BEGN-320 Fall-2017

Homework #3 Due: Nov. 22, 2017

Instructions: Please submit a **SINGLE DOCUMENT (PDF)** on Blackboard. You are allowed to scan in handwritten work, as long as it is legible, but please be sure to combine everything, including MATLAB scripts and plots, into a single document.
Points: [10+10+25] = 45

Q1): [10 points] Consider two linear time-invariant filters specified in terms of their impulse responses as: $h_1[n] = \delta(n) - \delta(n-1)$; and $h_2[n] = \delta(n) + \delta(n-1) + \delta(n-2)$. You design a new filter by connecting these two filters in cascade with each other. You can answer all of the following questions on paper and do not need MATLAB for it.

- **(i)** What is the expression for the transfer function of the overall (combined) filter (take the DTFT)? **[2 points]**
- **(ii)** What is the impulse response of this combined filter? **[2 points]**
- **(iii)** What is the gain (magnitude of transfer function) of the combined filter at DC, fs/4, fs/3 and fs/2, where fs represents the sampling frequency? (Note: fs/2 Hz corresponds to π rad/sample in the DTFT). **[4 points]**
- **(iv)** Deduce the overall nature of your system (lowpass, high-pass, band-pass or band-stop). Justify your selection. **[2 points]**

Q2.) [10 points] Sketch the asymptotic (Bode) magnitude plot for the system described by the transfer function:

$G(\omega) = \dfrac{0.1(j\omega+5)^2}{(j\omega+0.5)(j\omega+50)}$. Carefully label both the magnitude and frequency axis. Clearly annotate the magnitude at following frequencies $\omega = 0$, $\omega = 0.5$, $\omega = 5$, $\omega = 50$ and $\omega = 500$. Clearly mention the slopes of all your lines.

Q3). [25 points] Consider a signal "noisy_signal.mat" that has been corrupted by random additive noise. Your job is to filter out the signal using a moving average filter (of length M) which is described by the difference equation: $y[n] = \dfrac{1}{M}\sum_{k=0}^{M-1} x(n-k)$.

- **i)** What will be the impulse response of this system (in terms of M)? **[2 points]**
- **ii)** Plot the impulse response of this moving average filter for (a) M = 3, (b) M=7 and (c) M=15. **[3 points]**
- **iii)** Plot the frequency response (transfer function) magnitude of this moving average filter for (a) M = 3, (b) M=7 and (c) M=15. You can derive an expression for the

frequency response using the DTFT table and the impulse response found in part (ii) and then plot it against ω by defining ω over the range $0:\pi$ with intervals of $\pi/100$. **[4 points]**

iv) What can you deduce about the nature of this filter? **[2 points]**

v) What is the influence of increasing the value of M as illustrated by the frequency response magnitude? **[2 points]**

vi) Filter the signal "noisy_signal" through each of these filters independently for (a) M = 3, (b) M=7 and (c) M=15 and plot the filtered signals. Comment on the outputs in terms of the filter lengths M. **[4 points]**

vii) Consider that the output signal obtained by filtering through the M=15 moving average filter removes most of the noise, retrieve the "noise" by subtracting the filtered signal from the original input signal. Plot the probability distribution (histogram) of this noisy signal, what kind of distribution does this noise signal follow? **[3+1 points]**

viii) Plot the auto-correlation of the "noise" obtained in part (vii). What can you comment on the nature of this noise? **[2+2 points]**