Long-Run Confidence: Estimating Confidence Intervals when using Long-Run Multipliers

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Abstract

Researchers are often interested in the long-run relationship between variables where the dependent variable has dynamic properties. Though determining the long-run multiplier (LRM) for an independent variable is straightforward, correctly estimating the significance of the LRM is often difficult, especially when panel lengths are short and tests for series' stationarity are uncertain. We propose a Bayesian framework for estimating the LRM by using a bounded prior on the lagged dependent variable to constrain estimates for dynamic processes to the plausible range of values arising from either stationary or integrated series, and then taking draws of the posterior distribution to summarize the credible region. Doing so provides direct estimates of the LRM and its uncertainty, even for short time series. We highlight the advantages of this approach via Monte Carlo experiments and replicate several studies to show that our method clarifies long-run relationships that were inconclusive using existing techniques.

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Applied time series researchers often face difficult choices. It can be difficult to determine the best, let alone correct, specification. In most cases, the first step in this process is to test the stationarity of our series and make our modeling choices based on these diagnostics. Philips (2018), for instance, provides a remarkably useful flowchart of choices that one should make based on the results of a set of diagnostics. The difficulty is that many of the tests are low-powered, and our series frequently have a small number of observations. Too often, different tests will provide inconclusive or contradictory results. The applied researcher, then, has to do what she thinks is best and hope that readers and reviewers agree.

For a short period, a misreading of De Boef and Keele (2008) led some scholars to act as if a generalized error correction model (GECM) was a panacea for these problems. Grant and Lebo (2016) noted the difficulties with this approach and reiterated the need for effective diagnostics of the properties of time series. Grant and Lebo (2016) and Philips (2018) serve as useful summaries of the issues that time series analysts face and introduce the associated bounds testing procedure of Pesaran, Shin, and Smith (2001) (PSS) to political science. This approach recognizes the uncertainty we often have about the stationarity of our independent variables. Unfortunately, these early discussions still treat the diagnostics about the dependent variable as definitive. Philips (2018) has clear proscriptions for how to approach modeling time series if the dependent variable is stationary or non-stationary. If one can trust the knife-edged tests of stationarity, then the recommended approach is relatively straightforward, and one can simply follow the recipe that Philips provides.

Webb, Linn, and Lebo (2019) (WLL) reminded practitioners that these unit root tests are rarely certain. They advocate for a bounds approach that focuses on the long-run multiplier (LRM) that summarizes the relationship between the independent and dependent variables. They show that the significance of the LRM is a test of the presence of the long-run relationship (LRR) between the variables, regardless of stationarity. Importantly, they also provide the bounds of the t-test of the LRM that applied researchers can use to inform their conclusions about the significance of the relationship between the variables. It is difficult to overstate the importance of this result for applied time series researchers. One can use the test they provide regardless of the clarity of the stationarity tests for the dependent and independent variables. All that is needed is to estimate either a GECM or an autoregressive distributed lag model, calculate the LRM and an estimate of the uncertainty of the LRM, and calculate the ratio of these two.

This solution is straightforward and elegant. It has a single problem: the estimates of the uncertainty in the LRM are complicated. The LRM is a ratio of two coefficients, and there is "no simple formula for calculating the standard error of a ratio of coefficients" (Webb, Linn, and Lebo, 2019: 287). There are two methods for *approximating* the variance of the LRM—the delta method and the Bewley transformation, both of which are asymptotically equivalent to the direct estimation of the standard error of the LRM. WLL show that the distribution of the ratio of the LRM and its standard error is, however, not standard. Their solution is to run a series of dynamic simulations and develop critical values of the test statistic.

This is a smart approach, but it has one limitation for applied researchers. The bounds method that WLL use has a range of values where the hypothesis of a long-run relationship between X and y is rejected, a range where it is not rejected, and a range of values that is indeterminate. Their advice is to treat results that fall in this indeterminate range as failing to reject the null hypothesis of no relationship and be transparent about the lack of a definitive conclusion. This is likely to frustrate many applied researchers. An indeterminate answer to a research question is generally unsatisfying, even if it is intellectually honest.

This frustration can be mitigated somewhat. In this manuscript, we develop a very simple Bayesian estimator of the LRM that does not have this indeterminacy. We start by using a bounded, uniform prior for the estimated coefficient on the lagged DV that constrains the resulting dynamic relationship to the plausible range of values from either stationary or integrated series. We then take advantage of the well-known property of Markov chain Monte Carlo (MCMC) models where one can estimate and summarize the distribution of functions of parameters (e.g., ratios of coefficients) directly from the posterior distribution (Gelfand et al, 1990; Murr, Traunmüller, and Gill, 2023). This framework requires minimal additional assumptions over the approach suggested by WLL and is easy to estimate in most software. One could incorporate more information through the use of informative priors in the estimation, but this is not our intention here. We show that very diffuse priors enable the use of MCMC methods and the direct estimation of uncertainty of the LRM.

Bayesian estimation with a semi-informed prior to estimate uncertainty for the LRM offers a number of benefits. The semi-informed prior accommodates series of X and y where the dynamic properties of each series is unclear by limiting the range of the coefficient on y_{t-1} to their theoretical bounds but, by giving equal density to the values between these bounds, does not bias point estimates. The Bayesian inferential model, through its treatment of parameters as random, data as fixed, and use of sampling-based methods for estimation, helps overcome issues related to classification and low power. Together, the semi-informed prior and small sample properties result in more accurate and reliable estimates of uncertainty than alternatives that rely on asymptotic assumptions that may not hold.

In the next section of this paper, we revisit the results presented by WLL, demonstrating the importance of the significance tests of the LRM. Next, we provide the very simple MCMC approach to testing for the presence of an LRR. We use two Monte Carlo experiments to demonstrate key properties of our approach and compare them with the bounds approach. We then apply our approach to three empirical applications: two included in the original WLL (2019) work and one recent additional publication. These results show that the Bayesian approach to estimating the significance of the LRM and the presence of the LRR can reduce the indeterminacy that researchers face while still embracing and describing uncertainty in their estimates.

Long-Run Relationships and Hypothesis Testing

Most applied time series work in political science is intended to test for some relationship between one or more weakly exogenous independent variables, X, and a dependent variable y.[1] The key to these models is the existence of an LRR between X and y, which implies that there is a long-run equilibrium between the two. While the presence of an equilibrium means that the variables tend to not change over time, the practical implication is that it is the place where the variables tend to return to when they do deviate (Banerjee et al, 1993; Box-Steffensmeier et al, 2014; Burke, Hunter, and Canepa, 2017; Webb, Linn, and Lebo, 2019, 2020).

The particular nature of the equilibrium depends on the stationarity of the series. For a stationary series, the mean is the equilibrium. It will eventually revert to it when it deviates from this mean. The particular type of equilibrium depends on the relationship between X and y. As WLL note, if the equilibrium of y is a function of X, then there is a *conditional stationary equilibrium*, and if the equilibrium of y does not depend on X, then there is an *unconditional stationary equilibrium*. In contrast, a variable that is non-stationary, by definition, does not have an equilibrium level that it will tend to return to. The notion of the "random walk" is that this type of series will move randomly and not tend to move back to some mean level. This type of series, however, can have an equilibrium based on a relationship with X. If X also has a unit root, then a cointegrating equilibrium can exist between X and y, where y will tend to move together over time. In this case, there is a *cointegrating equilibrium*.

Traditionally, diagnosing the type of equilibrium is an essential step in testing for the LRR between X and y. The tests we use for our hypotheses and the critical values of those tests depend on these diagnostics. Getting the diagnostic wrong likely means that we will get the substantive conclusions wrong. This is the heart of an exchange on time series

[1] We follow WLL and denote X as a set of multiple regressors and x to indicate a single regressor. We also note that there are many reasons why a researcher might be interested in multiple dependent variables, but we are focusing our attention on models with a single dependent variable.

analysis in *Political Analysis*. Grant and Lebo (2016) demonstrate that if the researcher gets the diagnostics incorrect, or if they simply run a GECM without paying attention to the properties of the series, they can make remarkable errors in their hypothesis tests.

But how was an applied researcher supposed to move forward? If we knew the type of equilibrium possible for our variables, then we would know which model to use. The flowchart provided by Philips (2018) provides clear guidance on this. If X and y are all stationary, run an autoregressive distributed lag (ADL) model.[2] If the autoregressive distributed lag-bounds test (see Philips 2018) suggests cointegration, estimate a GECM. If there is not enough evidence to conclude that there is cointegration, difference the variables and then run an ADL. This advice is straightforward and helpful and the bounds approach created by PSS is an excellent step forward. Jordan and Philips (2018a,b, 2020) have also made packages in both R and Stata available to implement these suggested approaches, furthering their contribution to applied time series scholars.

The problem with this approach is, as WLL note, that it starts with the assumption that one can definitely diagnose if the dependent variable is stationary. This is often much harder than it sounds. Unit root tests have low power, particularly when we are working with the short time series that are common in political science (Lebo and Kraft, 2017). It gets more complicated because we have to make choices about trend, drift, and serial correlation that will change the test. Given the large number of decisions and tests available, all too often researchers end up with conflicting evidence from their diagnostics about the nature of the series. Therefore, they must hope that the results are robust enough to these numerous specification choices, that they would reach the same conclusion regardless of the chosen approach.

This is the motivation behind the work of WLL. They start with the error correction

[2]There is a fair amount of inconsistency in the use of acronyms for these models. PSS use "ARDL" to refer to a model like a GECM where a unit root y is differenced in the equation. The literature in political science has tended to use ADL to refer to the type of equation we are referencing here. We will use ADL to only refer to equations where the y variable is included in levels.

model (ECM) setup:

$$\Delta y_t = \alpha_0 + \alpha_1^*(y_{t-1} - \lambda x_{t-1}) + \beta_0^* \Delta x_t + \epsilon_t \tag{1}$$

where the LRM is represented as λ. This captures the total effect of a one-unit change in x on y summed over time. The y_{t-1} - λx_{t-1} piece of the equation is the long-run equilibrium relationship. The α_1^* term is the error correction that accounts for how fast the system returns to equilibrium after a shock. The actual estimation of this model is usually done as an ADL model:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \epsilon_t \tag{2}$$

or via the GECM setup:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \beta_0^* \Delta x_t + \beta_1^* x_{t-1} + \epsilon_t \tag{3}$$

where $\alpha_1^* = (1 - \alpha_1)$, $\beta_0^* = \beta_0$, and $\beta_1^* = \beta_0 + \beta_1$ in Equation 3. Which model is used is something of a matter of taste as they are mathematically equivalent to one another (Marriott and Newbold 1998: 327–328; De Boef and Keele, 2008: 189–190). Regardless of the specific model used, of particular interest is the LRM, which is calculated as $\frac{\beta_0+\beta_1}{\alpha_1-1}$ in the GECM, and as $\frac{\beta_0+\beta_1}{1-\alpha_1}$ for the ADL.

While recovering a point estimate of the long-run relation is relatively straightforward by simply inputting the estimated coefficient into the appropriate formula, calculating uncertainty is more complicated. As De Boef and Keele (2008) note, neither the ADL nor the GECM provide a direct estimate of the standard error of the LRM. Since the LRM is a ratio of coefficients, the calculation of the variance of the ratio of coefficients with known variances can be used. The formula is:

$$Var(\frac{a}{b}) = (\frac{1}{b^2})Var(a) + (\frac{a^2}{b^4})Var(b) - 2(\frac{a}{b^3})Cov(a,b). \tag{4}$$

There are two approaches used to *approximate* the variance of this ratio and estimate uncertainty in the long-run relationship. The first is to calculate the LRM from an ECM and use the Bewley (1979) transformation, which estimates the variance of the LRM directly. The Bewley transformation is:

$$y_t = \alpha_0\phi - \alpha_1\phi\Delta y_t + \phi(\beta_0 + \beta_1)x_t + \phi\beta_1\Delta x_t + \phi\epsilon_t \tag{5}$$

where $\phi = (\frac{1}{\alpha_1 - 1})$ and an instrument for Δy_t is calculated as the predicted values from the equation $\Delta y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 x_t + \gamma_3 \Delta x_t + \epsilon_t$. The LRM is the coefficient on x_t from equation 5. The second approach is to use the delta method. The delta method relies on expanding a random variable—in this case the LRM—via a Taylor series and calculating the resulting asymptotic variance of this estimate. It is fortunate that most statistics packages have made the estimation of the standard error straightforward, even if the intuition might be a touch complex.[3] Still, while these estimates of the standard error are *asymptotically* accurate, they may not be as appropriate in the small sample sizes typical in applied time series work.

WLL demonstrate convincingly that a clear test of the presence of a long-term relationship between X_t and y_t is captured by the significance of the LRM. As they note "*Thus, a nondegenerate, or valid, equilibrium relationship between y_t and x_t requires the LRM to be nonzero*" (Webb, Linn, and Lebo, 2019: 286-287, emphasis in original). Moreover, they demonstrate that this is true regardless of the stationarity of y_t, helping resolve much of the uncertainty in pre-analysis specification tests of the time series. This is a vitally important

[3]The `nlcom` command in Stata and the `deltamethod()` command in the `CAR` package in R both allow for estimation of the the variance of the LRM.

result. Additionally, because the LRM is calculated separately for each of the independent variables, this approach allows the researcher to know which of the variables have a significant long-run relationship with y_t. Other approaches, like the Granger-Engle two-step method or the autoregressive distributed lag-bounds approach, will only indicate that a cointegrating relationship exists between the dependent variable and *at least one* of the independent variables, but not which one.

What WLL also make clear, however, is that the interpretation of the specific parameters in the model depends entirely on the univariate properties of the individual series. Without knowing, with certainty, if these series are stationary, the traditional tests are indeterminate.

Given the importance of the LRM for specification testing, WLL empirically explore the appropriate distribution of the test statistics for the LRM based on the Bewley transformation. While the exact amount of information in the uncertainty estimates and the critical values for the LRM depend on the sample size and the degree of autoregression in y_t and x_t, in general they find that the critical values do no follow a standard distribution. Instead, they estimate these critical values via a stochastic simulation to determine the bounds of the test. Their conclusion is that most empirical tests of the LRM are likely overconfident in the hypothesis tests. More importantly, they develop the bounds for the hypothesis tests of the long-run relationship.[4] These bounds will guide a researcher to conclude whether or not there is a long-run relationship.

This is a tremendous step forward for applied time series. What may be unsatisfying for many researchers, however, is that the bounds have a relatively large range of middle values that are inconclusive. For many empirical research questions, a researcher may end up with the frustrating result of a test statistic between the bounds and an uncertain conclusion.

So how should an applied researcher interpret indeterminate results or results that are near the bounds? WLL de facto treat these cases as failing to reject the null hypothesis,

[4] Webb, Linn, and Lebo (2020) provide an expanded set of critical values used to denote upper and lower bounds, by both the number of observations [25, 50, 75, 150, 500, 1000] and α-level [.01, .05, .10].

whether or not a coefficient has cleared the lower bound and approaches the upper bound.[5] Some of the applied work that relies on WLL also treats results in this indeterminate region as if they fail to reject the null hypothesis (e.g., Wolak and Peterson, 2020). Yet, a long-run relationship may exist even if estimates fail to reach significance owing to a number of reasons, ranging from low power, a lack of precision, or issues related to equation balance (Keele, Linn, and Webb, 2016: 34–35).

Low power and a lack of precision are related but distinct issues. Low power may stem from a short time series or data that are truncated on either their starting or ending points owing to how they were collected.[6] A lack of precision may result from measurement error in one of the variables or from a time series simply being too short to return to equilibrium. In either case, when the number of periods in the time series T is small, for example, the estimate of α_1, the coefficient of the LRM, can be imprecise. In familiar cross-sectional models, these issues simply result in inflated standard errors and not many other issues. When dynamic processes are relevant, however, the inflation of standard errors is amplified through the LRM and is more problematic. For instance, if the confidence interval of α_1 includes 1, the variance calculation can produce nonsensical results: if the value of α_1 has probability mass at 1, the LRM is undefined for that point, while if there is mass where $\alpha_1 > 1$, the denominator will be negative. In each case, the calculation of the variance breaks down.

Given these various issues, which are all too common across the social sciences, applied researchers face numerous practical difficulties. Even if they meet theoretical standards of equation balance, their data may simply make it difficult to decisively demonstrate that it is stationary. Thus, treating any result that fails meet the more stringent upper bound criteria of the bounds test as the threshold for identifying a long-run relationship, whether the lower bound was exceeded or not, may be impractical for advancing our understanding

[5]See Webb, Linn, and Lebo (2019: 293, 299) and Webb, Linn, and Lebo (2020: 286).

[6]One could estimate a transformed-likelihood estimator, such as an orthogonal reparameterization estimator, for very short autoregressive panel data (Pickup and Hopkins, 2022).

of relationships for which a researcher's data simply do not play nice and clearly meet (or violate) stationarity assumptions, based on the usually battery of checks.

A Bayesian Approach

Our approach to this problem is to adopt a Bayesian framework.[7] We provide a brief overview of Bayesian inference, with a focus on how it relates to time series analysis. While a Bayesian approach provides some advantages in terms of overcoming the classification problem, our primary interest lies in using a semi-informed prior to formalize the range of plausible values of the dynamic relationship between an independent variable and the lagged dependent variable, particularly when the time series itself is relatively short. This formalization—whether paired with other informed- or non-informative priors on other parameters—helps applied researchers construct measures of uncertainty for an estimated LRM, whether or not one adopts a fully Bayesian theoretic and inferential perspective.

Bayesian statistics is based on Bayes' Law: $P(\theta|data) = \frac{P(\theta)P(data|\theta)}{P(data)}$, where $P(\theta|data)$ is the posterior probability, $P(\theta)$ is the prior probability, $P(data|\theta)$ is the probability density/mass function (e.g., a likelihood), and $P(data)$. Within this framework, the focus is on probabilistic statements using distributions on all facets: prior distribution, posterior distribution, and even the data itself. The data in the denominator, for example, have been

[7]Within political science, Bayesian tools have become a standard part of a researcher's toolkit and have been applied to a number of quantitative tasks, e.g., multilevel models (King and Gelman, 1991; Western, 1998), measurement models (Jackman, 1994, 2001; Quinn, 2004; Schnakenberg and Fariss, 2014), matching administrative data (Enamorado, Fifield, and Imai, 2019), text analysis and topics modeling (Grimmer, 2010; Roberts et al, 2014; Eshima, Imai, and Sasaki, 2024), high-order network dependencies (Hoff and Ward, 2007; Minhas, Hoff, and Ward, 2019), forecasting political events (Ward, Greenhill, and Bakke, 2010; Mueller and Rauh, 2018), model averaging (Montgomery and Nyhan, 2010), and missing data (Honaker and King, 2010; Hollenbach et al, 2021). Moreover, Bayesian methods are also frequently used within time series analysis, particularly for non-stationary data using change-point (Spirling, 2007; Park, 2010, 2011, 2012) or vector autoregressions models (Brandt and Freeman, 2006, 2009; Brandt and Sandler, 2012), and autoregressive processes with complex error structures in time-series cross-section data (Pang, 2010, 2014). Substantive research that applies Bayesian time series methods include election forecasting (Linzer, 2013) and voting behavior (Peterson, 2009), democratic backsliding (Knutsen et al, 2024), human security (Brandt and Sandler, 2010; Santifort, Sandler, and Brandt, 2013; Fariss, 2014), international conflict (Brandt, Colaresi, and Freeman, 2008; Park, 2010; Nieman, 2016), and major power competition (McGinnis and Williams, 1989; Thies and Nieman, 2017), among others. See Gill (2012) and Park and Shin (2020) for reviews.

observed and thus have a probability equal to one. Bayesian analysis, in other words, observes data and asks what distribution is the most likely to have produced it.[8] This view lends itself to a probabilistic treatment of parameter estimates and favors model comparison. In this respect, Bayesian analysis closely resembles maximum likelihood (ML), as random variables are treated as conditioned by the observed data to offer the 'most likely' outcomes (Gill, 2009: 39–44); in fact, Bayesian and ML models are asymptotically equivalent, regardless of the specified prior distribution (Gelman et al, 2004: 111, 247; Gill, 2009: 4, 60–61, 62–65).[9] Conversely, the frequentist approach treats the observed data as a random variable arising from a fixed parameter. That is, a distribution is assumed (the null hypothesis) and the question is whether the data differ from it. This view lends itself to null hypothesis testing, with models and parameters either rejecting or failing to reject the null.

This difference in how the parameters and data are treated has two key implications when applied to time series analysis. The first relates to the classification problem when determining the characteristics of the observed data, e.g., stationary, non-stationary, cointegration, etc. As described above, the uncertainty associated with the diagnostic pre-tests makes classifying y, X, and the equilibrium LRR a non-trivial task. The source of much of this difficulty arises from treating the data as a random variable, where the observed data are manifestations of an unobserved underlying data generation process. Hence, the data itself is uncertain, as are its properties.

From a Bayesian perspective, however, this is less problematic, as the data are known (Chan et al, 2020: 343–344). Therefore, pre-testing the data to reject or fail to reject a null hypothesis about their data generating process does not make sense: the data are known with a probability of 1 and it is the model parameters that are unknown. Rather than a pre-test for the presence or absence of a unit root, one could instead directly compare two competing model specifications, one with a unit root and one autoregressive, and make a

[8]See Gelman et al (2004), Gill (2009), and Chan et al (2020) for an introduction to Bayesian analysis.

[9]A key difference, however, is that ML estimation maximizes only the likelihood, while Bayesian estimation via MCMCs approximates the entire posterior distribution (Gill, 2009: 19, Miočević, Levy, and van de Schoot, 2020: 7); as we later discuss, this is a useful feature for estimating uncertainty in the LRM.

probabilistic assessment of their fit to the data using Bayes Factor—a ratio of the models' marginal likelihood—as evidence (Marriott and Newbold, 1998). Whereas pre-testing lacks clarity in the face of uncertain or conflicting diagnostic results, model comparison provides greater direction along with the degree of confidence with the selected choice.[10]

A second benefit of the Bayesian inferential model is that it works well for time series with small T. Whereas maximum likelihood and frequentist approaches rely on the central limit theorem, Bayesian analysis does not. Instead, Bayesian analysis typically applies sampling-based methods, such as MCMCs, where the quality of inference relies on the *number of samples* taken rather than the *size of the sample* (McNeish, 2016). This distinction is most pronounced in the estimates of uncertainty when the number of observations is small. This feature is useful for applied time series analysis, as many social science data sets contain short series samples and it is these series that are the cause of so many diagnostic problems.[11]

Our primary interest in applying a Bayesian framework, however, lies in the benefit of applying a semi-informed prior to help estimate uncertainty in the LRM. The prior can be specified so as to restrict the coefficient of the lagged DV so that it adheres to the theoretical range of the estimator: strictly between -2 and 0 in the GECM and -1 and 1 in the ADL.[12] We specify a diffuse, uniform prior that places equal probability on all values between these bounds. Alternatively, a more informed prior, which places more weight to values closer to 0 or either bound, may be practical in applied settings where one can incorporate existing knowledge.[13]

[10]Marriott and Newbold (1998) show that Bayes Factor is able to correctly distinguish unit root and autoregressive model specifications, even for short time series where α_1 takes on high values. Moreover, one could use Bayes Factor or BIC to apply a general-to-specific modeling approach. For instance, Bayes Factor is commonly used to differentiate models in applications using change-point models, e.g., Park (2010).

[11]Using a completely uninformative prior, however, may increase bias and reduce coverage in small samples, so weakly informed priors—e.g., that capture known theoretical characteristics, such as the one we propose—are recommended (McNeish, 2016; Veen and Egberts, 2020).

[12]The GECM in Eq 3 assumes that $-2 < \alpha_1 < 0$ and the ADL in Eq 2 assumes that $-1 < \alpha_1 < 1$. See Keele, Linn, and Webb (2016: Table 1) for a summary of error correction rates and long run equilibria.

[13]If one is estimating an ADL, for example, and has theoretical reasons to expect the coefficient on the lagged DV to approach 1, then a sharp prior where the pmf is massed near 1, such as $\mathcal{B}(5, .5)$, could be used (see Marriott and Newbold, 1998: 328–333). If, instead, a researcher believes that the coefficient on the lagged DV is positive, but has no other information, then a $\alpha_1 \sim \mathcal{B}(1, 1)$ prior would apply a uniform distribution between 0 and 1.

One way to think about the semi-informed prior is that it is simply the formalization of a plausible range of values on the dynamic relationship between the dependent variable and its lags that researchers are already making when they estimate a model like that in Equation 2 or 3. When a researcher treats a series as stationary, she assumes that the root of the characteristic equation of the time series is less than one. When the series is integrated, the root of the characteristic equation is exactly one. The use of this prior constrains the estimate of α_1 to be no greater than 1, precisely the implication of treating the dependent variable as stationary or integrated.

Using a bounded prior also keeps the estimates of uncertainty for the LRM firmly within their theoretical limits. If the confidence interval for α_1 nears or exceeds 1, then the denominator of the LRM can take very small, or even negative, values. In that case, the estimation of the variance of the LRM will be "mildly explosive"[14] (which is bad). The prior thus keeps estimates of uncertainty within the same theoretical bounds as the point estimate, providing more substantively plausible and theoretically-informed results.

The actual estimation of the Bayesian model is carried out via MCMCs. This allows us to calculate the distribution of the posterior for all of the coefficients directly, including the LRM. Once the model has converged, each simulation of the MCMC draws all of the parameters in the model from their joint probability distribution. The ratio of the parameters in the draw is, then, also a draw from the posterior of that ratio (Gelfand et al, 1990; Murr, Traunmüller, and Gill, 2023). Table 1 provides an illustration from an ADL model as in Equation 2. For the purposes of the illustration, assume that $\alpha_0 = 0$, $\alpha_1 = 0.5$, $\beta_0 = 0.25$, and $\beta_1 = 0.5$, resulting in the LRM, $\frac{0.5+0.25}{1-0.5} = 1.5$, and that the MCMC has 5,000 iterations. The table shows that for each individual simulation, we recover parameter estimates of the specified model. For each individual iteration, we are able to construct and calculate the LRM based on these coefficients.

As is standard with Bayesian estimation, the posterior distribution can be summarized to

[14]We borrow this phrase from Hill and Peng (2014: 293) and Hill, Li, and Peng (2016: 126)

Table 1: Hypothetical Parameter Estimates for MCMC Iterations.

Iteration	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\widehat{\text{LRM}}$
1	0.023	0.639	0.428	0.361	2.186
2	-0.054	0.379	0.400	0.150	0.886
3	-0.130	0.567	0.714	0.258	2.245
⋮	⋮	⋮	⋮	⋮	⋮
5000	-0.150	0.517	0.618	0.106	1.499

construct point estimates (e.g., median) and uncertainty (e.g., 95% credible region), as well as indicate the percent of individual draws above or below zero, for each model parameter. These summaries can also be constructed for the LRM. As such, we do not need to rely on an asymptotic equivalent to the confidence interval of the LRM, such as the Bewley transformation or delta method. In cases where a time series is relatively short, this direct estimate of the uncertainty of the LRM should be more accurate and reliable than approximations that rely on asymptotic properties that may not hold.

To demonstrate the usefulness of adopting a Bayesian approach with a semi-informed prior for estimating the LRM, we conduct two Monte Carlo experiments. The first experiment compares coverage rate estimates for the LRM—using both the bounds approach from a Bewley transformation of the ECM, and a Bayesian ECM with a semi-informed prior—under varying levels of univariate autocorrelation of x and y and for different time series lengths, when there is no long-run relationship. These conditions allow for assessing how well each estimation approach separates a spurious long-run bivariate relationship from actual univariate dynamics.

The second experiment illustrates how to apply the Bayesian approach in a more realistic (but still controlled) setting where the researcher is interested in testing and reporting the instantaneous and long-run effects of an independent variable. In this experiment, we report point and uncertainty estimates for x, a lagged y, and the LRM, where there is a moderate long-run relationship between x and y as well as moderate autocorrelation between x and its lag. This specification allows us to evaluate how well each approach captures a true long-run relationship under conditions that a researcher would experience in practice. Following these experiments, we compare the results and provide a general discussion for applied researchers, focusing on trade-offs and gains from each approach.

Monte Carlo Experiment #1

For the first Monte Carlo experiment, we generate our data replicating the dynamic simulation process used by WLL to identify bounds for the LRM. We begin by generating two independent autoregressive processes, such that $y_t = \rho_y y_{t-1} + \epsilon_y$ and $x_t = \rho_x x_{t-1} + \epsilon_x$, with the errors drawn from separate standard normal distributions and $T = 75$. While the error terms are each stationary, the values of ρ_y and ρ_x are set to be either 0 or 1—reflecting $I(0)$ and $I(1)$ for each variable. This gives four scenarios: one where $\rho_y = 0$ and $\rho_x = 0$, a second with $\rho_y = 0$ and $\rho_x = 1$, another where $\rho_y = 1$ and $\rho_x = 0$, and finally a case with $\rho_y = 1$ and $\rho_x = 1$. In the true data generating process there is no long-run relationship between y_t and x_t; therefore, any (mis)identified relationship is strictly due to the dynamics induced through the univariate autoregressive processes.

Using these data, we continue to follow WLL by estimating the LRM and its uncertainty using the Bewley transformation from Eq 5. We then also estimate a Bayesian ECM. As described above, one of the advantages of the Bayesian ECM is that uncertainty for each parameter, including those of functions, can be directly estimated from the posterior distribution of the MCMCs. That is, rather than an approximation for the uncertainty for the LRM that relies on asymptotic properties, as is the case with the Bewley transformation, we can calculate the LRM for each of the draws from the posterior and use the distribution of these draws to summarize the distribution of the LRM. The direct estimates from the Bayesian estimator should be more precise, particularly when T is relatively short and asymptotic properties are least likely to hold.

The Bayesian ECM is specified as in Eq 3, with diffuse priors of $\mathcal{N}(0, 20)$ for the constant and the coefficients associated with Δx_t and x_{t-1}, a prior of $\mathcal{U}(0,2)$ on the coefficient for lagged y, and the prior for the variance distributed $\mathcal{G}(1,10)$. Recall that the prior on the coefficient on y_{t-1} for an ECM formalizes the specification of the dynamic relationship between the dependent variable and its lagged values and prevents it from taking explosive values that do not return to the LRR equilibrium. Each Bayesian ECM is estimated using 5,000 MCMCs after a 2,500 burnin and thinning of 10.

For each combination of ρ_y, ρ_x from the data generating process, we estimate the LRM from the Bayesian and Bewley specification for two lengths of T, such that $T \in \{25, 75\}$.[15] The varying lengths for T allow us to look at both how well the estimates fair for short and moderately short time series that are common to social science data.

Table 2 reports summaries for estimates of the LRM from 20,000 simulations under each of the 8 scenarios (four ρ_y, ρ_x possibilities times two lengths of T).[16] For both the Bayesian and Bewley estimates of the LRM, we report a point estimate (posterior median for the Bayesian estimator, average point estimate for the Bewley estimator) and its coverage rate. The point estimate gives the expected LRM, while the coverage rates offer insights into how well the different estimation strategies perform in returning accurate estimates under varying conditions that are common in applied work (Hopkins et al, 2023). Lower coverage rates would suggest that, even if an estimator is unbiased on average, its results are less unreliable in any particular application. Coverage rates for the Bayesian ECM report how frequently the true value is within the estimated 95 percent credible intervals. For the Bewley model, we report the coverage rate using the bounds approach suggested by WLL; that is, we construct 95 percent confidence intervals for both the lower and upper bound at each length of T, based the appropriate t-statistics identified by Webb, Linn, and Lebo (2020: Table 2).[17] We also report the percent of indeterminate cases arising from incongruent outcomes between

[15] Both lengths of T are taken from the same original time series: when $T = 25$, the first 25 observations are used; when $T = 75$, all of the observations are used.

[16] To clarify, within each individual simulation of the first Monte Carlo experiment, the Bayesian ECM estimates are summaries based on its own 5,000 MCMCs following a 2,500 burnin. Wheels within wheels.

[17] In our experiment, with one X variable and $T = 25$, the t-statistic for the lower bound is 1.25 and for the upper bound is 3.79. When $T = 75$, the corresponding t-statistics are 1.06 and 3.68, respectively.

Table 2: LRM Estimates with Varying Autocorrelations and Sample Size.

	$T = 25$			
	$\rho_y = 0, \rho_x = 0$	$\rho_y = 0, \rho_x = 1$	$\rho_y = 1, \rho_x = 0$	$\rho_y = 1, \rho_x = 1$
Bayesian ECM				
Median	-0.001	-0.001	0.002	-0.001
Coverage	0.961	0.958	0.888	0.902
Bewley Transformation				
Mean	0.001	-0.001	-0.012	0.091
Lower Bound Coverage	0.763	0.749	0.780	0.648
Upper Bound Coverage	0.999	0.997	0.998	0.950
Indeterminate Range	0.235	0.248	0.218	0.303
	$T = 75$			
	$\rho_y = 0, \rho_x = 0$	$\rho_y = 0, \rho_x = 1$	$\rho_y = 1, \rho_x = 0$	$\rho_y = 1, \rho_x = 1$
Bayesian ECM				
Median	0.000	-0.000	-0.000	-0.003
Coverage	0.948	0.948	0.878	0.882
Bewley Transformation				
Mean	-0.000	-0.000	-0.002	-0.248
Lower Bound Coverage	0.702	0.697	0.763	0.583
Upper Bound Coverage	0.999	0.999	0.999	0.951
Indeterminate Range	0.297	0.302	0.237	0.369

Note: Bayesian estimates are median and 95% credible intervals from the posterior distribution. The LRM (and uncertainty) for the traditional ECM is estimated using the Bewley transformation. The coverage range for the Bewley transformed LRM is calculated using a t-statistic of 1.25 for the lower bound and 3.79 for the upper bound when $T = 25$, a t-statistic of 1.06 for the lower bound and 3.73 for the upper bound when $T = 75$ (Webb, Linn, and Lebo, 2019, 2020).

using lower and upper bound.

There are several notable results from the Monte Carlo experiment. Given our interest in estimates of uncertainty, we focus the coverage rates of each approach.[18] First, the Bayesian ECM recovers the true value in the overwhelming majority of cases, with rates of approximately 95% when $\rho_y = 0$ and a slightly lower 88% when $\rho_y = 1$. When using the bounds approach with the Bewley estimates, the upper bound recovers the true value in nearly all simulations, with only the scenario of $\rho_y = 1, \rho_x = 1$ being at 95% (which coincides, of course, with the aim of the WLL's bounds approach). The lower bound, however, performs much more poorly, with a high coverage rate of 78% and a low of 58%. The indeterminate range, where applied researchers cannot confidently reject the null hypothesis nor fail to reject it, is never less than 21% and reaches nearly 37% in one scenario.

[18]Each approach returns similar point estimates of the LRM, which is unsurprising given they both are estimated using OLS.

Second, when looking *within* each scenario of ρ_y, ρ_x across each T, the Bayesian ECM returns similar coverage rates. This reflects the fact, of course, that Bayesian statistics does not rely on large sample sizes (i.e., the central limit theorem) for calculating results (McNeish, 2016; van de Schoot and Miočević, 2020). In contrast, the Bewley coverage for the upper bound remains consistent, whereas that of the lower bound actually decreases as T increases. This stems from a smaller t-statistic being applied when using the bounds approach as the number of observations increases. As a result, the proportion of the indeterminate range between the lower and upper bounds increases with the size of the sample.

Third, the percent of cases in the indeterminate range for the bounds approach based on the Bewley estimates are greatest when $\rho_x = 1$. This result is substantively meaningful, as many covariates used as control (or even independent) variables included in panel data either do not change, or do not change very much, over time. For instance, it is well known in international relations that a country's regime type is typically relatively stable for long periods of time. Even in cases where these covariates do change, such as the GDP of a state, province, or country, they are often primarily a function of the prior value. Each of these are cases where ρ_x would approach 1. This is also something, of course, that can be evaluated and known prior to estimating a dynamic model.

Fourth, the Bayesian ECM provides the greatest coverage when $\rho_y = 0$, with a decrease of 5 to 7 percentage points when $\rho_y = 1$. This decrease in accuracy holds regardless of the length of the time series. This result may, however, reflect our use of a diffuse prior, which gives equal weight to all theoretically possible values of the lagged y, thus pulling it toward the lower and upper bounds. Conversely, the Bewley coverage rates are highest when $\rho_y = 1, \rho_x = 0$, even outperforming itself compared to when y was not dynamic. However, the Bewley approach performs its worst when $\rho_y = 1, \rho_x = 1$, returning coverage rates below 65% for the lower bound.

These results highlight key small sample properties of LRM estimates from a Bayesian ECM and applying the bounds approach with Bewley estimates under varying univariate dynamics when the true LRR is zero. While both are generally unbiased, coverage rates are impacted by the univariate dynamics. When y behaves nicely, then the benefits of the Bayesian approach are most evident: high coverage rates without the inconvenience of wide indeterminate ranges. When y is less well mannered, the coverage rates for the Bayesian approach drop slightly[19] while the indeterminate range for the bounds approach remains large.

Monte Carlo Experiment #2

For the second Monte Carlo experiment, we generate a simple dynamic model with a moderate long-run relationship between x and y, and mimic common features of real-world data by inducing a mild autoregressive process between x and its lag. More specifically, we generate the endogenous variable so that $y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 X_{t-1} + \epsilon_t$ where $\alpha_0 = 0$, $\alpha_1 = 0.5$, $\beta_0 = 0.5$, and $\beta_1 = 0.25$. We generate the exogenous variable so that $x_t = \gamma X_{t-1} + \eta_t$ where $\gamma = 0.5$. Both η_t and ϵ_t are drawn from a standard normal where $cov(\eta_t, \epsilon_t = 0)$. The LRM, given this specification, is $\frac{0.5+0.25}{1-0.5} = 1.5$. Since we are interested in the small sample properties of each approach, we set $T = 25$. This experiment illustrates how well each approach is able to help researchers make correct inferences when there is an LRR between their variables of interest.

We estimate both a Bayesian ECM and a traditional ECM to recover parameter estimates, specified as in Eq 3. The LRM is estimated directly from the posterior distribution in the case of the Bayesian ECM, and via the Bewley transformation for the traditional ECM. We give the Bayesian ECM diffuse priors of $\mathcal{N}(0, 20)$ for the constant and the coefficients associated with x_{t-1} and Δx, a semi-informed prior of $\mathcal{U}(-1, 0)$ on the coefficient for y_{t-1} such that the LRR cannot be negative (e.g., assumed to either be positive or have no effect), and a diffuse prior of $\mathcal{G}(1, 10)$ for the variance. Each individual Bayesian ECM is estimated from 5,000 MCMCs after a 2,500 burnin with a thinning of 10. Finally, the Bewley estimate

[19] These coverage rates, of course, would improve if a more informed prior were used—recall fn 13.

Table 3: Error Correction Model Parameter Estimates with a Moderate LRR and Univariate Autocorrelations, and $T = 25$.

	True Value	Bayesian ECM	ECM w/ Bewley Trans. & Bounds
Y_{t-1}	-0.5	-0.577	-0.594
		[-0.88, -0.25]	(0.172)
X_{t-1}	0.75	0.797	0.815
		[0.220, 1.368]	(0.273)
ΔX	0.5	0.494	0.495
		[0.011, 0.976]	(0.224)
Constant	0	-0.003	-0.003
		[-0.512, 0.506]	(0.239)
LRM	1.5	1.452	1.454
		[0.173, 3.469]	(0.543)
Bayesian Coverage Rate		0.683	
Lower Bound Coverage			0.940
Upper Bound Coverage			0.292
Percent in indeterminate Range			0.648

Note: Bayesian estimates are median and 95% credible intervals from the posterior distribution. The LRM (and uncertainty) for the traditional ECM is estimated using the Bewley transformation. The coverage range for the Bewley transformed LRM is calculated using a t-statistic of 1.25 for the lower bound and 3.79 for the upper bound (Webb, Linn, and Lebo, 2019, 2020).

of the LRM for the tradition ECM is specified as in Eq 5.

Table 3 reports summaries for each estimation approach based on 20,000 simulations. The first column reports the true value of the ECM estimates based on the data generating process described above;[20] the second column reports the median value, and the accompanying 95 percent credible interval, taken from the posterior distribution for the Bayesian ECM; and the last column reports the mean coefficient and standard error for the traditional ECM. Below the parameter for the instantaneous effects is the estimated LRM and its uncertainty; for the traditional ECM, these are obtained via the Bewley transformation. Finally, the bottom of the table reports the coverage rates from each estimator, with the lower and upper bounds based on the t-statistics identified by WLL used for the Bewley estimates, along with the range of indeterminate values where the lower and upper bounds give conflicting inferences.

As both the Bayesian and traditional ECM return similar parameter estimates—including for the LRM—we again focus attention on their coverage rates, where there is greater diver-

[20]Recall that the ADL and ECM specifications are mathematical equivalents of one another (Marriott and Newbold, 1998; De Boef and Keele, 2008).

gence. While the entirety of the Bayesian estimator's 95 percent credible interval range is greater than zero in 68.3% of the simulations, the 95 percent confidence intervals estimated using the bounds approach offer a more muddled conclusion.[21] Though the lower bound criteria returns estimates that are greater than zero for the entirety of the 95 percent confidence interval in 94% of the simulations, the more challenging upper bound criteria does so only 29.2% of the time. The result is that, in almost two-thirds of the simulations (64.8%), the bounds approach offers conflicting guidance on whether an LRR exists between x and y; this is despite the fact that a true LRR between x and y does exist by the construction of the experiment. Moreover, this type of scenario is where the bounds approach is most likely to be implemented by an applied researcher since the dynamics of x and y are neither $I(0)$ or $I(1)$, but in-between.

There are several takeaways from the two experiments. One is that the bounds approach—especially applying the stricter criteria of using only the upper bound—reduces the risk of a type 1 error. Conversely, this risk is slightly higher, under some conditions, when using the Bayesian approach. The cost of reducing this risk, however, is that the more stringent upper bound threshold dramatically increases the risk of a type 2 error. The Bayesian approach, on the other hand, is much better able to correctly recover moderate LRRs, even when the sample size is small. Another key finding is that, for the bounds approach, the size of the indeterminate range can be quite large, especially when sample sizes are small. This holds regardless of whether an actual LRR between X and y exists. This characteristic is likely to be especially unsatisfying for applied researchers.

So, what is an applied researcher to do? Given the insights from our Monte Carlo experiments, we think that the benefits of adopting a Bayesian approach are fairly strong, especially when the sample size is relatively small. At the small cost of a very slightly lower coverage rate when the LRR is null and the autocorrelation in y is very high—the latter a

[21] We report the coverage rates for the entirety of the credible range in order to make a more straightforward comparison between estimators in the current Monte Carlo experiment. In practice, however, one can calculate and report the percent of individual MCMC draws above/below zero to provide more precise confidence in the probability of an LRR between X and y. We demonstrate this in the next section.

condition that would be evident from summarizing the data and manageable with even a slighly-informed prior—one gains far more precision in making theoretical inferences.

Applications

In this section, we replicate three existing time series papers using our Bayesian approach with a semi-informed prior. We begin with the recent paper on aggregate levels of interest in politics in America by Peterson et al (2022). While this paper may be less familiar than the works replicated by WLL, it has a few features that allow us to illustrate the points we make above. We then proceed to the same replications as in the original WLL (2019) paper, replicating the work of Ferguson, Kellstedt, and Linn (2013) on the effect of policy on public mood and Lebo and O'Green (2011) on presidential success in Congress. These three applications demonstrate that the Bayesian approach can resolve some of the indeterminate results in existing work.

Replicating Peterson et al., 2022

Our first application explores the relationship between trust in government and interest in politics. Peterson et al (2022) argue that trust and interest trade-off in the electorate. When the electorate trusts the government more, the incentive to pay attention to politics lessens. Voters make a choice about whether or not to follow politics. For some, this is simply a habit or a hobby. For many people, however, it is something of a chore. The authors expect that when the electorate trusts the government to look out for the public, the incentive to actively monitor the government lessens. Some of the electorate chooses to spend that time on things they find more enjoyable than politics. In contrast, when the electorate believes that the government is untrustworthy, the need for monitoring increases. The electorate will need to hold government officials more accountable. Peterson et al (2022) argue that this creates a tradeoff of normative evaluations of government. In general, higher levels of trust and higher levels of interest are believed to be markers of a stronger democracy. With this tradeoff, however, it is difficult to have high levels of both.

The authors rely on existing macro-level measures of trust in government and use the technique developed by Stimson (2018) to construct a new aggregate measure of macrointerest. Peterson et al (2022) report that both of these macro series are integrated and that the Engle-Granger two-step method supports the hypothesis that they are cointegrated. Based on these results, they estimate the relationship as an ECM and find support for their theory. They also developed two other alternative hypotheses about the effect of presidential approval and consumer sentiment on macrointerest and found no evidence for either relationship. They do find that presidential campaigns heighten levels of political interest. Finally, they argue that scandals and other major events may cause Americans to become more interested in politics and include a long list of major events during their timeframe. September 11 is the only of these events to have a significant effect on macrointerest.

Peterson et al (2022) use the GECM setup for their model of macrointerest. A simplified version of their specification is:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \beta_1^* \Delta x_1 t + \beta_2^* x_{1t-1} + \beta_3^* \Delta x_2 t + \beta_4^* x_{2t-1} + \beta_5^* \Delta x_3 t + \beta_6^* x_{3t-1} + \epsilon_t \tag{6}$$

where Y_t is macrointerest at time t, x_1 is trust in government, x_2 is consumer sentiment, x_3 is presidential approval.[22]

The conclusion about the effect of trust on interest, however, is dependent on these specification tests and ignores the uncertainty in the stationarity and cointegration tests. If they are correct about the specification, then the evidence supports their theory. In the article, they report the confidence interval for their estimate of the LRM based on the delta method, but not the t-statistic. Replicating their published work, we find that the t-statistic for the LRM is -2.71, which lies between the bounds provided by WLL and the result should

[22]The model also includes several indicators for specific political events and a measure of presidential campaigns. None of these added controls are expected to have a LRR with macrointerest and we omit them from the equation for brevity, but do include them in the model to fully replicate the original work.

be seen as inconclusive. Based on the bounds method, then, the main conclusion of Peterson et al (2022) is not clearly determined. They do find a significant short-term effect of changes in trust on changes in interest, but the LRM is not significant if one is uncertain about the assumption that both trust and interest have unit roots.

We re-analyze this study using our Bayesian approach. Our approach is easy to implement in either R using the `brms` package (Bürkner, Bürkner) or Stata using the `bayesmh` command. For all of the applications in this paper, we estimate the model in R. The `brms` package excels at more complicated Bayesian models, but the specification of our model is quite straightforward. The `brm` command is specified as most other linear models in R, with the designation of the dependent and independent variables.[23] The `brm` command requires the specification of the family of the model (in this case, Gaussian), the number of chains for the MCMC (we use 4), the number of iterations (300,000), the burn-in (250,000), and the thinning rate of the MCMC sampler (5). These are all standard features of an MCMC model and the simplicity of the estimation makes these choices moderately trivial.

The only part of the model specification that requires any real choice is the that of the priors, but even this is straightforward. The `brm` command includes default priors for all parameters in the model. Any parameter not specified will automatically use the default priors. For this type of model, the priors for the β parameters with the independent variables are flat. The prior for the α_0 is specified as a Student-$t_{(3,0,2.5)}$ and the σ parameters is specified as a Half Student-$t_{(3,0,2.5)}$. Again, these are all the default parameters. The user can change them, but our approach does not require any changes. The only user-specified prior in the model is on the α_1 parameter, where we specify a $\mathcal{U}(-2, 0)$ prior.[24]

This creates a brms-class object in R, from which the summary command will return, among other things, the parameter estimates, their estimated error, and the 95 percent credible interval. Users should be sure to check the standard MCMC diagnostics to ensure

[23] `brms` does not have a direct way to lag and difference variables, so the data processing occurs prior to model specification.

[24] This is done with a single line of code: `prior = prior(uniform(-2, 0), coef = ldv)`, where ldv is the lagged dependent variable.

that the chains have converged on the posterior, but these models are so computationally easy that this should not be much of a problem. The object will also contain the draws from the posterior. The code `as_draws_df` will let the user define a new dataframe made up of these draws. The user can then construct the LRM of any independent variable of interest. The hypothesis tests are simple matters of summarizing this calculated LRM. In our applications, we calculate the 95 percent credible interval and the proportion of draws of the LRM that are either less than or greater than zero (depending on the direction of the hypothesis). The reporting of the proportion of draws, in particular, conveys a more precise and accurate level of confidence in whether a variable has a non-zero effect than is garnered from frequentist hypothesis testing.

The results, reported in Table 4, have the same pattern of significant results as the original article. The coefficients reported are medians from the posterior of the MCMC, while the numbers in the parentheses underneath are the 95 percent credible intervals from the posterior distribution. There is a negative relationship between trust and interest in both the long-run relationship and the short-term effect of changes in trust in government. Central for our application, the 95 percent credible interval of the LRM excludes zero, suggesting that there is a long-run relationship between the two. In fact, the estimate of the LRM is negative in 98.5 percent of the draws from the posterior. As a result, our analysis supports the main conclusion of the original article. We also replicate the results for the effect of September 11 and for presidential campaigns on macrointerest. Lastly, and consistent with Peterson et al (2022), we find no evidence that either the index of consumer sentiment or presidential approval are linked to macrointerest.

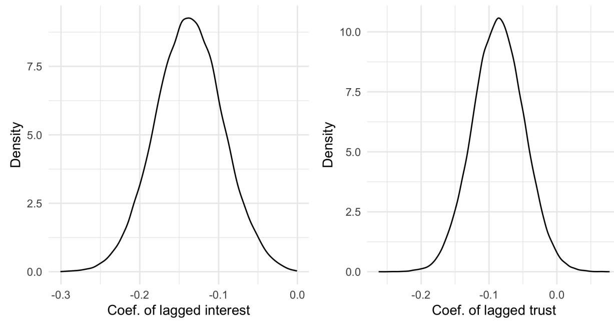
We can also plot the distribution of parameters to illustrate the "mildly explosive" potential of the variance estimates if the error correction rate is too slow. Figure 1 presents the density plot of the draws from the posterior for the coefficient on the lagged interest measure (the error correction rate) and the lagged trust measure. The posterior of the lagged interest measure is a normal distribution and the density plot looks like draws from a typical normal

Table 4: A Model of Macrointerest, 1973–2014.

Variable	X_{t-1}	ΔX_t	LRM
Interest	-0.14		
	(-0.22, -0.05)		
Trust	-0.09	-0.21	-0.62
	(-0.16, -0.01)	(-0.37, -0.04)	(-1.29, -0.13)
Consumer sentiment	0.00	0.00	0.01
	(-0.02, 0.02)	(-0.04, 0.04)	(-0.17, 0.16)
Presidential approval	0.01	0.00	0.04
	(-0.02, 0.03)	(-0.03, 0.03)	(-0.14, 0.28)
Presidential campaign	0.40		
	(0.23, 0.57)		
Watergate	0.55		
	(-0.66, 1.73)		
ABSCAM	1.52		
	(-0.96, 4.04)		
Jim Wright	-0.54		
	(-3.06, 2.05)		
Keating five	-0.23		
	(-2.72, 2.42)		
Clinton Impeachment	-0.29		
	(-2.13, 1.52)		
September 11	1.88		
	(0.01, 3.72)		
Hurricane Katrina	-0.11		
	(-2.50, 2.37)		
Invasion of Panama	-0.01		
	(-2.54, 2.45)		
Second Iraq War	-0.06		
	(-2.49, 2.38)		
Persian Gulf War	-0.96		
	(-3.48, 1.5)		
Constant	12.65		
	(4.77, 20.53)		
T	167		

distribution, with the peak centered at the mean/median. The one thing to note is that the posterior for interest does have a small number of draws that are very close to zero. The posterior of the lagged trust measure also looks as expected and simply visualizes the median and credible interval presented in Table 4. Some of the draws from the posterior are near

Figure 1: Estimated Posterior of the Lagged Values of Interest and Macrointerest.



Note: Results from a Bayesian regression specified as an ECM. Data from Peterson et al (2022).

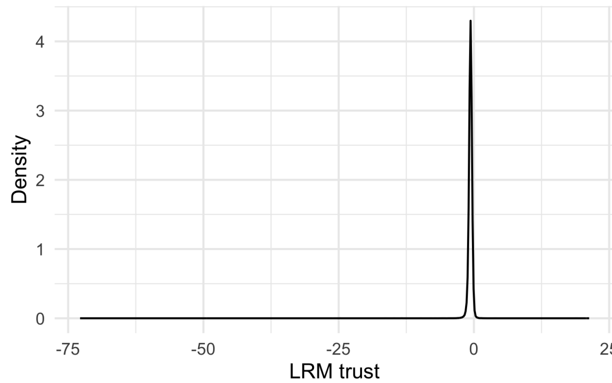
zero, but that will not matter much for the mildly explosive issue in the standard error of the LRM.

Next, Figure 2 presents the distribution of the LRM for the effect of trust on interest, the key hypothesis in the paper. Note the range on the x-axis. The posterior distribution has a handful of extreme values (both positive and negative). The vast majority of the mass of the distribution is around the median value of -0.62, but some are several standard deviations away from the median of the distribution. This is because a small number of the draws from the posterior of the α_1 parameter are very close to zero.

This illustrates the claim by Hill and Peng (2014, 293) and Hill, Li, and Peng (2016, 126) that if the variance of the α_1 parameter has probability mass at 0, the variance estimate of the LRM will be mildly explosive. That is, because α_1 is in the denominator of the LRM, if its value in the draw from the posterior is too close to zero, the absolute value of the LRM for that draw will be extraordinarily high, regardless of the coefficient on the independent variable in that same draw. In this application, there is a draw from the posterior of the LRM for trust that is -72.80, almost one hundred times the median estimate and more than seventy times the lower value of the credible interval. This is because the estimate of the α_1 in that draw is -0.00008. Even though the estimate of the coefficient for the lag of trust (-0.06) is below the median value of the posterior, the calculation of the LRM for that draw is a serious extreme value.

In this application, only a handful of the draws from the posterior of α_1 are close enough

Figure 2: Estimated Posterior of the Coefficient for the LRM for Trust



Note: Results from a Bayesian regression specified as an ECM. Data from Peterson et al (2022).

to zero for this issue to occur. Of course, as shown with the 95 percent credible interval reported in Table 4, these outliers are not common enough to result in extending the interval too large but do make it asymmetric and skewed downward. This result is intuitive and correctly reflects that, in this specific application, the error correction rate is slow and any shocks cause the series to return to equilibrium very slowly. If, however, the estimate of α_1 was closer to zero, meaning that the error correction rate in the process was even slower, then there would be an even longer tail and more skewed credible interval.

This also points to one of the advantages of a prior bounded at theoretically relevant limits: plotting substantive effects from a model with unbounded estimates of α_1, whether estimated using a frequentist or Bayesian model, could result in standard errors above their theoretical limits, longer tails, and substantive effects that include values of infinity among their draws—making their uncertainty intervals, or potentially even their point estimates, nonsensical. For example, when plotting substantive effects of the LRM estimated from a frequentist model using conventional simulation methods—i.e., drawn from a multivariate normal distribution—some draws would likely include values near or below zero in the denominator. Likewise, a Bayesian model without a semi-informed prior would face similar problems, just at the estimation stage when constructing the LRM from the MCMCs.

Replicating Ferguson, Kellstedt, and Linn, 2013

The second application is a test of how policy mood responds to public policy. Mood, in this context, is the electorate's preference for the size of government. When the public wants the federal government to expand, mood will be higher. When the electorate is more conservative, favoring a smaller government, mood will be lower (Stimson, 2018). The actual measure of policy mood is based on Stimson's (2018) dyad ratio algorithm that combines thousands of individual survey questions to develop a single time series of the preferences of the US electorate. It is one of the foundational concepts in the study of US macro politics. Durr (1993) was the first to argue that mood should respond to changes in the economy and the actual size of government. As the economy expands, Durr contended, the public would express support for a more expansive government because the stronger economy would make it easier to pay for the policies. When the economy contracts and finances become tighter, the public is likely to prefer lower taxes, leading mood to decrease. The second main antecedent of policy mood, public policy itself, has been a more robust predictor of changes in mood. This thermostatic model of public opinion holds that when policy moves in one direction, the public responds by moving in the opposite direction (Wlezien, 1995).

WLL chose the Ferguson, Kellstedt, and Linn (2013) paper as an application because the time series properties of policy mood are notoriously difficult to diagnose. As WLL note, several tests of the stationarity of policy mood give conflicting and ultimately confusing results. Thus, they continue, scholars have not reached a consensus on the stationary of mood. Across the literature, it seems like applied researchers have used almost every possible specification of mood's dynamic properties, and the tests are so inconsistent that no one really knows the correct specification. This is precisely the type of inconclusive results that bedevil applied time series researchers. They are usually forced to make a decision about the stationarity of policy mood and treat it as true. The approach advocated by WLL allows researchers to directly incorporate the uncertainty of the specification tests into the hypothesis testing.

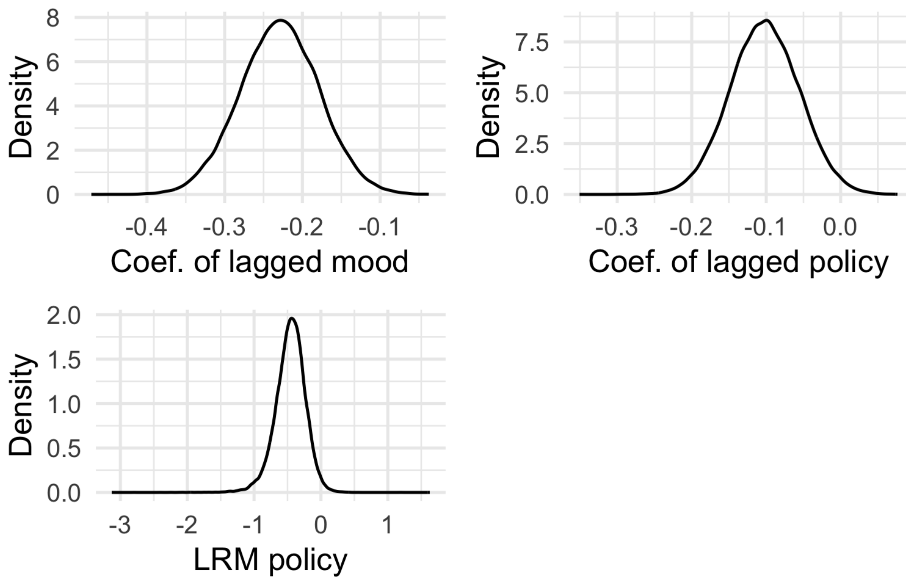
Table 5: A Model of Domestic Policy Mood: Quarter Two 1968 through Quarter Four 2010.

Variable	X_{t-1}	ΔX_t	LRM
Mood	-0.23		
	(-0.33, -0.13)		
Inflation	-0.12	-0.11	-0.51
	(-0.25, 0.01)	(-0.51, 0.29)	(-1.07, 0.06)
Unemployment	-0.08	0.93	-0.36
	(-0.28, 0.10)	(-0.02, 1.89)	(-1.49, 0.57)
Policy	-0.10	-0.17	-0.45
	(-0.20, -0.01)	(-0.58, 0.22)	(-0.95, -0.06)
Vietnam	1.82		
	(0.53, 3.11)		
Constant	19.57		
	(11.51, 27.83)		
T	169		

The conclusions that WLL reach are that the evidence is insufficient to support either the theory that policy mood responds to the economy or the thermostatic model of opinion. The effect of unemployment is unambiguously insignificant. The other two LRMs, however, are less clear. The t-statistics are between the upper and lower bounds. While these are technically indeterminate results, WLL suggest that the researcher should fail to reject the null hypothesis with this pattern of results unless the researcher is confident that mood is a stationary series. Again, the stationarity tests are so inconsistent that this does not seem like a warranted conclusion.

Using our Bayesian approach to the model, we reach different substantive conclusions. We report the result in Table 5. In this application, the MCMCs had four chains, each with a 250,000 iteration burnin, followed by 50,000 iterations and a thinning of 5. The results, then, are based on a total of 40,000 draws from the posterior. The point estimates of the coefficients are almost identical to the results in WLL using OLS. The main difference in the substantive conclusion is the significance of the LRM for the policy measure. In WLL, this fell between the bounds, leading to an indeterminate conclusion. In our Bayesian approach, the 95 percent credible interval excludes zero, suggesting that there is a significant long-run

Figure 3: Estimated Posteriors of the Coefficient for the Lagged Values of Mood and Policy, and the LRM for Policy.



Note: Results from a Bayesian regression specified as an ECM. Data from Ferguson, Kellstedt, and Linn (2013) .

relationship between public policy and public mood. In fact, in 98.7 percent of the draws from the posterior the estimated LRM for the policy measure are negative. In contrast, for the unemployment measure, only 78.2 percent are less than zero, while 96.3 percent of the draws from the LRM of inflation are below zero.

Figure 3 presents the posteriors of the coefficients for the lagged value of mood, the lagged value of the policy measure, and the LRM for the policy measure. Unlike the previous example, there are no draws of the posterior of the lagged dependent variable that approach zero. As a result, the LRM does not have the extreme values that we see in Figure 2. While the posterior of the LRM for policy does have some draws that are positive, the credible region is narrow enough that we conclude that there is a long-run relationship between the variables.

Replicating Lebo and O'Green, 2011

For our third application, we replicate the work of Lebo and O'Green (2011) that explores the predictors of presidential success in Congress. The dependent variable in this model is the percentage of times the president wins a vote in the U.S. House by year from 1953 to 2006. The basic question in the research is if presidential approval gives the president the ability to persuade members of Congress to vote along with the president's policy preferences. While there is a long history of work predicting that approval gave the president more policy leverage, Edwards (2009) argues that the institutional features of the time determine how successful the president will be. Instead of being able to persuade members of Congress to vote counter to their predispositions, presidential success is predetermined by the preferences of members of Congress themselves. When presidents appear successful, it is really due to the partisan balance of Congress.

The empirical application has three independent variables predicting presidential success. The main variable, and the key test of the presidential persuasion argument, is presidential approval. The other two variables capture the institutional balance of the U.S. House. The first is the percentage of House seats held by the president's party. The second is the conditional party government index (CPG) (Aldrich, Berger, and Rohde, 2002). WLL note that in these series, the unit root tests are ambiguous, making this work an apt choice for their bounds approach. The specification is also a straightforward ECM model.

In WLL's paper, they find a robust relationship between the share of the House held by the president's party and no evidence of a link between presidential approval and presidential success. The t-statistic capturing the relationship between CPG and success, however, fell between the bounds. Again, WLL conclude that there is not enough evidence to support the hypothesis that CPG predicts presidential success.

We estimate our model with the same MCMC specifications as the previous application. Our results are similar to those of WLL for two of these three variables. The point estimates of the coefficients and the LRMs reported in Table 6 are almost identical to the ones reported in WLL. The credible region for the LRMs also leads us to conclude that there is a relationship between the share of seats held by the president's party and the president's success. We also find no evidence of a relationship between approval and presidential success. Our

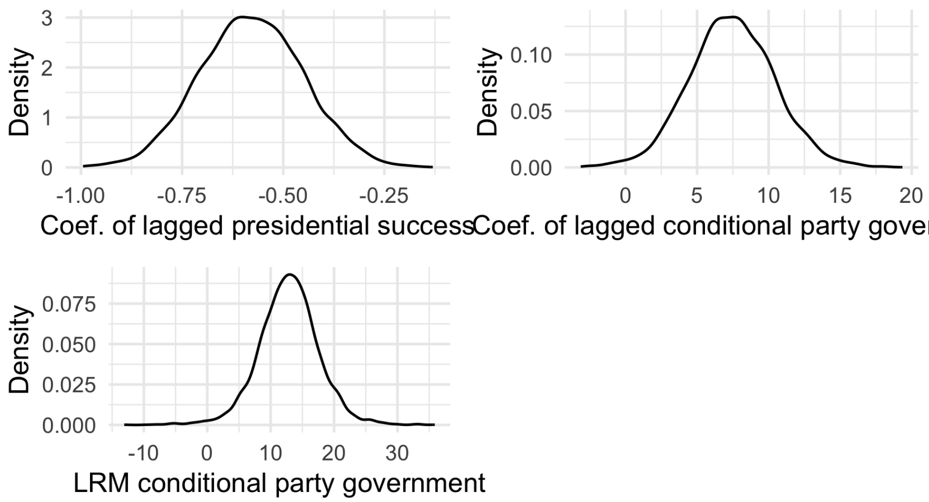
Table 6: A Model of Presidential Success, 1953–2006.

Variable	X_{t-1}	ΔX_t	LRM
Presidential success	-0.58		
	(-0.83, -0.33)		
Conditional party government	7.50	11.14	12.96
	(1.81, 13.36)	(5.54, 16.74)	(3.89, 22.51)
President's party House share	1.35	1.96	2.32
	(0.56, 2.14)	(1.41, 2.49)	(1.32, 3.29)
Presidential approval	0.09	0.30	0.15
	(-0.27, 0.43)	(-0.07, 0.67)	(-0.59, 0.73)
Constant	-34.64		
	(-73.05, 3.34)		
T	52		

results differ on the effect of CPG. The 95 percent credible region for the posterior estimate of the LRM excludes zero, indicating that there is a robust long-run relationship between the variables. With the MCMC approach, we have 4,000 draws from the posterior and can estimate the LRM for each of these iterations. For this variable, 99.6 percent of the draws from the posterior distribution are greater than zero. In comparison, the posterior of the LRM for the president's party House size is greater than zero in 99.95 percent of the draws from the posterior and only 69.3 percent of the draws for the LRM of presidential approval.

Finally, Figure 4 presents the posterior distributions for the coefficients for the lagged dependent variable and CPG, and the LRM for the CPG measure. The figures illustrate the same results as Table 6, but we see the value in presenting the visualization of the posteriors. Every draw from the posterior of the coefficient for the lagged dependent variable is negative, with the maximum value in the draws from the posterior being -0.13. The posterior distribution of the LRM has 99 percent of its mass to the right of zero, indicating that there is a robust long-run relationship between the CPG and presidential success.

Figure 4: Estimated Posterior of the Coefficient for the Lagged Values of Presidential Success and Conditional Party Government, and the LRM for Conditional Party Government.



Note: Results from a Bayesian regression specified as an ECM. Data from Lebo and O'Green (2011).

Conclusion

Applied time series work can be frustrating for researchers. The need to get the dynamic properties of the series correct can bedevil a project. The weak tests for stationarity and the often inconclusive results from differing tests make time series analysis more complicated than might be appreciated. We have the hunch that there are likely numerous studies that have been started and stopped, and eventually shoved in a digital file drawer because the researcher cannot be confident about the stationarity of the series they are working with. That is, time series researchers may not only be stymied by the same null results problem that we all face, but the uncertainty about which specification is the correct one can lead some to simply throw up their hands and give up on a project.

To this end, WLL provide a tremendous service by incorporating the uncertainty of the specification tests into their models and calculating a very clear set of bounds for when to conclude that there is an LRR between two variables. For some projects, this will be enough. If the results are clearly outside the bounds, the researchers know exactly what to do. But for many applied time series studies, the bounds approach will lead to inconclusive results. The necessity of the indeterminate zone of results, while intellectually honest, is likely to be unsatisfying for some.

In this paper, we show that a Bayesian approach that directly estimates the LRM from the posterior distribution is one way to address the indeterminate zone of results. Using non-informative priors on most parameters, but directly incorporating the limits on the dynamic parameters in the model, we get better coverage in our Monte Carlos for small sample sizes. This framework allows one to readily report uncertainty in the estimates of the LRM and their degree of confidence of a LRR, while also working well with the small T types of time series that are common with social science data.

While we focus on time series analysis, our approach should apply to other cases where small samples and multiplier effects are present. For example, spatial autoregressive models (Darmofal, 2015), along with their temporal and multiparametric counterparts (Franzese and Hays, 2007; Hays, Kachi, and Franzese, 2010; Cook, Hays, and Franzese, 2023), face some of the same theoretical and empirical constraints regarding stationarity and the potential for standard errors that exceed theoretical limits. Moreover, MCMCs are already frequently used in estimation in closely related fields, such as Markov random fields (Ward and Gleditsch, 2002; Chyzh and Kaiser, 2019) and exponential random graph models (Cranmer and Desmarais, 2011; Desmarais and Cranmer, 2012), with Bayesian extensions existing for each (Box-Steffensmeier, Christenson, and Morgan, 2018; Stundal et al, 2023). Future research could explore the benefits of incorporating semi-informed priors to these and other related areas.

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