Q1: $h_1[n] = \delta[n] - \delta[n-1]$

I: $h_2[n] = \delta[n] - \delta[n-1] - \delta[n-2]$

$$H_1(\omega) = 1 - e^{-j\omega} \qquad H_2(\omega) = 1 + e^{-j\omega} + e^{-2j\omega}$$

$$H(\omega) = H_1(\omega) \cdot H_2(\omega) = (1 - e^{-j\omega})(1 + e^{-j\omega} + e^{-2j\omega})$$

$$H(\omega) = 1 + \cancel{e^{-j\omega}} + \cancel{e^{-2j\omega}} - \cancel{e^{-j\omega}} - \cancel{e^{-2j\omega}} - e^{-3j\omega}$$

$$H(\omega) = 1 - e^{-3j\omega}$$

II: $H(\omega) \xrightarrow{DTFT} H[n] = \delta[n] - \delta[n-3]$

III: At $\frac{f_s}{4}$, $\omega = \frac{\pi}{2}$ $\qquad \frac{f_s}{4} = \frac{f_s}{2}\left(\frac{1}{2}\right)$, $\omega = \frac{\pi}{2}$

$$H(\omega)\Big|_{\omega = \frac{\pi}{2}} = 1 - e^{-3j\left(\frac{\pi}{2}\right)} = 1 - e^{-\frac{3\pi}{2}j}$$

$$1 - e^{-\frac{3\pi}{2}j} = 1 - \left(\cos\left(\frac{-3\pi}{2}\right) + j\sin\left(\frac{-3\pi}{2}\right)\right)$$

$$= 1 - (-j) = 1 + j$$

$$|H(\omega)| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

At $\frac{f_s}{3} = \frac{f_s}{2}\left(\frac{2}{3}\right)$; $\qquad \omega = \frac{2\pi}{3}$

$$H(\omega)\Big|_{\omega = \frac{2\pi}{3}} = 1 - e^{-3j\left(\frac{2\pi}{3}\right)} = 1 - e^{-2\pi j}$$

$$H(\omega) = 1 - e^{-2\pi j} = 1 - (\cos(-2\pi) + j\sin(-2\pi))$$

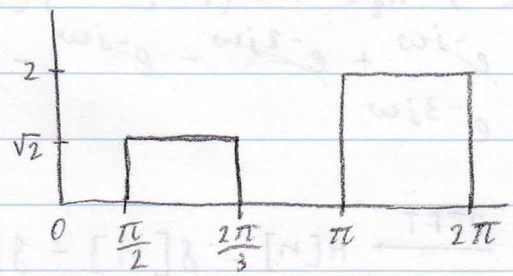
$$H(\omega) = 1 - (1) = 0$$

At $\frac{f_s}{2}$, $\omega = \pi$

$$H(\omega)\big|_{\omega=\pi} = 1 - e^{-3\pi j} = 1 - (\cos(-3\pi) + j\sin(-3\pi))$$

$$H(\omega) = 1 - (-1) = 2$$

IV $|H(\omega)|$



Bandstop

$$G(\omega) = \frac{0.1(\omega j + 5)^2}{(\omega j + 0.5)(j\omega + 50)}$$

Q2:

$$G(\omega) = \frac{0.1(5)\left(1 + \frac{j\omega}{5}\right)(5)\left(1 + \frac{j\omega}{5}\right)}{0.5\left(1 + \frac{\omega j}{0.5}\right)(50)\left(1 + \frac{\omega j}{50}\right)}$$

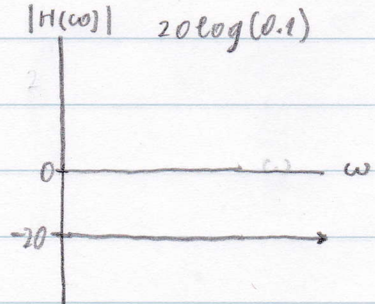
$$G(\omega) = 0.1\,\frac{\left(1 + \frac{j\omega}{5}\right)^2}{\left(1 + \frac{j\omega}{0.5}\right)\left(1 + \frac{j\omega}{50}\right)}$$

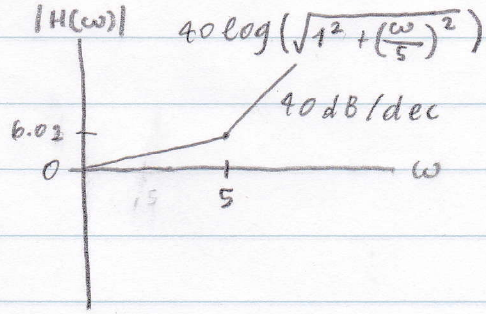
$$|G(\omega)| = 20\log\left(0.1\left(1 + \frac{j\omega}{5}\right)^2\right) -$$

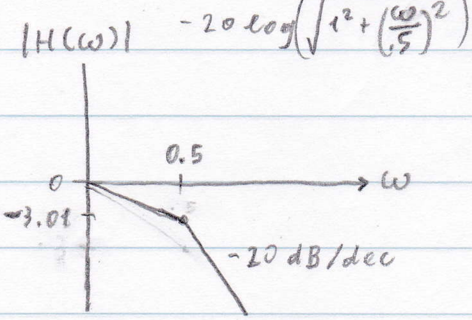
$$20\log\left(1 + \frac{j\omega}{0.5}\right) - 20\log\left(1 + \frac{j\omega}{50}\right)$$

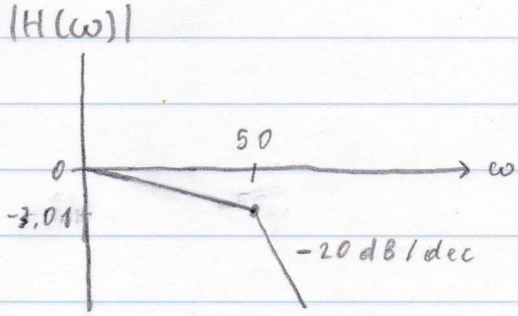
$$20\log(|G(\omega)|) = 20\log(0.1) + 40\log\left(\sqrt{1^2 + \left(\frac{\omega}{5}\right)^2}\right)$$

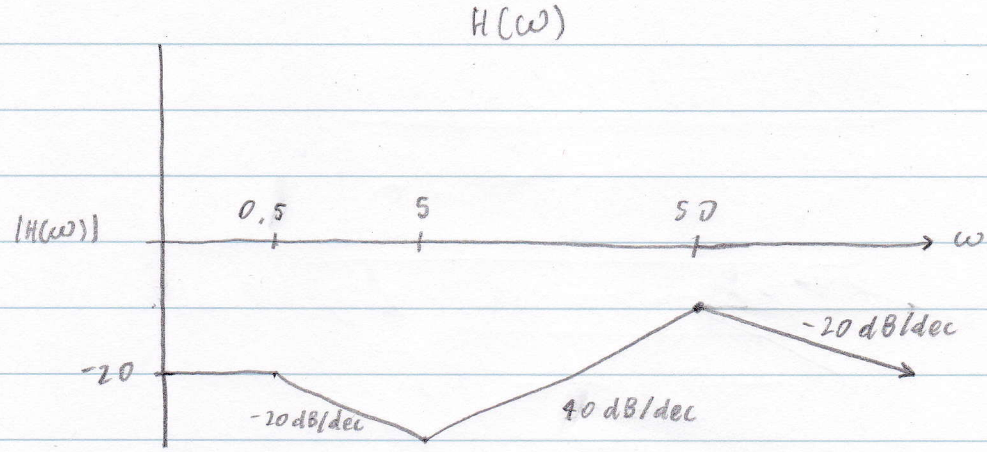
$$-20\log\left(\sqrt{1^2 + \left(\frac{\omega}{0.5}\right)^2}\right) - 20\log\left(\sqrt{1^2 + \left(\frac{\omega}{50}\right)^2}\right)$$







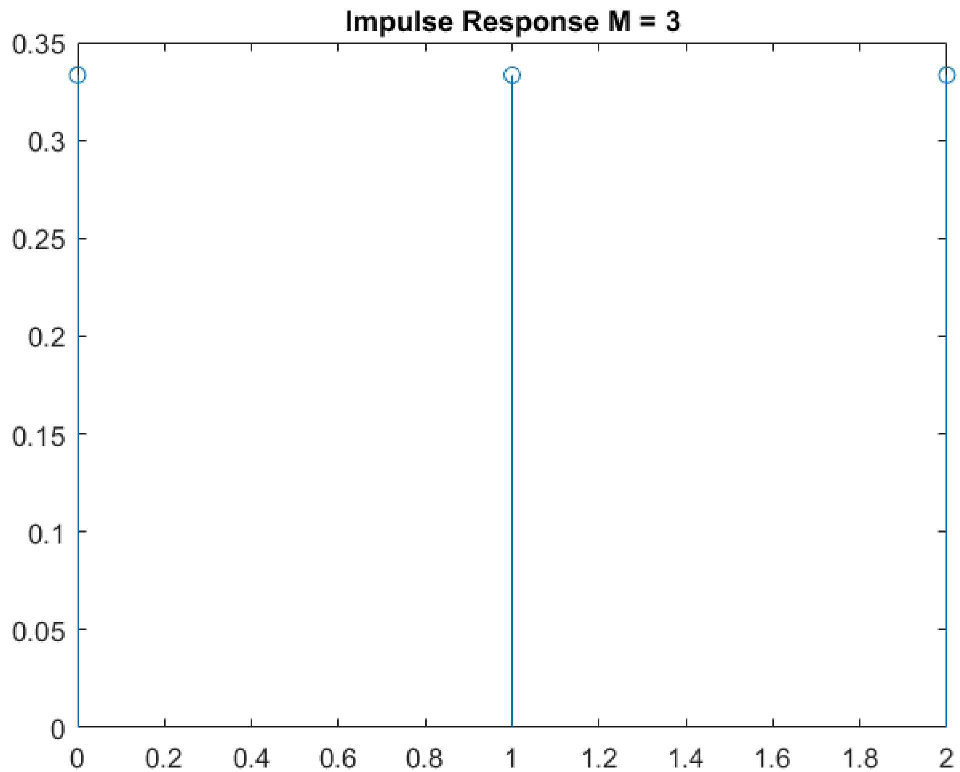


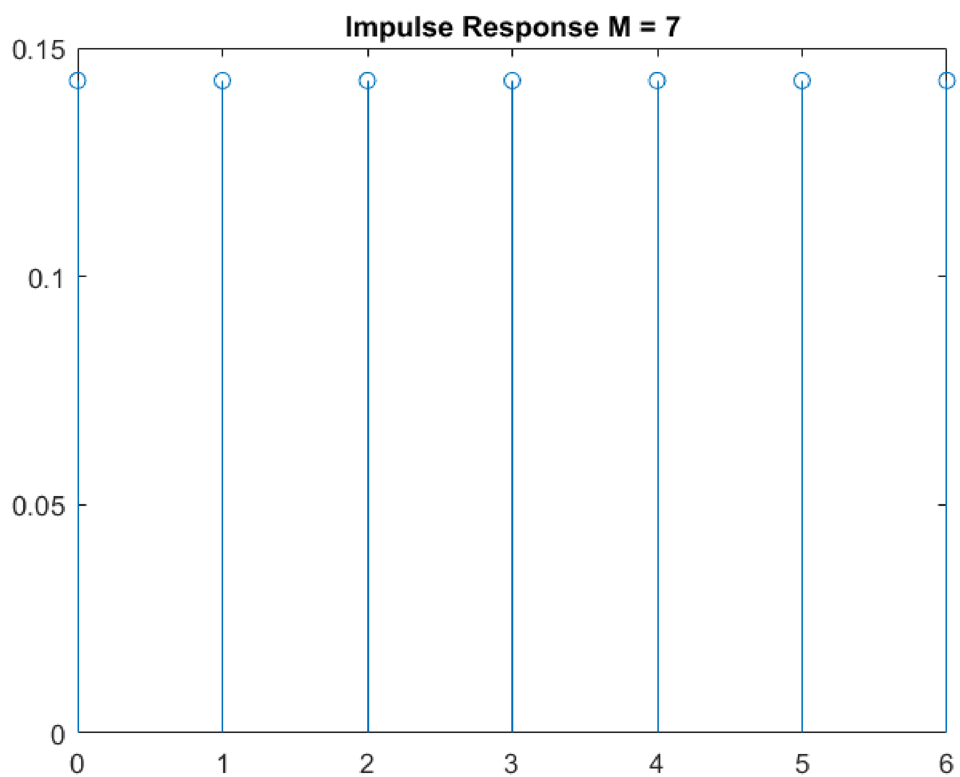


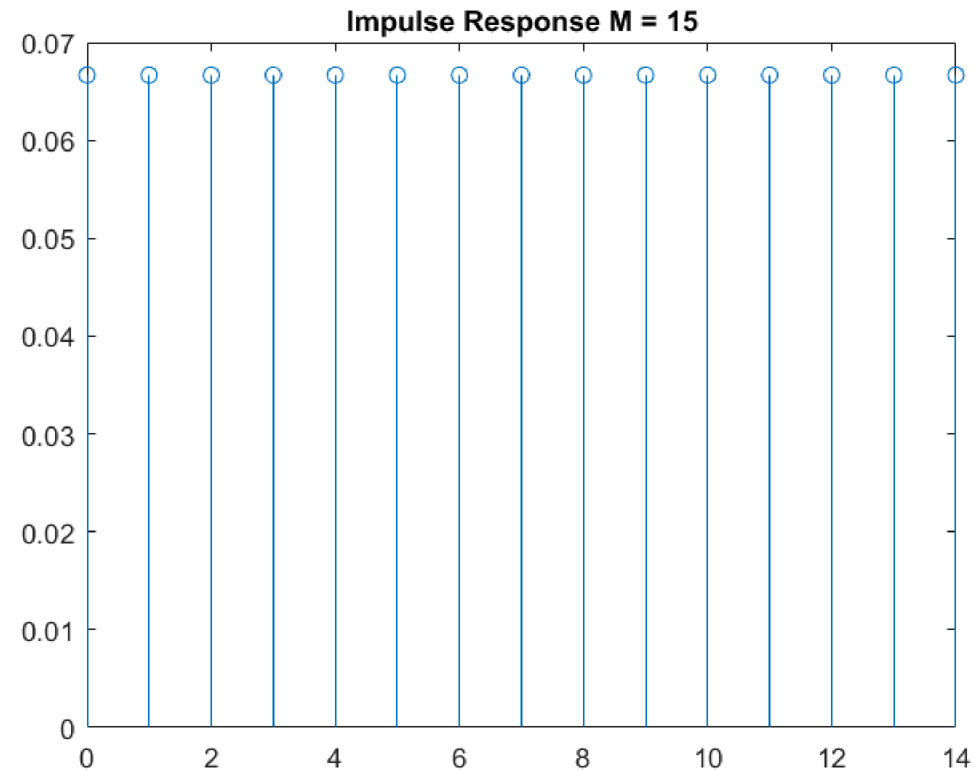
H(ω)
|H(ω)|
0.5
5
50
ω
-20
-20 dB/dec
40 dB/dec
-20 dB/dec

```
clear

A = load('C:\Users\Minh Quan Do\Desktop\GMU\year 3 contents\beng
 320\HW3\noisy_signal.mat');

sig = A.noisy_signal;
t = A.time;

% II
M3 = 3;
M7 = 7;
M15 = 15;

t1 = 0:1:2;
sig3 = ones(1,M3).*(1/M3);
t2 = 0:1:6;
sig7 = ones(1,M7).*(1/M7);
t3 = 0:1:14;
sig15 = ones(1,M15).*(1/M15);

figure(1)
stem(t1,sig3)
title('Impulse Response M = 3')

figure(2)
stem(t2,sig7)
title('Impulse Response M = 7')

figure(3)
stem(t3,sig15)
title('Impulse Response M = 15')

% III
j = sqrt(-1);
omega = 0:(pi/100):pi;
y3 = abs((1/M3)*(1-exp(-j*omega*M3))./(1-exp(-j*omega)));
y7 = abs((1/M7)*(1-exp(-j*omega*M7))./(1-exp(-j*omega)));
y15 = abs((1/M15)*(1-exp(-j*omega*M15))./(1-exp(-j*omega)));

figure(4)
plot(omega, y3)
title('Frequency Response M = 3')
xlabel('w (frequency)')
ylabel('|H(w)|')

figure(5)
plot(omega, y7)
title('Frequency Response M = 7')
xlabel('w (frequency)')
ylabel('|H(w)|')

figure(6)
```

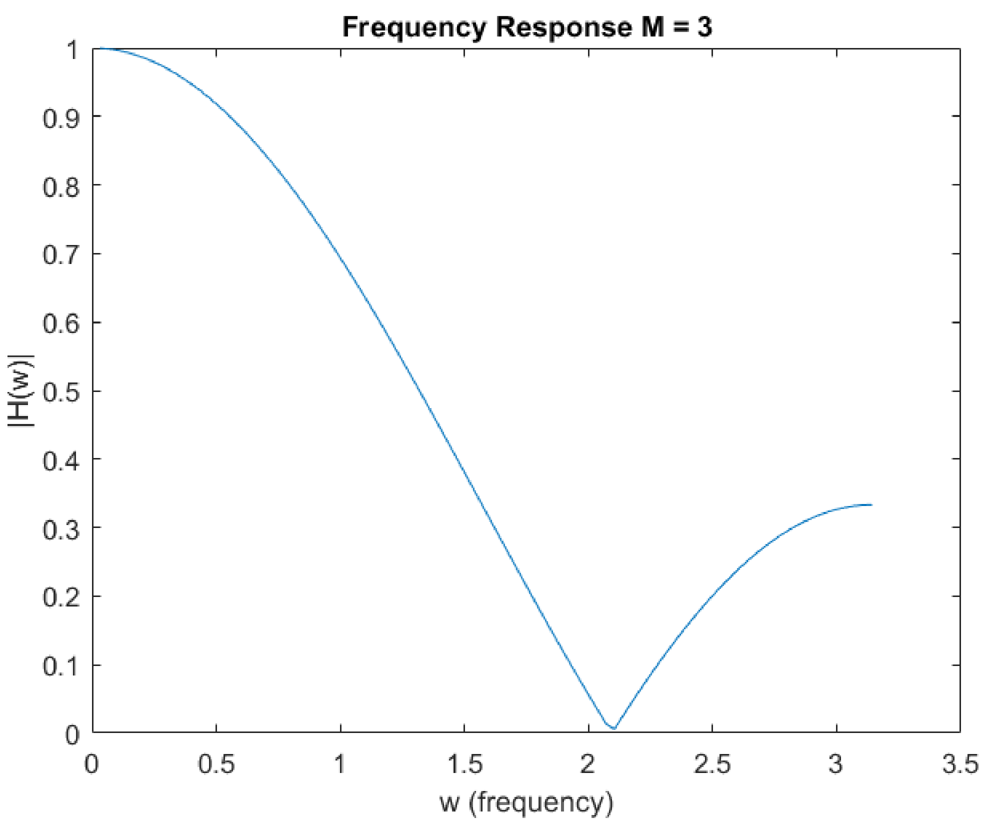
```
plot(omega, y15)
title('Frequency Response M = 15')
xlabel('w (frequency)')
ylabel('|H(w)|')

% IV
% the filters seems to be behaving like a low-pass filter

% V
% the larger the value of M, the more the peaks is in the graph

% VI
Y3f = filter(sig3,1,sig);
Y7f = filter(sig7,1,sig);
Y15f = filter(sig15,1,sig);

figure(7)
plot(t, Y3f)
title('Filtered Signal M = 3')

figure(8)
plot(t, Y7f)
title('Filtered Signal M = 7')

figure(9)
plot(t, Y15f)
title('Filtered Signal M = 15')
% the larger the value of M, the smoother the signal

% VII
noise = sig - Y3f;
figure(10)
histogram(noise)
title('Probability Distribution of Noise')
% noise follows a normal distribution

% VIII
x_noise = xcorr(noise, noise);
figure(11)
plot(x_noise)
title('Autocorrelation of noise')
axis tight
% noise is a random noise
```


Impulse Response M = 3
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

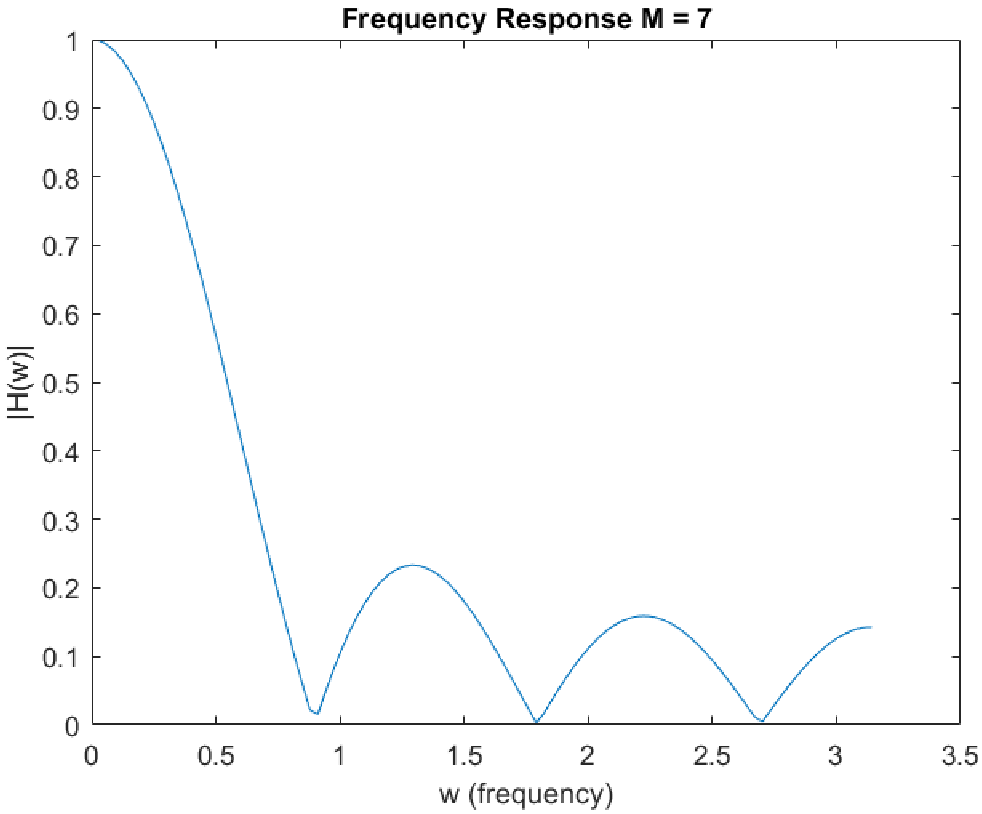

Impulse Response M = 7
0.15
0.1
0.05
0
0
1
2
3
4
5
6

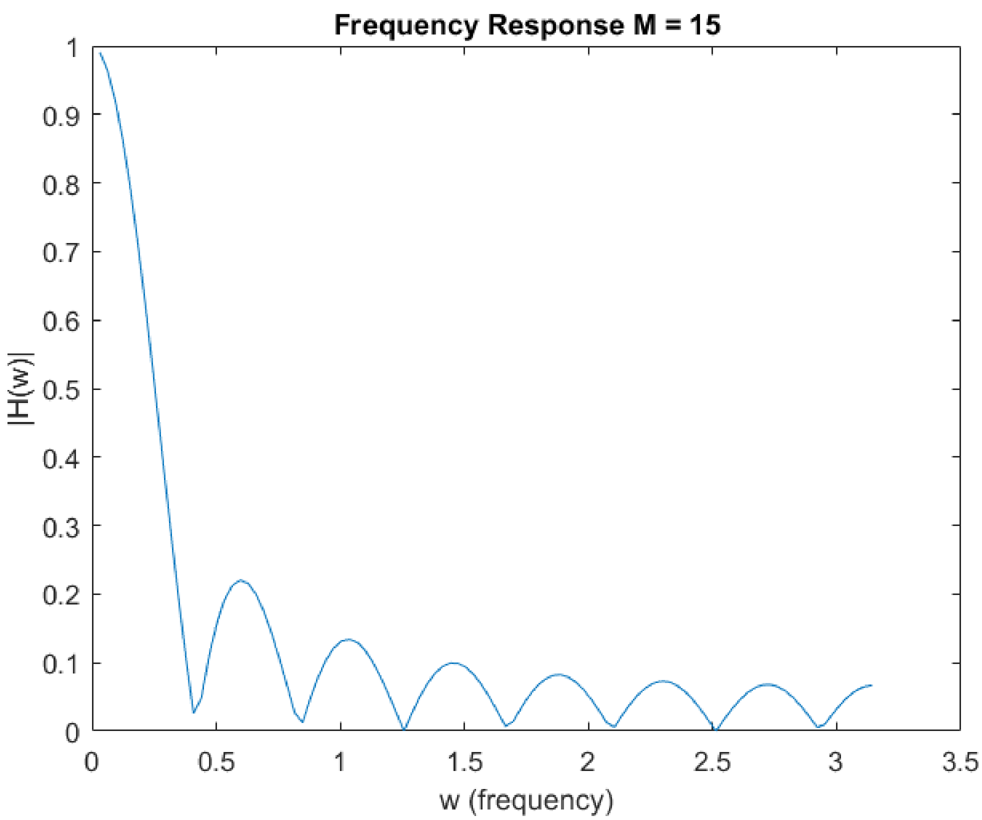


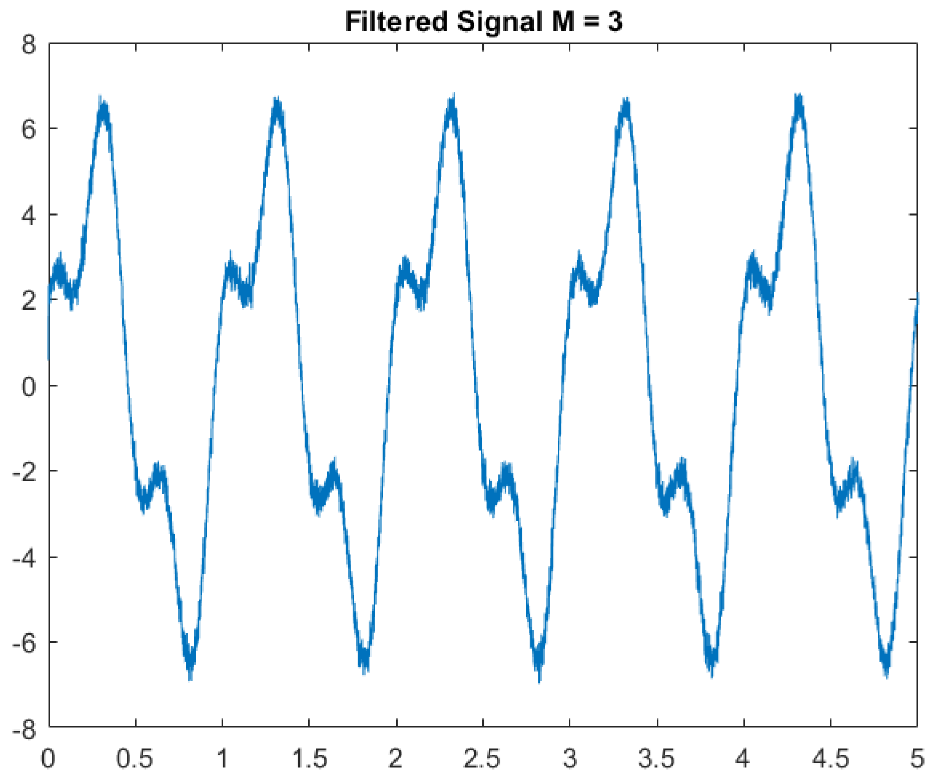
Impulse Response M = 15
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14

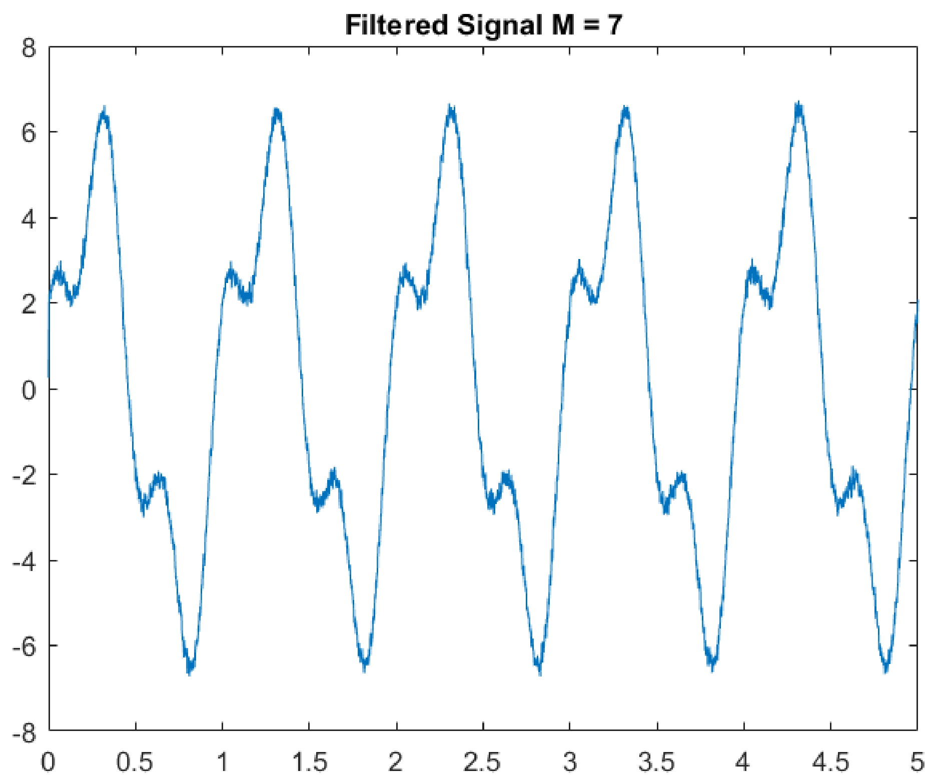


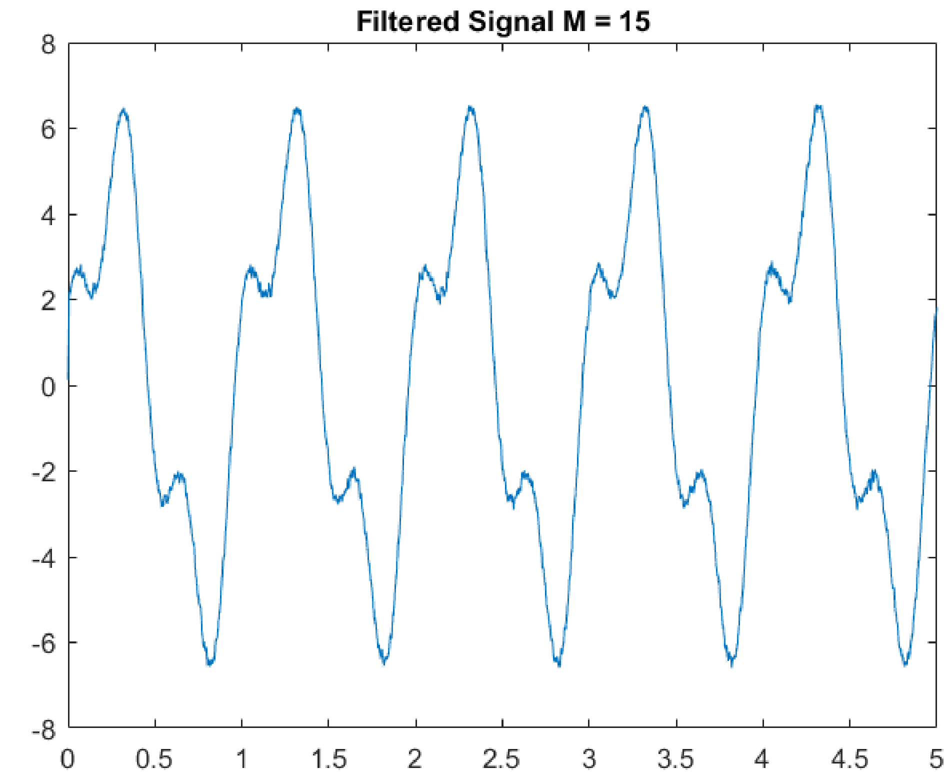
Frequency Response M = 3
|H(w)|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
w (frequency)

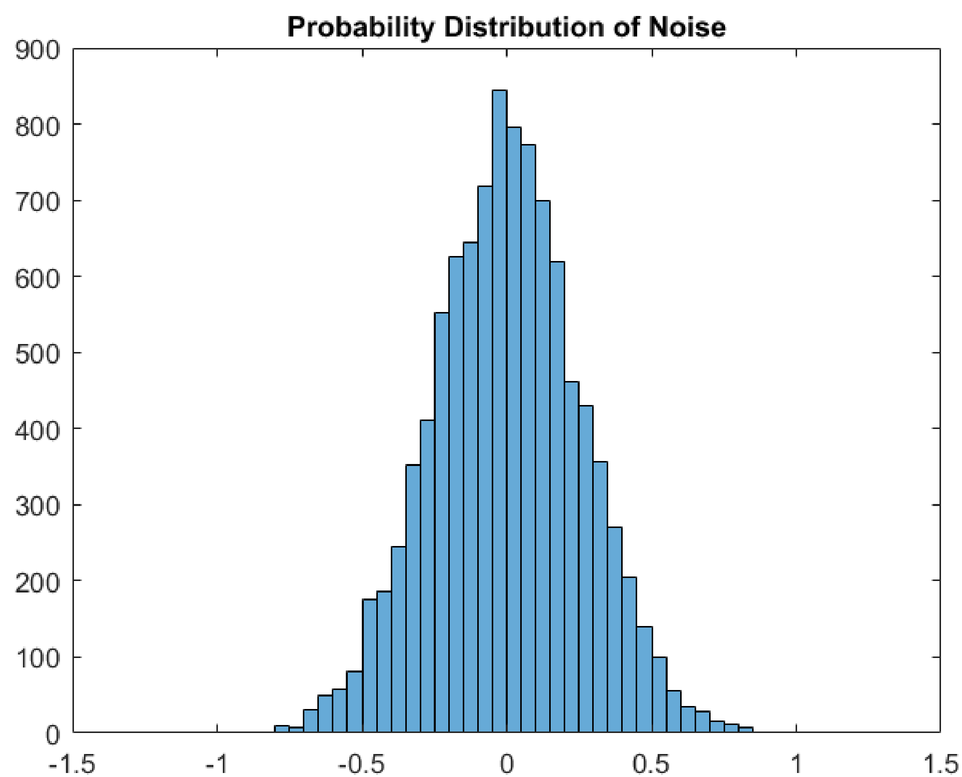

Frequency Response M = 7
|H(w)|
w (frequency)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5

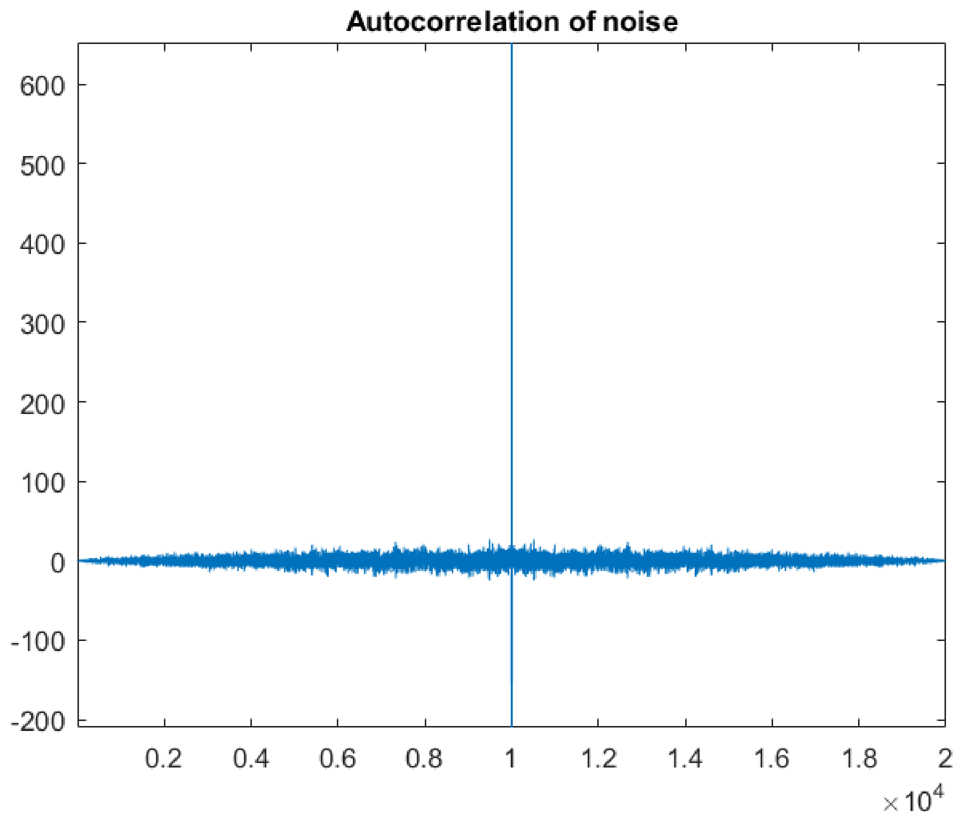

Frequency Response M = 15
|H(w)|
w (frequency)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5


Filtered Signal M = 3
8
6
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Filtered Signal M = 7
8
6
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Filtered Signal M = 15
8
6
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Probability Distribution of Noise
900
800
700
600
500
400
300
200
100
0
-1.5
-1
-0.5
0
0.5
1
1.5



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