value because the slope of the v_x–t graph is steeper.

Notice that the sudden changes in acceleration shown in Figure 2.8c are unphysical. Such instantaneous changes cannot occur in reality.

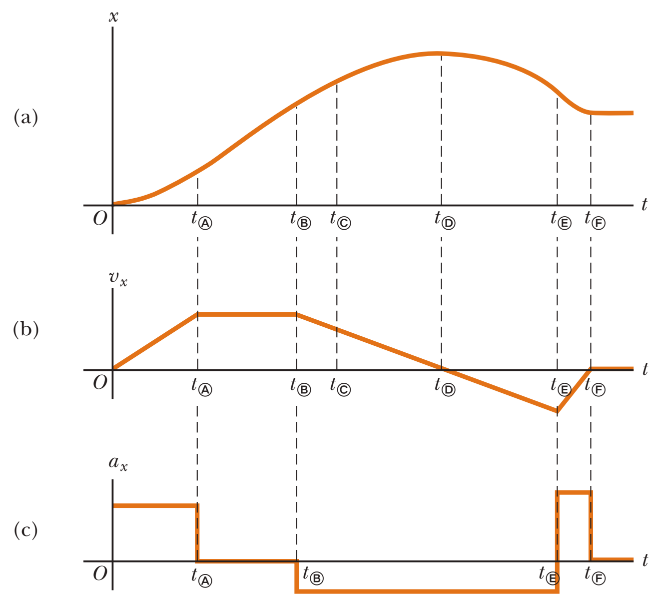


Figure 2.8 (Example 2.5) (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

EXAMPLE 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

SOLUTION

Think about what the particle is doing from the mathematical representation. Is it moving at $t = 0$? In which direction? Does it speed up or slow down? Figure 2.9 is a v_x–t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x–t curve is negative, we expect the acceleration to be negative.

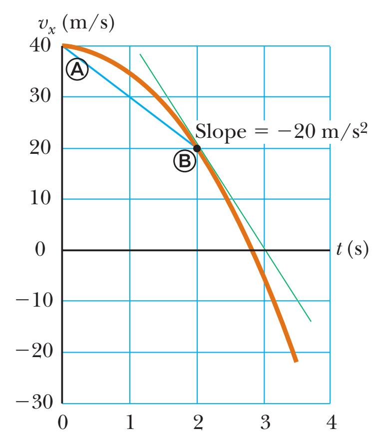


Figure 2.9 (Example 2.6) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

Find the velocities at $t_i = t_{Ⓐ} = 0$ and $t_f = t_{Ⓑ} = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xⒶ} = (40 - 5t_{Ⓐ}^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

$$v_{xⒷ} = (40 - 5t_{Ⓑ}^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

Find the average acceleration in the specified time interval $\Delta t = t_{Ⓑ} - t_{Ⓐ} = 2.0$ s:

$$a_{x,\text{ avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xⒷ} - v_{xⒶ}}{t_{Ⓑ} - t_{Ⓐ}} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}}$$

$$= \boxed{-10\text{ m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, represented by the slope of the line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

SOLUTION

Knowing that the initial velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s, find the velocity at any later time $t + \Delta t$:

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Find the change in velocity over the time interval Δt:

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\text{ m/s}$$

To find the acceleration at any time t, divide this expression by Δt and take the limit of the result as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t\text{ m/s}^2$$

Substitute $t = 2.0$ s:

$$a_x = (-10)(2.0)\text{ m/s}^2 = \boxed{-20\text{ m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.

Notice that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.9 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Notice also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.6.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. If you are familiar with calculus, you should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t, such as in the expression

$$x = At^n$$

where A and n are constants. (This expression is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

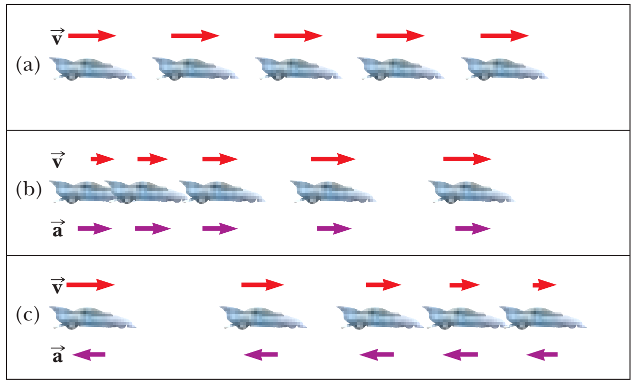
Applying this rule to Example 2.5, in which $v_x = 40 - 5t^2$, we quickly find that the acceleration is $a_x = dv_x/dt = -10t$.

2.5 Motion Diagrams

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. In forming a mental representation of a moving object, it is sometimes useful to use a pictorial representation called a *motion diagram* to describe the velocity and acceleration while an object is in motion.

A motion diagram can be formed by imagining a *stroboscopic* photograph of a moving object, which shows several images of the object taken as the strobe light flashes at a constant rate. Active Figure 2.10 (page 32) represents three sets of strobe photographs of cars moving along a straight roadway in a single direction, from left to right. The time intervals between flashes of the stroboscope are equal in each part of the diagram. So as to not confuse the two vector quantities, we use red for velocity vectors and violet for acceleration vectors in Active Figure 2.10. The vectors are shown at several instants during the motion of the object. Let us describe the motion of the car in each diagram.

In Active Figure 2.10a, the images of the car are equally spaced, showing us that the car moves through the same displacement in each time interval. This equal spacing is consistent with the car moving with *constant positive velocity* and *zero acceleration.*



ACTIVE FIGURE 2.10

(a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration is indicated by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

Sign in at www.thomsonedu.com and go to ThomsonNOW to select the constant acceleration and initial velocity of the car and observe pictorial and graphical representations of its motion.

We could model the car as a particle and describe it with the particle under constant velocity model.

In Active Figure 2.10b, the images become farther apart as time progresses. In this case, the velocity vector increases in length with time because the car's displacement between adjacent positions increases in time. These features suggest that the car is moving with a *positive velocity* and a *positive acceleration*. The velocity and acceleration are in the same direction. In terms of our earlier force discussion, imagine a force pulling on the car in the same direction it is moving: it speeds up.

In Active Figure 2.10c, we can tell that the car slows as it moves to the right because its displacement between adjacent images decreases with time. This case suggests that the car moves to the right with a negative acceleration. The length of the velocity vector decreases in time and eventually reaches zero. From this diagram we see that the acceleration and velocity vectors are *not* in the same direction. The car is moving with a *positive velocity*, but with a *negative acceleration*. (This type of motion is exhibited by a car that skids to a stop after applying its brakes.) The velocity and acceleration are in opposite directions. In terms of our earlier force discussion, imagine a force pulling on the car opposite to the direction it is moving: it slows down.

The violet acceleration vectors in parts (b) and (c) of Figure 2.10 are all of the same length. Therefore, these diagrams represent motion of a *particle under constant acceleration*. This important analysis model will be discussed in the next section.

Quick Quiz 2.5 Which one of the following statements is true? (a) If a car is traveling eastward, its acceleration must be eastward. (b) If a car is slowing down, its acceleration must be negative. (c) A particle with constant acceleration can never stop and stay stopped.

2.6 The Particle Under Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. A very common and simple type of one-dimensional motion, however, is that in which the acceleration is constant. In such a case, the average acceleration $a_{x,\text{ avg}}$ over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion. This situation occurs often enough that we identify it as an analysis model: the **particle under constant acceleration**. In the discussion that follows, we generate several equations that describe the motion of a particle for this model.

If we replace $a_{x,\text{ avg}}$ by a_x in Equation 2.9 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \quad \text{(for constant } a_x\text{)} \tag{2.13}$$

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity v_{xi} and its (constant) acceleration a_x. A velocity–time graph for this constant-acceleration motion is shown in Active Figure 2.11b. The graph is a straight line, the slope of which is the acceleration a_x; the (constant) slope is consistent with $a_x = dv_x/dt$ being a constant. Notice that the slope is positive, which indicates a positive acceleration. If the acceleration were negative, the slope of the line in Active Figure 2.11b would be negative. When the acceleration is constant, the graph of acceleration versus time (Active Fig. 2.11c) is a straight line having a slope of zero.

Because velocity at constant acceleration varies linearly in time according to Equation 2.13, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$v_{x,\text{ avg}} = \frac{v_{xi} + v_{xf}}{2} \quad \text{(for constant } a_x\text{)} \tag{2.14}$$

Notice that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.14 to obtain the position of an object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$ and recognizing that $\Delta t = t_f - t_i = t - 0 = t$, we find that

$$x_f - x_i = v_{x,\text{ avg}}\, t = \tfrac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \quad \text{(for constant } a_x\text{)} \tag{2.15}$$

◀ Position as a function of velocity and time

This equation provides the final position of the particle at time t in terms of the initial and final velocities.

We can obtain another useful expression for the position of a particle under constant acceleration by substituting Equation 2.13 into Equation 2.15:

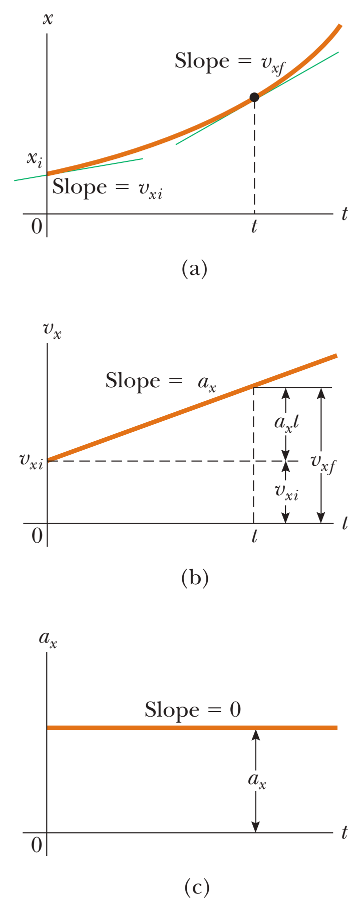
$$x_f = x_i + \tfrac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \quad \text{(for constant } a_x\text{)} \tag{2.16}$$

◀ Position as a function of time

This equation provides the final position of the particle at time t in terms of the initial velocity and the constant acceleration.

The position–time graph for motion at constant (positive) acceleration shown in Active Figure 2.11a is obtained from Equation 2.16. Notice that the curve is a parabola. The slope of the tangent line to this curve at $t = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity v_{xf} at that time.



ACTIVE FIGURE 2.11

A particle under constant acceleration a_x moving along the x axis: (a) the position–time graph, (b) the velocity–time graph, and (c) the acceleration–time graph.

Sign in at www.thomsonedu.com and go to ThomsonNOW to adjust the constant acceleration and observe the effect on the position and velocity graphs.

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation 2.13 into Equation 2.15:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^{\ 2} - v_{xi}^{\ 2}}{2a_x}$$

Velocity as a function of position ▶

$$v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x\,(x_f - x_i) \quad \text{(for constant } a_x) \qquad \textbf{(2.17)}$$

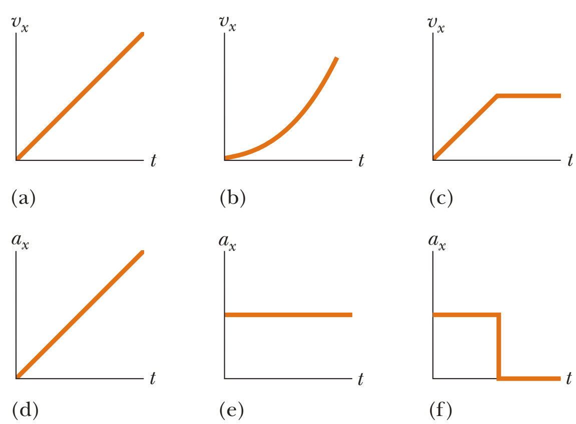
This equation provides the final velocity in terms of the initial velocity, the constant acceleration, and the position of the particle.

For motion at *zero* acceleration, we see from Equations 2.13 and 2.16 that

$$\left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f &= x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time. In terms of models, when the acceleration of a particle is zero, the particle under constant acceleration model reduces to the particle under constant velocity model (Section 2.3).

Quick Quiz 2.6 In Active Figure 2.12, match each v_x–t graph on the top with the a_x–t graph on the bottom that best describes the motion.



ACTIVE FIGURE 2.12

(Quick Quiz 2.6) Parts (a), (b), and (c) are v_x–t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

Sign in at www.thomsonedu.com and go to ThomsonNOW to practice matching appropriate velocity versus time graphs and acceleration versus time graphs.

Equations 2.13 through 2.17 are **kinematic equations** that may be used to solve any problem involving a particle under constant acceleration in one dimension. The four kinematic equations used most often are listed for convenience in Table 2.2. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. You should recognize that the quantities that vary during the motion are position x_f, velocity v_{xf}, and time t.

You will gain a great deal of experience in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x\,(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

EXAMPLE 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

SOLUTION

You might have seen movies or television shows in which a jet lands on an aircraft carrier and is brought to rest surprisingly fast by an arresting cable. Because the acceleration of the jet is assumed constant, we model it as a particle under constant acceleration. We define our x axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also notice that we have no information about the change in position of the jet while it is slowing down.

Equation 2.13 is the only equation in Table 2.2 that does not involve position, so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}}$$
$$= \boxed{-32\text{ m/s}^2}$$

(B) If the jet touches down at position $x_i = 0$, what is its final position?

SOLUTION

Use Equation 2.15 to solve for the final position:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s}) = \boxed{63\text{ m}}$$

If the jet travels much farther than 63 m, it might fall into the ocean. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of World War I. The cables are still a vital part of the operation of modern aircraft carriers.

What If? Suppose the jet lands on the deck of the aircraft carrier with a speed higher than 63 m/s but has the same acceleration due to the cable as that calculated in part (A). How will that change the answer to part (B)?

Answer If the jet is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.15 that if v_{xi} is larger, x_f will be larger.

EXAMPLE 2.8 Watch Out for the Speed Limit!

A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

SOLUTION

A pictorial representation (Fig. 2.13) helps clarify the sequence of events. The car is modeled as a particle under constant velocity, and the trooper is modeled as a particle under constant acceleration.

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_Ⓑ = 0$ as the time the trooper begins moving. At that

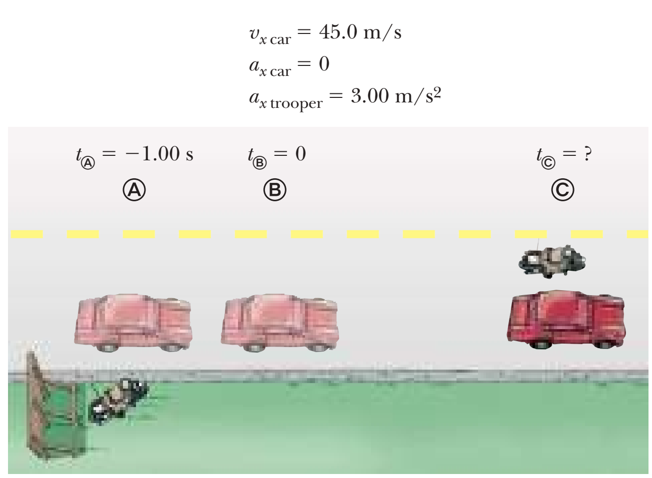


Figure 2.13 (Example 2.8) A speeding car passes a hidden trooper.

instant, the car has already traveled a distance of 45.0 m from the billboard because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Therefore, the initial position of the speeding car is $x_{Ⓑ} = 45.0$ m.

Apply Equation 2.7 to give the car's position at any time t:

$$x_{\text{car}} = x_{Ⓑ} + v_{x\,\text{car}}t = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

A quick check shows that at $t = 0$, this expression gives the car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_{Ⓑ} = 45.0$ m.

The trooper starts from rest at $t_{Ⓑ} = 0$ and accelerates at 3.00 m/s^2 away from the origin. Use Equation 2.16 to give her position at any time t:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$x_{\text{trooper}} = 0 + (0)t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2$$

Set the two positions equal to represent the trooper overtaking the car at position Ⓒ:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

Simplify to give a quadratic equation:

$$1.50t^2 - 45.0t - 45.0 = 0$$

The positive solution of this equation is $t = 31.0$ s.

(For help in solving quadratic equations, see Appendix B.2.)

What If? What if the trooper has a more powerful motorcycle with a larger acceleration? How would that change the time at which the trooper catches the car?

Answer If the motorcycle has a larger acceleration, the trooper will catch up to the car sooner, so the answer for the time will be less than 31 s.

Cast the final quadratic equation above in terms of the parameters in the problem:

$$\tfrac{1}{2}a_x t^2 - v_{x\,\text{car}}t - x_{Ⓑ} = 0$$

Solve the quadratic equation:

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_x x_{Ⓑ}}}{a_x} = \frac{v_{x\,\text{car}}}{a_x} + \sqrt{\frac{v_{x\,\text{car}}^2}{a_x^2} + \frac{2x_{Ⓑ}}{a_x}}$$

where we have chosen the positive sign because that is the only choice consistent with a time $t > 0$. Because all terms on the right side of the equation have the acceleration a_x in the denominator, increasing the acceleration will decrease the time at which the trooper catches the car.

PITFALL PREVENTION 2.6

***g* and g**

Be sure not to confuse the italic symbol g for free-fall acceleration with the nonitalic symbol g used as the abbreviation for the unit gram.

PITFALL PREVENTION 2.7

The Sign of *g*

Keep in mind that g is a *positive number*. It is tempting to substitute -9.80 m/s^2 for g, but resist the temptation. Downward gravitational acceleration is indicated explicitly by stating the acceleration as $a_y = -g$.

2.7 Freely Falling Objects

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the Greek philosopher Aristotle (384–322 BC) had held that heavier objects fall faster than lighter ones.

The Italian Galileo Galilei (1564–1642) originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the behavior of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments, he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration, which made it possible for him to make accurate measurements of the time intervals. By gradually increasing the slope of the incline, he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

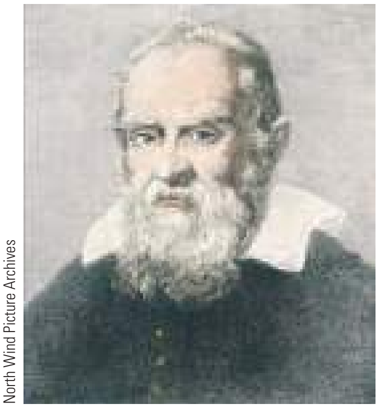
You might want to try the following experiment. Simultaneously drop a coin and a crumpled-up piece of paper from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free-fall* motion. If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, astronaut David Scott conducted such a demonstration on the Moon. He simultaneously released a hammer and a feather, and the two objects fell together to the lunar surface. This simple demonstration surely would have pleased Galileo!

When we use the expression *freely falling object*, we do not necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion**.

We shall denote the magnitude of the *free-fall acceleration* by the symbol g. The value of g near the Earth's surface decreases with increasing altitude. Furthermore, slight variations in g occur with changes in latitude. At the Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we shall use this value for g when performing calculations. For making quick estimates, use $g = 10$ m/s^2.

If we neglect air resistance and assume the free-fall acceleration does not vary with altitude over short vertical distances, the motion of a freely falling object moving vertically is equivalent to motion of a particle under constant acceleration in one dimension. Therefore, the equations developed in Section 2.6 for objects moving with constant acceleration can be applied. The only modification for freely falling objects that we need to make in these equations is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal direction (x) and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Therefore, we always choose $a_y = -g = -9.80$ m/s^2, where the negative sign means that the acceleration of a freely falling object is downward. In Chapter 13, we shall study how to deal with variations in g with altitude.

Quick Quiz 2.7 Consider the following choices: (a) increases, (b) decreases, (c) increases and then decreases, (d) decreases and then increases, (e) remains the same. From these choices, select what happens to **(i)** the acceleration and **(ii)** the speed of a ball after it is thrown upward into the air.



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GALILEO GALILEI
Italian physicist and astronomer (1564–1642)
Galileo formulated the laws that govern the motion of objects in free fall and made many other significant discoveries in physics and astronomy. Galileo publicly defended Nicolaus Copernicus's assertion that the Sun is at the center of the Universe (the heliocentric system). He published *Dialogue Concerning Two New World Systems* to support the Copernican model, a view that the Catholic Church declared to be heretical.

PITFALL PREVENTION 2.8
Acceleration at the Top of the Motion

A common misconception is that the acceleration of a projectile at the top of its trajectory is zero. Although the velocity at the top of the motion of an object thrown upward momentarily goes to zero, *the acceleration is still that due to gravity* at this point. If the velocity and acceleration were both zero, the projectile would stay at the top.

CONCEPTUAL EXAMPLE 2.9 The Daring Skydivers

A skydiver jumps out of a hovering helicopter. A few seconds later, another skydiver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both skydivers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?

SOLUTION

At any given instant, the speeds of the skydivers are different because one had a head start. In any time interval Δt after this instant, however, the two skydivers increase their speeds by the same amount because they have the same acceleration. Therefore, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Therefore, in a given time interval, the first skydiver covers a greater distance than the second. Consequently, the separation distance between them increases.

EXAMPLE 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14.

(A) Using $t_{Ⓐ} = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

SOLUTION

You most likely have experience with dropping objects or throwing them upward and watching them fall, so this problem should describe a familiar experience. Because the stone is in free fall, it is modeled as a particle under constant acceleration due to gravity.

Use Equation 2.13 to calculate the time at which the stone reaches its maximum height:

$$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad t = \frac{v_{yf} - v_{yi}}{a_y}$$

Substitute numerical values:

$$t = t_{Ⓑ} = \frac{0 - 20.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

(B) Find the maximum height of the stone.

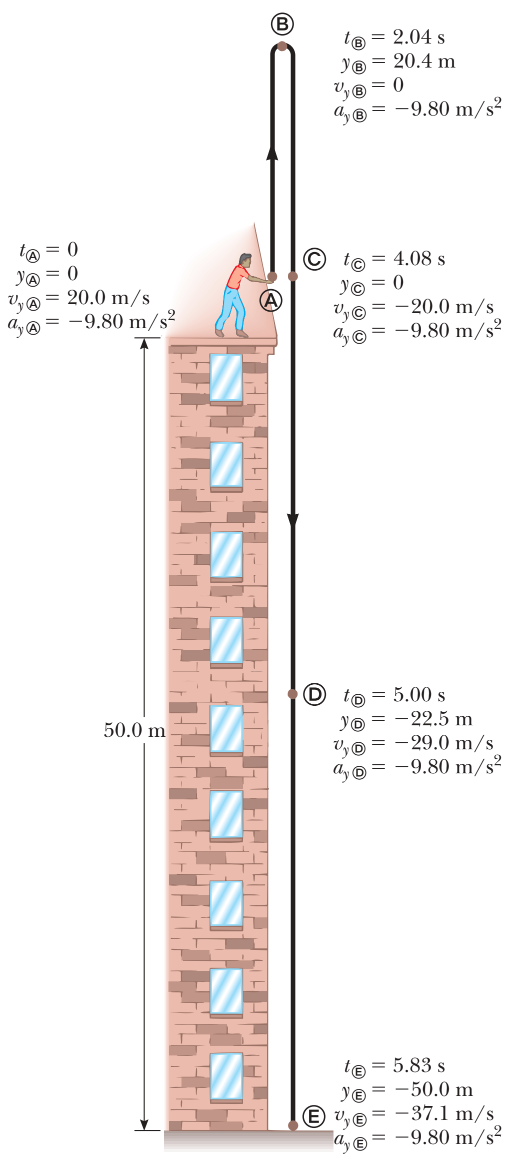

Figure 2.14 (Example 2.10) Position and velocity versus time for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s. Many of the quantities in the labels for points in the motion of the stone are calculated in the example. Can you verify the other values that are not?

SOLUTION

Set $y_Ⓐ = 0$ and substitute the time from part (A) into Equation 2.16 to find the maximum height:

$$y_{max} = y_Ⓑ = y_Ⓐ + v_{xⒶ}t + \tfrac{1}{2}a_y t^2$$

$$y_Ⓑ = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2 = \boxed{20.4 \text{ m}}$$

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

Substitute known values into Equation 2.17:

$$v_{yⒸ}^2 = v_{yⒶ}^2 + 2a_y(y_Ⓒ - y_Ⓐ)$$

$$v_{yⒸ}^2 = (20.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(0 - 0) = 400 \text{ m}^2/\text{s}^2$$

$$v_{yⒸ} = \boxed{-20.0 \text{ m/s}}$$

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point Ⓒ. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

(D) Find the velocity and position of the stone at $t = 5.00$ s.

Calculate the velocity at Ⓓ from Equation 2.13:

$$v_{yⒹ} = v_{yⒶ} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s}) = \boxed{-29.0 \text{ m/s}}$$

Use Equation 2.16 to find the position of the stone at $t_Ⓓ = 5.00$ s:

$$y_Ⓓ = y_Ⓐ + v_{yⒶ}t + \tfrac{1}{2}a_y t^2$$

$$= 0 + (20.0 \text{ m/s})(5.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$

$$= \boxed{-22.5 \text{ m}}$$

The choice of the time defined as $t = 0$ is arbitrary and up to you to select as the problem-solver. As an example of this arbitrariness, choose $t = 0$ as the time at which the stone is at the highest point in its motion. Then solve parts (C) and (D) again using this new initial instant and note that your answers are the same as those above.

What If? What if the building were 30.0 m tall instead of 50.0 m tall? Which answers in parts (A) to (D) would change?

Answer None of the answers would change. All the motion takes place in the air during the first 5.00 s. (Notice that even for a 30.0-m tall building, the stone is above the ground at $t = 5.00$ s.) Therefore, the height of the building is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height of the building into any equation.

2.8 Kinematic Equations Derived from Calculus

This section assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be obtained if its position as a function of time is known. Mathematically, the velocity equals the derivative of the position with respect to time. It is also possible to find the position of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative*. Graphically, it is equivalent to finding the area under a curve.

Suppose the v_x–t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of duration Δt_n. From the definition of average velocity we see that the displacement of the particle during any small interval, such as the one shaded in Figure 2.15, is

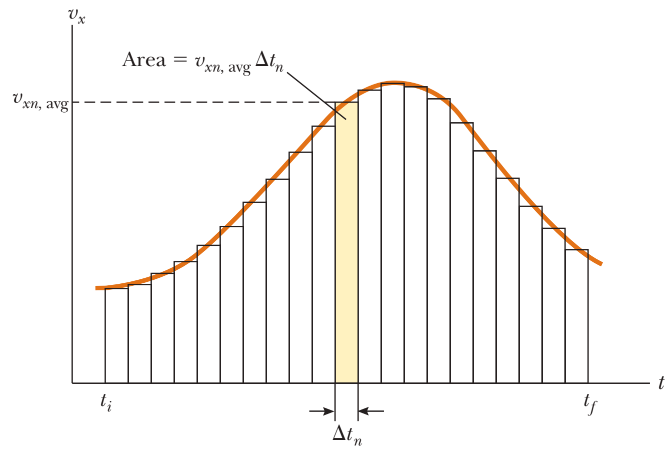

Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, whereas the total area under the curve is the total displacement of the particle.

given by $\Delta x_n = v_{xn,\text{ avg}}\,\Delta t_n$, where $v_{xn,\text{ avg}}$ is the average velocity in that interval. Therefore, the displacement during this small interval is simply the area of the shaded rectangle. The total displacement for the interval $t_f - t_i$ is the sum of the areas of all the rectangles from t_i to t_f:

$$\Delta x = \sum_n v_{xn,\text{ avg}}\,\Delta t_n$$

where the symbol Σ (uppercase Greek sigma) signifies a sum over all terms, that is, over all values of n. Now, as the intervals are made smaller and smaller, the number of terms in the sum increases and the sum approaches a value equal to the area under the velocity–time graph. Therefore, in the limit $n \to \infty$, or $\Delta t_n \to 0$, the displacement is

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn}\,\Delta t_n \tag{2.18}$$

Notice that we have replaced the average velocity $v_{xn,\text{ avg}}$ with the instantaneous velocity v_{xn} in the sum. As you can see from Figure 2.15, this approximation is valid in the limit of very small intervals. Therefore, if we know the v_x–t graph for motion along a straight line, we can obtain the displacement during any time interval by measuring the area under the curve corresponding to that time interval.

The limit of the sum shown in Equation 2.18 is called a **definite integral** and is written

Definite integral ▶

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn}\,\Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt \tag{2.19}$$

where $v_x(t)$ denotes the velocity at any time t. If the explicit functional form of $v_x(t)$ is known and the limits are given, the integral can be evaluated. Sometimes the v_x–t graph for a moving particle has a shape much simpler than that shown in Figure 2.15. For example, suppose a particle moves at a constant velocity v_{xi}. In this case, the v_x–t graph is a horizontal line, as in Figure 2.16, and the displacement of the particle during the time interval Δt is simply the area of the shaded rectangle:

$$\Delta x = v_{xi}\,\Delta t \quad (\text{when } v_x = v_{xi} = \text{constant})$$

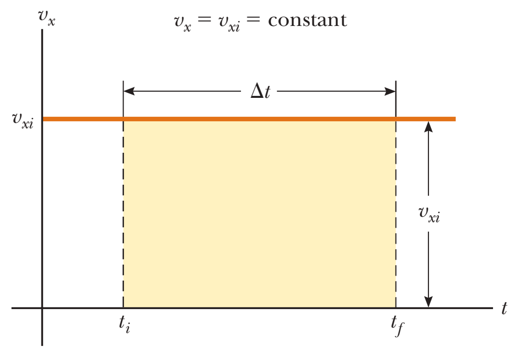

Figure 2.16 The velocity–time curve for a particle moving with constant velocity v_{xi}. The displacement of the particle during the time interval $t_f - t_i$ is equal to the area of the shaded rectangle.

Kinematic Equations

We now use the defining equations for acceleration and velocity to derive two of our kinematic equations, Equations 2.13 and 2.16.

The defining equation for acceleration (Eq. 2.10),

$$a_x = \frac{dv_x}{dt}$$

may be written as $dv_x = a_x\, dt$ or, in terms of an integral (or antiderivative), as

$$v_{xf} - v_{xi} = \int_0^t a_x\, dt$$

For the special case in which the acceleration is constant, a_x can be removed from the integral to give

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t - 0) = a_x t \qquad \textbf{(2.20)}$$

which is Equation 2.13.

Now let us consider the defining equation for velocity (Eq. 2.5):

$$v_x = \frac{dx}{dt}$$

We can write this equation as $dx = v_x\, dt$, or in integral form as

$$x_f - x_i = \int_0^t v_x\, dt$$

Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes

$$x_f - x_i = \int_0^t (v_{xi} + a_x t)dt = \int_0^t v_{xi}\, dt + a_x \int_0^t t\, dt = v_{xi}\,(t - 0) + a_x\left(\frac{t^2}{2} - 0\right)$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2$$

which is Equation 2.16.

Besides what you might expect to learn about physics concepts, a very valuable skill you should hope to take away from your physics course is the ability to solve complicated problems. The way physicists approach complex situations and break them into manageable pieces is extremely useful. The following is a general problem-solving strategy to guide you through the steps. To help you remember the steps of the strategy, they are *Conceptualize*, *Categorize*, *Analyze*, and *Finalize*.

GENERAL PROBLEM-SOLVING STRATEGY

Conceptualize

- The first things to do when approaching a problem are to *think about* and *understand* the situation. Study carefully any representations of the information (e.g., diagrams, graphs, tables, or photographs) that accompany the problem. Imagine a movie, running in your mind, of what happens in the problem.
- If a pictorial representation is not provided, you should almost always make a quick drawing of the situation. Indicate any known values, perhaps in a table or directly on your sketch.
- Now focus on what algebraic or numerical information is given in the problem. Carefully read the problem statement, looking for key phrases such as "starts from rest" ($v_i = 0$), "stops" ($v_f = 0$), or "falls freely" ($a_y = -g = -9.80 \text{ m/s}^2$).
- Now focus on the expected result of solving the problem. Exactly what is the question asking? Will the final result be numerical or algebraic? Do you know what units to expect?
- Don't forget to incorporate information from your own experiences and common sense. What should a reasonable answer look like? For example, you wouldn't expect to calculate the speed of an automobile to be 5×10^6 m/s.

Categorize

- Once you have a good idea of what the problem is about, you need to *simplify* the problem. Remove the details that are not important to the solution. For example, model a moving object as a particle. If appropriate, ignore air resistance or friction between a sliding object and a surface.
- Once the problem is simplified, it is important to *categorize* the problem. Is it a simple *substitution problem* such that numbers can be substituted into an equation? If so, the problem is likely to be finished when this substitution is done. If not, you face what we call an *analysis problem:* the situation must be analyzed more deeply to reach a solution.
- If it is an analysis problem, it needs to be categorized further. Have you seen this type of problem before? Does it fall into the growing list of types of problems that you have solved previously? If so, identify any analysis model(s) appropriate for the problem to prepare for the Analyze step below. We saw the first three analysis models in this chapter: the particle under constant velocity, the particle under constant speed, and the particle under constant acceleration. Being able to classify a problem with an analysis model can make it much easier to lay out a plan to solve it. For example, if your simplification shows that the problem can be treated as a particle under constant acceleration and you have already solved such a problem (such as the examples in Section 2.6), the solution to the present problem follows a similar pattern.

Analyze

- Now you must analyze the problem and strive for a mathematical solution. Because you have already categorized the problem and identified an analysis model, it should not be too difficult to select relevant equations that apply to the type of situation in the problem. For example, if the problem involves a particle under constant acceleration, Equations 2.13 to 2.17 are relevant.
- Use algebra (and calculus, if necessary) to solve symbolically for the unknown variable in terms of what is given. Substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

Finalize

- Examine your numerical answer. Does it have the correct units? Does it meet your expectations from your conceptualization of the problem? What about the algebraic form of the result? Does it make sense? Examine the variables in the problem to see whether the answer would change in a physically meaningful way if the variables were drastically increased or decreased or even became zero. Looking at limiting cases to see whether they yield expected values is a very useful way to make sure that you are obtaining reasonable results.
- Think about how this problem compared with others you have solved. How was it similar? In what critical ways did it differ? Why was this problem assigned? Can you figure out what you have learned by doing it? If it is a new category of problem, be sure you understand it so that you can use it as a model for solving similar problems in the future.

When solving complex problems, you may need to identify a series of subproblems and apply the problem-solving strategy to each. For simple problems, you probably don't need this strategy. When you are trying to solve a problem and you don't know what to do next, however, remember the steps in the strategy and use them as a guide.

For practice, it would be useful for you to revisit the worked examples in this chapter and identify the *Conceptualize, Categorize, Analyze,* and *Finalize* steps. In the rest of this book, we will label these steps explicitly in the worked examples. Many chapters in this book include a section labeled Problem-Solving Strategy that should help you through the rough spots. These sections are organized according to the General Problem-Solving Strategy outlined above and are tailored to the specific types of problems addressed in that chapter.

Summary

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DEFINITIONS

When a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \qquad (2.1)$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,\text{ avg}} \equiv \frac{\Delta x}{\Delta t} \qquad (2.2)$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

$$v_{\text{avg}} \equiv \frac{d}{\Delta t} \qquad (2.3)$$

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

The **instantaneous speed** of a particle is equal to the magnitude of its instantaneous velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurs:

$$a_{x,\text{ avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.9)$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.10)$$

CONCEPTS AND PRINCIPLES

When an object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Remembering that $F_x \propto a_x$ is a useful way to identify the direction of the acceleration by associating it with a force.

An object falling freely in the presence of the Earth's gravity experiences free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. Recall and apply the *Conceptualize, Categorize, Analyze,* and *Finalize* steps of the General Problem-Solving Strategy when you need them.

(*continued*)

ANALYSIS MODELS FOR PROBLEM-SOLVING

Particle Under Constant Velocity. If a particle moves in a straight line with a constant speed v_x, its constant velocity is given by

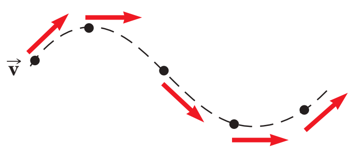
$$v_x = \frac{\Delta x}{\Delta t} \qquad \textbf{(2.6)}$$

and its position is given by

$$x_f = x_i + v_x t \qquad \textbf{(2.7)}$$

Particle Under Constant Speed. If a particle moves a distance d along a curved or straight path with a constant speed, its constant speed is given by

$$v = \frac{d}{\Delta t} \qquad \textbf{(2.8)}$$



Particle Under Constant Acceleration. If a particle moves in a straight line with a constant acceleration a_x, its motion is described by the kinematic equations:

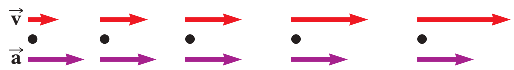
$$v_{xf} = v_{xi} + a_x t \qquad \textbf{(2.13)}$$

$$v_{x,\,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \qquad \textbf{(2.14)}$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \qquad \textbf{(2.15)}$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \textbf{(2.16)}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \qquad \textbf{(2.17)}$$



Questions

☐ denotes answer available in *Student Solutions Manual/Study Guide;* **O** denotes objective question

1. **O** One drop of oil falls straight down onto the road from the engine of a moving car every 5 s. Figure Q2.1 shows the pattern of the drops left behind on the pavement. What is the average speed of the car over this section of its motion? (a) 20 m/s (b) 24 m/s (c) 30 m/s (d) 100 m/s (e) 120 m/s

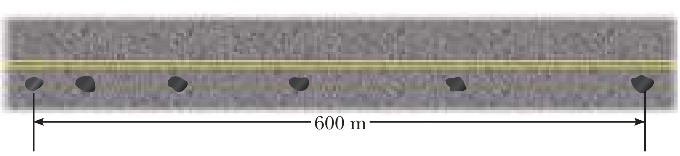


Figure Q2.1

$\boxed{\textbf{2.}}$ If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?

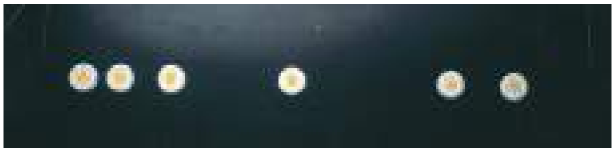
3. **O** Can the instantaneous velocity of an object at an instant of time ever be greater in magnitude than the average velocity over a time interval containing the instant? Can it ever be less?

4. **O** A cart is pushed along a straight horizontal track. (a) In a certain section of its motion, its original velocity is $v_{xi} = +3$ m/s and it undergoes a change in velocity of $\Delta v_x = +4$ m/s. Does it speed up or slow down in this section of its motion? Is its acceleration positive or negative? (b) In another part of its motion, $v_{xi} = -3$ m/s and $\Delta v_x = +4$ m/s. Does it undergo a net increase or decrease in speed? Is its acceleration positive or negative? (c) In a third segment of its motion, $v_{xi} = +3$ m/s and $\Delta v_x = -4$ m/s. Does it have a net gain or loss in speed? Is its acceleration positive or negative? (d) In a fourth time interval, $v_{xi} = -3$ m/s and $\Delta v_x = -4$ m/s. Does the cart gain or lose speed? Is its acceleration positive or negative?

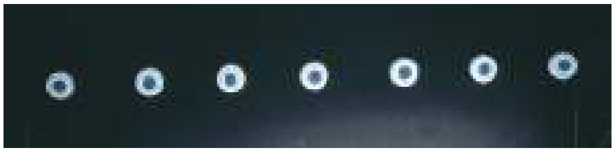
$\boxed{\textbf{5.}}$ Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does that mean that the acceleration of A is greater than that of B? Explain.

6. **O** When the pilot reverses the propeller in a boat moving north, the boat moves with an acceleration directed south. If the acceleration of the boat remains constant in magnitude and direction, what would happen to the boat (choose one)? (a) It would eventually stop and then remain stopped. (b) It would eventually stop and then start to speed up in the forward direction. (c) It would eventually stop and then start to speed up in the reverse direction. (d) It would never quite stop but lose speed more and more slowly forever. (e) It would never stop but continue to speed up in the forward direction.

7. **O** Each of the strobe photographs (a), (b), and (c) in Figure Q2.7 was taken of a single disk moving toward the right, which we take as the positive direction. Within each photograph, the time interval between images is constant. **(i)** Which photograph(s), if any, shows constant zero velocity? **(ii)** Which photograph(s), if any, shows constant zero acceleration? **(iii)** Which photograph(s), if any, shows constant positive velocity? **(iv)** Which photograph(s), if any, shows constant positive acceleration? **(v)** Which photograph(s), if any, shows some motion with negative acceleration?



(a)



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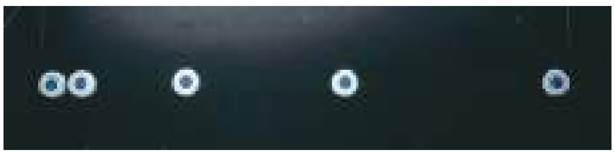


(c)

Figure Q2.7 Question 7 and Problem 17.

8. Try the following experiment away from traffic where you can do it safely. With the car you are driving moving slowly on a straight, level road, shift the transmission into neutral and let the car coast. At the moment the car comes to a complete stop, step hard on the brake and notice what you feel. Now repeat the same experiment on a fairly gentle uphill slope. Explain the difference in what a person riding in the car feels in the two cases. (Brian Popp suggested the idea for this question.)

9. O A skateboarder coasts down a long hill, starting from rest and moving with constant acceleration to cover a certain distance in 6 s. In a second trial, he starts from rest and moves with the same acceleration for only 2 s. How is his displacement different in this second trial compared with the first trial? (a) one-third as large (b) three times larger (c) one-ninth as large (d) nine times larger (e) $1/\sqrt{3}$ times as large (f) $\sqrt{3}$ times larger (g) none of these answers

10. O Can the equations of kinematics (Eqs. 2.13–2.17) be used in a situation in which the acceleration varies in time? Can they be used when the acceleration is zero?

11. A student at the top of a building of height h throws one ball upward with a speed of v_i and then throws a second ball downward with the same initial speed $|v_i|$. How do the final velocities of the balls compare when they reach the ground?

12. O A pebble is released from rest at a certain height and falls freely, reaching an impact speed of 4 m/s at the floor. **(i)** Next, the pebble is thrown down with an initial speed of 3 m/s from the same height. In this trial, what is its speed at the floor? (a) less than 4 m/s (b) 4 m/s (c) between 4 m/s and 5 m/s (d) $\sqrt{3^2 + 4^2}$ m/s = 5 m/s (e) between 5 m/s and 7 m/s (f) (3 + 4) m/s = 7 m/s (g) greater than 7 m/s **(ii)** In a third trial, the pebble is tossed upward with an initial speed of 3 m/s from the same height. What is its speed at the floor in this trial? Choose your answer from the same list (a) through (g).

13. O A hard rubber ball, not affected by air resistance in its motion, is tossed upward from shoulder height, falls to the sidewalk, rebounds to a somewhat smaller maximum height, and is caught on its way down again. This motion is represented in Figure Q2.13, where the successive positions of the ball Ⓐ through Ⓖ are not equally spaced in time. At point Ⓔ the center of the ball is at its lowest point in the motion. The motion of the ball is along a straight line, but the diagram shows successive positions offset to the right to avoid overlapping. Choose the positive y direction to be upward. **(i)** Rank the situations Ⓐ through Ⓖ according to the speed of the ball $|v_y|$ at each point, with the largest speed first. **(ii)** Rank the same situations according to the velocity of the ball at each point. **(iii)** Rank the same situations according to the acceleration a_y of the ball at each point. In each ranking, remember that zero is greater than a negative value. If two values are equal, show that they are equal in your ranking.

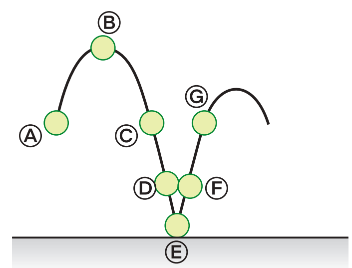


Figure Q2.13

14. O You drop a ball from a window located on an upper floor of a building. It strikes the ground with speed v. You now repeat the drop, but you ask a friend down on the ground to throw another ball upward at speed v. Your friend throws the ball upward at the same moment that you drop yours from the window. At some location, the balls pass each other. Is this location (a) *at* the halfway point between window and ground, (b) *above* this point, or (c) *below* this point?

Problems

WebAssign The Problems from this chapter may be assigned online in WebAssign.

ThomsonNOW™ Sign in at **www.thomsonedu.com** and go to ThomsonNOW to assess your understanding of this chapter's topics with additional quizzing and conceptual questions.

1, 2, 3 denotes straightforward, intermediate, challenging; □ denotes full solution available in *Student Solutions Manual/Study Guide*; ▲ denotes coached solution with hints available at **www.thomsonedu.com;** ▭ denotes developing symbolic reasoning; ● denotes asking for qualitative reasoning; ▣ denotes computer useful in solving problem

Section 2.1 Position, Velocity, and Speed

1. The position versus time for a certain particle moving along the x axis is shown in Figure P2.1. Find the average velocity in the following time intervals. (a) 0 to 2 s (b) 0 to 4 s (c) 2 s to 4 s (d) 4 s to 7 s (e) 0 to 8 s

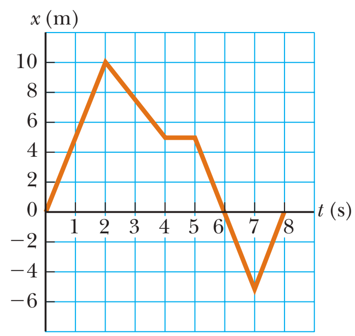


Figure P2.1 Problems 1 and 8.

2. The position of a pinewood derby car was observed at various moments; the results are summarized in the following table. Find the average velocity of the car for (a) the first 1-s time interval, (b) the last 3 s, and (c) the entire period of observation.

t (s)	0	1.0	2.0	3.0	4.0	5.0
x (m)	0	2.3	9.2	20.7	36.8	57.5

3. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. (a) What is her average speed over the entire trip? (b) What is her average velocity over the entire trip?

4. A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 s to 2.10 s.

Section 2.2 Instantaneous Velocity and Speed

5. ▲ A position–time graph for a particle moving along the x axis is shown in Figure P2.5. (a) Find the average velocity in the time interval t = 1.50 s to t = 4.00 s. (b) Determine the instantaneous velocity at t = 2.00 s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

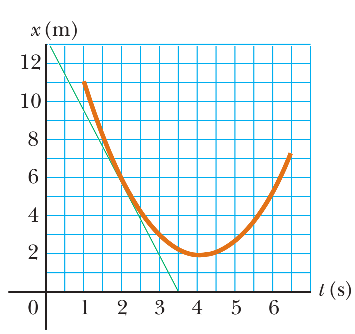


Figure P2.5

6. The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at t = 3.00 s and (b) at 3.00 s + Δt. (c) Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero to find the velocity at t = 3.00 s.

7. (a) Use the data in Problem 2.2 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from the graph, determine the average acceleration of the car. (d) What was the initial velocity of the car?

8. Find the instantaneous velocity of the particle described in Figure P2.1 at the following times: (a) t = 1.0 s (b) t = 3.0 s (c) t = 4.5 s (d) t = 7.5 s

Section 2.3 Analysis Models: The Particle Under Constant Velocity

9. A hare and a tortoise compete in a race over a course 1.00 km long. The tortoise crawls straight and steadily at its maximum speed of 0.200 m/s toward the finish line. The hare runs at its maximum speed of 8.00 m/s toward the goal for 0.800 km and then stops to tease the tortoise. How close to the goal can the hare let the tortoise approach before resuming the race, which the tortoise wins in a photo finish? Assume both animals, when moving, move steadily at their respective maximum speeds.

Section 2.4 Acceleration

10. A 50.0-g Super Ball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? *Note:* 1 ms = 10^{-3} s.

11. A particle starts from rest and accelerates as shown in Figure P2.11. Determine (a) the particle's speed at t = 10.0 s and at t = 20.0 s and (b) the distance traveled in the first 20.0 s.

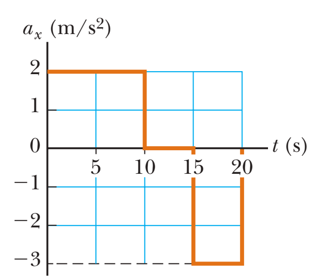


Figure P2.11

12. A velocity–time graph for an object moving along the x axis is shown in Figure P2.12. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals t = 5.00 s to t = 15.0 s and t = 0 to t = 20.0 s.

13. ▲ A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At t = 3.00 s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

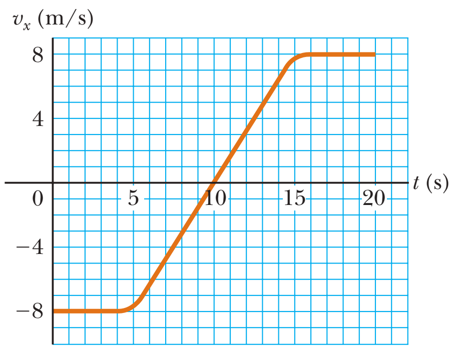


Figure P2.12

14. A child rolls a marble on a bent track that is 100 cm long as shown in Figure P2.14. We use x to represent the position of the marble along the track. On the horizontal sections from $x = 0$ to $x = 20$ cm and from $x = 40$ cm to $x = 60$ cm, the marble rolls with constant speed. On the sloping sections, the speed of the marble changes steadily. At the places where the slope changes, the marble stays on the track and does not undergo any sudden changes in speed. The child gives the marble some initial speed at $x = 0$ and $t = 0$ and then watches it roll to $x = 90$ cm, where it turns around, eventually returning to $x = 0$ with the same speed with which the child initially released it. Prepare graphs of x versus t, v_x versus t, and a_x versus t, vertically aligned with their time axes identical, to show the motion of the marble. You will not be able to place numbers other than zero on the horizontal axis or on the velocity or acceleration axes, but show the correct relative sizes on the graphs.

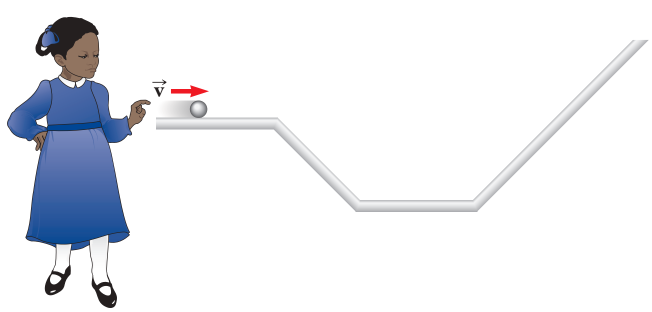


Figure P2.14

15. An object moves along the x axis according to the equation $x(t) = (3.00t^2 - 2.00t + 3.00)$ m, where t is in seconds. Determine (a) the average speed between $t = 2.00$ s and $t = 3.00$ s, (b) the instantaneous speed at $t = 2.00$ s and at $t = 3.00$ s, (c) the average acceleration between $t = 2.00$ s and $t = 3.00$ s, and (d) the instantaneous acceleration at $t = 2.00$ s and $t = 3.00$ s.

16. Figure P2.16 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

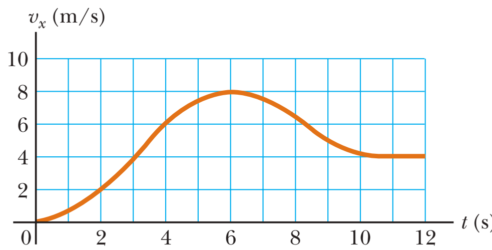


Figure P2.16

Section 2.5 Motion Diagrams

17. ● Each of the strobe photographs (a), (b), and (c) in Figure Q2.7 was taken of a single disk moving toward the right, which we take as the positive direction. Within each photograph the time interval between images is constant. For each photograph, prepare graphs of x versus t, v_x versus t, and a_x versus t, vertically aligned with their time axes identical, to show the motion of the disk. You will not be able to place numbers other than zero on the axes, but show the correct relative sizes on the graphs.

18. Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform; that is, if the speed were not changing at a constant rate?

Section 2.6 The Particle Under Constant Acceleration

19. ● Assume a parcel of air in a straight tube moves with a constant acceleration of -4.00 m/s^2 and has a velocity of 13.0 m/s at 10:05:00 a.m. on a certain date. (a) What is its velocity at 10:05:01 a.m.? (b) At 10:05:02 a.m.? (c) At 10:05:02.5 a.m.? (d) At 10:05:04 a.m.? (e) At 10:04:59 a.m.? (f) Describe the shape of a graph of velocity versus time for this parcel of air. (g) Argue for or against the statement, "Knowing the single value of an object's constant acceleration is like knowing a whole list of values for its velocity."

20. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

$\boxed{21.}$ ▲ An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

22. Figure P2.22 represents part of the performance data of a car owned by a proud physics student. (a) Calculate the total distance traveled by computing the area under the graph line. (b) What distance does the car travel between the times $t = 10$ s and $t = 40$ s? (c) Draw a graph of its acceleration versus time between $t = 0$ and $t = 50$ s. (d) Write an equation for x as a function of time for each phase of the motion, represented by (i) $0a$, (ii) ab, and (iii) bc. (e) What is the average velocity of the car between $t = 0$ and $t = 50$ s?

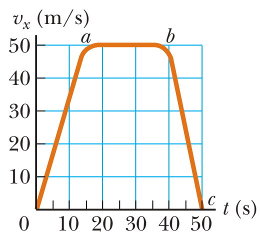


Figure P2.22

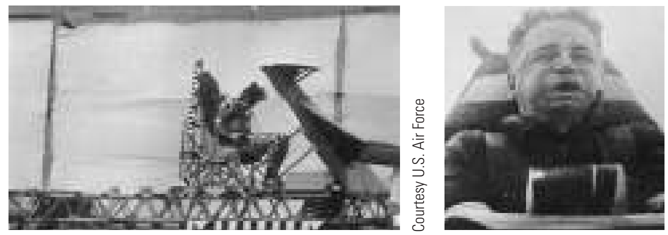
23. ● A jet plane comes in for a landing with a speed of 100 m/s, and its acceleration can have a maximum magnitude of 5.00 m/s^2 as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long? Explain your answer.

24. ● At $t = 0$, one toy car is set rolling on a straight track with initial position 15.0 cm, initial velocity -3.50 cm/s, and constant acceleration 2.40 cm/s^2. At the same moment, another toy car is set rolling on an adjacent track with initial position 10.0 cm, an initial velocity of $+5.50$ cm/s, and constant acceleration zero. (a) At what time, if any, do the two cars have equal speeds? (b) What are their speeds at that time? (c) At what time(s), if any, do the cars pass each other? (d) What are their locations at that time? (e) Explain the difference between question (a) and question (c) as clearly as possible. Write (or draw) for a target audience of students who do not immediately understand the conditions are different.

25. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of -5.60 m/s^2 for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

26. *Help! One of our equations is missing!* We describe constant-acceleration motion with the variables and parameters v_{xi}, v_{xf}, a_x, t, and $x_f - x_i$. Of the equations in Table 2.2, the first does not involve $x_f - x_i$, the second does not contain a_x, the third omits v_{xf}, and the last leaves out t. So, to complete the set there should be an equation *not* involving v_{xi}. Derive it from the others. Use it to solve Problem 25 in one step.

27. For many years Colonel John P. Stapp, USAF, held the world's land speed record. He participated in studying whether a jet pilot could survive emergency ejection. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at a speed of 632 mi/h. He and the sled were safely brought to rest in 1.40 s (Fig. P2.27). Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.



Courtesy U.S. Air Force

Photri, Inc.

Figure P2.27 (*Left*) Col. John Stapp on rocket sled. (*Right*) Stapp's face is contorted by the stress of rapid negative acceleration.

28. A particle moves along the x axis. Its position is given by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

29. An electron in a cathode-ray tube accelerates from a speed of 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) In what time interval does the electron travel this 1.50 cm? (b) What is its acceleration?

30. ● Within a complex machine such as a robotic assembly line, suppose one particular part glides along a straight track. A control system measures the average velocity of the part during each successive time interval $\Delta t_0 = t_0 - 0$, compares it with the value v_c it should be, and switches a servo motor on and off to give the part a correcting pulse of acceleration. The pulse consists of a constant acceleration a_m applied for time interval $\Delta t_m = t_m - 0$ within the next control time interval Δt_0. As shown in Figure P2.30, the part may be modeled as having zero acceleration when the motor is off (between t_m and t_0). A computer in the control system chooses the size of the acceleration so that the final velocity of the part will have the correct value v_c. Assume the part is initially at rest and is to have instantaneous velocity v_c at time t_0. (a) Find the required value of a_m in terms of v_c and t_m. (b) Show that the displacement Δx of the part during the time interval Δt_0 is given by $\Delta x = v_c\,(t_0 - 0.5t_m)$. For specified values of v_c and t_0, (c) what is the minimum displacement of the part? (d) What is the maximum displacement of the part? (e) Are both the minimum and maximum displacements physically attainable?

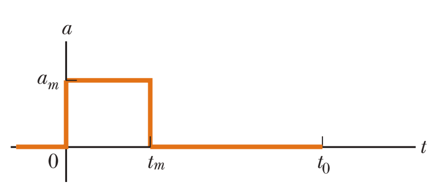


Figure P2.30

31. ● A glider on an air track carries a flag of length ℓ through a stationary photogate, which measures the time interval Δt_d during which the flag blocks a beam of infrared light passing across the photogate. The ratio $v_d = \ell/\Delta t_d$ is the average velocity of the glider over this part of its motion. Suppose the glider moves with constant acceleration. (a) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in space. (b) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in time.

32. ● Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? State how you decide. If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.

33. *Vroom, vroom!* As soon as a traffic light turns green, a car speeds up from rest to 50.0 mi/h with constant acceleration 9.00 mi/h · s. In the adjoining bike lane, a cyclist speeds up from rest to 20.0 mi/h with constant acceleration 13.0 mi/h · s. Each vehicle maintains constant velocity after reaching its cruising speed. (a) For what time interval is the bicycle ahead of the car? (b) By what maximum distance does the bicycle lead the car?

34. Solve Example 2.8 (Watch Out for the Speed Limit!) by a graphical method. On the same graph plot position versus time for the car and the police officer. From the intersection of the two curves read the time at which the trooper overtakes the car.

35. ● A glider of length 12.4 cm moves on an air track with constant acceleration. A time interval of 0.628 s elapses between the moment when its front end passes a fixed point Ⓐ along the track and the moment when its back end passes this point. Next, a time interval of 1.39 s elapses between the moment when the back end of the glider passes point Ⓐ and the moment when the front end of the glider passes a second point Ⓑ farther down the track. After that, an additional 0.431 s elapses until the back end of the glider passes point Ⓑ. (a) Find the average speed of the glider as it passes point Ⓐ. (b) Find the acceleration of the glider. (c) Explain how you can compute the acceleration without knowing the distance between points Ⓐ and Ⓑ.

Section 2.7 Freely Falling Objects

Note: In all problems in this section, ignore the effects of air resistance.

36. In a classic clip on *America's Funniest Home Videos*, a sleeping cat rolls gently off the top of a warm TV set. Ignoring air resistance, calculate (a) the position and (b) the velocity of the cat after 0.100 s, 0.200 s, and 0.300 s.

37. ● *Every morning at seven o'clock*
There's twenty terriers drilling on the rock.
The boss comes around and he says, "Keep still
And bear down heavy on the cast-iron drill

And drill, ye terriers, drill." And drill, ye terriers, drill.
It's work all day for sugar in your tea
Down beyond the railway. And drill, ye terriers, drill.

The foreman's name was John McAnn.
By God, he was a blamed mean man.
One day a premature blast went off
And a mile in the air went big Jim Goff. And drill . . .

Then when next payday came around
Jim Goff a dollar short was found.
When he asked what for, came this reply:
"You were docked for the time you were up in the sky."
And drill . . .

—American folksong

What was Goff's hourly wage? State the assumptions you make in computing it.

38. A ball is thrown directly downward, with an initial speed of 8.00 m/s, from a height of 30.0 m. After what time interval does the ball strike the ground?

39. ▲ A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

40. ● Emily challenges her friend David to catch a dollar bill as follows. She holds the bill vertically, as shown in Figure P2.40, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

George Semple

Figure P2.40

41. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) the ball's initial velocity and (b) the height it reaches.

42. ● An attacker at the base of a castle wall 3.65 m high throws a rock straight up with speed 7.40 m/s at a height of 1.55 m above the ground. (a) Will the rock reach the top of the wall? (b) If so, what is its speed at the top? If not, what initial speed must it have to reach the top? (c) Find the change in speed of a rock thrown straight down from the top of the wall at an initial speed of 7.40 m/s and moving between the same two points. (d) Does the change in speed of the downward-moving rock agree with the magnitude of the speed change of the rock moving upward between the same elevations? Explain physically why it does or does not agree.

43. ▲ A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The constant speed of the horse is 10.0 m/s, and the distance from the limb to the level of the saddle is 3.00 m. (a) What must the horizontal distance between the saddle and limb be when the ranch hand makes his move? (b) For what time interval is he in the air?

44. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

45. A freely falling object requires 1.50 s to travel the last 30.0 m before it hits the ground. From what height above the ground did it fall?

Section 2.8 Kinematic Equations Derived from Calculus

46. A student drives a moped along a straight road as described by the velocity-versus-time graph in Figure P2.46. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x–t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?

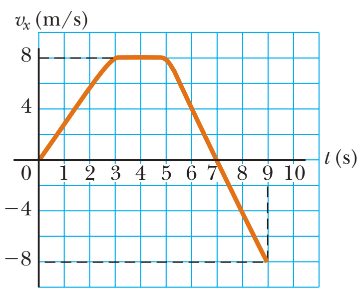


Figure P2.46

47. Automotive engineers refer to the time rate of change of acceleration as the "jerk." Assume an object moves in one dimension such that its jerk J is constant. (a) Determine expressions for its acceleration $a_x(t)$, velocity $v_x(t)$, and position $x(t)$, given that its initial acceleration, velocity, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^{\,2} = a_{xi}^{\,2} + 2J(v_x - v_{xi})$.

48. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by $v = (-5.00 \times 10^7)t^2 + (3.00 \times 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine the time interval over which the bullet is accelerated. (c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

Additional Problems

49. An object is at $x = 0$ at $t = 0$ and moves along the x axis according to the velocity–time graph in Figure P2.49. (a) What is the acceleration of the object between 0 and 4 s? (b) What is the acceleration of the object between 4 s and 9 s? (c) What is the acceleration of the object between 13 s and 18 s? (d) At what time(s) is the object moving with the lowest speed? (e) At what time is the object farthest from $x = 0$? (f) What is the final position x of the object at $t = 18$ s? (g) Through what total distance has the object moved between $t = 0$ and $t = 18$ s?

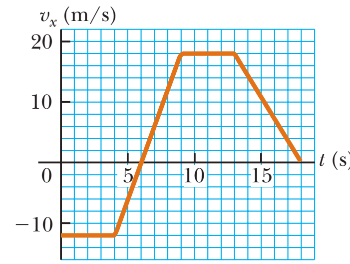


Figure P2.49

50. ● The Acela (pronounced ah-SELL-ah and shown in Fig. P2.50a) is an electric train on the Washington–New York–Boston run, carrying passengers at 170 mi/h. The carriages tilt as much as 6° from the vertical to prevent passengers from feeling pushed to the side as they go around curves. A velocity–time graph for the Acela is shown in Figure P2.50b. (a) Describe the motion of the train in each successive time interval. (b) Find the peak positive acceleration of the train in the motion graphed. (c) Find the train's displacement in miles between $t = 0$ and $t = 200$ s.

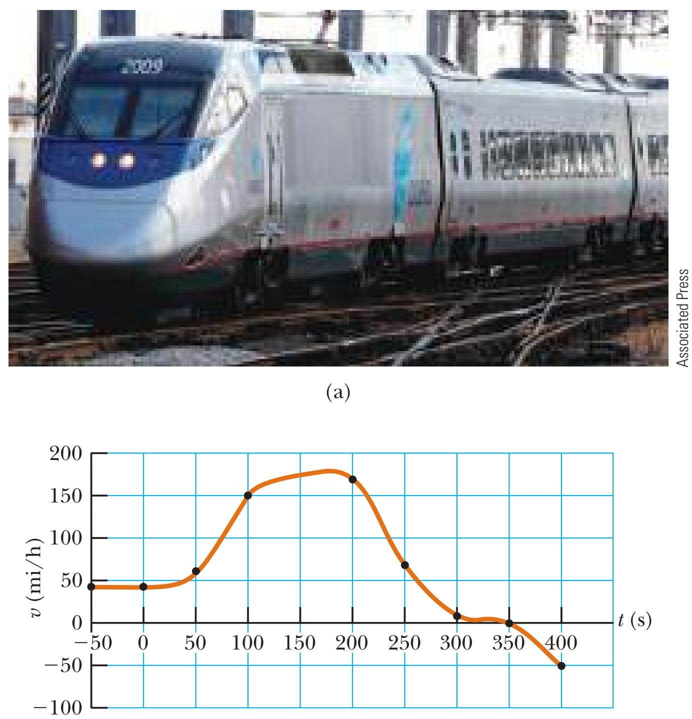


Figure P2.50 (a) The Acela: 1 171 000 lb of cold steel thundering along with 304 passengers. (b) Velocity-versus-time graph for the Acela.

51. A test rocket is fired vertically upward from a well. A catapult gives it an initial speed of 80.0 m/s at ground level.

Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of -9.80 m/s^2. (a) For what time interval is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion.)

52. ● In Active Figure 2.11b, the area under the velocity versus time curve and between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. Compute their areas and state how the sum of the two areas compares with the expression on the right-hand side of Equation 2.16.

53. Setting a world record in a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. Accelerating uniformly, Maggie took 2.00 s and Judy took 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark, and by how much?

54. ● *How long should a traffic light stay yellow?* Assume you are driving at the speed limit v_0. As you approach an intersection 22.0 m wide, you see the light turn yellow. During your reaction time of 0.600 s, you travel at constant speed as you recognize the warning, decide whether to stop or to go through the intersection, and move your foot to the brake if you must stop. Your car has good brakes and can accelerate at -2.40 m/s^2. Before it turns red, the light should stay yellow long enough for you to be able to get to the other side of the intersection without speeding up, if you are too close to the intersection to stop before entering it. (a) Find the required time interval Δt_y that the light should stay yellow in terms of v_0. Evaluate your answer for (b) $v_0 = 8.00$ m/s $= 28.8$ km/h, (c) $v_0 = 11.0$ m/s $= 40.2$ km/h, (d) $v_0 = 18.0$ m/s $= 64.8$ km/h, and (e) $v_0 = 25.0$ m/s $= 90.0$ km/h. **What If?** Evaluate your answer for (f) v_0 approaching zero, and (g) v_0 approaching infinity. (h) Describe the pattern of variation of Δt_y with v_0. You may wish also to sketch a graph of it. Account for the answers to parts (f) and (g) physically. (i) For what value of v_0 would Δt_y be minimal, and (j) what is this minimum time interval? *Suggestion:* You may find it easier to do part (a) after first doing part (b).

55. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. During rush hour the engineer minimizes the time interval Δt between two stations by accelerating for a time interval Δt_1 at a rate $a_1 = 0.100$ m/s^2 and then immediately braking with acceleration $a_2 = -0.500$ m/s^2 for a time interval Δt_2. Find the minimum time interval of travel Δt and the time interval Δt_1.

56. A Ferrari F50 of length 4.52 m is moving north on a roadway that intersects another perpendicular roadway. The width of the intersection from near edge to far edge is 28.0 m. The Ferrari has a constant acceleration of magnitude 2.10 m/s^2 directed south. The time interval required for the nose of the Ferrari to move from the near (south) edge of the intersection to the north edge of the intersection is 3.10 s. (a) How far is the nose of the Ferrari from the south edge of the intersection when it stops? (b) For what time interval is *any* part of the Ferrari within the boundaries of the intersection? (c) A Corvette is at rest on the perpendicular intersecting roadway. As the nose of the Ferrari enters the intersection, the Corvette starts from rest and accelerates east at 5.60 m/s^2. What is the minimum distance from the near (west) edge of the intersection at which the nose of the Corvette can begin its motion if the Corvette is to enter the intersection after the Ferrari has entirely left the intersection? (d) If the Corvette begins its motion at the position given by your answer to part (c), with what speed does it enter the intersection?

57. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What initial velocity must the second stone have if they are to hit simultaneously? (c) What is the speed of each stone at the instant the two hit the water?

58. ● A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Suppose the maximum depth of the dent is on the order of 1 cm. Compute an order-of-magnitude estimate for the maximum acceleration of the ball while it is in contact with the pavement. State your assumptions, the quantities you estimate, and the values you estimate for them.

59. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by racing with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2. Find (a) the time at which Kathy overtakes Stan, (b) the distance she travels before she catches him, and (c) the speeds of both cars at the instant she overtakes him.

60. A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is ignored, what percentage error is introduced when the depth of the well is calculated?

61. ● In a California driver's handbook, the following data were given about the minimum distance a typical car travels in stopping from various original speeds. The "thinking distance" represents how far the car travels during the driver's reaction time, after a reason to stop can be seen but before the driver can apply the brakes. The "braking distance" is the displacement of the car after the brakes are applied. (a) Is the thinking-distance data consistent with the assumption that the car travels with constant speed? Explain. (b) Determine the best value of the reaction time suggested by the data. (c) Is the braking-distance data consistent with the assumption that the car travels with constant acceleration? Explain. (d) Determine the best value for the acceleration suggested by the data.

Speed (mi/h)	Thinking Distance (ft)	Braking Distance (ft)	Total Stopping Distance (ft)
25	27	34	61
35	38	67	105
45	49	110	159
55	60	165	225
65	71	231	302

62. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time as given in the table in the next column. (a) Find the average velocity of the rock in the time interval between each measurement and the next. (b) Using these average velocities to approximate instantaneous velocities at the midpoints of the time intervals, make a graph of velocity as a function of time. Does the rock move with constant acceleration? If so, plot a straight line of best fit on the graph and calculate its slope to find the acceleration.

Time (s)	Height (m)	Time (s)	Height (m)
0.00	5.00	2.75	7.62
0.25	5.75	3.00	7.25
0.50	6.40	3.25	6.77
0.75	6.94	3.50	6.20
1.00	7.38	3.75	5.52
1.25	7.72	4.00	4.73
1.50	7.96	4.25	3.85
1.75	8.10	4.50	2.86
2.00	8.13	4.75	1.77
2.25	8.07	5.00	0.58
2.50	7.90		

63. Two objects, A and B, are connected by a rigid rod that has length L. The objects slide along perpendicular guide rails as shown in Figure P2.63. Assume A slides to the left with a constant speed v. Find the velocity of B when $\theta = 60.0°$.

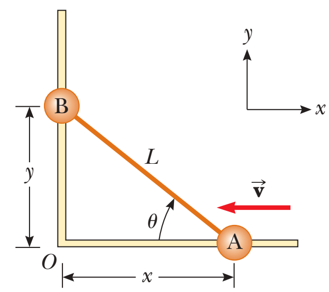


Figure P2.63

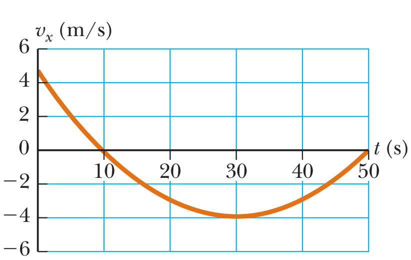
Answers to Quick Quizzes

2.1 (c). If the particle moves along a line without changing direction, the displacement and distance traveled over any time interval will be the same. As a result, the magnitude of the average velocity and the average speed will be the same. If the particle reverses direction, however, the displacement will be less than the distance traveled. In turn, the magnitude of the average velocity will be smaller than the average speed.

2.2 (b). Regardless of your speeds at all other times, if your instantaneous speed at the instant it is measured is higher than the speed limit, you may receive a speeding ticket.

2.3 (b). If the car is slowing down, a force must be pulling in the direction opposite to its velocity.

2.4 False. Your graph should look something like the following.



This v_x–t graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (= 11 mi/h), so the driver was not speeding.

2.5 (c). If a particle with constant acceleration stops and its acceleration remains constant, it must begin to move again in the opposite direction. If it did not, the acceleration would change from its original constant value to zero. Choice (a) is not correct because the direction of acceleration is not specified by the direction of the velocity. Choice (b) is also not correct by counterexample; a car moving in the $-x$ direction and slowing down has a positive acceleration.

2.6 Graph (a) has a constant slope, indicating a constant acceleration; it is represented by graph (e).

Graph (b) represents a speed that is increasing constantly but not at a uniform rate. Therefore, the acceleration must be increasing, and the graph that best indicates that is (d).

Graph (c) depicts a velocity that first increases at a constant rate, indicating constant acceleration. Then the velocity stops increasing and becomes constant, indicating zero acceleration. The best match to this situation is graph (f).

2.7 **(i)**, (e). For the entire time interval that the ball is in free fall, the acceleration is that due to gravity. **(ii)**, (d). While the ball is rising, it is slowing down. After reaching the highest point, the ball begins to fall and its speed increases.

These controls in the cockpit of a commercial aircraft assist the pilot in maintaining control over the velocity of the aircraft—how fast it is traveling and in what direction it is traveling—allowing it to land safely. Quantities that are defined by both a magnitude and a direction, such as velocity, are called *vector quantities*. (Mark Wagner/Getty Images)

3 Vectors

In our study of physics, we often need to work with physical quantities that have both numerical and directional properties. As noted in Section 2.1, quantities of this nature are vector quantities. This chapter is primarily concerned with general properties of vector quantities. We discuss the addition and subtraction of vector quantities, together with some common applications to physical situations.

Vector quantities are used throughout this text. Therefore, it is imperative that you master the techniques discussed in this chapter.

3.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. In Chapter 2, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. In two dimensions, this description is accomplished with the use of the Cartesian coordinate system, in which perpendicular axes intersect at a point defined as the origin (Fig. 3.1). Cartesian coordinates are also called *rectangular coordinates*.

Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ) as shown in Active Figure 3.2a (see page 54). In this *polar coordinate system*, r is the distance from the origin to the point having Cartesian coordinates (x, y) and θ is the angle between a fixed axis and a line drawn from the origin to the point. The fixed axis is often the positive x axis, and θ is usually measured counterclockwise from it. From the right triangle in Active Figure 3.2b,

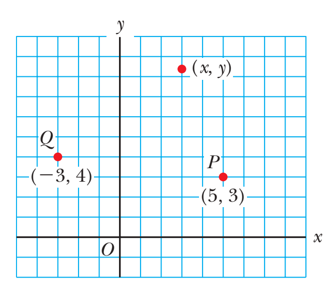
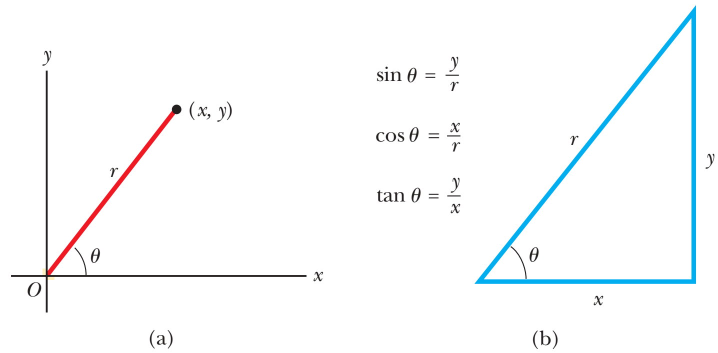


Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).



ACTIVE FIGURE 3.2

(a) The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ).

Sign in at www.thomsonedu.com and go to ThomsonNOW to move the point and see the changes to the rectangular and polar coordinates as well as to the sine, cosine, and tangent of angle θ.

we find that $\sin\theta = y/r$ and that $\cos\theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations

$$x = r\cos\theta \tag{3.1}$$

$$y = r\sin\theta \tag{3.2}$$

Furthermore, the definitions of trigonometry tell us that

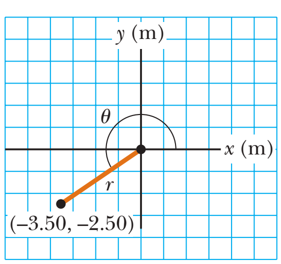
$$\tan\theta = \frac{y}{x} \tag{3.3}$$

$$r = \sqrt{x^2 + y^2} \tag{3.4}$$

Equation 3.4 is the familiar Pythagorean theorem.

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as shown in Active Figure 3.2a—in other words, when positive θ is an angle measured counterclockwise from the positive x axis. (Some scientific calculators perform conversions between Cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, the expressions relating the two sets of coordinates will change.

EXAMPLE 3.1 Polar Coordinates



ACTIVE FIGURE 3.3

(Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

Sign in at www.thomsonedu.com and go to ThomsonNOW to move the point in the xy plane and see how its Cartesian and polar coordinates change.

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m as shown in Active Figure 3.3. Find the polar coordinates of this point.

SOLUTION

Conceptualize The drawing in Active Figure 3.3 helps us conceptualize the problem.

Categorize Based on the statement of the problem and the Conceptualize step, we recognize that we are simply converting from Cartesian coordinates to polar coordinates. We therefore categorize this example as a substitution problem. Substitution problems generally do not have an extensive Analyze step other than the substitution of numbers into a given equation. Similarly, the Finalize step consists primarily of checking the units and making sure that the answer is reasonable. Therefore, for substitution problems, we will not label Analyze or Finalize steps.

Use Equation 3.4 to find r: $$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = \boxed{4.30\text{ m}}$$

Use Equation 3.3 to find θ: $$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = \boxed{216°}$$

Notice that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$, not $35.5°$.

3.2 Vector and Scalar Quantities

We now formally describe the difference between scalar quantities and vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit "degrees C" or "degrees F." Temperature is therefore an example of a *scalar quantity*:

> A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, speed, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a *vector quantity*:

> A **vector quantity** is completely specified by a number and appropriate units plus a direction.

Another example of a vector quantity is displacement, as you know from Chapter 2. Suppose a particle moves from some point Ⓐ to some point Ⓑ along a straight path as shown in Figure 3.4. We represent this displacement by drawing an arrow from Ⓐ to Ⓑ, with the tip of the arrow pointing away from the starting point. The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from Ⓐ to Ⓑ, such as shown by the broken line in Figure 3.4, its displacement is still the arrow drawn from Ⓐ to Ⓑ. Displacement depends only on the initial and final positions, so the displacement vector is independent of the path taken by the particle between these two points.

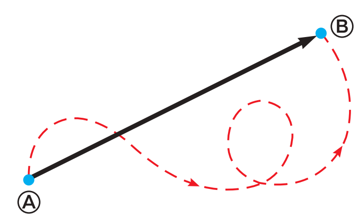


Figure 3.4 As a particle moves from Ⓐ to Ⓑ along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

In this text, we use a boldface letter with an arrow over the letter, such as $\vec{\mathbf{A}}$, to represent a vector. Another common notation for vectors with which you should be familiar is a simple boldface character: **A**. The magnitude of the vector $\vec{\mathbf{A}}$ is written either A or $|\vec{\mathbf{A}}|$. The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity. The magnitude of a vector is *always* a positive number.

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

3.3 Some Properties of Vectors

In this section, we shall investigate general properties of vectors representing physical quantities. We also discuss how to add and subtract vectors using both algebraic and geometric methods.

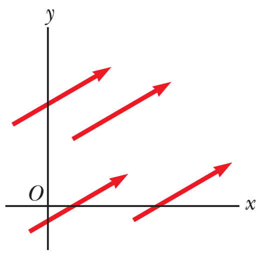


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

Equality of Two Vectors

For many purposes, two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ may be defined to be equal if they have the same magnitude and if they point in the same direction. That is, $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ only if $A = B$ and if $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

Adding Vectors

The rules for adding vectors are conveniently described by a graphical method. To add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$, first draw vector $\vec{\mathbf{A}}$ on graph paper, with its magnitude represented by a convenient length scale, and then draw vector $\vec{\mathbf{B}}$ to the same scale, with its tail starting from the tip of $\vec{\mathbf{A}}$, as shown in Active Figure 3.6. The **resultant vector** $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

A geometric construction can also be used to add more than two vectors as is shown in Figure 3.7 for the case of four vectors. The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$ is the vector that completes the polygon. In other words, **$\vec{\mathbf{R}}$ is the vector drawn from the tail of the first vector to the tip of the last vector**. This technique for adding vectors is often called the "head to tail method."

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, but as you will see in Chapter 11, the order is important when vectors are multiplied. Procedures for multiplying vectors are discussed in Chapters 7 and 11). This property, which can be seen from the geometric construction in Figure 3.8, is known as the **commutative law of addition:**

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}} \qquad \textbf{(3.5)}$$

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 3.9. This property is called the **associative law of addition:**

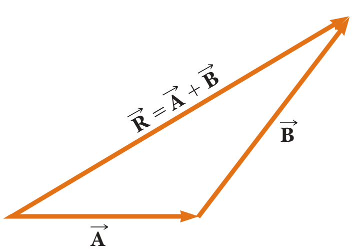
$$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} \qquad \textbf{(3.6)}$$

In summary, **a vector quantity has both magnitude and direction and also obeys the laws of vector addition** as described in Figures 3.6 to 3.9. When two or more vectors are added together, they must all have the same units and they must all be the same type of quantity. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because these vectors represent different physical quantities. The same

PITFALL PREVENTION 3.1

Vector Addition versus Scalar Addition

Notice that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$ is very different from $A + B = C$. The first equation is a vector sum, which must be handled carefully, such as with the graphical method. The second equation is a simple algebraic addition of numbers that is handled with the normal rules of arithmetic.



ACTIVE FIGURE 3.6

When vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$, the resultant $\vec{\mathbf{R}}$ is the vector that runs from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

Sign in at www.thomsonedu.com and go to ThomsonNOW to explore the addition of two vectors.

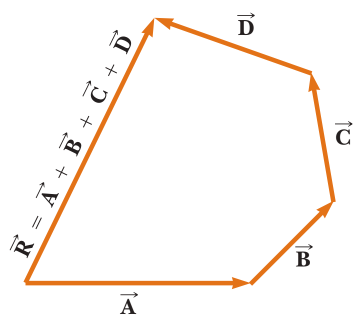


Figure 3.7 Geometric construction for summing four vectors. The resultant vector $\vec{\mathbf{R}}$ is by definition the one that completes the polygon.

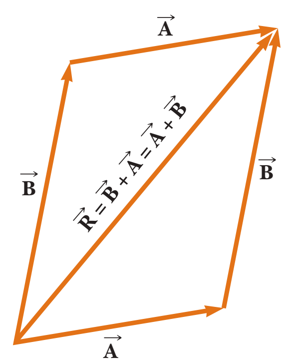


Figure 3.8 This construction shows that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$ or, in other words, that vector addition is commutative.

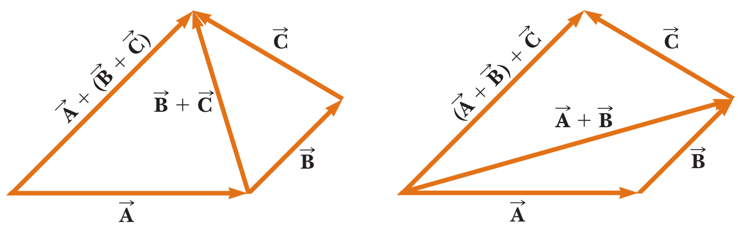


Figure 3.9 Geometric constructions for verifying the associative law of addition.

rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

Negative of a Vector

The negative of the vector $\vec{\mathbf{A}}$ is defined as the vector that when added to $\vec{\mathbf{A}}$ gives zero for the vector sum. That is, $\vec{\mathbf{A}} + (-\vec{\mathbf{A}}) = 0$. The vectors $\vec{\mathbf{A}}$ and $-\vec{\mathbf{A}}$ have the same magnitude but point in opposite directions.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ as vector $-\vec{\mathbf{B}}$ added to vector $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}}) \tag{3.7}$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.10a.

Another way of looking at vector subtraction is to notice that the difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is what you have to add to the second vector to obtain the first. In this case, as Figure 3.10b shows, the vector $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ points from the tip of the second vector to the tip of the first.

Multiplying a Vector by a Scalar

If vector $\vec{\mathbf{A}}$ is multiplied by a positive scalar quantity m, the product $m\vec{\mathbf{A}}$ is a vector that has the same direction as $\vec{\mathbf{A}}$ and magnitude mA. If vector $\vec{\mathbf{A}}$ is multiplied by a negative scalar quantity $-m$, the product $-m\vec{\mathbf{A}}$ is directed opposite $\vec{\mathbf{A}}$. For example, the vector $5\vec{\mathbf{A}}$ is five times as long as $\vec{\mathbf{A}}$ and points in the same direction as $\vec{\mathbf{A}}$; the vector $-\frac{1}{3}\vec{\mathbf{A}}$ is one-third the length of $\vec{\mathbf{A}}$ and points in the direction opposite $\vec{\mathbf{A}}$.

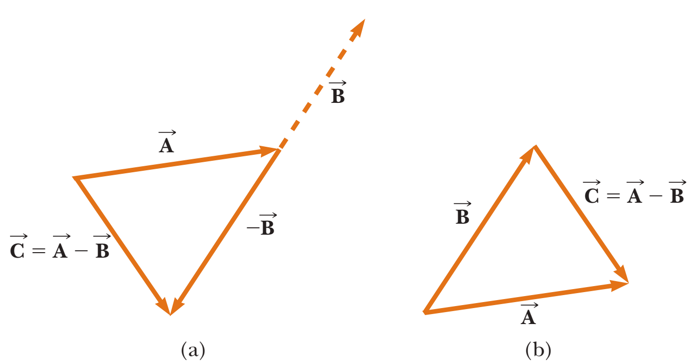


Figure 3.10 (a) This construction shows how to subtract vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$. The vector $-\vec{\mathbf{B}}$ is equal in magnitude to vector $\vec{\mathbf{B}}$ and points in the opposite direction. To subtract $\vec{\mathbf{B}}$ from $\vec{\mathbf{A}}$, apply the rule of vector addition to the combination of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$: first draw $\vec{\mathbf{A}}$ along some convenient axis and then place the tail of $-\vec{\mathbf{B}}$ at the tip of $\vec{\mathbf{A}}$, and $\vec{\mathbf{C}}$ is the difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$. (b) A second way of looking at vector subtraction. The difference vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ is the vector that we must add to $\vec{\mathbf{B}}$ to obtain $\vec{\mathbf{A}}$.

Quick Quiz 3.2 The magnitudes of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are $A = 12$ units and $B = 8$ units. Which of the following pairs of numbers represents the *largest* and *smallest* possible values for the magnitude of the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers

Quick Quiz 3.3 If vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$, which *two* of the following choices must be true for the resultant vector to be equal to zero? (a) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in the same direction. (b) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in opposite directions. (c) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitude. (d) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular.

EXAMPLE 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in Figure 3.11a. Find the magnitude and direction of the car's resultant displacement.

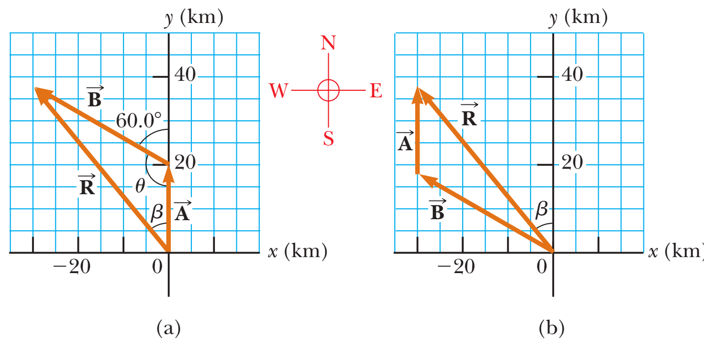


Figure 3.11 (Example 3.2) (a) Graphical method for finding the resultant displacement vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (b) Adding the vectors in reverse order ($\vec{\mathbf{B}} + \vec{\mathbf{A}}$) gives the same result for $\vec{\mathbf{R}}$.

SOLUTION

Conceptualize The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ drawn in Figure 3.11a help us conceptualize the problem.

Categorize We can categorize this example as a simple analysis problem in vector addition. The displacement $\vec{\mathbf{R}}$ is the resultant when the two individual displacements $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added. We can further categorize it as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

Analyze In this example, we show two ways to analyze the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of $\vec{\mathbf{R}}$ and its direction in Figure 3.11a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of $\vec{\mathbf{R}}$ can be obtained from the law of cosines as applied to the triangle (see Appendix B.4).

Use $R^2 = A^2 + B^2 - 2AB\cos\theta$ from the law of cosines to find R:

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Substitute numerical values, noting that $\theta = 180° - 60° = 120°$:

$$R = \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$

$$= \boxed{48.2\text{ km}}$$

Use the law of sines (Appendix B.4) to find the direction of $\vec{\mathbf{R}}$ measured from the northerly direction:

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = \boxed{38.9°}$$

The resultant displacement of the car is 48.2 km in a direction 38.9° west of north.

Finalize Does the angle β that we calculated agree with an estimate made by looking at Figure 3.11a or with an actual angle measured from the diagram using the graphical method? Is it reasonable that the magnitude of $\vec{\mathbf{R}}$ is larger than that of both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$? Are the units of $\vec{\mathbf{R}}$ correct?

Although the graphical method of adding vectors works well, it suffers from two disadvantages. First, some people find using the laws of cosines and sines to be awkward. Second, a triangle only results if you are adding two vectors. If you are adding three or more vectors, the resulting geometric shape is usually not a triangle. In Section 3.4, we explore a new method of adding vectors that will address both of these disadvantages.

What If? Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

Answer They would not change. The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant. Graphically, Figure 3.11b shows that the vectors added in the reverse order give us the same resultant vector.

3.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector or its **rectangular components**. Any vector can be completely described by its components.

Consider a vector $\vec{\mathbf{A}}$ lying in the xy plane and making an arbitrary angle θ with the positive x axis as shown in Figure 3.12a. This vector can be expressed as the sum of two other *component vectors* $\vec{\mathbf{A}}_x$, which is parallel to the x axis, and $\vec{\mathbf{A}}_y$, which is parallel to the y axis. From Figure 3.12b, we see that the three vectors form a right triangle and that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$. We shall often refer to the "components of a vector $\vec{\mathbf{A}}$," written A_x and A_y (without the boldface notation). The component A_x represents the projection of $\vec{\mathbf{A}}$ along the x axis, and the component A_y represents the projection of $\vec{\mathbf{A}}$ along the y axis. These components can be positive or negative. The component A_x is positive if the component vector $\vec{\mathbf{A}}_x$ points in the positive x direction and is negative if $\vec{\mathbf{A}}_x$ points in the negative x direction. The same is true for the component A_y.

From Figure 3.12 and the definition of sine and cosine, we see that $\cos\theta = A_x/A$ and that $\sin\theta = A_y/A$. Hence, the components of $\vec{\mathbf{A}}$ are

$$A_x = A\cos\theta \quad \textbf{(3.8)}$$

$$A_y = A\sin\theta \quad \textbf{(3.9)}$$

◀ Components of the vector $\vec{\mathbf{A}}$

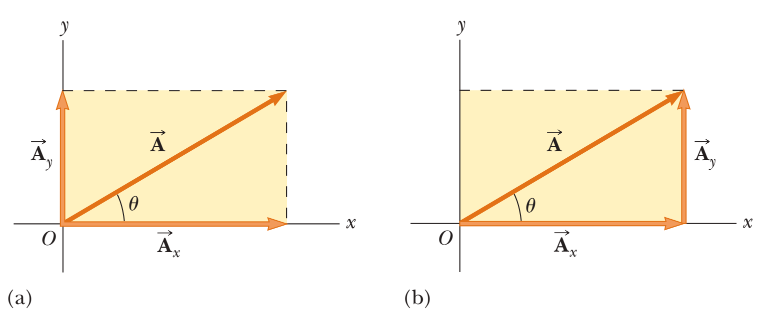


Figure 3.12 (a) A vector $\vec{\mathbf{A}}$ lying in the xy plane can be represented by its component vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$. (b) The y component vector $\vec{\mathbf{A}}_y$ can be moved to the right so that it adds to $\vec{\mathbf{A}}_x$. The vector sum of the component vectors is $\vec{\mathbf{A}}$. These three vectors form a right triangle.

PITFALL PREVENTION 3.2

Component Vectors versus Components

The vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the *component vectors* of $\vec{\mathbf{A}}$. They should not be confused with the quantities A_x and A_y, which we shall always refer to as the *components* of $\vec{\mathbf{A}}$.

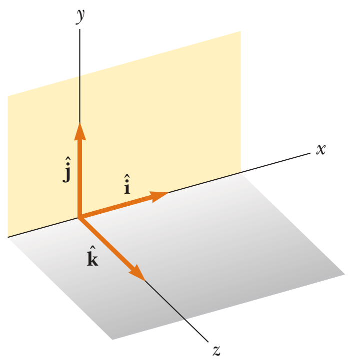
PITFALL PREVENTION 3.3

***x* and *y* Components**

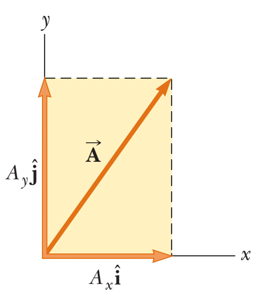
Equations 3.8 and 3.9 associate the cosine of the angle with the x component and the sine of the angle with the y component. This association is true *only* because we measured the angle θ with respect to the x axis, so do not memorize these equations. If θ is measured with respect to the y axis (as in some problems), these equations will be incorrect. Think about which side of the triangle containing the components is adjacent to the angle and which side is opposite and then assign the cosine and sine accordingly.

A_x negative	A_x positive
A_y positive	A_y positive
A_x negative	A_x positive
A_y negative	A_y negative

Figure 3.13 The signs of the components of a vector $\vec{\mathbf{A}}$ depend on the quadrant in which the vector is located.



(a)



(b)

ACTIVE FIGURE 3.14

(a) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are directed along the x, y, and z axes, respectively. (b) Vector $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ lying in the xy plane has components A_x and A_y.

Sign in at www.thomsonedu.com and go to ThomsonNOW to rotate the coordinate axes in three-dimensional space and view a representation of vector $\vec{\mathbf{A}}$ in three dimensions.

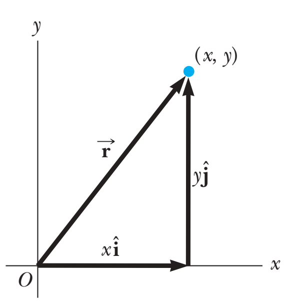


Figure 3.15 The point whose Cartesian coordinates are (x, y) can be represented by the position vector $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

The magnitudes of these components are the lengths of the two sides of a right triangle with a hypotenuse of length A. Therefore, the magnitude and direction of $\vec{\mathbf{A}}$ are related to its components through the expressions

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} \tag{3.10}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3.11}$$

Notice that **the signs of the components A_x and A_y depend on the angle** θ. For example, if $\theta = 120°$, A_x is negative and A_y is positive. If $\theta = 225°$, both A_x and A_y are negative. Figure 3.13 summarizes the signs of the components when $\vec{\mathbf{A}}$ lies in the various quadrants.

When solving problems, you can specify a vector $\vec{\mathbf{A}}$ either with its components A_x and A_y or with its magnitude and direction A and θ.

Suppose you are working a physics problem that requires resolving a vector into its components. In many applications, it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but that are still perpendicular to each other. For example, we will consider the motion of objects sliding down inclined planes. For these examples, it is often convenient to orient the x axis parallel to the plane and the y axis perpendicular to the plane.

Quick Quiz 3.4 Choose the correct response to make the sentence true: A component of a vector is (a) always, (b) never, or (c) sometimes larger than the magnitude of the vector.

Unit Vectors

Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a bookkeeping convenience in describing a direction in space. We shall use the symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ to represent unit vectors pointing in the positive x, y, and z directions, respectively. (The "hats," or circumflexes, on the symbols are a standard notation for unit vectors.) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ form a set of mutually perpendicular vectors in a right-handed coordinate system as shown in Active Figure 3.14a. The magnitude of each unit vector equals 1; that is, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$.

Consider a vector $\vec{\mathbf{A}}$ lying in the xy plane as shown in Active Figure 3.14b. The product of the component A_x and the unit vector $\hat{\mathbf{i}}$ is the component vector $\vec{\mathbf{A}}_x = A_x\hat{\mathbf{i}}$, which lies on the x axis and has magnitude $|A_x|$. Likewise, $\vec{\mathbf{A}}_y = A_y\hat{\mathbf{j}}$ is the component vector of magnitude $|A_y|$ lying on the y axis. Therefore, the unit–vector notation for the vector $\vec{\mathbf{A}}$ is

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{3.12}$$

For example, consider a point lying in the xy plane and having Cartesian coordinates (x, y) as in Figure 3.15. The point can be specified by the **position vector** $\vec{\mathbf{r}}$, which in unit–vector form is given by

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \tag{3.13}$$

This notation tells us that the components of $\vec{\mathbf{r}}$ are the coordinates x and y.

Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$ in Equation 3.12, where vector $\vec{\mathbf{B}}$ has components B_x and B_y. Because of the bookkeeping convenience of the unit vectors, all we do is add the x and y components separately. The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is

$$\vec{\mathbf{R}} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

or

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \tag{3.14}$$

Because $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$\begin{aligned} R_x &= A_x + B_x \\ R_y &= A_y + B_y \end{aligned} \tag{3.15}$$

The magnitude of $\vec{\mathbf{R}}$ and the angle it makes with the x axis from its components are obtained using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \tag{3.16}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \tag{3.17}$$

We can check this addition by components with a geometric construction as shown in Figure 3.16. Remember to note the signs of the components when using either the algebraic or the graphical method.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ both have x, y, and z components, they can be expressed in the form

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \tag{3.18}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \tag{3.19}$$

The sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}} \tag{3.20}$$

Notice that Equation 3.20 differs from Equation 3.14: in Equation 3.20, the resultant vector also has a z component $R_z = A_z + B_z$. If a vector $\vec{\mathbf{R}}$ has x, y, and z components, the magnitude of the vector is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. The angle θ_x that $\vec{\mathbf{R}}$ makes with the x axis is found from the expression $\cos\theta_x = R_x/R$, with similar expressions for the angles with respect to the y and z axes.

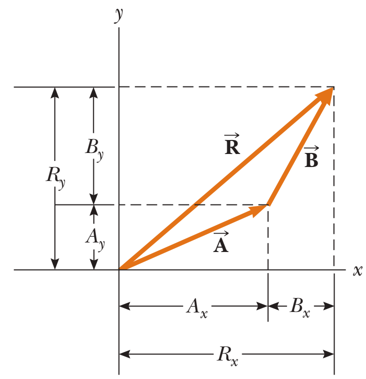


Figure 3.16 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant $\vec{\mathbf{R}}$ and the components of the individual vectors.

PITFALL PREVENTION 3.4

Tangents on Calculators

Equation 3.17 involves the calculation of an angle by means of a tangent function. Generally, the inverse tangent function on calculators provides an angle between $-90°$ and $+90°$. As a consequence, if the vector you are studying lies in the second or third quadrant, the angle measured from the positive x axis will be the angle your calculator returns plus 180°.

Quick Quiz 3.5 For which of the following vectors is the magnitude of the vector equal to one of the components of the vector? (a) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ (b) $\vec{\mathbf{B}} = -3\hat{\mathbf{j}}$ (c) $\vec{\mathbf{C}} = +5\hat{\mathbf{k}}$

EXAMPLE 3.3 The Sum of Two Vectors

Find the sum of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the xy plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \qquad \text{and} \qquad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

SOLUTION

Conceptualize You can conceptualize the situation by drawing the vectors on graph paper.

Categorize We categorize this example as a simple substitution problem. Comparing this expression for $\vec{\mathbf{A}}$ with the general expression $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, we see that $A_x = 2.0$ m and $A_y = 2.0$ m. Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m.

Use Equation 3.14 to obtain the resultant vector $\vec{\mathbf{R}}$: $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m}$

Evaluate the components of $\vec{\mathbf{R}}$: $R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$

Use Equation 3.16 to find the magnitude of $\vec{\mathbf{R}}$: $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0 \text{ m})^2 + (-2.0 \text{ m})^2} = \sqrt{20} \text{ m} = 4.5 \text{ m}$

Find the direction of $\vec{\mathbf{R}}$ from Equation 3.17: $\tan\theta = \frac{R_y}{R_x} = \frac{-2.0 \text{ m}}{4.0 \text{ m}} = -0.50$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\theta = 333°$

EXAMPLE 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm, and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find the components of the resultant displacement and its magnitude.

SOLUTION

Conceptualize Although x is sufficient to locate a point in one dimension, we need a vector $\vec{\mathbf{r}}$ to locate a point in two or three dimensions. The notation $\Delta\vec{\mathbf{r}}$ is a generalization of the one-dimensional displacement Δx in Equation 2.1. Three-dimensional displacements are more difficult to conceptualize than those in two dimensions because the latter can be drawn on paper.

For this problem, let us imagine that you start with your pencil at the origin of a piece of graph paper on which you have drawn x and y axes. Move your pencil 15 cm to the right along the x axis, then 30 cm upward along the y axis, and then 12 cm *perpendicularly toward you away* from the graph paper. This procedure provides the displacement described by $\Delta\vec{\mathbf{r}}_1$. From this point, move your pencil 23 cm to the right parallel to the x axis, then 14 cm parallel to the graph paper in the $-y$ direction, and then 5.0 cm perpendicularly away from you toward the graph paper. You are now at the displacement from the origin described by $\Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2$. From this point, move your pencil 13 cm to the left in the $-x$ direction, and (finally!) 15 cm parallel to the graph paper along the y axis. Your final position is at a displacement $\Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3$ from the origin.

Categorize Despite the difficulty in conceptualizing in three dimensions, we can categorize this problem as a substitution problem because of the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way, as we see below.

To find the resultant displacement, add the three vectors:

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3 \\ &= (15 + 23 - 13)\hat{\mathbf{i}} \text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}} \text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}} \text{ cm} \\ &= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}}) \text{ cm}\end{aligned}$$

Find the magnitude of the resultant vector:

$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2 + R_z^2} \\ &= \sqrt{(25 \text{ cm})^2 + (31 \text{ cm})^2 + (7.0 \text{ cm})^2} = 40 \text{ cm}\end{aligned}$$

EXAMPLE 3.5 Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

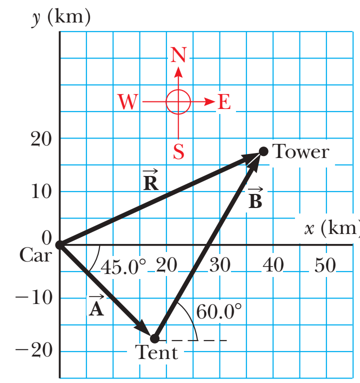


Figure 3.17 (Example 3.5) The total displacement of the hiker is the vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

(A) Determine the components of the hiker's displacement for each day.

SOLUTION

Conceptualize We conceptualize the problem by drawing a sketch as in Figure 3.17. If we denote the displacement vectors on the first and second days by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.17.

Categorize Drawing the resultant $\vec{\mathbf{R}}$, we can now categorize this problem as one we've solved before: an addition of two vectors. You should now have a hint of the power of categorization in that many new problems are very similar to problems we have already solved if we are careful to conceptualize them. Once we have drawn the displacement vectors and categorized the problem, this problem is no longer about a hiker, a walk, a car, a tent, or a tower. It is a problem about vector addition, one that we have already solved.

Analyze Displacement $\vec{\mathbf{A}}$ has a magnitude of 25.0 km and is directed 45.0° below the positive x axis.

Find the components of $\vec{\mathbf{A}}$ using Equations 3.8 and 3.9:

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = -17.7\text{ km}$$

The negative value of A_y indicates the hiker walks in the negative y direction on the first day. The signs of A_x and A_y also are evident from Figure 3.17.

Find the components of $\vec{\mathbf{B}}$ using Equations 3.8 and 3.9:

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

SOLUTION

Use Equation 3.15 to find the components of the resultant displacement $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

Write the total displacement in unit–vector form:

$$\vec{\mathbf{R}} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}$$

Finalize Looking at the graphical representation in Figure 3.17, we estimate the position of the tower to be about (38 km, 17 km), which is consistent with the components of $\vec{\mathbf{R}}$ in our result for the final position of the hiker. Also, both components of $\vec{\mathbf{R}}$ are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.17.

What If? After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

Answer The desired vector $\vec{\mathbf{R}}_{\text{car}}$ is the negative of vector $\vec{\mathbf{R}}$:

$$\vec{\mathbf{R}}_{\text{car}} = -\vec{\mathbf{R}} = (-37.7\hat{\mathbf{i}} - 16.9\hat{\mathbf{j}})\text{ km}$$

The heading is found by calculating the angle that the vector makes with the x axis:

$$\tan\theta = \frac{R_{\text{car},y}}{R_{\text{car},x}} = \frac{-16.9\text{ km}}{-37.7\text{ km}} = 0.448$$

which gives an angle of $\theta = 204.1°$, or 24.1° south of west.